# L-27 Eular Formula

Monday, April 11, 2022 10:01 AM

**Theorem 1. EULER'S FORMULA**

Euler Formula :-

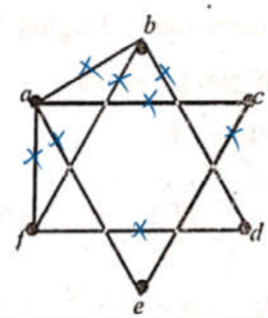
Let G = (V, E) be a connected planar graph and let R be the number of regions defined by any planar depiction of G. Then

$$\boxed{R = |E| - |V| + 2}$$

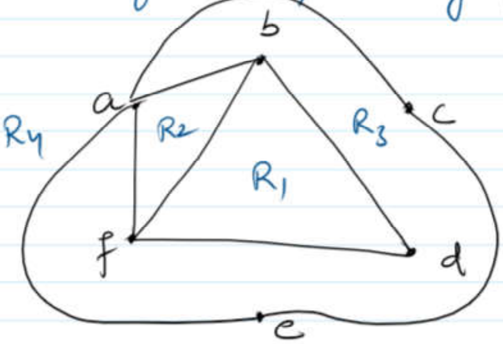
$$\boxed{r = e - v + 2}$$

r = No. of Region
e = No. of Edges
v = No. of Vertices

$$\boxed{e = r + v - 2} \qquad \boxed{v = e - r + 2}$$




(Ex) Verify Euler's formula for the planar graph.

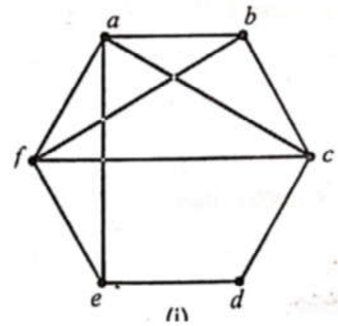



$v = 6$
$e = 8$
$r = 4$

$R.H.S = e - v + 2 = 8 - 6 + 2 = 4 = r$

$\therefore \boxed{r = e - v + 2}$

(Ex) How Many Regions are there in the planar graph




(a) 4 (b) 5 (c) 6 (d) 7

$r = e - v + 2 = 10 - 6 + 2 = 6$ $v = 6$
$e = 10$

$\boxed{r = 6}$

$2e = 2 \times 10 = 20$
$3r = 3 \times 6 = 18$
$20 > 18$

**Theorem 2.** Let G = (V, E) be a simple, connected Planar graph with more than one edge, then the following inequalities holds.

(*ii*) $2|E| \geq 3R$ (*ii*) $|E| \leq 3|V| - 6$

(*iii*) There is a vertex $v$ of G such that $\deg(v) \leq 5$.

(iv) $|E| \leq 2|V| - 4$ if No cycle of length 3 is there.

(1) $2e \geq 3r$

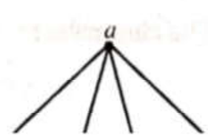
(2) $e \leq 3v - 6$

(4) $e \leq 2v - 4$

No Cycle of length 3

(Ex) Prove that $K_5$ is a Non-planar graph.

Sol $e = 10$ $v = 5$




$\frac{n(n-1)}{2}$

It Contains cycle of length 3

Sol $e = 10$ $v = 5$

It Contain cycle of length 3.

Let $K_5$ is a planar graph

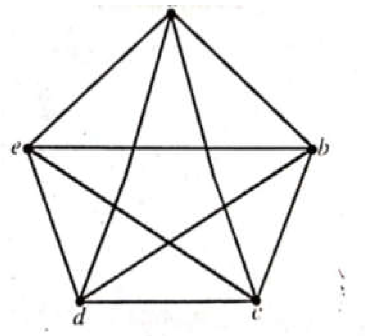
$\therefore e \leq 3v - 6$

$\Rightarrow 10 \leq 3 \times 5 - 6$

$\Rightarrow 10 \leq 15 - 6$

$\Rightarrow 10 \leq 9$ which is Not True.

$\therefore$ Our Supposition is wrong $\therefore$ $K_5$ is a Non-planar graph.



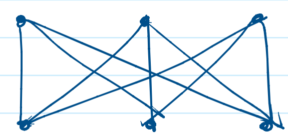

$\frac{n(n-1)}{2}$

$= \frac{5 \times 4}{2} = 10$

Ex $K_{3,3}$ is a Non-planar

Sol Suppose $K_{3,3}$ is a planar graph

$e = 9$ $v = 6$ No cycle of length 3.



$\therefore e \leq 2v - 4$

$\Rightarrow 9 \leq 2 \times 6 - 4$

$\Rightarrow 9 \leq 8$ which is Not True $\therefore$ $K_{3,3}$ is Non-planar

Note **Kuratowski's Theorem**

A graph is nonplanar if and only if it contains a subgraph homeomorphic to $K_{3,3}$ or $K_5$.

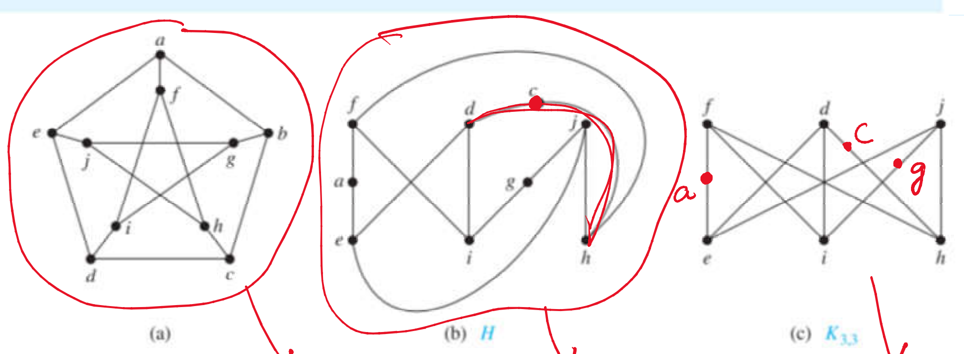
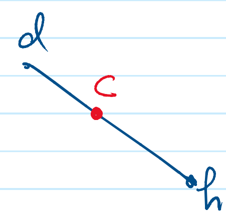
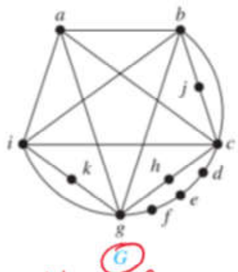
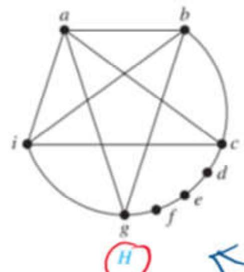
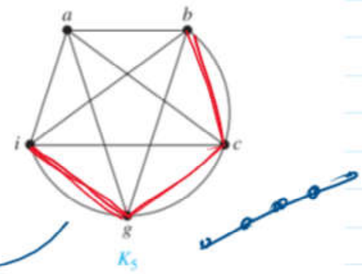



**FIGURE 14** **(a) The Petersen Graph, (b) a Subgraph $H$ Homeomorphic to $K_{3,3}$, and (c) $K_{3,3}$.**





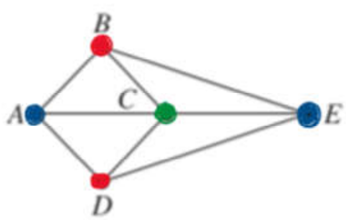




Non-planar

## Graph Coloring

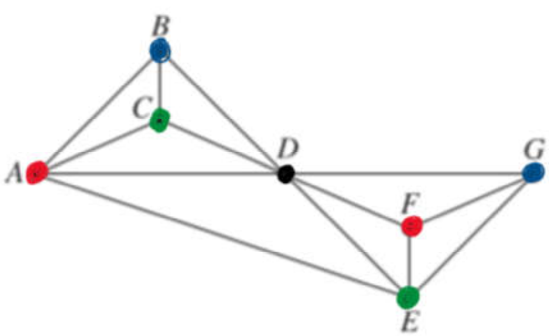
A *coloring* of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.

The *chromatic number* of a graph is the least number of colors needed for a coloring of this graph. The chromatic number of a graph $G$ is denoted by $\chi(G)$. (Here $\chi$ is the Greek letter *chi*.)




Chromatic Number

$\chi(G) = 3$




Four Colours are Needed