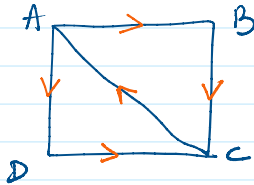


A simple path in a graph  $G$  is called Euler path if it traverses every edge of the graph exactly once.

An Euler path (or chain) through a graph is a path whose edge list contains each edge of the graph exactly once.

Euler Circuit  $\rightarrow$  closed Euler path.

Euler Graph Eulerian Graph: A graph which contains either Euler path or Euler circuit is called Euler graph.

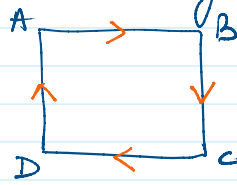


A-B-C-A-D-C

A-B-C-D-A-C

Euler Path

Euler graph



(A)-B-C-D-(A)

Euler Circuit (Closed Euler path)

Euler Graph

- ① A connected graph  $G$  is an Euler graph iff the degree of every vertex is even, and it contains an Euler circuit.
- ② A connected graph  $G$  is an Euler graph, having Euler path iff exactly two vertices have odd degree.

Hamilton Path / Hamiltonian Path

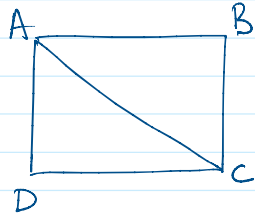
A Hamilton path in a connected graph  $G$  is the path which contains every vertex of the graph exactly once.

Hamilton Circuit: closed H-Path.

Hamiltonian Graph: A graph is said to be H-Graph if it contains either H-path or H-Circuit.

- Note:
- ① If  $G$  has  $n$ -vertices then H-Circuit will contain  $n$ -Edges.
  - ② " " " " " " H-path will "  $(n-1)$  Edges.
  - ③ There may be more than one H-path & H-Circuit in graph.

③ There may be more than One H-path & H-Circuit in graph.



$n=4$

$B \xrightarrow{1} C \xrightarrow{2} D \xrightarrow{3} A$  — H-path

Length = 3

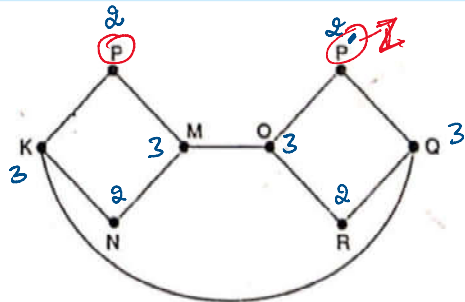
No. of Edges = 3 = 4 - 1 (n-1)

$C \xrightarrow{1} D \xrightarrow{2} A \xrightarrow{3} B \xrightarrow{4} C$  — H-Circuit

Length = 4

No. of Edges = 4 = n

④ If No. of Edges in a graph  $G < (n-1)$ ,  $n$  = No. of Vertices then the graph is Not an H-graph.



More than 2 odd Vertices  
Not Euler

① Hamilton ✓

② Euler.

$P-M-N-K-Q-R-O-P'$   
Z

**Theorem V.** Prove that a simple graph with  $k$ -components and  $n$  vertices can have at the most of  $\frac{(n-k)(n-k+1)}{2}$  edges.

(P.T.U., M.C.A. Dec. 2006)

Let  $G = \{V, E\}$  be a connected graph, then cardinality of cut set of  $G$  is called edge connectivity of graph  $G$ .

The edge connectivity of a connected graph cannot be more than the smallest degree of a vertex in the graph. It is denoted as  $\lambda(G)$

### **Vertex connectivity**

Let  $G$  be a connected graph. Vertex connectivity of a graph is the least number of vertices whose removal disconnects the graph. It is written as  $K(G)$  and is given by

$$K(G) = n - 1$$

for a complete graph with  $n$  vertices