T-7 G2

Thursday, March 3, 2022 11:00 AM

Lattice

$(L, \leq)$ (A, R)

Lattice :→ A poset (Partially Ordered Set) $(L, \leq)$ is said to be lattice if every two elements in the set L has unique least upper bound (lub, Sup) and a unique greatest lower bound (glb, Inf)

← Every pair of element in L has lub & glb

The Poset $(L, \leq)$ is a lattice if for every $a, b \in L$, $Sup\{a,b\}$ and $Inf\{a,b\}$ exist in L i.e

$$\left[\begin{array}{l} Sup\{a,b\} = a \vee b = \text{'a joint b'} \\ Inf\{a,b\} = a \wedge b = \text{'a meet b'} \end{array}\right]$$

$\forall\, a, b \in L$ $lub\{a,b\}$, $glb\{a,b\}$ exist in L

$lub\{a,b\} = Sup\{a,b\} = \text{"a join b"} \quad a \vee b$

$glb\{a,b\} = Inf\{a,b\} = \text{"a meet b"} \quad a \wedge b$

Note The other Notations for $\left[\begin{array}{l} Sup\{a,b\} = a \cup b \text{ or } a+b \\ Inf\{a,b\} = a \cap b \text{ or } a \cdot b \end{array}\right]$

$\cup$ — OR, $+$

$\cap$ — and, $\cdot$

Note: The set N of natural numbers Under divisibility relation '/' formed a lattice in which

$lub\{a,b\} = a \vee b = lcm\{a,b\} \in N$

$glb\{a,b\} = a \wedge b = gcd\{a,b\} \in N$

Set — Natural Number

Element of set are Natural Numbers

Relation : Divisibility

$lub\{a,b\} = LCM\{a,b\}$

$glb\{a,b\} = GCD/HCF\{a,b\}$

Q: If A = {2,3,5,6,10,15,30,45}and a R b iff a/b,lub of 2 & 5 is ___

a) 10 b) 6 c) 30 d) All of the above

a/b R : Divisibility

$lub\{2,5\} = LCM\{2,5\}$

$glb\{2,5\} = gcd\{2,5\} = 1$

∵ 2 & 5 are prime

(a) 2

(b) 5

(c) 10

(d) None of these

The Least Upper Bound for the set {1,2,4,5,10} is

a) 12, 20
b) 2, 5
c) 20
d) 12,20,25

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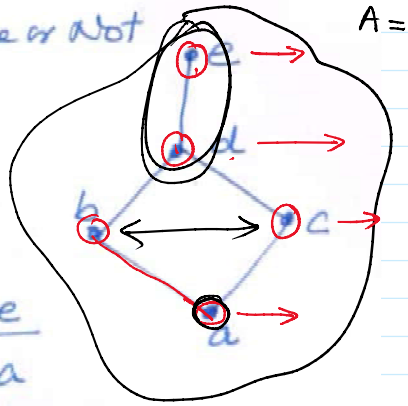
Ex) Determine whether the following a Lattice or Not

Sol: Construct the Closure table for lub and glb of the given problem.



lub/Sup

	a	b	c	d	e
a	a	b	c	d	e
b	b	b	d	d	e
c	c	d	c	d	e
d	d	d	d	d	e
e	e	e	e	e	e

Every pair has lub.

glb/Inf

	a	b	c	d	e
a	a	a	a	a	a
b	a	b	a	b	b
c	a	a	c	c	c
d	a	b	c	d	d
e	a	b	c	d	e

(e, d)

d | b, c, a

lower {d, e} upper

Since Each subset of two elements has lub and glb

{ g, a, c, b, d, f }

lowerbound

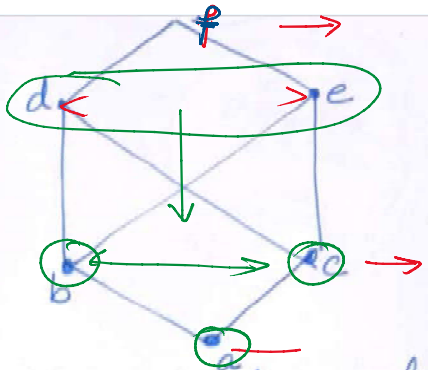
upperbound

Ex)

lub/Sup

	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	b	?	d	e	f
c	c	c	c	d	e	f
d	d	d	d	d	f	f
e	e	e	e	f	f	f
f	f	f	f	f	f	f

b, c



d, e, f

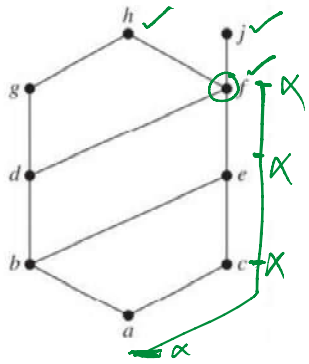
{b, c}

Side 'b' and 'c' has no least upper bound. Since the upper bound of b & c are d, e, f

but none of these three element precedes the ~~poset~~ other two with respect to the ordering of this poset.

∴ It is not a Lattice

The Lower Bound of $\{a, c, d, f\}$ in the following Poset is ?



a) a
b) c, a
c) f, j
d) a, b, c