

$\forall x P(x)$ Universal $P(x): x > 2 \quad x \in \mathbb{R} \quad (-\infty, \infty)$

All

Every

$-\infty$ $+\infty$
 We can choose any value
 x can have only value
 from $-\infty$ to $+\infty$

For every value of x
 $P(x)$ is True

is $P(x)$ is True?

Is it
No

 $x=2$ $P(2): 2 > 2$ $x=1.9$ $P(1.9): 1.9 > 2$ False

Counter Example

 E_1 $\exists x P(x)$: There exist at least one value of x such that $P(x)$ is True $x=3$ $x=2.1$ $P(2.1): 2.1 > 2$ True

Nested Quantifier: \rightarrow If we use a quantifier that appears within the scope of another quantifier is called Nested quantification. It is also called Multiple quantification.

 E_1 $\forall a$ $\exists b$ $P(a,b)$ where $P(a,b): a+b=2 \quad a,b \in \mathbb{R}$ E_2

$\forall a$ $\forall b$ $\forall c$ $P(a,b,c)$ $P(a,b,c): a.(b.c) = (a.b).c$
 $a+(b+c) = (a+b)+c$ } Associated Laws

Note: $\forall a \exists b P(a,b) \neq \exists a \forall b P(a,b)$ E_3

"All Rabbits are Faster than All Tortoises"

Domain $R = \{\text{Rabbits}\} \quad T = \{\text{Tortoises}\}$ $x \in R$
 x is a Rabbit $y \in T$
 y is Tortoise $P(x,y)$: Rabbit x is Faster than Tortoise y $\forall x \in R \forall y \in T P(x,y)$ $\forall x \forall y P(x,y)$ E_4

"There Exist a Rabbit which is faster than all Tortoise"

 $\exists x \in R \forall y \in T; P(x,y)$

$$\exists x \in R \forall y \in T; P(x, y)$$

$P(x, y)$: Rabbit x is faster than Tortoise y

Negating Quantified Expressions

"Every student in your class has taken a course in calculus."

T	F
F	T

There exist a student in your class who has not taken a course in Calculus.

$$\left[\begin{array}{l} \neg \forall x P(x) \\ \exists x \neg P(x) \end{array} \right. \quad P(x): x \text{ student in your class has } \dots$$

$$\boxed{\sim [\forall x P(x)] = \exists x \sim P(x)}$$

$$\sim (\exists x P(x)) \equiv \forall x \sim P(x)$$

$$\begin{array}{l} \textcircled{1} \\ \textcircled{2} \end{array} \left[\begin{array}{l} \sim (\forall x P(x)) = \exists x, \sim P(x) \\ \sim (\exists x P(x)) = \forall x \sim P(x) \end{array} \right] \text{ D. Morgan's Laws}$$

"Not (all) Science Students Study Hard"

$$\sim (\forall x P(x)) = \exists x, \sim P(x)$$

$x \in \text{Science}$
 $\hookrightarrow \text{Student}$

P : Study Hard

= There exist at least one science student who does not study hard.