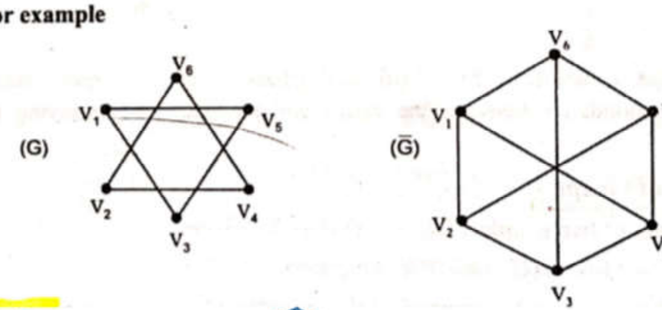
L-23 Walk

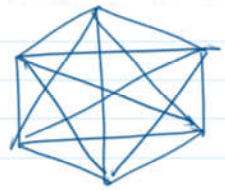
Wednesday, March 30, 2022 10:06 AM

***(iii)* Complement of a graph :** The compliment of a graph G is denoted by (and is defined as the simple graph with the vertex set same as the vertex set of G togeth with the edge set satisfying the property that there is an edge between two vertices in $\overline{G}$ when there is no edge between these vertices in G.

For example



Note ↦ $G \cup \overline{G} =$



= complete graph.

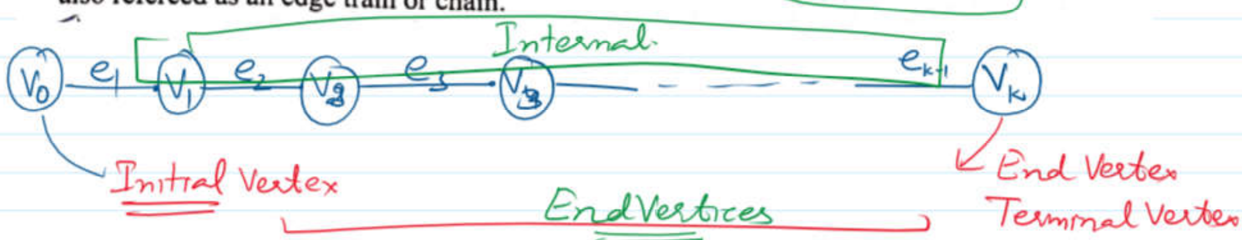
But Not Always

Walk :-

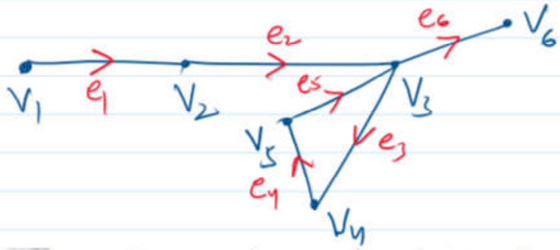
Walk : A walk in a graph G is finite sequence

$$W = V_0, e_1, V_1, e_2, \ldots\ldots, V_{k-1}, e_{k-1}, V_k$$

whose terms are alternatively vertices and edges such that for $1 \leq i \leq k-1$, the edge e_i has end vertices v_{i-1} and v_i. The vertex V_0 is called the initial and the vertex V_k is called terminal of the walk W. Vertices $V_1, V_2, \ldots, V_{k-1}$ are called internal vertices. A walk is also refereed as an edge train or chain.



Note :- **Remark.** (*i*) Each edge can appear only once in a walk, however vertices may appear more than once.



$V_1\ e_1\ V_2\ e_2\ V_3\ e_3\ V_4\ e_4\ V_5\ e_5\ V_3\ e_6\ V_6$

open Walk

Open Walk : If a walk begin and end with the different vertices, it is called an open walk.

Closed Walk : If the initial and terminal vertices of a walk are same, it is called a closed walk.

Note **Remark.** A walk containing no edge and has length zero is called a **Trivial walk.**

Path :→ A walk in which Vertex appear Only Once.

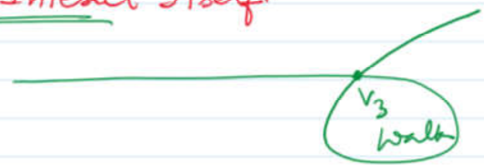
PATH : An open walk in which no vertex appear more than once is called a path or

Path :-> A walk in which vertex appear only once.

PATH : An open walk in which no vertex appear more than once is called a path or simple path.

Every Path is a Walk but Converse May or May Not be true.

Note A path do not Intesect Itself.



Length of path : The number of edges appearing in the sequence of the path is called the length of the path.

For example. Consider the following graph

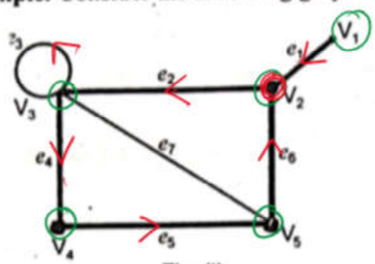


Fig. (i)

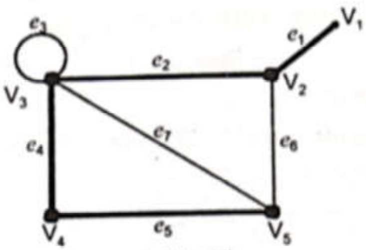


Fig. (ii)

$W = V_1, e_1, V_2, e_2, V_3, e_3, V_3, e_4, V_4, e_5, V_5, e_6, V_2.$ different Edges

V_2 – Rep 2 time.
V_3 — 2

Then W is a walk of length 6 as shown by the bold line in fig. (*i*). The above walk is not a path as the vertices V_3 and V_2 appear twice in the walk W. However the walk

$W' = V_1, e_1, V_2, e_2, V_3, e_4, V_4, e_5, V_5$ →

is a path of length 4 as shown by the bold line in fig. (*ii*). Moreover, the above walk W and W′ are open walk as their terminus vertices are different.

But the walk

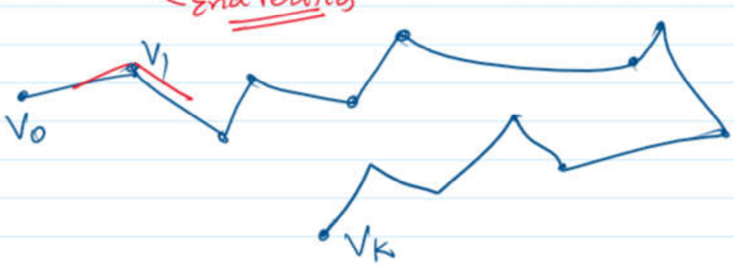
$W'' = V_1, e_1, V_2, e_3, V_3, e_3, V_3, e_4, V_4, e_5, V_5, e_6, V_2, e_1, V_1$ Closed

is a closed walk as the terminus vertices are same.

Note :-> **Remark.** (*i*) An edge which is not a self loop is a path of length 1.

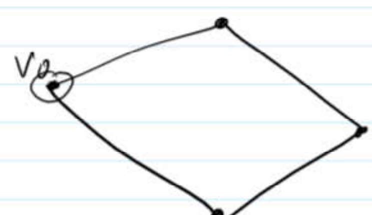
(*ii*) A self loop can be included in a walk but not in a path.

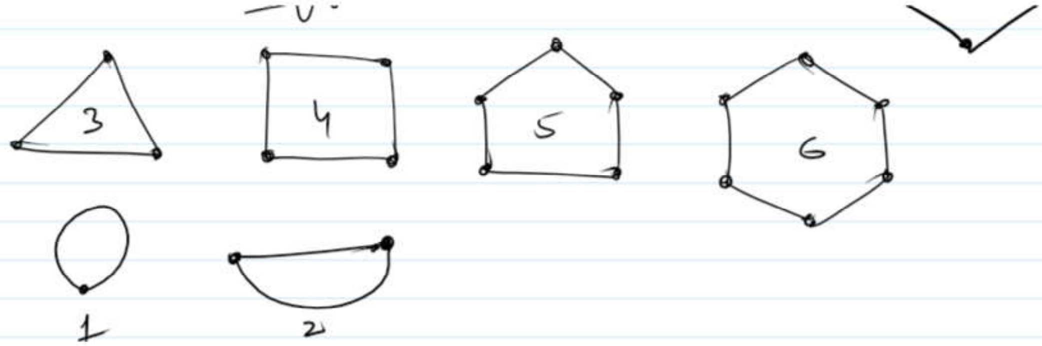
(*iii*) The terminus (End Vertices) vertices of a path are of degree 1 and the internal vertices of the ~~walk~~ Path are of degree 2.



$d(V_0) = 1$
$d(V_K) = 1$] End vertices
$d(V_1) = 2$

Closed Path / Circuit / Cycle :→
Polygon





CONNECTED GRAPHS, DISCONNECTED GRAPHS, AND COMPONENTS

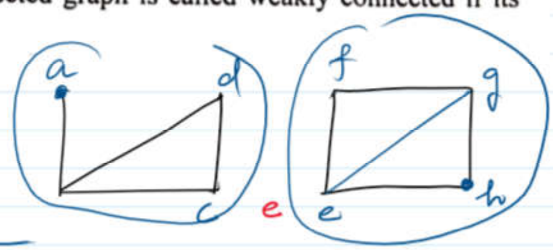
Every Pair

Connectivity : An undirected graph is said to be connected, if for any pair of vertices of the graph the two vertices are reachable from one another. If there is a path.

Strongly Connected : If any pair of vertices of the digraph both the vertices of the pair are reachable from another, then graph is strongly connected.

Unilaterally Connected : A simple directed graph is said to unilaterally connected if for any pair of vertices of the graph, at least one of the vertices of the pair is reachable from other vertex.

Weakly Connected Digraph : A directed graph is called weakly connected if its undirected graph is connected.



a—b—c—e—

a—b—d—c—e—f—g—h