Week 12 Practice Problem Set - Laplace Transforms

Practice the questions in the following problem set and **submit only Qn No. 1a and 2a from Part 1 and Qn No. 2 from Part 2 as a single scanned pdf along with z-Transforms in Moodle.**

Part 1 **Questions:**

1. Determine the Laplace Transform and the associated region of convergence and pole zero plot for each of the following functions of time:

   (a) $x_1(t) = e^{2t}u(t) + e^{-3t}u(t)$

   (b) $x_2(t) = |t|e^{-2|t|}$

2. Determine the function of time $x(t)$, for each of the Laplace Transforms and their associated region of convergence.

   (a) $\frac{s^2-s+1}{(s+1)^2}$, $Re\{s\} > -1$

   (b) $\frac{s+1}{s^2+5s+6}$, $-3 < Re\{s\} < -2$

3. Consider the signal $x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$ and denote its Laplace Transform by $X(s)$. What are the constraints placed on the real and imaginary parts of β if the region of convergence of $X(s)$ is $Re\{s\} > -3$?

Part 2

1. A causal LTI system S with impulse response $h(t)$ has it's input $x(t)$ and output $y(t)$ related through a linear constant coeffiecient differential equation of the form,

$$\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(1+\alpha)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

   (a) If $g(t) = \frac{dh(t)}{dt} + h(t)$, how many poles does $G(s)$ have?

   (b) For what real parameter α is S guaranteed to be stable?

2. Consider the RL circuit shown below:

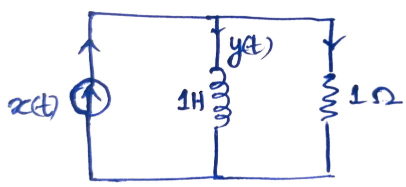

Figure 1:

   (a) Determine the zero state response of the circuit when input current is $x(t) = e^{-2t}u(t)$.

(b) Determine the zero-input response of the circuit for $t > 0^-$, given that $y(0^-) = 1$.

(c) Determine the output of the circuit when the input current is $x(t) = e^{-2t}u(t)$ and the initial condition is same as the one specified in part (b)

3. Consider an LTI system with input $x(t) = e^{-t}u(t)$ and impulse response $h(t) = e^{-2t}u(t)$.

   (a) Determine the Laplace transforms of $x(t)$and $h(t)$. Further, using convolution property, determine the LT $Y(s)$ of the output $y(t)$.

   (b) From Y(s) obtained in part (a), determine $y(t)$.