Weak Coin Flipping beyond bias 1/6

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Problem Statement

QKD: Two trusting parties protect against adversaries.
Two party secure: Two distrustful parties wish to collaborate.
E.g. MS wants to use IBM's QC.

Coin Flipping (CF): Establish a random bit among two mutually distrustful, physically separated players without a trusted third party.

A, B → friends →  → fight → coin flip to decide

Weak CF: Preferences are known. E.g. A and B both want the car.

Strong CF: Preferences are unknown.

Scenarios: Both honest (easy)
One honest other cheats (non-trivial; bias analysis)
Both cheat (independent of the protocol)

Bias: Smallest ϵ s.t. prob(heads), prob(tails) $\leq \frac{1}{2}+\epsilon$.

NB: $0 \leq \epsilon \leq \frac{1}{2}$.

Prior Art

Classically: $\epsilon = \frac{1}{2}$ viz. at least one player can always cheat and win.

Quantumly: Strong CF: $\epsilon \geq \frac{1}{\sqrt{2}} - \frac{1}{2}$, best known $\epsilon = \frac{1}{4}$.

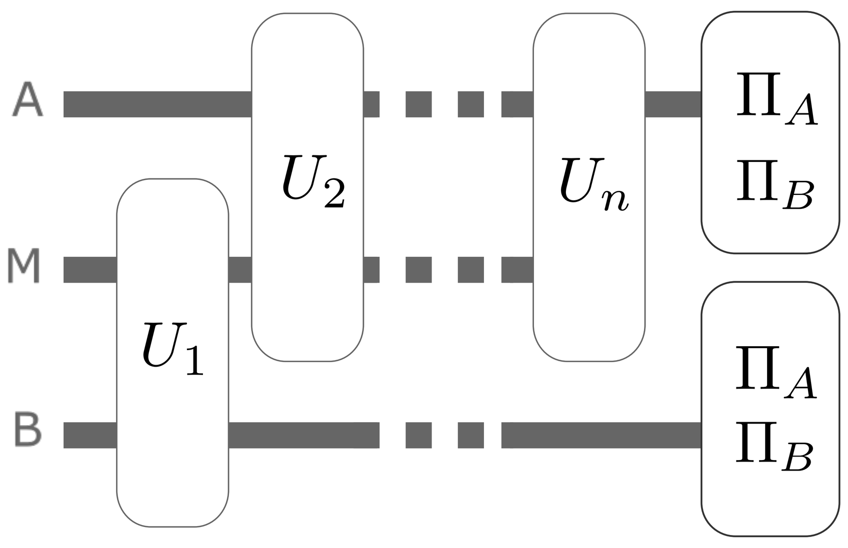
Weak CF: $\epsilon \rightarrow 0$, best known $\epsilon = \frac{1}{6}$.

Kitaev's Frameworks

e.g. Flip and declare protocol $P_A = P_B = \frac{1}{2}$

$$P_A^* = 1, P_B^* = \frac{1}{2} \implies \epsilon = \frac{1}{2}.$$

General Protocol:



P_A^* is an SDP in ρ_B:

$\max P_A^* = \mathrm{tr}[\Pi_A \rho_B]$

s.t. the honest player (Bob) follows the protocol.

Similarly for P_B^*.

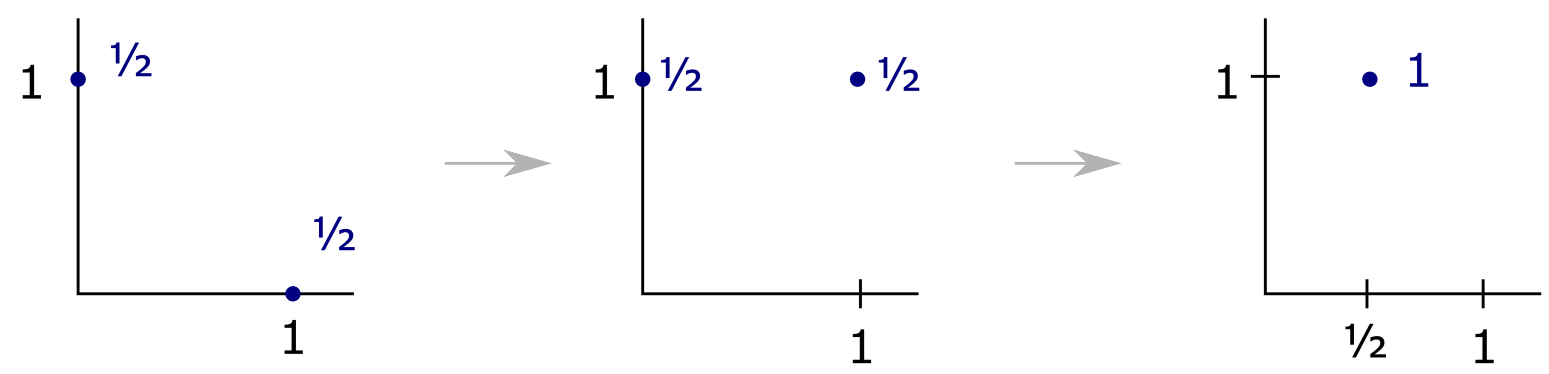
Dual: $\rho \leftrightarrow Z, \max \leftrightarrow \min, P^* = \max \leftrightarrow P^* \leq$certificate

Time Dependent Point Game (TDPG):

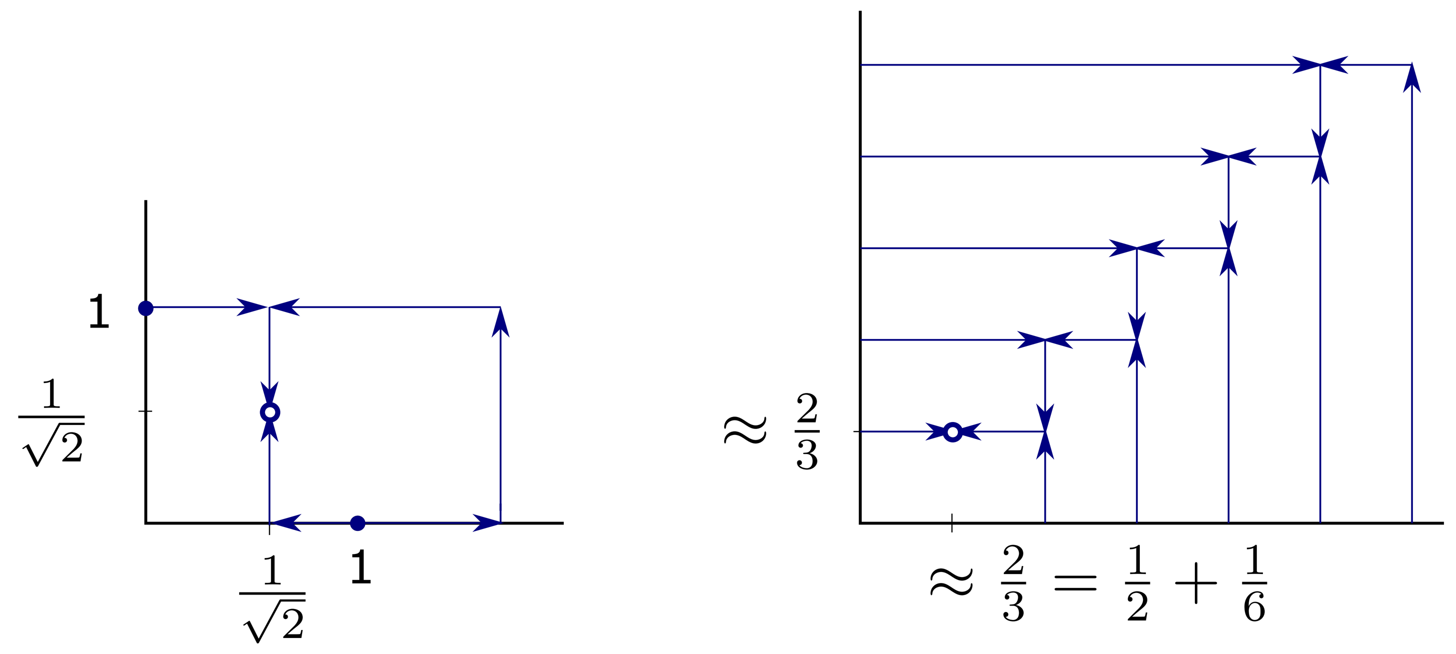
Sequence of frames (frame = points on a plane) s.t.

1. Start and end frames are fixed.
2. Consecutive: $\sum_z \frac{\lambda z}{\lambda+z} p_z \leq \sum_z \frac{\lambda z'}{\lambda+z'} p'_z$ along a line. $(\forall \lambda \geq 0)$

e.g. merge: weighted average; raise
split: harmonic average



Protocols Re-expressed



Time Independent Point Game (TIPG):

Weight can be negative; $h(x,y), v(x,y)$ s.t.

$h+v =$ final - initial frame; h, v satisfy a similar eqn.

Mochon's Breakthrough

Family of TIPGs yield $\epsilon = \frac{1}{4k+2}$

$2k = \#$ points (non-trivial step).

ρ, Z, U $\xrightarrow{\text{easy}}$ TDPG $\xrightarrow{\text{easy}}$ TIPG

TIPG $\xrightarrow{\text{doable}}$ TDPG $\xrightarrow{\text{hard but exists}}$ ρ, Z, U

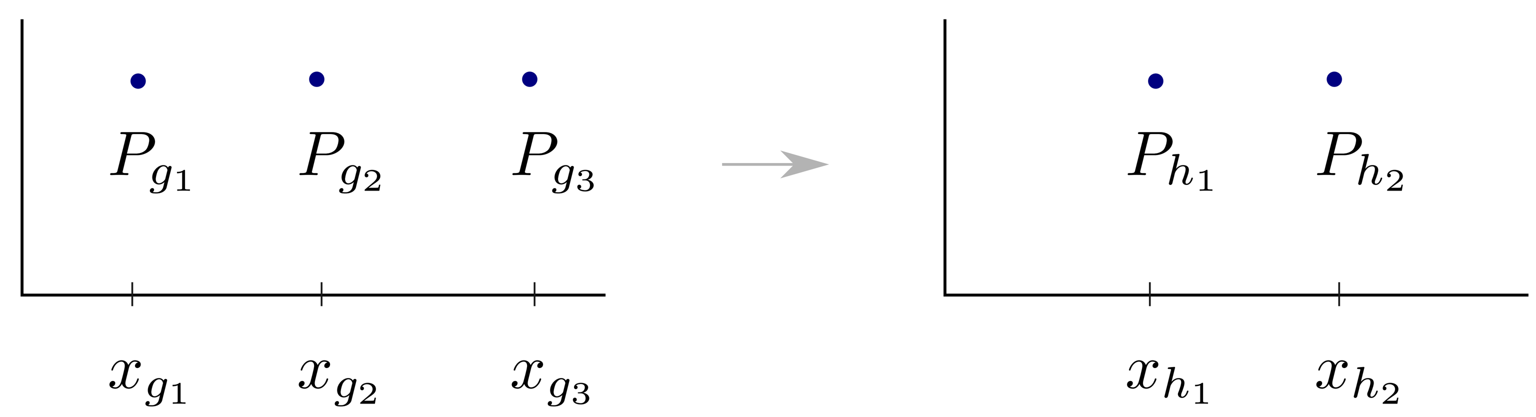
Contribution

Framework: A TDPG $\rightarrow \rho, Z, U$ if
for each "TDPG move" one can construct a U s.t.

$$\sum x_{h_i} |h_i\rangle \langle h_i| - \sum x_{g_i} E_h U |g_i\rangle \langle g_i| U^\dagger E_h \geq 0$$

and

$$U \underbrace{\sum \sqrt{P_{g_i}} |g_i\rangle}_{|v\rangle} = \underbrace{\sum \sqrt{P_{h_i}} |h_i\rangle}_{|w\rangle}.$$



E.g.: For the 1/6 protocol, U to implement the following are needed:
(a) split: $1 \rightarrow n$ (b) merge: $n \rightarrow 1$

Claim: $U_{\text{blink}} = |w\rangle\langle v| + |v\rangle\langle w| + \mathbb{1}_{\text{else}}$ can perform both.

E.g.: For the 1/10 protocol, U to implement the following are needed in addition to the split and merge:
(a) $3 \rightarrow 2$ (b) $2 \rightarrow 2$

Claim: $U_{3\rightarrow 2}$ and $U_{2\rightarrow 2}$ constructed (not pretty).

NB: Better than the current best.

Future: Construct a systematic scheme for constructing Us.

References, Affiliation, PDF and related | QR

ULB QuIC

QIP 2018
Delft, January 13-19