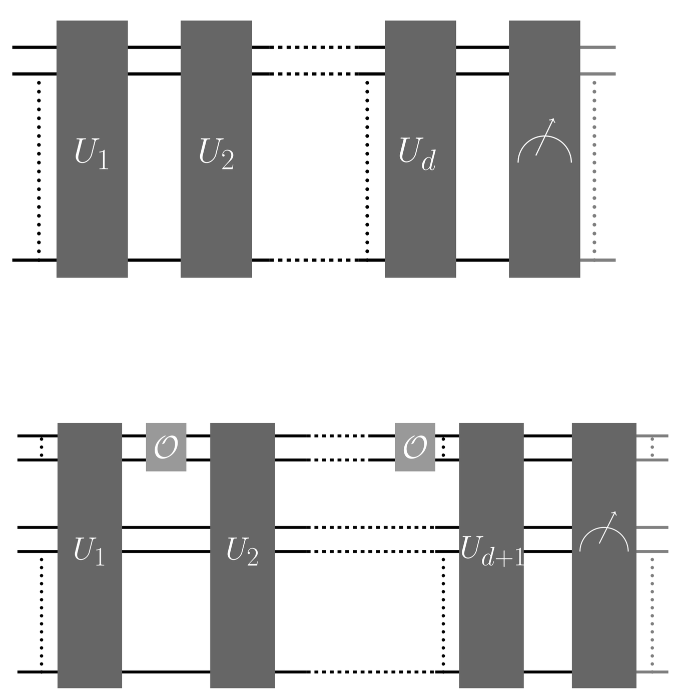
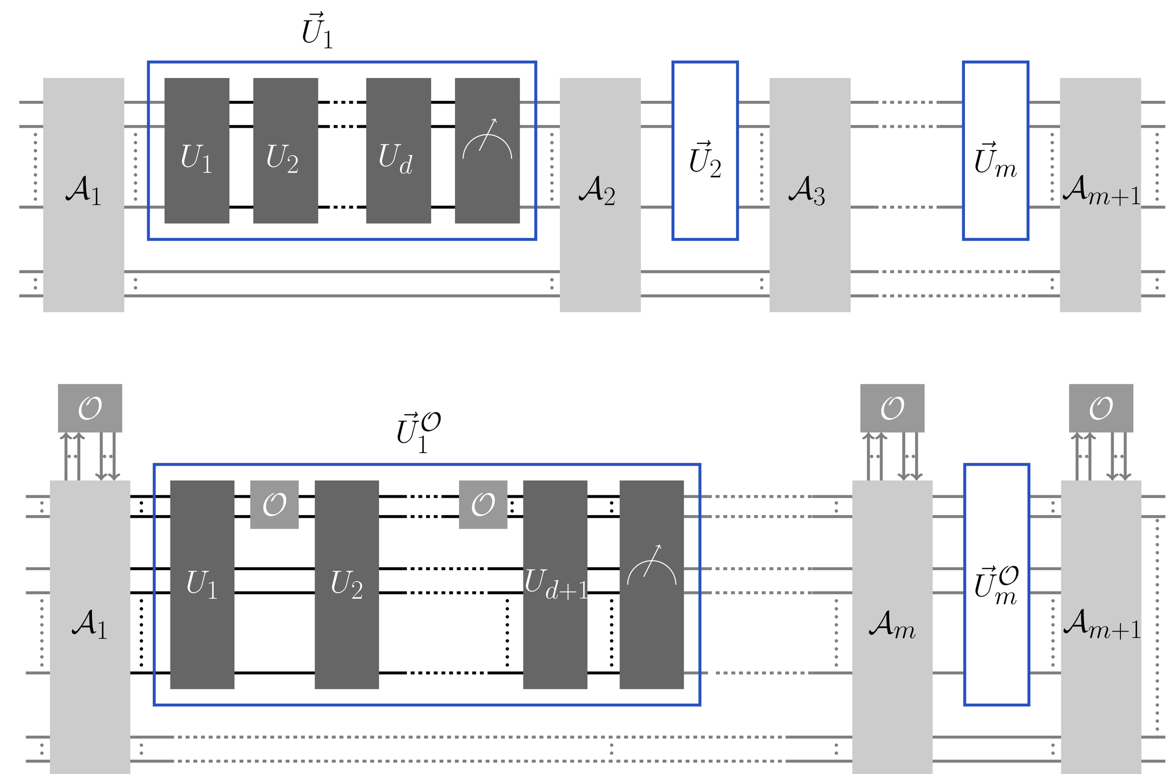
Oracle separations of Hybrid Quantum-Classical circuits

Atul Singh Arora* Uttam Singh Alexandru Gheorghiu

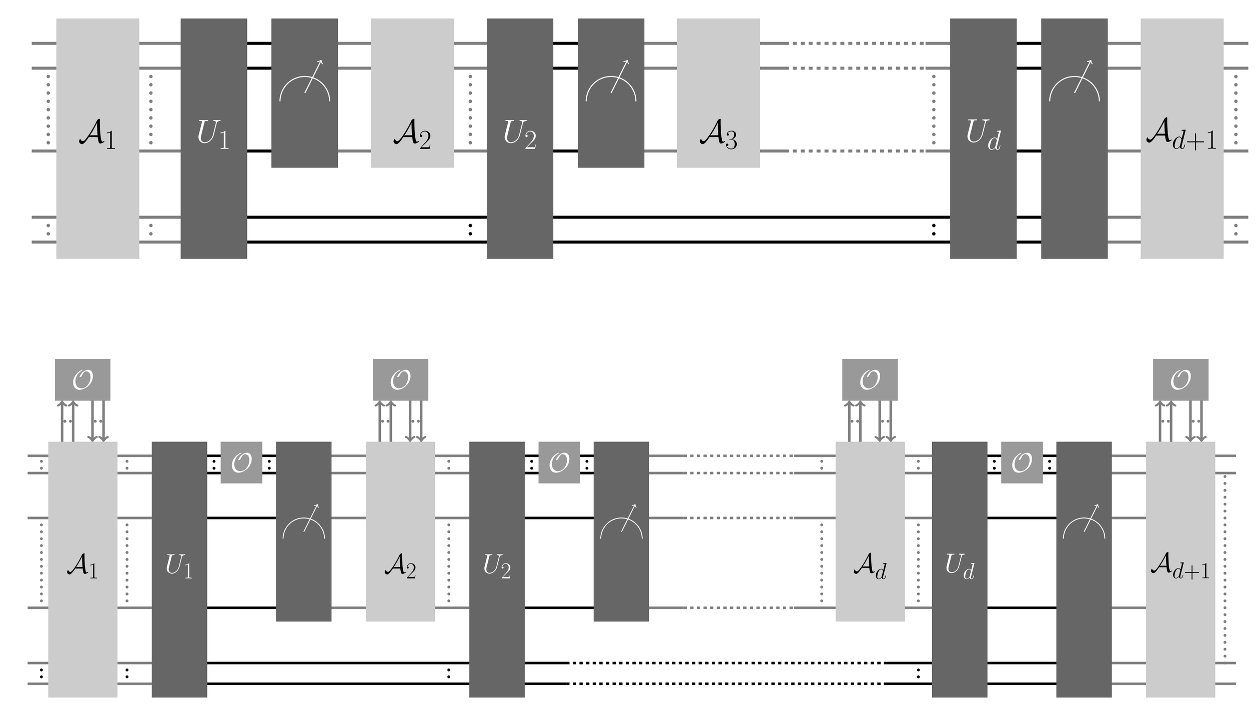
The Models of Computation



QNC_d ~ QNC when $d = \log^{O(1)}(n)$



CQ_d or $\text{BPP}^{\text{QNC}_d}$



QC_d or "$\text{QNC}_d^{\text{BPP}}$"

Motivation

Belief: BPP ⊂ BQP

Why? Period Finding,
Integer Factoring, Discrete Log

But. These can be solved QNC with
poly-time classical pre-post-processing

Extended Josza's conjecture*:

$\text{BPP}^{\text{QNC}} = \text{QNC}^{\text{BPP}} = \text{BQP}$

Results (and Prior Art)

	QC_d	CQ_d	Oracle	
d'-SeS	$d'+1 \le d \le 2d'+2$	$d \le 1$	Standard	This work
d'-SCS	$d \le 4$	$d'+1 \le d \le d'+5$	Stochastic	This work
d'-SSP	$d'+1 \le d \le 2d'+2$	$d'+1 \le d \le 2d'+1$	Standard	CCL*
d'-SS		$d'+1 \le d \le 2d'+1$	Standard	This work

wrt to an oracle

$\text{BPP}^{\text{QNC}} \subset \text{BQP}$

$\text{QNC}^{\text{BPP}} \subset \text{BQP}$

Prior Art (CCL* and CM)

$\text{BPP}^{\text{QNC}} \nsubseteq \text{QNC}^{\text{BPP}}$

$\text{BPP}^{\text{QNC}} \nsupseteq \text{QNC}^{\text{BPP}}$

This work

Background

Recall | Simon's Problem

Given a Simon's function $f: \{0,1\}^n \to \{0,1\}^n$, i.e. a two-to-one function s.t. $f(x) = f(x \oplus s)$ for some hidden period s, find the period s.

Recall | Simon's Algorithm

$$|0^n\rangle_X |0^n\rangle_Y \overset{H}{\mapsto} \sum_x |x\rangle |0\rangle$$

$$\overset{O}{\mapsto} \sum_x |x\rangle |g(x)\rangle$$

$$\overset{\Pi_Y}{\mapsto} (|x\rangle + |x \oplus s\rangle) |y\rangle$$

$$\overset{H}{\mapsto} \sum_d (-1)^{x \cdot d} (1 + (-1)^{s.d}) |d\rangle |y\rangle$$

Repeat, obtain equations $s \cdot d = 0$ and solve to obtain s.

d-Shuffler

Consider d random permutations, $f'_0, \cdots, f'_{d-1}$ from $\{0,1\}^{2n} \to \{0,1\}^{2n}$.

Define f'_d to be such that $f'_d(\cdots f'_0(x)) = f(x)$ for $x \in \{0,1\}^n$.

Define $f_i = \begin{cases} f'_i(\cdots f'_0(x)) & \text{if } x \in \{0,1\}^n \\ \perp & \text{otherwise} \end{cases}$.

$(f_i)_{i=0}^d$ is called a d-Shuffler and we denote it as Ξ.

Stochastic Oracle

Let
X, Y be finite sets,
$\mathbb{F}_Y$ be some distribution over Y,
$g(x, y) : X \times Y \to Z$ be a function.

An *intrinsically stochastic oracle* $\mathscr{S}$ wrt $\mathbb{F}_Y$ corresponding to g acts on each query as:
Samples $y \leftarrow \mathbb{F}_Y$ and on input $|x\rangle|z\rangle$ produces $|x\rangle |z \oplus g(x, y)\rangle$.

The Problems

d-Serial Simon's Problem

Sample $d+1$ random Simon's functions $\{f_i\}_{i=0}^d$ with periods $\{s_i\}$.
The problem: to find the period s_d of the last Simon's function.

However, only access to f_0 is given directly. Access to f_i, for $i \ge 1$, is given via a function L_{f_i} which outputs $f_i(x)$ if the input is (s_{i-1}, x) and $\perp$ otherwise.

NB: To access the ith Simon's function, one needs the period of the $(i-1)$th Simon's function.

d-Shuffled Collisions to Simon's Problem

Uniformly sample f from all 2-to-1 functions, g from all Simon's functions, and h from all 1-to-1 functions.

Let p be some canonical bijection which maps colliding pairs of f to those of g (and p_{inv} be the inverse).

Let p' be such that $p'(h(f(x)), x) = p(x)$ and Ξ is a d-Shuffler encoding h.

The problem: given p', p'_{inv}, Ξ and a *stochastic* oracle $\mathscr{S}$ for f, find the period of g.

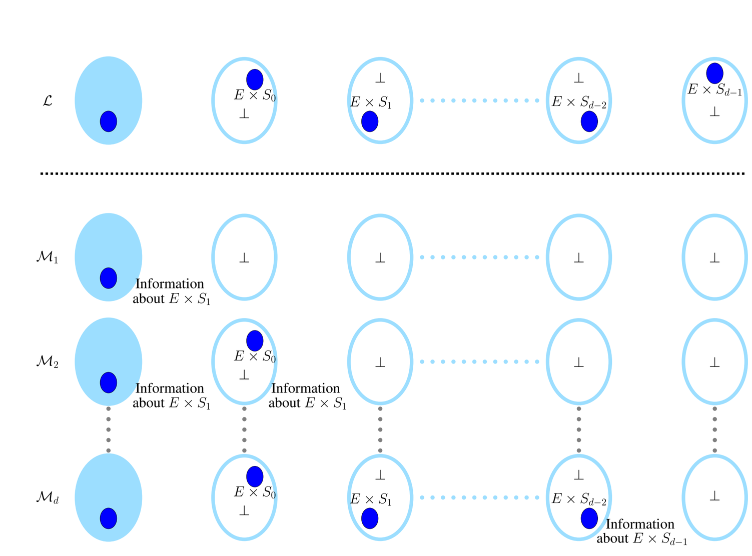
Bounds

CQ_1 solves d-SeS

CQ_1 can make polynomially many oracle calls to QNC_d.

Thus, CQ_1 can parallely unlock the periods s_i, hence the Simon function f_d, and can find s_d.

QC_d cannot



QC_4 solves d- SCS

1. Apply the stochastic oracle $\mathscr{S}$ on inputs $(|0\rangle + |1\rangle)_Q |0\rangle_{RR'} \mapsto (|0\rangle_Q |x_0\rangle_R + |1\rangle_Q |x_1\rangle_R) |y\rangle_{R'}$ where $y = f(x_0) = f(x_1)$ is random.
2. Classically, compute, $h(y)$ using Ξ.
3. Quantumly, use p' with $h(y)$ to get $|0\rangle|x_0\rangle|p(x_0)\rangle + |1\rangle|x_1\rangle|p(x_1)\rangle$
4. Proceed as in Simon's to get $p(x_0) \oplus p(x_1) = s$ the period.

CQ_d cannot

Oracle access is to a generic 2-to-1 function f instead of the Simon function g. Superpositions over colliding pairs are no longer related by the period s. To obtain any information about s, query to the bijection p is needed. But in CQ_d the quantum subroutines must measure their states completely before invoking the classical subroutines. Only the classical subroutines can obtain access to the bijection as the shuffler can only be invoked by a circuit of depth at least d.

References, Affiliation, PDF and related | QR

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