Introduction to Angle

Part - 01

Trigonometry

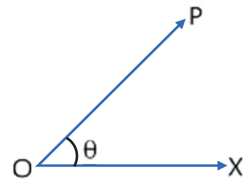
Introduction to Angle

Consider a revolving line OP.

Suppose that it revolves in anticlockwise direction starting from its initial position OX .

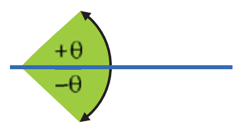
The angle is defined as the amount of revolution that the revolving line makes with its initial position.

From fig. the angle covered by the revolving line OP is $\theta = \angle POX$



The angle is taken positive if it is traced by the revolving line in anticlockwise direction.

The angle is taken negative if it is covered in clockwise direction.



$1° = 60'$ (minute)

$1' = 60''$ (second)

1 right angle = 90° (degrees) also 1 right angle = $\frac{\pi}{2}$ rad (radian)

One radian is the angle subtended at the centre of a circle by an arc of the circle, whose length is equal to the radius of the circle. 1 rad = $\frac{180°}{\pi} \approx 57.3°$

Units of Angle

Practical units : degrees (°)

$1° = 60'$(minute)

$1' = 60''$(second)

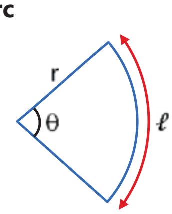
To convert an angle from degree to radian multiply it by $\frac{\pi}{180°}$

To convert an angle from radian to degree multiply it by $\frac{180°}{\pi}$

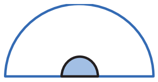
Relation between Angle and Arc

$$\text{Angle } (\theta) = \frac{\text{Arc}}{\text{Radius}} = \frac{\ell}{r}$$

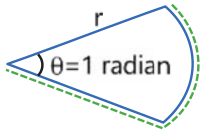
SI UNIT → Radian



Radian



π rad $= 180°$



$1\, rad = \frac{180°}{\pi} \approx 57.3°$

$2\pi = 360°$	$\pi = 180°$	$\frac{\pi}{2} = 90°$
$\frac{\pi}{3} = 60°$	$\frac{\pi}{4} = 45°$	$\frac{\pi}{6} = 30°$
$\frac{2\pi}{3} = 120°$	$\frac{3\pi}{4} = 135°$	$\frac{5\pi}{6} = 150°$

Illustration 1.

Convert the given angles in desired units.

(i) 5° to minutes

(ii) 6' to seconds

(iii) 120" to minutes

Solution.

(i) 1° = 60'

5° × 60' = 300'

(ii) 1' = 60"

6' × 60" = 360"

(iii) 60" = 1'

$\frac{120''}{60''} = 2'$

Illustration 2.

Convert the given angles in desired units.

1. Convert 45° to radians

2. Convert $\frac{5\pi}{6}$ rad to degree

Solution.

1. $45° \times \frac{\pi}{180°} = \frac{\pi}{4}$ radians

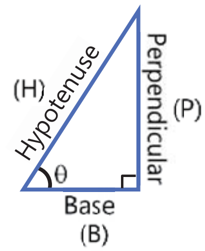
2. $\frac{5\pi}{6} \times \frac{180°}{\pi} = 150°$

Pythagoras Theorem and Trigonometric Ratio

Part - 02

Pythagoras Theorem

$P^2 + B^2 = H^2$



Pythagorean Triplets

3, 4, 5 ($3^2 + 4^2 = 5^2$)

6, 8, 10 ($6^2 + 8^2 = 10^2$)

7, 24, 25 ($7^2 + 24^2 = 25^2$)

12, 16, 20 ($12^2 + 16^2 = 20^2$)

Remember for fast calculations in Physics!!

Trigonometric Ratios (or T ratios)

$\sin\theta = \frac{P}{H}$ $\quad$ $\operatorname{cosec}\theta = \frac{H}{P}$

$\cos\theta = \frac{B}{H}$ $\quad$ $\sec\theta = \frac{H}{B}$

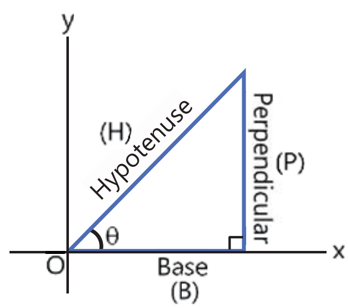
$\tan\theta = \frac{P}{B}$ $\quad$ $\cot\theta = \frac{B}{P}$

It can be easily proved that :

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$



Trigonometric Identities

$\sin^2\theta + \cos^2\theta = 1$

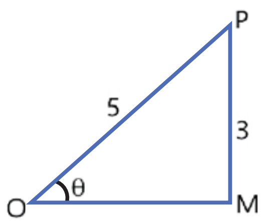
$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \mathrm{cosec}^2\theta$

Illustration 1.

Given $\sin\theta = 3/5$. Find all the other T-ratios, if θ lies in the first quadrant.

Solution.



In ΔOMP, $\sin\theta = \frac{3}{5}$

so, MP = 3 and OP = 5

$\because$ $OM = \sqrt{(5)^2 - (3)^2} = \sqrt{25-9} = \sqrt{16} = 4$

Now, $\cos\theta = \frac{OM}{OP} = \frac{4}{5}$ $\quad \tan\theta = \frac{MP}{OM} = \frac{3}{4}$

$\cot\theta = \frac{OM}{MP} = \frac{4}{3}$ $\quad \sec\theta = \frac{OP}{OM} = \frac{5}{4}$ $\quad \mathrm{cosec}\theta = \frac{OP}{MP} = \frac{5}{3}$

Table : The T-ratios of a few standard angles ranging from 0° to 90°

Angle(θ)	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ (not defined)

Quadrant Theory and Trigonometric Formulae

Part - 03

Quadrants & ASTC Rule

In first quadrant, all trigonometric ratios are positive.
In second quadrant, only sinθ and cosecθ are positive.
In third quadrant, only tanθ and cotθ are positive.
In fourth quadrant, only cosθ and secθ are positive

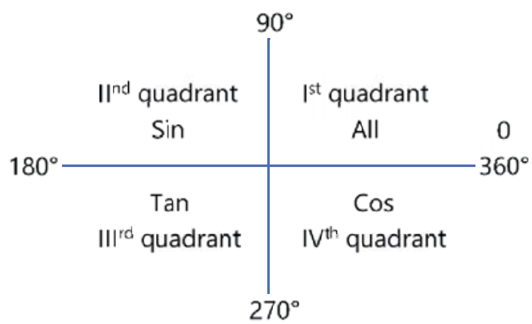
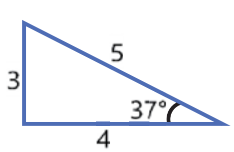
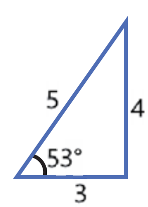


Table to Remember

Angle(θ)	0°	30°	45°	60°	90°
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanθ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ (not defined)

T-Ratios of Special Angles





$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

T-ratios of angles greater than 90°

STEP-1	Decide sign according to quadrant.	
STEP-2	$\alpha = A \pm \theta$	A : Integral multiple of 90° θ : Acute angle
STEP-3	If A is odd multiple of 90°	$\sin\theta \rightleftharpoons \cos\theta$ $\text{cosec}\theta \rightleftharpoons \sec\theta$ $\tan\theta \rightleftharpoons \cot\theta$
	If A is Even multiple of 90°	No Change

Trigonometrical Ratios of General Angles (Reduction Formulae)

(i) Trigonometric function of an angle $(2n\pi + \theta)$ where n=0, 1, 2, 3,.... will be remain same.

$\sin(2n\pi + \theta) = \sin\theta$ $\quad$ $\cos(2n\pi+\theta) = \cos\theta$ $\quad$ $\tan(2n\pi+\theta)=\tan\theta$

(ii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will remain same if n is even and sign of trigonometric function will be according to value of that function in quadrant.

$\sin(\pi-\theta) = +\sin\theta$ $\quad$ $\cos(\pi-\theta) = -\cos\theta$ $\quad$ $\tan(\pi-\theta)= -\tan\theta$

$\sin(\pi+\theta)= -\sin\theta$ $\quad$ $\cos(\pi+\theta)= -\cos\theta$ $\quad$ $\tan(\pi+\theta)= +\tan\theta$

$\sin(2\pi-\theta)= -\sin\theta$ $\quad$ $\cos(2\pi-\theta)= +\cos\theta$ $\quad$ $\tan(2\pi-\theta)= -\tan\theta$

(iii) Trigonometric function of an angle $\left(\frac{n\pi}{2}+\theta\right)$ will be changed into co-function if n is odd and sign of trigonometric function will be according to value of that function in quadrant.

$\sin\left(\frac{\pi}{2}+\theta\right)=+\cos\theta$ $\quad$ $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$ $\quad$ $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$

$\sin\left(\frac{\pi}{2}-\theta\right)=+\cos\theta$ $\quad$ $\cos\left(\frac{\pi}{2}-\theta\right)=+\sin\theta$ $\quad$ $\tan\left(\frac{\pi}{2}-\theta\right)=+\cot\theta$

(iv) Trigonometric function of an angle $-\theta$ (negative angles)

$\sin(-\theta) = -\sin\theta$ $\quad$ $\cos(-\theta) = +\cos\theta$ $\quad$ $\tan(-\theta) = -\tan\theta$

Sum property for sine function

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

Sum property for cosine function

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

Sum property for tan function

$\tan(A + B) = \dfrac{\tan A+\tan B}{1-\tan A\tan B}$

$\tan(A - B) = \dfrac{\tan A-\tan B}{1+\tan A\tan B}$

Double angle property

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$ $\quad$ $\cos 2A = 2\cos^2 A - 1$; $\cos 2A = 1 - 2\sin^2 A$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Illustration 1.

Write the function in terms of acute angle θ

(1) $\cos(270° - \theta)$ = ?

(2) $\cos(90° + \theta)$ = ?

(3) $\cos\left(\frac{\pi}{2}-\theta\right)$ = ?

(4) $\sin\left(\frac{\pi}{2}-\theta\right)$ = ?

Solution.

(1) $\cos(270° - \theta) = \cos(\frac{3\pi}{2}-\theta) = -\sin\theta$

(2) $\cos(90° + \theta) = \cos(\frac{\pi}{2}+\theta) = -\sin\theta$

(3) $\cos(\frac{\pi}{2}-\theta) = +\sin\theta$

(4) $\sin(\frac{\pi}{2}-\theta) = +\cos\theta$

Illustration 2.

Evaluate :

(a) sin120°   (b) tan150°   (c) cos330°

Solution.

(a) $\sin 120° = \sin(90° + 30°) = \sin(\frac{\pi}{2}+30°) = +\cos 30° = \frac{\sqrt{3}}{2}$

(b) $\tan 150° = \tan(90° + 60°) = \tan(\frac{\pi}{2}+60°) = -\cot 60° = -\frac{1}{\sqrt{3}}$

(c) $\cos 330° = \cos(360° - 30°) = \cos(2\pi - 30°) = +\cos 30° = \frac{\sqrt{3}}{2}$

Illustration 3.

Evaluate :

(a) cos(–30°)   (b) sin(–45°)

Solution.

(a) $\cos(-30°) = \cos(30°) = \frac{\sqrt{3}}{2}$

(b) $\sin(-45°) = -\sin(45°) = -\frac{1}{\sqrt{2}}$

Illustration 4.

Evaluate :

(a) sin105°   (b) cos75°

Solution.

(a) $\sin(105°) = \sin(60° + 45°) = \sin60°\cos45° + \cos60°\sin45° = \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}+\frac{1}{2}\times\frac{1}{\sqrt{2}}$

(b) $\cos(75°) = \cos(45° + 30°) = \cos45°\cos30° - \sin45°\sin30° = \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\times\frac{1}{2}$

Illustration 5.

Evaluate $\cos74°$

Solution.

(a) $\cos74° = \cos(2 \times 37°)$

$= \cos^2(37°) - \sin^2(37°)$

$= \frac{16}{25}-\frac{9}{25}=\frac{7}{25}$

Range of Trigonometric Functions

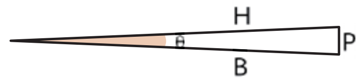
Part - 04

Small angle approximation

$\sin\theta \approx \theta$

$\tan\theta \approx \theta$

$\cos\theta \approx 1$



Here θ must be in radians

Illustration 1.

Find :

1. sin2°　　　2. tan1°　　　3. sinπ°

Solution.

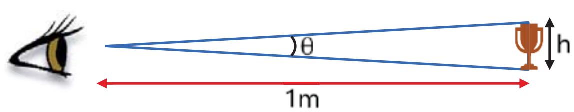
1. $\sin 2^\circ = 2^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{90}$

2. $\tan 1^\circ = 1^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{180}$

3. $\sin \pi^\circ = \pi \times \frac{\pi}{180}$

Illustration 2.

A normal human eye can see an object making an angle 1.8° at the eye. What is the minimum height of object which can be seen by an eye from 1 m distance.



Solution.

θ is very small

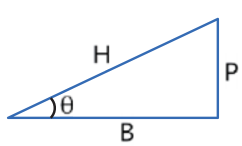
$\therefore \tan\theta \approx \theta$

$\theta = \frac{1.8^\circ \times \pi}{180^\circ} = \frac{\pi}{100}\text{rad}$

$\frac{h}{1} = \frac{\pi}{100}$

$\therefore h = 0.031$ m

Range of Trigonometric Functions



$\sin\theta = \frac{P}{H}$ $\longrightarrow$ $-1 \le \sin\theta \le 1$

$\cos\theta = \frac{B}{H}$ $\longrightarrow$ $-1 \le \cos\theta \le 1$

$\tan\theta = \frac{P}{B}$ $\longrightarrow$ $-\infty < \tan\theta < -\infty$

Illustration 3.

Find the maximum value of y = (sinx) (cosx)

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $\sqrt{2}$

Solution.

$y = \sin x\cos x = \frac{1}{2}(\sin 2x) = \frac{1}{2}$ $\{\sin 2A = 2\sin x\cos x\}$

Important result

Range of function : "asinθ + bcosθ"

$-\sqrt{a^2+b^2} \le a\sin\theta + b\cos\theta \le \sqrt{a^2+b^2}$

Illustration 4.

If y = –3sinθ + 4 cosθ then find y_{max} and y_{min}

Solution.

$-\sqrt{(-3)^2+(4)^2} \le y \le \sqrt{(-3)^2+(4)^2}$

$y_{max} = 5$

$y_{min} = -5$

Co-ordinate Geometry

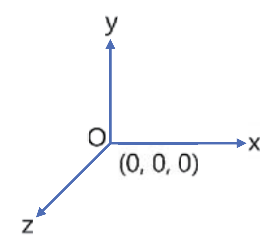
Part - 05

Co-ordinate Geometry

To specify the position of a point in space, we use right handed rectangular axes coordinate system. This system consists of (i) origin (ii) axis or axes. If a point is known to be on a given line or in a particular direction, only one coordinate is necessary to specify its position, if it is in a plane, two coordinates are required, if it is in space three coordinates are needed.

- **Origin**
  This is any fixed point which is convenient to you. All measurements are taken w.r.t. this fixed point.

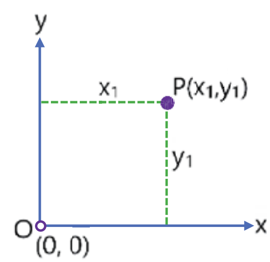
- **Axis or Axes**
  Any fixed direction passing through origin and convenient to you can be taken as an axis. If the position of a point or position of all the points under consideration always happen to be in a particular direction, then only one axis is required. This is generally called the x-axis. If the positions of all the points under consideration are always in a plane, two perpendicular axes are required. These are generally called x and y-axis. If the points are distributed in a space, three perpendicular axes are taken which are called x, y and z-axis.



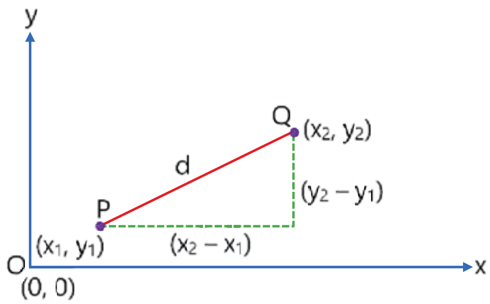
Position of a point

x_1 = Abscissa : Distance of point from y axis

y_1 = Ordinate : Distance of point from x axis

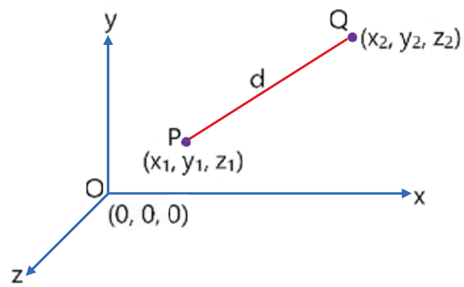


Distance Formula in plane



In a plane : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance Formula in space



In space : $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Illustration 1.

Find distance between two points A (1, 2, 5) and B (3, 4, 6).

Solution.

$A(x_1\ y_1\ z_1) = A(1, 2, 5)$

$B(x_2\ y_2\ z_2) = B\ (3, 4, 6)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(3-1)^2 + (4-2)^2 + (6-5)^2} = \sqrt{4+4+1} = 3$$

Illustration 2.

Find possible values of a if distance between the points (–9 cm, a cm) and (3 cm, 3 cm) is 13 cm.

Solution.

$$13 = \sqrt{(3+9)^2 + (3-a)^2}$$

$\Rightarrow\ 169 = 144 + (3 - a)^2$

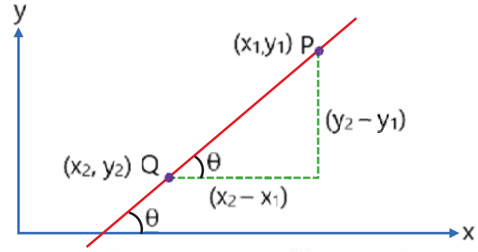
$\Rightarrow\ \pm 5 = (3 - a)$

$\Rightarrow\ +5 = 3 - a$ or $-5 = 3 - a$

$a = -2$cm or $a = 8$cm

Slope of Line joining Two points

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$



Angle θ is measured from x axis.

Illustration 4.

Find slope of a line passing through points A(2, 4) and B(3, 8)

Solution.

$A(x_1, y_1) = A(2, 4)$

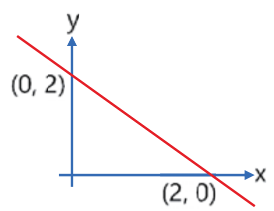
$B(x_2, y_2) = B(3, 8)$

$$\tan\theta = \frac{8-4}{3-2} = \frac{4}{1}$$

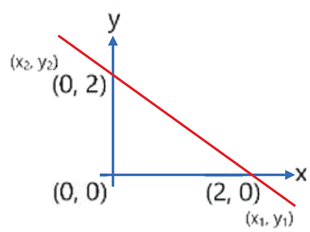
so, slope = 4

Illustration 5.

Calculate slope of the shown line and its angle with x axis.



Solution.



$$\tan\theta = \frac{2-0}{0-2}$$

$\tan\theta = -1$

So, slope = −1

angle with x-axis 135°.

Equation of Straight Line

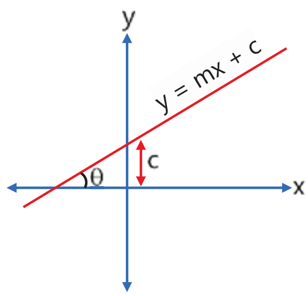
Part - 06

Equation of Straight Line

Genera equation of straight line

$y = mx + c$

Slope or gradient $= \tan\theta$ — Intercept on y-axis



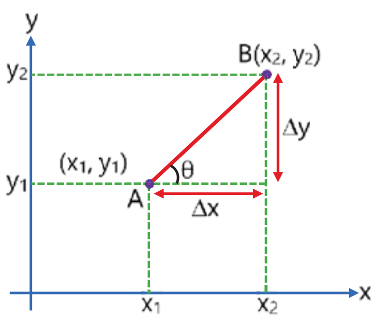
Slope of A Line

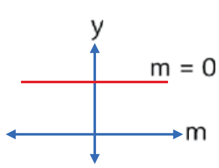
The slope of a line joining two points A(x_1, y_1) and B(x_2, y_2) is denoted by m and is given by

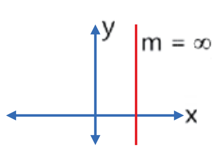
$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$ [If both axes have identical scales]

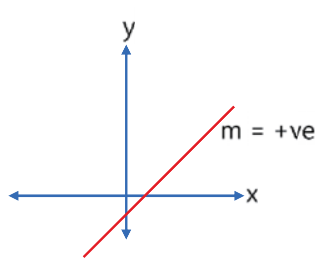
Here θ is the angle made by line with positive x-axis.

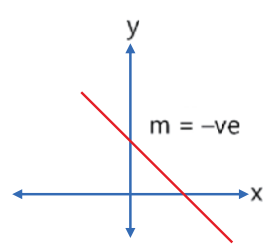
Slope of a line is a quantitative measure of inclination.



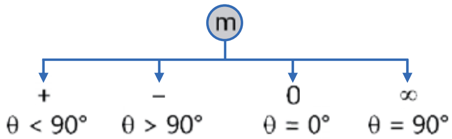








Slope of a Straight line



Intercept of A Line

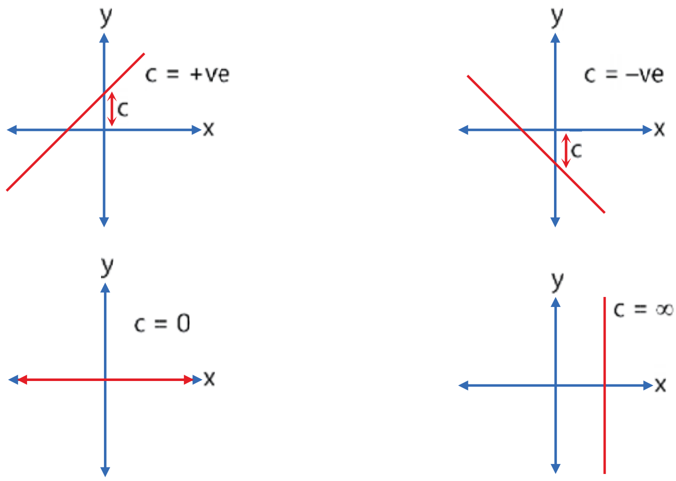
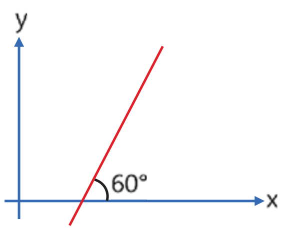


Illustration 1.

Find slope of the line.



Solution.

slope = $\tan\theta$

= $\tan 60°$

= $\sqrt{3}$

Illustration 2.

Find slope and intercept of a line $y = 3x + 2$, also draw the line.

Solution.

$y = mx + c$ (general equation) ...(i)

$y = 3x + 2$ (given equation) ...(ii)

comparing equation (1) & (2)

$m = \text{slope} = 3;\ c = 2$

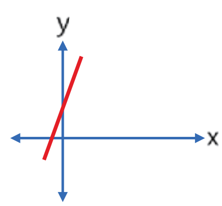


Illustration 3.

Find slope and intercept of a line 4y + 3x = 8, also draw the line.

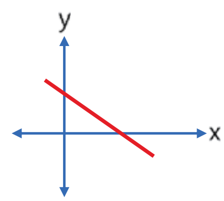
Solution.

$y = mx + c$ (general equation) ...(i)

$4y + 3x = 8$ (given equation)

$4y = -3x + 8$

$y = -\frac{3}{4}x + 2$ ...(ii)

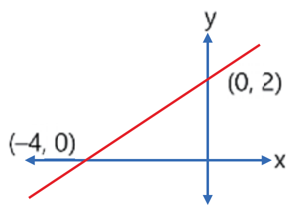


comparing equation (i) & (ii)

$m = \text{slope} = -\frac{3}{4}$; $c = 2$

Illustration 4.

Write equation of the line drawn



Solution.

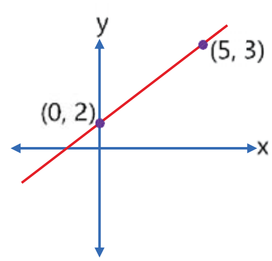
$\text{slope} = \tan\theta = \frac{2-0}{0+4} = \frac{1}{2}$

$c = 2$

$\therefore$ equation of straight line $y = \frac{1}{2}x + 2$

Illustration 5.

Write equation of the line drawn



Solution.

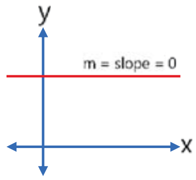
$\text{slope} = \tan\theta = \frac{3-2}{5-0} = \frac{1}{5}$

$c = 2$

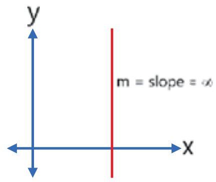
$\therefore$ equation of straight line $y = \frac{1}{5}x + 2$

Special Cases

1. Straight line parallel to x-axis



2. Straight line parallel to y-axis



Introduction to Differentiation and Concept of Slope

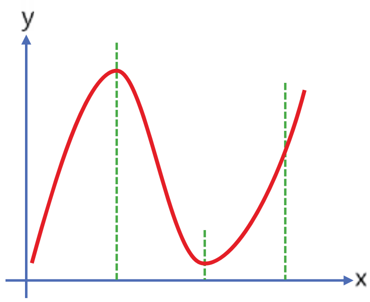
Part - 07

Function

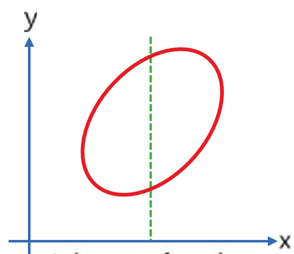
Constant : A quantity, whose value remains unchanged during mathematical operations, is called a constant quantity. The integers, fractions like π ,e etc are all constants.

Variable : A quantity, which can take different values, is called a variable quantity. A variable is usually represented as x, y, z, etc.

Function : A quantity y is called a function of a variable x, if corresponding to any given value of x, there exists a single definite value of y. The phrase 'y is function of x' is represented as y = f (x)



(A) It is a function



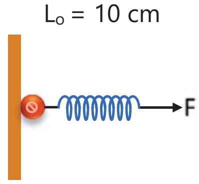
(B) It is not a function

Finite change (Δx)

If change in a quantity is comparable to its initial value, change is said to be finite.

For example, if length of a spring is 10 cm, then a change of 2 cm in length is considered as finite change in length.

L_o = 10 cm



L_o = 10 cm

ΔL = 2 cm

Infinitesimal change

If change in a quantity is not comparable to its initial value, change is said to be infinitesimal.

- A very small change in y is called as dy

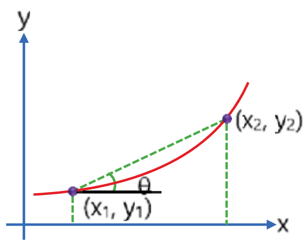
  for example y_2 = 100 and y_1 = 99.999999999.............

  $y_2 - y_1$ = dy = 0.0000000000.............001
- A very small change in x is called as dx
- A very small change in z is called as dz

- A very small change in t is called as dt

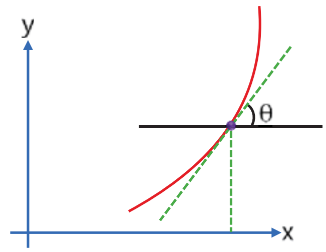
Slope of A Curve

Average slope of curve
(Between two different points)



$$m_{avg} = \frac{\Delta y}{\Delta x} = \tan\theta$$

Instantaneous slope of curve
(At a Single point)



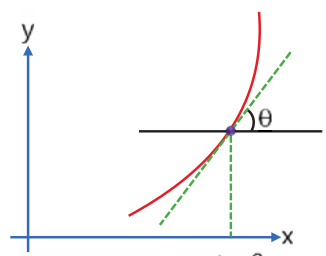
$$m_{instantaneous} = \tan\theta$$

Instantaneous slope of curve

(At a Single point)

To find the slope we require the tool called

DIFFERENTIATION



$$m_{instantaneous} = \tan\theta$$

Definition of Differentiation/Derivative

At a point :

$\frac{dy}{dx}$ = "instantaneous rate of change of y w.r.t. x"

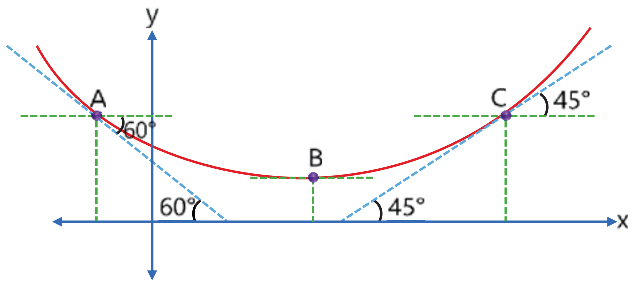
If y is a function of x : y = f(x)

Then derivative of y "w.r.t "x is given by :

$$y' = f'(x) = \frac{dy}{dx}$$

Illustration 1.

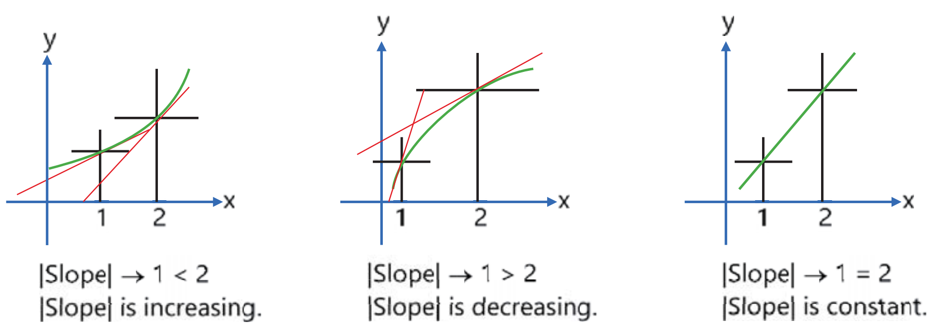
Find slope at A, B & C



Solution.

at A	at B	at C
slope = $\tan\theta$	slope = $\tan\theta$	slope = $\tan\theta$
$= \tan(120^0)$	$= \tan(0°)$	$= \tan(45°)$
$= \tan(90° + 30°)$	$= 0$	$= 1$
$= -\cot(30°)$		
$= -\sqrt{3}$		

Increasing functions



Decreasing functions

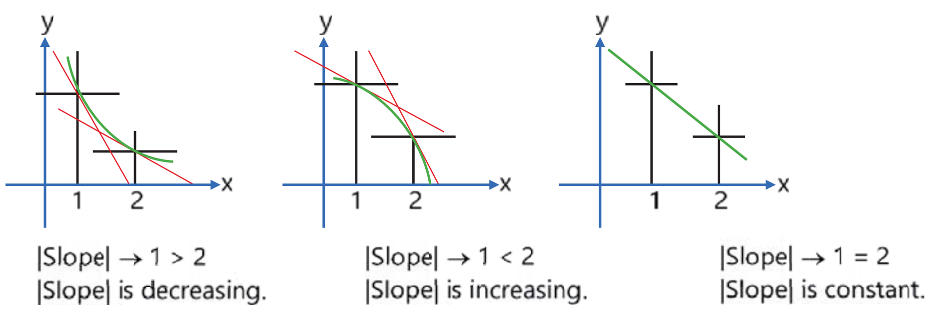
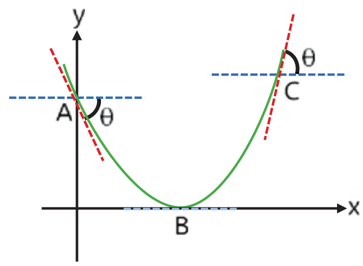


Illustration 2.

Comment on slope and its magnitude from A to B & B to C



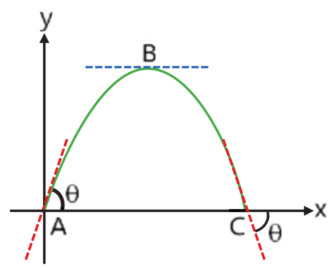
Solution.

A $\rightarrow$ B slope increasing — A $\rightarrow$ B magnitude decreasing

B $\rightarrow$ C slope increasing — B $\rightarrow$ C magnitude increasing

Illustration 3.

Comment on slope and its magnitude from A to B & B to C



Solution.

A $\rightarrow$ B slope decreasing — A $\rightarrow$ B magnitude decreasing

B $\rightarrow$ C slope decreasing — B $\rightarrow$ C magnitude increasing

Differentiation of Standard Functions

Part - 08

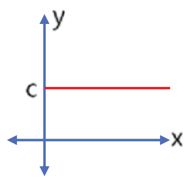
Physical meaning of $\frac{dy}{dx}$

- The ratio of small change in the function y and the variable x is called the average rate of change of y w.r.t. x. For example, the velocity of a body changes by a small amount Δv in small time Δt, then average acceleration of the body, $a_{av} = \frac{\Delta v}{\Delta t}$
- When $\Delta x \to 0$ The limiting value of $\frac{\Delta y}{\Delta x}$ is $\lim\limits_{\Delta t \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

Derivative of Constant Functions

y = f (x) = constant, then it's derivative is ZERO.

$$\frac{dy}{dx} = \frac{dc}{dx} = 0$$



Derivative of Algebraic Functions

y = f (x) = x^n, then it's derivative is :

$$\frac{dy}{dx} = \frac{dx^n}{dx} = nx^{(n-1)}$$

Illustration 1.

Find the derivative of given functions w.r.t x

(1) x^4 (2) x^{-2}

Solution.

(1) $\frac{d}{dx}(x^4) = 4x^{4-1} = 4x^3$

(2) $\frac{d}{dx}(x^{-2}) = -2x^{(-2-1)} = -2x^{-3}$

Illustration 2.

Find value of $\frac{dy}{dx}$

(1) y = x^6 (2) $y = (x)^{\frac{3}{4}}$ (3) $y = \frac{1}{x}$

Solution.

(1) $\frac{d}{dx}(x^6) = 6x^{6-1} = 6x^5$

(2) $\frac{d}{dx}(x^{\frac{3}{4}}) = \frac{3}{4}x^{\frac{3}{4}-1} = \frac{3}{4}x^{-\frac{1}{4}}$

(3) $\frac{d}{dx}(x^{-1}) = -1x^{-1-1} = -\frac{1}{x^2}$

Formulae for Differentiation of Trigonometric Functions

y = sinx $\longrightarrow$ $\frac{d(\sin x)}{dx} = \cos x$

y = cosx $\longrightarrow$ $\frac{d(\cos x)}{dx} = -\sin x$

y = tanx $\longrightarrow$ $\frac{d(\tan x)}{dx} = \sec^2 x$

y = cosecx $\longrightarrow$ $\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x \cot x$

y = secx $\longrightarrow$ $\frac{d(\sec x)}{dx} = \sec x \tan x$

y = cotx $\longrightarrow$ $\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

Formulae for Differentiation of Exponential Functions

y = e^x $\longrightarrow$ $\frac{d(e^x)}{dx} = e^x$

Formulae for Differentiation of Logarithmic Functions

y = $\log_e x$ or lnx $\longrightarrow$ $\frac{d(\ln x)}{dx} = \frac{1}{x}$

Rules of Differentiation - Basic

Part - 09

Constant Multiple Rule

If y = cf(x) = cU, then :

$$\frac{dy}{dx}=\frac{d(cU)}{dx}=c\frac{dU}{dx}$$

Illustration 1.

Find the derivative of given functions w.r.t x

(1) $7x^6$ (2) $3x^{-2}$

Solution.

(1) $\frac{d}{dx}(7x^6)=7\frac{d}{dx}(x^6)=7\times 6x^{6-1}=42x^5$

(2) $\frac{d}{dx}(3x^{-2})=3\frac{d}{dx}(x^{-2})=3\times(-2x^{-2-1})=-6x^{-3}$

Illustration 2.

Find the derivative of given functions w.r.t x

(1) 3(sinx) (2) 4π(tanx)

Solution.

(1) $\frac{d}{dx}\big(3(\sin x)\big)=3\frac{d}{dx}(\sin x)=3\cos x$

(2) $\frac{d}{dx}\big(4\pi(\tan x)\big)=4\pi\frac{d}{dx}(\tan x)=4\pi\sec^2 x$

Illustration 3.

Find the derivative of given functions w.r.t x

(1) 20(lnx) (2) $0.6(e^x)$

Solution.

(1) $\frac{d}{dx}\big(20(\ell n x)\big)=20\frac{d}{dx}(\ell nx)=\frac{20}{x}$

(2) $\frac{d}{dx}\big(0.6(e^x)\big)=0.6\frac{d}{dx}(e^x)=0.6e^x$

Addition/Subtraction Rule

If U and V are functions of x and y is sum of functions U and V :

y = U ± V

$$\frac{d(U\pm V)}{dx}=\frac{dU}{dx}\pm\frac{dV}{dx}$$

Illustration 4.

Find value of $\frac{dy}{dx}$

(1) $y = x^2 + 2x$ (2) $y = x^3 - x^{-2} + 1$

Solution.

(1) $\frac{d}{dx}(x^2+2x)=\frac{d}{dx}(x^2)+\frac{d}{dx}(2x)=2x+2$

(2) $\frac{d}{dx}(x^3-x^{-2}+1)=\frac{d}{dx}(x^3)-\frac{d}{dx}(x^{-2})+\frac{d}{dx}(1)=3x^2+2x^{-3}+0$

Illustration 5.

Find value of $\frac{dx}{dt}$ if : $x = 4t^2 + 3t + 2$

Solution.

$\frac{d}{dt}(4t^2+3t+2)=\frac{d}{dx}(4t^2)+\frac{d}{dx}(3t)+\frac{d}{dx}(2)=8t+3$

Illustration 6.

Find value of $\frac{dy}{dx}$ if : $y=\sqrt{x}+\frac{1}{\sqrt{x}}+1$

Solution.

$y=x^{\frac{1}{2}}+x^{-\frac{1}{2}}+1$

$\frac{d}{dx}(x^{\frac{1}{2}}+x^{-\frac{1}{2}}+1)=\frac{d}{dx}(x^{\frac{1}{2}})+\frac{d}{dx}(x^{-\frac{1}{2}})+\frac{d}{dx}(1)$

$=\frac{1}{2}x^{\frac{1}{2}-1}-\frac{1}{2}x^{-\frac{1}{2}-1}+0$

$=\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}$

Illustration 7.

Find value of $\frac{dy}{dx}$

(1) $y = 3\sin x + \cos x$ (2) $y = 2\tan x - x^3$

Solution.

$\frac{d}{dx}(3\sin x+\cos x)=\frac{d}{dx}(3\sin x)+\frac{d}{dx}(\cos x)=3\cos x-\sin x$

$\frac{d}{dx}(2\tan x-x^3)=\frac{d}{dx}(2\tan x)-\frac{d}{dx}x^3=2\sec^2 x-3x^2$

Illustration 8.

Find $\frac{dy}{dx}$ for the following

(i) $y=x^{\frac{7}{2}}$ (ii) $y = x^{-3}$ (iii) $y = x$ (iv) $y = x^5 + x^3 + 4x^{1/2} + 7$

(v) $y = 5x^4 + 6x^{3/2} + 9x$ (vi) $y = ax^2 + bx + c$ (vii) $y = 3x^5 - 3x - \frac{1}{x}$

Solution.

(1) $\frac{d}{dx}(x^{\frac{7}{2}})=\frac{7}{2}x^{\frac{7}{2}-1}=\frac{7}{2}x^{\frac{5}{2}}$

(2) $\frac{d}{dx}(x^{-3})=-3x^{-3-1}=-3x^{-4}$

(3) $\frac{d}{dx}(x)=1$

(4) $\frac{d}{dx}(x^5+x^3+4x^{\frac{1}{2}}+7)=\frac{d}{dx}(x^5)+\frac{d}{dx}(x^3)+\frac{d}{dx}(4x^{\frac{1}{2}})+\frac{d}{dx}(7)$

$=5x^4+3x^2+\frac{2}{\sqrt{x}}+0$

(5) $\frac{d}{dx}(5x^4+6x^{\frac{3}{2}}+9x)=\frac{d}{dx}(5x^4)+\frac{d}{dx}(6x^{\frac{3}{2}})+\frac{d}{dx}(9x)$

$=20x^3+9x^{\frac{1}{2}}+9$

(6) $\frac{d}{dx}(ax^2+bx+c)=\frac{d}{dx}(ax^2)+\frac{d}{dx}(bx)+\frac{d}{dx}(c)=2ax+b$

(7) $\frac{d}{dx}\left(3x^5-3x-\frac{1}{x}\right)=\frac{d}{dx}(3x^5)-\frac{d}{dx}(3x)-\frac{d}{dx}\left(\frac{1}{x}\right)$

$=15x-3+\frac{1}{x^2}$

Rules of Differentiation - Product Rule and Quotient Rule

Part - 10

Product Rule

If we need to differentiate the product of two functions, then we apply product rule.

$$\frac{d(UV)}{dx} = V\frac{dU}{dx} + U\frac{dV}{dx}$$

Illustration 1.

Find value of $\frac{dy}{dx}$

(1) y = x lnx (2) y = e^xcosx

Solution.

(1) $\frac{d}{dx}(x\ell nx) = \ell nx\frac{d}{dx}(x) + x\frac{d}{dx}(\ell nx)$

$= \ell nx(1) + x\left(\frac{1}{x}\right) = \ell nx + 1$

(2) $\frac{d}{dx}(e^x \cos x) = \cos x\frac{d}{dx}(e^x) + e^x\frac{d}{dx}\cos x$

$= \cos x e^x + e^x$

Quotient Rule

If we need to differentiate a function which is the ratio of two functions, then we apply Quotient Rule.

$$\frac{d\left(\frac{U}{V}\right)}{dx} = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$$

Illustration 2.

Find derivative of $y = \frac{4x}{x-7}$ w.r.t. x

Solution.

$$\frac{d\left(\frac{4x}{x-7}\right)}{dx} = \frac{(x-7)\frac{d}{dx}(4x) - 4x\frac{d}{dx}(x-7)}{(x-7)^2}$$

$$= \frac{(x-7)(4) - 4x(1)}{(x-7)^2} = -\frac{28}{(x-7)^2}$$

Illustration 3.

Find value of $\frac{dy}{dx}$

(1) y = x^2cosx (2) y = e^xsinx

Solution.

(1) $\frac{d}{dx}(x^2\cos x) = \cos x\frac{d}{dx}(x^2) + x^2\frac{d}{dx}(\cos x) = (2x)(\cos x) - x^2\sin x$

(2) $\frac{d}{dx}(e^x\sin x) = \sin x\frac{d}{dx} + e^x\frac{d}{dx}(\sin x)$

$= e^x\sin x + e^x\cos x$

Illustration 4.

Find value of $\frac{dy}{dx}$

(1) $y = \frac{x^2}{\cos x}$ (2) $y = \frac{\cot x}{x^3}$

Solution.

$$\frac{\cos x\frac{d}{dx}(x^2) - x^2\frac{d}{dx}(\cos x)}{(\cos x)^2} = \frac{(\cos x)(2x) + (x^2)(\sin x)}{\cos^2 x}$$

Illustration 5.

Find value of $\frac{dI}{dt}$ if : $I = t^3(t-2)$

Solution.

$I = t^4 - 2t^3$

$\frac{d}{dt}(t^4 - 2t^3) = 4t^3 - 6t^2$

Illustration 6.

Find the derivative of q w.r.t. t if : $q = (t+5)^3(t+2)^4$

Solution.

$\frac{dq}{dt} = \frac{d}{dt}((t+5)^3(t+2)^4)$

$= (t+2)^4(3)(t+5)^2(1) + (t+5)^3(4)(t+2)^3(1)$

$= (t+2)^3(t+5)^2[3(t+2) + 4(t+5)]$

Rules of Differentiation - Chain Rule

Part - 11

Chain Rule

The Chain Rule tells us how to find the derivative of a composite function.

If y = f(g(x)) then $\frac{dy}{dx}$ will be given by :

$$\frac{dy}{dx}=\frac{d}{dx}[f(g(x))]=f'(g(x))g'(x)$$

Another way to represent the Chain Rule is :

If y = f(U) then $\frac{dy}{dx}$ will be given by :

$$\frac{dy}{dx}=\frac{df(U)}{dU}\times\frac{dU}{dx}$$

Illustration 1.

Find value of $\frac{dy}{dx}$

(1) $y = \cos(2x + 3)$ (2) $y = \sin(x^2 + x^3)$

Solution.

(1) Let $u = 2x + 3$ $\therefore$ $y = \cos(u)$

$$\frac{du}{dx}=\frac{d}{dx}(2x+3)=2 \qquad \frac{dy}{du}=\frac{d}{du}(\cos(u))=-\sin u$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=-(\sin u)(2)=-2\sin(2x+3)$$

(2) Let $u = x^2 + x^3$ $\therefore$ $y = \sin(u)$

$$\frac{du}{dx}=\frac{d}{dx}(x^2+x^3)=2x+3x^2 \qquad \frac{dy}{du}=\frac{d}{du}(\sin(u))=\cos u$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\cos u(2x+3x^2)=\cos(x^2+x^3)(2x+3x^2)$$

Illustration 2.

Find derivative of y w.r.t. x if : $y = \ln(x^3 + 4)$

Solution.

Let $u = x^3 + 4$ $\therefore$ $y = \ln(u)$

$$\frac{du}{dx}=\frac{d}{dx}(x^3+4)=3x^2 \qquad \frac{dy}{du}=\frac{d}{du}(\ln(u))=\frac{1}{u}$$

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx}=\left(\frac{1}{u}\right)(3x^2)=\frac{3x^2}{x^3+4}$$

Illustration 3.

Find value of $\frac{dy}{dx}$

$y = e^{(3x-6)}$

Solution.

Let $u = 3x - 6$ $\therefore$ $y = e^u$

$\frac{du}{dx} = \frac{d}{dx}(3x-6) = 3$ $\frac{dy}{du} = \frac{d}{du}(e^u) = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = e^u \cdot 3 = 3e^{3x-6}$$

Illustration 4.

Find value of $\frac{dy}{dt}$

$y = A\sin(\omega t + \phi)$

Solution.

Let $u = \omega t + \phi$ $\therefore$ $y = A\sin u$

$\frac{du}{dx} = \frac{d}{dx}(\omega t + \phi) = \omega$ $\frac{dy}{du} = \frac{d}{du}(A\sin u) = A\cos u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (A\cos u)(\omega) = A\omega\cos(\omega t + \phi)$$

Illustration 5.

Find value of $\frac{dy}{dt}$ if : $y = e^{\sin t}$

Solution.

Let $u = \sin t$ $\therefore$ $y = e^u$

$\frac{du}{dx} = \frac{d}{dx}(\sin t) = \cos t$ $\frac{dy}{du} = \frac{d}{du}(e^u) = e^u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = (e^u)(\cos t) = e^{\sin t}\cos t$$

Power Chain Rule

If $y = [f(x)]^n$ then $\frac{dy}{dx}$ will be given by :

$$\frac{dy}{dx} = n[f(x)]^{(n-1)}\frac{d[f(x)]}{dx}$$

$$\frac{dy}{dx} = n[f(x)]^{(n-1)}f'(x)$$

If $y = U^n$ then $\frac{dy}{dx}$ will be given by :

$$\frac{dy}{dx} = nU^{(n-1)}\frac{dU}{dx}$$

Illustration 6.

Find the derivative of $y = (x^2 + 3)^6$ w.r.t. x

Solution.

Let $u = x^2 + 3$ $\quad\therefore\quad$ $y = u^6$

$$\frac{du}{dx} = \frac{d}{dx}(x^2+3) = 2x \qquad \frac{dy}{du} = \frac{d}{du}(u^6) = 6u^5$$

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = (6u^5)(2x) = 6(x^2+3)^5(2x)$$

Illustration 7.

Find the derivative of : $x = \sqrt{t^2+2}$ w.r.t. t

Solution.

Let $u = t^2 + 2$ $\quad\therefore\quad$ $x = u^{\frac{1}{2}}$

$$\frac{du}{dt} = \frac{d}{dt}(t^2+2) = 2t \qquad \frac{dx}{du} = \frac{d}{du}(u^{\frac{1}{2}}) = \frac{1}{2}u^{-\frac{1}{2}} = \frac{1}{2\sqrt{u}}$$

$$\frac{dx}{dt} = \frac{dx}{du}\times\frac{du}{dt} = \frac{1}{2\sqrt{u}}(2t) = \frac{t}{\sqrt{t^2+2}}$$

Illustration 8.

Find the derivative of y w.r.t. x

(1) $y = \cos^2 x$ $\qquad$ (2) $y = \sin(x^2)$

Solution.

(1) $\cos 2x = 2\cos^2 x - 1$

$$\Rightarrow \frac{1+\cos 2x}{2} = \cos^2 x$$

$$\therefore\quad y = \frac{1+\cos 2x}{2}$$

$$= \frac{1}{2} + \frac{\cos 2x}{2}$$

$$\frac{dy}{dx} = 0 + \left(-\frac{\sin 2x}{2}\right)(2) = -\sin(2x)$$

(2) Let $u = x^2$ $\quad\therefore\quad$ $y = \sin(u)$

$$\frac{du}{dx} = \frac{d}{dx}(x^2) = 2x \qquad \frac{dy}{du} = \frac{d}{du}(\sin u) = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx} = \cos(u)(2x) = \cos(x^2)(2x)$$

Applications of Derivatives

Part - 12

Application of Derivatives

Instantaneous rate of change of a quantity "w.r.t." another quantity

$$\vec{v}_{inst} = \frac{d\vec{x}}{dt}$$

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt} = \frac{d^2x}{dt^2}$$

$$\vec{F}_{inst} = \frac{d\vec{p}}{dt}$$

Illustration 1.

If height of magical tree depends on time as h = $3t^2$ + 5t + 2 m. Find out :-

(1) Rate of change of height at t = 3 sec.

(2) Rate of change of height from t = 0 to t = 3 sec.

Solution.

(1) $\frac{dh}{dt} = \frac{d}{dt}(3t^2+5t+2) = 6t+5$

$\frac{dh}{dt}$ = rate of change of height = 6t + 5

$\therefore$ rate of change of height at t = 3

= 6(3) + 5 = 23 m

(2) Height at t = 3, h = $3(3^2)$ + 5(3) + 2 = 44 m

height at t = 0, h = $3(0^2)$ + 5(0) + 2 = 2 m

$$\frac{\Delta h}{\Delta t} = \frac{h_f - h_i}{t_f - t_i} = \frac{44-2}{3-0} = 14m$$

Illustration 2.

If position of particle is given by x = ($3t^2$ + 4t – 1) m. Find its initial velocity and initial acceleration.

Solution.

$$\frac{dx}{dt} = \frac{t}{dt}(3t^2+4t-1)$$

= 6t + 4

$\Rightarrow$ Velocity = $\frac{dx}{dt}_{(t\,=\,0)} = 6(0)+4 = 4m/s$

$$\frac{d^2x}{dt^2} = 6$$

$\Rightarrow$ Acceleration = 6m/s^2 (constant)

Illustration 3.

If position of particle is given by $x = (t^3 - 36t^2 + 30t - 1)$ m. Find its velocity when acceleration becomes zero.

Solution.

Velocity $= \frac{dx}{dt} = 3t^2 - 72t + 30\ m/s^2$ ...(i)

acceleration $= \frac{d^2x}{dt^2} = 6t - 72\ m/s^2$ ...(ii)

acceleration $= 6t - 72\ m/s^2 = 0$

$\therefore\ t = 12$ s

Velocity at t = 12 s,

velocity $= 3(12)^2 - 72(12) + 30 = -402$ m/s

Illustration 4.

The area A of a circle is related to its radius by the equation $A = \pi r^2$. How fast is the area changing with respect to the radius when the radius is 10 m?

Solution.

$A = \pi r^2$

$\frac{dA}{dr} = \frac{d}{dr}(\pi r^2) = \pi(2r)$

$= \pi(2)(10) = 20\pi$ m

Illustration 5.

If side of a cube is changing by a rate of 4 m/s find rate of change of its volume w.r.t. time when side length is 2m.

Solution.

$V = a^3$

$\frac{dV}{dt} = \frac{d}{dt}(a^3) = \frac{3a^2 da}{dt} = 3(2)^2(4) = 48\ m^3/s$

Illustration 6.

The area of a block of ink is growing such that after t second its area is given by $A = (3t^2 + 7)\ cm^2$. Calculate the rate of increase of area at = 5second.

Solution.

$\frac{dA}{dt} = \frac{d}{dt}(3t^2 + 7) = 6t$

at t = 5, $\frac{dA}{dt} = 6(5) = 30\ cm^2/s$

Application of Derivatives

Slope of a curve at a given point

$m_{inst} = \tan\theta = \frac{dy}{dx}$

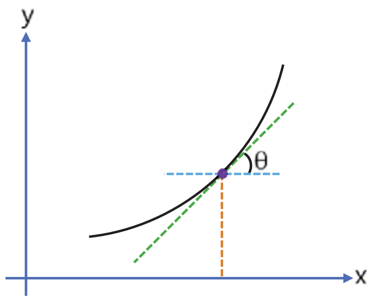


Illustration 7.

Find the slope of the tangent of a curve $y = x^2 + 2x + 4$ at $x = 0$ and $x = -1$

Solution.

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 2x + 4) = \frac{d}{dx}(x^2) + \frac{d}{dx}(2x) + \frac{d}{dx}(4) = 2x + 2$$

Slope of the tangent at $x = 0$, $\frac{dy}{dx} = 2(0) + 2 = 2$

Slope of the tangent at $x = -1$, $\frac{dy}{dx} = 2(-1) + 2 = 0$

Concept of Maxima and Minima

Part - 13

Double Differentiation

$$y = f(x) \xrightarrow{\text{Differentiation}} \frac{dy}{dx} \xrightarrow{\text{Differentiation}} \frac{d\left(\frac{dy}{dx}\right)}{dx} = \frac{d^2y}{dx^2}$$

Function — First Derivative — Second Derivative

Maxima and Minima

Suppose a quantity y depends on another quantity x in a manner shown in the figure.

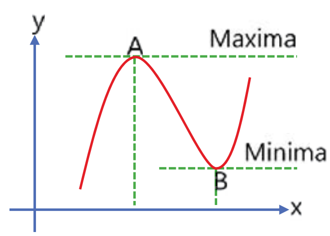
First derivative

$\frac{dy}{dx}$ = rate of change of y w.r.t. x = slope

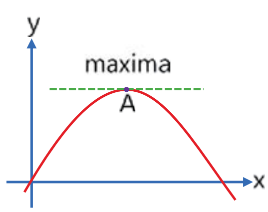
Second derivative

$\frac{d^2y}{dx^2}$ = rate of change of slope

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\text{slope})$$



Maxima

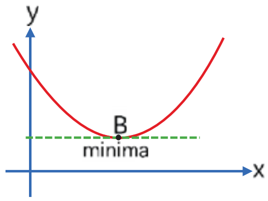


Condition for maxima are :

$\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$

in figure at point A (maxima)

Minima



Condition for minima are :

$\frac{dy}{dx}=0$ and $\frac{d^2y}{dx^2}>0$

in figure at point B (minima)

Summary

How to check Maxima & Minima of a function Y

Step-1 : Calculate $\frac{dy}{dx}$ and put it equal to zero.

Step-2 : Find value of x from above equation.

Step-3 : Find $\frac{d^2y}{dx^2}$ to check for maxima and minima

- If $\frac{d^2y}{dx^2}>0$ "then minima"
- If $\frac{d^2y}{dx^2}<0$ then maxima
- If $\frac{d^2y}{dx^2}=0$ then neither maxima, nor minima

Illustration 1.

Find maximum or minimum value for given equation $y = x^2 - 4x + 8$

Solution.

Step-1 : $\frac{dy}{dx}=2x-4$ ...(i)

Step-2 : $2x - 4 = 0$

$x = 2$

Step-3 : $\frac{d^2y}{dx^2}=2$ ...(ii)

$\frac{d^2y}{dx^2}>0$

So, minima at $x = 2$

$\therefore$ minimum value of given equation at $x = 2$; $y = (2)^2 - 4(2) + 8 = 4$

Illustration 2.

Find local maximum and minimum value for $y = x^3 + 2x^2 - 4x + 2$

Solution.

Step-1 : $\frac{dy}{dx}=3x^2+4x-4$

Step-2 : $3x^2 + 4x - 4 = 0$

$\Rightarrow$ (3x – 2)(x + 2) = 0

$x=\frac{2}{3}$ or x = –2

Step-3 : $\frac{d^2y}{dx^2}=6x+4$

at $x=\frac{2}{3}$; $\frac{d^2y}{dx^2}=8>0$ (minima)

$y_{minimum}$ = $\left(\frac{2}{3}\right)^3+2\left(\frac{2}{3}\right)^2-4\left(\frac{2}{3}\right)+2=\frac{14}{27}$

at x = –2 ; $\frac{d^2y}{dx^2}=-8<0$ (maxima)

$y_{maximum}$ = $(-2)^3 + 2(-2)^2 - 4(-2) + 2 = 10$

Indefinite Integration

Part - 14

Integration

In integral calculus, the differential coefficient of a function is given. We are required to find the function.

Integration is basically used for summation . $\sum$ is used for summation of discrete values, while $\int$ sign is used for continous function.

Reverse process of differentiation

If $F'(x)=f(x)$, then $\int f(x)dx=F(x)+c$

Here, c is called constant of integration or arbitrary constant

$$\int f(x)dx = F(x) + c$$

Integration symbol

Integrand

Variable of Integration

Constant of Integration

Types of Integration

1. Indefinite Integration
2. Definite Integration

1. Indefinite Integration

$$\int f'(x)dx = f(x)+c$$

2. Definite Integration

$$\int_a^b f'(x)dx = [f(x)]_a^b = [f(b)-f(a)]$$

Formulae of Integration for Algebraic function

$$\int x^n dx = \frac{x^{n+1}}{n+1}+c$$ (Provided $n \neq -1$)

$$\int \frac{1}{x}dx = \ln x + c$$

$$\int k dx = kx + c$$

$$\int e^x dx = e^x + c$$

$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

Illustration 1.

Evaluate :

(i) $\int x^6 dx$ (ii) $\int x^3 dx$

Solution.

(i) $\int x^6 dx = \frac{x^{6+1}}{6+1} + c$

$= \left(\frac{x^7}{7}\right) + c$

(ii) $\int x^3 dx = \frac{x^{3+1}}{3+1}$

$= \frac{x^4}{4} + c$

Illustration 2.

Find $\int \frac{1}{\sqrt{x}} dx$

Solution.

$$I = \int \frac{1}{x^{\frac{1}{2}}} dx = \int x^{-\frac{1}{2}} dx$$

$$= \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{x} + c$$

Rules For Integration

Rule 1 $\int (u \pm v) dx \Rightarrow \int u dx \pm \int v dx$

Rule 2 $\int cf(x) dx \Rightarrow c\int f(x) dx$

Illustration 3.

$\int (4x^2 - 6x + 2) dx$

Solution.

$$I = \int (4x^2 - 6x + +2) dx = \int 4x^2 dx - \int 6x dx + \int 2 dx$$

$$= 4\left[\frac{x^{2+1}}{2+1}\right] - 6\left[\frac{x^{1+1}}{1+1}\right] + 2\left[\frac{x^{0+1}}{0+1}\right] = \frac{4}{3}x^3 - 3x^2 + 2x + c$$

Illustration 4.

$$\int\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 dx$$

Solution.

$$I=\int\left(x^{\frac{1}{2}}+\frac{1}{x^{\frac{1}{2}}}\right)^2 dx=\int\left[\left(x^{\frac{1}{2}}\right)^2+\left(\frac{1}{x^{\frac{1}{2}}}\right)^2+2\left(x^{\frac{1}{2}}\right)\left(\frac{1}{x^{\frac{1}{2}}}\right)\right]$$

$$=\int\left(x+\frac{1}{x}+2\right)dx=\int x\,dx+\int\frac{1}{x}dx+\int 2dx$$

$$=\frac{x^2}{2}+\ln x+2x+c$$

Illustration 5.

$$\int(\cos\theta-\sin\theta+3)d\theta$$

Solution.

$$I=\int(\cos\theta-\sin\theta+3)d\theta=\int\cos\theta d\theta-\int\sin\theta d\theta+\int 3d\theta$$

$$= \sin\theta + \cos\theta + 3\theta + c$$

Illustration 6.

$$\int(e^x+x^e+e^e)dx$$

Solution.

$$I=\int(e^x+x^e+e^e)dx=\int e^x dx+\int x^e dx+\int e^e dx$$

$$= e^x+\frac{x^{e+1}}{e+1}+e^e x + c$$

Illustration 7.

Integrate y w.r.t x, where $y=e^x-\frac{1}{x}+4$

Solution.

$$I=\int\left(e^x-\frac{1}{x}+4\right)dx=\int e^x dx-\int\frac{1}{x}dx+\int 4dx$$

$$=e^x - \ln x + 4x + c$$

Linear Substitution in Integration Algebraic function

$$\int(ax+b)^n dx=\frac{1}{a}\frac{(ax+b)^{n+1}}{(n+1)}+c$$

$$\int\frac{1}{ax+b}dx=\frac{1}{a}\ln(ax+b)+c$$

$$\int \sin(ax+b)dx = -\frac{\cos(ax+b)}{a}+c$$

$$\int \cos(ax+b)dx = \frac{\sin(ax+b)}{a}+c$$

$$\int e^{ax+b}dx = \frac{1}{a}e^{ax+b}+c$$

Illustration 8.

(i) $\int \cos(2x+4)dx$ (ii) $\int \frac{1}{4t-2}dt$

Solution.

(i) $I = \int \cos(2x+4)dx = \frac{\sin(2x+4)}{2}+c$

(ii) $I = \int \frac{1}{4t-2}dt = \frac{\ln(4t-2)}{4}+c$

Illustration 9.

$$\int e^{(-4x+3)}dx$$

Solution.

$$I = \int e^{(-4x+3)}dx = \frac{e^{(-4x+3)}}{-4}+c$$

Illustration 10.

$$\int (3x-4)^4 dx$$

Solution.

$$I = \int (3x-4)^4 dx = \frac{(3x-4)^{4+1}}{4+1}\cdot\frac{1}{3} = \frac{1}{15}(3x-4)^5$$

Illustration 11.

$$\int (\sin\omega t)dt$$

Solution.

$$I = \int (\sin\omega t)dt = -\frac{\cos(\omega t)}{\omega}+c$$

Definite Integration

Part - 15

Definite Integration

When a function is integrated between a lower limit and an upper limit, it is called a definite integral.

If $\frac{d}{dx}(f(x)) = f'(x)$, then

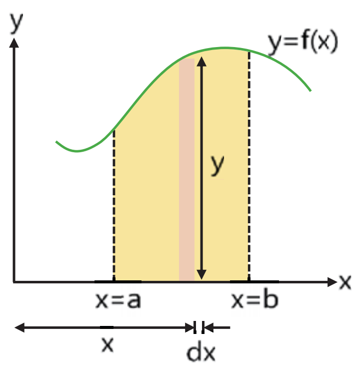
$\int f'(x)dx$ is called indefinite integral and $\int_a^b f'(x)dx$ is called definite integral

Here, a and b are called lower and upper limits of the variable x.

After carrying out integration, the result is evaluated between upper and lower limits as explained below :

$$\int_a^b f'(x)dx = |f(x)|_a^b = [f(b) - f(a)]$$

Area under a curve and definite integration



Area of small shown darkly shaded element = ydx = f(x) dx

If we sum up all areas between x=a and x= b then

$\int_a^b f(x)dx$ = shaded area between curve and x-axis.

$$\int_a^b f(x)dx = [F(x)]_a^b = [F(b) - F(a)]$$

Illustration 1.

Find value of $\int_3^4 6xdx$

Solution.

$$I = \int_3^4 6x\,dx = 6\int_3^4 x\,dx = 6\left[\frac{x^2}{2}\right]_3^4$$

$$= 6\left[\frac{(4)^2}{2} - \frac{(3)^2}{2}\right] = 6\left[\frac{7}{2}\right] = 21$$

Illustration 2.

Find value of $\int_1^2 (10x^2 - 4x + 4)dx$

Solution.

$$I = \int_1^2 (10x^2 - 4x + 4)dx = \int_1^2 10x^2 dx - \int_1^2 4x\,dx + \int_1^2 4dx$$

$$= 10\left[\frac{(x^3)}{3}\right]_1^2 - 4\left[\frac{x^2}{2}\right]_1^2 + 4[x]_1^2$$

$$= 10\left[\frac{2^3}{3} - \frac{1}{3}\right] - 4\left[\frac{2^2}{2} - \frac{1}{2}\right] + 4[2-1] = \frac{128}{6}$$

Illustration 3.

Find value of $\int_0^{\frac{\pi}{3}} \cos x\,dx$

Solution.

$$I = \int_0^{\frac{\pi}{3}} \cos x\,dx = [+\sin x]_0^{\frac{\pi}{3}} = \left(\sin\frac{\pi}{3} - \sin 0\right)$$

$$= \frac{\sqrt{3}}{2}$$

Illustration 4.

Find value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\,dx$

Solution.

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\,dx = [-\cos x]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \left[-\cos\left(\frac{\pi}{2}\right) - \left(-\cos\left(-\frac{\pi}{2}\right)\right)\right] = 0$$

Illustration 5.

Find value of $\int_{\infty}^{0} e^{-2t} dt$

Solution.

$$I = \int_{\infty}^{0} e^{-2t} dt = \left[\frac{e^{-2t}}{-2}\right]_{\infty}^{0} = -\frac{1}{2}\left[e^{-2(0)} - e^{-2(\infty)}\right] = -\frac{1}{2}$$

Illustration 6.

Find value of $\int_{0}^{\frac{\pi}{6}} \sin 2\theta d\theta$

Solution.

$$I = \int_{0}^{\frac{\pi}{6}} \sin 2\theta d\theta = \left[-\frac{\cos 2\theta}{2}\right]_{0}^{\frac{\pi}{6}}$$

$$= -\frac{1}{2}\left[\cos\frac{2\pi}{6} - \cos 0\right] = \frac{1}{4}$$

Applications of Integration - Analytical

Part - 16

Application of Integration

There are many applications of integration such as :

(a) Displacement/Change in position $\Delta x = x_2 - x_1$

(b) Change in velocity $\Delta v = v_2 - v_1$

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt}$$

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} vdt \qquad \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} adt$$

(Change in position) $x_2 - x_1 = \int_{t_1}^{t_2} vdt$ (Change in velocity) $v_2 - v_1 = \int_{t_1}^{t_2} adt$

Illustration 1.

Initial position of a particle is x = 20 m and its velocity is $v = (2t^2 - 4t)$ m/s. Find position of the particle at t = 3 sec.

Solution.

$$v = \frac{dx}{dt} \Rightarrow \int_{20}^{x} dx = \int_{t=0}^{t=3} vdt$$

$$\Rightarrow x - 20 = \int (2t^2 - 4t)dt$$

$$x - 20 = \left[\frac{2t^3}{3} - 2t^2\right]_0^3$$

$x - 20 = 18 - 18$

$x = 20$

Illustration 2.

Initial velocity of a particle is '$2u_0$' and acceleration is a = kt. Find velocity at time t.

Solution.

$$a = \frac{dv}{dt}$$

$$dv \int_{2u_0}^{v} dv = \int adt = \int_{o}^{t} ktdt$$

$$[v]_{2u_0}^{v} = k\left[\frac{t^2}{2}\right]_0^t$$

$$v - 2u_0 = k\frac{t^2}{2}$$

$$v = \frac{kt^2}{2} + 2u_0$$

Illustration 3.

Find change in momentum from t = 1 to t = 2s if a force F = $4t^2 - 6$ N acts on a particle. (Use $\Delta p = \int_{t_1}^{t_2} F dt$)

Solution.

$$\Delta P = \int_{t_1}^{b_2} F dt$$

$$= \int_1^2 (4t^2 - 6) dt$$

$$= \int_1^2 (4t^2) dt - \int_1^2 6 dt$$

$$= \left[\frac{4t^3}{3}\right]_1^2 - [6t]_1^2$$

$$= \left[\frac{32}{3} - \frac{4}{3}\right] - [12 - 6]$$

$$= \frac{28}{3} - 6$$

$$= \frac{10}{3}$$

Applications of Integration - Graphical

Part - 17

Area Under The Curve

Area of shaded element small shown darkly = ydx = f(fx) dx

If we sum all areas between x = a and x = b then

$$\int_a^b f(x)dx = F(b) - F(a)$$

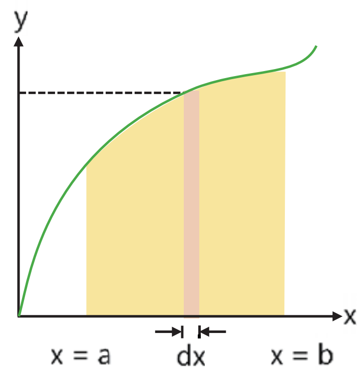
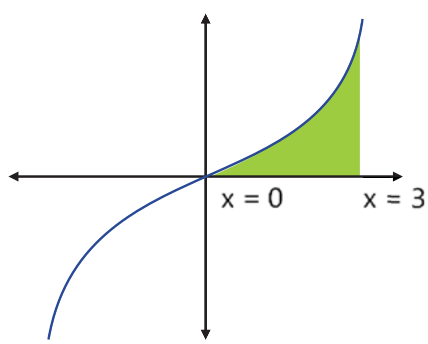


Illustration 1.

Find area between the curve $y = x^3$ and x axis from x = 0 to x = 3.



Solution.

Area under the curve = $\int ydx$

$$= \int_{x=0}^{x=3} x^3 dx$$

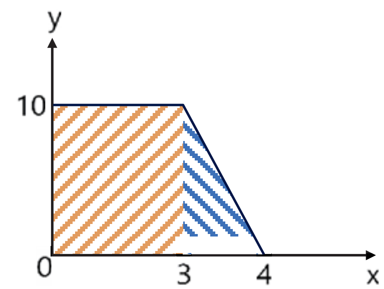
$$= \left[\frac{x^4}{4}\right]_0^3$$

$$= \frac{81}{4} \text{ unit}$$

Illustration 2.

Find the values of

(a) $\int_0^3 y dx$ (b) $\int_3^4 y dx$



Solution.

(a) $\int_0^3 y dx$ = Area under the curve from x = 0 to x = 3

= 10 × 3 = 30 unit

(b) $\int_3^4 y dx$ = Area under the wave from x = 3 to x = 4

$= \frac{1}{2} \times 1 \times 10 = 5$ unit

Illustration 3.

Find area under the curve y = $2x^2 - 4x + 6$, from x = 2 to x = 4.

Solution.

Area under the curve = $\int_{x_1}^{x_2} y dx = \int_2^4 (2x^2 - 4x + 6) dx$

$= \int_2^4 2x^2 dx - \int_2^4 4x dx + \int_2^4 6 dx$

$= \left[\frac{2x^3}{3}\right]_2^4 - \left[2x^2\right]_2^4 + \left[6x\right]_2^4$

$= \left(\frac{112}{3}\right) - (24) + 12$

$= \frac{76}{3}$ unit

Illustration 4.

Find area under the curve y = cos x, from x = 0 to x = $\frac{\pi}{2}$.

Solution.

Area under of the curve = $\int y dx$

$= \int_0^{\frac{\pi}{2}} (\cos x) dx$

$= [\sin x]_0^{\frac{\pi}{2}}$

$= \left[\sin\frac{\pi}{2} - \sin 0\right] = 1$

Average Value of Function

Part - 18

Average Value of Function

Average value of a function y = f(x), over an interval a $\le x \le$ b is given by $<y> = \frac{\int_a^b y\,dx}{\int_a^b dx} = \frac{\int_a^b y\,dx}{b-a}$

Suppose there is a function y = f(x)

Then average value of y = f(x) from x_1 = a to x_2 = b is

$$<y> = \frac{\int_{x_1}^{x_2} f(x)dx}{\int_{x_1}^{x_2} dx}$$

$$<y> = \frac{\int_a^b f(x)dx}{\int_a^b dx}$$

Average Value of Function

In case of Graphical section : - The average value of y = f(x) from x_1 = a to x_2 = b is

$$<y> \frac{\text{area under } y - x \text{ curve}}{\text{range of } x\ (b-a)}$$

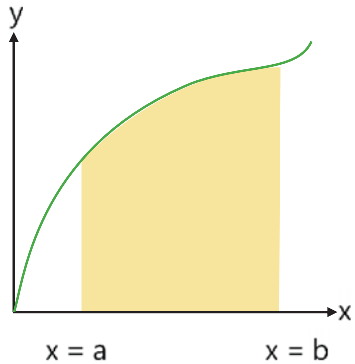


Illustration 1.

What is the average value of the function x^3 on the interval [0, 4] ?

Solution.

Average value = $\frac{\int_0^4 x^3 dx}{4-0} = \frac{\left[\frac{x^4}{4}\right]_0^4}{4}$

$= \frac{256}{16} = 16 \text{ unit}$

Illustration 2.

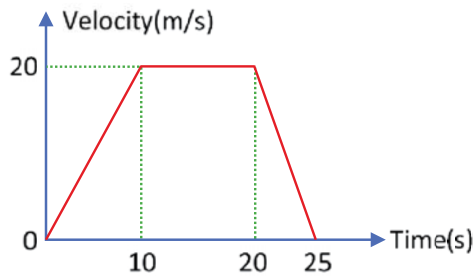
Find average value of a function I = $i_0$cost in interval of time $[0, \frac{\pi}{2}]$

Solution.

$$<I>=\frac{\int_0^{\pi/2}(I_0\cos t)dt}{\frac{\pi}{2}-0}=\frac{I_0\int_0^{\pi/2}\cos t dt}{\frac{\pi}{2}}=\frac{I_0}{\frac{\pi}{2}}=\frac{2I_0}{\pi}\text{ unit}$$

Illustration 3.

The velocity-time graph of a car moving along a straight road is shown in figure. The average velocity of the car in first 25 seconds is –



Solution.

Average velocity = $\frac{\int_0^{25} v dt}{25-0}=\frac{\text{Area of v-t graph between t=0 to t = 25 s}}{25}$

$$=\frac{1}{25}\left[\left(\frac{25+10}{2}\right)(20)\right]=14 m/s$$

Illustration 4.

A particle is moving with velocity, v = $(3t^2 + 4t^3 + 4)$m/s. Find <v> for interval 0 to 2 sec.

Solution.

$$<v>=\frac{\int_0^2 v dt}{2-0}=\frac{\int_0^2(3t^2+4t^3+4)}{2-0}=\frac{\left(\frac{3t^3}{3}+\frac{4t^4}{4}+4t\right)_0^2}{2}$$

$$=\frac{8+16+8}{2}=16\text{ unit}$$

Illustration 5.

What is the average value of the function f(t) = cosπt in the interval [0, 1]?

Solution.

$$\langle f(t)\rangle=\frac{\int_0^1 f(t)dt}{1-0}=\frac{\int_0^2(\cos\pi t)}{1}=\frac{[\sin(\pi t)]_0^1}{\pi}=\frac{[\sin\pi-\sin(0)]}{\pi}=0$$

Quadratic Equation and Binomial Theorem

Part - 19

Quadratic Equation

An algebraic equation of second order (Highest power of the variable is equal to 2) is called a quadratic equation.

$ax^2 + bx + c = 0$ is the general Quadratic Equation.

where $a \neq 0$

Roots of Quadratic Equation

The general solution of the quadratic equation or it's roots are:

$$x = \frac{-b \pm \sqrt{D}}{2a} \quad \begin{cases} x_1 = \dfrac{-b + \sqrt{D}}{2a} \\ x_2 = \dfrac{-b - \sqrt{D}}{2a} \end{cases}$$

Where $D = b^2 - 4ac$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Condition for Real and Imaginary Roots

For Real Roots	For Imaginary Roots
$D \geq 0$	$D < 0$
$b^2 - 4ac \geq 0$	$b^2 - 4ac < 0$

Sum and Product of Roots

Sum of the roots	Product of the roots
$x_1 + x_2 = -\frac{b}{a}$	$x_1 x_2 = \frac{c}{a}$

Illustration 1.

Solve the equation to $x^2 + 3x - 18 = 0$

Solution.

$$x^2 + 3x - 18 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(-18)}}{2(1)} = \frac{-3 \pm \sqrt{81}}{2}$$

$$x_1 = \frac{-3+9}{2} = 3\ ;\ x_2 = \frac{-3-9}{2} = -6$$

Illustration 2.

Solve for x : $pqx^2 - (p^2 + q^2)x + pq = 0$

Solution.

$pqx^2 - (p^2 + q^2)x + pq = 0$

$$x = \frac{-\left(-\left(p^2+q^2\right)\right) \pm \sqrt{\left(-\left(p^2+q^2\right)\right)^2 - 4(pq)(pq)}}{2(pq)} = \frac{\left(p^2+q^2\right) \pm \left(p^2-q^2\right)}{2pq}$$

$$x_1 = \frac{p}{q} \quad ; \quad x_2 = \frac{q}{p}$$

Illustration 3.

In quadratic equation $ax^2 + bx + c = 0$, if discriminant is $D = b^2 - 4ac$, then roots of the quadratic equation are : (choose the correct alternative)

(1) Real and distinct, if $D > 0$

(2) Real and equal (i.e., repeated roots), if $D = 0$.

(3) Non-real (i.e. imaginary), if $D < 0$

(4) All of the above are correct

Solution.

(4) All of the above are correct

Binomial Theorem

An algebraic expression containing two terms is called a binomial expression.

For example : $(a + b)$, $(a + b)^3$, $(2x - 3y)^{-2}$ etc.

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2\times 1}x^2 + \frac{n(n-1)(n-2)}{3\times 2\times 1}x^2 + \ldots\ldots$$

Binomial Approximation

If x is very small, compared to 1, then terms containing higher powers of x can be neglected so $(1+x)^n \approx 1 + nx$.

If $|x| << 1$ $\xrightarrow{\text{Binomial Approximation}}$

- $(1+x)^n \approx 1 + nx$
- $(1-x)^n \approx 1 - nx$
- $(1+x)^{-n} \approx 1 - nx$
- $(1-x)^{-n} \approx 1 + nx$

Illustration 4.

Find the value of

(i) $(1.01)^4$ (ii) $(0.997)^2$ (iii) $\sqrt{0.99}$

Solution.

(i) $(1.01)^4 = (1+0.01)^4 \approx 1 + 4(0.01) \approx 1.04$

(ii) $(0.997)^2 = (1 - 0.003)^2 \approx 1 - 2(0.003) \approx 0.994$

(iii) $\sqrt{0.99} = (1 - 0.01)^{1/2} \approx 1 - \frac{1}{2}(0.01) \approx 1 - 0.005 \approx 0.995$

Illustration 5.

Given that $g=\dfrac{GM}{(R+h)^2}$, find the value of g if h << R

Solution.

$$g = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} = \frac{GM}{R^2}\left(1+\frac{h}{R}\right)^{-2}$$

If h << R

$$g = \frac{GM}{R^2}\left(1-\frac{2h}{R}\right)$$

Logarithm and Progressions

Part - 20

Progression

Arithmetic Progression (AP)

General form: a, (a + d), (a + 2d), ..., [a + (n – 1) d]

Here First Term : a

Common Difference : d

n_{th} Term : $\left[a+(n-1)d\right]$

Sum of first n terms: $S_n = \frac{n}{2}\left[1^{st}\text{ term} + n^{th}\text{ term}\right]$

$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

Illustration 1.

Find the fifth term of given Arithmetic Progression:

5, 7, 9, . . .

Solution.

a = 5

d = 7 – 5 = 9 – 7 = 2

Fifth term = (a + 4d) = (5 + 4 (2)) = 13

Illustration 2.

Find the sum of first ten terms of given Arithmetic Progression:

2, 4, 6, . . .

Solution.

n = 10

d = 4 – 2 = 6 – 4 = 2

$S_n = \frac{10}{2}\left[2(2)+(10-1)(2)\right]$

= 5 [4 + 18] = 110

Illustration 3.

Find the sum of given series:

4 + 8 + 12 + ... + 64

Solution.

First term = 4

Last term = 64

n^{th} term = a + (n – 1) d

64 = 4 + (n – 1) 4

n = 16

$S_n = \frac{16}{2}\left[4+64\right] = 544$

Illustration 4.

Find the sum of first 20 natural numbers:

Solution.

First 20 natural numbers = 1, 2, 3, ... 20

a = 1, d = 1

$$S_n = \frac{20}{2}[2(1)+(19)(1)] = 210$$

Geometric Progression (GP)

General form : a, ar, ar^2, ... $ar^{(n-1)}$

First Term : a

Common Ration : r

n^{th} Term : $ar^{(n-1)}$

Sum of first n terms : $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum of all the terms of an infinite GP : $S_\infty = \frac{a}{(1-r)}$; Only when |r| < 1

Illustration 6.

Find the sixth term of 1, 2, 4, ...

Solution.

Sixth term = ar^5

a = 1, r = $\frac{2}{1} = \frac{4}{2} = 2$

∴ Sixth term = $1(2)^5$ = 32

Illustration 7.

Find sum of all the terms of an infinite GP : $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.........\infty$

Solution.

a = 1, r = $\frac{1/2}{1} = \frac{1/4}{1/2} = \frac{1}{2}$

S_n = $\frac{a}{1-r} = \frac{1}{1-1/2} = 2$

Formulae to Remember

Sum of first n natural numbers:

1 + 2 + 3 + 4 + 5 + ... + n = $\frac{n(n+1)}{2}$

Sum of squares of first n natural numbers:

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

Sum of cubes of first n natural numbers :

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Logarithm

The exponent or power to which a base must be raised to yield a given number.

Expressed mathematically, x is the logarithm of n to the base b

$x = \log_b n$ (if $b^x = n$, exponential form)

If base is 10 it is called **Standard log** $\Rightarrow \log_{10} n$

If base is e then it is called **Natural log** $\Rightarrow \log_e n$

$e \approx 2.71$

Common Formulae of Logarithm

Product Formula $\Rightarrow \log(mn) = \log m + \log n$

Quotient Formula $\Rightarrow \log\left(\frac{m}{n}\right) = \log m - \log n$

Power Formula $\Rightarrow \log(m^n) = n \log m$

Standard Values of Logarithm

Base Changing Formula $\Rightarrow \log_e m = 2.303 \log_{10} m$

For any Base $\Rightarrow \log_b 1 = 0$

For Base a $\Rightarrow \log_a a = 1$

Standard Values to remember

Base 10	Base e
log 2 = 0.301	ln 2 = 0.693
log 3 = 0.477	ln 3 = 1.09
log 5 = 0.699	ln 5= 1.6

Illustration 8.

Find the value of :

(i) $\ln e^5$ (ii) $\ln e^{2/3}$

Solution.

(i) $\ell n e^5 = 5 \ell n e = 5 \log_e^e = 5$

(ii) $\ell n e^{2/3} = \frac{2}{3} \ell n_e^e = \frac{2}{3}$

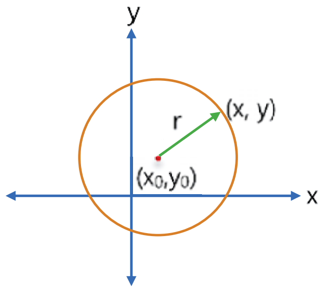
Graphs - Ellipse, Circle

Part - 21

Circle

Assume that (x, y) are the coordinates of a point on the circle shown, the centre is at (x_0, y_0) and the radius is r.

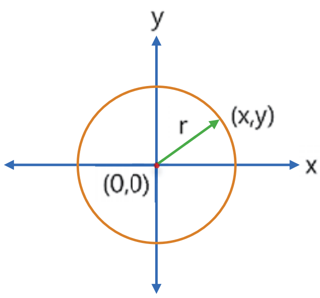
Equation of circle = $(x - x_0)^2 + (y - y_0)^2 = r^2$



Equation of Circle with center at Origin

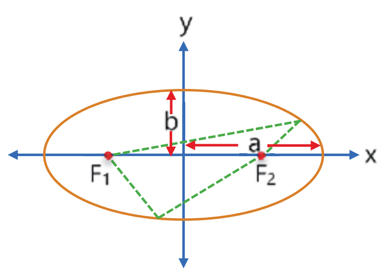
$(x - 0)^2 + (y - 0)^2 = r^2$

$x^2 + y^2 = r^2$



Ellipse

An ellipse is the locus of points in a plane, the sum of whose distances from two fixed points is a constant value. The two fixed points are called the **foci** of the ellipse.



In this diagram: -

a = semi major axis ; b = semi minor axis

F_1 and F_2 = foci of the ellipse.

Equation of Ellipse with center at Origin

The equation of ellipse is written in terms of it's semi-major axis and semi-minor axis as:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Illustration 1.

What is the radius of the circle given by:

$x^2 + y^2 = 49$

Solution.

$x^2 + y^2 = 49$ (given equation) ......(1)

$x^2 + y^2 = r^2$ (general equation) ......(2)

Comparing both equations : $r^2 = 49$

Radius of the circle = r = 7

Illustration 2.

What is the value of c if the radius of the circle is 9 and centre is at origin:

$x^2 + y^2 = c$

Solution.

$x^2 + y^2 = c$ (given equation) ......(1)

$x^2 + y^2 = r^2$ (general equation) ......(2)

Comparing both equations : $c = r^2$

Radius of the circle = c = 81

Illustration 3.

Find length of major axis and minor axis for ellipse

$$\frac{x^2}{16}+\frac{y^2}{36}=1$$

Solution.

$\frac{x^2}{16}+\frac{y^2}{36}=1$ (given equation) ......(1)

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (general equation) ......(2)

Comparing both equations :

a = 4 (semi minor axis)

b = 6 (semi major axis)

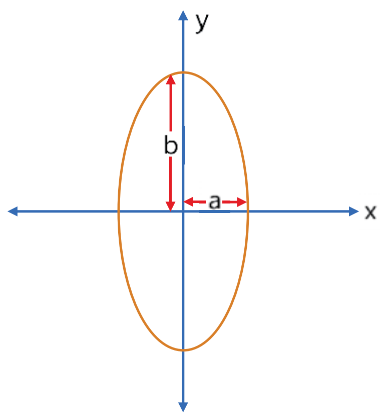


Illustration 4.

If the length of major axis is 5 and minor axis is 3 then write the equation of ellipse centered at origin.

Solution.

a = 5 ; b = 3

$$\frac{x^2}{5^2}+\frac{y^2}{3^2}=1$$

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

Graphs - Parabola, Rectangular Hyperbola, Exponential Functions

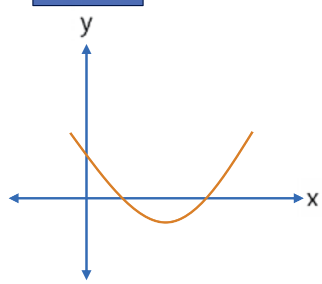
Part - 22

Parabola

The equation of parabola is given by:

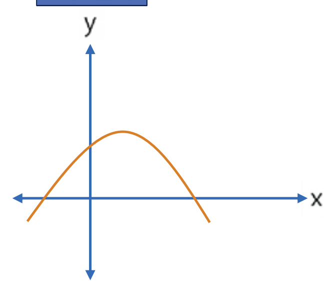
$$y = ax^2 + bx + c$$

If a > 0



Upward Opening Parabola

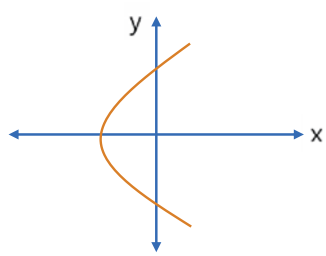
If a < 0



Downward Opening Parabola

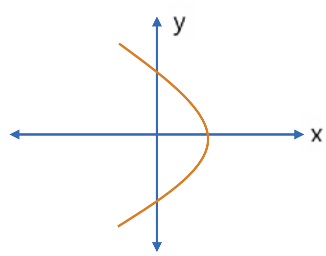
$$x = ay^2 + by + c$$

If a > 0



Rightward Opening Parabola

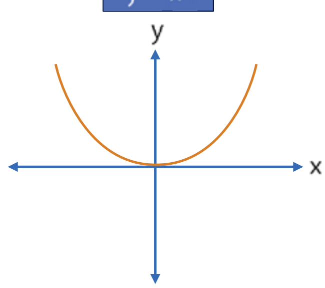
If a < 0



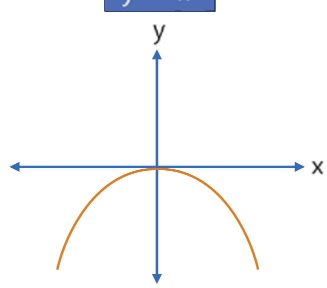
Leftward Opening Parabola

Some standard Parabola

$y = x^2$



$y = -x^2$



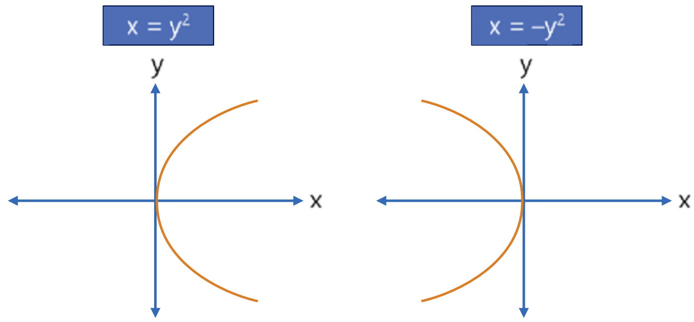


Illustration 1.

A particle of mass m is moving with speed v. Draw the graph of K.E vs v

Solution.

K.E. = $\frac{1}{2}mv^2$

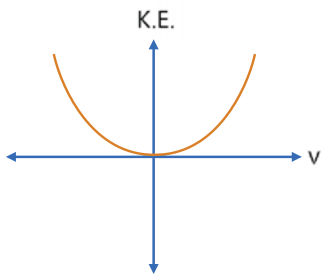


Illustration 2.

If $x = 9t^2$ and $y = 3t$ represents the coordinate of a particle, then its path will be?

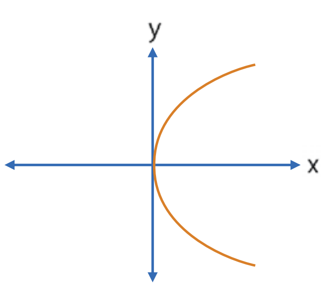
Solution.

$x = 9t^2$ ......(1)

$y = 3t$ ......(2)

From equation (2) $t = \frac{y}{3}$ ......(3)

Now, from equation (1) and (3) $x = 9\,\frac{y^2}{9} = y^2$



Rectangular Hyperbola

The equation of Rectangular Hyperbola is given by:

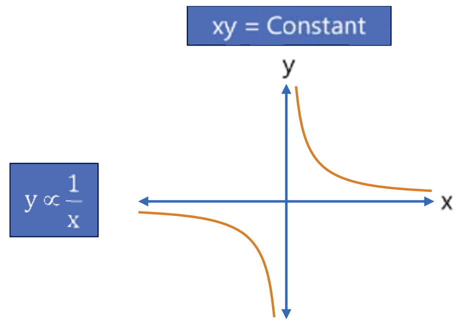


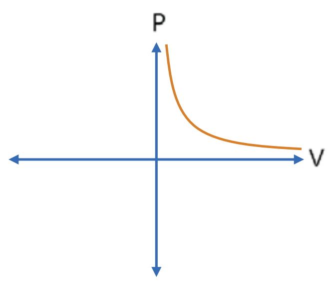
Illustration 3.

Draw graph between pressure and volume for an ideal gas at constant temperature (PV = Constant)

Solution.

PV = nRT

PV = constant

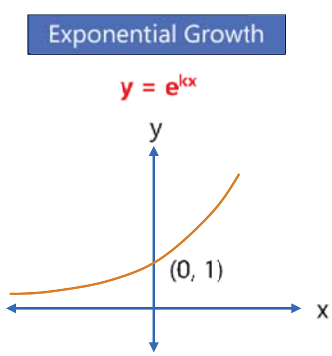


Exponential Graphs

There are two types of exponential graphs:

(i) Exponential Growth

(ii) Exponential Decay



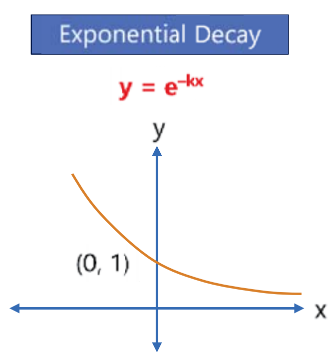


Illustration 4.

A particle moves along path $y = 9x^2 - 2x + 4$, then its path will be?

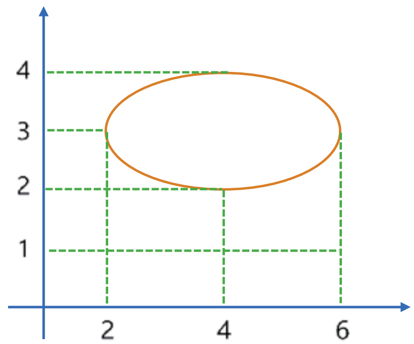
Solution.

$y = ax^2 - bx + c$

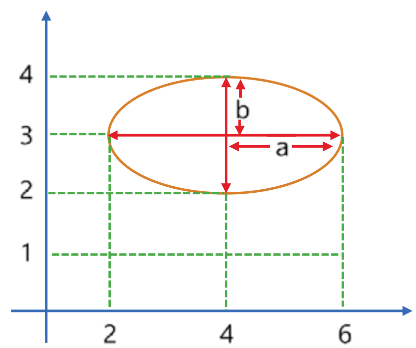
$\therefore$ path will be parabola

Illustration 5.

Calculate the area enclosed by shown ellipse



Solution.



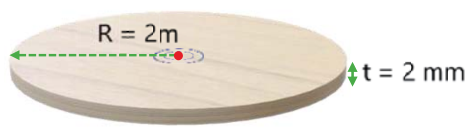
Shaded area = Area of ellipse = πab

Here $a = 6 - 4 = 2$ and $b = 4 - 3 = 1$

$\Rightarrow$ Area = $\pi \times 2 \times 1 = 2\pi$ units

Illustration 6.

Calculate the volume of given disc.



Solution.

Volume = (Area) (thickness)

Volume = $\pi R^2 t = (3.14)(2)^2 (2 \times 10^{-3}) = 25.12 \times 10^{-3}\, m^3$

Basic Maths Test (NEET Pattern) 01:00 Hr

Important Instructions

This test contains **45** questions. Each question carries **4 marks**. For each **correct** response the candidate will get **4 marks**. For each **incorrect** response, **one mark will be deducted** from the total scores. The maximum marks are **180**.

1. As θ increases from 0° to 90°, the value of $\sin\theta$:-
(1) Increases
(2) Decreases
(3) Remains constant
(4) First decreases then increases.

2. If $\sin\ \theta = \frac{\sqrt{2}}{\sqrt{3}}$ and θ lies in the first quadrant, the value of $\tan\ \theta$ is :
(1) $\sqrt{2}$
(2) $\frac{1}{\sqrt{2}}$
(3) $\frac{\sqrt{3}}{\sqrt{2}}$
(4) $\frac{1}{2}$

3. Find θ for which $\sin\theta = \cos\theta$, if $180° < \theta < 360°$
(1) 135°
(2) 315°
(3) 225°
(4) 150°

4. If $\sin\theta_1+\sin\theta_2+\sin\theta_3=3$
then value of $\cos\theta_1+\cos\theta_2+\cos\theta_3$ is :-
(1) 3
(2) 0
(3) –3
(4) 1

5. If $\tan(2A+B)=\sqrt{3}$ and $\cot(3A-B)=\sqrt{3}$. Find A and B.
(1) 18°, 24°
(2) 24°, 18°
(3) 20°, 20°
(4) 18°, 36°

6. Value of $\sin^2 15° + \sin^2 30° + \sin^2 45° + \sin^2 60° + \sin^2 75°$ is :-

(1) 1

(2) $\frac{3}{2}$

(3) $\frac{5}{2}$

(4) 3

7. Value of $\sin(45°+\theta)\cos(15°+\theta) - \cos(45°+\theta)\sin(15°+\theta)$ is :-

(1) 1

(2) $\frac{3}{2}$

(3) $\frac{1}{2}$

(4) $-\frac{1}{2}$

8. Value of $\sin(-420°)\cos(390°) + \cos(-660°)\sin(330°)$ is :-

(1) 0

(2) -1

(3) 1

(4) $\frac{3}{2}$

9. Value of $(\tan 1° \tan 2° \tan 3° ....\tan 89°)$ is :-

(1) 0

(2) 1

(3) 2

(4) $\frac{1}{2}$

10. The greatest value of the function $7\sin\theta - 24\cos\theta$ is -

(1) 12

(2) 13

(3) 25

(4) 17

11. The length of hypotenuse of a right angle triangle exceeds the length of its base by 2 cm and exceeds twice the length of altitude by 1 cm. Find length of each side of the triangle.

(1) 6, 8, 10

(2) 7, 24, 25

(3) 8, 15, 17

(4) 7, 40, 41

12. If $x = at^4$ and $y = bt^3$. Find $\frac{dy}{dx}$ -

(1) $\frac{4a}{3bt}$

(2) $\frac{3b}{4at}$

(3) $\frac{3a}{4bt}$

(4) $\frac{4b}{3at}$

13. A metallic disc is being heated. Its area at any time t is given by $A = 5t^2 + 4t + 8$. Calculate rate of increase in area at t = 3s.

(1) 30 m^2/s

(2) 34 m^2/s

(3) 28 m^2/s

(4) 20 m^2/s

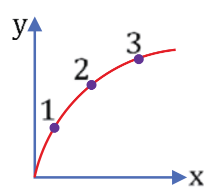
14. The side of a square is increasing at the rate of 0.1 cm/s. The rate of increase of perimeter w.r.t. time is :

(1) 0.2 cm/s
(2) 0.4 cm/s
(3) 0.6 cm/s
(4) 0.8 cm/s

15. A particle moves along the straight line 3y = x + 5. Which coordinate changes at a faster rate ?

(1) x–coordinate
(2) y–coordinate
(3) Both x and y coordinates
(4) Data insufficient

16. The slope of graph as shown in figure at points 1, 2 and 3 is m_1, m_2 and m_3 respectively, then



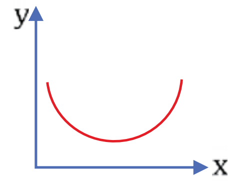
(1) $m_1 > m_2 > m_3$

(2) $m_1 < m_2 < m_3$

(3) $m_1 = m_2 = m_3$

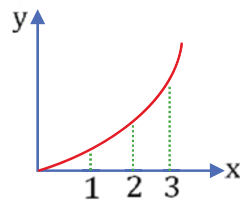
(4) $m_1 = m_3 > m_2$

17. Magnitude of slope of the shown graph.



(1) First increases then decreases
(2) First decreases then increases
(3) Increases
(4) Decreases

18. Calculate the area enclosed under the curve $f(x) = x^2$ between the limits x = 2 and x = 3



(1) 5
(2) $\frac{19}{3}$
(3) $\frac{17}{3}$
(4) 8

19. Find the value of $\int_0^4 |(1-x)|.dx$

(1) zero
(2) 1
(3) 4
(4) 5

20. The equation of a curve is given as $y = x^2 + 2 - 3x$. The curve intersects the y-axis at
(1) (1, 0)
(2) (2, 0)
(3) (0, 2)
(4) No where

21. Two particles A and B are moving in XY-plane. Their positions vary with time t according to relation :

$x_A(t) = 3t, x_B(t) = 6$

$y_A(t) = t, \ y_B(t) = 2+3t^2$

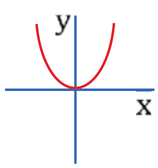
Find distance between A and B at t = 2 sec.
(1) 12
(2) 13
(3) 5
(4) $\sqrt{12}$

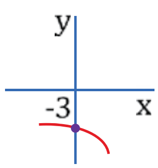
22. The distance between points (a + b, b + c) and (a – b, c – b) is :-
(1) $2\sqrt{a^2+b^2}$
(2) $2\sqrt{b^2+c^2}$
(3) $2\sqrt{2}b$
(4) $\sqrt{a^2-c^2}$

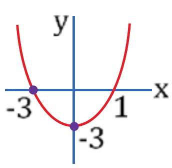
23. A dog is at point A(0, 3, 4)m and cat is at B(5,3,–8)m. The dog is free to move but cat is fixed. The minimum distance travelled by dog to catch the cat is :-
(1) 25m
(2) 12m
(3) 13m
(4) 20m

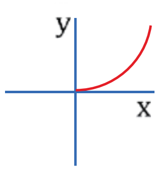
24. A particular straight line passes through origin and a point whose abscissa is equal to ordinate. The equation of such straight line is :
(1) y = x
(2) y = 2x
(3) y = –4x
(4) $y = -\frac{x}{4}$

25. If $y = x^2 + 2x - 3$, then y-x graph is :-

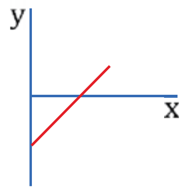
(1) 

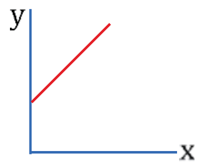
(2) 

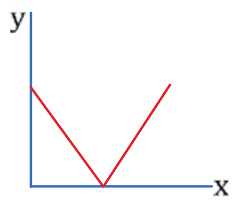
(3) 

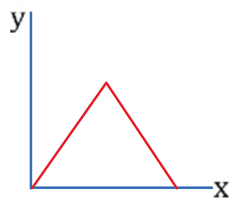
(4) 

26. If y =|x–1|, then y-x graph is :-

(1) 

(2) 

(3) 

(4) 

27. The coordinates of a particle moving in XY–plane vary with time $x = a\cos\omega t$, $y = a\sin\omega t$. The locus of the particle is :-

(1) Straight line

(2) Circle

(3) Parabola

(4) Ellipse

28. Frequency f of a simple pendulum depends on its length ℓ and acceleration g due to gravity according to the following equation $f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$. Graph between which of the following quantities is a parabola ?

(1) f on the ordinate and $1/\ell$ on the abscissa

(2) f on the ordinate and $\sqrt{\ell}$ on the abscissa

(3) f^2 on the ordinate and ℓ on the abscissa

(4) f^2 on the ordinate and $1/\ell$ on the abscissa

29. The sum of the series $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\ldots\infty$ is -

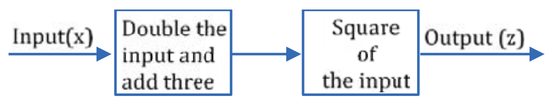
(1) $\frac{8}{7}$

(2) $\frac{6}{5}$

(3) $\frac{2}{3}$

(4) $\frac{3}{2}$

30. In the given figure, each box represents a function machine. A function machine illustrates what it does with the input.



Which of the following statements is correct ?

(1) $z=(2x+3)^2$

(2) $z=2(x+3)$

(3) $z=\sqrt{2x+3}$

(4) $z=\sqrt{2(x+3)}$

31. Given $s=t^2+5t+3$, find $\frac{ds}{dt}$.

(1) 2t+5

(2) $\frac{t^3}{3}+5t^2+3t$

(3) t+5

(4) None

32. The minimum value of $y=5x^2-2x+1$ is -

(1) $\frac{1}{5}$

(2) $\frac{2}{5}$

(3) $\frac{4}{5}$

(4) $\frac{3}{5}$

33. Evaluate the integrals :- $\int_{-\pi/2}^{\pi/2} \cos x\,dx$ -

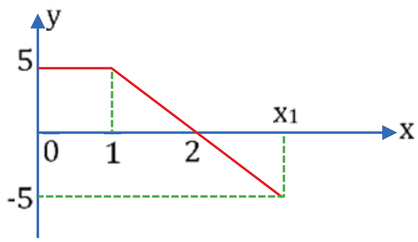
(1) 0

(2) 2

(3) −2

(4) 1

34. Find the value of x_1, so that $\int_0^{x_1} y\,dx = 5$

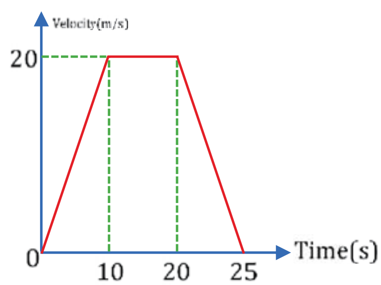


(1) 2
(2) 7
(3) 3
(4) 5

35. Value $\int_0^2 3x^2 dx + \int_0^{\pi/2} \sin x\,dx$ is -

(1) 8
(2) 7
(3) 9
(4) 10

36. The velocity-time graph of a car moving along a straight road is shown in figure. The average velocity of the car in first 25 seconds is -



(1) 20 m/s
(2) 14 m/s
(3) 10 m/s
(4) 17.5 m/s

37. The speed (v) of a particle moving along a straight line is given by $v = t^2 + 3t - 4$ where v is in m/s and t in seconds. Find time t at which the particle will momentarily come to rest.
(1) t = 4s
(2) t = 1s
(3) t = 2s
(4) t = 0s

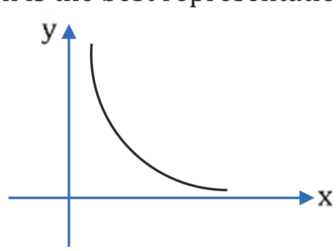
38. The mass m of a body moving with a velocity v is given by $m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where m_0 = rest mass of body = 10 kg and c = speed of light = $3\times10^8 m/s$. Find the value of m at v = $3\times10^7 m/s$.
(1) 10 kg
(2) 9.95 kg
(3) 9.99 kg
(4) 10.05 kg

39. Find the value of $\log_{25}^{3\sqrt{5}}$ -
(1) $\frac{1}{3}$
(2) $\frac{1}{2}$
(3) $\frac{1}{6}$
(4) $\frac{3}{5}$

40. Solve for x :-
log(3x+2) — log(3x–2) = log5
(1) –1
(2) 1
(3) $\frac{2}{3}$
(4) $-\frac{2}{3}$

41. The slope of straight line $\sqrt{3}y=3x+4$ is -
(1) 3
(2) $\sqrt{3}$
(3) $\frac{1}{\sqrt{3}}$
(4) $\frac{1}{3}$

42. Which of the following equation is the best representation of the given graph?



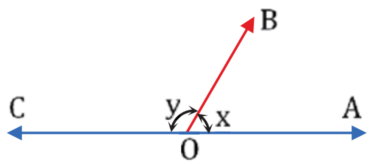
(1) $y = e^{-x}$

(2) $y = e^{x}$

(3) $y = \frac{1}{x}$

(4) None of these

43. If y-x = 80° then find the values of x and y –



(1) 50°, 130°
(2) 60°, 120°
(3) 70°, 110°
(4) 80°, 100°

44. A function has the form f(x)=ax+b, where a and b are constants. If f(2)=1 and f(-3)=11, the function is defined by -

(1) f(x)=2x+5
(2) f(x)=2x–5
(3) f(x)=–2x+5
(4) f(x)= –2x–5

45. If x = $\sqrt{2}$ -1 then find the value of $\left(\frac{1}{x} - x\right)^3$:-

(1) $2\sqrt{2}+1$

(2) $2\sqrt{2}-4$

(3) 8

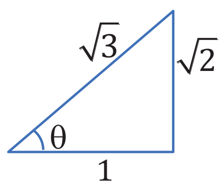
(4) 27

Answer Key

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**1**	**1**	**3**	**2**	**1**	**3**	**3**	**2**	**2**	**3**	**3**	**2**	**2**	**2**	**1**
Question	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Answer	**1**	**2**	**2**	**4**	**3**	**1**	**3**	**3**	**1**	**3**	**3**	**2**	**1**	**4**	**1**
Question	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
Answer	**1**	**3**	**2**	**3**	**3**	**2**	**2**	**4**	**3**	**2**	**2**	**3**	**1**	**3**	**3**

SOLUTIONS

2.



$\sin\theta = \frac{\sqrt{2}}{\sqrt{3}}$

$\left(\sqrt{3}\right)^2 = \left(\sqrt{2}\right)^2 + (\text{Base})^2$

Base = 1

$\tan\theta = \frac{P}{B} = \frac{\sqrt{2}}{1}$

3. If $\sin\theta = \cos\theta$

$\tan\theta = 1$

For $180° < \theta < 360° \Rightarrow \theta = 225°$

4. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

Then $\theta_1 = \theta_2 = \theta_3 = 90°$

So $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

5. If $\tan(2A + B) = \sqrt{3}$

$2A + B = 60°$ ....(1)

and $\cot(3A - B) = \sqrt{3}$

$3A - B = 30°$ ....(2)

From eq^n (1) & eq^n (2)

$A = 18°$ and $B = 24°$

6. $\sin^2(15) + \sin^2(30) + \sin^2(45) + \sin^2(60) + \sin^2(75)$

$$=\left[\frac{1-\cos(30^\circ)}{2}\right]+\left(\frac{1}{4}\right)+\left(\frac{1}{2}\right)+\left(\frac{3}{4}\right)+\left[\frac{1-\cos(150^\circ)}{2}\right]$$

$$=\left(\frac{1}{2}\right)-\left(\frac{\sqrt{3}}{4}\right)+\left(\frac{1}{4}\right)+\left(\frac{1}{2}\right)+\left(\frac{3}{4}\right)+\left(\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{4}\right)$$

$$=2+\frac{1}{2}=\frac{5}{2}$$

7. $\sin(45^\circ+\theta)\cos(15^\circ+\theta) - \cos(45^\circ+\theta)\sin(15^\circ+\theta)$

$= \sin(45^\circ+\theta-15^\circ-\theta)$

$= \sin(30^\circ) = \dfrac{1}{2}$

8. $\sin(-420^\circ)\cos(390^\circ) + \cos(-660^\circ)\sin(330^\circ)$

$= -\sin(360^\circ+60^\circ)\cos(360^\circ+30^\circ) + \cos(720^\circ-60^\circ)\sin(360^\circ-30^\circ)$

$$= -\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)=-\frac{3}{4}-\frac{1}{4}=-1$$

9. $\tan 1^\circ \tan 2^\circ \tan 3^\circ \,.......\, \tan 89^\circ$

$= \tan 1^\circ \tan 89^\circ \tan 2^\circ \tan 88^\circ \,.......\, \tan 44^\circ \tan 46^\circ \tan 45^\circ$

$= \tan 1^\circ \tan(90^\circ-1^\circ) \tan 2^\circ \tan(90^\circ-2^\circ) \,.......\, \tan 44^\circ \tan(90^\circ-44^\circ) \tan 45^\circ$

$= \tan 1^\circ \cot 1^\circ \tan 2^\circ \cot 2^\circ \,.......\, \tan 44^\circ \cot 44^\circ \tan 45^\circ$

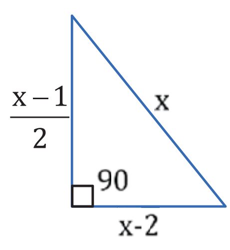
$= \tan 45^\circ = 1$

10. Max. value = $\sqrt{7^2+(-24)^2}=\sqrt{49+576}=25$

11. Hypotenuse (H) = x

Base (B) = x – 2

Perpendicular (P) =



$$x^2=(x-2)^2+\left(\frac{x-1}{2}\right)^2$$

$$4x^2=4(x-2)^2+(x-1)^2$$

$$x^2-18x+17=0$$

x = 17 cm and x = 1 cm (not considerable)

So, H = 17 cm

B = 17 – 2 = 15 cm

$P = \dfrac{17-1}{2} = 8$ cm

12. $x = a t^4$; $y = b t^3$

$\frac{dx}{dt} = 4a t^3$ & $\frac{dy}{dt} = 3b t^2$

So $\frac{dy}{dt} = \frac{3bt^2}{4at^3} = \frac{3b}{4at}$

13. $A = 5t^2 + 4t + 8$

$\frac{dA}{dt} = 10t + 4$

At $t = 3$ sec

$\frac{dA}{dt} = 34\ m^2/s$

14. If side = a, then rate of increase of perimeter w.r.t.

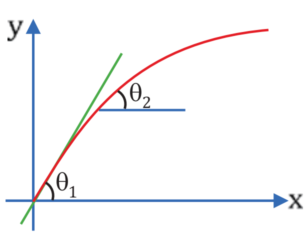
time = $4\left(\frac{da}{dt}\right) = 4(0.1) = 0.4\,cm/s$

15. $3y = x + 5$

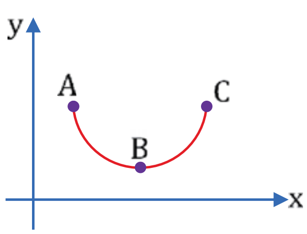
$\frac{3dy}{dt} = \frac{dx}{dt}$

x changes at a faster rate

16. $\theta_1 > \theta_2 > \theta_3 \Rightarrow m_1 > m_2 > m_3$



17.



From A to B : θ is obtuse and increases, so slope (tan θ) will decreases.

From B to C : θ is acute and increases

So, B to C magnitude will increases.

18. Area under the curve $\int_2^3 f(x)dx = \int_2^3 x^2 dx$

$$= \frac{1}{3}[x^3]_2^3 = \frac{19}{3}$$

19. $\int_0^4 |1-x| dx = \int_0^1 (1-x)dx + \int_1^4 -(1-x)dx$

$$= [x]_0^1 - \left[\frac{x^2}{2}\right]_0^1 - [x]_1^4 + \left[\frac{x^2}{2}\right]_1^4$$

$$= 1 - \frac{1}{2} - 3 + \frac{15}{2} = 5$$

20. $y = x^2 + 2 - 3x$

on y-axis, $x = 0$

$\Rightarrow y = 2$

The curve will intersects y axis at (0, 2)

21. At t = 2 $x_A = 6,\ y_A = 2$

$x_B = 6,\ y_B = 2 + 3(2)^2 = 14$

Distance = $\sqrt{(6-6)^2 + (14-2)^2} = 12$

22. Distance d = $\sqrt{\{a-b-a-b\}^2 + \{c-b-b-c\}^2}$

$$d = \sqrt{4b^2 + 4b^2} = 2\sqrt{2}b$$

23. Minimum distance = $\sqrt{(5-0)^2 + (3-3)^2 + (-8-4)^2}$

$$= \sqrt{5^2 + 12^2} = 13m$$

25. $\because$ $y = x^2 + 2x - 3$

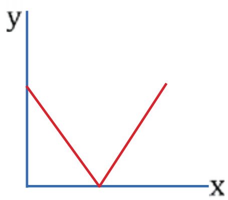
$\therefore$ $y = (x + 3)(x - 1)$

For $y = 0 \Rightarrow x = -3$ and $x = 1$

For $x = 0 \Rightarrow y = -3$

26. For $0 \le x \le 1$, $y = -(x-1) = -x + 1$

For $x \ge 1$, $y = x-1$



27. $x = a\cos\omega t,\ y = a\sin\omega t$

$\Rightarrow \quad x^2 + y^2 = a^2 \Rightarrow$ circle

28. $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{\ell}}$

$f^2 = \dfrac{1}{4\pi^2}\dfrac{g}{\ell}$

$\Rightarrow f^2 \propto \dfrac{1}{\ell}$ graph between f & $\dfrac{1}{\ell}$ is parabolic

29. $1 + \dfrac{1}{3} + \dfrac{1}{9} + \dfrac{1}{27} + ...\infty$

So, $\text{sum} = \dfrac{a}{1-r} = \dfrac{1}{1-1/3} = \dfrac{3}{2}$

30.

$\xrightarrow{x} \boxed{2x+3} \longrightarrow \boxed{(2x+3)^2} \xrightarrow{z=(2x+3)^2}$

31. $\dfrac{ds}{dt} = \dfrac{d}{dt}(t^2 + 5t + 3)$

$= 2t+5$

32. For maximum/minimum value $\dfrac{dy}{dx} = 0 \Rightarrow 5(2x) - 2(1) + 0 = 0 \Rightarrow x = \dfrac{1}{5}$.

Now at $x = \dfrac{1}{5}, \dfrac{d^2y}{dx^2} = 10$ which is positive

so y has minimum value at $x = \dfrac{1}{5}$. Therefore $y_{min} = 5\left(\dfrac{1}{5}\right)^2 - 2\left(\dfrac{1}{5}\right) + 1 = \dfrac{4}{5}$

33. $= [+\sin x]_{-\pi/2}^{\pi/2} = \left[\sin\pi/2 - (\sin(-\pi/2))\right]$

$= [\sin\pi/2 + \sin\pi/2] = 2$

34. Area under the curve = $\displaystyle\int_0^{x_1} y\,dx$

$\Rightarrow 1\times5 + \dfrac{1}{2}\times1\times5 - \dfrac{1}{2}(x_1 - 2)\times5 = 5$

$\Rightarrow \dfrac{1}{2}\times1\times5 = \dfrac{1}{2}(x_1 - 1)(5)$

$\Rightarrow x_1 = 3$

35. $=\left[3\dfrac{x^3}{3}\right]_0^2+\left[-\cos x\right]_0^{\pi/2}$

$=[8-0]+\left[-\cos\left(\dfrac{\pi}{2}\right)-\left(-\cos(0)\right)\right]$

$=8+[0+1]=9$

36. Average velocity $=\dfrac{\int_0^{25} v\,dt}{25-0}=\dfrac{\text{Area of v-t graph between t=0 to t=25 s}}{25}$

$=\dfrac{1}{25}\left[\left(\dfrac{25+10}{2}\right)(20)\right]=14\text{m}/\text{s}$

37. When particle comes to rest, v = 0

So $t^2+3t-4=0$ $\Rightarrow t=\dfrac{-3\pm\sqrt{9-4(1)(-4)}}{2(1)}\Rightarrow$ t = 1 or -4

Neglect negative value of t=–4,
Hence t = 1 s

38. $m=m_0\left(1-\dfrac{v^2}{c^2}\right)^{-1/2}=10\left[1-\left(\dfrac{3\times10^7}{3\times10^8}\right)^2\right]^{-1/2}=10\left[1-\dfrac{1}{100}\right]^{-1/2}\approx10\left[1-\left(-\dfrac{1}{2}\right)\left(\dfrac{1}{100}\right)\right]$

$=10+\dfrac{10}{200}\approx10.05\text{kg}$

39. $\log_{5^2}^{(5)^{1/3}}=\dfrac{1}{2\times3}\log_5^5$ $\{\therefore\log_a^a=1\}$

$=\dfrac{1}{6}$

40. $\log\left(\dfrac{3x+2}{3x-2}\right)=\log5$

$\dfrac{3x+2}{3x-2}=5\Rightarrow x=1$

41. $y=\dfrac{3}{\sqrt{3}}x+\dfrac{4}{\sqrt{3}}$ $\{\therefore y=mx+c\}$

$y=\sqrt{3}x+\dfrac{4}{\sqrt{3}}$

$\therefore m=\sqrt{3}$

42. xy = constant
So, Given graph is rectangular hyperbola

43. Given

$y-x = 80°$ .....(i)

From Diagram

$y+x = 180°$ ...(ii)

from equation (i) and (ii)

$x = 50°; y = 130°;$

44. $f(2) = 2a+b = 1$ ....(i)

$f(-3) = -3a + b = 11$ ....(ii)

from equation (i) and (ii)

$a = -2; b = 5$

so, $f(x) = -2x+5$

45. $$=\left(\frac{1}{x}-x\right)^3=\left(\frac{1-x^2}{x}\right)^3$$

$$=\left(\frac{1-\left(\sqrt{2}-1\right)^2}{\left(\sqrt{2}-1\right)}\right)^3$$

$$=\left(\frac{2\left(\sqrt{2}-1\right)}{\left(\sqrt{2}-1\right)}\right)^3=8$$

DAILY PRACTICE PROBLEM

Range of Trigonometric Functions DPP - 01

1. **If $\sec\theta = \frac{5}{3}$, find $\sin\theta$ and $\tan\theta$?**

(1) $\frac{3}{5}, \frac{3}{4}$

(2) $\frac{4}{5}, \frac{4}{3}$

(3) $\frac{5}{4}, \frac{4}{3}$

(4) $\frac{4}{5}, \frac{3}{4}$

2. **Find the values of :**

(i) tan (-30°) (ii) cos 150° (iii) sin 210°

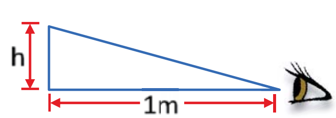
(1) $\frac{1}{\sqrt{3}}, \frac{\sqrt{3}}{2}, \frac{1}{2}$

(2) $-\frac{1}{\sqrt{3}}, -\frac{\sqrt{3}}{2}, -\frac{1}{2}$

(3) $-\frac{1}{\sqrt{3}}, -\frac{\sqrt{3}}{2}, \frac{1}{2}$

(4) $-\frac{1}{\sqrt{3}}, +\frac{\sqrt{3}}{2}, -\frac{1}{2}$

3. **A normal human eye can see an object making an angle of 1.8° at the eye. What is the**



approximate height of object which can be seen by an eye placed at a distance of 1 m from the object.

(1) 0.031 m

(2) π m

(3) 0.031 cm

(4) 0.31 m

4. **Convert the angle from degree to radian :**

(a) 210° **(b) 315°**

(1) $\frac{7\pi}{6},\frac{7\pi}{4}$

(2) $\frac{5\pi}{6},\frac{5\pi}{4}$

(3) $\frac{\pi}{6},\frac{\pi}{4}$

(4) $\frac{3\pi}{6},\frac{3\pi}{4}$

5. **Convert the following angle from radian to degree**

(a) $\frac{3\pi}{4}$**rad** **(b)** $\frac{7\pi}{6}$**rad**

(1) 135°, 210°

(2) 210°, 135°

(3) 225°, 240°

(4) 135°, 225°

6. **Find the value of the following :-**

(a) $\cot\left(\frac{3\pi}{4}\right)$ **(b)** $\cos\left(\frac{7\pi}{3}\right)$

(1) $-1,-\frac{1}{2}$

(2) $+1,\frac{1}{2}$

(3) $-1,\frac{1}{2}$

(4) $+1,-\frac{1}{2}$

7. **The maximum and minimum values of expression $(4 - 2\cos\theta)$ respectively are**

(1) 4 and 0

(2) 4 and 2

(3) 6 and 0

(4) 6 and 2

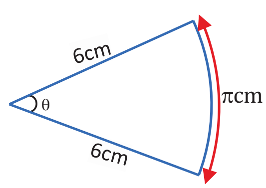
8. **A circular arc is of length π cm. Find angle subtended by it at the centre in radian and degree.**

(1) 60°

(2) 30°

(3) 90°

(4) 15°



9. **What is value of expression $2(\sin 15^\circ + \sin 75^\circ)^2$?**

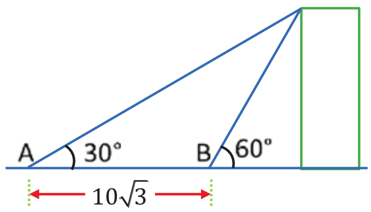
(1) 3/2

(2) 1/2

(3) 2

(4) 3

10. **Angle of elevation is the angle which line of sight makes with the horizontal. Angle of elevation of the top of a tall building is 30° from a place A and becomes 60° from another place B that is 10Ö3 m from A towards the building as shown in the figure. Height of the building is close to**



(1) 7.5 m

(2) 10 m

(3) 12.5 m

(4) 15 m

11. **Value of sin (37°) cos (53°) is -**

(1) $\frac{9}{25}$

(2) $\frac{12}{25}$

(3) $\frac{16}{25}$

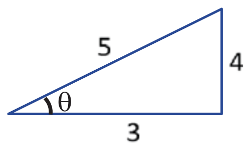
(4) $\frac{3}{5}$

Answer key

Question	1	2	3	4	5	6	7	8	9	10	11
Answer	**2**	**2**	**1**	**1**	**1**	**3**	**4**	**2**	**4**	**4**	**1**

SOLUTIONS

1. **(2)**



$\sec\theta = \frac{5}{3}$

$\therefore \cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$; $\tan\theta = \frac{4}{3}$

2. **(2)**

(i) $\tan(-30°) = -\tan 30° = -1/\sqrt{3}$

(ii) $\cos(150°) = \cos(90°+60°) = -\sin(60°) = -\sqrt{3}/2$

(iii) $\sin(210°) = \sin(180°+30°) = -\sin(30°) = -1/2$

3. **(1)**

θ is very small

$\therefore \tan\theta \approx \theta$

$\theta = \frac{1.8° \times \pi}{180°} = \frac{\pi}{100}\text{rad}$

$\frac{h}{1} = \frac{\pi}{100}$

$\therefore$ h = 0.031 m

4. **(1)**

(a) $\frac{210° \times \pi}{180°} = \frac{7\pi}{6}$ (b) $\frac{315° \times \pi}{180°} = \frac{7\pi}{4}$

5. **(1)**

(a) $\frac{3\pi}{4} \times \frac{180°}{\pi} = 135°$ (b) $\frac{7\pi}{6} \times \frac{180°}{\pi} = 210°$

6. **(3)**

(a) $= \cot\left(\frac{3\pi}{4}\right) = \cot\left(\pi - \frac{\pi}{4}\right) = -\cot\frac{\pi}{4} = -1$

(b) $= \cos\left(2\pi + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

7. **(4)**

Maximum value of $\cos\theta = 1$

Minimum value of $\cos\theta = -1$

$\therefore$ Maximum value of given function is = $(4-2(-1)) = 6$

$\therefore$ Minimum value of given function is = $(4-2(1)) = 2$

8. **(2)**

$$\theta = \frac{s}{r} = \frac{\pi cm}{6cm} = \frac{\pi}{6} rad = 30°$$

9. **(4)**

= $2(\sin 15° + \sin 75°)^2$

= $2(\sin 15° + \cos 15°)^2$ $(\because \sin\theta = \cos(\pi/2 - \theta))$

= $2[\sin^2 15° + \cos^2 15° + 2\sin 15° \cos 15°]$

= $2[1 + \sin 30°]$ $(\because 2\sin\theta\cos\theta = \sin 2\theta)$

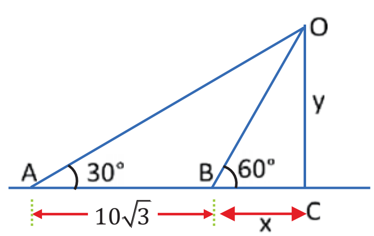
= 3

10. **(4)**

For ΔBCO

$\tan 60° = \frac{y}{x}$

$x = \left(\frac{y}{\sqrt{3}}\right)$ ...(i)



For ΔACO

$$\tan 30° = \frac{y}{(x + 10\sqrt{3})}$$

$$\frac{1}{\sqrt{3}} = \frac{y}{\left(\frac{y}{\sqrt{3}} + 10\sqrt{3}\right)}$$

y + 30 = 3y

y = 15m

11. **(1)**

$$\sin 37° = \frac{3}{5}$$

$$\cos 53° = \frac{3}{5}$$

$\Rightarrow \sin(37°)\cos(53°) = \frac{9}{25}$

DPP

DAILY PRACTICE PROBLEM

Equation of Straight Line DPP - 02

1. Distance between two points (8, – 4) and (0, a) is 10. All the values are in the same unit of length. Find the positive value of a.

(1) –10

(2) +2

(3) –2

(4) +10

2. Calculate the distance between two points (0, –1, 1) and (3, 3, 13).

(1) 12

(2) 9

(3) 16

(4) 13

3. The slope of straight line 2y = 3x+5;

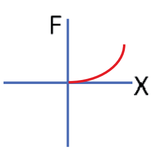
(1) 3

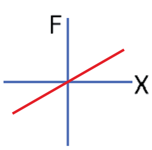
(2) 1

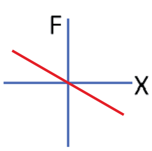
(3) $\frac{3}{2}$

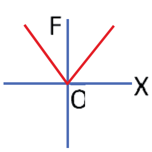
(4) $\frac{5}{2}$

4. The spring force is given by F = -kx, here k is a constant and x is the deformation of spring. The F-x graph is -

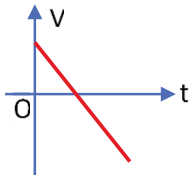
(1) 

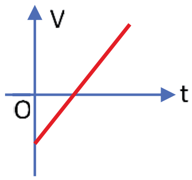
(2) 

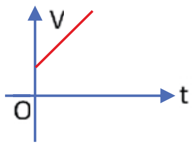
(3) 

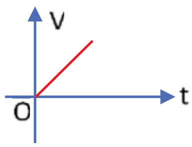
(4) 

5. If velocity v varies with time(t) as v = 2t – 3, then the plot between v and t is best represented by :

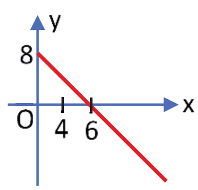
(1) 

(2) 

(3) 

(4) 

6. The equation of straight line shown in figure is :



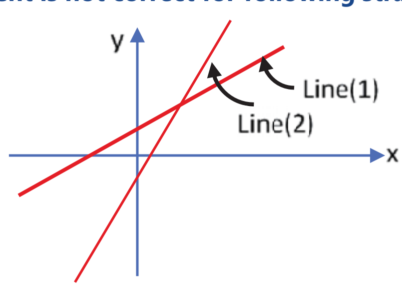
(1) $6x + 8y = 15$

(2) $4x + 3y = 18$

(3) $2y + 6x = 7$

(4) $3y + 4x = 24$

7. Which of the following statement is not correct for following straight line graph :-



(1) Line (2) has negative y intercept

(2) Line (1) has positive y intercept

(3) Line (2) has positive slope

(4) Line (1) has negative slope

Answer key

Question	1	2	3	4	5	6	7
Answer	**2**	**4**	**3**	**3**	**2**	**4**	**4**

SOLUTIONS

1. **(2)**

Let P(8, –4) and Q(0, a) be two points, distance PQ is 10.

$\therefore$ According to distance formula

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0-8)^2 + (a+4)^2}$$

$$\Rightarrow PQ = \sqrt{64 + (a^2 + 16 + 8a)}$$

$$\Rightarrow 10 = \sqrt{80 + a^2 + 8a}$$

$\Rightarrow$ $a^2 + 8a + 80 = 100$ $\Rightarrow$ $a^2 + 8a = 20$

$\Rightarrow$ $a^2 + 10a - 2a - 20 = 0$

$\Rightarrow$ $(a - 2)(a + 10) = 0$ $\Rightarrow$ $(a = 2)$ and $(a = -10)$

$\therefore$ a = 2

2. **(4)**

$P(x_1, y_1, z_1) = (0, -1, +1)$

and $Q(x_2, y_2, z_2) = (3, 3, 13)$

using distance formula

$$\Rightarrow PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

3. **(3)**

2y = 3x + 5 (given equation) ....(i)

y = mx + c (general equation) ....(ii)

Comparing equation (i) and (ii)

slope = m = $\frac{3}{2}$

4. **(3)**

F = -kx (given equation) ...(i)

y = mx + C (general equation) ...(ii)

Comparing Both equations

Slope = m = -K-; Negative slope

Intercept = C = 0; So line passing through origin

5. **(2)**

v = 2t – 3

y = mx + c

m = 2; c = –3

6. **(4)**

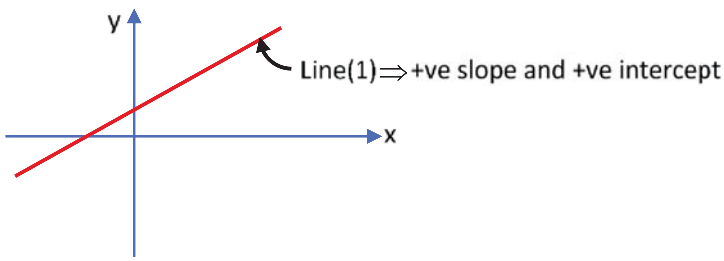
$C = 8,\ m = -\frac{8}{6} = -\frac{4}{3}$

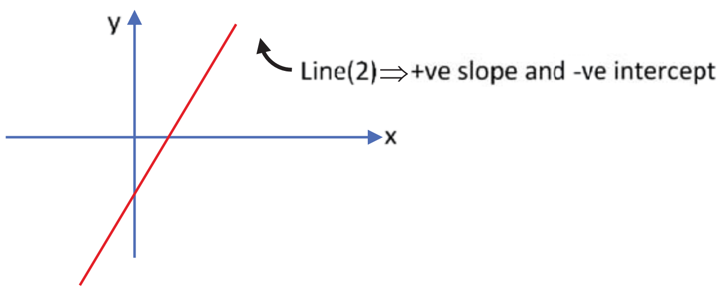
y = mx + c

$y = -\frac{4}{3}x + 8$

$\therefore 4x + 3y = 24$

7. **(4)**





DPP
DAILY PRACTICE PROBLEM

Rules of Differentiation - Basic DPP - 03

1. Find derivative of $y = 8$, w.r.t x

(1) 8x
(2) 0
(3) can't find
(4) None

2. Differentiate with respect to x –

$$\frac{d}{dx}\left(\frac{4}{x^3}\right)$$

(1) $\frac{x^{-2}}{-2}$
(2) $12x^4$
(3) $\frac{-12}{x^4}$
(4) $\frac{4}{x^2}$

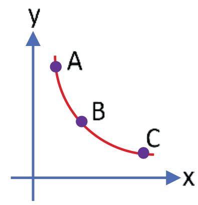
3. If $y = \log_e x + \sin x + e^x$ then $\frac{dy}{dx}$ is -

(1) $\frac{1}{x}+\sin+e^x$
(2) $\frac{1}{x}-\cos x+e^x$
(3) $\frac{1}{x}+\cos x+e^x$
(4) $\frac{1}{x}-\sin x$

4. Find derivative of $y=x^3+\frac{4}{3}x^2-5x+1$

(1) $\frac{x^4}{4}+\frac{4x^3}{9}-\frac{5x^2}{2}+x$
(2) $3x^2+\frac{8}{3}x-5$
(3) x^2+x-5
(4) None

5. The slope of graph in figure at point A, B and C is m_A, m_B and m_C respectively, then :



(1) $m_A > m_B > m_C$

(2) $m_A < m_B < m_C$

(3) $m_A = m_B = m_C$

(4) $m_A = m_C < m_B$

6. If y = a sin x + b cos x, then $y^2 + \left(\frac{dy}{dx}\right)^2$ is a -

(1) Function of x

(2) Function of y

(3) Function of x and y

(4) Constant

7. Find differentiation of y w.r.t. x –
If $y = 4\ell nx + \cos x$

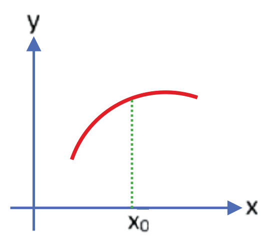
(1) $\frac{4}{x} + \sin x$

(2) $4\ell nx - \sin x$

(3) $\frac{4}{x} - \sin x$

(4) $\frac{4}{x} + \cos x$

8. Which of the following statements are true based on graph of y-versus x as shown below?



(1) Slope at x_0 is positive and non-zero in graph

(2) Slope is constant in graph

(3) Slope at x_0 is negative in graph

(4) None

Answer key

Question	1	2	3	4	5	6	7	8
Answer	2	3	3	2	2	4	3	1

SOLUTIONS

1. **(2)**

$$\frac{d}{dx}(8)=0$$

$$\left\{\frac{d(c)}{dx}=0;\ \text{where c is constant}\right\}$$

2. **(3)**

$$\frac{d}{dx}\left(\frac{4}{x^3}\right)=\frac{d}{dx}(4x^{-3})$$

$$=4(-3)x^{-4}=\frac{-12}{x^4}$$

3. **(3)**

$$\frac{dy}{dx}=\frac{d}{dx}(\log_e x)+\frac{d}{dx}(\sin x)+\frac{d}{dx}(e^x)$$

$$=\frac{1}{x}+\cos x+e^x$$

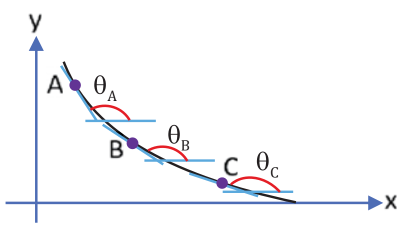
4. **(2)**

$$\Rightarrow \frac{d}{dx}\left(x^3+\frac{4}{3}x^2-5x+1\right)$$

$$\Rightarrow dx(x^3)+\frac{d}{dx}\left(\frac{4}{3}x^2\right)-\frac{d}{dx}(5x)+\frac{d}{dx}(1)$$

$$=3x^2+\frac{8}{3}x-5$$

5. **(2)**



$\because\ \theta_A<\theta_B<\theta_C$ (all are obtuse)

$\Rightarrow \tan\theta_A<\tan\theta_B<\tan\theta_C$

$\Rightarrow m_A<m_B<m_C$

6. **(4)**

y = a sin x + b cos x ....(i)

$$\frac{dy}{dx}=\frac{d}{dx}(a\sin x+b\cos x)$$

$$=a\cos x-b\sin x \quad ....(ii)$$

Now, $y^2+\left(\frac{dy}{dx}\right)^2=(i)^2+(ii)^2$

$$=a^2\sin^2 x+b^2\cos^2 x+2a\sin x b\cos x+a^2\cos^2 x+b^2\sin^2 x-2a\cos x b\sin x$$

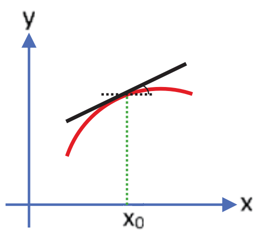
$=a^2+b^2$ = constant

7. **(3)**

$$\frac{dy}{dx}=\frac{d}{dx}(4\ell nx+\cos x)$$

$$=\frac{d}{dx}(4\ell nx)+\frac{d}{dx}(\cos x)=4\left(\frac{1}{x}\right)-\sin x$$

8. **(1)**



DAILY PRACTICE PROBLEM

Rules of Differentiation - Chain Rule DPP - 04

1. **Find value of $\frac{dy}{dx}$, If y = (x-1) (2x+5)**

(1) 4x+5

(2) 3

(3) 4x+3

(4) x+3

2. **Find the derivative of $y=\frac{3x+4}{4x+5}$**

(1) $\frac{-1}{(4x+5)^2}$

(2) $\frac{1}{(4x+5)^2}$

(3) $\frac{24x+31}{(4x+5)^2}$

(4) $\frac{-24x-31}{(4x+5)^2}$

3. **Find the first derivative of y = xsinx**

(1) -xcos+sinx

(2) -xcosx+x^2sinx

(3) xsinx+cosx

(4) xcosx+sinx

4. **Find value of $\frac{dy}{dx}$, If y = 2cos$\left(\sqrt{x}\right)$**

(1) $\frac{-1}{2\sqrt{x}}\sin(\sqrt{x})$

(2) $\frac{1}{\sqrt{x}}\sin(\sqrt{x})$

(3) $\frac{1}{2\sqrt{x}}\sin(\sqrt{x})$

(4) $\frac{-1}{\sqrt{x}}\sin(\sqrt{x})$

5. $\frac{d}{dx}(e^{100}) =$ ....

(1) e^{100}

(2) 0

(3) $_{100}e^{999}$

(4) None of these

6. If $y = x^3\cos x$ then $\frac{dy}{dx} =$ ....

(1) $x^2(3\cos x - x\sin x)$

(2) $x^2(3\cos x + x\sin x)$

(3) $3x^2\cos x + x^3\sin x$

(4) None of these

7. If $y = \sin x$ & $x = 3t$ then $\frac{dy}{dt}$ will be

(1) $3\cos(x)$

(2) $\cos x$

(3) $-3\cos(x)$

(4) $-\cos x$

8. If $y = \frac{3x}{\tan x}$ then $\frac{dy}{dx}$ will be -

(1) $\frac{3}{\sec^2 x}$

(2) $\frac{3\tan x - 3x\sec^2 x}{\tan^2 x}$

(3) $\frac{3\tan x + 3x\sec^2 x}{\tan^2 x}$

(4) $\frac{3x\sec^2 x - 3\tan x}{\tan^2 x}$

9. If $y = 2\sin(\omega t + \phi)$ where ω and ϕ constants then $\frac{dy}{dt}$ will be -

(1) $2\cos(\omega + \phi)$

(2) $2\omega\sin(\omega t + \phi)$

(3) $2\omega\cos(\omega t + \phi)$

(4) $-2\omega\cos(\omega t + \phi)$

10. If $y = \tan x \cdot \cos^2 x$ then $\frac{dy}{dx}$ will be -

(1) $1+2\sin^2 x$

(2) $\sin^2 x - \cos^2 x$

(3) $\sin^2 x + \cos^2 x$

(4) $1-2\sin^2 x$

11. **If $y = 4e^{x^2-2x}$ then $\frac{dy}{dx}$ will be -**

(1) $(8x-8)(e^{x^2-2x})$

(2) $(2x-2)(e^{x^2-2x})$

(3) $(8x-8)(e^{2x-2})$

(4) $4e^{x^2-2x}$

12. **Find the derivative of y = 4sin3x**

(1) 4cos3x

(2) 12cos(3x)

(3) $\frac{4}{3}$ cos(3x)

(4) None

13. **Find the derivative of y, w.r.t. t**

$y = \ell n(t^2+t)$

(1) $\frac{1}{t^2+t}$

(2) $\frac{1}{2t+1}$

(3) $\frac{2t+1}{t^2+t}$

(4) $\frac{1}{(2t+1)(t^2+t)}$

14. **Find derivative of y = $(x^3+1)^2$**

(1) $(x^3+1)(3x^2)$

(2) $2(x^3+1)$

(3) $2(3x^2)$

(4) $2(x^3+1)(3x^2)$

Answer key

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Answer	**3**	**1**	**4**	**4**	**2**	**1**	**1**	**2**	**3**	**4**	**1**	**2**	**3**	**4**

SOLUTIONS

1. **(3)**

$$\frac{dy}{dx}=\frac{d}{dx}(x-1)(2x+5)$$

$$=(x-1)[2]+(2x+5)(1)$$

$$=2x-2+2x+5=4x+3$$

2. **(1)**

$$\frac{dy}{dx}=\frac{d(3x+4)}{dx(4x+5)}$$

$$=\frac{(4x+5)\frac{d}{dx}(3x+4)-(3x+4)\frac{d}{dx}(4x+5)}{(4x+5)^2}$$

$$=\frac{(4x+5)(3)-(3x+4)(4)}{(4x+5)^2}=\frac{-1}{(4x+5)^2}$$

3. **(4)**

$$\frac{dy}{dx}=\frac{d}{dx}(x\sin x)$$

$$=x\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(x)$$

$$=x(\cos x)+\sin x$$

4. **(4)**

Let $U=x^{1/2}$; $\quad y=2\cos(U)$

$$\frac{dv}{dx}=\frac{1}{2}x^{-1/2} \qquad \frac{dy}{dv}=-2\sin u$$

$$\frac{dy}{dx}=\frac{dv}{dx}\times\frac{dy}{dv} \qquad =\frac{1}{2}x^{-1/2}\times(-2\sin v)$$

$$=-\frac{1}{\sqrt{x}}\sin(\sqrt{x})$$

5. **(2)**

$\frac{d}{dx}$ (constant) = 0

6. **(1)**

$y = x^3 \cos x$

apply product rule

$$\frac{dy}{dx}=3x^2\cos x-x^3\sin x$$

$= x^2(3 \cos x - x \sin x)$

7. **(1)**

y = sinx x = 3t

$\frac{dy}{dx}=\cos x$ $\frac{dx}{dt}=3$

$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=3\cos x$

8. **(2)**

$\frac{dy}{dx}=\frac{d(3x)}{dx(\tan x)}$

$=\frac{\tan x\frac{d}{dx}(3x)-(3x)\frac{d}{dx}(\tan x)}{(\tan^2 x)}$

$=\frac{3\tan x-3x\sec^2 x}{\tan^2 x}$

9. **(3)**

$u=\omega t+\phi;$ y = 2sin(u)

$\frac{du}{dt}=\omega;$ $\frac{dy}{du}=2\cos u$

$\frac{dy}{dt}=\frac{du}{dt}.\frac{dy}{du}$

$\frac{dy}{dt}=(\omega)(2\cos u)=2\omega\cos(\omega t+\phi)$

10. **(4)**

y = tanx . $\cos^2 x$ = $\frac{\sin x}{\cos x}.\cos^2 x$ =sinx cosx

$\frac{dy}{dx}=\sin x\frac{d}{dx}(\cos x)+\cos x\frac{d}{dx}(\sin x)$

$=\sin x(-\sin x)+\cos x(\cos x)$

$=-\sin^2 x+\cos^2 x=1-2\sin^2 x$

11. **(1)**

$u=x^2-2x;$ $y=4e^u$

$\frac{du}{dx}=2x-2$ $\frac{dy}{du}=4e^u$

$\frac{dy}{dx}=\frac{du}{dx}\times\frac{dy}{du}=(2x-2)(4e^u)$

$=4(2x-2)(e^{x^2-2x})=(8x-8)(e^{x^2-2x})$

12. **(2)**

u = 3x y = 4sin(u)

$\frac{du}{dx}=3$ $\frac{dy}{du}=4\cos u$

$\frac{dy}{dx}=\frac{du}{dx}*\frac{dy}{du}$ = 3(4cosu)

=12cos(3x)

13. **(3)**

$u = t^2 + t \qquad y = \ell n(u)$

$\frac{dy}{dt} = \frac{du}{dt} \times \frac{dy}{du} \qquad = (2t+1)\left(\frac{1}{u}\right)$

$= \frac{2t+1}{t^2+t}$

14. **(4)**

$u = (x^3 + 1) \qquad y = (u^2)$

$\frac{du}{dx} = 3x^2 \qquad \frac{dy}{du} = 2u$

$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du} \qquad = (2u)(3x^2)$

$= 2(x^3+1)(3x^2)$

DAILY PRACTICE PROBLEM

Concept of Maxima and Minima DPP - 05

1. For a straight line $3y=\sqrt{3}x+3$. Choose correct alternative(s)

(1) $\frac{dy}{dx}=\tan 30°$

(2) $\frac{dx}{dy}=\cot 30°$

(3) y–intercept is 1

(4) All correct

2. If radius of a spherical bubble starts to increase with time t as r = 0.5t. What is the rate of change of volume of the bubble with time t = 4s ?

(1) 8π units/s

(2) 4π units/s

(3) 2π units/s

(4) π units/s

3. The slope of the tangent to the curve

y = ln (sinx) at x = $\frac{3\pi}{4}$ is

(1) 1

(2) –1

(3) $\ell n\sqrt{2}$

(4) $\frac{1}{\sqrt{2}}$

4. The charge flowing through a conductor beginning with time t=0 is given by the formula $q=2t^2+3t+1$ (coulombs). Find the current i = $\frac{dq}{dt}$ at the end of the 5th second.

(1) 23

(2) 66

(3) $\frac{31}{6}$

(4) 5

5. A metallic disc is being heated. Its area (in m^2) at any time t (in sec) is given by $A = 4t^2 + 2t$. Calculate the rate of increase in area at t = 4sec.

(1) 72 m^2/sec

(2) 72 m^2

(3) 34 m^2/sec

(4) 34 m^2

6. A car moves along a straight line whose equation of motion is given by
$s = 12t + 3t^2 - 2t^3$
where s is in metres and t is in seconds. The velocity of the car at start will be :–

(1) 7 m/s
(2) 9 m/s
(3) 12 m/s
(4) 16 m/s

7. If y = sinx + cosx then $\frac{d^2y}{dx^2}$ is :-

(1) sinx – cosx
(2) cosx – sinx
(3) –(sinx + cosx)
(4) None of these

8. Given that $y = \frac{10}{\sin x + \sqrt{3}\cos x}$. Minimum value of y is

(1) zero
(2) 2
(3) 5
(4) $10/(1+\sqrt{3})$

9. Find maxima and minima of function $y = x^3 - 18x^2 + 96x$

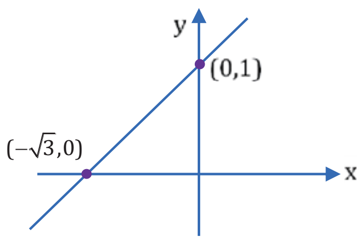
(1) 8,4
(2) 4,8
(3) 4,0
(4) 0,8

Answer key

Question	1	2	3	4	5	6	7	8	9
Answer	4	1	2	1	3	3	3	3	2

SOLUTIONS

1. (4)



$3y = \sqrt{3}x + 3$

(i) $\frac{dy}{dx} = \frac{1}{\sqrt{3}} = \tan 30°$

(ii) $\frac{dx}{dy} = \frac{\sqrt{3}}{1} = \cot 30°$

(iii) y–intercept means (x = 0) $\Rightarrow$ y = 1

2. (1)

Given r = 0.5 t

$\frac{dr}{dt} = 0.5$

$v = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

at t = 4 sec

r = 0.5(4) $\Rightarrow$ r = 2

So $\left(\frac{dv}{dt}\right)_{t=4} = 4\pi(2)^2(0.5)$

$= 8\pi$

3. (2)

$y = \ell n(\sin x)$

$\frac{dy}{dx} = \frac{1}{\sin x}(\cos x) = \cot x$

$x = \frac{3\pi}{4}$

$\frac{dy}{dx} = \cot\left(\frac{3\pi}{4}\right) = -1$

4. (1)

q = $2t^2 + 3t + 1$

$\frac{dq}{dt} = 4t + 3$

$i_{t=5} = 4(5) + 3$

$i_{t=5} = 23$

5. (3)

A = $4t^2 + 2t$

$\frac{dA}{dt} = 8t + 2$

at t = 4 sec.

$\frac{dA}{dt} = 8(4) + 2 = 34\ m^2/sec$

6. (3)

$v = \frac{ds}{dt}$

v = $12 + 6t - 6t^2$

At t = 0 $\Rightarrow$ V = 12m/s

7. (3)

y = sin x + cos x

$\frac{dy}{dx} = \cos x - \sin x$

$\frac{d^2y}{dx^2} = -\sin x - \cos x$

= −(sin x + cos x)

8. (3)

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{10}{\sin x + \sqrt{3}\cos x}\right)$$

$$= \frac{\sin x + \sqrt{3}\cos x \frac{d}{dx}(10) - 10\frac{d}{dx}(\sin x + \sqrt{3}\cos x)}{(\sin x + \sqrt{3}\cos x)^2} = 0$$

$$\Rightarrow 0 - 10(\cos x + \sqrt{3}(-\sin x)) = 0$$

$$\Rightarrow \cos x - \sqrt{3}\sin x = 0$$

$$\Rightarrow \frac{\cos x}{\sin x} = \sqrt{3}$$

$$\Rightarrow \cot x = \sqrt{3}$$

$$\Rightarrow x = 30°$$

$$\therefore y = \frac{10}{\sin(30°) + \sqrt{3}\cos(30°)} = \frac{10}{\frac{1}{2} + \left(\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right)} = \frac{10}{2} = 5$$

9. (2)

Step-1

$y = x^3 - 18x^2 + 96x$

$\frac{dy}{dx} = 3x^2 - 36x + 96$

$= x^2 - 12x + 32$

Step-2

$x^2 - 12x + 32 = 0$

$(x-8)(x-4) = 0$

$\therefore$ $x = 8$ or $x = 4$

Step-3

$\frac{d^2y}{dx^2} = (2x - 12)$

at $x = 8$ $\frac{d^2x}{dx^2} > 0$ at $x = 4$ $\frac{d^2y}{dx^2} < 0$

at $x = 8$, minima at $x = 4$ maxima

DAILY PRACTICE PROBLEM

Definite Integration DPP - 06

1. **Evaluate the indefinite integral $\int \frac{dx}{(4x+5)}$** -

(1) $\log_e(4)$

(2) $\log_e(4x+5)+c$

(3) $\frac{1}{4}\log_e(4x+5)+c$

(4) None

2. **Evaluate $\int \frac{dx}{\sqrt[3]{x}}$** -

(1) $\frac{3}{2}x^{-2/3}+c$

(2) $\frac{2}{3}x^{-3/2}+c$

(3) $\frac{2}{3}x^{3/2}+c$

(4) $\frac{3}{2}x^{2/3}+c$

3. **Integrate $\int (2x^3 - x^2 + 1)dx$** -

(1) 6x-2x+c

(2) $6x^2$-2x+1+c

(3) $\frac{x^4}{2}-\frac{x^3}{3}+x+c$

(4) None

4. **Evaluate $\int \left(x^2 - \cos x + \frac{1}{x}\right)dx$** -

(1) $x^3 - \sin x + \ell nx + c$

(2) $2x + \sin x + \ell nx + c$

(3) $\frac{x^3}{3} + \sin x + \ell nx + c$

(4) $\frac{x^3}{3} - \sin x + \ell nx + c$

5. $\int \cos^2 x dx$ -

(1) $\frac{x}{2}+\frac{\sin 2x}{4}+c$

(2) $\frac{x}{2}+\frac{\sin 2x}{2}$ +c

(3) $-2\sin x$ +c

(4) $2\sin x$ +c

6. **The value of integral** $\int_0^{\pi/2} \cos x dx$ -

(1) 0

(2) 1

(3) -1

(4) None

7. **The value of** $\int_2^3 (x^3-4x^2+5x-10)dx$ -

(1) $\frac{74}{12}$

(2) 464

(3) –464

(4) $\frac{-74}{12}$

8. **The value of** $\int_0^{\pi/4} (\cos x-\sin x)dx$ -

(1) $\sqrt{2}-1$

(2) $\sqrt{2}+1$

(3) $1-\sqrt{2}$

(4) $1+\sqrt{2}$

9. **Value of** $\int_0^2 4x^3 dx+\int_0^{\pi/2} \cos x dx$ **is -**

(1) 16

(2) 15

(3) 17

(4) None

10. **The value of integral** $\int_0^{\pi/2} \sin(2x)dx$ -

(1) 0

(2) –1

(3) 1

(4) None

Answer key

Question	1	2	3	4	5	6	7	8	9	10
Answer	3	4	3	4	1	2	4	1	3	3

SOLUTIONS

1. (3)

$$\int \frac{dx}{(4x+5)} = \frac{1}{4}\log_e(4x+5)+c$$

2. (4)

$$\int \frac{dx}{\sqrt[3]{x}} = \int \frac{dx}{(x)^{1/3}} = \int x^{-1/3}dx$$

$$\frac{x^{-1/3+1}}{-\frac{1}{3}+1}+c = \frac{3}{2}(x)^{\frac{2}{3}}+c$$

3. (3)

$$(2x^3-x^2+1)dx = \int = \int 2x^3dx - \int x^2dx + \int dx$$

$$=2\frac{x^4}{4}-\frac{x^3}{3}+x+c$$

4. (4)

$$=\int x^2dx - \int \cos x dx + \int \frac{1}{x}dx$$

$$=\frac{x^{2+1}}{2+1}-\sin x+\ell nx+c$$

$$=\frac{x^3}{3}-\sin x+\ell nx+c$$

5. (1)

$$\int \cos^2 x dx = \int \frac{1+\cos 2x}{2}dx = \int \frac{1}{2}dx + \int \frac{\cos 2x}{2}$$

$$=\frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x$$

$$=\frac{x}{2}+\frac{1}{2}\frac{\sin 2x}{2}+c$$

6. (2)

$$\int_0^{\pi/2} \cos x dx = [\sin x]_0^{\pi/2}$$

$$=\left[\sin\frac{\pi}{2}-\sin(0)\right]=1$$

7. (4)

$$\int_2^3 x^3dx-\int_2^3 4x^2dx+\int_2^3 5xdx-\int_2^3 10dx$$

$$\left[\frac{x^4}{4}\right]_2^3-\left[\frac{4x^3}{3}\right]_2^3+\left[\frac{5x^2}{2}\right]_2^2-[10x]_2^3$$

$$\left[\frac{81}{4}-\frac{16}{4}\right]-\left[\frac{108}{3}-\frac{32}{3}\right]+\left[\frac{45}{2}-\frac{20}{2}\right]-[30-20]=\frac{-74}{12}$$

8. (1)

$$\int_0^{\pi/4}\cos xdx-\int_0^{\pi/4}\sin xdx$$

$$=[\sin x]_0^{\pi/4}-[-\cos x]_0^{\pi/4}$$

$$=\left[\sin\frac{\pi}{4}-\sin 0\right]+\left[\cos\frac{\pi}{4}-\cos 0\right]$$

$$=\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}-1\right)=\left(\sqrt{2}-1\right)$$

9. (3)

$$\int_0^2(4x^3)dx+\int_0^{\pi/2}(\cos x)dx$$

$$=4\left(\frac{x^4}{4}\right)_0^2+(\sin x)_0^{\pi/2}$$

$$=\left[(2)^4-0^4\right]+\left[\sin\left(\frac{\pi}{2}\right)-\sin 0°\right]$$

$= 16 + 1 = 17$

10. (3)

$$\int_0^{\pi/2}\sin(2x)dx=\left(\frac{-\cos(2x)}{2}\right)_0^{\pi/2}$$

$$=\left[\frac{-\cos(2\times\frac{\pi}{2})}{2}+\frac{\cos(2\times 0)}{2}\right]$$

$$=\frac{1}{2}[+1+1]=1$$

DAILY PRACTICE PROBLEM

Average Value of Function DPP - 07

1. **If the velocity of a particle moving along x-axis is given as v = $(3t^2 - 2t)$ and at t = 0, x = 0 then calculate position of the particle at t = 2sec.**

(Hint :- change in position $\Delta x = \int v dt$)

(1) 8

(2) –8

(3) –4

(4) +4

2. **Area bounded by curve y = sin x, with x-axis, when x varies from 0 to $\frac{\pi}{2}$ is :-**

(1) 1 unit

(2) 2 units

(3) 3 units

(4) 0

3. **Kinetic energy of a particle executing S.H.M. is $K = \frac{1}{2}m\omega^2(a^2 - x^2)$ calculate average value of kinetic energy from x = 0 to x = a.**

(1) $\frac{1}{2}m\omega^2a^2$

(2) $\frac{1}{4}m\omega^2a^2$

(3) $\frac{1}{3}m\omega^2a^2$

(4) $\frac{1}{6}m\omega^2a^2$

4. **Evaluate the $\int_{r_1}^{r_2} -\left(K\frac{q_1q_2}{r^2}\right)dr$**

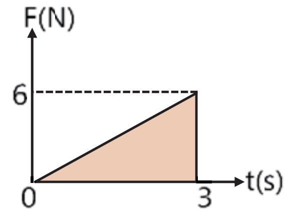
(1) $kq_1q_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$

(2) $kq_1q_2\left(\frac{1}{r_2} + \frac{1}{r_1}\right)$

(3) $kq_1q_2\left(\frac{1}{r_2^2} + \frac{1}{r_1^2}\right)$

(4) $kq_1q_2\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right)$

5. The figure shows an estimate force time graph for a baseball stuck by a bat. From the curve determine impulse delivered to the ball. (If $I = \int F dt$)



(1) 18 N-s
(2) 4.5 N-s
(3) 9 N-s
(4) None

6. The average value of alternating current I = $I_0 \sin \omega t$ in time interval $\left[0, \frac{\pi}{\omega}\right]$ is -

(1) $\frac{2I_0}{\pi}$
(2) $2I_0$
(3) $\frac{4I_0}{\pi}$
(4) $\frac{I_0}{\pi}$

7. If acceleration of a particle at any time is given by : a = 2t + 5, calculate the velocity after 5 s, if it starts from rest :
(1) 50 m/s
(2) 25 m/s
(3) 100 m/s
(4) 75 m/s

8. Find the area under wave for y = 2x between x = 0; and x = 10
(1) 200 unit
(2) 100 unit
(3) 50 unit
(4) 20 unit

Answer key

Question	1	2	3	4	5	6	7	8
Answer	4	2	3	1	3	1	1	1

SOLUTIONS

1. (4)

$v = 3t^2 - 2t$

$$\frac{dx}{dt} = 3t^2 - 2t$$

$$\int dx = \int (3t^2 - 2t)dt$$

When $t = 0 \longrightarrow x = 0$

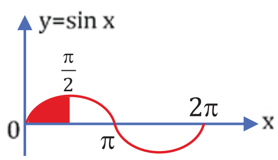
$t = 2 \longrightarrow x = ?$

$$\int_0^x dx = \int_0^2 (3t^2 - 2t)dt$$

$$x = \left[t^3 - t^2\right]_0^2$$

$x = 4$

2. (2)



$$\text{Area} = \int_{\pi}^{2\pi} + \sin x = (-\cos x)_{\pi}^{2\pi}$$

$$= -[\cos 2\pi - \cos \pi]$$

$= -[1 + 1] = -2$

3. (3)

$$\bar{K} = \frac{\int_0^a K dx}{\int_0^a dx} = \frac{\int_0^a \frac{1}{2} m\omega^2 (a^2 - x^2) dx}{a} = \frac{1}{3} m\omega^2 a^2$$

4. (1)

$$-\frac{kq_1q_2}{r} \int_{r_1}^{r_2} r^{-2} dr = -kq_1q_2 \left(\frac{r^{-1}}{-1}\right)_{r_1}^{r_2}$$

$$= K_1 q_1 q_2 \left[\frac{1}{r}\right]_{r_1}^{r_2}$$

$$= Kq_1q_2 \left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

5. (3)

$I = \int F dt$ = impulse will be area under the curve

$= \frac{1}{2} \times 6 \times 3 = 9N - s$

6. (1)

$$I_{av} = \frac{\int_0^{\pi/\omega} I dt}{\frac{\pi}{\omega} - 0} = \frac{\omega}{\pi} \int_0^{\pi/\omega} I_0 \sin\omega t dt = \frac{\omega}{\pi}\left[\frac{I_0(-\cos\omega t)}{\omega}\right]_0^{\pi/\omega}$$

$$= -\frac{\omega}{\pi}\frac{I_0}{\omega}[\cos\pi - \cos 0] = -\frac{I_0}{\pi}[-1-1] = \frac{2I_0}{\pi}$$

7. (1)

a = (2t + 5)

$$\frac{dv}{dt} = (2t+5) \Rightarrow \int_0^v dv = \int_0^5 (2t+5)dt$$

$$v = 2\left(\frac{t^2}{2}\right)_0^5 + 5(t)_0^5$$

= 25 + 25 = 50 m/sec

8. (1)

$$\int_0^{10} y dx = \int_0^{10} (2x) dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^{10}$$

= 100 units

DAILY PRACTICE PROBLEM

Logarithm and Progressions DPP-08

1. If $y^2 - 2y - 3 = 0$, find the value of y: -

(1) 3,1

(2) –3, –1

(3) 3, –1

(4) –3,1

2. Which of the following is Quadratic equation?

(1) $x + 1 = 0$

(2) $x^2 (2x + 3) = 0$

(3) $x (x^2 + 1) + 2$

(4) $(x - 2)^2 + 1 = 0$

3. Sum of the roots of equations, $2x^2 - 4x + 5 = 0$ is

(1) –2

(2) 2

(3) –4

(4) 4

4. Find the value of $\log_{10} 1000 - \log_{10} 100$ = ...?

(1) 3

(2) 2

(3) 1

(4) 10

5. Solve for x : $\log (3x + 2) - \log (3x - 2) = \log 5$

(1) –1

(2) 1

(3) $\frac{2}{3}$

(4) $-\frac{2}{3}$

6. Find the sum of 50 Natural Numbers: -

(1) 1250

(2) 1350

(3) 1225

(4) 1275

7. **Find $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+..............\infty$**

(1) 2

(2) 1

(3) $\frac{2}{3}$

(4) ∞

8. **Find sum of first ten terms of given Arithmetic progression: 1+3+5+7……. Ten terms.**

(1) 100

(2) 80

(3) 95

(4) 200

9. **Find approximate value of : -**

$(1.005)^{12}$

(1) 1.005

(2) 1.060

(3) 1.025

(4) 1.020

10. **If $B_{axis} = B_{centre}\left(\frac{R^3}{\left(R^2+x^2\right)^{3/2}}\right)$, find $\frac{B_{axis}}{B_{centre}}$ if $x << R$**

(1) $\left[1-\frac{3}{2}\frac{x^2}{R^2}\right]$

(2) $\left[1+\frac{3}{2}\frac{x^2}{R^2}\right]$

(3) $\left[1+\frac{3}{2}\frac{x}{R}\right]$

(4) $\left[1-\frac{3}{2}\frac{x}{R}\right]$

Answer Key

Question	1	2	3	4	5	6	7	8	9	10
Answer	**3**	**4**	**2**	**3**	**2**	**4**	**3**	**1**	**2**	**1**

SOLUTIONS DPP-08

1. **(3)**

$y^2 - 2y - 3 = 0$

$$y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-3)}}{2(1)} = \frac{2 \pm \sqrt{4+12}}{2}$$

$$y = \frac{2+4}{2} \text{ or } \frac{2-4}{2}$$

$y = 3$ or -1

2. **(4)**

$(x-2)^2 + 1 = 0$

$x^2 - 4x + 5 = 0$

Highest power of the variable equal to 2.

3. **(2)**

$ax^2 + bx + c = 0$

Sum of roots = $-\frac{b}{a}$

From given equation $(2x^2 - 4x + 5) = 0$

Sum of roots = $-\frac{(-4)}{2} = 2$

4. **(3)**

$\Rightarrow \log_{10}^{10^3} - \log_{10}^{10^2}$

$\Rightarrow 3\log_{10}^{10} - 2\log_{10}^{10}$ $\qquad \{\log_a^a = 1\}$

$\Rightarrow 3 - 2 = 1$

5. **(2)**

$$\log\left(\frac{3x+2}{3x-2}\right) = \log 5$$

$$\left\{\log(A) - \log(B) = \log\left(\frac{A}{B}\right)\right\}$$

Comparing both sides

$$\frac{3x+2}{3x-2} = 5$$

$3x + 2 = 15x - 10$

$x = 1$

6. **(4)**

Sum first n Natural numbers

$$S_n = \frac{n(n+1)}{2}$$

$$S_n = \frac{50(51)}{2} = 1275$$

7. **(3)**

$$S_\infty = \frac{a}{1-r} \qquad \begin{cases} a = 1 \\ r = -\frac{1/2}{2} = -\frac{1/4}{1/2} = -\frac{1}{2} \end{cases}$$

$$S_\infty = \frac{1}{1-\left(-\frac{1}{2}\right)} = \frac{1}{1+\frac{1}{2}}$$

$$= \frac{2}{3}$$

8. **(1)**

a = 1, d = 3 – 1 = 5 – 3 = 2, n = 10

$$S_n = \frac{10}{2}[2(1)+(10-1)(2)]$$

= 5 [2 + 18] = 100

9. **(2)**

$(1.005)^{12} = (1+0.005)^{12}$ $\{(1+x)^n = 1+nx \text{ ; when } x << R\}$

$\therefore (1 + 0.005)^{12} = 1 + 12(0.005)$

$= 1.060$

10. **(1)**

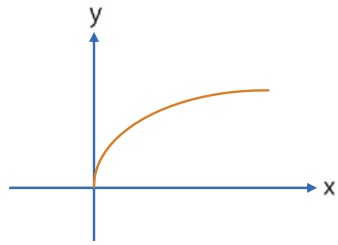
$$\frac{B_{axis}}{B_{centre}} = \frac{R^3}{(R^2+x^2)^{3/2}} = \frac{R^3}{R^3\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

$$\left(1+\frac{x^2}{R^2}\right)^{-3/2} = \left[1+\left(-\frac{3}{2}\right)\frac{x^2}{R^2}\right] \qquad \left(\frac{x}{R} << 1\right)$$

DPP
DAILY PRACTICE PROBLEM

Graphs - Parabola, Rectangular Hyperbola, Exponential Functions DPP-09

1. Which of the following equation is the best representation of the given graph?



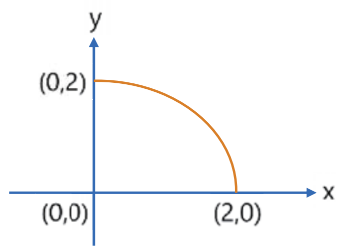
(1) $y = 2x^2$

(2) $x = 2y^2$

(3) $y = -2x^2$

(4) $x = -2y^2$

2. Which of the following equation is the best representation of the given graph?



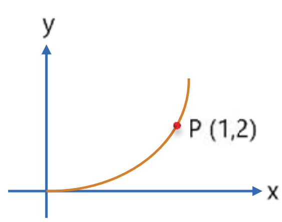
(1) $x + y = 2$

(2) $x^2 + y^2 = 4$

(3) $x^2 + y^2 = 2$

(4) $x^2 + y = 2$

3. The equation of graph shown in figure is $y = 3x^2$. The slope of graph at point P is:



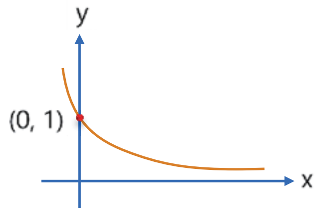
(1) 1

(2) 2

(3) 3

(4) 6

4. Which of the following equation is the best representation of the given graphs?



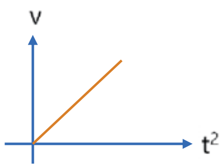
(1) $x = \frac{1}{y}$

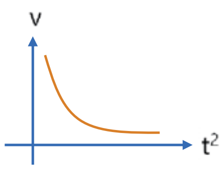
(2) $y = e^{-x}$

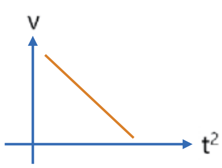
(3) $y = e^{x}$

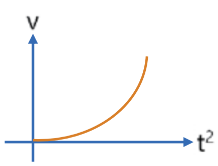
(4) $y = \log_e x$

5. If velocity v varies with time t as $v = 2t^2$, then the plot between v and t^2 will be given as: -

(1)


(2)


(3)


(4)


6. $\frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$ is a equation of: -

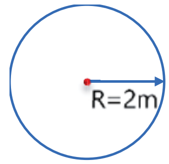
(1) Ellipse

(2) Circle

(3) Parabola

(4) Rectangular Hyperbola

7. Find area of given circle: -



(1) 2π

(2) 4π

(3) $\frac{\pi}{4}$

(4) $\frac{\pi}{2}$

8. Find the surface area of sphere and volume of sphere of radius, r = 2m.

(1) $16\pi m^2, \frac{32\pi}{3} m^3$

(2) $\frac{32\pi}{3} m^2, 16\pi m^3$

(3) $16\pi m^2, \frac{16\pi}{3} m^3$

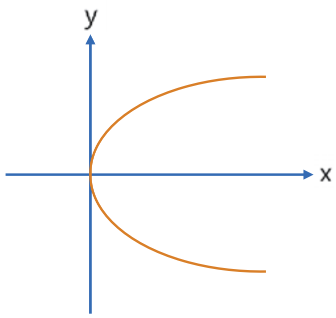
(4) $\frac{16\pi}{3} m^2, \frac{32\pi}{3} m^3$

Answer Key

Question	1	2	3	4	5	6	7	8
Answer	**2**	**2**	**4**	**2**	**1**	**1**	**2**	**1**

SOLUTIONS DPP-09

1. **(2)**



Equation of these type of parabolas are $y^2 = x$

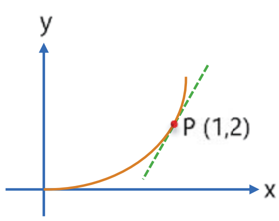
2. **(2)**

Equation of circle $\Rightarrow$ $x^2 + y^2 = r^2$ {when centre of circle is (0, 0)}

In given diagram radius of circle, r = 2

$\therefore$ Equation of circle $\Rightarrow$ $x^2 + y^2 = 4$

3. **(4)**



$y = 3x^2$

$\frac{dy}{dx} = 6x$

Point P (1, 2)

$\frac{dy}{dx} = 6(1)$

$= 6$

4. **(2)**

$y = e^{-x}$

5. **(1)**

$\therefore$ $V = 2t^2$ (given eq^n)

$y = m(x)$ (general eq^n)

$\therefore$ Straight line

6. **(1)**

$\because \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is an equation of ellipse.

7. **(2)**

Area of circle = πr^2

$= \pi(2)^2 = 4\pi$

8. **(1)**

Area = $4\pi r^2$

$= 4\pi(2)^2$

$= 16\pi m^2$

Volume = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(2)^3$

$= \frac{32\pi}{3} m^3$