L-10 Posets

Monday, February 14, 2022 9:53 AM

Poset : Partially ordered Set

Partially ordered Relation $R : A \to A$ R is said to be Partially order if ① Reflexive ② Antisymmetric ③ Transitive.

Partially Ordered Sets: — A relation R on a set A is called Partially ordering if it is Reflexive, Antisymmetric and transitive. i.e if

(i) $aRa, \forall a \in A$

(ii) aRb and $bRa \Rightarrow a = b$

(iii) aRb and $bRc \Rightarrow aRc$

then the set 'A' with partial order Relation R is a partially ordered set or 'Poset' and is denoted by (A,R)

(A, R)

① $\forall a \in A;\ aRa$
$(a,a) \in R$

② If aRb & $bRa \Rightarrow a = b$

③ $aRb,\ bRc \Rightarrow aRc$

Ex Show that the relation $\geq$ is a partial Ordering on set of Integers Z

Relation R is $\geq$

Sol Set is Integer Z

$$R = \{(x,y) : x, y \in Z \text{ and } x \geq y\}$$

xRy, $(x,y) \in R$] $x \geq y$

① Reflexive :→ $\forall x \in Z$ we have $x \geq x$

$\Rightarrow xRx$ i.e $(x,x) \in R$

$\therefore \forall x \in Z \Rightarrow (x,x) \in R$

$\therefore$ R is Reflexive.

$x \geq y$

$2 \geq 2$ $\quad -10 \geq -10$

$-1 \geq -1$

$5 \geq 5$

$6 \geq 6$

(ii) Antisymmetric : for $x, y \in Z$

we have if $x \geq y$ and $y \geq x$ then we have $x = y$

$\therefore$ If xRy and $yRx \Rightarrow x = y$

$\therefore$ R is Antisymmetric.

$6 \geq 5$ and $5 \geq 6$

Is it possible

$6 \geq 6$, $6 \geq 6$ Equality hold

(iii) Transitive :→ for $x, y, z \in Z$

If $x \geq y,\ y \geq z$ then $x \geq z$

for $x, y, z \in Z$, $xRy,\ yRz \Rightarrow xRz$

$\therefore$ R is Transitive

$6 \geq 5,\ 5 \geq 4$

$\Rightarrow 6 \geq 4$

∴ R is Reflexive, Antisymmetric & Transitive ∴ R is Partially Ordering Relation

∴ The set Z with R is a Poset

Consider the binary relation R = {(x, y), (x, z), (z, x), (z, y)} on the set {x, y, z}.
Which one of the following is TRUE?
(A) R is symmetric but NOT antisymmetric
(B) R is NOT symmetric but antisymmetric
(C) R is both symmetric and antisymmetric
(D) R is neither symmetric nor antisymmetric

$(x,y) \in R \Rightarrow (y,x) \in R$

Note:– A partial ordering Relation R is often denoted by the symbol $\preceq$

Now $x \preceq y$ Means x precedes y

$x \prec y$ " x Strictly precedes y

$\leqslant$

$\preccurlyeq$

$x \leq y$ x is less than or $\leq$ y

$x \preceq y$

x y

Comparable Two Elements a & b in a poset $(S, \preceq)$ are said to be Comparable if either $a \preceq b$ or $b \preceq a$.
If neither $a \preceq b$ nor $b \preceq a$ then a & b are Called Incomparable.

Ex $(Z, /)$ Relation is "Divisibility"

a/b "a divides b"

$\hookrightarrow \frac{b}{a} = \text{Integer}$

$\frac{a}{b}$

3, 9 ⟶ are Comparable $3/9$ ∵ 3 divides 9

5, 7 $5 \nmid 7$ also $7 \nmid 5$

neither 5 divides 7 nor 7 divides 5

∴ 5, 7 are Incomparable element of the Poset