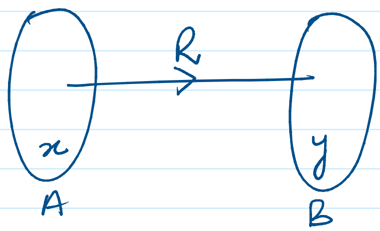
T-4 G2

Thursday, February 10, 2022 10:57 AM

Relation:– Let A and B be two non-empty sets. A Relation R from set A to set B is a subset of the Cartesian product set $A \times B$. The subset is derived by describing a relationship between the first Element and the second element of the ordered pairs in $A \times B$

- $R \subseteq A \times B = \{(x, y) ; x \in A \text{ and } y \in B\}$

if $(x, y) \in R$ then such ordered pair is written as xRy and read as "x is related to y" by relation R.



$R \subseteq A \times B$

$R = \{(x, y) ; x \in A, y \in B\}$

$(x, y) \in R$ — xRy

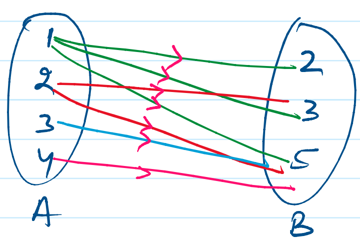
element $x \in A$ is related to $y \in B$ Under the relation R

Ex

$A = \{1, 2, 3, 4\}$ $\quad B = \{2, 3, 5\}$

$R : \{(x, y) : x \in A, y \in B, x < y\}$ $\quad$ R: "Lessthan"

$R = \{(1, 2) (1, 3) (1, 5) (2, 3) (2, 5) (3, 5) (4, 5)\}$



$D_R = \{1, 2, 3, 4\} = A$

$R_R = \{2, 3, 5\} = B$

- **Domain and Range:–** if $R = \{(x, y) : x \in A \ \& \ y \in B\}$ is a Relation from set A to set B then

Domain of $R = \{x \in A : (x, y) \in R \text{ for some } y \in B\}$

Range of $R = \{y \in B : (x, y) \in R \text{ for some } x \in A\}$

$D_R \subseteq A$

$R_R \subseteq B$

$A = \{1, 2, 3, 4, 5, 6\}$ $\quad B = \{1, 2, 3, 4\}$ $\quad$ R: "<"

$R = \{(1, \ ) - - - (2, 3) (2, 4) (3, 4)\}$

$D_R = \{1, 2, 3\} \subset A$ $\quad R_R = \{1, 2, 3, 4\} = B$

Types of Relation:

$D_R = \{1, 2, 3\} \subseteq A \qquad R_R = \{\ldots\} = B$

Types of Relation:

① ① **Reflexive Relation:**– A relation R on a set A is called reflexive if $aRa \ \forall a \in A$ i.e $(a,a) \in R \ \forall a \in A$ i.e if every element of set A is related with itself.

(Ex) if $A = \{1, 2, 3\}$ and $R = \{(1,1) (1,2) (2,2) (2,3) (3,1) (3,3) (3,2)\}$

then R is Reflexive because $(1,1), (2,2), (3,3) \in R$

A

$\forall a \in A,\ (a,a) \in R$ or aRa $\qquad A = \{1, 2, 3\}$

$$R = \{(1,1) \ldots (2,2) \ldots (3,3)\}$$

R: "$\leq$" $\quad R = \{(1,1)\ (1,2)\ (1,3)\ (2,2)\ (2,3)\ (3,3)\}$

$\geq$ $\longrightarrow$ R is reflexive

R: $<$, $>$ — Not Reflexive $\quad R = \{(1,2)\ (1,3)\ (2,3)\}$

$R = \{(1,1)\ (2,1)\ (2,2)\ (2,3)\ (3,2)\}$ — Not Reflexive

└── But $(3,3)$ is not there

Every Element (All) $\forall a \in A;\ aRa$

② **Irreflexive Relation:** A relation R which is not ~~self~~ reflexive is called Irreflexive Relation. i.e $\forall a \in A,\ (a,a) \notin R$ i.e there is ~~some~~ no $a \in A$ such that ~~aRa or (a,a)∈R~~ $(a,a) \in R$

Reflexive — All

Irreflexive — No

Non-Ref — Some

③ **Non-Reflexive:**– A relation R on set A is non-reflexive if R is neither reflexive nor Irreflexive. i.e $(a,a) \in R$ for some $a \in A$ is true and for some $a \in A$ is false.

$A = \{1, 2, 3\}$

$R = \{(1,1)\ (1,2)\ (2,1)\ (2,2)\ (3,3)\}$ — All Reflexive

$R = \{(1,1)\ (1,2)\ (2,1)\ (3,2)\}$ — Some Non-Reflexive

$R = \{(1,2)\ (2,1)\ (2,3)\ (3,2)\}$ — No — Irreflexive

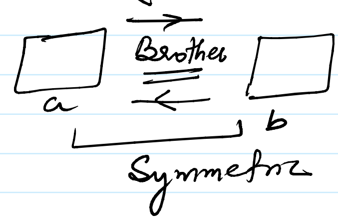
Symmetric Relation: → A relation R on a set A is called symmetric if $(a,b) \in R \Rightarrow (b,a) \in R$

Ex: if $R = \{(1,1), (1,2), (1,3), (2,2), (2,1), (3,1)\}$ on $A = \{1,2,3\}$
R is symmetric.

$aRb \Rightarrow bRa$ for $a, b \in A$

$(a,b) \in R \Rightarrow (b,a) \in R$

$a \rightleftarrows b$



$(2,3) \in R \Rightarrow (3,2) \in R$
$(1,3) \in R \Rightarrow (3,1) \in R$
$(2,5) \in R \Rightarrow (5,2) \in R$

Asymmetric Relation: — A relation R on a set A is called Asymmetric relation if $(a,b) \in R \Rightarrow (b,a) \notin R$

Ex: if $R = \{(1,1)\ (1,2)\ (2,2)\ (1,3)\}$
here $(1,2) \in R \Rightarrow (2,1) \notin R$
$(1,3) \in R$ But $(3,1) \notin R$

$a \longrightarrow b$

$aRb \Rightarrow b \not R a$

$(a,b) \in R \Rightarrow (b,a) \notin R$

Antisymmetric Relation: A relation R on a set A is called Antisymmetric if $\forall a, b \in A$

~~(a,b) ∧ (b,a)~~ aRb and $bRA \Rightarrow a = b$

(a,b) and $(b,a) \Rightarrow a = b$

$(a,b) \in R$ and $(b,a) \in R \Rightarrow \boxed{a = b}$

$x \leq y$ & $y \leq x \Rightarrow x = y$

Transitive Relation: A relation R on set A is called transitive if $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$ i.e. aRb and $bRc \Rightarrow aRc$

Ex: If R is '<' relation (Less than)
Now if $a<b$ and $b<c$ then $a<c$

$aRb,\ bRc \Rightarrow aRc$

$aRb,\ bRc,\ cRd,\ dRe \Rightarrow aRe$

$a<b,\ b<c,\ c<d,\ d<e \Rightarrow a<e$

Equivalence Relation: A relation on a set A is called equivalence relation if it is Reflexive, Symmetric and Transitive.

" " " Non-Empty $= 2^{mn}-1$

Number of different relation from a set with n elements to a set with m elements is 2^{mn} 2^{nm}

Number of Reflexive Relations on a set with n elements : $2^{n(n-1)}$. $2^{n(n-1)}$

. Number of Symmetric Relations on a set with n elements : $2^{n(n+1)/2}$. $(2)^{\frac{n(n+1)}{2}}$

Number of Anti-Symmetric Relations on a set with n elements: $2^n\ 3^{n(n-1)/2}$. $2^n \cdot 3^{\frac{n(n-1)}{2}}$

Number of Asymmetric Relations on a set with n elements : $3^{n(n-1)/2}$. $3^{\frac{n(n-1)}{2}}$

Irreflexive Relations on a set with n elements : $2^{n(n-1)}$.

Reflexive and symmetric Relations on a set with n elements : $2^{n(n-1)/2}$.

Ex

If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of relations from A to B is

$A = \{2,4,6\}$ $B = \{2,3,5\}$

$n(A)=3$ $n(B)=3$

No. of Relation $= 2^{mn} = 2^{3\cdot 3} = 2^9$

If A is the set of even natural numbers less than 8 and B is the set of prime numbers less than 7, then the number of Non-empty relations from A to B is

(a) 2^9 (b) 9^2 (c) 3^2 (d) 2^9-1