1-2-22 Predicates

Tuesday, February 1, 2022 10:01 AM

Note :→ A predicate is a Sentence that Contain a finite number of Variables and become a Statement when specific Value are Substituted for the Variables. the **Domain** of the predicate Variable is the set of all Values that may be substituted in place of Variable.

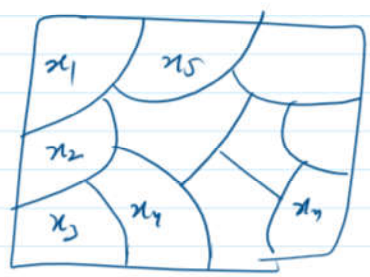
Note: A predicate with Variable is Called an Atomic Formula Can be ~~mable~~ made a proposition by applying one of the following two operation
(i) Assign a Value to the Variable
(ii) Quantify the Variable using a Quantifiers.

Note: If $P(x)$ is predicate and x has domain D, the Truth Set of $P(x)$ is the set of all elements in D that makes $P(x)$ true when they are substituted for x & denoted by: $\{x \in D;\ P(x)\}$.

When all the elements in the domain can be listed—say, $x_1, x_2, \ldots, x_n$—it follows that the universal quantification $\forall x\, P(x)$ is the same as the conjunction

$$P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n),$$

because this conjunction is true if and only if $P(x_1), P(x_2), \ldots, P(x_n)$ are all true.



Domain

$\forall x\, P(x)$:

$$\left[P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge - - - - \wedge P(x_n) \right]$$

Ex

What is the truth value of $\forall x P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

$P(x):\ x^2 < 10$ D: Positive Integer not Exceeding 4 $x = 1, 2, 3, 4$

$P(1):\ (1)^2 < 10$ T

$P(2):\ (2)^2 < 10$ T

$P(3):\ (3)^2 < 10$ T

$P(4):\ (4)^2 < 10$ F

$\forall x\, P(x):\ P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

$T \wedge T \wedge T \wedge F = F$

$\therefore\ \forall x\, P(x)$ is False

② Existential Quantifier : (Some , Few)

The *existential quantification* of $P(x)$ is the proposition

"There exists an element x in the domain such that $P(x)$."

We use the notation $\exists x P(x)$ for the existential quantification of $P(x)$. Here $\exists$ is called the *existential quantifier*.

$\exists x, P(x)$

There Exist some x ; $P(x)$ is True

$\exists$ There Exist Existential quantifier

$\exists x$ Means atleast One Object is there in the domain of x for Which $P(x)$ is True.

There Exist at least one x such that $P(x)$

Ex

Let $P(x)$ denote the statement "$x > 3$." What is the truth value of the quantification $\exists x P(x)$, where the domain consists of all real numbers?

$P(x): x > 3$; $x \in R$

① $\forall x, P(x) \longrightarrow$ False $x=2$ $P(2): 2>3$ Which is False

$\therefore \forall x P(x)$ is Not True or False.

② $\exists x. P(x)$ — True If $x=4$ or $x=3.00001 \in R$

$P(4): 4>3$ True $P(3.0001): 3.0001>3$ True

the existential Quantificati. $\exists x P(x)$ is True

Let $Q(x)$ denote the statement "$x = x + 1$." What is the truth value of the quantification $\exists x Q(x)$, where the domain consists of all real numbers?

$Q(x): x = x+1$; $x \in R$ $\exists x Q(x) =$ False

$\because x$ is Reas there is no real Number x Such that $x = x+1$

$Q_1(x): x < x+1$ $x > x+1$

When all elements in the domain can be listed—say, $x_1, x_2, \ldots, x_n$— the existential quantification $\exists x P(x)$ is the same as the disjunction

$x_1, x_2, x_3 - - - x_n$

$$P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n),$$

$\exists x P(x) \approxeq P(x_1) \vee P(x_2) \vee P(x_3) -- \vee P(x_n)$

because this disjunction is true if and only if at least one of $P(x_1), P(x_2), \ldots, P(x_n)$ is true.

Ex

What is the truth value of ~~$\forall x P(x)$~~ $\exists x P(x)$, where $P(x)$ is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?

$P(x): x^2 < 10$ $x = 1, 2, 3, 4$

$\exists x P(x) \cong P(1) \vee P(2) \vee P(3) \vee P(4) = T \vee T \vee T \vee F = T$

The truth value of $\exists x P(x)$ is True

Ex: "Someone loves Me"

x: Subject (variable) P: loves Me.

$P(x)$: x loves me

$\boxed{\exists x P(x)}$ There exis x that loves me.

There is x, that loves me

"Every One loves Me" $\underline{\forall x} P(x)$