AI Physics Notes: Work

Master Notes: Work

Welcome, future engineers and doctors! Let's conquer the fundamental concept of "Work" in physics. This topic is crucial for understanding energy, power, and various applications in mechanics. Pay close attention to the nuances and vector nature of forces and displacements.

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1. Summary

In physics, work is defined as the energy transferred to or from an object by means of a force acting on that object over a displacement. It's a scalar quantity, meaning it only has magnitude, not direction. Work is done only when a force causes a displacement, and the force (or a component of it) must be in the direction of the displacement.

Key Idea: Work links force and displacement.

Scalar Nature: Magnitude only.

Unit: Joule (J) in SI system.

Types: Positive, Negative, Zero.

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2. Key Formulas

Here are the essential formulas you need to master for NEET/JEE:

1. Work done by a Constant Force ($\vec{F}$) over a Displacement ($\vec{s}$):

   $W = \vec{F} \cdot \vec{s} = Fs \cos\theta$

   where:

   F is the magnitude of the force.

   s is the magnitude of the displacement.

   θ is the angle between the force vector ($\vec{F}$) and the displacement vector ($\vec{s}$).

2. Work done by a Variable Force in One Dimension (1D):

   When the force F is a function of position x, work done from x_1 to x_2 is:

   $W = \int_{x_1}^{x_2} F(x) dx$

3. Work done by a Variable Force in Three Dimensions (3D):

   When the force $\vec{F}$ is a function of position $\vec{r}$, work done along a path from $\vec{r_1}$ to $\vec{r_2}$ is:

   $W = \int_{\vec{r_1}}^{\vec{r_2}} \vec{F} \cdot d\vec{r}$

   If $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$ and $d\vec{r} = dx \hat{i} + dy \hat{j} + dz \hat{k}$, then

   $W = \int (F_x dx + F_y dy + F_z dz)$

4. Work-Energy Theorem:

   The net work done by all forces acting on an object equals the change in its kinetic energy.

   $W_{net} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

5. Work Done by a Spring Force:

For a spring with spring constant k:

Work done by the spring when stretched/compressed from x_i to x_f (where x is displacement from equilibrium):

$W_{spring} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

Work done by an external force to stretch/compress a spring from x_i to x_f:

$W_{external} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$

(Note: If the spring is initially at equilibrium $x_i=0$, then $W_{spring} = -\frac{1}{2}kx_f^2$ and $W_{external} = \frac{1}{2}kx_f^2$)

6. Work Done by Gravity:

When an object of mass m changes its vertical height by h:

Work done by gravity when moving downwards: $W_g = +mgh$

Work done by gravity when moving upwards: $W_g = -mgh$

(Note: The displacement s here is h, and θ is 0° for downwards, 180° for upwards, relative to the force of gravity)

7. Work Done by Kinetic Friction:

For a kinetic friction force f_k acting over a displacement s:

$W_{friction} = -f_k s$

(Note: Friction always opposes motion, so the angle between friction and displacement is 180°, hence the negative sign)

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3. Graphical Interpretation

The work done by a force can be elegantly represented and calculated using graphs, especially the Force-Displacement (F-s) graph.

Work as Area Under the F-s Curve:

For a constant force, the F-s graph is a horizontal line. The work done is simply the area of the rectangle formed by the force magnitude and the displacement: $W = F \times s$.

For a variable force, the F-s graph is a curve. The work done is equal to the area under the F-s curve from the initial position to the final position.

If the force is positive, and displacement is positive, the area is above the s-axis, contributing positive work.

If the force is negative (e.g., opposing motion), or if the displacement is in the negative direction, the area might be below the s-axis, contributing negative work.

Example: A force varying linearly with displacement (like a spring force $F=kx$) results in a triangular area. The area under the $F=kx$ curve from 0 to x is $\frac{1}{2} (x)(kx) = \frac{1}{2}kx^2$. This is the work done by the external agent to stretch the spring.

Important Note: The area is calculated with respect to the displacement axis (horizontal axis). Areas above the axis are positive, and areas below are negative.

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4. Conceptual Explanation

4.1. Definition and Conditions for Work

In physics, work has a very specific meaning. Work is done only if a force acts on an object and causes a displacement of the object in the direction of the force or a component of the force.

Conditions for Work to be Done:

1. A force must be applied to the object.
2. The object must undergo a displacement.
3. The force (or its component) must have a component along the direction of the displacement.

If any of these conditions are not met, no work is done in the physical sense.

4.2. Scalar Quantity

Work is a scalar quantity. It only has magnitude (e.g., 10 Joules) and no direction. This is because work is the dot product of two vectors (Force and Displacement), and the dot product of two vectors is always a scalar.

4.3. Units and Dimensions

SI Unit: Joule (J). One Joule is the work done when a force of one Newton displaces an object by one meter in the direction of the force. $1 \text{ J} = 1 \text{ N} \cdot \text{m}$.

CGS Unit: Erg. $1 \text{ erg} = 1 \text{ dyne} \cdot \text{cm}$.

Conversion: $1 \text{ J} = 10^7 \text{ erg}$.

Other Units: Electron-volt (eV), calorie (cal), kilowatt-hour (kWh).

Dimensions: From $W = Fs = (ma)s = (MLT^{-2})L = [ML^2T^{-2}]$. These are the same dimensions as energy, which is no coincidence as work is a form of energy transfer.

4.4. Types of Work

Work can be positive, negative, or zero, depending on the angle θ between the force vector ($\vec{F}$) and the displacement vector ($\vec{s}$).

1. Positive Work ($\theta < 90^\circ$):

Occurs when the force (or a component of it) acts in the same general direction as the displacement.

The object gains energy.

Examples:

Pushing a box across the floor (force and displacement are in the same direction, $\theta=0^\circ$).

Gravity doing work on a falling object ($\theta=0^\circ$).

A spring pulling a mass towards equilibrium (if the mass is moving towards equilibrium, $\theta=0^\circ$).

2. Negative Work ($\theta > 90^\circ$ and $\theta \le 180^\circ$):

Occurs when the force (or a component of it) acts in the opposite general direction to the displacement.

The object loses energy.

Examples:

Work done by friction on a sliding object ($\theta=180^\circ$).

Gravity doing work on an object being lifted ($\theta=180^\circ$).

Work done by air resistance on a moving object ($\theta=180^\circ$).

A spring compressing a mass (if the mass is moving into the compression, opposite to spring's pushing force).

3. Zero Work ($\theta = 90^\circ$ or $s=0$ or $F=0$):

Occurs when:

The force is perpendicular to the displacement ($\theta=90^\circ$, so $\cos 90^\circ = 0$).

There is no displacement ($s=0$).

There is no force ($F=0$).

Examples:

Work done by the centripetal force on an object moving in a circular path (force is towards the center, displacement is tangential).

Work done by the normal force on a block sliding on a horizontal surface (normal force is vertical, displacement is horizontal).

Work done by holding a heavy bag stationery (no displacement, $s=0$).

Work done by gravity on an object moving purely horizontally (gravity is vertical, displacement is horizontal).

4.5. Work Done by Various Forces

Constant Force: Straightforward application of $W = Fs \cos\theta$.

Variable Force: Requires integration ($W = \int F(x) dx$ or $W = \int \vec{F} \cdot d\vec{r}$).

Gravitational Force: Is a conservative force. Work done depends only on the initial and final positions, not the path taken. $W_g = - \Delta U_g = -(mgh_f - mgh_i)$.

Spring Force: Also a conservative force. Work done depends only on initial and final extensions/compressions. $W_{spring} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$.

Frictional Force: Is a non-conservative force. Work done depends on the path taken. Always negative when motion occurs. $W_f = -f_k s$.

Normal Force & Tension (in ideal strings): Often do zero work if perpendicular to displacement. However, tension can do work if it causes displacement along its direction (e.g., pulling a block).

4.6. Work-Energy Theorem (Revisited)

This theorem is a cornerstone of mechanics. It connects the net work done by all forces to the change in the kinetic energy of the object.
$W_{net} = \Delta K$
This means if positive net work is done, kinetic energy increases. If negative net work is done, kinetic energy decreases. If zero net work is done, kinetic energy remains constant. This theorem is incredibly powerful as it allows us to analyze motion without directly dealing with acceleration and time in many cases.

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5. NEET/JEE-level Examples

Let's apply these concepts to typical problems you might encounter.

Example 1: Work done by multiple constant forces

A block of mass 2 kg is pulled across a horizontal floor by a force of 10 N acting at an angle of 37° above the horizontal. The block moves a distance of 5 m. The coefficient of kinetic friction between the block and the floor is 0.2. Take $g = 10 \text{ m/s}^2$, $\cos 37^\circ = 0.8$, $\sin 37^\circ = 0.6$.
Calculate the work done by:

(a) The applied force

(b) The force of kinetic friction

(c) The normal force

(d) Gravity

(e) The net force

Solution:

First, let's identify all forces and their components.

Applied force $F = 10 \text{ N}$ at $\theta = 37^\circ$.

Mass $m = 2 \text{ kg}$. Displacement $s = 5 \text{ m}$.

Coefficient of kinetic friction $\mu_k = 0.2$.

Free Body Diagram Analysis:

Vertical forces:

Normal force N (upwards)

Weight $mg = 2 \times 10 = 20 \text{ N}$ (downwards)

Vertical component of applied force $F_y = F \sin 37^\circ = 10 \times 0.6 = 6 \text{ N}$ (upwards)

Since there's no vertical acceleration, $N + F_y - mg = 0 \Rightarrow N = mg - F_y = 20 - 6 = 14 \text{ N}$.

Horizontal forces:

Horizontal component of applied force $F_x = F \cos 37^\circ = 10 \times 0.8 = 8 \text{ N}$ (forward)

Kinetic friction $f_k = \mu_k N = 0.2 \times 14 = 2.8 \text{ N}$ (backward)

Now, calculate work done by each force:

(a) Work done by the applied force (W_F):

$W_F = Fs \cos\theta = 10 \text{ N} \times 5 \text{ m} \times \cos 37^\circ = 10 \times 5 \times 0.8 = 40 \text{ J}$

(b) Work done by the force of kinetic friction (W_f):

The friction force opposes the displacement ($\theta = 180^\circ$).

$W_f = f_k s \cos 180^\circ = 2.8 \text{ N} \times 5 \text{ m} \times (-1) = -14 \text{ J}$

(c) Work done by the normal force (W_N):

The normal force is perpendicular to the displacement ($\theta = 90^\circ$).

$W_N = Ns \cos 90^\circ = N \times 5 \times 0 = 0 \text{ J}$

(d) Work done by gravity (W_g):

The gravitational force is perpendicular to the horizontal displacement ($\theta = 90^\circ$).

$W_g = mg s \cos 90^\circ = 20 \times 5 \times 0 = 0 \text{ J}$

(e) Work done by the net force (W_{net}):

$W_{net} = W_F + W_f + W_N + W_g = 40 + (-14) + 0 + 0 = 26 \text{ J}$

Alternatively, calculate net force and then work:

Net horizontal force $F_{net,x} = F_x - f_k = 8 - 2.8 = 5.2 \text{ N}$.

$W_{net} = F_{net,x} \times s = 5.2 \times 5 = 26 \text{ J}$.

This is consistent with the Work-Energy Theorem. If the block started from rest, its final kinetic energy would be 26 J.

Example 2: Work done by a variable force

A particle moves along the x-axis under the influence of a force $F(x) = (3x^2 - 2x + 1) \text{ N}$.

Calculate the work done by this force when the particle moves from $x=0$ to $x=2 \text{ m}$.

Solution:

Since the force is variable and depends on position, we use integration:

$W = \int_{x_1}^{x_2} F(x) dx$

Given $F(x) = 3x^2 - 2x + 1$, $x_1 = 0$, $x_2 = 2 \text{ m}$.

$W = \int_{0}^{2} (3x^2 - 2x + 1) dx$

$W = \left[\frac{3x^3}{3} - \frac{2x^2}{2} + x \right]_{0}^{2}$

$W = \left[x^3 - x^2 + x \right]_{0}^{2}$

Now, substitute the limits:

$W = ((2)^3 - (2)^2 + 2) - ((0)^3 - (0)^2 + 0)$

$W = (8 - 4 + 2) - (0)$

$W = 6 \text{ J}$

Example 3: Work done by a spring and Work-Energy Theorem

A 0.5 kg block is attached to a horizontal spring with spring constant $k = 50 \text{ N/m}$. The block is pulled 0.1 m from its equilibrium position ($x=0$) and released from rest. Calculate:

(a) The work done by the spring force when the block moves from $x=0.1 \text{ m}$ to $x=0.05 \text{ m}$.

(b) The speed of the block when it passes through $x=0.05 \text{ m}$.

Solution:

Given: $m = 0.5 \text{ kg}$, $k = 50 \text{ N/m}$.

Initial position $x_i = 0.1 \text{ m}$. Initial speed $v_i = 0$.

Final position $x_f = 0.05 \text{ m}$.

(a) Work done by the spring force (W_{spring}):

$W_{spring} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$

$W_{spring} = \frac{1}{2}(50)(0.1)^2 - \frac{1}{2}(50)(0.05)^2$

$W_{spring} = \frac{1}{2}(50)(0.01) - \frac{1}{2}(50)(0.0025)$

$W_{spring} = 25 \times 0.01 - 25 \times 0.0025$

$W_{spring} = 0.25 - 0.0625$

$W_{spring} = 0.1875 \text{ J}$

(This is positive work because the spring force is pulling the block towards equilibrium, and the block is moving in that direction.)

(b) Speed of the block at $x=0.05 \text{ m}$ (v_f):

We use the Work-Energy Theorem: $W_{net} = \Delta K$.

In this case, the only force doing work is the spring force (assuming horizontal surface and no friction).

So, $W_{spring} = K_f - K_i$

$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.5)(0)^2 = 0 \text{ J}$ (since released from rest)

$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(0.5)v_f^2 = 0.25 v_f^2$

$0.1875 = 0.25 v_f^2 - 0$

$v_f^2 = \frac{0.1875}{0.25} = 0.75$

$v_f = \sqrt{0.75} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \text{ m/s} \approx 0.866 \text{ m/s}$

Example 4: Graphical Interpretation

A particle is subjected to a force F that varies with its position x as shown in the graph below.

!\[F-x graph description](https://i.stack.imgur.com/K1n0t.png)

(Self-correction: I cannot actually display an image here. I will describe the graph and its points for calculation.)

Graph Description:

The force F (in N) is plotted against position x (in m).

From $x=0$ to $x=2$: F is constant at $+3 \text{ N}$.

From $x=2$ to $x=4$: F is constant at 0 N.

From $x=4$ to $x=6$: F is constant at -3 N.

Calculate the net work done by the force as the particle moves from $x=0$ to $x=6 \text{ m}$.

Solution:

The work done is the area under the F-x graph. We can divide the graph into three segments.

1. Work done from $x=0$ to $x=2 \text{ m}$ (W_1):

   This is a rectangle with height $F = 3 \text{ N}$ and width $\Delta x = 2 - 0 = 2 \text{ m}$.

   $W_1 = \text{Area}_1 = F \times \Delta x = 3 \text{ N} \times 2 \text{ m} = 6 \text{ J}$

2. Work done from $x=2$ to $x=4 \text{ m}$ (W_2):

   Here, the force $F = 0 \text{ N}$.

   $W_2 = \text{Area}_2 = 0 \times (4-2) = 0 \text{ J}$

3. Work done from $x=4$ to $x=6 \text{ m}$ (W_3):

   This is a rectangle with height $F = -3 \text{ N}$ and width $\Delta x = 6 - 4 = 2 \text{ m}$.

   $W_3 = \text{Area}_3 = F \times \Delta x = (-3) \text{ N} \times 2 \text{ m} = -6 \text{ J}$

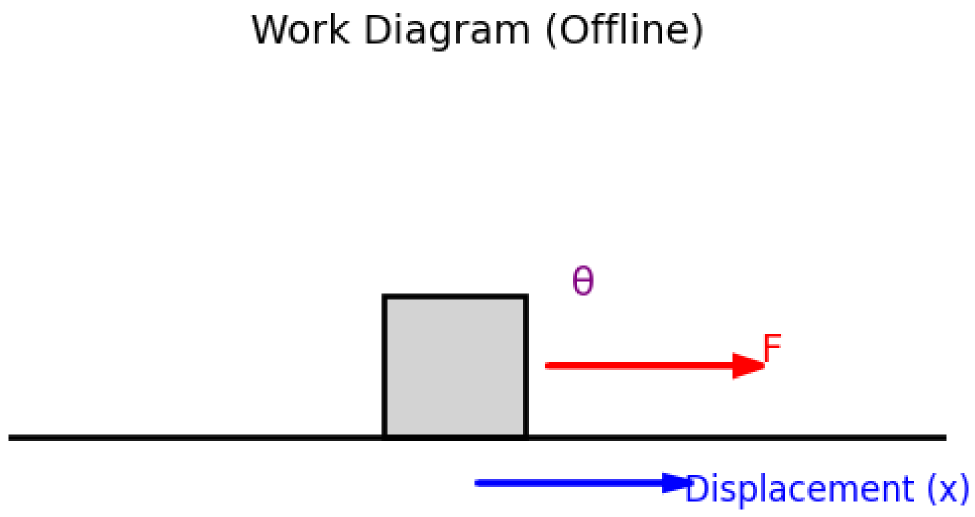
Net work done (W_{net}):

$W_{net} = W_1 + W_2 + W_3 = 6 \text{ J} + 0 \text{ J} + (-6) \text{ J} = 0 \text{ J}$

Even though the force acted, the net work done over the entire displacement from $x=0$ to $x=6 \text{ m}$ is zero. According to the Work-Energy Theorem, this implies that the particle's kinetic energy at $x=6 \text{ m}$ is the same as its kinetic energy at $x=0 \text{ m}$.

Keep practicing these concepts and problems. Work is a foundational topic, and a strong understanding here will greatly benefit you in related topics like energy conservation, power, and rotational dynamics. All the best!

Concept Diagram



Graphical Interpretation