Assignment 1 EE5609

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***Abstract*—This assignment finds the vector triple product.**

Download python code from

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https://github.com/Atul191/EE−5609−
    Assignment_1/blob/master/vectors.py
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1 Problem Statement

1.1 Find the value of:

$$\begin{pmatrix}1\\0\\0\end{pmatrix}^T\left(\begin{pmatrix}0\\1\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix}\right)+\begin{pmatrix}0\\1\\0\end{pmatrix}^T\left(\begin{pmatrix}1\\0\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix}\right)+\begin{pmatrix}0\\0\\1\end{pmatrix}^T\left(\begin{pmatrix}1\\0\\0\end{pmatrix}\times\begin{pmatrix}0\\1\\0\end{pmatrix}\right)$$

1.2 Solution

Equate these matrices as linearly independent vectors as a, b and c such as:

$$\mathbf{a}=\begin{pmatrix}1\\0\\0\end{pmatrix},\mathbf{b}=\begin{pmatrix}0\\1\\0\end{pmatrix},\mathbf{c}=\begin{pmatrix}0\\0\\1\end{pmatrix} \tag{1}$$

Using scalar triple product property we deduce

$$\mathbf{a}^T\left(\mathbf{b}\times\mathbf{c}\right)=\mathbf{b}^T\left(\mathbf{c}\times\mathbf{a}\right)=\mathbf{c}^T\left(\mathbf{a}\times\mathbf{b}\right) \tag{2}$$

Note: Cross product is given by:

$$\begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix}\times\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix}=\begin{pmatrix}0&-a_3&a_2\\a_3&0&-a_1\\-a_2&a_1&0\end{pmatrix}\begin{pmatrix}b_1\\b_2\\b_3\end{pmatrix} \tag{3}$$

1.3 Step 1

Equating (1) with problem statement we deduce the following:

$$\mathbf{a}^T\left(\mathbf{b}\times\mathbf{c}\right)+\mathbf{b}^T\left(\mathbf{a}\times\mathbf{c}\right)+\mathbf{c}^T\left(\mathbf{a}\times\mathbf{b}\right) \tag{4}$$

As Cross Product is anti-commutative we get:

$$\mathbf{a}^T\left(\mathbf{b}\times\mathbf{c}\right)-\mathbf{b}^T\left(\mathbf{c}\times\mathbf{a}\right)+\mathbf{c}^T\left(\mathbf{a}\times\mathbf{b}\right) \tag{5}$$

$$=\mathbf{a}^T\left(\mathbf{b}\times\mathbf{c}\right)-\mathbf{c}^T\left(\mathbf{a}\times\mathbf{b}\right)+\mathbf{c}^T\left(\mathbf{a}\times\mathbf{b}\right) \tag{6}$$

$$=\mathbf{a}^T\left(\mathbf{b}\times\mathbf{c}\right) \tag{7}$$

1.4 Step 2

So instead of calculating each step we just calculate one iteration by referring (3) and (7) i.e.

$$\begin{pmatrix}0\\1\\0\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix} \tag{8}$$

$$\implies\begin{pmatrix}1&0&0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}=\begin{pmatrix}1\end{pmatrix} \tag{9}$$

1.5 Step 3 (Answer)

As the terms in the given problem are equal. so we have computed only one term as in step 2 and the solution to the problem statement looks like $\begin{pmatrix}1\end{pmatrix}+\begin{pmatrix}-1\end{pmatrix}+\begin{pmatrix}1\end{pmatrix}=\begin{pmatrix}1\end{pmatrix}$