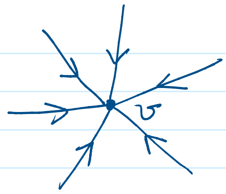
L-19 Graphs

Monday, March 21, 2022 10:06 AM

Sink



It has only Indegree

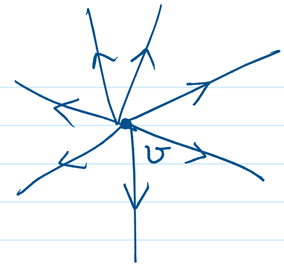
Outdegree $= 0$

$\boxed{d^-(v) = 0}$

Source

Indegree $= 0$

$\boxed{d^+(v) = 0}$



Even or odd (Parity) of a Vertex : The vertex V is said to be even or odd according as deg.(V) is even or odd.

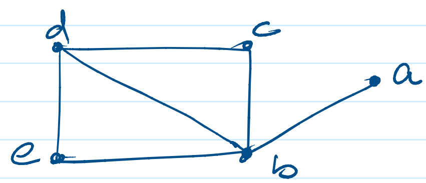
Even Parity / Vertex — degree = Even No.

odd Vertex $\Rightarrow$ degree = odd number

Pendent vertex (End vertex)

A vertex whose degree in a graph is one is called pendent vertex.

$\boxed{deg(v) = 1}$



$deg(a) = 1$

'a' is Pendent Vertex

Isolate Vertex :→ v is Isolate Vertex $d(v) = 0$

Regular Graph :→ If the degree of Every Vertex is Same.

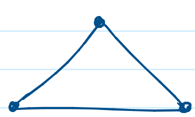
K-Regular graph :→ " " " " " " Equal to K

Definition Regular Graph : A graph in which all the vertices are of same degree is called a regular graph.

Definition *k*-Regular Graph : A graph in which all the vertices have the same degree equal to *k*, is called a *k*-Regular graph.

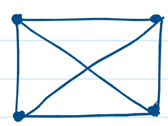
1-Regular

K_2



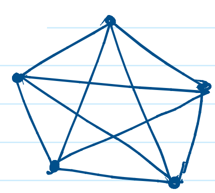
2-Regular

K_3



3-Regu.

K_4



4-Reg.

K_5

Note :- K_n (Complete graph with n-vertices) is a $(n-1)$-Regular graph

degree of Every Vertex in $K_n = (n-1)$

Theorem 1. First Theorem on Graph Theory (Handshaking Theorem)

The sum of the degrees of all the vertices in *a* graph G is equal to twice the number of edges in G.

OR The Sum of degree of all the Vertices in a graph G

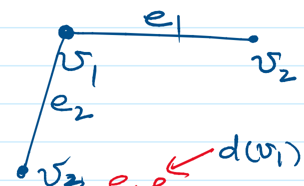
The sum of the degrees of all the vertices in *a* graph G is equal to twice the number of **edges** in G.

OR The Sum of degree of all the Vertices in a graph G is Even.

Sol :–

$d(v_1) = 1 \quad d(v_2) = 1$

$d(v_1) + d(v_2) = 1 + 1 = 2 = 2 \cdot 1$



e_1 is Counted 2 time [one with v_1, one " v_2]

e_2 " " " " [—— v_1 & —— v_3]

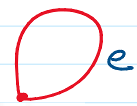
$e_1 e_2 \rightarrow d(v_1) = 2$

$e_1 \rightarrow d(v_2) = 1$

$e_2 \rightarrow d(v_3) = 1$

$= 4 = 2 \times 2 = 2e$

degree of loop = 2

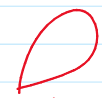


Proof : Let *e* be any edge in graph between two vertices V_1 and V_2.

Now, when we count degree of all vertices *e* is counted twice, once in degree of V_1 and again in degree of V_2.

Also, if V_1 and V_2 are identical, again *e* will be counted twice.

($\because$ *e* is self-loop)

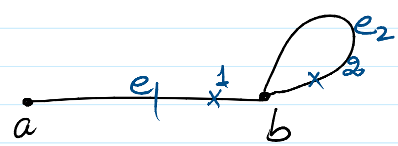


Hence every edge is counted twice.

So total degree is twice number of edges.

or $$\boxed{\sum_{i=1}^{n} \deg(v_i) = 2e} \quad \text{Sum of degree of all Vertices} = 2 \text{ No. of Edges}$$

$2e \rightarrow$ Even No.



Theorem 2. Prove that in a graph the number of vertices of odd degree is even. in Number

$\deg(a) = 1 \quad \deg(b) = 3$

$\sum d(v) = 1 + 3 = 4 = 2 \times 2 = 2e$

$$\sum d(v) = 2e = \text{Even No.}$$

$$\Rightarrow \sum_{\text{Even}} d(v) + \sum_{\text{odd}} d(v) = \text{Even}$$

$$\Rightarrow \text{Even No.} + \sum_{\text{odd}} d(v) = \text{Even (No)}$$

$$\Rightarrow \sum_{\text{odd}} d(v) = \text{Even No.}$$

$$\Rightarrow \text{Sum of Odd Number} = \text{Even.}$$

$2 + 3 + 7 + 8 + 2 + 4 = \text{Even}$

$(2 + 8 + 2 + 4) + (3 + 7) = —$

$4 - 2 = 2$

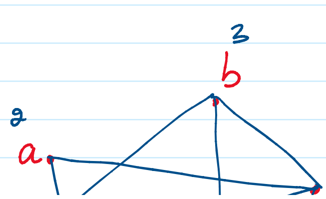
$8 - 4 = 4$

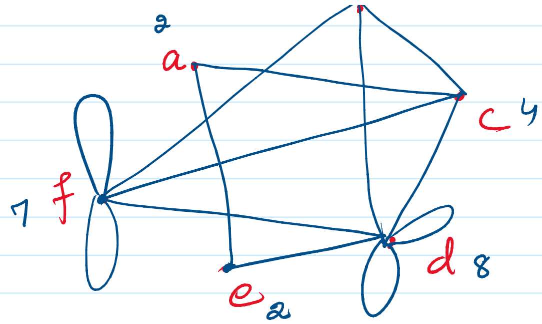
$8 - 6 = 2$

$3 + 1 = 4$

$3 + 3 + 1 = 7$

$3 + 3 + 1 + 1 = 8$





$3+3+1 = 7$

$3+3+1+1 = 8$

$1+3+3+1+5 = 13$