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Explicit Quantum Weak Coin Flipping

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Problem Statement

Quantum Key Distribution: Two trusting parties protect against adversaries.
Two party secure: Two distrustful parties wish to collaborate.
E.g. MS wants to use IBM's QC.

Coin Flipping (CF): Establish a random bit among two mutually distrustful, physically separated players without a trusted third party.

A, B → friends →  → fight → coin flip to decide

Weak CF: Preferences are known. E.g. A and B both want the car.
Strong CF: Preferences are unknown.

Both honest	pr(A wins) $= P_A$	pr(B wins) $= P_B$
Only A/B cheats	pr(A/B wins) $= P^*_{A/B}$	pr(B/A wins) $= 1 - P^*_{A/B}$

Bias: Smallest ϵ s.t. $P^*_A, P^*_B \leq \frac{1}{2} + \epsilon$.

NB. $0 \leq \epsilon \leq \frac{1}{2}$.

Prior Art

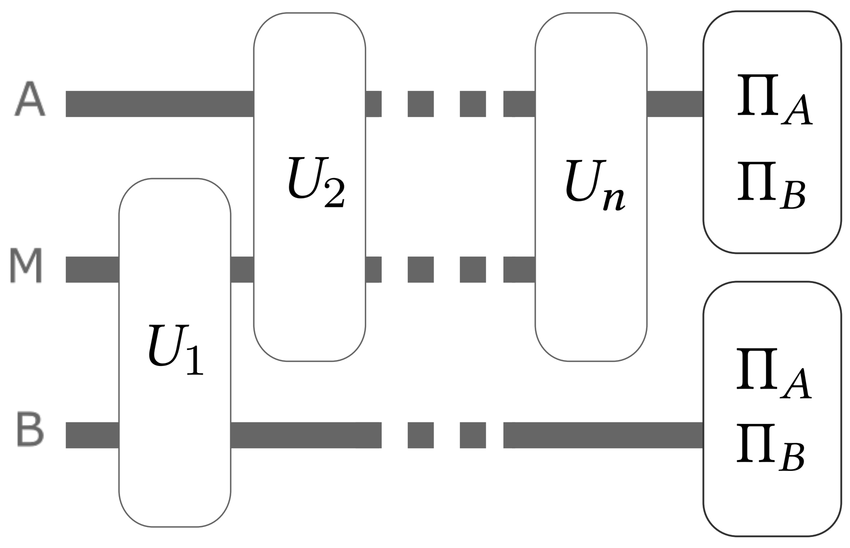
Classically: $\epsilon = \frac{1}{2}$ viz. at least one player can always cheat and win
(unless computational assumptions are made)

Quantumly: Strong CF: $\epsilon \geq \frac{1}{\sqrt{2}} - \frac{1}{2}$, best known $\epsilon = \frac{1}{4}$.

Weak CF: $\epsilon \to 0$, best known $\epsilon = \frac{1}{10}$,
numerically (using EMA) $\epsilon \to 0$.

Kitaev's Frameworks

General Protocol: ρ, U, Z



P^*_A is an SDP in ρ_B:

$$\max P^*_A = \mathrm{tr}[\Pi_A \rho_B]$$

s.t. the honest player (Bob) follows the protocol.

Similarly for P^*_B.

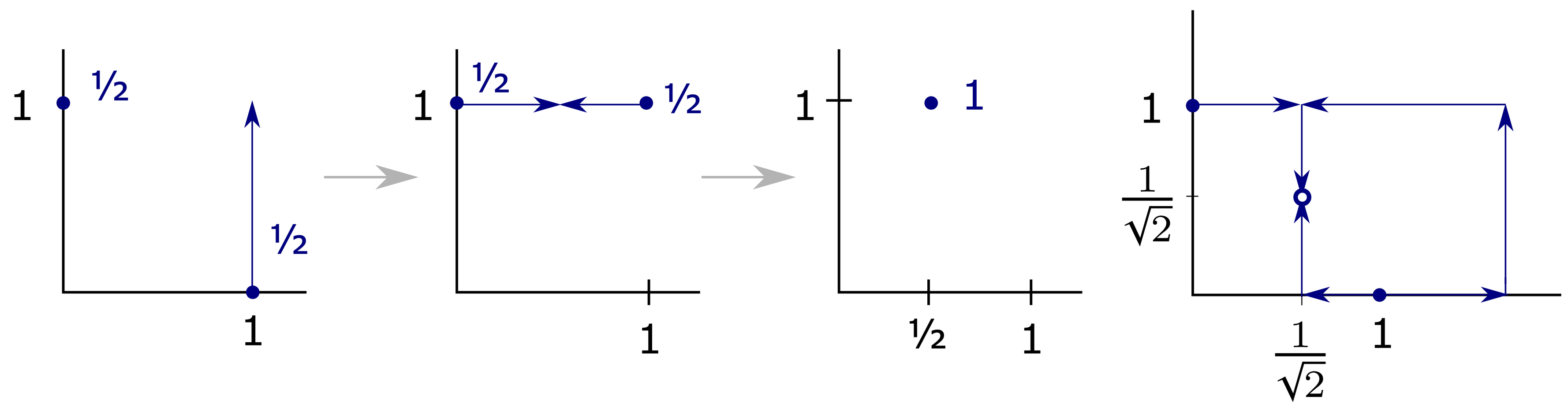
Dual: $\rho \leftrightarrow Z$, $\max \leftrightarrow \min$, $P^* = \max \leftrightarrow P^* \leq \text{certificate}$

Time Dependent Point Game (TDPG):

Sequence of frames (frame = points on a plane) s.t.

1. Start and end frames are fixed.
2. Consecutive: $\sum_z \frac{\lambda z}{\lambda + z} p_z \leq \sum_{z'} \frac{\lambda z'}{\lambda + z'} p'_{z'}$ along a line. $(\forall \lambda \geq 0)$

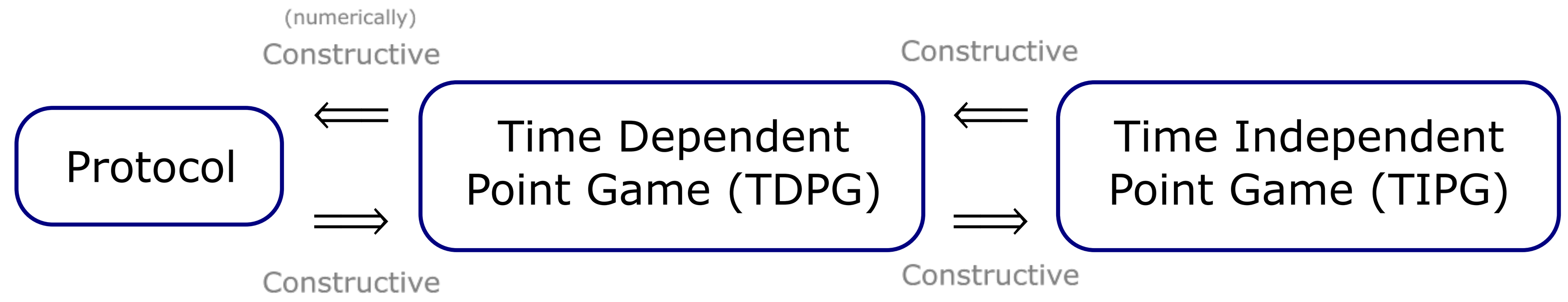
e.g. merge: weighted average; raise
split: harmonic average



Time Independent Point Game (TIPG):

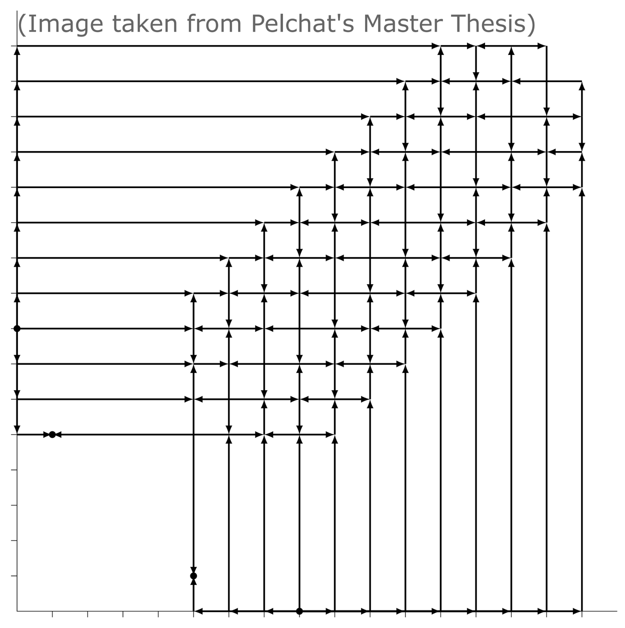
Weight can be negative; $h(x,y), v(x,y)$ s.t.

$h + v =$ final - initial frame; h, v satisfy a similar eqn.



Mochon's Breakthrough

(Image taken from Pelchat's Master Thesis)



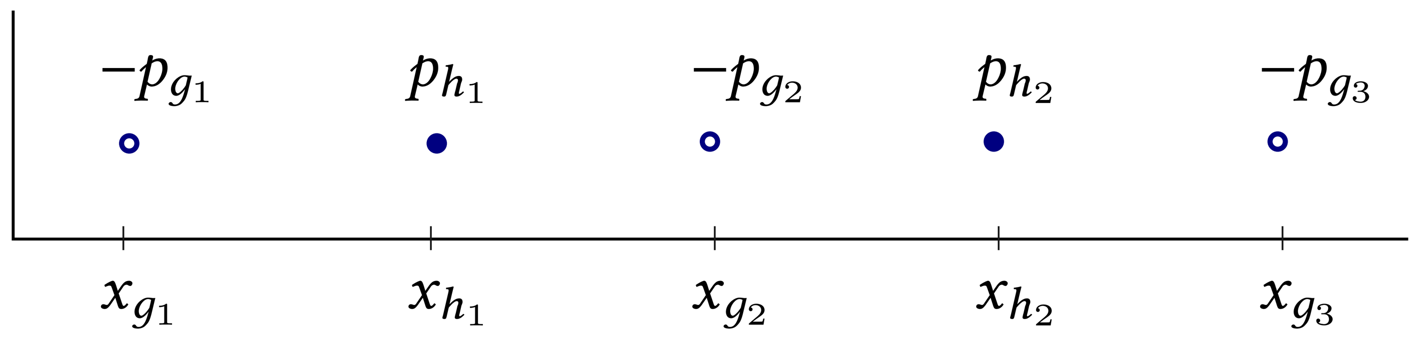
Family of TIPGs yield $\epsilon = \frac{1}{4k+2}$

$2k = \#$ points (non-trivial step).

Framework: A TDPG → Protocol if for each "TDPG move" one can construct a U s.t.

$$\sum x_{h_i} |h_i\rangle\langle h_i| - \sum x_{g_i} E_h U |g_i\rangle\langle g_i| U^\dagger E_h \geq 0$$

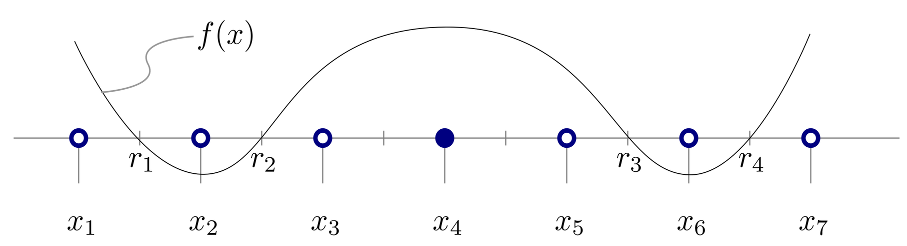
$$U \underbrace{\sum \sqrt{p_{g_i}} |g_i\rangle}_{|v\rangle} = \underbrace{\sum \sqrt{p_{h_i}} |h_i\rangle}_{|w\rangle}.$$

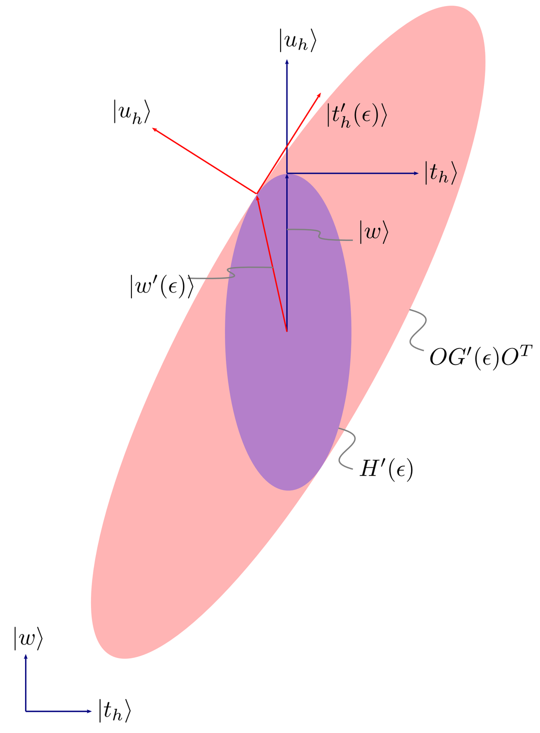


Contributions

Problem: Find the unitaries for Mochon's f-assignments:

$p(x_i) = \frac{-f(x_i)}{\prod_{j \neq i} x_j - x_i}$ where $f(x)$ is a polynomial satisfying $f(-\lambda) \geq 0 \;\forall\; \lambda \geq 0$





Special case: Mochon's m-assignments

$$p(x_i) = \frac{-c(-x_i)^k}{\prod_{j \neq i} x_j - x_i} \quad c > 0$$

For $k = 0$

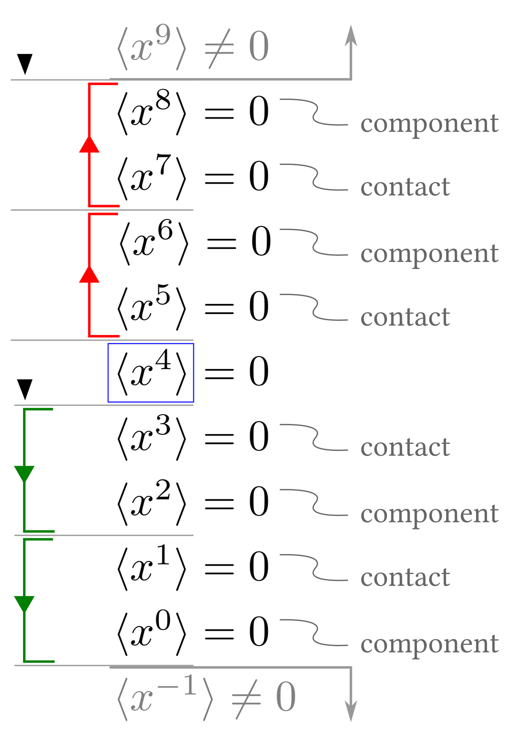
$$O = \sum_i |u_{h_i}\rangle\langle u_{g_i}|$$

$\langle x^l \rangle = 0$

For a general k (essence)

$$O = \sum_{i \in S} |u_{h_i}\rangle\langle u_{g_i}| + \sum_{j \in S'} |u'_{h_j}\rangle\langle u'_{g_j}|$$

$A > B > 0 \iff A^{-1} < B^{-1}$



Solution:

f-assignments = Sum of m-assignments

References, Affiliation, PDF and related | QR

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