27-1-22 T G2

Thursday, January 27, 2022 10:57 AM

Logical Equal / Equivalence

$P(p,q,r,--) = Q(\text{———})$

- **Logical Equivalence :–** Two proposition $P(p,q,r,--)$ and $Q(p,q,r--)$ where $p,q,r,\ldots$ are propositional variables, are called Logical Equivalence or Equal if they have same truth value in every possible case. and written as $P \approxeq Q$. $P = Q$ $P \approxeq Q$

Working Rule

Note :→ To test whether the proposition P and Q are logically Equivalence the following steps are followed.

Step-I : Construct the truth table for P. (L.H.S)

- Step II : Construct the truth table for Q (R.H.S) using the same propositional variable.

Step III : Check each combinations of truth values of the propositional variable to see whether the value of P is the same as truth value of Q.

OR

If in each row the truth value of P is the same as that of Q. Then P and Q are logically Equivalent.

Ex. Show that $\sim(p \vee q) = \sim p \wedge \sim q$ (and)

	p	q	$\sim p$	$\sim q$	$p \vee q$	P: $\sim(p \vee q)$	Q: $\sim p \wedge \sim q$
1	T ✓	T ✓	F ✓	F ✓	T	F	F
2	T ✓	F ✓	F ✓	T ✓	T	F	F
3	F ✓	T ✓	T ✓	F ✓	T	F	F
4	F ✓	F ✓	T ✓	T ✓	F	T	T
	I	II	III	IV	V	VI	VII

∵ the VI^{th} & VII^{th} Column of T.V. Table are Identical

∴ $\boxed{\sim(p \vee q) = \sim p \wedge \sim q}$

Ex. $p \rightarrow q = \sim p \vee q$

$p \rightarrow q$	$\sim p \vee q$
T	F V T = T
F	F V F = F
T	T V T = T
T	T V F = T

→ Contingency

$T \vee T = T$
$T \vee F = T$ → Contingency

Note:→ $P \approx Q$ iff $P \Leftrightarrow Q$ is a Tautology

① Tautology ② Contradiction ③ Contingency

Tautologies and Contradiction:— A Compound proposition that is always true for all possible truth value of its variable is called a Tautology.
A Compound proposition that is always False (F) for all possible Truth Value is called Contradiction.
Note:— A proposition that is neither a Tautology (T) nor Contradiction (F) is called a Contingency

T, T, T, T → Tautology F, F, F, F → Contradiction F, T, T, F → Contingency

Note: Number of Rows in the truth table of a Compound Statement $= 2^{(\text{No. of prop. Variable})}$

p, q No. of prop. Variable = 2 $n=2$
No. of Rows in T.V. Table $= 2^n = 2^2 = 4$

Note $n=2$ No. of Rows = 4 p: T T F F
q: T F T F

$n=3$ " " $= 2^3 = 8$
p: T T T T F F F F
q: T T F F T T F F
r: T F T F T F T F

$\frac{8}{2} = 4$
$\frac{4}{2} = 2$
$\frac{2}{2} = 1$

$n=4$ No. of Rows $= 2^4 = 16$
p: T T T T T T T T F F F F F F F F
q: T T T T F F F F T — — — —
r: T T F F T T F F — — — —

$\frac{16}{2} = 8$
$\frac{8}{2} = 4$
$\frac{4}{2} = 2$
$\frac{2}{2} = 1$

q: T T T T F F F F

r: T T F F T T F F

s: T F T F T F

$\frac{4}{2}=2$

$\frac{2}{2}=1$

$\sim p \vee q \neq p \to \sim q$

F	T	T
F	F	F
T	T	T
T	F	T

T → F		F
T → T		T
F → F		T
F → T		T

$T \to F = F$

Laws of Prop. Logic

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

$\sim$ — Negation

$\vee$ — Disjunction

$\wedge$ — Conjunction

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\sim(p \wedge q) = \sim p \vee \sim q$

$x \cdot (y+z) = (x \cdot y) + (x \cdot z)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

TABLE 7 Logical Equivalences Involving Conditional Statements.
$p \rightarrow q \equiv \neg p \vee q$
$p \rightarrow q \equiv \neg q \rightarrow \neg p$
$p \vee q \equiv \neg p \rightarrow q$
$p \wedge q \equiv \neg(p \rightarrow \neg q)$
$\neg(p \rightarrow q) \equiv p \wedge \neg q$
$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
$(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
$(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
$(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$