31-1-22

Monday, January 31, 2022 9:34 AM

Predicates and Quantifiers

$\sim, \wedge, \vee, \Rightarrow, \Leftrightarrow$

"Every computer connected to the university network is functioning properly."

Every person who is 18 year or Older, is eligible to Vote

quantity

Predicate logic :→ " x is greater than 5 "

I st :→ Variable x is Called subject

II nd → "greater than 5" is called predicate

$P(x)$: x is greater than 5

P : denote the predicate " is greater than 5"

x : Variable (Subject)

$P(\underline{x})$ " x is greater than 5 "

the predicate P Can be Considered as a function of the Subject 'x'

It tells the truth Value of the statement P(x) at 'x', Once a Value is assigned to the Variable x. then the statement P(x) become a proposition. and has a truth Value.

$\underbrace{x+3=7}_{\text{Not prop.}}$ $\underbrace{x+3>7}_{\text{Not prop.}}$

$x=2$

$\left.\begin{array}{c}2+3=7\\5=7\end{array}\right]$ False

$x=5$

$\left.\begin{array}{c}5+3>7\\8>7\end{array}\right]$ True

$P(x) : x+3=7$ $P(x) : x+3>7$

$P(x,y) : x+y=7$ $x+y>5$

$P(x,y,z) : x+y+z=0$

$x+y=z$

In general a Statement with n Variables $x_1, x_2, x_3 --- x_n$

$P(x_1, x_2, x_3 --- x_n)$: P is called n-place Predicate

n-ary predicate

Let $P(x)$ denote the statement "$x > 3$." What are the truth values of $P(4)$ and $P(2)$?

$P(x): x > 3$

$P(4): 4 > 3$ True

$P(2): 2 > 3$ False

Let $Q(x, y)$ denote the statement "$x = y + 3$." What are the truth values of the propositions $Q(1, 2)$ and $Q(3, 0)$?

$Q(x, y): x = y + 3$ (Not a prop.)

$Q(1, 2): 1 = 2 + 3$, $1 = 5$] False → Proposition

$Q(3, 0): 3 = 0 + 3$, $3 = 3$] True → Proposition.

Quantifiers :→ We use Quantifier along with Predicate.

These are the words that refer to a Quantity " Some" "All" and Tell how many elements a given $P(x)$ are True.

Ie Express the Extent to which a predicate is True over the Range of elements (Domain)

Using the quantifier to Create such proposition is called Quantification.

Types of Quantifiers :

① Universal ("All", "Every") For All

The *universal quantification* of $P(x)$ is the statement

"$P(x)$ for all values of x in the domain."

The notation $\forall x P(x)$ denotes the universal quantification of $P(x)$. Here $\forall$ is called the **universal quantifier.** We read $\forall x P(x)$ as "for all $x P(x)$" or "for every $x P(x)$." An element for which $P(x)$ is false is called a **counterexample** of $\forall x P(x)$.

$\forall$ For all, For Every

$[\forall x, P(x)]$

$\forall x P(x)$ is True

↳ Universal Quantifier

$\forall x, P(x)$: For every x, $P(x)$ is True

Let $P(x)$ be the statement "$x + 1 > x$." What is the truth value of the quantification $\forall x P(x)$, where the domain consists of all real numbers?

$P(x): x + 1 > x$

$\forall x, P(x)$

$x \in R$

[$\forall x$. $P(x)$ is True] ✓

Let $Q(x)$ be the statement "$x < 2$." What is the truth value of the quantification $\forall x Q(x)$, where the domain consists of all real numbers?

Let $Q(x)$ be the statement "$x < 2$." What is the truth value of the quantification $\forall x\, Q(x)$, where the domain consists of all real numbers?

$Q(x): x < 2$ $\qquad\qquad$ $\boxed{\forall x\, Q(x)}$

$Q(x)$ is Not True $\forall x \in R$

if $x = 3$ $\quad Q(x): 3 < 2$ which is False

$\therefore \forall x\, Q(x)$ is False