Hall Effect

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Abstract

The basic aim of the experiment was to find the hall coefficient along with some other physical properties for certain N-type, P-type and un-doped Germanium and Silicon semiconductors. Majority carrier's sheet density, mobility and the band gap for the laminar metalloids could also be found using the technique suggested by Van der Pauw[1].

Using Van der Pauw method I found out the sheet resistance which was used to find the sheet density. The sheet density gives the sheet hall coefficient and the electron/hole mobility. The electron/hole mobility agrees fairly with their accepted values. Further, the band gap for Germanium was found using sheet resistances at different temperatures and by fitting the conductance vs. temperature curve to an exponential. The justification for ignoring the variations due to the polynomial term being that the exponential curve dominates over the range of temperatures we deal with.

Introduction

Semicomductors

Solids are classified as either conductors, insulators or semiconductors on the basis of their ability to conduct electricity through their mass. A material that conducts electricity easily through it is called a metal, whereas one that does not conduct at all are called insulators. Semi-conductors are materials that show an intermediate behavior. Semi-conductors posses some remarkable properties as compared with metals such as its conductivity increases with temperature. This is because the band gap, i.e. the energy difference between conduction band and the valence band is of the order of eV and so when the temperature increases, more and more electrons get excited to the valence band resulting in an increase in conductivity. This is not so for metals because the valence band and the conduction band are indifferentiable, and with the increase in temperature only the scattering of conducting electrons due to lattice vibrations increases, resulting in a decrease in the conductivity.

In this experiment we found out, for each sample, the sheet hall coefficient, which was used to find the (majority) sheet carrier density. Further using this and sheet resistance, we found out the majority carrier (hole for P-type and electron for N-type) mobility. Using resistivity measurements at different temperatures Germanium lattice band gap was found.

Hall effect

When a moving electric charge is subjected to a magnetic field that is not along its direction of motion, the charge drifts and the drift is given by the Lorenz force

$$F = q(\vec{E} + \vec{v} \times \vec{B})$$

Similarly when a current carrying material is subjected to a magnetic field perpendicular to the direction of current, the current carriers are drifted in the direction perpendicular to both current and the magnetic field. Soon, the potential developed across the width of the current carrying material balances the magnetic force acting on current carriers and is called Hall voltage (V_H) which is related to current (I) and magnetic field (B) by

$$V_H = (\frac{w}{epA})IB \tag{1}$$

where w is the width(along the hall voltage), e the electronic charge, p- carrier concentration and A- Area of cross section. Hall coefficient is defined as

$$R_H = \frac{E}{JB} = \frac{1}{ep} = \frac{1}{ewp_s}. \quad (2)$$

where E is the electric field along the hall voltage and J the current density. p is replaced with negative n for a N-type semiconductor.

Apparatus

The apparatus involved is:-

- Square shaped N-doped, P-doped and Undoped laminar samples of Germanium and Silicon. These samples satisfied the requirements put forth by Van der Pauw for using his method as given in the section below on Method and Results.
- A Van der Pauw power supply which has four alike knobs and a current adjusting knob. The four knobs are basically used to adjust about which of the four terminal is the current supplied or the voltage measured.
- A (milli)voltmeter and an (milli)ammeter
- A Heater
- An Electromagnet and/or a Permanent magnet. In this experiment both were used.
- Bell 620 Gaussmeter probe to measure the magnetic field produced by the magnets.

Method and Results

As suggested by Van der Pauw in 1958[1][3] the Hall coefficient and electrical resistivity of a laminar material can be found using his method. The Van der Pauw(VdP) method requires the following conditions to be satisfied:-

- Sample should be flat, i.e laminar
- Sample should not have any holes
- Sample should be homogeneous and isotropic
- Four electrodes should be contacted at the perimeter of the sheet.

Hall coefficient and Carrier concentration

With these conditions satisfied, the hall coefficient can be found by taking voltage readings across opposite terminals when current is supplied across other two terminals. Since we used square homogeneous samples, by symmetry we expect voltages $V_{13} = V_{24} = V_{31} = V_{42}$ (Where the subscripts indicate the terminals across which voltage was measured). Therefore the average of these is taken as the Hall voltage. This is used to find the sheet Hall coefficient and the sheet carrier concentration.

There is a little caveat I would like to mention. The voltages V_{13} in general may not equal V_{31} because of the 'background' hall voltage. This is an intrinsic voltage that appears across the width even in the absence of any magnetic field. The method employed by VdP takes care of this by taking the average of both the potentials, which cancels any 'background' hall voltage.

The data taken from the experimental setup was used to calculate the Hall coefficient and carrier sheet density for Ge-P-doped, Ge-N-Doped, Ge- Undoped and Si-N-doped by taking the plot of V_H vs I and using Eq. (1) and (2).

For the Si-Undoped samples V_H vs B curves was taken. As Eq. (1) suggests, this curve would also lead to the Hall coefficient. The calculated data is shown in Table 1.

Here I should mention that the definition of p_s(or n_s) is a little ambiguous for undoped samples because in them we expect the concentration of both holes and electrons to be almost equal. Because both are the carriers in this case, one would expect a net 0 hall effect but rather we observed a slight N-doped behavior, i.e. electrons conduct more electricity. The explanation for this could be that since (as expected) electrons have a larger mobility, they are more conducting as compared to holes.

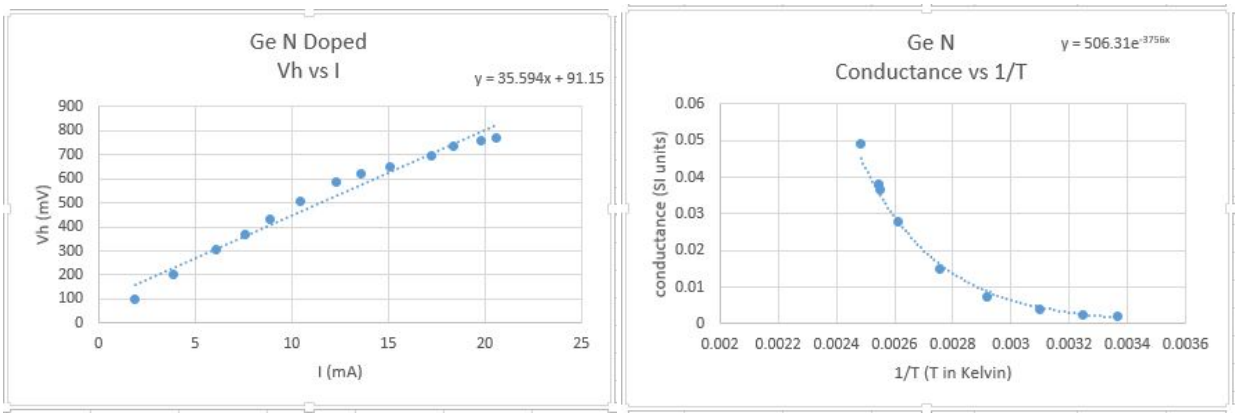

Figure 1: *Two curves fitted. Left: To find the Hall coefficient the slope of Vh vs I curve was found. Right: For finding the band gap the conductance vs 1/T scatter was fitted exponentially*

Table 1: *Calculated results*

Sample	$n_s(m^{-2})$	R_h/t	mobility e/h
Ge-P	$6.56 \times 10^17, h$	9.5	0.1875,h
Ge-N	$6.84 \times 10^16, e$	-91.3	0.224,h
Ge-Un	$6.58 \times 10^20, e, h$	-0.01	0.001,e,h
Si-N	$4.1 \times 10^18, h$	-1.52	0.127,e
Si-Un	$3.13 \times 10^14, h$	2000	167,e,h

Mobility

Mobility of a carrier(μ) is related to its conductivity(σ) by

$$\sigma = ne\mu \tag{3}$$

where n is the carrier density and e electronic charge. Eq. (3) can be translated to

$$\mu = \frac{1}{R_s n_s e} \tag{4}$$

where R_s is the sheet Hall coefficient. Therefore, to find the mobility, we found out the sheet resistance which was calculated using the VdP method.

According to VdP method,

$$e^{-\frac{\pi R_{12,34}}{R_s}} + e^{-\frac{\pi R_{23,41}}{R_s}} = 1$$

where $R_{12,34} = \frac{V_{34}}{I_{12}}$. Giving symmetry argument, one can show this leads to

$$e^{-\frac{\pi R_v}{R_s}} + e^{-\frac{\pi R_h}{R_s}} = 1$$

where

$$R_v = \frac{R_{12,34} + R_{34,12}}{2}, R_h = \frac{R_{23,41} + R_{41,23}}{2}$$

and due to symmetry between vertical and horizontal axes $R_v = R = R_h$

$$\therefore 2e^{-\frac{\pi R}{R_s}} = 1$$

$$R_s = \frac{\pi R}{ln2} \tag{5}$$

Using which the mobility μ was calculated using Eq. (4).

Band Gap

The band gap for a material is related to its conductivity and temperature by

$$\sigma = \mu e(n_{extrinsic} + n_{intrinsic})$$

$$= CT^B(n_{extrinsic} + (\frac{kT}{2\pi\hbar^2})^{\frac{3}{2}}(m_e m_h)^{\frac{3}{4}} e^{(\frac{-\Delta E}{2kT})}) \tag{6}$$

The constants $C, B, and m_h$ are unknown but since these only vary constants of multiplication, an exponential fit to the scatter of σ vs. T will give us the ΔE.

Fitting the curve by an exponential is justified because for smaller ranges of temperature, the exponential term dominates and gives essentially the same result as it's product with the polynomial term would give.

By VdP method we found the R_s values for some temperatures. This was done using samples Ge-N, Ge-Un, Ge-P,Si-N, Si-P and Si-Un doped samples.

The Ge-Un and Ge-N doped samples showed an exponential dependence as expected and was used to get definitive results for the Band Gap. $\Delta E = 0.70eV \pm 0.06eV$ was obtained for Germanium which is in close agreement with the presently accepted value.

Other conductance vs. temperature curves showed remarkable behavior. The Si-P and Ge-P samples showed a dip in the curve. Both these curves rise until $70°C$. This indicates that until $T = 70°C$ the increase in the number of intrinsic carriers dominates over the increase in scattering due to lattice vibrations. Hence the conductivity increases. But after $70°C$ the lattice vibrations start dominating and there is a net decrease in the conductivity.

For Si-N and Un-doped samples we observe a decreasing σ vs. T curves, indicating the lattice vibration scattering are dominant from a very low temperature.

Acknowledgement

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References

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