L-14 Counting

Monday, February 28, 2022 9:58 AM

The final value of x where $x = 10! - 9!$ is

(a) $10! \times 9$ (b) $9! \times 10$ (c) $9! \times 9$ (d) none of these

$$10 \cdot \lfloor 9 - \lfloor 9 = (10-1)\lfloor 9$$
$$= 9 \cdot \lfloor 9$$

The value of $\frac{n!}{(n-r)!}$ is, when $n = 9, r = 5$

(a) 15130 (b) 26510 (c) 28120 (d) 15120

$$\frac{\lfloor 9}{\lfloor 9-5} = \frac{\lfloor 9}{\lfloor 4} = \frac{9.8.7.6.5.\cancel{\lfloor 4}}{\cancel{\lfloor 4}}$$
$$= 9.8.7.6.5$$
$$= 9.8.210$$
$$= 9 \times 1680$$
$$=$$

(1) How many 3-letter words can be formed by using the letters of the words ORIENTAL

$n = 8 \quad r = 3$

Total No. of words $= {}^nP_r = {}^8P_3$

$$= \frac{\lfloor 8}{\lfloor 8-3} = \frac{\lfloor 8}{\lfloor 5} = \frac{8.7.6.\cancel{\lfloor 5}}{\cancel{\lfloor 5}} = 8.7.6 = \underline{336}$$

n → Different

$${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$\lfloor 2 = 2$ $\lfloor 6 = 720$
$\lfloor 3 = 6$ $\lfloor 7 = 5040$
$\lfloor 4 = 24$ $\lfloor 8 = 40320$
$\lfloor 5 = 120$

How many numbers of 4 letter words, with or without meaning, can be formed out of the letters of the word 'WATCH' ?

(a) 25 (b) 240 (c) 160 (d) 120

Example 14 Find the number of different 8-letter arrangements that can be made from the letters of the word DAUGHTER so that

(i) all vowels occur together (ii) all vowels do not occur together.

Total No. of Words / Arrg. $= {}^8P_8 = \lfloor 8 = 40320$

Vowels A, U, E $\quad \underbrace{(AUE)}_{1}, \underset{2}{D}, \underset{3}{G}, \underset{4}{H}, \underset{5}{T}, \underset{6}{R}$

Arrg. All 6 objects taking all at a time $= {}^6P_6 = \lfloor 6 = 720$

(AUE), (UAE), (EAU), (EUA)

No. of ways that the Vowels be arrange themselves $= {}^3P_3 = \lfloor 3 = 6$

∴ F.P.C $\quad 720 \times 6 = \underline{4320}$

(2) No all Vowels occur together = Total Arrg − (Vowels occur Together)

$= 40320 - 4320$

(3) No Two Vowels Comes Together   AUE   DGHTR

(5) $^5P_5 = \lfloor 5 = 120$

$* D * G * H * T * R *$ $^6P_3 = \frac{\lfloor 6}{\lfloor 6-3} = \frac{\lfloor 6}{\lfloor 3} = \frac{720}{6} = 120$

F.P.C   Total No. of words $= 120 \times 120$

$= 14400$

Ex. $\{1, 2, 3, 5, 6, 7, 8\}$ No. of digits $= 7$

How Many 4-digit Number are there

(1) With Repetation

T H T U

7 7 7 7

No. of ways of filling Unit place $= 7$

— — — — — — 10th " $= 7$

— — — — — — Hund " $= 7$

— — — — — — Thous " $= 7$

F.P.C   Total 4-digit Number $= 7 \times 7 \times 7 \times 7$

$= 7^4$

$= 2401$

Without Repetation

T H T U

4 5 6 7

———————— 7

———————— 6

———————— 5

———————— 4

F.P.C $7 \times 6 \times 5 \times 4 = 840$

Ex. $\{\cancel{0}, 1, 2, 3, \cancel{5}, \cancel{6}, 7, \cancel{8}\}$ No. of digits $= 8$

With Rep

T H T U

7 8 8 8

4-digit $7 \times 8 \times 8 \times 8$

$= 3584$

Without Rep

T H T U

7 ? 6 5

7

$7 \times 7 \times 6 \times 5 = 1470$

$\boxed{6508}$

Permutation when the Things are not all different.

Permutation when the Things are not all different:

- The Number of Permutation / Arrangement of n things taken all at a time when p of them are of one kind, q of them are of second kind and 'r' of them are of third kind are given by

$$\boxed{\text{No. of Permutations} = \frac{\lfloor n}{\lfloor p \, \lfloor q \cdot \lfloor r}}$$

DAUGHTER

MATHEMATICS

M occurs — 2 time
T " — 2 "
A " — 2 "

$$\text{Total No. of Arrg} = \frac{\lfloor 11}{\lfloor 2 \, \lfloor 2 \, \lfloor 2}$$

The number of permutations of the letters of the word 'FILL' is
(a) 8 (b) 12 (c) 16 (d) 24

In how many different ways can the letters of the word 'LEADING' be arranged in such a way that the vowels always come together?
(a) 720 (b) 360 (c) 480 (d) 5040

$\lfloor 3 = 6$

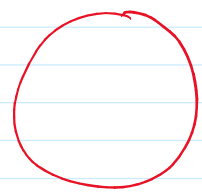
(AEI) L D N G

$\lfloor 5 = 120$

$= 120 \times 6$
$= 720$

Circular Permutations:-

1. No. of ways in which 'n' things / Person can be arranged in a circle $= \lfloor n-1$



2. In case of Necklaces with Beads or Garlands with different flowers.
Total No. of Circular permutations $= \frac{1}{2} \lfloor n-1$ $\quad \frac{\lfloor n-1}{2}$

In how many ways can 8 things be arranged in
(i) a straight line ② in a Circle.

$^8P_8 = \lfloor 8$
$= 40320$

$\lfloor n-1$
$= \lfloor 8-1$
$= \lfloor 7 = 5040$

In how many ways can 8 beads of different colour form a necklace.

$n=8$ $\quad \frac{\lfloor n-1}{2} = \frac{\lfloor 7}{2} = \frac{5040}{2} = 2520$

Two ways

Two ways

one Necklace

Rank of a word:

RAMI

A, I, M, R

$$\underbrace{A}_{1} \times \underbrace{\times \times}_{\lfloor 3} \quad 1 \times \lfloor 3 = 6$$

$$\underbrace{I}_{1} \times \times \times = 6 \quad 1 \times \lfloor 3$$

$$M \times \times \times = 6$$

RAIM — 1

RAMI

$6+6+6+1=19$

Rank of RAMI = 20