

[50] **Homework 2.** *Language of Mathematics*

Each problem is worth 10 points

[10] Prove that for any sets A and B

$$\overline{A \cup B} = \bar{A} \cap \bar{B}$$

[10] Let x and y be *integers*. Determine whether the following relations are reflexive, symmetric, antisymmetric, or transitive:

- $x \equiv y \pmod{5}$;
- $xy \leq 1$;
- $x = y^3$.

Justify your statements.

Finally, determine which of the above relations are equivalence and partial order relations. For equivalence relations, construct the equivalence classes.

[10] Determine whether the following function is bijection from \mathbf{R} to $f(\mathbf{R})$:

- $f(x) = x^4$
- $f(x) = \cos^2(x)$
- $f(x) = \frac{x+3}{x+1}$.

[10] Let $g(x) = \lfloor x \rfloor$. Find

- $g^{-1}(\{0\})$;
- $g^{-1}(\{x : 0 < x < 1\})$

[10] What are the values of the following:

$$\begin{aligned} & \sum_{i=1}^{500} 6^{i+1}, \\ & \sum_{i=1}^2 \sum_{j=1}^3 (i+j) \\ & \sum_{j=0}^8 j \cdot 3^j. \end{aligned}$$