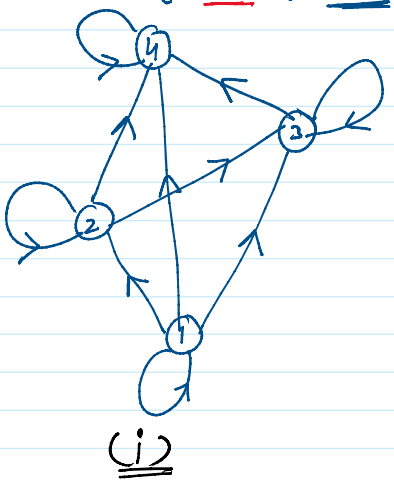
L-11 Hasse Diagram

Monday, February 21, 2022 9:44 AM

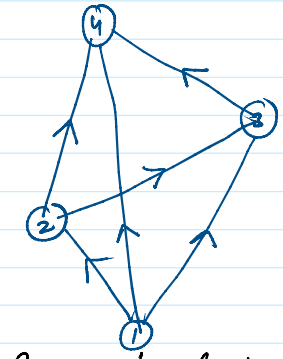
Ex) Construct the Hasse diagram for Poset $(\{1,2,3,4\}; \leq)$ where set $A = \{1,2,3,4\}$ and Relation is $\leq$

Sol ↦ $A = \{1,2,3,4\}$ $R = \{(x,y) : x,y \in A,\ x \leq y\}$

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

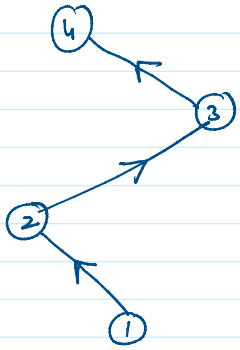


(i)



Remove the loops

(ii)



(iii)

Transitive Elem.

$(1,2)(2,3) \Rightarrow (1,3)$

$(1,2), (2,4) \Rightarrow (1,4)$

$(2,3), (3,4) \Rightarrow (2,4)$

Transitive $aRb, bRC \Rightarrow aRC$

(iv)

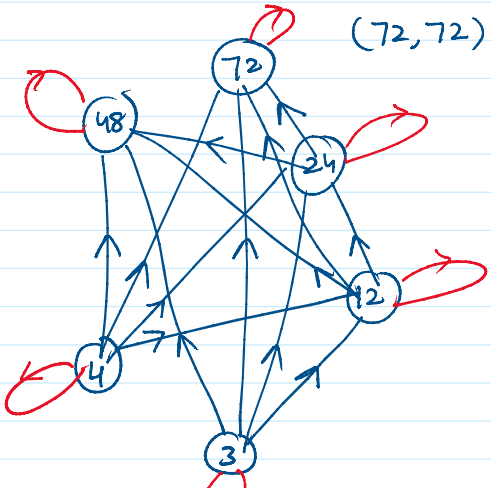
Hasse Diagram

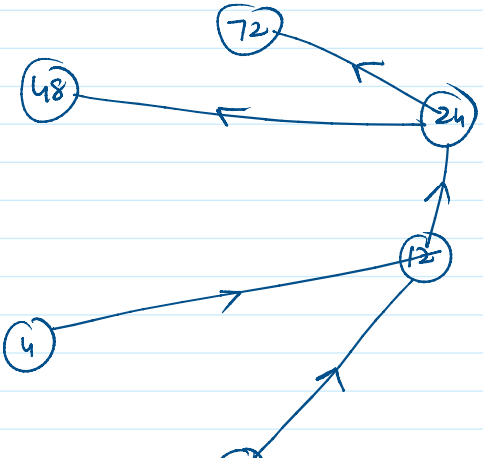
Ex) Draw the Hasse diagram of $(A, \leq)$ where $A = \{3,4,12,24,48,72\}$ and the relation $\leq$ be such that $a \leq b$ if "a divides b"

$a \mid b$ $a \nmid b$?

$R = \{(a,b) : a,b \in A,$ "a divides b" $\}$

$R = \{(3,12)(3,24)(3,48), (3,72)(3,3), (4,4)(4,12)(4,24)(4,48)(4,72)$
$(12,12)(12,24)(12,48)(12,72)(24,24)(24,48)(24,72)(48,48)$
$(72,72)\}$





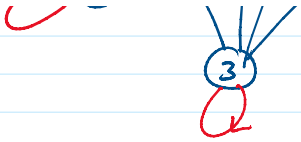
$(3,12)(12,24) \Rightarrow (3,24)$

$(3,12)(12,48) \Rightarrow (3,48)$

$(3,12)(12,72) \Rightarrow (3,72)$

$(4,12)(12,24) \Rightarrow (4,24)$

$(4,12)(12,72)$



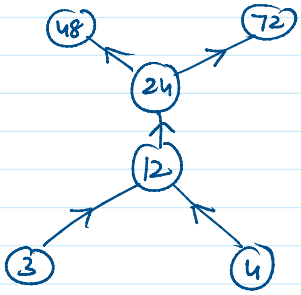
$(12,24)(24,48) \Rightarrow (12,48)$

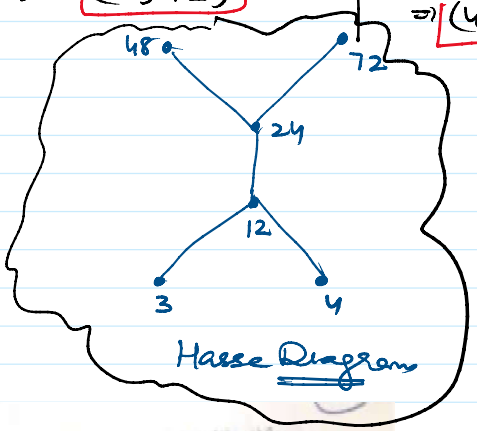
$(12,24)(24,72) \Rightarrow (12,72)$

$(4,24) \Rightarrow (4,24)$

$(4,12)(12,72) \Rightarrow (4,72)$

$(4,12)(12,48) \Rightarrow (4,48)$





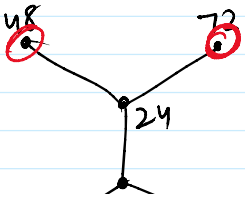
Q.1 Ex Construct the Hasse diagram of the relation 'S' defined as "divides" on set $A = \{2,3,4,6,12,36,48\}$ i.e $a \leq b$ if "a divides b"

Q.2 t $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation "a divides b". [R.T.U.]

Maximal Element and Minimal Element :→ An Element of a poset is called Maximal if it is not less than any element of the poset. That is 'a' is Maximal element in poset $(A, \leq)$ if there is no $b \in A$ such that $a \leq b$ / $a < b$: Similarly an Element elemd of a poset is called Minimal if it is not greater than any element of the poset. ie an element 'a' is minimal if there exist no $b \in A$ such that $b < a$.

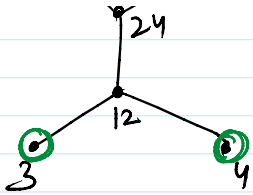
Note : There will be No Successor of Maximal Elements
" " " Predecessor " Minimal "

In the Hasse Diagram The Top & Bottom elements are Maximal and Minimal elements.



Maximal : 48, 72

Minimal : 3, 4



1st minimal : 3, 4