L-31 Tree

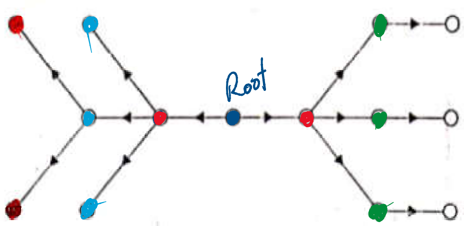
Tuesday, April 19, 2022 10:06 AM

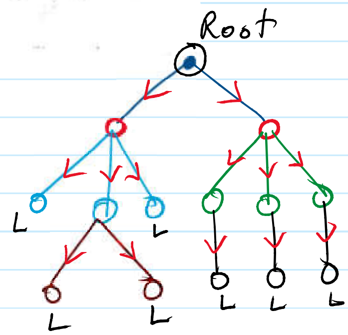
SubTree :- A part of a Tree is called SubTree.

Forest :- A forest is an Undirected graph whose Components are all tree.

12.3. DIRECTED TREES

A directed tree is an acyclic directed graph. It has one node with indegree 1, while all other nodes have indegree 1 as shown in Figs. 12.2 and 12.3.

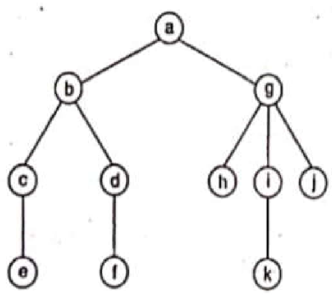




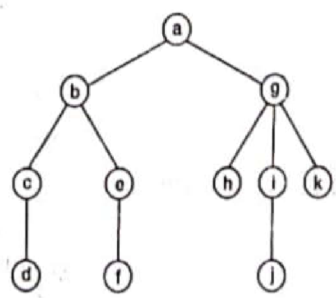
12.4. ORDERED TREES

If in a tree at each level, an ordering is defined, then such a tree is called an ordered tree.

e.g., the trees shown in Figs. 12.4 and 12.5 represent the same tree but have different orders.



(12.4)



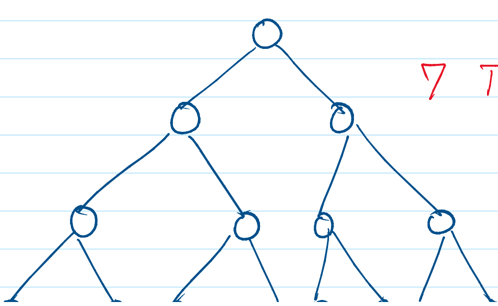
(12.5)

12.5. ROOTED TREES

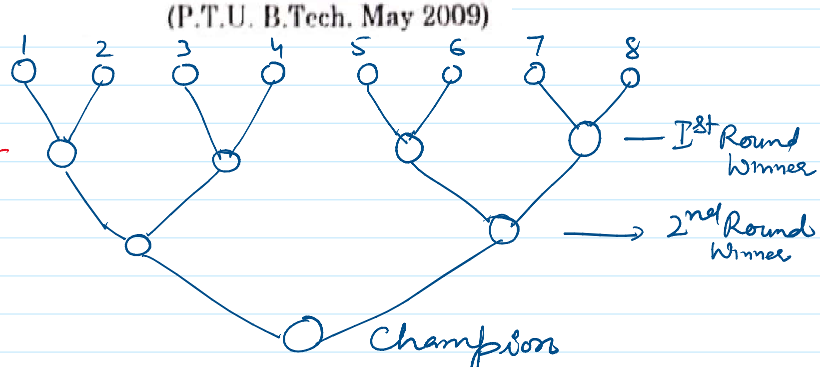
If a directed tree has exactly one node or vertex called root whose incoming degree is 0 and all other vertices have incoming degree one, then the tree is called rooted tree.

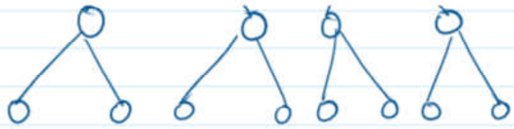
* A tree with no nodes is a rooted tree (the empty tree).
* A single node with no children is a rooted tree.

Example. *Suppose 8 people enter a Badminton tournament use a rooted tree model of the tournament to determine how many games must be played to determine a champion if a player is eliminated after one loss.* (P.T.U. B.Tech. May 2009)



7 Tournament





Champions

12.6. PATH LENGTH OF A VERTEX

The path length of a vertex in a rooted tree is defined to be the number of edges in the path from the root to the vertex.

For example, we find the path lengths of the nodes b, f, l, q in Fig. 12.7a.

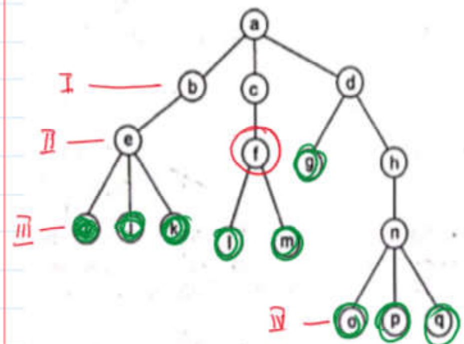


Fig. 12.7a

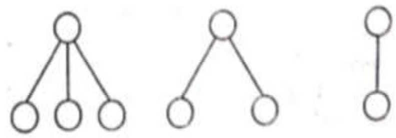
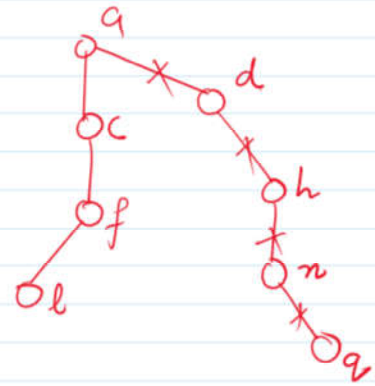


Fig. 12.7b



The path length of node *b* is one.
The path length of node *f* is two.
The path length of node *l* is three.
The path length of node *q* is four.

Theorem I. *Prove that there is one and only one path between every pair of vertices in a tree T.*

12.7. FOREST

If the root and the corresponding edges connecting the nodes are deleted from a tree, we obtain a set of disjoint trees. This set of disjoint trees is called a forest. (Fig. 12.7b)

12.8. BINARY TREE

0, 1, 2

If the outdegree of every node is less than or equal to 2, in a directed tree then the tree is called a binary tree. A tree consisting of no nodes (empty tree) is also a binary tree.

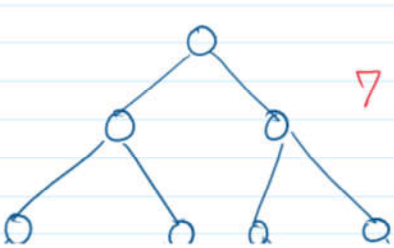
Outdegree — Child
atmost 2 Child
less than or equal ⇒ Atmost

m-Tree / m-ary Tree :→ If the Outdegree of every Node is atmost m.

If m=2 — Binary Tree

Complete m-Tree :→ If Outdegree of Every Node is Exactly m

Complete-Binary Tree :→ " " " " " 2.

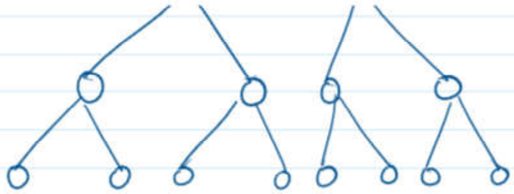


7 To

No. of Nodes	Level
1	0
2	1
4	2

1, 2, 4, 8, 16, 32, 64, - - - -
It is a G.P with c.Ratio = 2

Number of Nodes at n^{th} level of Comp. Binary
n



4 2

8 3

Number of Nodes at n^{th} level of Comp. Binary Tree $= 2^n$

Note: Number of Nodes at n^{th} level of a Complete m-Tree $= m^n$

12.9. BASIC TERMINOLOGY

(*a*) **Root.** A binary tree has a unique node called the root of the tree.

(*b*) **Left Child.** The node to the left of the root is called its left child.

(*c*) **Right Child.** The node to the right of the root is called its right child.

(*d*) **Parent.** A node having left child or right child or both is called parent of the nodes.

(*e*) **Siblings.** Two nodes having the same parent are called siblings.

(*f*) **Leaf.** A node with no children is called a leaf. The number of leaves in a binary tree can vary from one (minimum) to half the number of vertices (maximum) in a tree.

(*g*) **Ancestor.** If a node is the parent of another node, then it is called ancestor of that node. The root is an ancestor of every other node in the tree.

(*h*) **Descendent.** A node is called descendent of another node if it is the child of the node or child of some other descendent of that node. All the nodes in the tree are descendents of the root.

(*i*) **Left Subtree.** The subtree whose root is the left child of some node is called the left subtree of that node.

(*j*) **Right Subtree.** The subtree whose root is the right child of some node is called the right subtree of that node.

(*k*) **Level of a Node.** The level of a node is its distance from the root. The level of root is defined as zero. The level of all other nodes is one more than its parent node. The maximum number of nodes at any level N is 2^N.

(*l*) **Depth or Height of a Tree.** The depth or height of a tree is defined as the maximum number of nodes in a branch of tree. This is one more than the maximum level of the tree *i.e.*, the depth of root is one. The maximum number of nodes in a binary tree of depth d is $2^d - 1$, where $d \geq 1$.

(*m*) **External Nodes.** The nodes which has no children are called external nodes or terminal nodes.

(*n*) **Internal Nodes.** The nodes which has one or more than one children are called internal nodes or non-terminal nodes.