L-12 Hasse Diag.

Tuesday, February 22, 2022 10:19 AM

1) **Greatest and Least Element** :→ An element $a \in A$ is said to be the Greatest Element of poset $(A, \leq)$ if $\forall x \in A ; x \leq a$.

$\forall x \in A \quad x \leq a$

An element $a \in A$ is called the least Element of Poset if $a \leq x \ \forall x \in A$

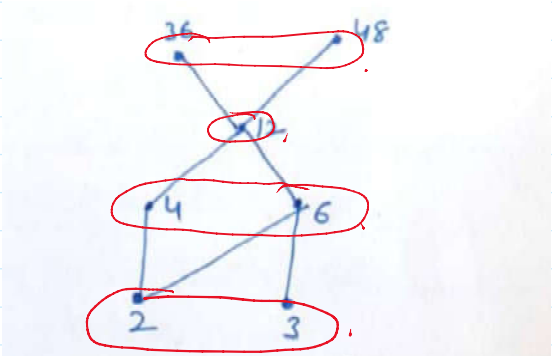
$a \leq x$

Note: the least element is also called the first Element or Zero element and if it exist is Unique only one

② the Greatest Element is also called the last Element or Unit Element, if it exist is Unique

③ Least Element is denoted by 0

Greatest " " " 1

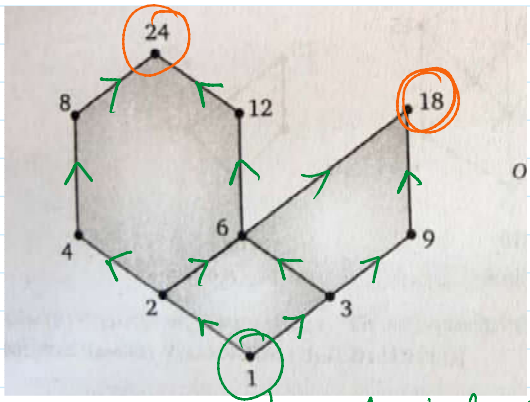


Maximal = 36, 48

Minimal = 2, 3

$x \leq 36$, $x \leq 48$ ✗ Greatest element

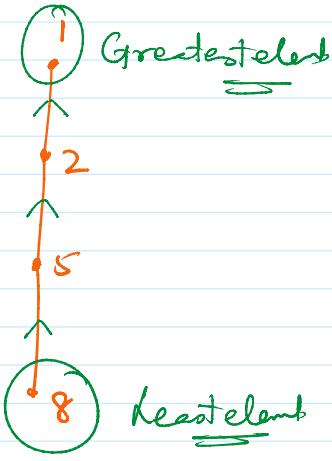
$\forall x$ $3 \leq x$, $2 \leq x$ ✗ Least Element



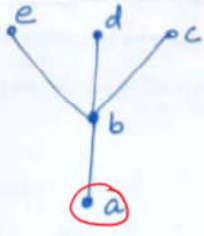
Least Element = 1

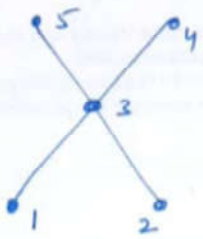
Greatest Element = None.

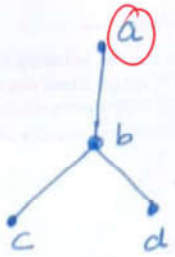
→ Least element

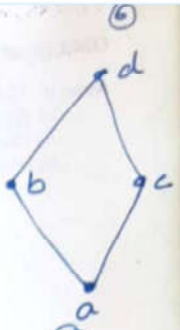


Ex 2









	①	②	③	④
Least Element	a ✓	No least Elem ✓	No least Elem	a ✓
Greatest Element	No Greatest Element	No Greatest Element ✓	a	d ✓

① Upper Bound and least upper Bound :— Let $(A, \leq)$ be a poset and Let S be a subset of A then an element $x \in A$ is called an Upper Bound of S if $a \leq x$ $\forall a \in S$

$(A, \leq)$

An element $x \in A$ is said to be Least upper Bound or lub or Supremum of S if x is an upper bound of S and $x \leq y$ for all upper bounds y of S.

the least upperbound if it exist is Unique

$a \leq x$

$A = \{ \ldots -10, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 \ldots \}$ with S = $[-2, 3]$

$\forall a \in S$
$a \leq x$

$3.5 \in A$

3.5

$a \in S \quad a \leq 3.5$

(3.6) $\quad a \leq 3.6$

3.7 $\quad a \leq 3.7$