L-21 Isomorphic

Monday, March 28, 2022 9:39 AM

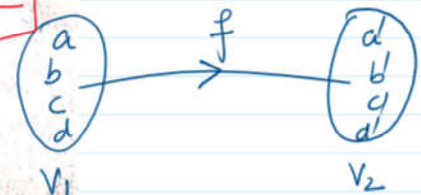
Art-5. Isomorphic Graphs

$G_1(V_1, E_1)$

$G_2(V_2, E_2)$

Let $G = (V, E)$ and $G' = (V', E')$ be two graphs. Then G is isomorphic to G' written as $G \cong G'$ if there exists a bijection f, from V onto V' such that $(v_i, v_j) \in E$, if and only if $(f(v_i), f(v_j)) \in E'$.

In other words, two graphs are isomorphic if there exists a one-one correspondence between their vertices and edges such that incidence relationship is preserved.



Remark. (*a*) Two graphs which are isomorphic will have

(*i*) same number of vertices

(*ii*) same number of edges

(*iii*) an equal number vertices with given degrees Degree Sequence

(*b*) The converse of (*a*) need not be true.

$f : V_1 \to V_2$ one-one & Onto

$V_2' = f(V_1)$

$(a, b) \in E_1$

a ——— b

$f(a) = c'$

$f(b) = a'$

$f(c) = b'$

$f(d) = d'$

$G_1 \cong G_2$

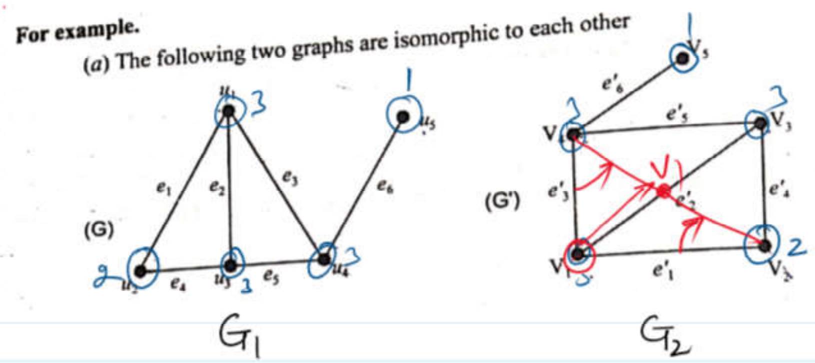
c' ——— a'

$(c' \, a') \in E_2$

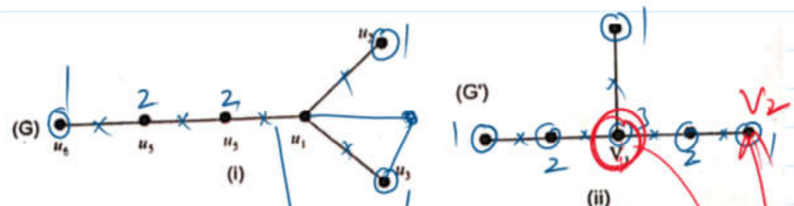
$(f(a), f(b)) \in E_2$

For example.

(*a*) The following two graphs are isomorphic to each other



	G_1	G_2
Vertices	5	5
Edges	6	6
Degree Seq	1, 2, 3, 3, 3	1, 2, 3, 3, 3



No

Incidence Relation is No there

$V = 6$

$E = 5$

1, 1, 1, 2, 2, 3

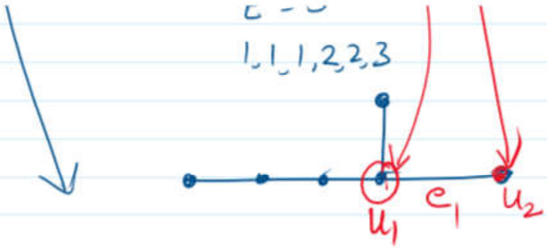
$V = 6$

$E = 5$

1, 1, 1, 2, 2, 3

$E = 5$
$1,1,1,2,2,3$

$E = 5$
$1,1,1,2,2,3$



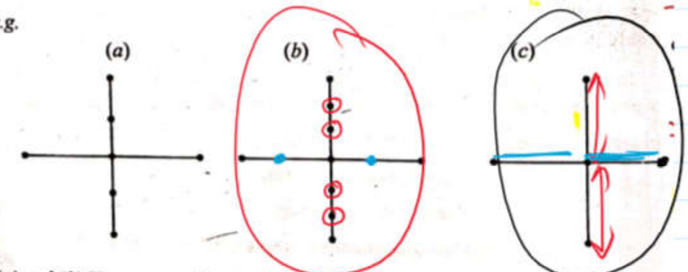
17) Which of the following is false?

a) Single vertex with no edge is a graph b) there is no graph with 3 vertices each with degree 3 c) handshaking lemma is not applicable on simple graphs d) degree of a loop is 2

Homeomorphic Graphs

Given any graph G, obtain a new graph by dividing an edge of G with additional vertices.

e.g.



(a) and (b) Homeomorphic obtained from (c).

b is Homeomorphic to c

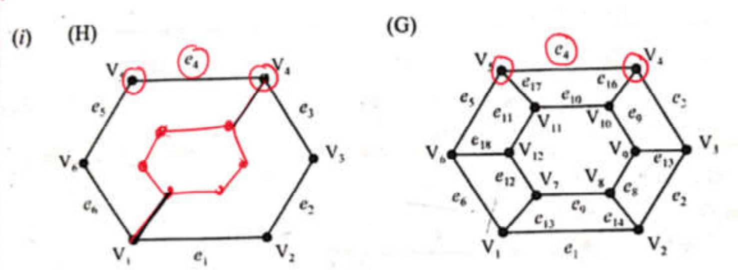
Subgraph :→ A part of a graph.

Let G and H be two graphs with vertex sets V(H), V(G) and edge sets E(H) and E(G) respectively such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$, then we call H as a **Subgraph** of G (or G as a supergraph of H).

If $V(H) \subset V(G)$ and $E(H) \subset E(G)$, then H is a **Proper subgraph** of G and if $V(H) = V(G)$ and $E(H) \subset (G)$ then we say that H is a **spanning subgraph** of G.

In other words, a graph H is said to be a subgraph of G if all the vertices and all the edges of H are in G, and each edge of H has the same end vertices in H as in G.

$V(H) \subseteq V(G)$
$E(H) \subseteq E(G)$
H Subgraph of G



H is Subgraph of G

H_1 is Spanning Subgraph. G
Same Vertex Less Edges