Probing homogeneity of the Cosmos using Quasars

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by

Atul Kedia

Roll Number 120260004

Physics Department, IIT Bombay

Under the guidance of

Prof. Subhabrata Majumdar

Department of Theoretical Physics, TIFR

and co-guide

Prof. Vikram Rentala

Department of Physics, IIT Bombay

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Acceptance Certificate

Department of Physics
Indian Institute of Technology, Bombay

The Project Report entitled 'Probing homogeneity of the Cosmos using Quasars' submitted by Mr. Atul Kedia (Roll Number 120260004) may be accepted for evaluation.

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(Supervisor : Prof. Subhabrata Majumdar)

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Abstract

Modern cosmology is built upon the premise that Cosmological Principle holds true on large length scales. The Cosmological principle claims that the universe is homogeneous at large length scales. Testing the principle observationally is crucial for our understanding of the universe since most of our modern cosmology is built upon this principle. In this work we try to observe whether the universe is indeed homogeneous at large length scales or is it fractal like structured or completely inhomogeneous by using fractal dimension definition. Further random catalogs have to be generated, which are homogeneous by definition, and similar calculations for fractal dimensions will be performed over them. Homogeneity will be said to have achieved when quasar data agrees closely with random data generated.

SDSS III BOSS: Data Release 10 provides with quasar distribution deep in the night sky which enables us to probe for homogeneity at length scales never done before. All the calculations have been performed using the software package MATLAB.

Contents

Chapter 1

Introduction

1.1 The Cosmological Principle

Modern cosmology is built upon the premise that the universe is homogeneous and isotropic on very large length scales. This is called the Cosmological Principle. The cosmological principle implies that although at small length scales our universe has both big localized lumps of masses, like planets, stars and galaxies, and vast void expanses, like inter-galactic space, at large enough lengths we should observe that these non-uniformity are indistinguishable from others and for an observer it appears the same in any point in space and in any direction.

Hence at a large enough length scale, one would observe the galaxies, quasars(AGNs) and other celestial objects to be spread out randomly and on an average with a uniform density.

1.2 Current understanding of the Large-Scale Universe

There have been several attempts to understand the homogeneity and isotropy of the universe. Since any theoretical calculations cannot prove the postulate, we are left with observation as the only way to confirm.

The universe, as we know now, has an ever increasing size of bodies from planet-moon systems, to star-planet systems, to galactic center(possibly a black hole)-star systems, to supercluster-cluster systems and so on. This pattern of larger

and larger systems leads to an obvious and interesting question - 'is the universe fractal structured instead of being homogeneous and isotropic with such systems going on forever?' If true, the cosmological principle would break down and our modern understanding of the cosmos and its physics would have to be reframed.

Some works have attempted to confirm the cosmological principle. The Cosmic Microwave Background radiation(CMB) shows almost uniform temperature distribution in all directions giving evidence for isotropy with anisotropy of about 1 part in a 100,000 (Penzias & Wilson, 1965, Smoot et al., 1992, Planck Collaboration et al., 2014). Certain other probes for isotropy involve x-ray background and the celestial distribution of radio sources across the sky.

Other attempts to analyse the homogeneity of the universe has lead to some different results. Pietronero, 1987 found that the distribution of galaxies is fractal like to arbitrarily large scales. While many other works have found a fractal like structure of the universe, works by Yadav et al. 2005, Hogg et al. 2005, Sarkar et al. 2009 and Scrimgeour et al. 2012 showed using different data and different analytical techniques that although the universe is inhomogeneous at small length scales, homogeneity is achieved at a larger scale of about at 70 h^{-1} Mpc - 80 h^{-1} Mpc. Still some other works have found the universe to be inhomogeneous at very large length scales. Apart from these, the presence of the Huge-LQG (Clowes et al. 2013), a humongous pseudo-structure of 73 quasars and about 4 billion light years across, leaves the topic far from settled.

1.3   Motivation

With the immense expanse of the universe in mind, it becomes essential for computational methods to be employed to understand and prove or disprove the cosmological principle. With the present day advancements in observational techniques and high computational power it has becomes plausible for such calculations to be made and to conclusively say whether the universe is homogeneous, fractal like or inhomogeneous. If the universe is indeed homogeneous we would like to know the cosmological length-scales at which this is achieved. If the universe is structured like a fractal we would like to know its dimensionality.

This work is aimed to test whether the universe achieves homogeneity or not using Quasar data collected by Sloan Digital Sky Survey. Previous works have primarily been performed on galaxy data and Luminous Red Galaxy data with no work on quasars, which is being done in this work.

Chapter 2

Quasar Data from SDSS

2.1 SDSS III BOSS Data Release 10

Sloan Digital Sky Survey(SDSS) (York et al., 2000) is a wide field photometric and spectroscopic sky survey which is based at Apache Point Observatory, New Mexico, USA. It has surveyed about a-third of the night sky in 5 different broad wavelength bands (u, g, r, i and z) and has about $2,72,863$ quasars ranging to up to a redshift of 7. This gives us the unique opportunity to probe for homogeneity at distances so large which has never been done before. SDSS uses its survey coordinates (λ, η) for cataloging while it could be converted to a more natural set of celestial coordinate system used in astronomy (α, δ)(Called right-ascension and declination or ra-dec) by the conversion given in Stoughton et al., 2002.

2.2 Data

The quasar data used in this analysis was collected from SDSS III - BOSS Data Release 10 released in July of 2013. It contains quasar data in three different patches in the sky. Two of these patches are too small for any analysis to be performed, hence they are neglected leaving quasars only within the main survey region $-67 < \lambda < 67$ and $-40 < \eta < 40$ (see Figure 2.2). The (λ, η) coordinates given by SDSS are corrected and can be used directly as spherical polar coordinates while creating a 3-D map in cartesian coordinate system.
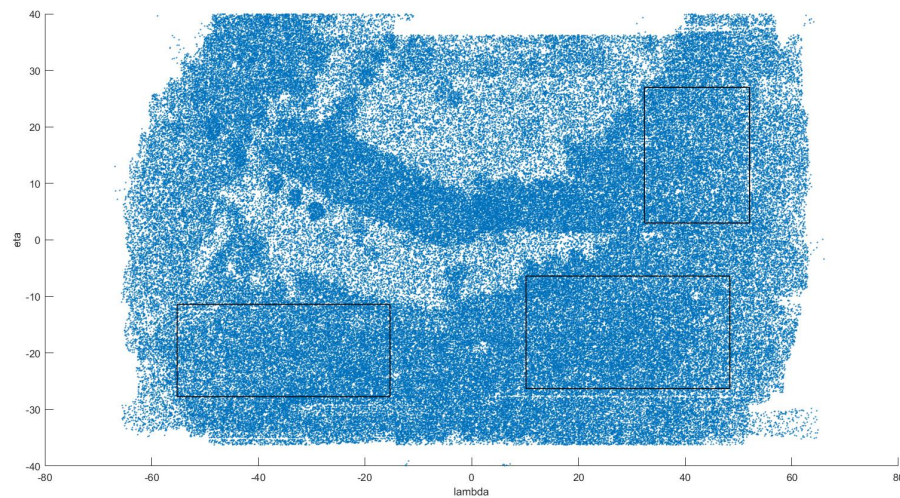
FIGURE 2.1: Central region from SDSS III BOSS: DR10 in survey coordinates $(\lambda - \eta)$ space. Each point is a Quasar. The Boxes indicate the subsample regions.
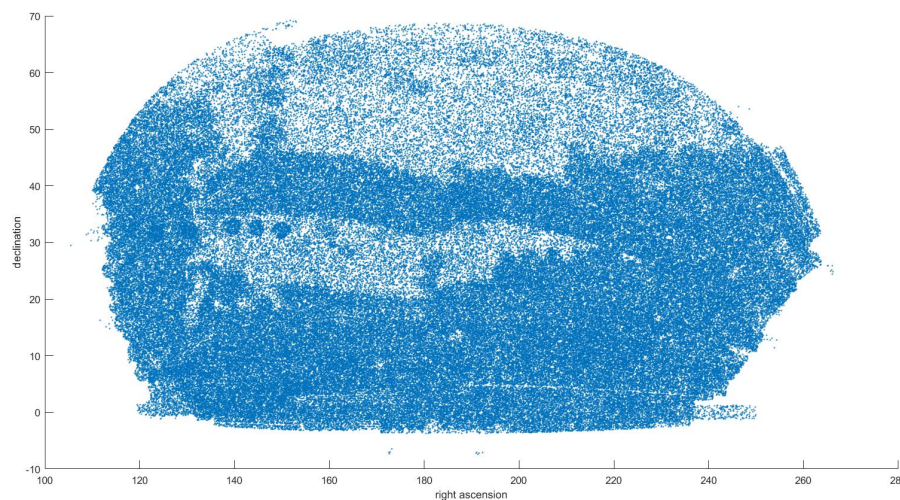

FIGURE 2.2: The same data in the celestial coordinates system $\alpha - \delta$

Chapter 3

Method of Analysis

The methods to calculate transition to homogeneity or fractal structure, as explored in previous works by Scrimgeour et al., 2012 and Sarkar et al., 2009 involves calculating the Scaled number counts for quasars followed by calculating it's fractal dimensions. These are then also produced for 10 randomly generated samples which, by definition, are homogeneous. Homogeneity is said to have achieved if the two closely match at large length scales else the fractal dimension can be obtained from the Minkowski-Bouligand dimension definition.

3.1   Scaled Number Counts

This is the primary test for homogeneity where the number of quasars (also called the count-in-spheres $n_i(< R)$) within a distance R from any one quasar is calculated. Then it is averaged over all quasars to give $N_A(< R)$ and then it's trend is observed. To calculate $n_i(< R)$ we construct an imaginary sphere of radius R around each quasar and count the number of other quasars within this sphere. This is then done for all quasars and the average is calculated to give $N_A(< R)$. A caveat here is that for a particular radius R, $n_i(< R)$ is calculated for only those quasars for which the constructed sphere entirely lies within the region volume. Hence quasars close to the boundary are avoided in calculating $N_A(< R)$ for radii below certain level, depending upon their distance from each boundary.

This $N_A(< R)$ measures the homogeneity since at homogeneity one would observe $N_A(< R)$ to have very little deviation, whereas if the distribution is inhomogeneous $N_A(< R)$ would have a large standard deviation. Further one would expect $N_A(<$

R) to scale as

$$N_A(< R) \propto R^D \tag{3.1}$$

where D is the dimension in which the quasars are distributed. At homogeneity, D is expected to be 3. Hence if for the quasar data we obtain such scaling, we can safely say that homogeneity is achieved.

Scaled number count $N(< R)$ is a useful estimator calculated by

$$\mathcal{N}(< R) = \frac{\sum\limits_{i=1}^{N_c} \rho_i{}^2 \left(\frac{N_A(<R)}{N_{Ri}(<R)} \right)}{\sum\limits_{i=1}^{N_c} \rho_i{}^2}, \tag{3.2}$$

where $N_{R_i}(< R)$ is the number of quasars($n_i(< R)$) within a distance R averaged over all quasars for random samples and ρ_i is the ratio of number of quasars in the random sample to the number of quasars in the data sample being used for the particular radius R. The Scaled number count $N(< R)$ at homogeneity is expected to reduce to unity.

3.2 Multifractal Analysis and Minkowski-Bouligand Dimension

The fractal like structure of the universe, as described earlier, motivates calculation of its fractal dimension. Their are multiple fractal dimension definitions as given by Borgani, 1995, of which Box-counting dimension and Minkowski-Bouligand dimensions can easily be used for describing finite galactic or quasar distributions. In this work I would aim to calculate the Minkowski-Bouligand dimension D_q for the quasar distributions. Minkowski-Bouligand dimension D_q is defined as the *log* derivative of the Correlation Integral $C_q(R)$ as defined below.

Using the count-in-spheres $n_i(< R)$, the correlation integral $C_q(R)$ can be calculated by

$$C_q(R) = \frac{1}{MN} \sum_{i=1}^{M} [n_i(< R)]^{(q-1)} \tag{3.3}$$

where M is the total number of quasars for which $n_i(< R)$ is calculated, i.e. neglecting the ones that are within R distance from the survey boundaries.

Thus the generalised Minkowski-Bouligand dimension is given by

$$D_q(R) = \frac{1}{(q-1)} \frac{\mathrm{d} \log C_q(R)}{\mathrm{d} \log r} \qquad (3.4)$$

Here q represents different moments of the quasar counts. Higher values of $q(>1)$ weighs overdense regions of quasars over underdense regions, i.e. regions were large number of quasars are packed (larger $n_i(< R)$) will be weighed higher as compared to regions of large voids and absence of quasars (small $n_i(< R)$). Likewise, $q < 1$ probes underdense regions of the space, with vast voids over regions with abundance of quasars. For calculating D_q from C_q cubic-spline interpolation method will be used, followed by the interpolated function's differentiation.

In the following chapter, I introduce how the samplings have been performed along with some intermediate steps and calculations.

Chapter 4

Sampling and Calculations

4.1 Subsampling

The survey does not observe all regions in the sky uniformly and the data collected from some regions of (λ, η) are more dense than others. This can be seen on Figure 2.2. Here the central and top regions can be seen to have relatively lesser density of quasars, suggesting incomplete surveys in these region. Hence it makes sense to select patches of the sky with complete survey instead of taking the entire region for the analysis. For this purpose, three subsamples were selected from the (λ, η) scatter as shown in Figure 2.2. The table 4.1 provides essential details about these regions. The three subsamples are analysed separately by the analysis methods employed and will be combined for the final calculation.

Region	No. of quasars	λ_{min}	λ_{max}	η_{min}	η_{max}
Region 1	13,483	10.2	48.4	-26.3	-6.4
Region 2	6,651	32.4	52.1	3	27
Region 3	10,715	-55.2	-15.3	-27.7	-11.4

TABLE 4.1: Regions and their corresponding λ and η ranges.

The original data collected from SDSS had the λ, η positions over the survey coordinate space, α, δ coordinates, the redshifts and the apparent magnitudes in the u, g, r, i, and z pass bands. All the calculations mentioned below have been performed for all these three regions separately. All the calculations and manipulations mentioned further on were performed using MATLAB, 2015b software package. From here on I will keep prompting about any new information extracted using the information available.

4.2 Comoving Distance Calculation and positions in 3-D space

In order to map the quasars using their redshifts obtained from the SDSS survey to actual 3-D space, needed to calculate the count-in-spheres and fractal dimensions, we need to calculate the comoving distance of quasars w.r.t Earth. Comoving distance is defined as the distance between two objects in the universe at a given epoch. When calculated for all quasars, we get a map of their distances w.r.t us at the same instant. Since the universe keeps expanding and the expansion rate increases with distance, the light that we observe here on earth from two different quasars at different redshifts left the quasars at different absolute times(epoch). Therefore using comoving distance becomes essential to create a map of the universe at an instant of time.

As expressed by Hogg, 2000, the comoving distance (D_C) can be calculated from the equation,

$$D_C = D_H \int_0^\infty \frac{dz'}{E(z')} \tag{4.1}$$

where z is the redshift, D_H the Hubble distance

$$D_H = \frac{c}{H_0} \tag{4.2}$$

and $E(z)$ given by

$$E(z) = \sqrt{\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda} \tag{4.3}$$

where Ω_M, Ω_Λ and Ω_K are cosmological parameters. Ω_M is the total Matter + Dark matter density in the universe, Ω_Λ is the Dark energy density and Ω_K measures the curvature of space. The values of H_0, Ω_M and Ω_Λ for calculating D_C obtained by the Planck Collaboration et al., 2014 is given in Table 4.2.

Parameter	Planck Value
H_0	67.80 h^{-1} Mpc
Ω_M	0.3083
Ω_Λ	0.692
Ω_K	-0.0003

TABLE 4.2: Cosmological Constants from Planck Collaboration et al., 2014

Further on, mapping the quasars to a 3-D space which is constructed for a particular epoch of time becomes easy with the quasars' celestial coordinates and
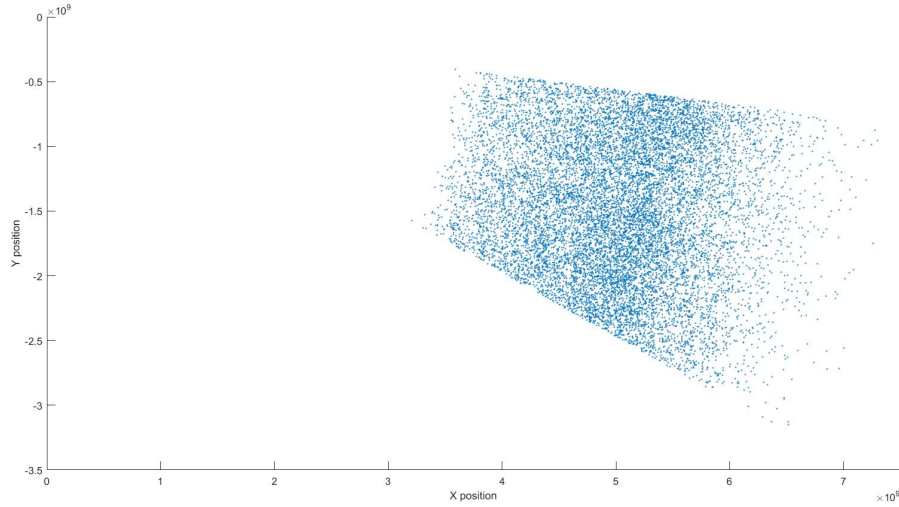
FIGURE 4.1: (x, y) projection of the 3-D map of quasars in region 1 with Absolute Magnitude $2 < M < 4.51$

comoving distances know. A simple trigonometric calculation was performed to obtain the cartesian coordinates and the (x,y) plane projection of Region 1 can be seen in Figure 4.1.

At this stage, along with the information about angular position, redshift and apparent magnitudes, we have calculated the comoving distances of quasars w.r.t us and a map of their (x,y,z) positions.

4.3 Absolute Magnitude and K-correction

The magnitudes obtained from the SDSS survey gives us the information about quasars' apparent brightness at Earth. A more standard way to quantify brightness of celestial bodies is Absolute Magnitude. Absolute magnitude is the magnitude of a celestial body observed at a cosmic distance of $10pc$ independent of its distance from any observers. It can be calculated using the Apparent magnitude available to the observer and the distance to the body.

Following calculations as described in Hogg, 2000 lead us to the Absolute magnitude (M) of quasars using Luminosity distance(D_L) and Distance modulus(DM).

$$D_L = (1 + z)D_H \frac{1}{\sqrt{|\Omega_K|}} sin\left[\sqrt{|\Omega_K|}\frac{D_C}{D_H}\right] \qquad (4.4)$$

$$DM = 5 * log\left(\frac{D_L}{10pc}\right) \qquad (4.5)$$

$$M = m - DM - K \tag{4.6}$$

where m is the apparent magnitude observed and K is the k-correction. The k-correction is necessary to be made in the observed Magnitude as described in Oke & Sandage, 1968. Ross et al., 2013 calculated k-corrections for i-band in the redshift range of $2 < z < 4.51$. Hence rest of the data had to be discarded and the remaining data to be taken for further analysis.

Now we have, the absolute magnitude in i-band(M), luminosity distance(D_L) and distance modulus(DM) of quasars, apart from the earlier data available for analysis.

4.4 Volume Limited Sample

Quasars observed by SDSS have a large redshift which makes them particularly useful to probe large distances. Due to their high brightness quasars at large distances can be observed easily. Still, the presence of low brightness quasars at small distances(redshift) suggests that although there are quasars at larger redshifts, they are not being detected by the survey due to their low apparent magnitude Baugh & Efstathiou, 1993. This is called the radial-selection effect and volume limited sampling is a useful technique that helps rectify this. A volume limited sample restricts data selection so that the number density, i.e. number of quasars per unit volume remains constant with increasing radius. We construct a volume limited sample by selecting quasars of absolute magnitude smaller than a value depending on it's redshift. This was only possible for quasars within a redshift of $2.15 < z < 2.85$. Hence volume limited samples with uniform density were arrived at for each regions. A combined volume limited sample of the three regions can be seen in Figure 4.2 and table 4.3 gives all the details about the data left in the three regions.
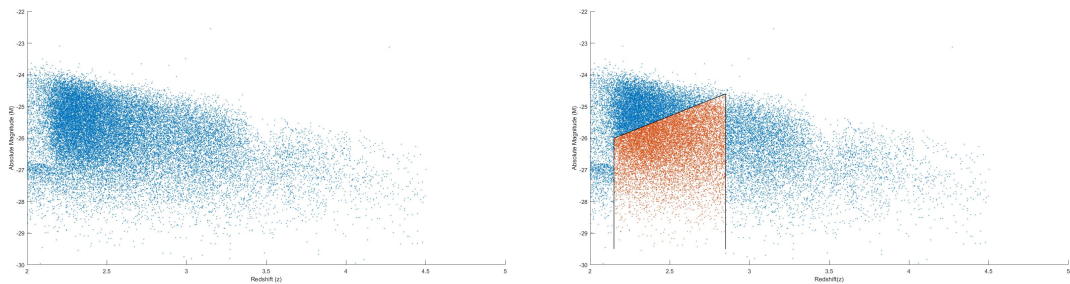

FIGURE 4.2: The quasar Absolute Magnitude in i-band and the volume limited sample (in orange).

Region	No. of quasars	z_{min}	z_{max}	M_{min}	M_{max}
Region 1	5,660	2.15	2.85	-29.399	-24.629
Region 2	2,705	2.15	2.85	-29.546	-24.712
Region 3	4,406	2.15	2.85	-29.433	-24.674

TABLE 4.3: A table of newly constructed volume limited samples.

4.5 Count-in-spheres

Now we calculate the count-in-spheres $n_i(< R)$. For this we need to know the distance of each quasar from all six of the boundaries. Two of the boundaries are along the radial distance form Earth to the quasar. Two lie on constant λ_{min} and λ_{max}, and two others on η_{min} and η_{max} boundaries. After having calculated the maximum allowed radius for each quasar we calculate the number of quasars within the distance of R from every quasar where R runs from 1 h^{-1} Mpc to 300 h^{-1} Mpc.

Figure 4.3 shows a plot of average count-in-spheres $n_i(< R)$ of all quasars for which the boundaries are farther than 300 h^{-1} Mpc. It is a monotonically increasing function as expected. The number of quasars within a radius could never decrease as the radius increases.

Count-in-spheres have been calculated for all quasars in all three regions.

4.6 Fractal dimension(D_q) calculation and error-bars

The fractal dimension calculation is straightforward and follows a very simple mathematical expression given above 3.4. Some D_qs can be seen in 4.4. D_qs show a lot of fluctuation figure 4.4. To see whether this data approaches the expected value of $D_q = 3$ we find the standard deviation in the calculated D_q using bootstrap resampling technique (described in a later chapter).

The D_qs calculated show large fluctuations and hence we employ new techniques and find the averaged out effect. We calculate the moving average of D_q for 11 consecutive values to show a smoothed out effect(Fig. 4.5).

We can see the D_qs approaching the expected value 3 by around 200 h^{-1} Mpc. This is one of the signs that the universe is homogeneous at lengths greater than 200
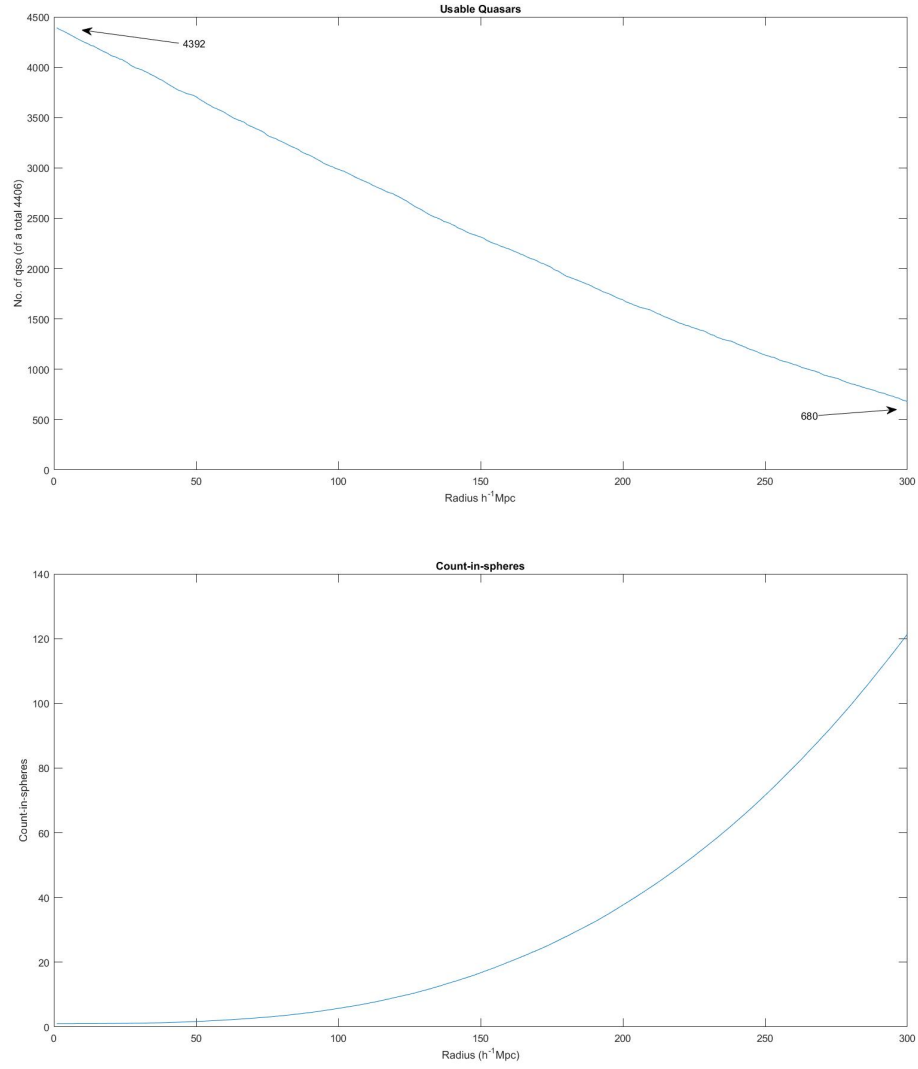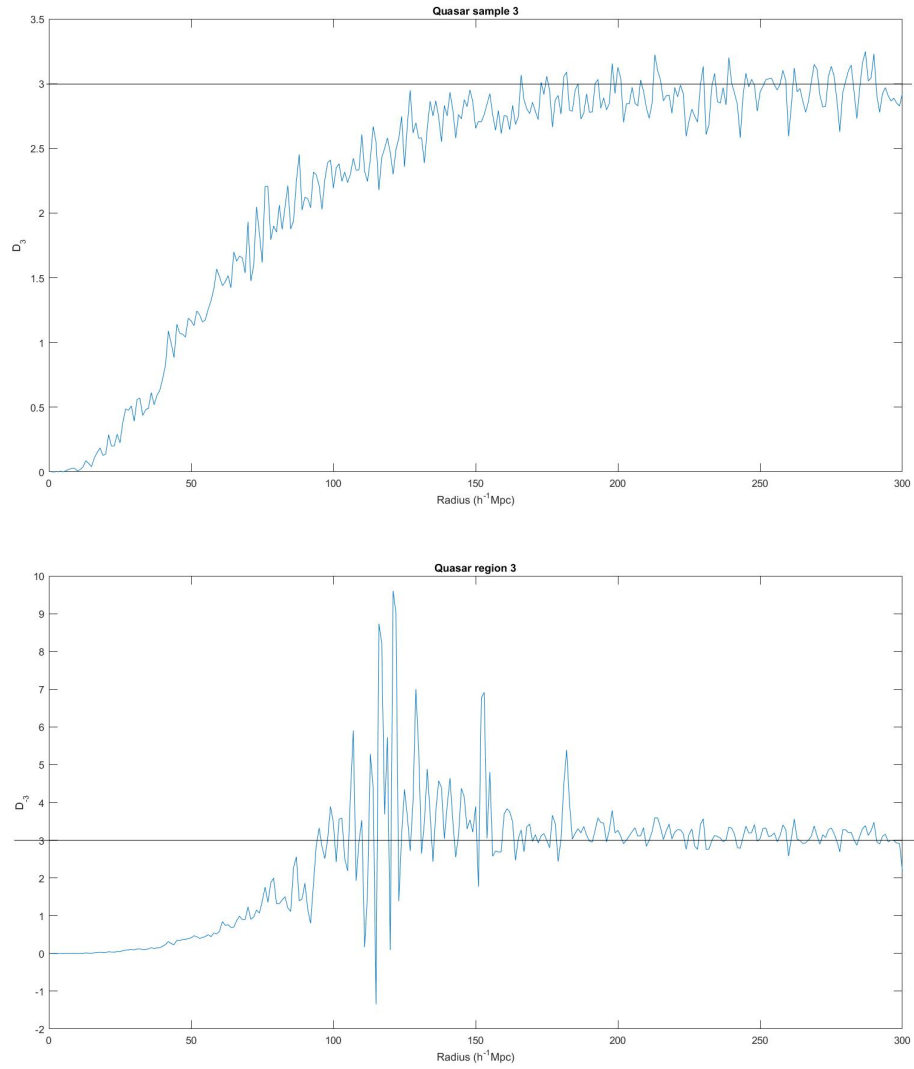
FIGURE 4.3: The First figure shows usable quasars for each radius in redion 3. The Second figure shows the average count-in-spheres for quasar in all three regions. As one will notice, the maximum allowed radii for these quasars are greater than 300 h^{-1} Mpc.

h^{-1} Mpc. The three data sets all show similar results and approach homogeneity at around 200 h^{-1} Mpc.

FIGURE 4.4: D_3 and D_{-3} calculated for region 3

4.7   Scaled-number-counts

The scaled number counts calculations were performed using the relation given 3.2. A plot of this is shown in fig. 4.6. This is against what was expected for homogeneity since it deviates away from unity and also contradicts the dimension calculation results.

A possible explanation for this behaviour is the relatively small sample size. The deviation from unity in other works is much larger than the deviation(of about 0.16) we see here.
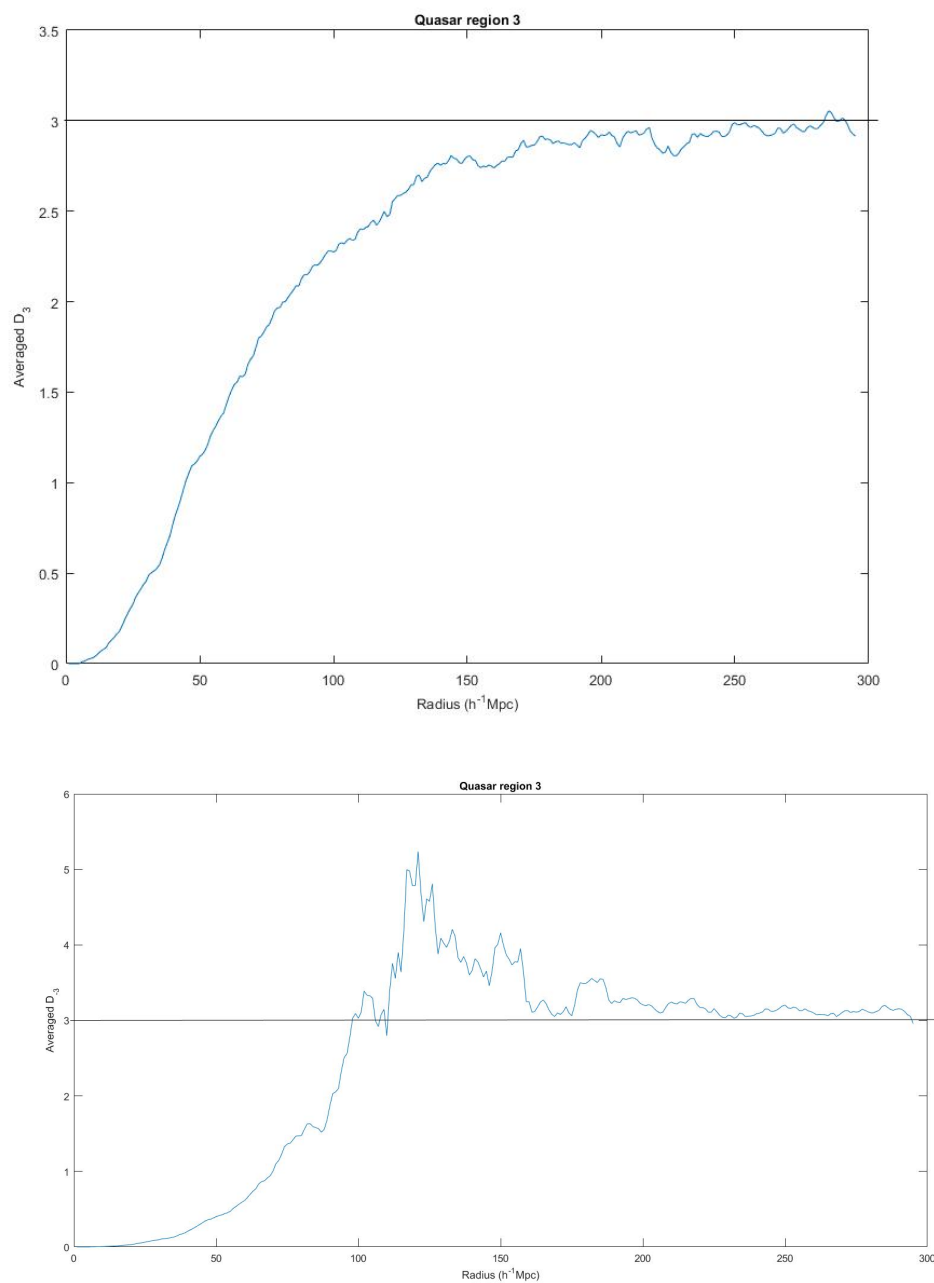
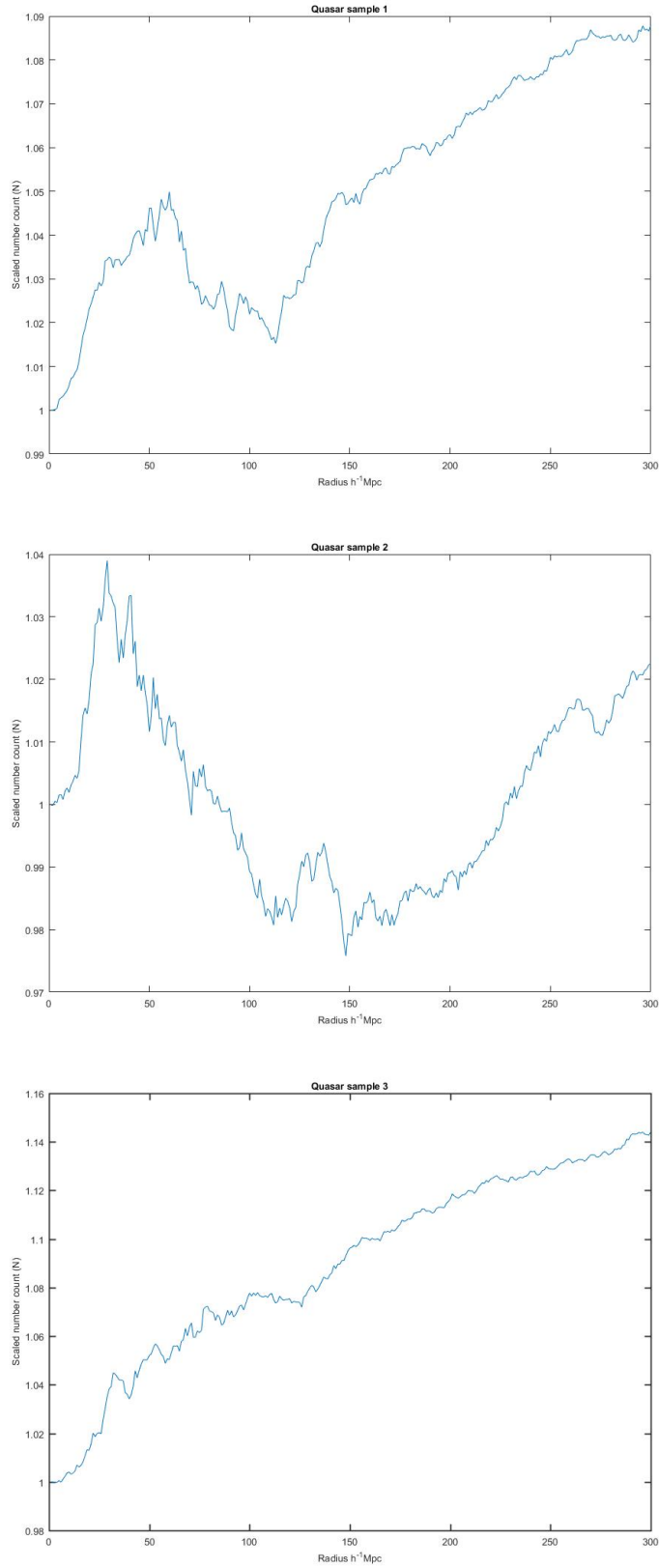FIGURE 4.5: Averaged D_3 and D_{-3} calculated for region 3

FIGURE 4.6: Scaled number count calculated for regions 1,2 and 3.

Chapter 5

Bootstrap resample

Bootstrap resampling technique is used to find the statistical variance in the quasar sample distribution. We construct 10 random samples in each region with number of quasars equal to as there were in each region. We follow the same calculations to find correlation integral (C_q), then Minkowski-Bouligand dimension (D_q) to find out the standard deviation in them. We dont need to calculate the maximum radius allowed for each quasar now as they have been calculated before and can be carried forward.

Plots of fractal dimension are shown in fig. 5.1. Homogeneity is achieved when the calculated dimension is within errorbars away from the expected value 3. This is achieved at about $R = 200h^{-1}\,\mathrm{Mpc}$. Hence the system is homogeneous above this length scale.

The plot D_q v. q(fig. 5.2) at different R values shows how closely they match. The D_q values at $R = 150h^{-1}\,\mathrm{Mpc}$ are far from $D_q = 3$ for large q. Whereas at $R = 210h^{-1}\,\mathrm{Mpc}$, $D_q = 3$ within statistical variations, and hence is homogeneous at such length scale.
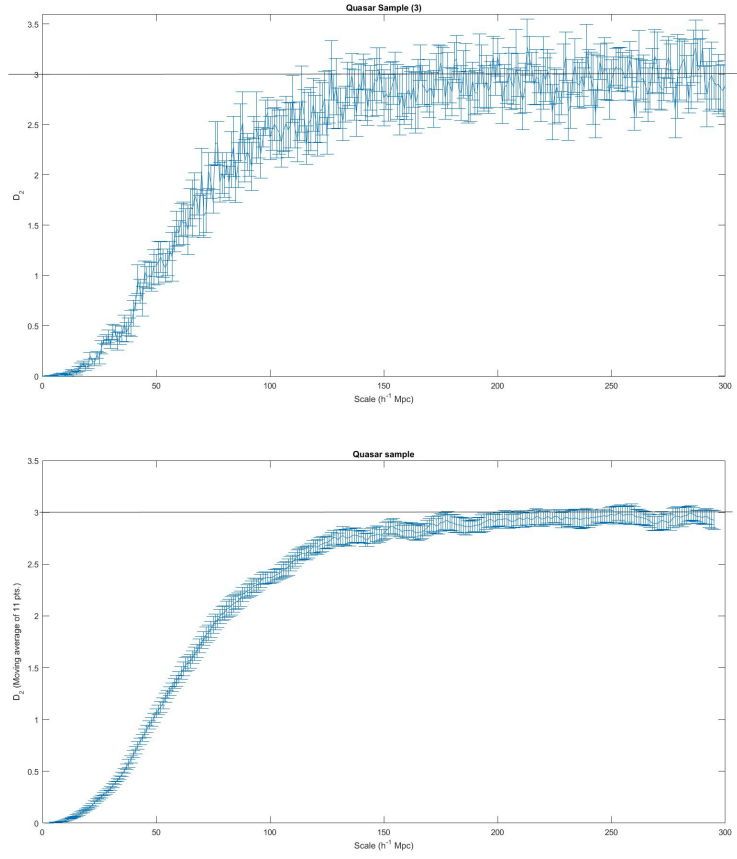
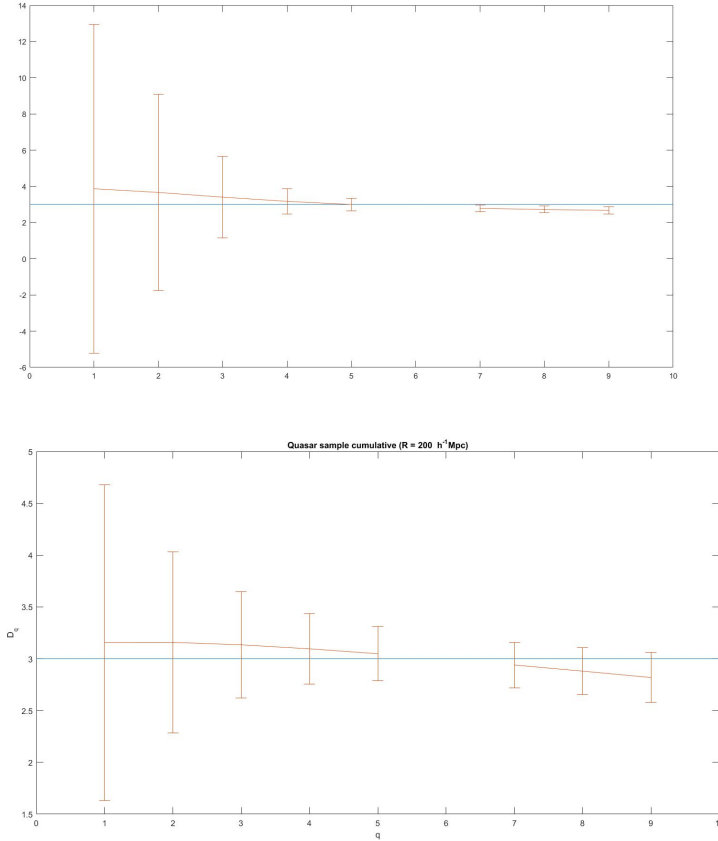FIGURE 5.1: These figures show D_3 and averaged D_3 calculated for region 3 along with their error bars.

FIGURE 5.2: D_q for $R = 150h^{-1}\,\mathrm{Mpc}$ and for $R = 200h^{-1}\,\mathrm{Mpc}$ for the cumulative of the three regions with their error bars.
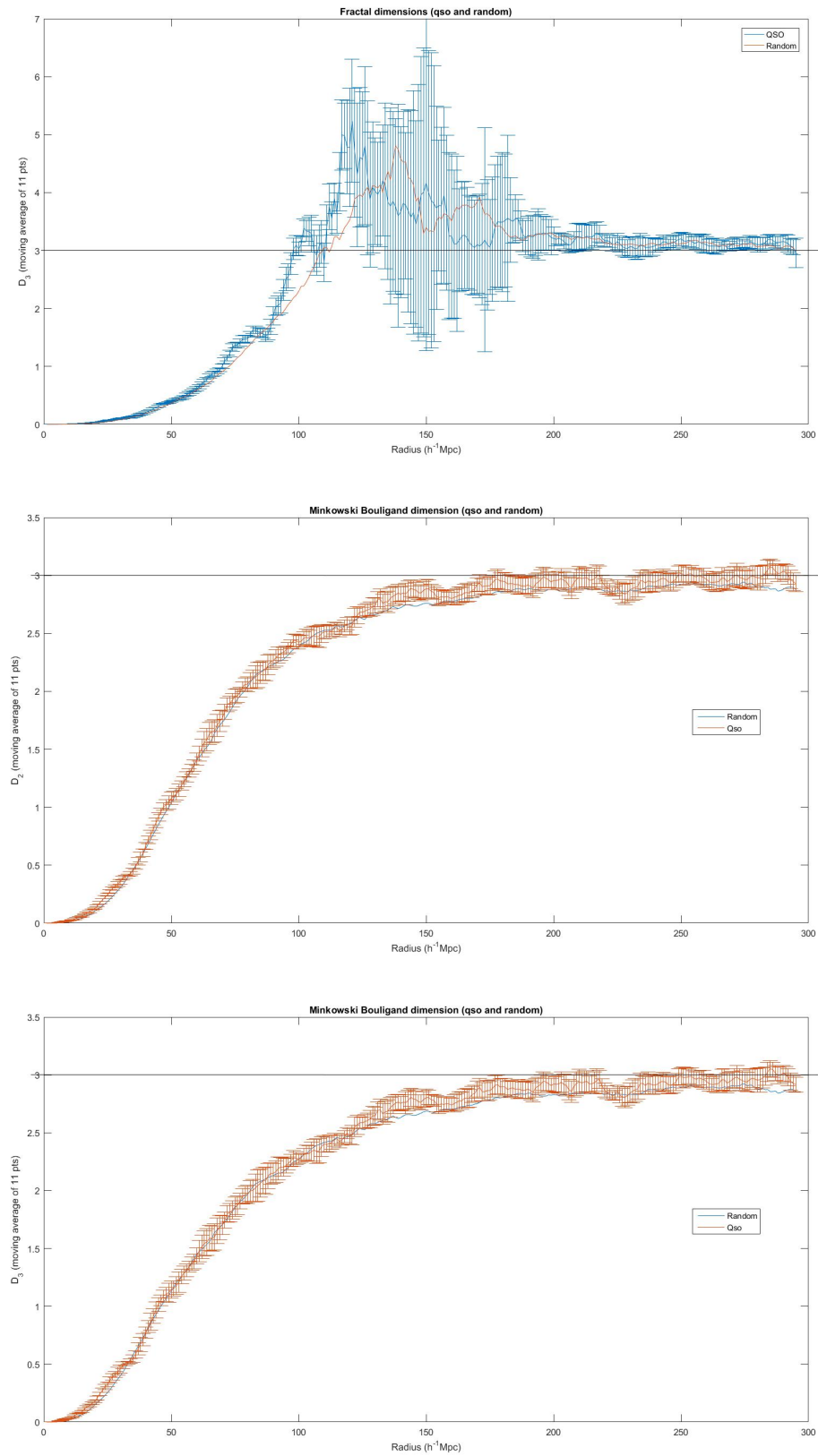
Chapter 6

Random sample

We construct random samples and follow the calculation D_q and compare it with the quasar data D_q. Random samples are created because they, by definition, are homogeneous and show similar trends in fractal dimension. We expect that homogeneous samples like the random samples we generate will approach dimensional $= 3$ at length scales smaller than the quasar sample due to its truly homogeneous nature.

As can be seen, all the quasar datasets and the random datasets approach homogeneity at length scales of around $200 \ h^{-1}$ Mpc. This indicates that homogeneity is achieved at this length scale.

In our calculations random samples do not approach homogeneity at a smaller length scale compared to quasar samples. Reason for this is unknown though we speculate that it is due to the small size of our datasets used for calculations. As shown earlier, each region has $2,700 - 5,700$ quasars, which is small enough for statistical variations to creep into calculations at small lengths.

FIGURE 6.1: D_{-3}, D_2 and D_3 for quasars and random samples.

Chapter 7

Conclusion

The analysis we follow for understanding the homogeneity properties of the universe are described and performed. Off the two methods employed, Minkowski-Bouligand dimension show that the universe is homogeneous at length scales greater than 200 h^{-1} Mpc whereas the scaled-number-counts leaves the subject questionable. Though the deviations are small, they show contradictory trends.

This work was aimed at finding whether the universe is homogeneous at large length scales. We obtained two measures of homogeneity of which one shows clear signs of homogeneity whereas the other still questionable.

Starting with a huge set of quasars spread out in about a-third of the night sky we narrow down our workable dataset and perform intricate calculations to find their spatial distribution, comoving distance, absolute magnitude, correlation integral, Minkowski-Bouligand dimension and scaled-number-count. All this to find whether the universe obey cosmological principle.

It is rather commendable that we humans could even dare to observe such cosmological phenomenon. With the aid of computers, we humans aim to understand the universe, our only home.

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