

Graph

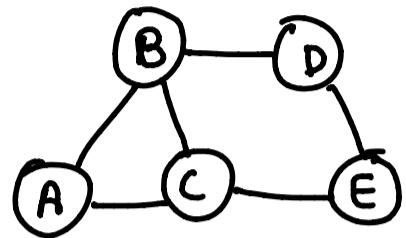
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Graphs

graph G is a pair (V, E) where V is set of vertices & E is set of edges. $n = |V|$ & $e = |E|$

Eg



$$V = \{A, B, C, D, E\} \quad n = 5$$

$$E = \{AB, AC, BC, BD, CE, DE\} \quad e = 6$$

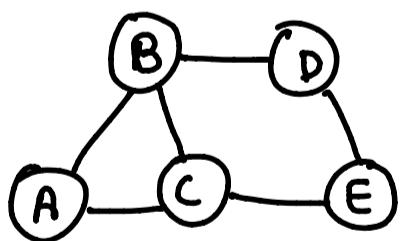
Applications →

Google maps → To find shortest route

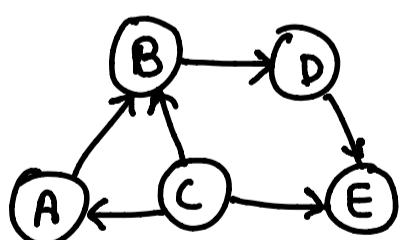
Social network → user, connection
 ↑
 vertex edge

Types →

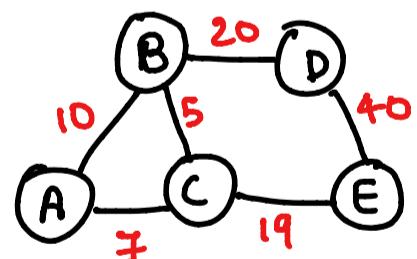
1) Undirected



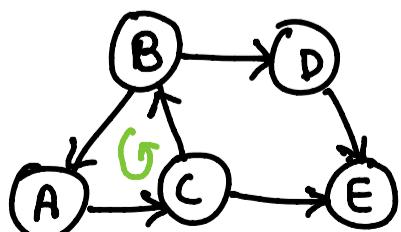
2) Directed



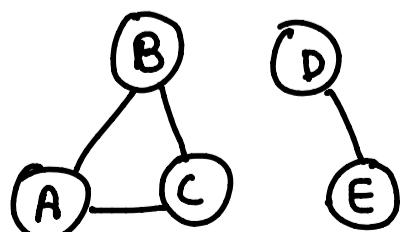
3) Weighted



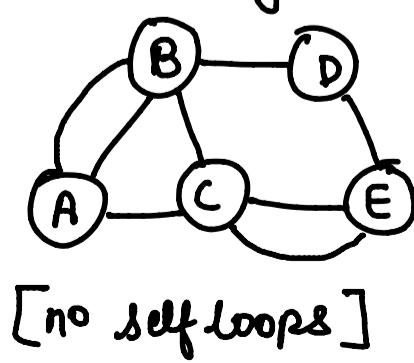
4) Cyclic



5) Disconnected



6) Multigraph



Representation →

adj. matrix

$$sc \rightarrow O(n^2)$$

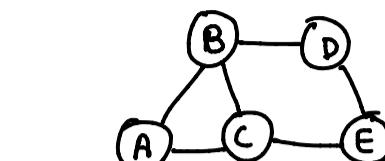
adj. list

$$sc \rightarrow O(n+E)$$

Graph Traversal

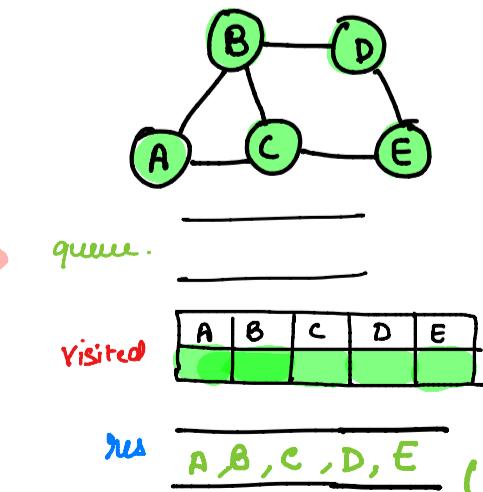
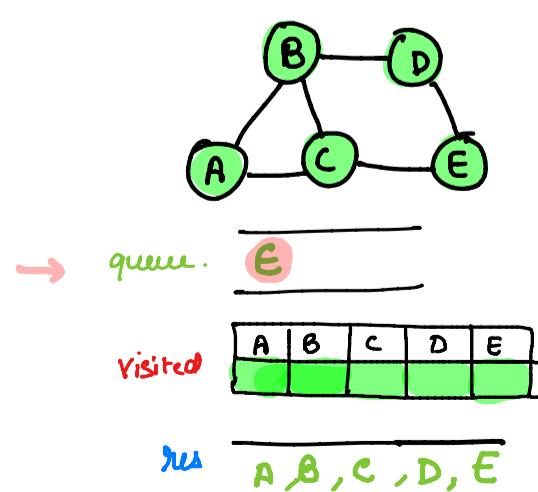
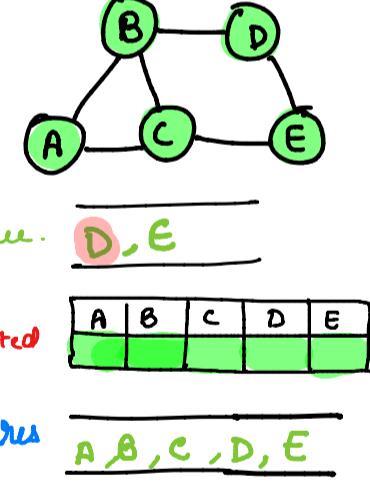
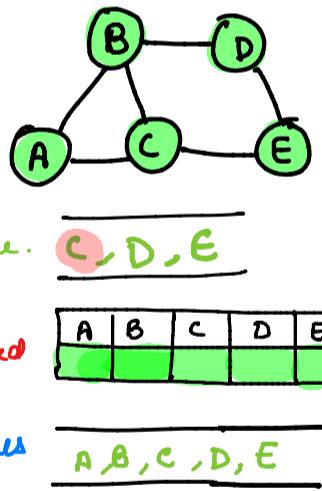
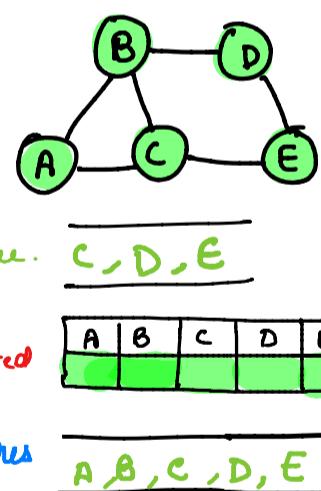
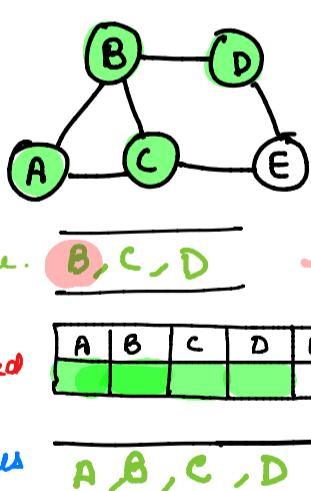
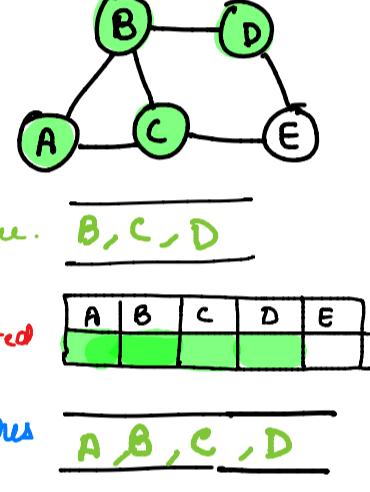
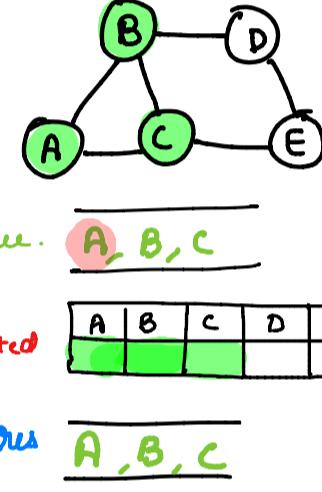
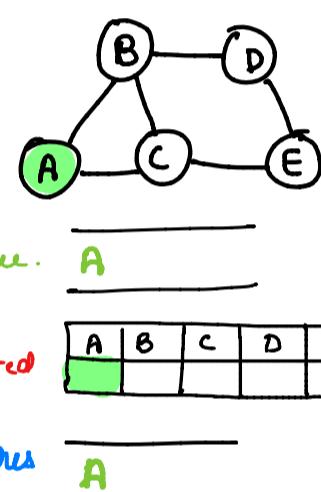
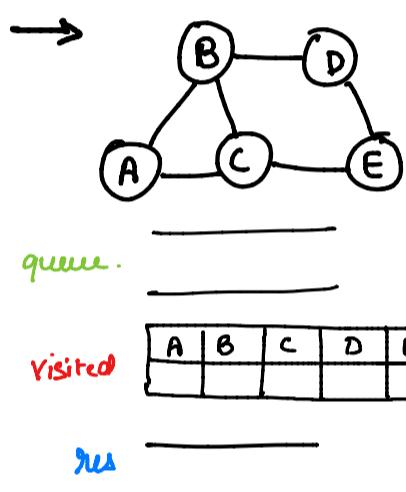
(a) BFS → visit each and every vertex in a defined order.

- select node
- visit its unvisited neighbour nodes
- mark it as visited & push into result
- push it into queue
- if no neighbours then pop.
- repeat till queue is empty



queue. _____

Visited	A	B	C	D	E
---------	---	---	---	---	---



TC $\rightarrow O(V+E)$

SC $\rightarrow O(V)$

↳ Return res .

Code

```
class Solution {
public:

    vector<int> bfsOfGraph(int v, vector<int> adj[]) {
        vector<int> ans;
        vector<int> vis(v, 0);
        queue<int> q;
        q.push(0);

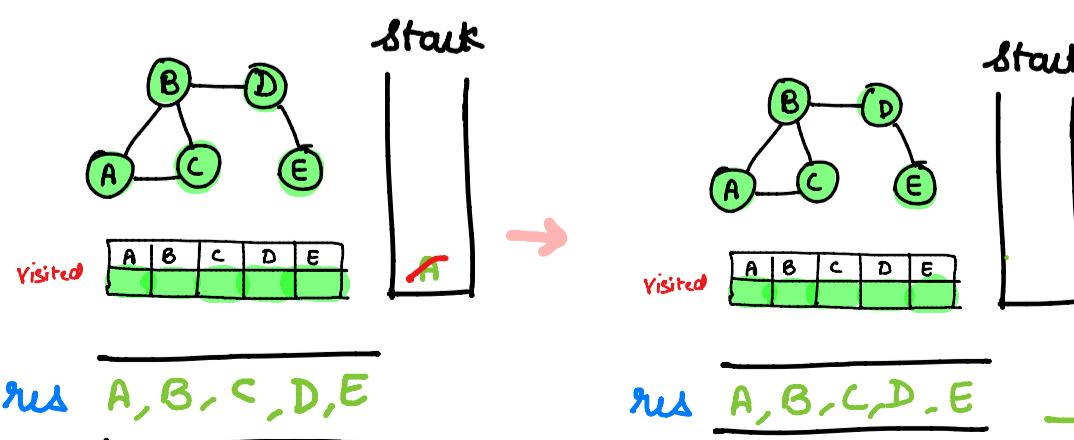
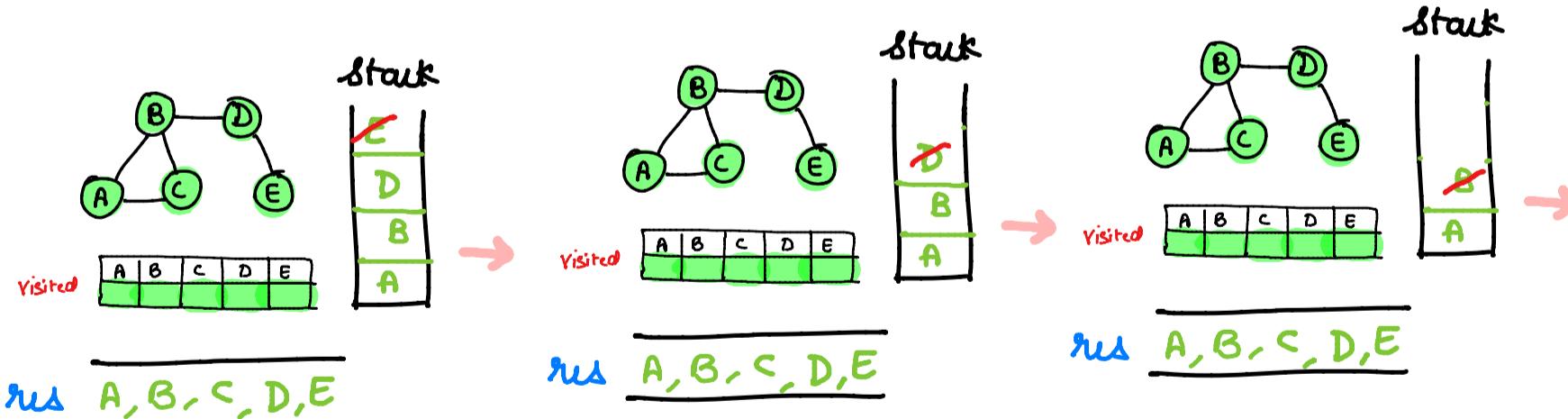
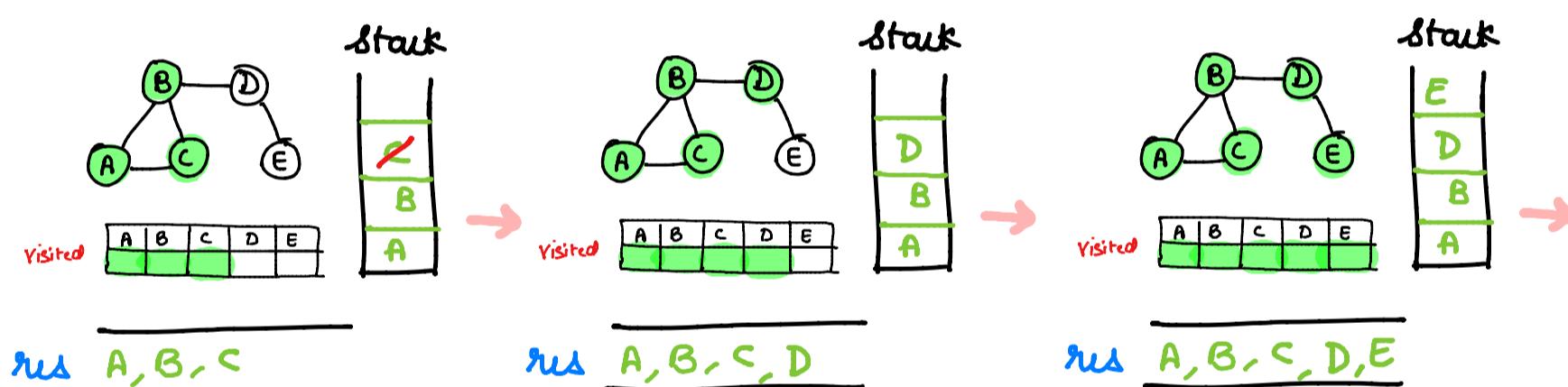
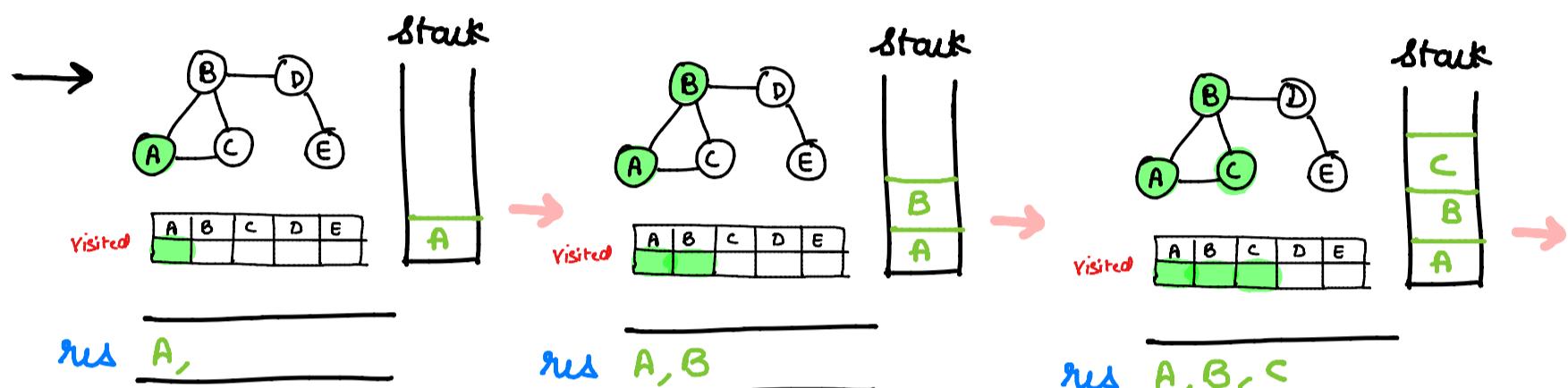
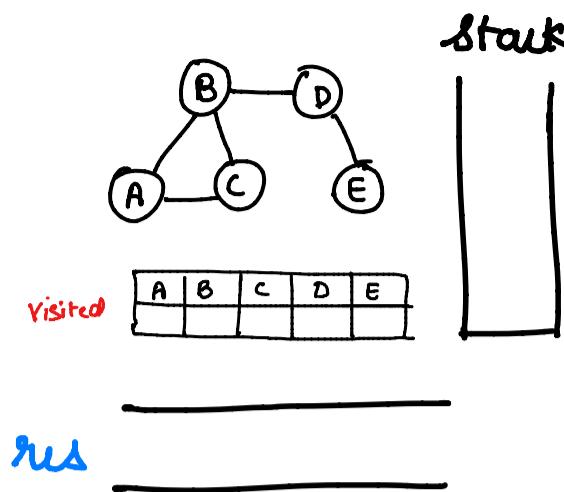
        while(!q.empty()){
            int curr = q.front();
            q.pop();
            vis[curr] = 1;
            ans.push_back(curr);
            for(auto it:adj[curr]){
                if(vis[it]==0){
                    vis[it] = 1;
                    q.push(it);
                }
            }
        }
        return ans;
    }
};
```

Applications → [BFS]

1. shortest path
2. Min. spanning tree for unweighted graph
3. cycle detection
4. GPS
5. social network.

⑥ DFS →

- select node
- visit its unvisited neighbour nodes
- mark it as visited & push into result
- push it into stack
- if no neighbours then pop.
- repeat till stack is empty



TC $\rightarrow O(V+E)$

SC $\rightarrow O(V)$

→ Return res.

Code

```
class Solution {
public:

    void dfs(vector<int>&ans, vector<int>&vis, int node, vector<int>adj[]){
        vis[node] = 1;
        ans.push_back(node);
        for(auto it:adj[node]){
            if(!vis[it]){
                vis[it] = 1;
                dfs(ans, vis, it, adj);
            }
        }
    }
    vector<int> dfsOfGraph(int V, vector<int> adj[]) {
        vector<int> ans;
        vector<int> vis(V, 0);
        for(int i=0; i<V; i++){
            if(vis[i]==0)
                dfs(ans, vis, i, adj);
        }
        return ans;
    }
};
```

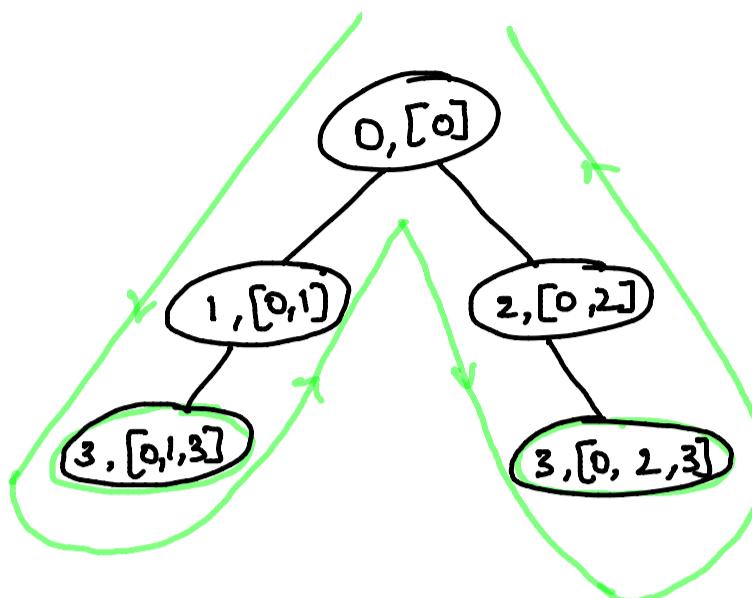
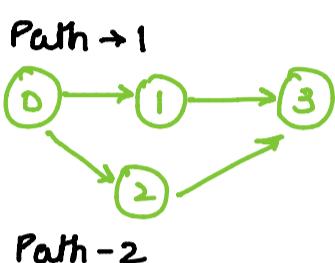
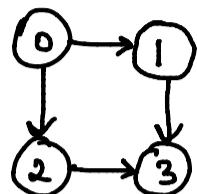
Applications → [DFS]

1. Path finding
2. Cycle detection
3. Topological sort
4. Finding strongly connected components.

① All paths from src to target

Given a directed acyclic graph, return all paths from node 0 to node n-1.

Eg



Code →

```
1 class Solution {
2 public:
3     void findAllPaths(vector<vector<int>>&graph, int currNode, vector<bool>&visited,
4                        int n, vector<int> &currPath, vector<vector<int>>&res){
5
6         if(currNode==n-1){
7             res.push_back(currPath);
8             return;
9         }
10
11         if(visited[currNode]==true) return;
12
13         // backtrack for every node
14         visited[currNode] = true;
15
16         for(auto neighbour: graph[currNode]){
17             currPath.push_back(neighbour);
18             findAllPaths(graph, neighbour, visited, n, currPath, res);
19             currPath.pop_back();
20         }
21
22         visited[currNode] = false;
23     }
24
25     vector<vector<int>> allPathsSourceTarget(vector<vector<int>>& graph) {
26         vector<vector<int>> res;
27         vector<int> currPath;
28         int n = graph.size();
29         vector<bool> visited(n);
30
31         // traversing from 0 node
32         currPath.push_back(0);
33
34         findAllPaths(graph, 0, visited, n, currPath, res);
35         return res;
36     }
37 }
```

T_C → O(V+E)

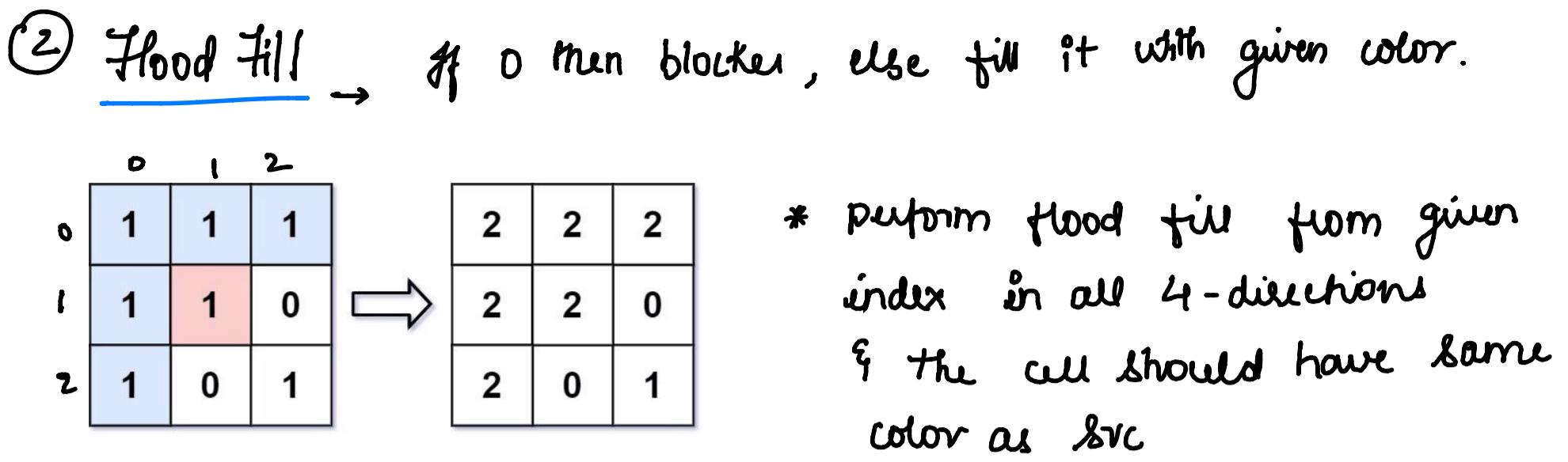
V → Vertices

E → edges

S_C →

Recursive stack

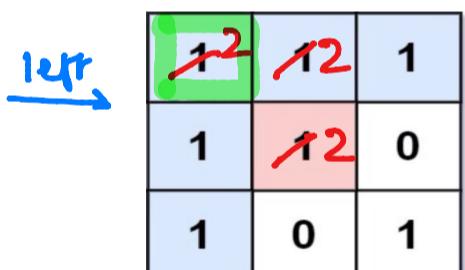
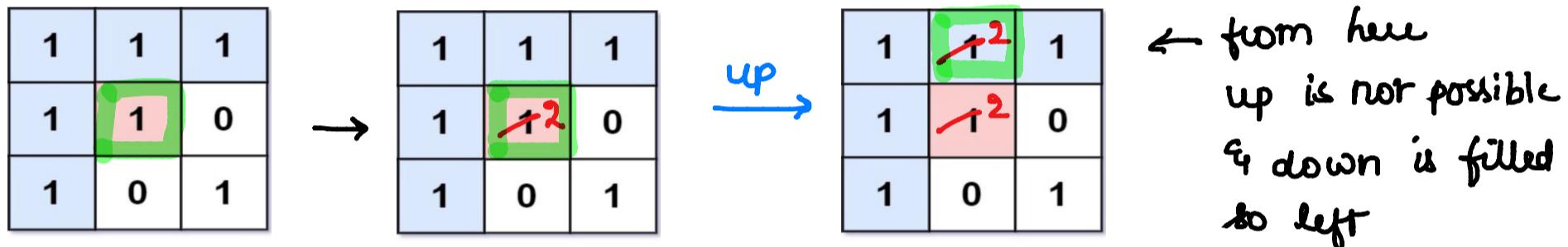
+ Result



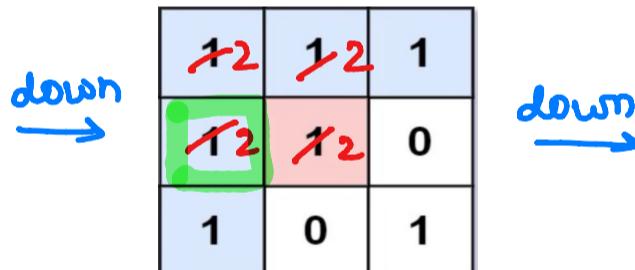
✓ let's follow the order to fill → UP, DOWN, LEFT, RIGHT

Eg In above case starting point is (1,1) & value = 1 so

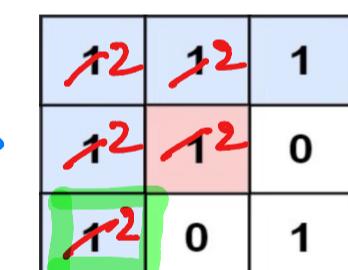
*



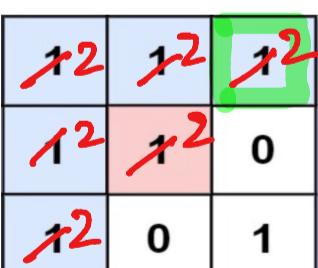
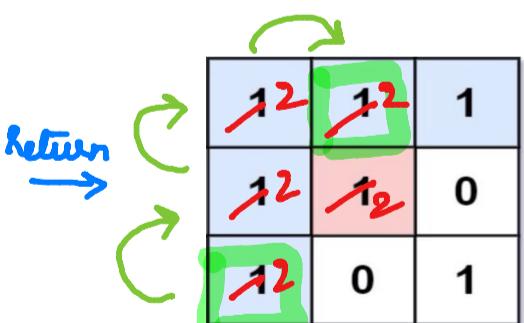
from here up nor possible so down



from here up nor possible so down



→ no direction is possible so return



no other way possible



Code

```
● ● ●  
1 class Solution {  
2 public:  
3     void floodFiller(vector<vector<int>>& image, int i, int j,  
4     int m, int n, int currColor, int newColor)  
5     {  
6         if(i<0 || i>=m || j<0 || j>= n || image[i][j] == newColor  
7             || image[i][j] != currColor)  
8             return;  
9  
10        image[i][j] = newColor;  
11        floodFiller( image, i-1, j, m, n, currColor, newColor);  
12        floodFiller( image, i+1, j, m, n, currColor, newColor);  
13        floodFiller( image, i, j-1, m, n, currColor, newColor);  
14        floodFiller( image, i, j+1, m, n, currColor, newColor);  
15    }  
16  
17    vector<vector<int>> floodFill(vector<vector<int>>& image, int sr,  
18    int sc, int newColor)  
19    {  
20        int m = image.size();  
21        int n = image[0].size();  
22        int currColor = image[sr][sc];  
23        floodFiller(image, sr, sc, m, n, currColor, newColor);  
24        return image;  
25    }  
26};
```

$$Tc \rightarrow O(mn)$$

$$Sc \rightarrow O(h)$$

↳ recursive stack

③ Number of islands → Given grid of 1 (land) & 0 (water), return no. of islands.

Eg
 0 $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$
 1 $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$
 2 $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$
 3 $\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix}$
 4 $\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

- Always start dfs only if value = 1 & change its value to 0, so it cannot be visited again
- if initial value = 0 then skip.
- initially ans = 0

• let start from (0,0) & try moving U,D,L,R

→ the traversal goes in this order

(0,0) → (1,0) → (1,1) → (0,1) i.e

& update ans.

$\begin{bmatrix} [1,1,0,0,0] \\ [1,1,0,0,0] \\ [0,0,1,0,0] \\ [0,0,0,1,1] \end{bmatrix}$
 ans = 1.

→ now grid becomes

0 $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$
 1 $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
 2 $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
 3 $\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

- now, we can skip every entry from (1,0) to (2,1) as they are 0s.
 - now start from (2,2), as U,D,L,R is not possible, set its value = 0 & update ans.
- ans = 1.

→ now grid becomes

0 $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \end{bmatrix}$
 1 $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
 2 $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
 3 $\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

- now, we can skip every entry from (2,3) to (3,2) as they are 0s.
- now start from (3,3), it goes as follows
 $(3,3) \rightarrow (3,4)$
- further traversal from (3,4) is not possible

ans = 3. ans = 3

Code

```
1 class Solution {
2 public:
3     void countIsland(vector<vector<char>>& grid, int currRow, int currCol, int row, int col){
4         if(currRow<0 || currRow>=row || currCol<0 || currCol>=col || grid[currRow][currCol]=='0')
5             return;
6
7         grid[currRow][currCol] = '0';
8         countIsland(grid, currRow-1, currCol, row, col);
9         countIsland(grid, currRow+1, currCol, row, col);
10        countIsland(grid, currRow, currCol-1, row, col);
11        countIsland(grid, currRow, currCol+1, row, col);
12    }
13
14    int numIslands(vector<vector<char>>& grid) {
15        int ans = 0;
16        int row = grid.size();
17        int col = grid[0].size();
18
19        for(int currRow = 0; currRow < row; currRow++)
20            for(int currCol = 0; currCol < col; currCol++)
21                if(grid[currRow][currCol]=='1'){
22                    ans++;
23                    countIsland(grid, currRow, currCol, row, col);
24                }
25
26        return ans;
27    }
28};
```

$T_c \rightarrow O(mn)$ Avg case
 $O(m^2n^2)$ Worst case

④ Max Area of the Island

- * Intuition is same as previous problem.
- * Minor Tweak to count number of 1s in island.
- * Once entire island traversal is done,
compute for max area of island.

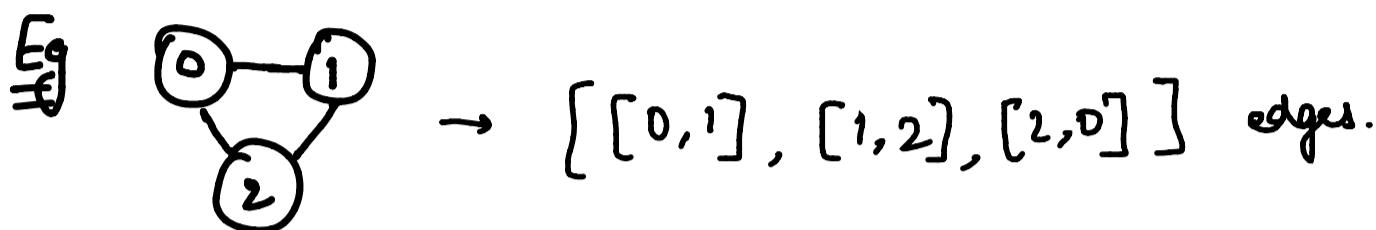
T_C → O(mn) Avg case.

code →

```
● ● ●
1 class Solution {
2 public:
3     int findArea(vector<vector<int>>& grid, int currRow, int currCol, int m, int n){
4         if(currRow<0 || currCol<0 || currRow>=m || currCol>=n || grid[currRow][currCol]==0)
5             return 0;
6
7         grid[currRow][currCol]=0;
8
9         // this is for single cell where we started traversing
10        int count = 1;
11        count += findArea(grid, currRow-1, currCol, m, n);
12        count += findArea(grid, currRow+1, currCol, m, n);
13        count += findArea(grid, currRow, currCol-1, m, n);
14        count += findArea(grid, currRow, currCol+1, m, n);
15        return count;
16    }
17    int maxAreaOfIsland(vector<vector<int>>& grid) {
18        int m = grid.size();
19        int n = grid[0].size();
20        int ans = 0;
21        for(int currRow = 0; currRow<m; currRow++)
22            for(int currCol = 0; currCol<n; currCol++){
23                if(grid[currRow][currCol]==1){
24                    ans = max(ans, findArea(grid, currRow, currCol, m, n));
25                }
26            }
27        return ans;
28    }
29};
```

5) Find if path exist in graph.

Given src, dest, no. of nodes & set of edges, find if path exist b/w src & dest.

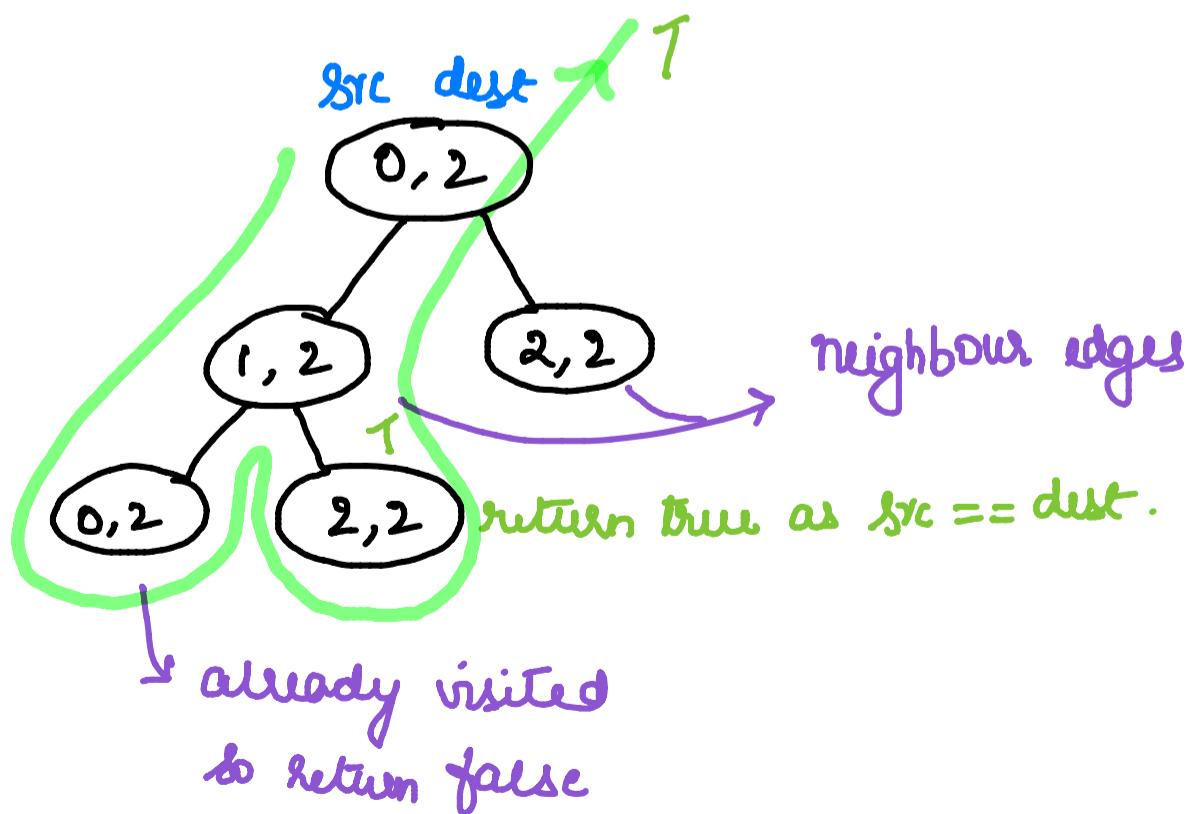


$n = 3$ edges = $[[0,1], [1,2], [2,0]]$ src = 0, dest = 2.

- 1) Create a graph using adj list rep. $[[1,2], [0,2], [1,0]]$
- 2) Perform dfs

$[[1,2], [0,2], [1,0]]$

0	1	2
T	F	I



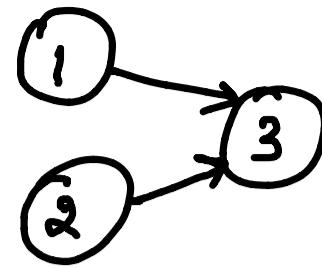
Code →



```
1 class Solution {
2 public:
3     bool validPath(int n, vector<vector<int>>& edges, int src, int dest) {
4
5         vector<vector<int>>graph(n);
6         for(int i=0;i<edges.size();i++)
7         {
8             int v1 = edges[i][0];
9             int v2 = edges[i][1];
10            graph[v1].push_back(v2);
11            graph[v2].push_back(v1);
12
13        }
14        vector<bool>vis(n,false);
15        return pathExist(src, dest, graph, vis);
16    }
17
18    bool pathExist(int src , int dest,vector<vector<int>>&graph,vector<bool>&vis){
19
20        if(src==dest) return true;
21
22        vis[src]=true;
23
24        for(int i=0;i<graph[src].size();i++)
25            if(vis[graph[src][i]]==false)
26                if(pathExist(graph[src][i],dest,graph,vis)==true)
27                    return true;
28
29        return false;
30    }
31};
```

⑥ Find the town judge

$n = 3$, trust = $\begin{bmatrix} [1, 3], [2, 3] \end{bmatrix}$



* In degree of town judge = $n - 1$

& Outdegree = 0.

✓ Create 2 arrays

outdegree	<table border="1"> <tr> <td>0</td><td>0</td><td>1</td><td>0</td></tr> <tr> <td>0</td><td>1</td><td>2</td><td>3</td></tr> </table>	0	0	1	0	0	1	2	3
0	0	1	0						
0	1	2	3						
indegree	<table border="1"> <tr> <td>0</td><td>0</td><td>0</td><td>2</td></tr> <tr> <td>0</td><td>1</td><td>2</td><td>3</td></tr> </table>	0	0	0	2	0	1	2	3
0	0	0	2						
0	1	2	3						

for $[1, 3]$

indegree of 1 ↑
Outdegree of 3 ↑

for $[2, 3]$

indegree of 2 ↑
Outdegree of 3 ↑

→ traverse both indegree & outdegree

if $\text{indegree} == 0 \ \&\ \&$

$\text{outdegree} == n - 1$

then return that vertex

code

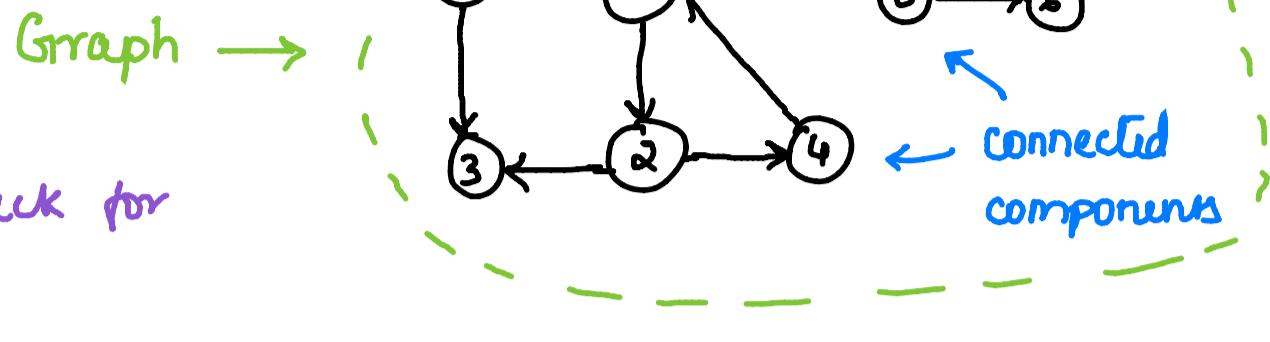
```
● ● ●  
1 class Solution {  
2 public:  
3     int findJudge(int n, vector<vector<int>>& trust) {  
4         vector<int>indegree(n+1,0);  
5         vector<int>outdegree(n+1,0);  
6         for(int i=0;i<trust.size();i++)  
7         {  
8             int v1 = trust[i][0];  
9             int v2 = trust[i][1];  
10            outdegree[v1]+=1;  
11            indegree[v2]+=1;  
12        }  
13        for(int i=1;i<=n;i++)  
14        {  
15            if(outdegree[i]==0 && indegree[i]==n-1)  
16                return i;  
17        }  
18        return -1;  
19    }  
20};
```

7 Detect cycle in a directed graph

Consider a graph with 'n' vertices labelled as $[0..n-1]$

Eg $n=7 [0, 1, 2, 3, 4, 5, 6]$

Graph \rightarrow



* To detect cycle, check for backedge.

Let's start dfs from 0 vertex.

* At every vertex, check if it's already visited, if already visited then check if it is present in recursive stack.

If present, then it indicates back edge \rightarrow Returns True

* If vertex is not visited then mark it in visited array & recursive stack

Visited $\rightarrow \{0, 1, 2, 3, 4\}$

Recursive stack $\rightarrow \{0, 1, 2, 3, 4\}$

* At 3 vertex, there's no neighbour

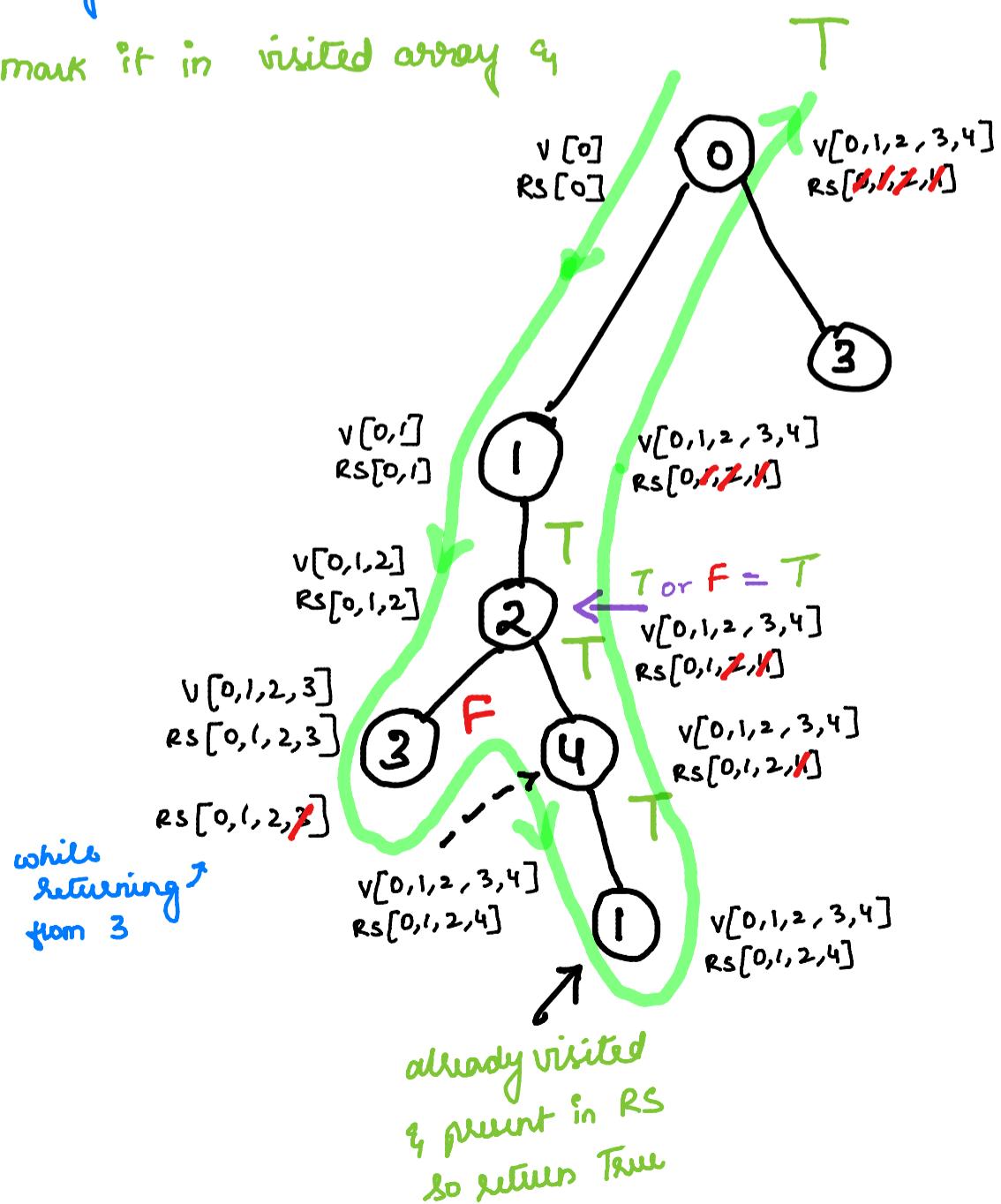
& no cycle is detected so return F.

Before returning, undo change made in Recursive stack by popping it.

Visited $\rightarrow \{0, 1, 2, 3, 4, 1\} \text{ q}$

Recursive stack $\rightarrow \{0, 1, 2, 3, 4\}$

1 is already present in recursive stack so return true.



Code

$T_C \rightarrow O(V+E)$

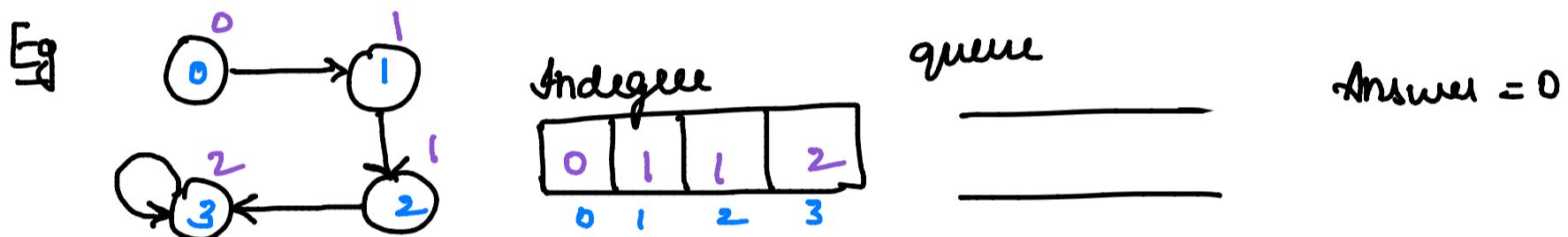
$S_C \rightarrow O(V)$

```
1 class Solution {
2     public:
3         bool dfs(int node, vector<int>&vis, vector<int>&rs, vector<int> adj[])
4         {
5             vis[node]=1;
6             rs[node]=1;
7             for(auto it:adj[node])
8             {
9                 if(vis[it]==0){
10                     if(dfs(it,vis,rs,adj))
11                         return true;
12                 }
13                 else if(rs[it]==1)
14                     return true;
15             }
16             rs[node]=0;
17             return false;
18         }
19         bool isCyclic(int V, vector<int> adj[]) {
20
21             vector<int>vis(V,0);
22             vector<int>rs(V,0);
23
24             for(int i=0;i<V;i++)
25             {
26                 if(vis[i]==0)
27                     if(dfs(i,vis,rs,adj))
28                         return true;
29             }
30             return false;
31         }
32     };
```

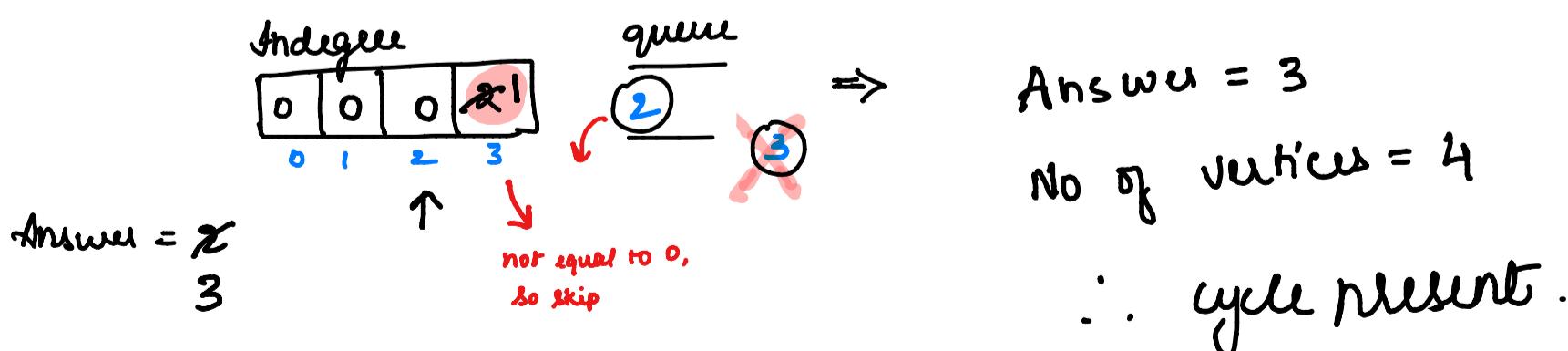
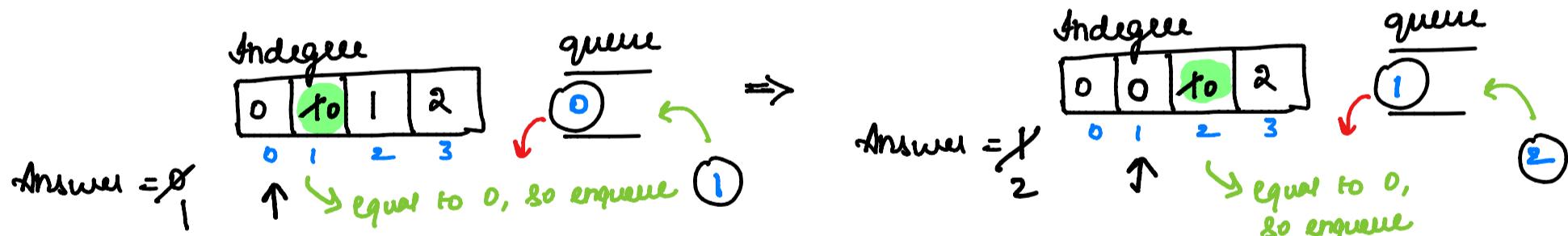
* Kahn's Algorithm → To find topological Ordering

↓
can be used to find cycle using BFS.

- ① Find indegree of every vertex in graph & answer = 0
- ② If indegree of vertex is 0, then push into queue & do bfs till queue is not empty & while doing bfs decrease the indegree of neighbour by 1.
if indegree of neighbour = 0, then enqueue & increment answer by 1
- ③ If answer != no. of vertices then cycle is present.



→ As indegree of 0 is 0, we push into queue & do bfs till queue is not empty.



code

```
1 class Solution{
2     public:
3         bool isCyclic(int V, vector<int> adj[]) {
4
5             vector<int>indegree(V,0);
6             for (int i = 0; i <V; i++)
7                 for(int it : adj[i])
8                     indegree[it]++;
9
10            queue<int>q;
11            int ans = 0;
12            unordered_set<int>vis;
13
14            for (int i=0;i<V;i++)
15            {
16                if(indegree[i]==0){
17                    q.push(i);
18                    ans+=1;
19                }
20            }
21
22            while(!q.empty())
23            {
24                int currvertex = q.front();
25                q.pop();
26                if(vis.find(currvertex)!=vis.end())
27                    continue;
28                vis.insert(currvertex);
29                for(int neighbour:adj[currvertex])
30                {
31                    indegree[neighbour]-=1;
32                    if(indegree[neighbour]==0)
33                    {
34                        q.push(neighbour);
35                        ans+=1;
36                    }
37                }
38            }
39            if(ans==V)  return false;
40            return true;
41        }
42    };
```

⑧ Topological sort

→ use Kahn's algorithm. & add node to result while performing dfs.

Code →

$T\bar{C} \rightarrow O(V + E)$

$S\bar{C} \rightarrow O(V)$

```
● ● ●  
1 class Solution  
2 {  
3     public:  
4     vector<int> topoSort(int V, vector<int> adj[]){  
5         vector<int> indegree(V, 0), res;  
6  
7         for(int i=0; i<V; i++)  
8             for(auto it:adj[i])  
9                 indegree[it]++;  
10  
11         queue<int> q;  
12         int ans = 0;  
13         unordered_set<int> vis;  
14  
15         for(int i=0; i<V; i++)  
16         {  
17             if(indegree[i]==0){  
18                 q.push(i);  
19                 ans+=1;  
20             }  
21         }  
22  
23         while(!q.empty())  
24         {  
25             int curr = q.front();  
26             q.pop();  
27  
28             // add to res  
29             res.push_back(curr);  
30  
31             if(vis.find(curr)!=vis.end())  
32                 continue;  
33  
34             vis.insert(curr);  
35  
36             for(int neighbour: adj[curr])  
37             {  
38                 indegree[neighbour]-=1;  
39                 if(indegree[neighbour]==0)  
40                 {  
41                     q.push(neighbour);  
42                     ans+=1;  
43                 }  
44             }  
45         }  
46     }  
47  
48     return res;  
49 }  
50 };
```

⑨ Course Schedule → can be solved using Kahn's algo.

$$Tc \rightarrow O(V + E)$$

$$Sc \rightarrow O(V + E)$$

Code →

```
● ● ●

1 class Solution {
2 public:
3     vector<vector<int>> createGraph(int n, vector<vector<int>>& pre){
4         vector<vector<int>> graph(n);
5         for(auto it:pre){
6             int v = it[1];
7             int u = it[0];
8             graph[v].push_back(u);
9         }
10        return graph;
11    }
12
13    bool canFinish(int n, vector<vector<int>>& pre) {
14        vector<vector<int>> graph = createGraph(n, pre);
15        vector<int> indegree(n, 0);
16        for(int i=0; i<n; i++)
17            for(int it: graph[i])
18                indegree[it]++;
19
20        queue<int> q;
21        int ans = 0;
22        unordered_set<int> vis;
23
24        for(int i=0; i<n; i++)
25            if(indegree[i]==0){
26                q.push(i);
27                ans++;
28            }
29
30        while(!q.empty()){
31            int currvertex = q.front();
32            q.pop();
33            if(vis.find(currvertex)!=vis.end())
34                continue;
35            vis.insert(currvertex);
36            for(int neighbour: graph[currvertex]){
37                indegree[neighbour]--;
38                if(indegree[neighbour]==0){
39                    q.push(neighbour);
40                    ans++;
41                }
42            }
43        }
44        if(ans==n) return true;
45        return false;
46    }
47};
```

10 Course Schedule - II

pre → edges $[v, u]$

" u should be completed before v "

$n \rightarrow$ no. of courses [vertices]

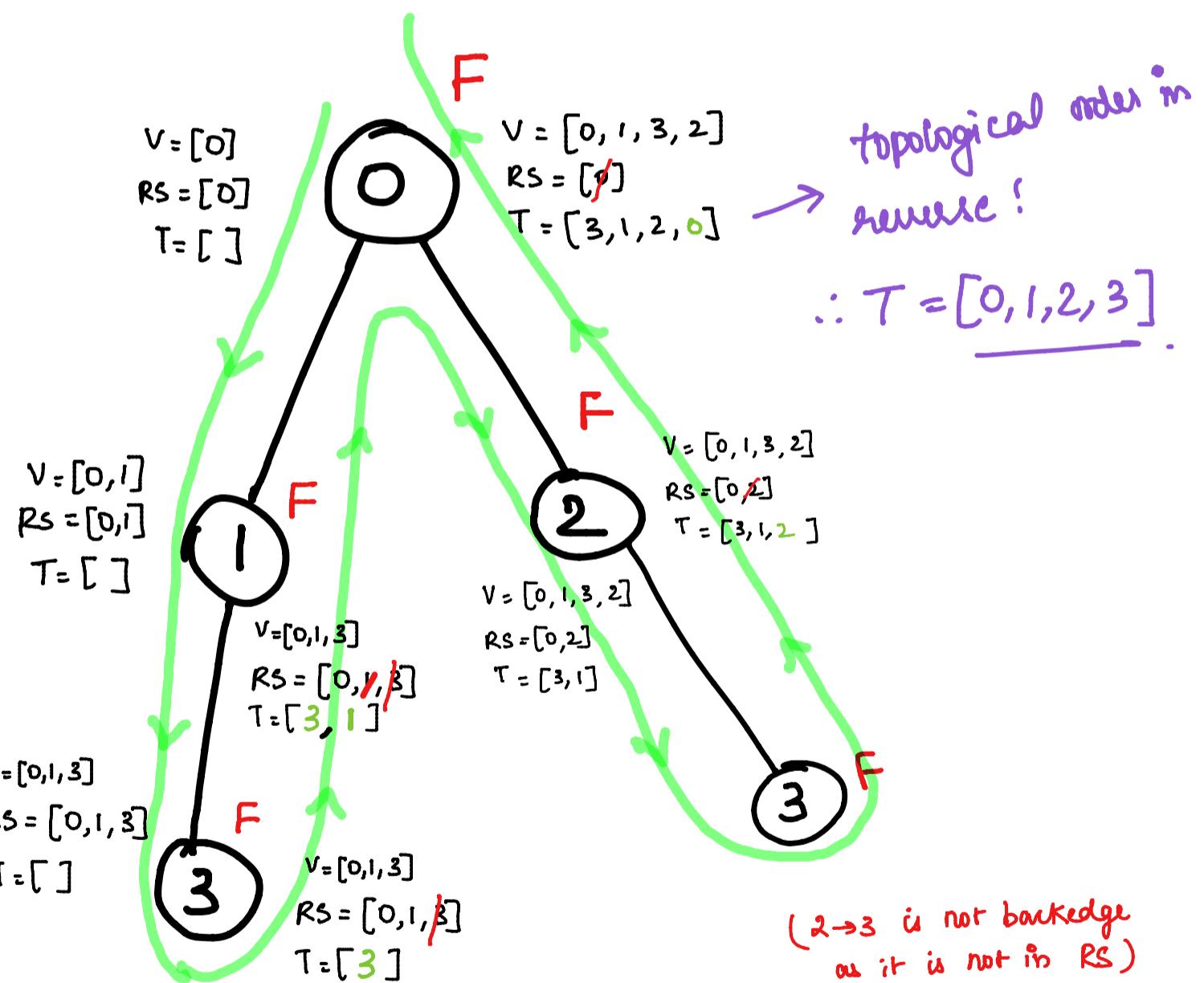
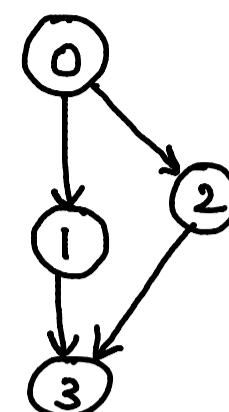
Topological sort only for DAG.

Eg $n \rightarrow 4 (0, 1, 2, 3)$

pre → $\{[1, 0], [2, 0], [3, 1], [3, 2]\}$

Initially

$V = []$, $RS = []$, $traversal = []$



while returning from $3 \uparrow$
pop 3 & push into traversal array.
returns F, as no cycle is found

Code →

$$Tc \rightarrow O(v + E)$$

$$Sc \rightarrow O(v + E)$$

```
● ● ●  
1 class Solution {  
2 public:  
3     bool dfs(vector<vector<int>>&graph, int i, vector<int> &vis,  
4             vector<int> &rs, vector<int> &traversal){  
5  
6         vis[i] = 1;  
7         rs[i] = 1;  
8         for(int neighbour: graph[i]){  
9             if(vis[neighbour]==0){  
10                 if(dfs(graph, neighbour, vis, rs, traversal))  
11                     return true;  
12             }  
13             else if(rs[neighbour]==1)    return true;  
14         }  
15         traversal.push_back(i);  
16         rs[i]=0;  
17         return false;  
18     }  
19  
20     vector<vector<int>> createGraph(int n, vector<vector<int>>& pre){  
21         vector<vector<int>> graph(n);  
22         for(auto it:pre){  
23             int v = it[1];  
24             int u = it[0];  
25             graph[v].push_back(u);  
26         }  
27         return graph;  
28     }  
29  
30     vector<int> findOrder(int n, vector<vector<int>>& pre) {  
31         vector<vector<int>> graph = createGraph(n, pre);  
32         vector<int> vis(n,0), rs(n,0), traversal;  
33         for(int i=0; i<n; i++){  
34             if(vis[i]==0)  
35                 if(dfs(graph, i, vis, rs, traversal)) return {};  
36         }  
37         reverse(traversal.begin(), traversal.end());  
38         return traversal;  
39     }  
40 };
```