[50] Homework 4. *Proof Techniques*

Each problem is worth 10 points

[10] Show that $\sqrt[3]{3}$ is irrational.

[10] Let A be a set of cardinality n. Let $P(A)$ be the power set, that is, the set of *all* subsets of A. Prove by induction that cardinality of $P(A)$ is 2^n, that is,

$$|P(A)| = 2^n.$$

[10] Prove by induction on $n \geq 1$

$$\sum_{i=1}^{n} i \cdot i! = (n+1)! - 1.$$

[10] The *harmonic number* H_n is defined as for $n \geq 1$

$$H_n = \sum_{k=1}^{n} \frac{1}{k}.$$

Prove by induction that

$$H_{2^n} \geq 1 + \frac{n}{2}$$

whenever n is a nonnegative natural number.

[10] Derive an explicit formula for the following recurrence for $n \geq 2$

$$a_n = \frac{n-1}{3} a_{n-1}$$

with $a_1 = 1$.