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KISHOR CAREER POiNT

Std: XI

Physics

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2027

Name of Student: ....

-: Chapters :-

NEWTON'S LAWS OF MOTION (NLM)

WORK, POWER, ENERGY (WPE)

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NEWTON'S LAWS OF MOTION

Inertia

It is the inability of a body to change its state of rest or of uniform motion or its direction by itself. Mass is a measure of inertia in translatory motion. Heavier the mass, larger the inertia & vice versa.

Types of inertia

There are three types of inertia

(i) Inertia of rest

(ii) Inertia of motion and

(iii) Inertia of direction.

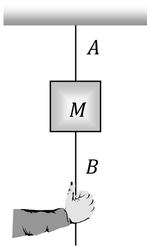
Inertia of rest:

It is the inability of a body to change its state of rest by itself.

Ex: (i) When a bus is at rest and starts suddenly moving forward the passangers inside it will fall back.

(ii) In the arrangement shown in the figure :

(a) If the string B is pulled with a sudden jerk then it will experience tension while due to inertia of rest of mass M this force will not be transmitted to the string A and so the string B will break.



(b) If the string B is pulled steadily the force applied to it will be transmitted from string B to A through the mass M and as tension in A will be greater than in B by Mg (weight of mass M) the string A will break.

Inertia of motion:

It is the inability of a body to change its state of uniform motion by itself.

Ex: Passengers in a moving bus fall forward, when brakes are applied suddenly.

Inertia of direction:

It is the inability of a body to change its direction of motion by itself.

Ex: When a bus takes a turn, passengers in it experience an outward force.

- A person sitting in a moving train, throws a coin vertically upwards, then i) it falls behind him, if the train is accelerating.

  ii) it falls in front of him, if the train is retarding

  iii) it falls into the hand of the person, if the train is moving with uniform velocity.

  iv) It falls into the hand of the person if the train is at rest.

Newton's First Law (Law of Inertia)

Every body continues to be in its state of rest (or) uniform motion in a straight line unless it is acted upon by a net external force to change its state.

It defines inertia, force and mechanical equilibrium.

If the net external force on an object is zero, then acceleration of object is zero.

Linear momentum

Linear momentum is the product of the mass of a body and its velocity.

$$\vec{p} = m\vec{v}$$

Linear momentum is a vector. It has the same direction as the direction of velocity of the body.

SI unit: kg m s^{-1},

CGS unit: g cm s^{-1}

Dimensional Formula: [MLT^{-1}]

Change in momentum of a body in different cases:

Consider a body of mass m moving with velocity $\vec{v}_i$ and momentum $\vec{p}_i$. Due to a collision (or) due to the action of a force on it suppose its velocity changes to $\vec{v}_f$ and momentum changes to $\vec{p}_f$ in a small time interval Δt.

Change in momentum of body

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

Where P_i = initial momentum

P_f = final momentum

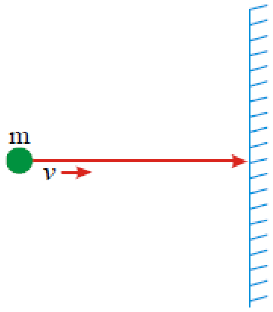
$$\Delta\vec{p} = m\,\vec{v}_f - m\,\vec{v}_i$$

$$|\vec{p}_f - \vec{p}_i| = \sqrt{P_f^2 + P_i^2 - 2P_fP_i cos\theta}$$

where θ= angle between $\vec{p}_f$ and $\vec{p}_i$

Consider a body of mass 'm' moving with velocity '$\vec{v}$' along a straight line

Case (i) : If it hits a wall and comes to rest, Change in momentum of the body

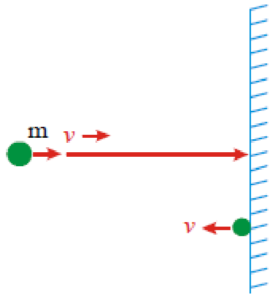


$$\Delta\vec{P} = \vec{P_f} - \vec{P_i} = 0 - (m\vec{v})\hat{\imath} = -m\vec{v}\hat{\imath};$$

$$|\Delta\vec{\boldsymbol{P}}| = \boldsymbol{mv}$$

along the normal and away from the wall.

Case(ii): If the body rebounds with same speed 'v' then θ= 180^0



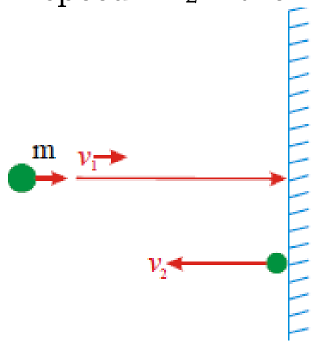
$$\Delta\vec{P} = \vec{P_f} - \vec{P_i} = [-(m\vec{v})\hat{\imath}] - [(m\vec{v})\hat{\imath}]$$

$$\Delta\vec{P} = -(2m\vec{v})\hat{\imath}$$

$$\therefore |\Delta\vec{\boldsymbol{P}}| = \boldsymbol{2mv}$$

along the normal and away from the wall.

Case (iii) : If the body hits a rigid wall normally with speed v_1 and rebounds with speed v_2 then θ = 180^0,



$$\Delta\vec{P} = \vec{P_f} - \vec{P_i} = [-(m\vec{v_2})\hat{\imath}] - [(m\vec{v_1})\hat{\imath}],$$

$$|\Delta\vec{\boldsymbol{P}}| = \boldsymbol{m(v_2 + v_1)}$$

along the normal and away from the wall.

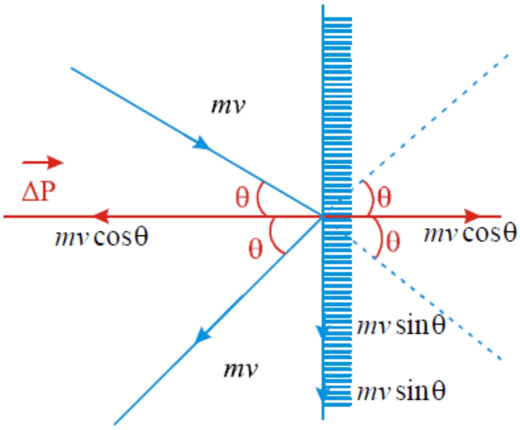
Case (iv) : A body of mass 'm' moving with speed 'v' hits a rigid wall at an angle of incidence and rebounds with same speed 'v', then $\Delta\vec{P}$ is along the normal, away from the wall

$$\Delta\vec{P}_x = -mv\cos\theta\, i - mv\cos\theta\, i$$

$$\Delta\vec{P}_y = mv\sin\theta\, j - mv\sin\theta\, j$$

$$\Delta\vec{P} = \Delta\vec{P}_x + \Delta\vec{P}_y = 2mv\cos\theta\,(-i)$$

$$|\Delta\vec{P}| = 2mv\cos\theta$$

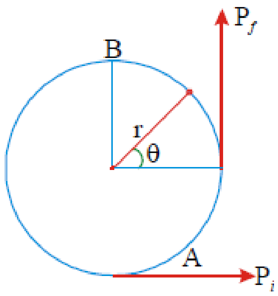


Case(v) : In the above case if θ is the angle made with wall then

$$|\Delta\vec{P}| = 2mv\sin\theta$$

along the normal and away from the wall.

Case(vi): A particle of mass 'm' is moving uniformly with a speed 'v' along a circular path of radius 'r'. As it moves from a point A to another point B, such that the arc AB subtends an angle θ at the centre, then the magnitude of change in momentum is 2mv $\sin(\theta/2)$ and is directed towards the centre of the circle.



Newton's second law

The rate of change of momentum of a body is directly proportional to the resultant (or) net external force acting on the body and takes place along the direction of force.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

$$\therefore \vec{F}_{net} = \frac{d(m\vec{v})}{dt}$$

In a system, if only velocity changes and mass remain constant,

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt}$$

$$\vec{F}_{net} = m\vec{a}$$

In a system, if only mass changes and velocity remians constant,

$$\vec{F}_{net} = \vec{v}\frac{dm}{dt}$$

Force is a vector and the acceleration produced in the body is in the direction of net force,

SI unit : newton (N). CGS unit :dyne.

One newton = 10^5 dyne

D.F=MLT^{-2}

Gravitational units of force:

Kilogram weight (kg wt) and gram weight (gm wt) are gravitational units of force.

1 kg.wt = 9.8 N,

1 gm.wt= 980 dyne

- A metallic plate of mass 'M' is kept held in mid air by firing 'n' bullets in 't' seconds each of mass 'm' with a velocity 'v' from below.

(a) If the bullet falls dead after hitting the plate, then

$$\frac{mnv}{t} = Mg$$

(b) If the bullet rebounds after hitting the plate with same velocity, then

$$\frac{2mnv}{t} = Mg$$

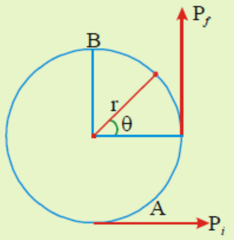
Problem No 1 - A force produces an acceleration16 ms^{-2} in a mass 0.5 kg and an acceleration 4 ms^{-2} in an unknown mass when applied separately. If both the masses are tied together, what will be the acceleration under same force?

Sol. Force F=ma=0.5×16=8N when both masses are joined and same force acts, acceleration is given by $a^1 = \frac{F}{m+m^1} = \frac{8}{0.5+(8/4)} = 3.2ms^{-2}$

Problem No 2 - When the forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remain stationary. If the force

F_1 is now removed, then find the acceleration of the particle .

Sol. If mass 'm' is stationary under three forces,



$$\vec{F}_1+\vec{F}_2+\vec{F}_3=0$$
$$\vec{F}_1=-\left(\vec{F}_2+\vec{F}_3\right)$$
$$\sqrt{F_2^2+F_3^2}=F_1$$

Obviously if F_1 is removed then the mass will have

acceleration, $a=\dfrac{\sqrt{F_2^2+F_3^2}}{m}$ (or) $a=\dfrac{F_1}{m}$

Problem No 3 - A body of mass m=3.513 kg is moving along the x-axis with a speed of $5ms^{-1}$.The magnitude of its momentum is recorded as

Sol. $m=3.513kg, v=5ms^{-1}$ momentum,
$p=mv=3.513\times5=17.565kgms^{-1}$

Problem No 4 - A very flexible chain of length L and mass M is vertically suspended with its lower end just touching the table. If it is released so that each link strikes the table and comes to rest. What force the chain will exert on the table at the moment 'y' part of length falls on the table?

Sol. Since chain is uniform , the mass of 'y' part of the chain will be $\left(\dfrac{M}{L}y\right)$. When this part reaches the table, its total force exerted must be equal to the weight of y part resting on table + Force due to the momentum imparted

$$F=\frac{M}{L}yg+\frac{\left(\frac{M}{L}dy\right)\sqrt{2gy}}{dt}=\frac{Mg}{L}y+\frac{M}{L}v.\sqrt{2gy}$$

$$\left(\because\frac{dy}{dt}=v\right)\quad=\frac{Mg}{L}y+\frac{M}{L}\sqrt{2gy}.\sqrt{2gy}\quad=3\frac{My}{L}g$$

Problem No 5 - A body of mass 8kg is moved by a force F =(3x)N, where x is the distance covered. Initial position is $x=2m$ and final position is $x=10m$. If initially the body is at rest, find the final speed.

Sol: F=ma $\Rightarrow$ F=m$\dfrac{dv}{dt}$ $\Rightarrow$ $3x$=m$\dfrac{dv}{dx}\dfrac{dx}{dt}$

$$3x=8\frac{dv}{dx}v\Rightarrow 3xdx=8vdv$$

$$3\int_2^{10}xdx=8\int_0^v vdv\Rightarrow 3\left[\frac{x^2}{2}\right]_2^{10}=8\left[\frac{v^2}{2}\right]_0^v$$

$$3[100-4]=8v^2\Rightarrow v^2=\frac{3\times96}{8}=36\Rightarrow v=6ms^{-1}$$

Problem No 6 - Sum of magnitudes of the two forces acting at a point is 16 N. If their resultant is normal to the smaller force, and has a magnitude 8N, then the forces are

Sol. $F_1+F_2=16$ ——(1) Resultant force is perpendicular to F_1, then $F_2^2-F_1^2=F^2$

$$F_2{}^2-F_1{}^2=8^2\Rightarrow(F_2-F_1)(F_2+F_1)=64$$

$$(F_2-F_1)\times16=64 \text{ ——— (2)}$$

Solving(1) &(2),we get $F_1=6N, F_2=10N$

Problem No 7 - A particle is at rest at $x=a$. A force $\vec{F}=-\frac{b}{x^2}\vec{i}$ begins to act on the particle. The particle starts its motion, towards the origin, along X–axis. Find the velocity of the particle, when it reaches a distance x from the origin.

Sol. $F=-\dfrac{b}{x^2}\Rightarrow\dfrac{d}{dt}(p)=-\dfrac{b}{x^2}$

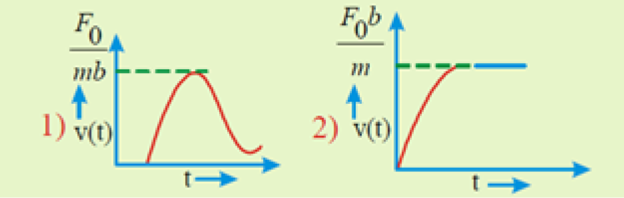
$$\frac{d}{dt}(mv)=-\frac{b}{x^2}\Rightarrow m\frac{dv}{dx}\frac{dx}{dt}=-\frac{b}{x^2}$$

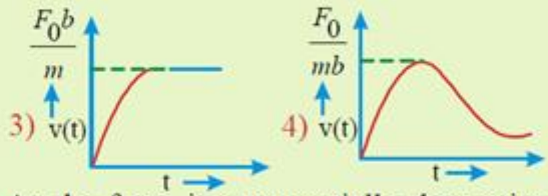
$$mv\,dv=-\frac{b}{x^2}dx\Rightarrow vdv=-\frac{b}{mx^2}dx$$

$$\int_0^v v\,dv=\int_a^x-\frac{b}{mx^2}dx\Rightarrow\frac{v^2}{2}=\frac{b}{m}\left[\frac{1}{x}\right]_a^x$$

$$\frac{v^2}{2}=\frac{b}{m}\left[\frac{1}{x}-\frac{1}{a}\right]\qquad\therefore v=\sqrt{\frac{2b}{m}\left(\frac{a-x}{xa}\right)}$$

Probem No 8 - A particle of mass m is at rest at the origin at time $t=0$. It is subjected to a force F(t)= F_0e^{-bt} in the X-direction. Its speed V(t) is depicted by which of the following curves.





Sol: As the force is exponentially decreasing, its acceleration, i.e.rate of increase of velocity will decrease with time.Thus, the graph of velocity will be an increasing curve with decreasing slope with time.

$$a=\frac{F}{m}=\frac{F_0}{m}e^{-bt}\Rightarrow\frac{dv}{dt}=\frac{F_0}{m}e^{-bt}\Rightarrow\int_0^v dv=\int_0^t\frac{F_0}{m}e^{-bt}dt$$

$$\Rightarrow v=\left[\frac{F_0}{m}\left(\frac{1}{-b}\right)e^{-bt}\right]_0^t=\left[\frac{F_0}{m}\left(\frac{1}{b}\right)e^{-bt}\right]_t^0$$

$$=\frac{F_0}{mb}\left(e^0-e^{-bt}\right)=\frac{F_0}{mb}\left(1-e^{-bt}\right)$$

So, velocity increases continuously and attains a maximum value, $v_{max}=\frac{F_0}{mb}$ **Ans: 3**

Problem No 9 - A bus moving on a level road with a velocity v can be stopped at a distance of x , by the application of a retarding force F. The load on the bus is increased by 25% by boarding the passengers. Now, if the bus is moving with the same speed and if the same retarding force is applied, the distance travelled by the bus before it stops is

Sol : By using equations of motion $v^2-u^2=2as$

$$v^2-u^2=-2\left(\frac{F}{m}\right)s \quad -u^2=-2\left(\frac{F}{m}\right)s$$

$$u^2=2\frac{Fs}{m}\Rightarrow m=\frac{2Fs}{u^2}\Rightarrow m\propto s\Rightarrow\frac{m_1}{m_2}=\frac{s_1}{s_2}$$

Given $s_1=x$,$m_1=m$, and

$$m_2=m+\frac{25}{100}(m)=m+\frac{m}{4}=\frac{5m}{4}$$

$$\Rightarrow\frac{m}{5m/4}=\frac{x}{s_2}\Rightarrow s_2=\frac{5x}{4}=(1.25x)\text{m}$$

Applications of variable mass

- When a machine gun fires 'n' bullets each of mass 'm' with a velocity v in a time interval 't' then force needed to hold the gun steadily is

$$\boldsymbol{F=\frac{nmv}{t}}$$

- When a jet of liquid coming out of a pipe strikes a wall normally and falls dead, then force exerted by the jet of liquid on the wall is

$$\boldsymbol{F=A\rho v^2}$$

Where - A = Area of cross section of the pipe

v = Velocity of jet

ρ = density of the liquid

- If the liquid bounces back with the same velocity then the force exerted by the liquid on the wall is

$$\boldsymbol{F=2A\rho v^2}$$

- If the liquid bounces back with velocity v' then the force exerted on the wall is

$$\boldsymbol{F=A\rho v(v+v')}$$

- When a jet of liquid strikes a wall by making an angle 'θ' with the wall with a velocity 'v' and rebounds with same velocity then force exerted by the water jet on wall is

$$\boldsymbol{F=2A\rho v^2}\sin\theta$$

- If gravel is dropped on a conveyor belt at the rate of $\left(\frac{dm}{dt}\right)$, an extra force required to keep the belt moving with constant velocity 'u' is

$$\boldsymbol{F=u\left(\frac{dm}{dt}\right)}$$

Problem No 10 - A gardener is watering plants at the rate 0.1litre/sec using a pipe of cross-sectional area 1 cm^2. What additional force he has to exert if he desires to increase the rate of watering two times?

Sol : $F=Adv^2=\frac{(Av)^2d}{A}$. If rate of watering of plant (Av) is doubled, it means that the amount of water poured/sec is doubled which is possible only if velocity is doubled. Hence, force is to be made 4 times.

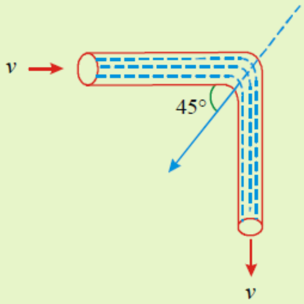
$\therefore$ Additional force = 3 times initial force

$$=3Adv^2=3\frac{(Av)^2}{A}d$$

$$=\frac{3\times0.1\times0.1\times10^3}{10^{-4}}=3\times10^5N$$

Problem No 11 - A liquid of density r flows along a horizontal pipe of uniform cross – section A with a velocity v through a right angled bend as shown in Fig. What force has to be exerted at the bend to hold the pipe in equilibrium?

Sol : Change in momentum of mass Δm of liquid as it passes through the bend



$$dP = P_f - P_i = \sqrt{2}d\Delta mv$$

$$F = \frac{dP}{dt} = \left(\sqrt{2}\right)v\frac{dm}{dt};\left[as\ dm = \rho AdL\right]$$

$$F = \sqrt{2}v\frac{(\rho.AdL)}{dt};\left[as\ dL/dt = v\right]$$

$$F = \sqrt{2}\rho Av^2$$

Problem No 12 - A flat plate moves normally with a speed v_1 towards a horizontal jet of water of uniform area of cross section. The jet discharges water at the rate of volume V per second at a speed of v_2. The density of water is r. Assume that water splashes along the surface of the plate at right angles to the original motion. The magnitude of the force acting on the plate due to the jet is

Sol. Force acting on the plate $F = \frac{dp}{dt} = u_r\frac{dm}{dt}$

Since $Av_2 = V \Rightarrow \frac{dm}{dt} = A(v_1+v_2)\rho = \frac{V}{v_2}(v_1+v_2)\rho$

($u_r = v_1 + v_2 =$ velocity of water coming out of jet w.r.t plate)

$$F = (v_1+v_2).\frac{V}{v_2}(v_1+v_2)\rho = \frac{V}{v_2}(v_1+v_2)^2\rho \text{ N}$$

Impulse $(\vec{J})$

It is the product of impulsive force and time of action that produces a finite change in momentum of body.

$$J = Ft = m(v-u)$$

$$\therefore J = change\ in\ momentum$$

SI unit: Ns (or) Kg ms^{-1};

Dimensional Formula: [MLT^{-1}]

It is a vector directed along the force

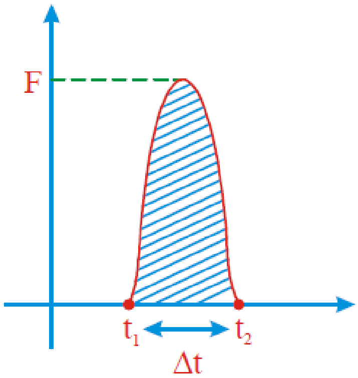
change in momentum and Impulse are always in the same direction.

- For constant force, J = Ft
- If impulsive force is variable, then

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$J = \int_{t_1}^{t_2} Fdt$$

- The ***area*** bounded by the force-time graph measures ***Impulse.***



Applications of Impulse:

(a) shock absorbers are used in vehicles to reduce the magnitude of impulsive force.

(b) A cricketer lowers his hands, while catching the ball to reduce the impulsive force.

Problem No 13 - Find the impulse due to the force $\vec{F} = a\hat{\imath} + bt\hat{\jmath}$, where a = 2N and b = 4Ns^{-1} if this force acts from t_i = 0 to t_f = 0.3s

Sol: $J = \int_{t_i}^{t_f} \vec{F}dt = \int_0^{0.3}(a\hat{i} + bt\hat{j})dt$

$$J = a\int_0^{0.3} dt\,\hat{i} + b\int_0^{0.3} t\,dt\,\hat{j} = a[t]_0^{0.3}\hat{i} + b\left[\frac{t^2}{2}\right]_0^{0.3}\hat{j}$$

$$= 2\times 0.3\times\hat{i} + 4\times\frac{(0.3)^2}{2}\times\hat{j} = 0.6\hat{i} + 0.18\hat{j}\ \text{NSec}$$

Problem No 14 - A ball falling with velocity $\vec{v}_i = (-0.65\hat{\imath} - 0.35\hat{\jmath})$ ms-1 is subjected to a net impulse $\vec{I} = (0.6\hat{\imath} + 0.18\hat{\jmath})$ Ns. If the ball has a mass of 0.275kg, Calculate its velocity immediately following the impulse.

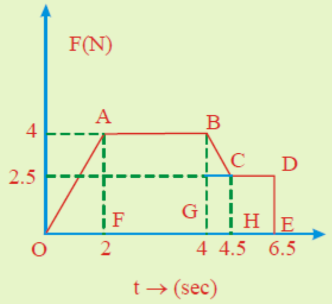
Sol: $m\vec{v}_f - m\vec{v}_i = \vec{I}$; $\vec{v}_f = \vec{v}_i + \frac{\vec{I}}{m}$

$$\vec{v}_f = -0.65\hat{i} - 0.35\hat{j} + \frac{0.6\hat{i} + 0.18\hat{j}}{0.275}$$

$$\vec{v}_f = -0.65\hat{i} - 0.35\hat{j} + 2.18\hat{i} + 0.655\hat{j}$$

$$\vec{v}_f = (1.53\hat{i} + 0.305\hat{j})ms^{-1}$$

Problem No 15 - A body of mass 2kg has an initial speed 5 ms-1. A force acts on it for some time in the direction of motion. The force–time graph is shown in figure. Find the final speed of the body.



Sol. Area of $OAF = \frac{1}{2} \times 2 \times 4 = 4$

Area of ABG F = 2 × 4 = 8

Area of $BGHC = \frac{1}{2}(4 + 2.5) \times 0.5 = 1.625$

Area of CDEH = 2 × 2.5 = 5

Total area under F-t graph = Change in momentum

$\Rightarrow$ m(v – u) = 18.625

$$\Rightarrow v = \frac{18.625}{2} + 5 = 14.25ms^{-1}$$

Problem No 16 - A bullet is fired from a gun. The force on a bullet is, F = 600 – 2 x 10^5 t newton. The force reduces to zero just when the bullet leaves barrel. Find the impulse imparted to the bullet.

Sol. F = 600 – 2 × 10^5 t , F becomes zero as soon as the bullet leaves the barrel.

$0 = 600 - 2 \times 10^5 t \Rightarrow 600 = 2 \times 10^5 t$

$t = 3 \times 10^{-3}$ s $\Rightarrow$ Impulse $= \int_0^t F dt$

$$= \int_0^t (600 - 2 \times 10^5 t)\, dt = \left[600t - 2 \times 10^5 \frac{t^2}{2}\right]_0^{3\times10^{-3}}$$

$= 600 \times 3 \times 10^{-3} - 10^5 \times 9 \times 10^{-6} = 0.9Ns$

Equilibrium

The necessary and sufficient conditions for the translational equilibrium of the rigid body are,

$$\sum \boldsymbol{F} = \boldsymbol{0}$$

$$\sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$

For rotational equilibrium,

$$\sum \boldsymbol{\tau} = \boldsymbol{0}$$

$$\sum \tau_x = 0, \sum \tau_y = 0, \sum \tau_z = 0$$

As for, $\vec{F} = 0$, $m\vec{a} = 0$ (or) m(dv/dt) = 0

as $m \neq 0, \frac{d\bar{v}}{dt} = 0$,(or) $\vec{v}$ = constant or zero

If a body is in translatory equilibrium it will be either at rest or in uniform motion. If it is at rest, the equilibrium is called static, otherwise dynamic.

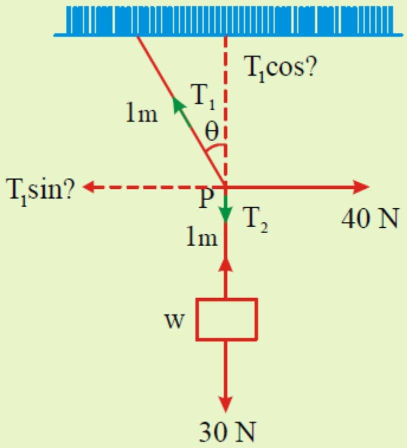
- If 'n' coplanar forces of equal magnitudes acting simultaneously on a particle at a point, with the angle between any two adjacent forces is 'θ' and keep it in equilibrium, then $\theta = \frac{360}{n}$

Problem No 17 - A mass of 3kg is suspended by a rope of length 2m from the ceiling. A force of 40N in the horizontal direction is applied at midpoint P of the rope as shown. What is the angle the rope makes with the vertical in equilibrium and the tension in part of string attached to the ceiling? (Neglect the mass of the rope)

Sol : Resolving the tension T_1 into two mutually perpendicular components, we have

$T_1 \cos\theta = W = 30N \qquad T_1 \sin\theta = 40N$

$\therefore \tan\theta = \frac{4}{3}$ (or) $\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53^0$

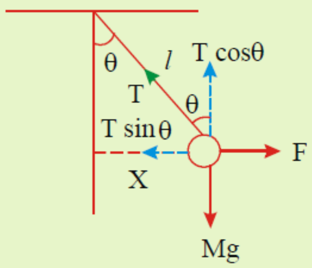


The tension in part of string attached to the ceiling

$T_1 = \sqrt{W^2 + F^2} = \sqrt{30^2 + 40^2} = 50N$

Problem No 18 - A mass M is suspended by a weightless string. The horizontal force required to hold the mass at 60^0 with the vertical is

Sol :



$F = T\sin\theta \qquad ----- (1)$

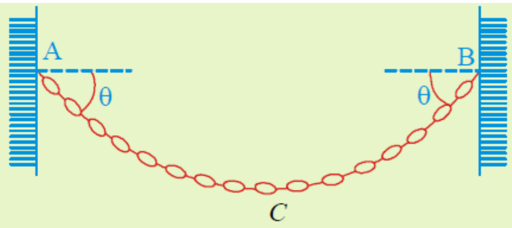
$Mg = T\cos\theta \qquad ----- (2)$

Dividing Eq.(1) and Eq.(2)

$\frac{F}{Mg} = \frac{T\sin\theta}{T\cos\theta}$; $F = Mg\tan\theta$

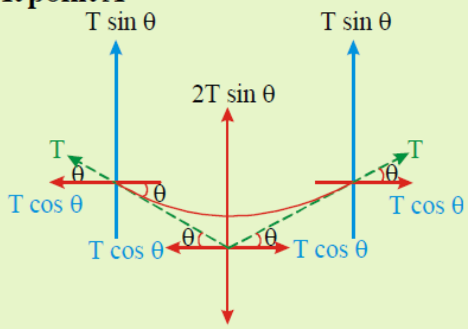
$F = Mg\tan 60^0$; $F = \sqrt{3}Mg$

Problem No 19 - A chain of mass 'm' is attached at two points A and B of two fixed walls as shown in the figure. Find the tension in the chain near the walls at point A and at the mid point C.



Sol.

i) At point A



$2T\sin\theta = mg \Rightarrow T = \frac{1}{2} mg\,\mathrm{cosec}\theta$

ii) Tension along horizontal direction is same everywhere

$\because$ (no external force is acting on it in horizontal direction.)

At point C

$T^1 = T\cos\theta = \frac{mg\cos\theta}{2\sin\theta} = \frac{mg\cot\theta}{2}$

Newton's Third Law

It states that, *"For every action there is always an equal and opposite reaction."*

Action and reaction do not act on the same body and they act on different bodies at same instant of time.

Action and reaction, known as pair of forces, are equal in magnitude and opposite in directions acting on different bodies in interaction. So they never cancel each other.

Newton's third law is not applicable to pseudo forces.

Newton's third law defines nature of force and gives the law of conservation of linear momentum.

Examples:

1) When we walk on a road we push the road backwards and road applies equal (in magnitude) and opposite force on us, so that we can move forward.

2) When we swim in water, we push water backward and water applies equal (in magnitude) and opposite force on us,

so that we can move forward.

3) A bird is in a *wire cage* hanging from a spring balance. When the bird starts flying in the cage, the reading of the balance decreases.

4) If the bird is in a *closed cage* (or) air tight cage and it hovers in the cage the reading of the spring balance does not change.

In the closed cage if the bird accelerates upward the reading of the balance is

$$R = W_{bird} + ma$$

Limitations of Newton's third law:

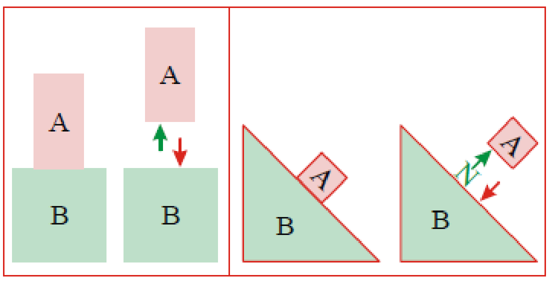
1) Newton's third law is not strictly applicable for the interaction between two bodies separated by large distances, of the order of astronomical units.

2) It does not apply strictly when the objects move with velocity nearer to that of light.

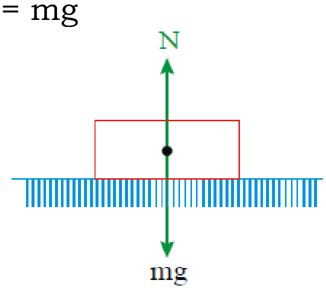
3) It does not apply where the gravitational field is strong.

Normal Reaction/Force:

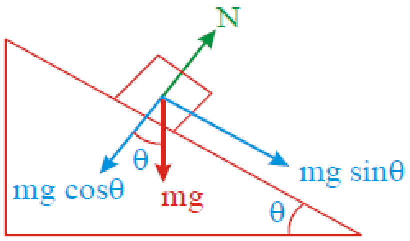
Normal force acts perpendicular to the surfaces in contact when one body tries to press on the surface of the second body. In this way second body tries to push away the first body.



When the body lies on a horizontal surface N = mg



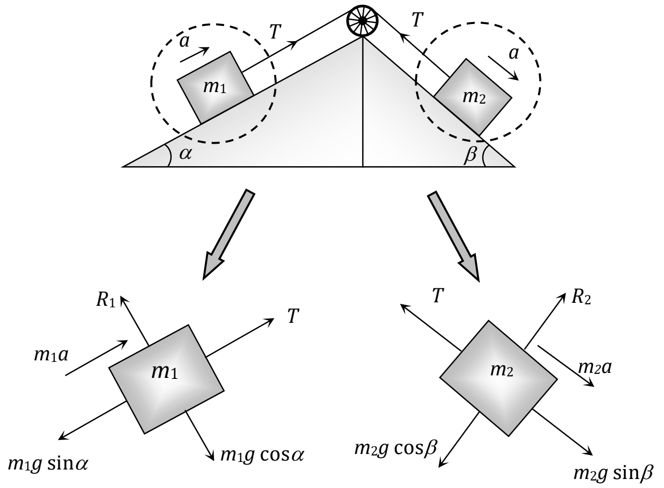
When the body lies on an inclined surface N = mgcosθ



Free Body Diagram (FBD)

In this diagram the object of interest is isolated from its surrounding and the interactions between the object and the surrounding are represented in terms of forces.

Example :



Free body diagram of mass m_1

Free body diagram of mass m_2

Frames of Reference

A system of coordinate axes which defines the position of a particle or an event in two or three dimensional space is called a frame of reference. There are two types of frames of reference,

a) inertial or unaccelerated frames of reference.

b) non-inertial or accelerated frames of reference.

Inertial frames of reference:

- Frames of reference in which Newton's Laws of motion are applicable are called inertial frame.
- Inertial frames of reference are either at rest or move with uniform velocity with respect to a fixed imaginary axis.
- In inertial frame, acceleration of a body

is caused by real forces.

- Equation of motion of mass 'm' moving with acceleration 'a' relative to an observer in an inertial frame is,

$$\sum \vec{F}_{real} = ma$$

Examples:

1) A lift at rest,

2) Lift moving up (or) down with constant velocity,

3) Car moving with constant velocity on a straight road.

Real Force :

Force acting on an object due to its interaction with another object is called a real force.

Ex: Normal force, Tension, weight, spring force, muscular force etc.

a) All fundamental forces of nature are real.

b) Real forces form action-reaction pair.

Non-Inertial frames :

- Frames of reference in which Newton's Laws are not applicable are called non-inertial frames.
- Accelerated frames move with either uniform acceleration or non-uniform acceleration.
- All the accelerated and rotating frames are non-inertial frames of reference.

Examples:

1) Accelerating car on a road.

2) Merry go round.

3) Artificial satellite around the earth.

Pseudo force:

a) In non-inertial frame, Newton's second law is not applicable. In order to make Newton's second law applicable in non-inertial frame a pseudo force is introduced.

b) If $\vec{a}$ is the acceleration of a non-inertial frame, the pseudo force acting on an object of mass m, as measured by an observer in the given non-inertial frame is

$$\vec{F}_{Pseudo} = -m\vec{a}$$

i.e. Pseudo force acts on an object opposite to the direction of acceleration of the non-inertial frame.

c) Pseudo forces exist for observers only in non-inertial frames, such forces have no existence relative to an inertial frame.

d) Equation of motion relative to non-inertial frame is

$$\sum \vec{F}_{real} + \vec{F}_{Pseudo} = m\vec{a}$$

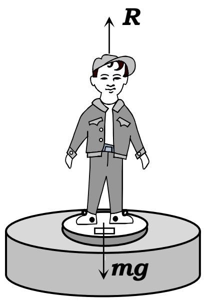
Where $\vec{a}$ is the acceleration of a body as measured in non-inertial frame.

e) Earth is an inertial frame for an observer on the earth but it is an accelerated frame for an observer at centre of earth (or) in a satellite.

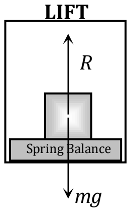
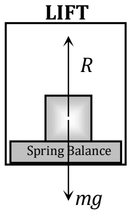
Apparent Weight of a Body in a Lift (Elavator)

When a body of mass m is placed on a weighing machine which is placed in a lift, then actual weight of the body is mg.

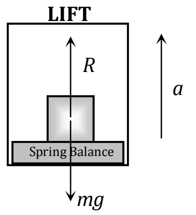
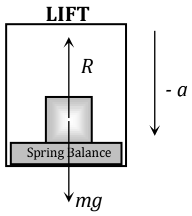
This acts on a weighing machine which offers a reaction R given by the reading of weighing machine. This reaction exerted by the surface of contact on the body is the *apparent weight* of the body.



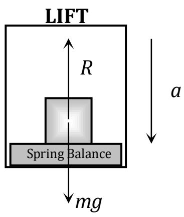
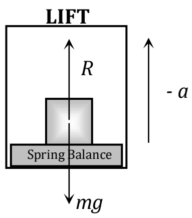
Expressions for apparent weight in a lift in different situations are given in following table:

<table>
<tr><th>Condition</th><th>Figure</th><th>Reaction
(Apparent weight)</th><th>Conclusion</th></tr>
<tr><td>Lift is at rest
(a = 0)</td><td></td><td>$R - mg = 0$
$\therefore$ $R = mg$</td><td>Apparent weight
= Actual weight</td></tr>
<tr><td>Lift moving upward or downward with constant velocity (a = 0)</td><td></td><td>$R - mg = 0$
$\therefore$ $R = mg$</td><td>Apparent weight
= Actual weight</td></tr>
<tr><td>Lift moving upward with acceleration ‘a’

OR

Lift moving downward with retardation ‘a’ ($a < g$)</td><td>
OR
</td><td>$R - mg = ma$
$\therefore$$R = m(g + a)$</td><td>Apparent weight
> Actual weight</td></tr>
<tr><td>Lift moving downward with acceleration ‘a’

OR

Lift moving upward with retardation ‘a’ ($a < g$)</td><td>
OR
</td><td>$mg - R = ma$
$\therefore$ $R = m(g - a)$</td><td>Apparent weight
< Actual weight</td></tr>
</table>

Problem No 20 - A mass of 1kg attached to one end of a string is first lifted up with an acceleration 4.9m/s^2 and then lowered with same acceleration. What is the ratio of tension in string in two cases?

Sol : When mass is lifted up with acceleration $4.9 m/s^2$

$T_1 = m(g+a) = 1\ (9.8+4.9) = 14.7 N$

When mass is lowered with same acceleration

$T_2 = m(g-a) = 1(9.8-4.9) = 4.9\ N$

$$\therefore \frac{T_1}{T_2} = \frac{14.7}{4.9} = 3:1$$

Problem No 21 -The apparent weight of a man in a lift is W_1 when lift moves upwards with some acceleration and is W_2. when it is accelerating down with same acceleration. Find the true weight of the man and acceleration of lift.

Sol : (a) $W_1 = m(g+a), W_2 = m(g-a)$

$$W_1 + W_2 = 2mg \Rightarrow W_1 + W_2 = 2W (\because W = mg)$$

$$\Rightarrow \frac{W_1 + W_2}{2} = W$$

(b) $$\frac{W_1}{W_2} = \frac{m(g+a)}{m(g-a)} = \frac{g+a}{g-a}$$

$$\frac{g}{a} = \frac{W_1 + W_2}{W_1 - W_2} \Rightarrow a = g\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

MOTION OF CONNECTED BODIES

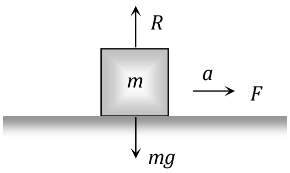
Acceleration of Block on a horizontal smooth surface

1) When a pull is horizontal

$R = mg$

and $F = ma$

$\therefore\ a = F/m$



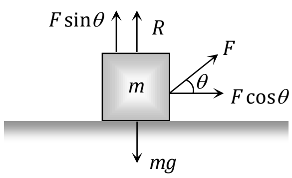
2) When a pull is acting at an angle (θ) to the horizontal (upward)

$R + F\sin\theta = mg$

$\Rightarrow R = mg - F\sin\theta$

and $F\cos\theta = ma$

$$\therefore\ a = \frac{F\cos\theta}{m}$$

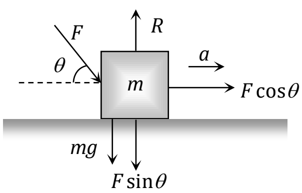


3) When a push is acting at an angle (θ) to the horizontal (downward)

$R = mg + F\sin\theta$

and $F\cos\theta = ma$

$$a = \frac{F\cos\theta}{m}$$



Acceleration of Block on a Smooth Inclined Plane

1) When inclined plane is at rest

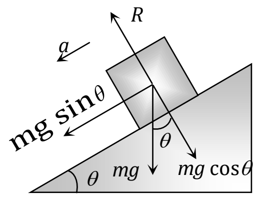
Normal reaction $R = mg\cos\theta$

Force along a inclined plane

$F = mg\sin\theta$;

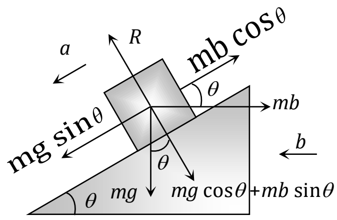
$ma = mg\ \sin\theta$

$\therefore a = g\sin\theta$



2) When a inclined plane is given a horizontal acceleration '*b*'

Since the body lies in an accelerating frame, an inertial force (mb) acts on it in the opposite direction.



Normal reaction $R = mg\cos\theta + mb\sin\theta$

and $ma = mg\ \sin\theta - mb\cos\theta$

$\therefore\quad a = g\sin\theta - b\cos\theta$

Note:

The condition for the body to be at rest relative to the inclined plane :

$$a = g\sin\theta - b\cos\theta = 0$$

$$\therefore \boldsymbol{b = g\tan\theta}$$

Table: Motion of Blocks in Contact

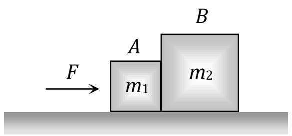
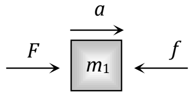
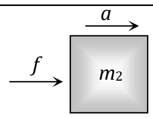
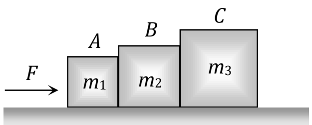
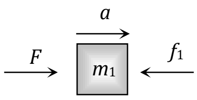
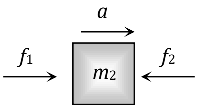
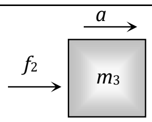
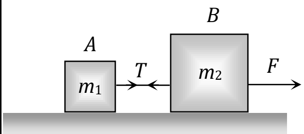
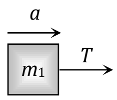
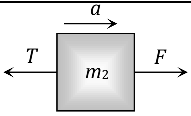
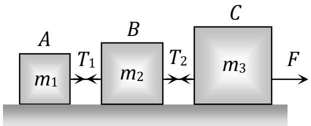
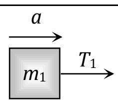
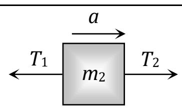
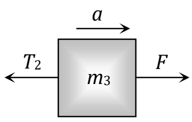
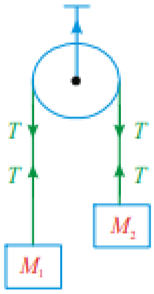
Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \dfrac{m_2 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \dfrac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \dfrac{m_3 F}{m_1 + m_2 + m_3}$

Table: Motion of Blocks Connected by Mass Less String

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \dfrac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \dfrac{m_1 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \dfrac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \dfrac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \dfrac{(m_1 + m_2)F}{m_1 + m_2 + m_3}$

Atwood's Machine

Consider the masses M_1 and M_2 ($M_1 > M_2$) are tied to a string, which passes over a frictionless light pulley. The string is massless and inextensible.



Acceleration of the system,

$$a = \frac{(M_1 - M_2)g}{M_1 + M_2}$$

Tension in the string,

$$T = \left(\frac{2M_1M_2}{M_1 + M_2}\right)g$$

Thrust on the pulley,

$$F = 2T = \left(\frac{4M_1M_2}{M_1 + M_2}\right)g$$

- If the pulley begins to move with acceleration $\bar{a}$ then,

i) If the pulley accelerates upward, then

$$a_{net} = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)(g + a)$$

$$T_{net} = \left(\frac{2M_1M_2}{M_1 + M_2}\right)(g + a)$$

ii) If the pulley accelerates downward, then

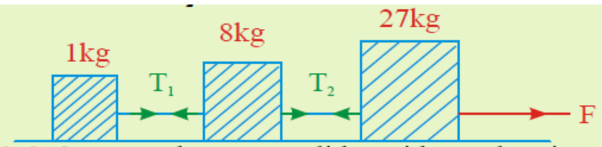
$$a_{net} = \left(\frac{M_1 - M_2}{M_1 + M_2}\right)(g - a)$$

$$T_{net} = \left(\frac{2M_1M_2}{M_1 + M_2}\right)(g - a)$$

iii) Thrust on the pulley when it comes downward with acceleration 'a' is

$$F = \frac{4M_1M_2}{(M_1 + M_2)}(g - a)$$

Problem No. 22 - Three blocks connected together by strings are pulled along a horizontal surface by applying a force F. If F= 36N, What is the tension T_2?



Sol : Suppose the system slides with acceleration 'a'.

$m_1 = 1kg, m_2 = 8kg, m_3 = 27kg$

$F - T_2 = m_3 a, T_2 - T_1 = m_2 a, T_1 = m_1 a$

Solving the above equations, we get

$$a = \frac{F}{m_1 + m_2 + m_3}$$

$$= \frac{36}{1+8+27} = \frac{36}{36} = 1\ ms^{-2}$$

From the above equation, $T_2 = F - m_3 a$

$T_2 = 36 - 27 \times 1 = 9$ N

Problem No 23 - The maximum tension a rope can withstand is 60 kg-wt. The ratio of maximum acceleration with which two boys of masses 20kg and 30kg can climb up the rope at the same time is,

Sol. $m_1 = 20kg, m_2 = 30kg, T = 60kgwt = 600N$

For 'm_1'; $T - m_1 g = m_1 a_1$

$600 - 20 \times 10 = 20 \times a_1 \Rightarrow a_1 = 20ms^{-2}$

For 'm_2'; $T - m_2 g = m_2 a_2$

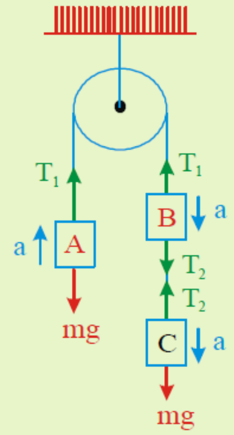
$600 - 30 \times 10 = 30 \times a_2 \Rightarrow a_2 = 10ms^{-2}$

$a_1 : a_2 = 20 : 10 = 2 : 1$

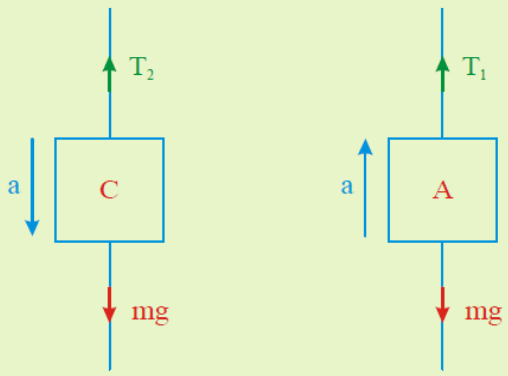
Problem No 24 - Figure shows three blocks of mass 'm' each hanging on a string passing over a pulley. Calculate the tension in the string connecting A to B and B to C ?

Sol. Net pulling force $= 2mg - mg = mg$

Total mass $= m + m + m = 3m$



Acceleration, $a = \frac{mg}{3m} = \frac{g}{3}$



Considering block A,

$T_1 - mg = ma$; $T_1 = mg + ma$

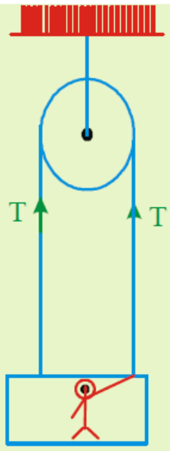
$T_1 = mg + m\left(\frac{g}{3}\right) \Rightarrow T_1 = \frac{4}{3}mg$

Considering block C,

$mg - T_2 = ma \Rightarrow T_2 = mg - ma$

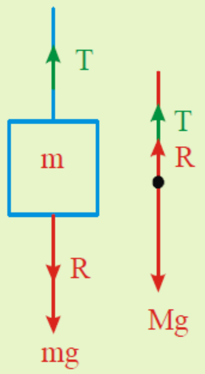
$\Rightarrow T_2 = mg - \frac{mg}{3} = \frac{2}{3}mg$.

Problem No 25 - A man of mass 60 kg is standing on a weighing machine kept in a box of mass 30 kg as shown in the diagram. If the man manages to keep the box stationary, find the reading of the weighing machine.



Sol. we know that Normal reaction = scale reading

For man , $T = Mg - R$

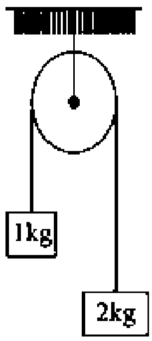


For box : $T = mg + R$

$Mg - R = mg + R$; $2R = (M - m)g$

$R = \frac{(60-30)\times 10}{2} = 150N$

Problem No 26 - Two unequal masses are connected on two sides of a light string passing over a light and smooth pulley as shown in figure. The system is released from rest. The larger mass is stopped for a moment, 1sec after the system is set into motion. Find the time elapsed before the string is tight again. (g = 10 m/s²)



Sol. Net pulling force = 2g – 1g = 10N Mass being pulled = 2 + 1 = 3 kg

Acceleration of the system is $a = \frac{10}{3} m/s^2$ Velocity of both the blocks at t = 1 s will be

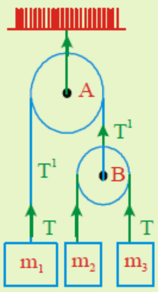
$v_0 = at = \left(\frac{10}{3}\right)(1) = \frac{10}{3}$ m/s. Now, at this moment velocity of 2kg block becomes zero, while that of 1kg block is $\frac{10}{3}$ m/s upwards. Hence, string becomes tight again when displacement of 1 kg block = displacement of 2 kg block.

$v_0 t - \frac{1}{2}gt^2 = \frac{1}{2}gt^2 \Rightarrow gt^2 = v_0 t$

$t = \frac{v_0}{g} = \frac{(10/3)}{10} = \frac{1}{3}s$

Problem No 27 - In the figure, if m1 is at rest, find the relation among m₁, m₂ and m₃?

Sol. m_1 is at rest ⇒ point B does not move, m_2 and m_3 move with acceleration

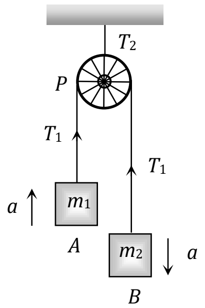
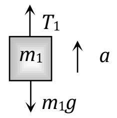
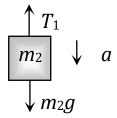
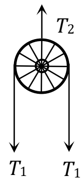
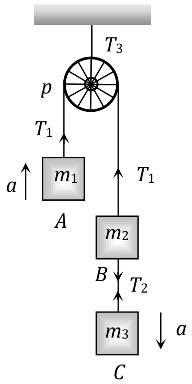
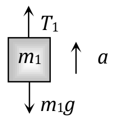
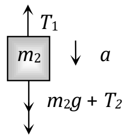
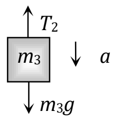
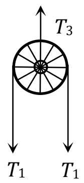
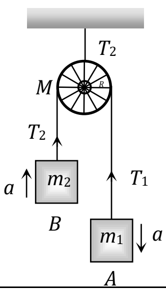
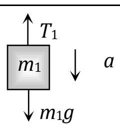
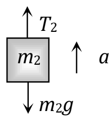


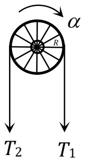
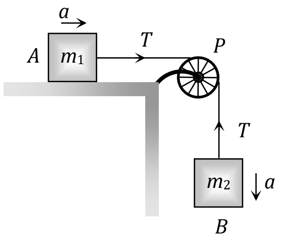
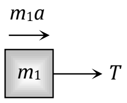
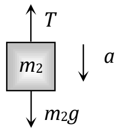
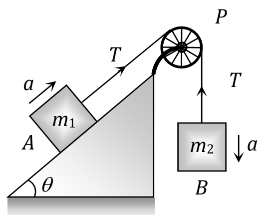
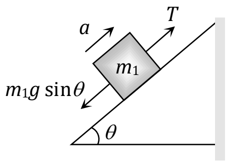
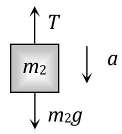
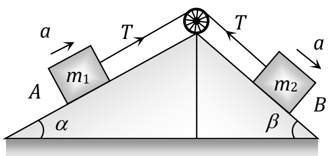
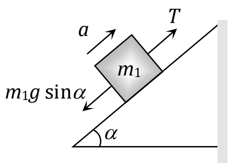
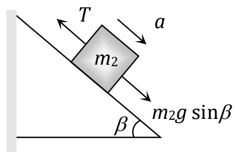
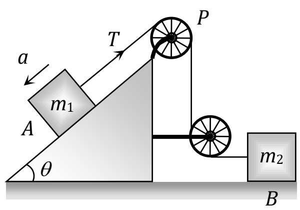
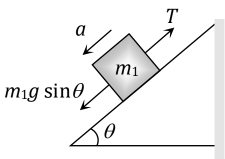
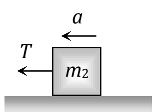
$a = \left(\frac{m_3 - m_2}{m_2 + m_3}\right)g$; $m_3 > m_2$

$T = \frac{2m_2 m_3 g}{m_2 + m_3}$; $T^1 = 2T = \frac{4m_2 m_3 g}{m_2 + m_3}$

$m_1 \cancel{g} = \frac{4m_2 m_3 \cancel{g}}{m_2 + m_3}$ $\boxed{\frac{4}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}}$

Table: Motion of Connected Blocks over a Pulley

Condition	Free body diagram	Equation	Tension and acceleration
		$m_1a = T_1 - m_1g$	$T_1 = \frac{2m_1m_2}{m_1+m_2}g$
		$m_2a = m_2g - T_1$	$T_2 = \frac{4m_1m_2}{m_1+m_2}g$
		$T_2 = 2T_1$	$a = \left[\frac{m_2-m_1}{m_1+m_2}\right]g$
		$m_1a = T_1 - m_1g$	$T_1 = \frac{2m_1[m_2+m_3]}{m_1+m_2+m_3}g$
		$m_2a = m_2g + T_2 - T_1$	$T_2 = \frac{2m_1m_3}{m_1+m_2+m_3}g$
		$m_3a = m_3g - T_2$	$T_3 = \frac{4m_1[m_2+m_3]}{m_1+m_2+m_3}g$
		$T_3 = 2T_1$	$a = \frac{[(m_2+m_3)-m_1]g}{m_1+m_2+m_3}$
When pulley have a finite mass *M* and radius *R* then tension in two segments of string are different 		$m_1a = m_1g - T_1$	$a = \frac{m_1-m_2}{m_1+m_2+\frac{M}{2}}$
		$m_2a = T_2 - m_2g$	$T_1 = \frac{m_1\left[2m_2+\frac{M}{2}\right]}{m_1+m_2+\frac{M}{2}}g$

<table>
<tr><td></td><td></td><td>Torque $=(T_1-T_2)R=I\alpha$
$(T_1-T_2)R=I\dfrac{a}{R}$
$(T_1-T_2)R=\dfrac{1}{2}MR^2\dfrac{a}{R}$
$T_1-T_2=\dfrac{Ma}{2}$</td><td>$T_2=\dfrac{m_2\left[2m_1+\dfrac{M}{2}\right]}{m_1+m_2+\dfrac{M}{2}}g$</td></tr>
<tr><td rowspan="2"></td><td></td><td>$T=m_1a$</td><td>$a=\dfrac{m_2}{m_1+m_2}g$</td></tr>
<tr><td></td><td>$m_2a=m_2g-T$</td><td>$T=\dfrac{m_1m_2}{m_1+m_2}g$</td></tr>
<tr><td rowspan="2"></td><td></td><td>$m_1a=T-m_1g\sin\theta$</td><td>$a=\left[\dfrac{m_2-m_1\sin\theta}{m_1+m_2}\right]g$</td></tr>
<tr><td></td><td>$m_2a=m_2g-T$</td><td>$T=\dfrac{m_1m_2(1+\sin\theta)}{m_1+m_2}g$</td></tr>
<tr><td rowspan="2"></td><td></td><td>$T-m_1g\sin\alpha=m_1a$</td><td>$a=\dfrac{(m_2\sin\beta-m_1\sin\alpha)}{m_1+m_2}g$</td></tr>
<tr><td></td><td>$m_2a=m_2g\sin\beta-T$</td><td>$T=\dfrac{m_1m_2(\sin\alpha+\sin\beta)}{m_1+m_2}g$</td></tr>
<tr><td rowspan="2"></td><td></td><td>$m_1g\sin\theta-T=m_1a$</td><td>$a=\dfrac{m_1g\sin\theta}{m_1+m_2}$</td></tr>
<tr><td></td><td>$T=m_2a$</td><td>$T=\dfrac{2m_1m_2}{4m_1+m_2}g$</td></tr>
</table>

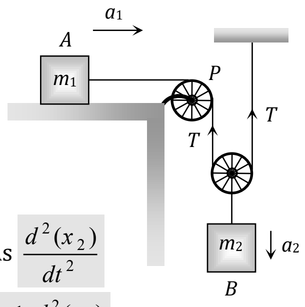
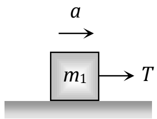
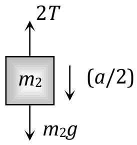
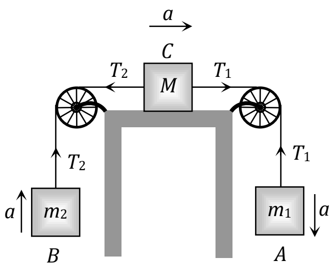
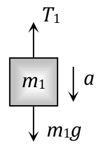
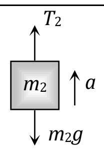
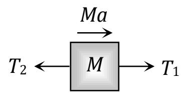
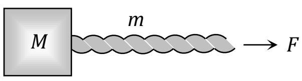
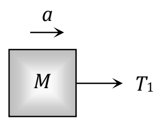
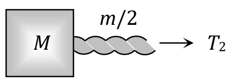
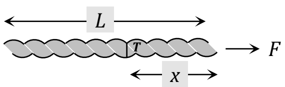
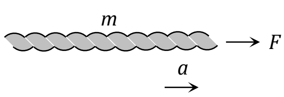
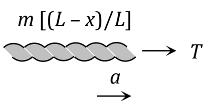
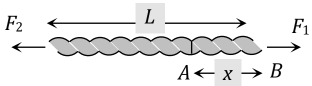
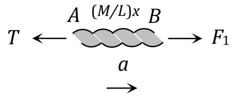
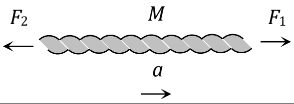
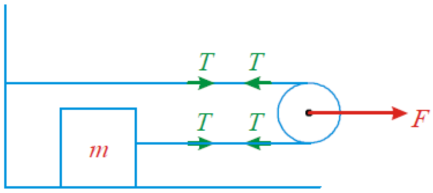
 As $\frac{d^2(x_2)}{dt^2} = \frac{1}{2}\frac{d^2(x_1)}{dt^2}$ $\therefore a_2 = \frac{a_1}{2}$ a_1 = acceleration of block A a_2 = acceleration of block B		$T = m_1 a$	$a_1 = a = \frac{2m_2 g}{4m_1 + m_2}$ $a_2 = \frac{m_2 g}{4m_1 + m_2}$ $T = \frac{2m_1 m_2 g}{4m_1 + m_2}$
		$m_2 \frac{a}{2} = m_2 g - 2T$	
		$m_1 a = m_1 g - T_1$	$a = \frac{(m_1 - m_2)}{[m_1 + m_2 + M]} g$
		$m_2 a = T_2 - m_2 g$	$T_1 = \frac{m_1(2m_2 + M)}{[m_1 + m_2 + M]} g$
		$T_1 - T_2 = Ma$	$T_2 = \frac{m_2(2m_2 + M)}{[m_1 + m_2 + M]} g$

Table: Motion of Massive String

Condition	Free body diagram	Equation	Tension and acceleration
	 T_1 = force applied by the string on the block	$F = (M + m)a$ $T_1 = Ma$	$a = \frac{F}{M + m}$ $T_1 = M\frac{F}{(M + m)}$ $T_2 = \frac{(2M + m)}{2(M + m)} F$
	 T_2 = Tension at mid point of the rope	$T_2 = \left(M + \frac{m}{2}\right) a$	
 m = Mass of string T = Tension in string at a distance x from the end where the force is applied		$F = ma$	$a = F/m$ $T = \left(\frac{L - x}{L}\right) F$
		$T = m\left(\frac{L - x}{L}\right) a$	

 M = Mass of uniform rod L = Length of rod		$F_1 - T = \frac{Mxa}{L}$	$a = \frac{F_1 - F_2}{M}$
		$F_1 - F_2 = Ma$	$T = F_1\left(1 - \frac{x}{L}\right) + F_2\left(\frac{x}{L}\right)$

- A force F is applied on the massless pulley as shown in the figure and string is connected to the block on smooth horizontal surface. Then



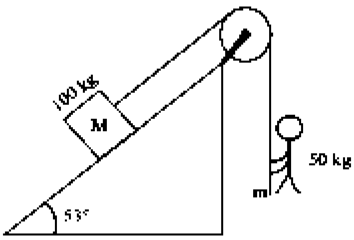
$$\boldsymbol{F = 2T \quad and \quad T = ma_{block}}$$

If the block moves a distance 'x' the pulley moves x/2 (Total length of the string remains constant)

Therefore acceleration of the pulley

$$a_{pulley} = \frac{a_{block}}{2} = \frac{T}{2m} = \frac{F/2}{2m} = \frac{F}{4m}$$

Problem No 28 - By what acceleration the boy must go up so that 100 kg block remains stationary on the wedge. wedge is fixed and is smooth. (g = 10m/s²)



Sol : For the block to remain stationary,

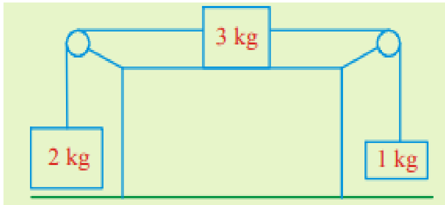
$T = Mg\sin\theta = 100 \times 10 \times \sin 53$

$= 100 \times 10 \times \frac{4}{5} = 800\text{N}$

For man ; $T - mg = ma$

$T = m(g + a) \Rightarrow 800 = 50(10 + a) \quad a = 6\text{m/s}^2$

Problem No 29 - The system as shown in fig is released from rest. Calculate the tension in the strings and force exerted by the strings on the pulley, assuming pulleys and strings are massless.



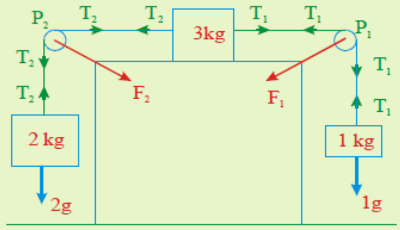
Sol: $T_1 - 1g = 1a$ — (1)

$T_2 - T_1 = 3a$ — (2)

$2g - T_2 = 2a$ —(3)

Solving the above equations,

we get , $a = \frac{g}{6} m/s^2$



$T_1 = \frac{7g}{6} N, T_2 = \frac{5g}{3} N$

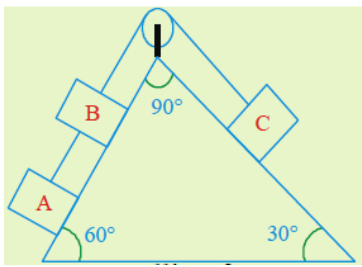
Force on pulley P_1 is $F_1 = \sqrt{T_1^2 + T_1^2}$

$= \sqrt{2}T_1 = \frac{7g}{3\sqrt{2}}$ N

Force on pulley P_2 is $F_2 = \sqrt{T_2^2 + T_2^2}$

$= \sqrt{2}T_2 = \frac{5\sqrt{2}g}{3}$ N

Problem No 30 - In the adjacent fig, masses of A, B and C are 1kg, 3kg and 2kg respectively. Find a) the acceleration of the system b) tension in the string (g =10m/s²)



Sol .a) In this case net pulling force

$= m_A g\sin 60^0 + m_B g\sin 60^0 - m_C g\sin 30^0$

$=(1)(10)\frac{\sqrt{3}}{2}+(3)(10)\left(\frac{\sqrt{3}}{2}\right)-(2)(10)\left(\frac{1}{2}\right)=24.64\text{N}$

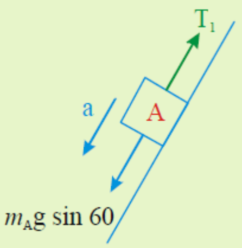
Total mass $= 1+3+2 = 6\text{kg}$

$\therefore$ Acceleration of the system

$a=\frac{24.64}{6}=4.1\,m/s^2$

b) For the tension in the string between A and B.

FBD of Body A



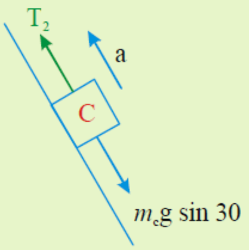
$m_A g\sin 60 - T_1 = m_A a$

$T_1 = m_A g\sin 60 - m_A a = m_A\left(g\sin 60 - a\right)$

$T_1=(1)\left(10\times\frac{\sqrt{3}}{2}-4.1\right)=4.56N$

For the tension in the string between B and C

FBD of body C



$T_2 - m_C g\sin 30^0 = m_C a\,;\; T_2 = m_C\left(g\sin 30^0 + a\right)$

$T_2=2\left(10\left(\frac{1}{2}\right)+4.1\right)=18.2N$

Constrained Motion

(a) Constraint : Restriction to the free motion of a body in any direction is called constraint.

(b) Constrained Body : A body whose displacement in space is restricted by other bodies, either connected to or in contact with it, is called a constrained body.

(c) Kinematic Constraints : These are equations that relate the motion of two or more particles.

Types of Constraints

i) General constraints

ii) Pulley constraints

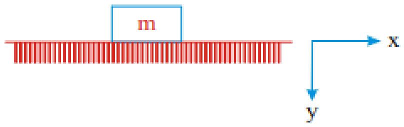
iii) Wedge constraints

iv) Mixed constraints

General Constraints

i) A body placed on floor:

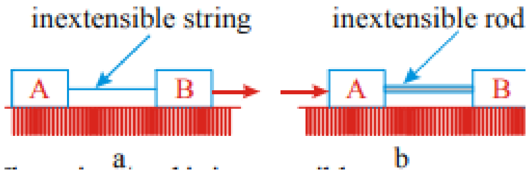
The floor acting as a constraint restricts the kinematical quantities in the downward direction such that



y =0 ; $v_y = 0$,

and $a_y = 0$ for body placed on the floor.

ii) Two bodies connected with a string or rod.



The string / rod is inextensible.

$\therefore$ Displacements of A and B are equal in horizontal direction $\Rightarrow S_A = S_B$

Differentiating w.r.t. time,

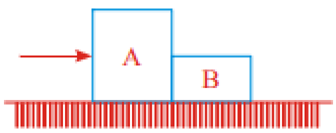
$\frac{ds_A}{dt}=\frac{ds_B}{dt}\Rightarrow v_A = v_B$

Again differentiating

$\frac{dv_A}{dt}=\frac{dv_B}{dt}\Rightarrow a_A = a_B$

iii) Two bodies in contact with each other

Displacement of A and B are equal in horizontal direction.



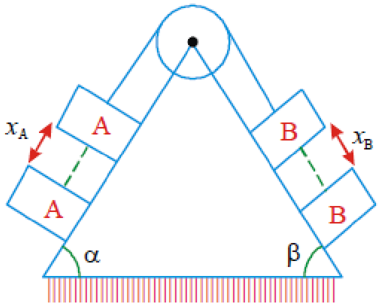
$S_A = S_B$

By differentiating, we will get

$v_A = v_B$ and $a_A = a_B$ in horizontal direction

Pulley Constraints

For example, the motion of block A is downwards along the inclined plane in fig. will cause a corresponding motion of block B up the other inclined plane. Assuming string AB length is inextensible, i.e., length of AB is constant.



The displacements of A (x_A) and B (x_B) are equal

$$\therefore x_A = x_B$$

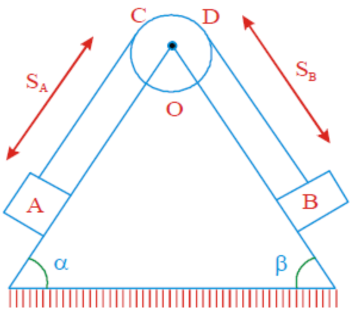
Differentiating w.r.t. time $\Rightarrow v_A = v_B$

Once again differentiating w.r.t. time,

$$a_A = a_B$$

i.e., if one body (A) moves down the inclined plane with certain acceleration, then the other body will move up inclined plane with an equal acceleration (magnitude).

Alternate Method: First specify the location of the blocks using position co-ordinates S_A and S_B.



From the fig. the position co-ordinates are related by the equation

$$S_A + l_{CD} + S_B = \text{L}$$

where l_{CD} = the length of the string over arc, CD = constant, L = total length of the string = constant

Differentiating w.r.t. time, we get

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0$$

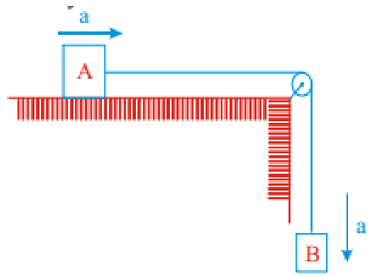
$$\therefore v_B = -v_A$$

The negative sign indicates that when block A has a velocity downward, i.e., in the direction of positive S_A, it causes a corresponding upward velocity of block B, i.e., B moves in the negative S_B direction.

Again differentiating w.r.t. time,

$$\frac{dv_B}{dt} + \frac{ds_A}{dt} = 0 \Rightarrow a_B = -a_A$$

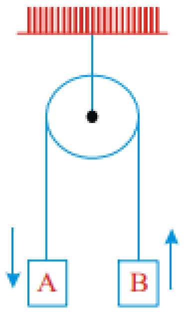
Similarly,



$$x_A = x_B$$

$$\Rightarrow v_A = v_B$$

$$\Rightarrow a_A = a_B$$



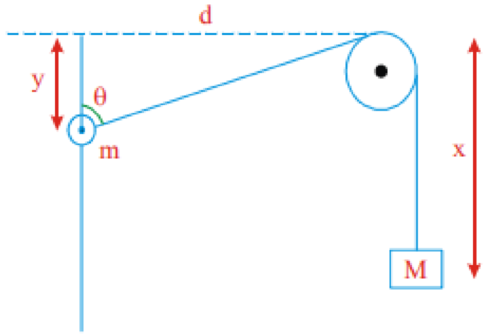
$$x_A = x_B \Rightarrow v_A = v_B \Rightarrow a_A = a_B$$

Mixed Constraints

Ring sliding on a smooth rod:

Consider a ring of mass m connected through a string of length L with a block of mass M. If the ring is moving up with acceleration a_m and a_M is the acceleration of block. As the length of the string is constant,

$$\text{L} = \sqrt{d^2 + y^2} + \text{x}$$



Since, L is constant, differentiating with respect to time t, we get

$$\frac{dL}{dt} = \frac{1}{2}\frac{2y}{(d^2+y^2)^{\frac{1}{2}}}\left(\frac{dy}{dt}\right) + \frac{dx}{dt} = 0$$

Since $\frac{dy}{dt} = v_m$ and $\frac{dx}{dt} = v_M$

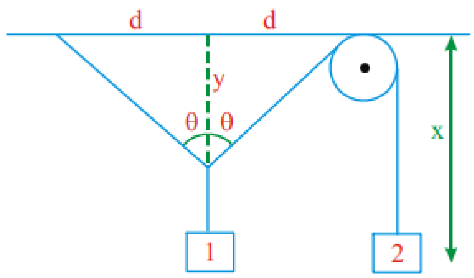
and Cos θ $\frac{y}{\sqrt{d^2+y^2}}$

so $v_M = -v_m \cos\theta$

By differentiating, relation between a_m and a_M can be obtained, however, while doing so remember that cos θ is not constant, but it is variable.

Two blocks connected with pulley:

If the blocks are connected as shown in fig, then the length of the string is



$$L = \sqrt[2]{d^2 + y^2} + x$$

Since, L is constant, differentiating with respect to time t, we get

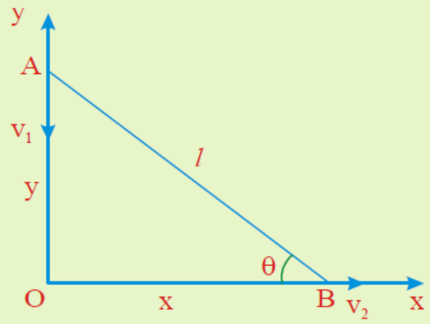
$$\frac{dL}{dt} = \frac{2\times 2y}{2(d^2+y^2)^{\frac{1}{2}}}\left(\frac{dy}{dt}\right) + \frac{dx}{dt} = 0$$

$$\Rightarrow 2v_1 \cos\theta + v_2 = 0;$$

$$v_2 = -v_1 \cos\theta$$

Problem No 31 - A rod of length ‘l’is inclined at an angle ‘q ’with the floor against a smooth vertical wall.If the end A moves instantaneously with velocity v_1,what is the velocity of end B at the instant when rod makes ‘θ’ angle with the horizontal.

Sol: Let at any instant,end B and A are at a distance x and y respectively from the point ‘O’.



Thus we have, $x^2 + y^2 = l^2$ ....(1)

Here l is the length of the rod,which is constant.
Differentiating eq (1) with respect to time,we get

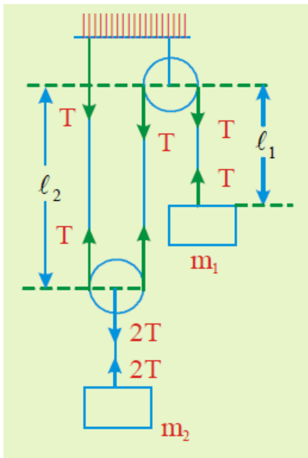
$$\frac{d}{dt}(x^2+y^2) = \frac{d}{dt}(l^2); \quad 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

If $\frac{dx}{dt} = v_2$ and $\frac{dy}{dt} = -v_1$

$$x(v_2) + y(-v_1) = 0$$

$$\Rightarrow v_2 = \left(\frac{y}{x}\right)v_1 = v_1 \tan\theta$$

Problem No 32 - In the fig, find the acceleration of mass m_2



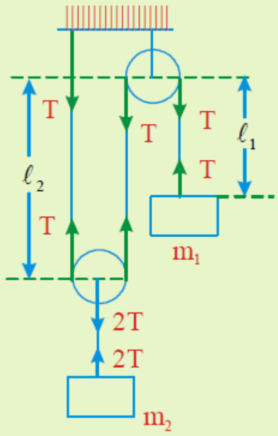
Sol: $\ell_1 + 2\ell_2 = \text{constant}$

on differentiating

$$v_1 + 2v_2 = 0$$

Again differentiating

$$a_1 + 2a_2 = 0 \Rightarrow a_1 = -2a_2$$

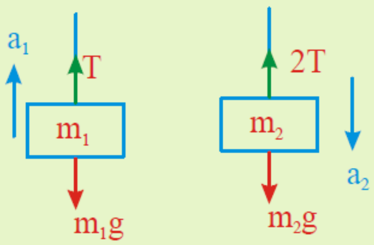


'- ve' sign indicates that the accelerations are in opposite direction.Suppose acceleration of m_2 is a_2 downward and then acceleration of m_1 will be a_1 upwards.

$$T - m_1 g = m_1 a_1$$

$$T = m_1 g + m_1 a_1$$

$$m_2 g - 2T = m_2 a_2$$



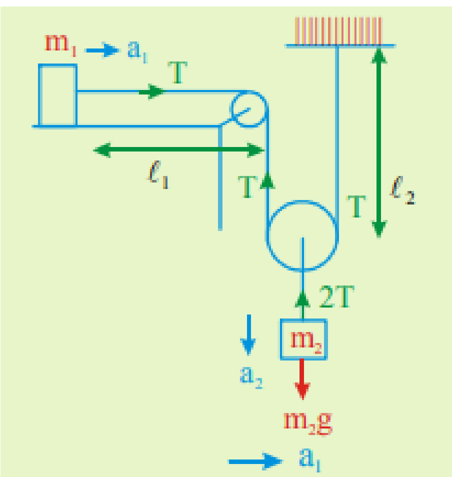
$$m_2 g - 2(m_1 g + m_1 a_1) = m_2 a_2$$

$$m_2 g - 2m_1 g = m_2 a_2 + 4m_1 a_2 \quad (\because a_1 = 2a_2)$$

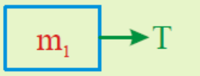
'–' sign should not be substituted

$$a_2 = \frac{(m_2 - 2m_1)g}{4m_1 + m_2} \quad ms^{-2}$$

Problem No 33 - In the fig, find the acceleration of m_1 and m_2



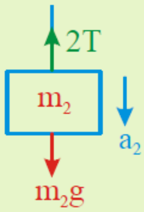
Sol. $\ell_1 + 2\ell_2$ = constant



$$a_1 = 2a_2 \; ; \; T = m_1 a_1$$

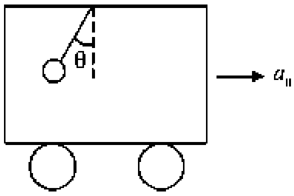
$$m_2 g - 2T = m_2 a_2 \; ; \; m_2 g = 2m_1 a_1 + m_2 a_2$$

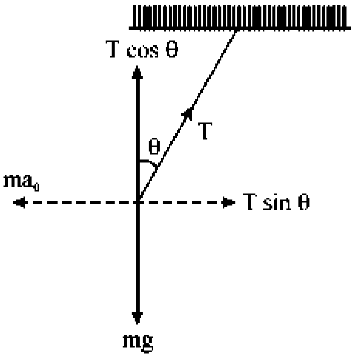
$$m_2 g = 2m_1(2a_2) + m_2 a_2$$



$$a_2 = \frac{m_2 g}{4m_1 + m_2} \quad , \quad a_1 = \frac{2m_2 g}{4m_1 + m_2}$$

Problem No 34 - A pendulum is hanging from the ceiling of a car having an acceleration a0 with respect to the road. Find the angle made by the string with vertical at equilibrium. Also find the tension in the string in this position.





Sol : $T\sin\theta = ma_0 ......(i) ; T\cos\theta = mg......(ii)$

dividing (i)&(ii) $\tan\theta = \dfrac{a_0}{g}$

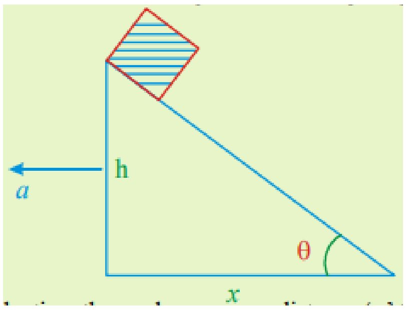
$\therefore$ The string is making an angle $\theta = \tan^{-1}\left(\dfrac{a_0}{g}\right)$ with the vertical at equilibrium

Squaring and adding (i) and (ii)

$$T^2\sin^2\theta + T^2\cos^2\theta = m^2\left(a_0^2 + g^2\right)$$

$$T = m\sqrt{a_0^2 + g^2}$$

Problem No 35 - For what value of 'a' the block falls freely?



Sol : In the time the wedge moves a distance 'x' towards left with an acceleration a the block falls from a height 'h' with acceleration 'g'

$$x=\frac{1}{2}at^2,\ h=\frac{1}{2}gt^2 \Rightarrow \frac{x}{h}=\frac{a}{g},\ \Rightarrow \cot\theta=\frac{a}{g}\ \Rightarrow a=g\cot\theta$$

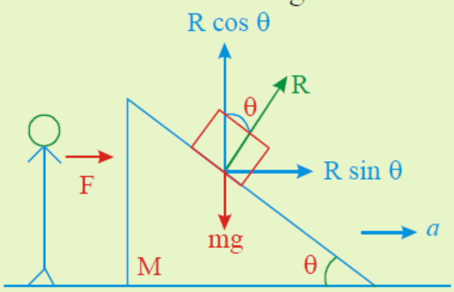
Problem No 36 - A block of mass m is placed on a smooth wedge of inclination. The whole system is accelerated horizontally so that the block does not slip on the wedge. Find the

i) Acceleration of the wedge

ii) Force to be applied on the wedge

iii) Force exerted by the wedge on the block.

Sol. **(i)**. For an observer on the ground :



$R\sin\theta = ma,\ R\cos\theta = mg$

$\Rightarrow a = g\tan\theta$

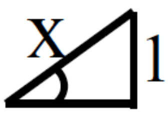
ii) F = (M + m)a = (M + m) g tan

iii) Force exerted by the wedge on the block

$\Rightarrow R=\dfrac{mg}{\cos\theta}$ or $R = mg\sec\theta$

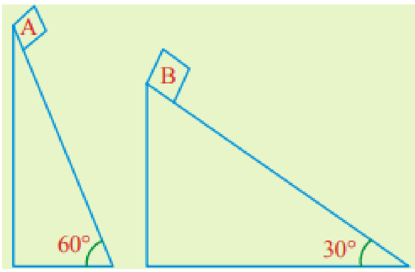
Note : If inclination is given as 1 in x, $\sin\theta=\dfrac{1}{x}$

$\tan = \dfrac{1}{\sqrt{x^2-1}}$



$\Rightarrow$ Acceleration $a = g\tan = \dfrac{g}{\sqrt{x^2-1}}$

Problem No 37 - Two fixed frictionless inclined planes making an angles 30^0 and 60^0 with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



Sol: $mg\sin\theta = ma \Rightarrow a = g\sin\theta$

where a is along the inclined plane

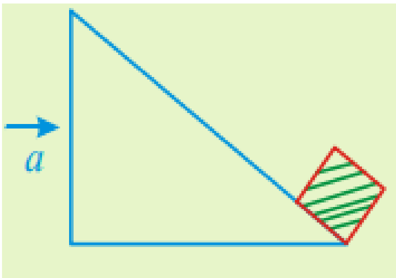
$\therefore$ vertical component of acceleration is $g\sin^2\theta$

$a_r = a_{AB} = a_A - a_B$

$\therefore$ relative vertical acceleration of A with respect to B is $g\left(\sin^2 60^0 - \sin^2 30^0\right)=\dfrac{g}{2}=4.9ms^{-2}$

(in vertical direction)

Problem No 38 - For what value of 'a' block slides up the plane with an acceleration 'g' relative to the inclined plane.



Sol.

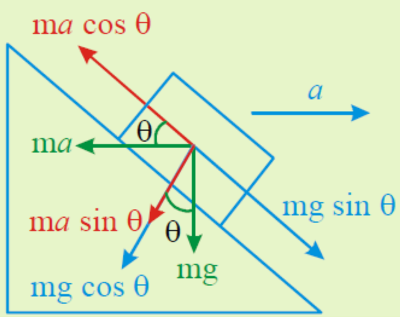
$F_{net} = ma\cos\theta - mg\sin\theta$

$ma' = ma\cos\theta - mg\sin\theta$

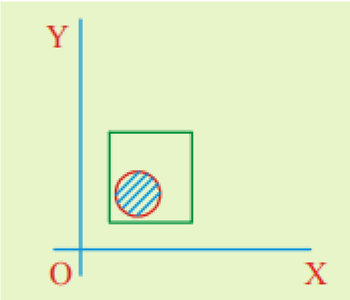
If $a' = g$, $mg = ma\cos\theta - mg\sin\theta$

$a\cos\theta = g + g\sin\theta \Rightarrow a = g\left(\dfrac{1+\sin\theta}{\cos\theta}\right)$

$\Rightarrow a = g(\sec\theta + \tan\theta)$

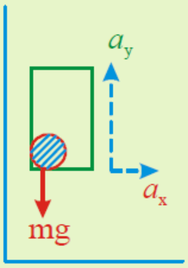


Problem No 39 - A solid sphere of mass 2kg rests inside a cube as shown. The cube is moving with velocity $\vec{v}$ = $(5t\hat{\imath} + 2t\hat{\jmath})ms^{-1}$ where 't' is in sec and 'υ' is in m/s. What force does sphere exert on cube?



Sol. As given, $\vec{\upsilon} = 5t\hat{i} + 2t\hat{j}$;

$$\therefore a_x = \frac{d\upsilon_x}{dt} = 5,\ a_y = \frac{d\upsilon_y}{dt} = 2$$



When cube is moving with above accelerations along x and y-axes, the forces that exert on cube are

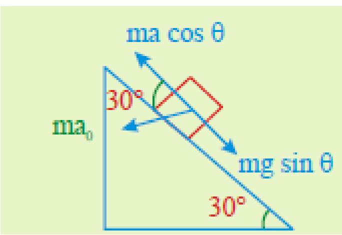
$$F_x = -ma_x = -2 \times 5 = -10N$$

$$F_y = -\left(mg + ma_y\right) = -(20 + 2 \times 2) = -24N$$

Net force, $F = \sqrt{(F_x)^2 + (F_y)^2}$

$$= \sqrt{(10)^2 + (24)^2} = 26N$$

Problem No 40 - A block is placed on an inclined plane moving towards right with an acceleration a_0 = g. The length of the inclined plane is 0 l. All the surfaces are smooth. Find the time taken by the block to reach from bottom to top.



Sol. $ma = ma_0 \cos 30^0 - mg \sin 30^0$

$$a = \frac{ma_0 \cos 30^0 - mg \sin 30^0}{m}$$

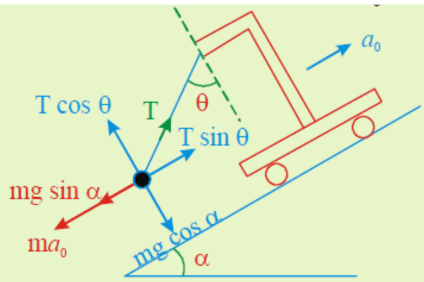
$$a = \frac{ma_0 \frac{\sqrt{3}}{2} - mg\frac{1}{2}}{m} \quad = g\left(\frac{\sqrt{3}-1}{2}\right)$$

from $s = ut + \frac{1}{2}at^2$; $\quad \ell_0 = \frac{1}{2}at^2$

$$\ell_0 = \frac{1}{2}g\left(\frac{\sqrt{3}-1}{2}\right)t^2 \Rightarrow t = \sqrt{\frac{4\ell_0}{g\left(\sqrt{3}-1\right)}} \text{ sec}$$

Problem No 41 - A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination a with acceleration a_0 is

Sol.



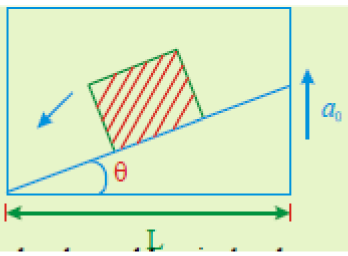
$T \sin \quad = ma_0 + mg \sin$ -------------(1)

$T \cos \quad = mg \cos$ -------------(2)

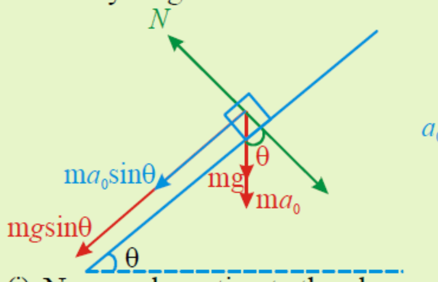
$\frac{①}{②} \Rightarrow \tan \quad = \frac{a_0 + g \sin}{g \cos}$

$$\boxed{= \tan^{-1}\left[\frac{a_0 + g\sin}{g\cos}\right]}$$

Problem No 42 - A block slides down from top of a smooth inclined plane of elevation q fixed in an elevator going up with an acceleration a0.The base of incline has length L. Find the time taken by the block to reach the bottom.



Sol. Let us solve the problem in the elevator frame. The free body diagram is shown. The forces are



(i) N normal reaction to the plane,

(ii) mg acting vertically downwards,

(iii) ma_0 (pseudo force).acting vertically down

If a is acceleration of the body with respect to inclined plane, taking components of forces parallel to the inclined plane.

$mg\sin\theta + ma_0\sin\theta = ma \quad \therefore \quad a = (g + a_0)\sin\theta$

This is the acceleration with respect to the elevator

The distance travelled is $\frac{L}{\cos\theta}$. If 't' is the time for reaching the bottom of inclined plane

$$\frac{L}{\cos\theta} = 0 + \frac{1}{2}(g + a_0)\sin\theta.t^2$$

$$t = \left[\frac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{\frac{1}{2}} = \left[\frac{4L}{(g + a_0)\sin 2\theta}\right]^{\frac{1}{2}}$$

Law of Conservation of Momentum

When the resultant external force acting on a system is zero, the total momentum (vector sum) of the system remains constant. This is called "law of conservation of linear momentum".

Newton's third law of motion leads to the law of conservation of linear momentum.

Walking, running, swimming, jet propulsion, motion of rockets, rowing of a boat, recoil of a gun etc., can be explained by Newton's third law of motion.

Explosions, disintegration of nuclei, recoil of gun, collisions etc., can be explained on the basis of the law of conservation of linear momentum.

Applications:

- When a shot is fired from a gun, while the shot moves forwards, the gun moves backwards. This motion of gun moves backwards. This motion of gun is called **recoil of the gun**. When a gun of mass 'M' fires a bullet of mass 'm' with a muzzle velocity 'v', the gun recoils with a velocity 'V' given by

  V = mv/M

- When a bullet of mass 'm' moving with a velocity 'v' gets embedded into a block of mass M at rest and free to move on a smooth horizontal surface, then their common velocity

  V = mv/(M + m)

- A boy of mass 'm' walks a distance 's' on a boat of mass 'M' that is floating on water and initially at rest. If the boat is free to move, it moves back a distance,

  d = ms/(M + m)

Explosion of Bomb:

- A shell of mass 'M' at rest explodes into two fragments and one of masses 'm' moves out with a velocity 'v', then other piece of mass (M – m) moves in opposite direction with a velocity of

  V = mv/(M – m)

- Suppose a shell of mass m at rest explodes into three pieces of masses m_1, m_2 and m_3, moving with velocities $\vec{v_1}$, $\vec{v_2}$ and $\vec{v_3}$ respectively, then

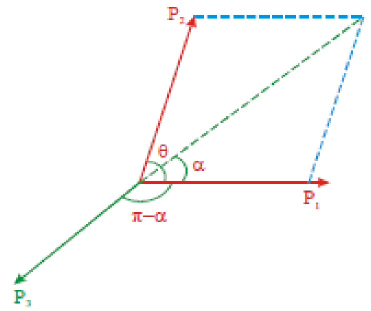
$$m_1\vec{v_1} = \vec{p_1};$$
$$m_2\vec{v_2} = \vec{p_2};$$
$$m_3\vec{v_3} = \vec{p_3}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = m_1\vec{v_1} + m_2\vec{v_2} + m_3\vec{v_3} = 0$$

(as shell is at rest initially)

$$\therefore \ \vec{p_3} = -(\vec{p_1} + \vec{p_2})$$

So the third piece moves with the same magnitude of the resultant momentum of the other two pieces but in opposite direction.



$P_3 = \sqrt{P_1^2 + P_2^2 + 2P_1P_2COS\ \theta}$

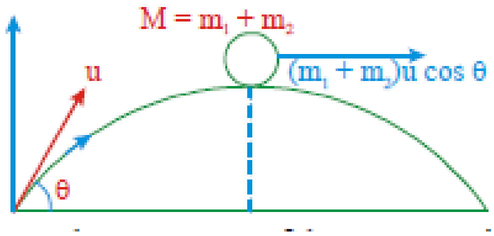
θ = angle between $\vec{P_1}$, $\vec{P_2}$

(π - α) = angle between $\vec{P_3}$, $\vec{P_1}$

$$\tan\alpha = \frac{P_2 \sin\theta}{P_1 + P_2\cos\theta}$$

Explosion of a shell travelling in a parabolic path at its highest point (into two fragments):

Consider a shell of mass M as a projectile with velocity u and angle of projection θ. Suppose the shell breaks into two fragments at maximum height and their initial velocities are $\vec{v_1}$ and $\vec{v_2}$



Total momentum of the two parts is constant just before and just after the explosion.

$$[m_1 + m_2]\ u\cos\theta\vec{i} = m_1\ \vec{v_1} + m_2\ \vec{v_2}$$

Case: (i) If the fragments travel in opposite direction after explosion then

$$(m_1 + m_2)\ u\cos\theta\vec{i} = m_1\ v_1\vec{i} - m_2\ v_2\vec{i}$$

Case: (ii) If one fragment retraces its path and falls at the point of projection

$$(m_1 + m_2)u\cos\theta\vec{i} = -m_1 u\cos\theta\vec{i} + m_2\ \vec{v_2}$$

Case: (iii) If one fragment falls freely after explosion

$$(m_1 + m_2)u\cos\theta\vec{i} = m_1 0 + m_2\ \vec{v_2}$$

$$(m_1 + m_2)u\cos\theta\vec{i} = m_2\ \vec{v_2}$$

Rocket propulsion:

The initial momentum of the rocket on its launching pad is zero. When it is fired from the launching pad, the exhaust gases rush downward at a high speed and to conserve momentum, the rocket moves upwards.

Let m = mass of rocket at any instant 't' (instantaneous mass), u = velocity of exhaust gases,

$\frac{dm}{dt}$ = rate of change of mass of rocket

= rate of fuel consumption

= rate of ejection of the fuel (gases).

(a) Thrust on the rocket:

$$\boldsymbol{F = -\left(u\frac{dm}{dt} - mg\right)}$$

Here negative sign indicates that direction of thrust is opposite to the direction of escaping gases.

$F = -u\frac{dm}{dt}$ (if effect of gravity is neglected)

(b) Acceleration of the rocket:

$$\boldsymbol{a = \frac{u}{m}\frac{dm}{dt} - g}$$

and if effect of gravity is neglected

$$a = \frac{u}{m}\frac{dm}{dt}$$

(c) Instantaneous velocity of the rocket:

$$\boldsymbol{v = u\log_e\left(\frac{m_0}{m}\right) - gt}$$

and if effect of gravity is neglected

$$v = u\log_e\left(\frac{m_0}{m}\right) = 2.303u\log_{10}\left(\frac{m_0}{m}\right)$$

(d) Burnt out speed of the rocket :

$$\boldsymbol{v_b = v_{max} = u\log_e\left(\frac{m_0}{m_r}\right)}$$

The speed attained by the rocket when the complete fuel gets burnt is called burnt out speed of the rocket. It is the maximum speed acquired by the rocket.

Problem No 43 - A bomb moving with velocity(40i+50j-25k) m/s explodes into two pieces of mass ratio1:4. After explosion the smaller piece moves away with velocity (200i+70j+15k)m/s. The velocity of larger piece after explosion is

Sol: From Law of conservation of linear momentum

$Mu = m_1v_1 + m_2v_2; M = 5x, m_1 = x, m_2 = 4x$

$u = 40\hat{i} + 50\hat{j} - 25\hat{k}ms^{-1};$

$v_1 = \left(200\hat{i} + 70\hat{j} + 15\hat{k}\right)ms^{-1}$

here v_2 is the velocity of the larger piece

$5x\left(40\hat{i} + 50\hat{j} - 25\hat{k}\right) =$

$x\left(200\hat{i} + 70\hat{j} + 15\hat{k}\right) + 4x\left(v_2\right)$

On simplification, we get $v_2 = 45\hat{j} - 35\hat{k}$

Problem No 44 - A particle of mass 4 m explodes into three pieces of masses m, m and 2m. The equal masses move along X-axis and Y- axis with velocities 4ms-1 and 6 ms^{-1} respectively. The magnitude of the velocity of the heavier mass is

Sol: M=4 m ,u=0, $m_1 = m, \quad m_2 = m, \quad m_3 = 2m$

$v_1 = 4ms^{-1}, \quad v_2 = 6ms^{-1}, v_3 = ?$

According to law of conservation of momentum,

$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$

$\vec{P}_3 = -\left(\vec{P}_1 + \vec{P}_2\right), \left|\vec{P}_3\right| = \left|\vec{P}_1 + \vec{P}_2\right|$

$P_3 = \sqrt{P_1^2 + P_2^2 + 2P_1P_2Cos\theta}$

P_1 and P_2 are perpendicular to each other

$P_3 = \sqrt{P_1^2 + P_2^2}$, $m_3v_3 = \sqrt{(m_1v_1)^2 + (m_2v_2)^2}$

$2mv_3 = \sqrt{(m \times 4)^2 + (m \times 6)^2}$

$2v_3 = \sqrt{16+36} \Rightarrow v_3 = \sqrt{13}ms^{-1}$

Problem No 45 - A rifle of 20kg mass can fire 4 bullets/s. The mass of each bullet is 35´10-3 kg and its final velocity is 400ms-1. Then, what force must be applied on the rifle so that it does not move backwards while firing the bullets?

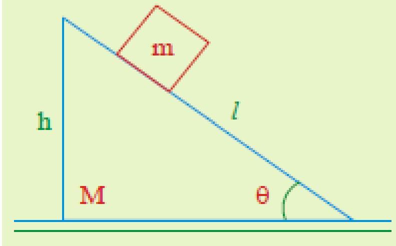
Sol : Law of conservation of momentum MV + 4mv = 0

$\Rightarrow V = -\frac{4mv}{M} = -\frac{4 \times 35 \times 10^{-3} \times 400}{20} = -2.8\, ms^{-1}$

Force applied on the rifle

$F = \frac{MV}{t} = -\frac{20 \times 2.8}{1} = -56$ N

Problem No 46 - All surfaces are smooth. Find the horizontal displacements of the block and the wedge when the block slides down from top to bottom.



Sol : When the block slides down on the smooth wedge, the wedge moves backwards. In the horizontal direction there is no external force ; $\vec{F}_x = 0$

$\therefore \vec{P}_x$ =constant

$\vec{P}_f = \vec{P}_i$ (along x-axis) ; $\quad m\vec{u} + M\vec{V} = \vec{0}$

x_1 = forward distance moved by the block along X-axis.

x_2 = backward distance moved by the wedge along X-axis.

$m\vec{u} = -M\vec{V}$;

$m\frac{x_1}{t} = M\frac{x_2}{t}$

$mx_1 = Mx_2$, $x_1 = \frac{ML}{M+m} = \frac{M\ell\cos\theta}{M+m}$

$x_2 = \frac{mL}{M+m} = \frac{m\ell\cos\theta}{M+m}$

Problem No 47 - A bomb of 1 kg is thrown vertically up with speed 100 m/s. After 5 seconds, it explodes into two parts. One part of mass 400gm goes down with speed 25m/s. What will happen to the other part just after explosion?

Sol : After 5 sec, velocity of the bomb,

$v = u + at$

$\vec{v}$ = u $\hat{j}$ – gt $\hat{j}$ = (100 – 10 x 5) $\hat{j}$ = 50 $\hat{j}$ m/s

$m = 1kg, m_1 = 0.4kg, m_2 = 0.6kg, v_1 = 25ms^{-1}$

According to law of conservation of momentum $m\bar{v} = m_1\bar{v}_1 + m_2\bar{v}_2$

$1 \times 50\hat{j} = -0.4 \times 25\hat{j} + 0.6\bar{v}_2$

$\Rightarrow v_2 = 100\hat{j} = v_2 = 100ms^{-1}$,vertically upwards

Problem No 48 - A particle of mass 2m is projected at an angle 450 with horizontal with a velocity of $20\sqrt{2}$ m/s. After 1sec, explosion takes place and the particle is broken into two equal pieces. As a result of explosion one part comes to rest. The maximum height attained by the other part from the ground is (g = 10m/s²

Sol : $M = 2m, \theta = 45^0, u = 20\sqrt{2}ms^{-1}$

$$u_x = u\cos\theta = 20\sqrt{2}\times\frac{1}{\sqrt{2}} = 20ms^{-1}$$

$$u_y = u\sin\theta = 20\sqrt{2}\times\frac{1}{\sqrt{2}} = 20ms^{-1}$$

But height attained before explosion, H_1

$$= ut - \frac{1}{2}gt^2 = 20\times1 - \frac{1}{2}\times10\times1^2 = 15m$$

After 1sec, $v_x = 20ms^{-1}$

$$v_y = u_y - gt = 20 - 10 = 10ms^{-1}$$

Due to explosion one part comes to rest,

$$m_1 = m_2 = m, v_1 = 0$$

$$M(v_x i + v_y j) = m_1\overline{v_1} + m_2\overline{v_2}$$

$$2m(20i + 10j) = m(0) + m\overline{v_2}$$

$$v_2 = 40i + 20j$$

$$v_y{}^1 = 20ms^{-1}$$

Height attained after explosion =

$$H_2 = \frac{(v_y{}^1)^2}{2g} = \frac{20\times20}{2\times10} = 20m$$

$$H_{TOT} = H_1 + H_2 = 15 + 20 = 35m$$

FRICTION

Introduction

If we slide or try to slide a body over another surface, the motion of the body is resisted by bonding between the body and the surface. This resistance is called friction.

The force of friction is parallel to the contact surfaces and *opposite to the direction of intended or relative motion.*

There are three types of frictional forces

i. Static friction

ii. Dynamic friction

iii. Rolling friction

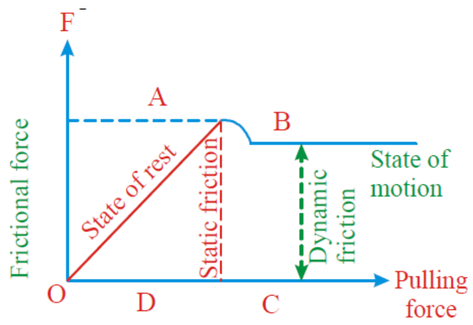
If a body is at rest and *no pulling force is acting* on it, force of friction on it is zero.

If a force is applied to move the body and it does not move, the friction developed is called **static friction**, which is equal in magnitude and opposite in direction to the applied force (static friction is self-adjusting force).

If a force is applied to move the body and it moves, then the friction developed is called **dynamic or kinetic friction**.

When a body rolls on the surface of another body friction developed is called as **rolling friction**.

It is due to the deformation at the point of contact and depends on area of contact.



- If you are walking due east, then the friction on the feet is due east and the friction on the surface is due west.
- Engine is connected to rear wheels of a car. When the car is accelerated, direction of frictional force on the rear wheels will be in the direction of motion and on the front wheels in the opposite direction of motion.
- In cycling, the force exerted by rear wheel on the ground makes the force of friction to act on it in the forward direction. Front wheel moving by itself experience force of friction in backward direction.

If the pedaling cycle is accelarating on the horizontal surface, then the forward friction on the rear wheel is greater than the backward friction on the front wheel.

When pedaling is stopped, the frictional force is in backward direction for both the wheels.

Laws of Friction

1) Friction is directly proportional to the *normal reaction* acting on the body.

   The law of static friction may thus be written as

   $$f_s \propto N$$

   $$(\boldsymbol{f_s})_{max} = \boldsymbol{f_l} = \boldsymbol{\mu_s N}$$

   Generally,

   $$0 \le \text{static friction} \le f_l$$

   Where the dimensionless constant μ_s is called the coefficient of static friction and N is the magnitude of the normal force. f_l = Limiting friction.

   Coefficient of static friction (μ_s) depends on the nature of the two surfaces in contact and is independent of the area of contact.

2) Static friction is independent of the area of contact between the two surfaces

3) Coefficient of kinetic friction

   $$\mu_k = \frac{f_k}{N}$$

   It is independent of velocity of the body.

4) Coefficient of rolling friction

   $$\mu_R = \frac{f_R}{N}$$

   Rolling friction depends on the area of the surfaces in contact.

   Note:

   $$\mu_s > \mu_k > \mu_R$$

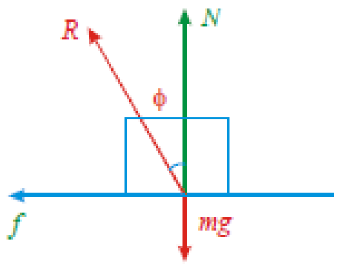
   Friction depends on the nature of the two surfaces in contact i.e., nature of materials, surface finish, temperature of the two surfaces etc.

Angle of Friction

Angle made by the resultant of f and N with the normal reaction N is called angle of friction.

Friction is parallel component of contact force to the surfaces.

Normal force is the perpendicular component of contact force to the surfaces.



$$\boldsymbol{R = \sqrt{f^2 + N^2}}$$

When the block is static

$$\tan \emptyset = \frac{f}{N}; \qquad \emptyset \le \emptyset_s$$

When the block is in impending state,

$$\tan\emptyset_s = \frac{\mu_s N}{N} = \mu_s$$

Where $\emptyset_s \rightarrow$ maximum angle of friction.

When block is sliding,

$$\tan\emptyset_k = \frac{\mu_k N}{N} = \mu_k$$

Since $\mu_s > \mu_k$, it follows that $\emptyset_s > \emptyset_k$

$$F_R = \sqrt{f_1^2 + N^2}$$

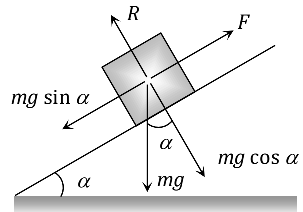
$$F_R = \sqrt{(\mu_s N)^2 + N^2} = N\sqrt{\mu_s^2 + 1}$$

$$F_R = \text{mg}\sqrt{\tan^2 \emptyset_s + 1} \quad (\because \mu_s = \tan \emptyset_s)$$

$$F_R = \text{mg} \sec \emptyset_s$$

Angle of Repose

The angle of inclined plane with the horizontal such that a body placed on it just begins to slide is called angle of repose.



In figure, α is angle of repose.

In limiting condition,

$$F = mg\sin\alpha$$

and $R = mg\cos\alpha$ So $\frac{F}{R} = \tan\alpha$

But, $\frac{F}{R} = \mu = \tan\theta$

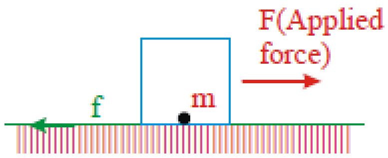
$$\therefore \quad \boldsymbol{\tan\alpha = \frac{F}{R} = \mu = \tan\theta}$$

Thus the coefficient of limiting friction is equal to the tangent of angle of repose.

Motion on a Horizontal Rough Surface

Consider a block of mass 'm' placed on a horizontal surface with normal reaction N.

Case (i) : If applied force F = 0, then the force of friction is also zero.



Case (ii) : If applied force F < $(f_s)_{max}$, the block does not move and the force of friction is f_s = F

Case (iii) : If applied force F = $(f_s)_{max}$ block just ready to slide and frictional force $(f_s)_{max} = f_l = \mu_s N$

$$F = \mu_s mg$$

$$(\because N = mg); \text{ (at time t = 0)}$$

Case(iv) : If the above applied force continues to act (t > 0), the body gets motion, static friction converts into kinetic friction and body possesses acceleration,

$$a = \frac{F_{ext} - f_k}{m} = \frac{f_l - f_k}{m} = (\mu_s - \mu_k)\, g$$

Case (v) : If the applied force is greater than limiting friction the body starts moving and gets acceleration,

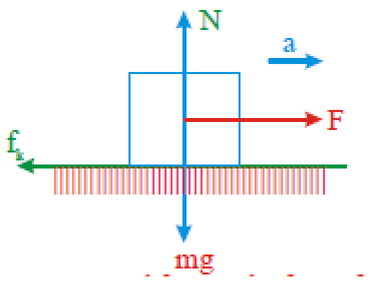
$$a = \frac{F'_{ext} - f_k}{m}$$

Here, $F'_{ext} > f_l$

If the block slides with an acceleration 'a' under the influence of applied force 'F', $F_R = F - f_k$;

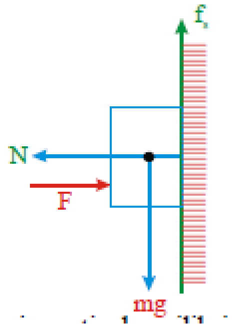
$$ma = F - f_k$$

$$\therefore a = \frac{F - f_k}{m} = \frac{F - \mu_k mg}{m} \quad (f_k = \mu_k N = \mu_k mg)$$



Bodies in contact with vertical surfaces

A block of mass m is pressed against a wall without falling, by applying minimum horizontal Force F, then



As the body is in vertical equilibrium

$$f_s = mg; \qquad \mu_s N = mg$$

$$\mu_s F = mg \quad (\because N = F)$$

$$\boldsymbol{F = \frac{mg}{\mu_s}}$$

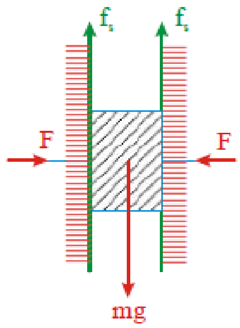
A block is pressed between two hands without falling, by applying minimum horizontal force

'F' by each hand. Then

W = 2 f_s;

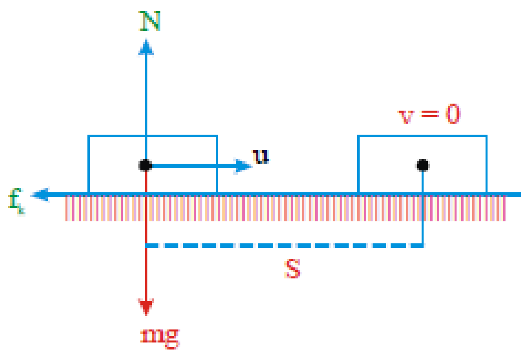
mg = $2\mu_s F$

$$\boldsymbol{F = \frac{mg}{2\mu_s}}$$



Note : Here in the above two cases, by applying any amount of horizontal force 'F', the frictional force f_s can never be greater than 'mg'

Sliding block on a horizontal rough surface coming to rest



a) The acceleration of the block is

$$a = -\mu_k g$$

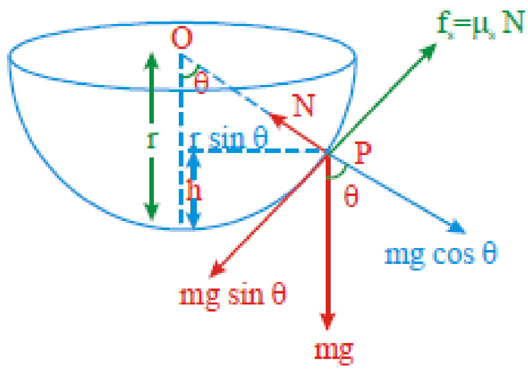
b) Distance travelled by the block before coming to rest is

$$S = \frac{u^2}{2\mu_k g}$$

c) Time taken by the block to come to rest is

$$t = \frac{u}{\mu_k g}$$

- An insect is crawling in a hemispherical bowl of radius 'r'. Maximum height upto which it can crawl is

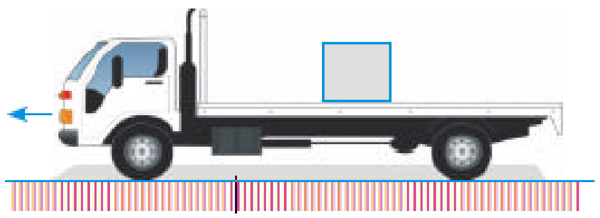


$$H = r(1 - \cos\theta) = r\left(1 - \frac{1}{\sqrt{\mu_s^2 + 1}}\right)$$

Maximum angular displacement upto which it can crawl is θ.

Then, $\mu_s = \tan\theta$

- A block is placed on rear horizontal surface of a truck moving along the horizontal with an acceleration 'a'. Then



The maximum acceleration of the truck for which block does not slide on the floor of the truck is

$$a = \mu_s g$$

If $a < \mu_s g$ block does not slide and frictional force on the block is

$$f = ma$$

If $a > \mu_s g$ block slips or slides on the floor. The acceleration of the block relative to the truck is

$$a' = a - \mu_k g$$

If ℓ is the distance of the block from rear side of the truck, time taken by the block to cover a distance ℓ,

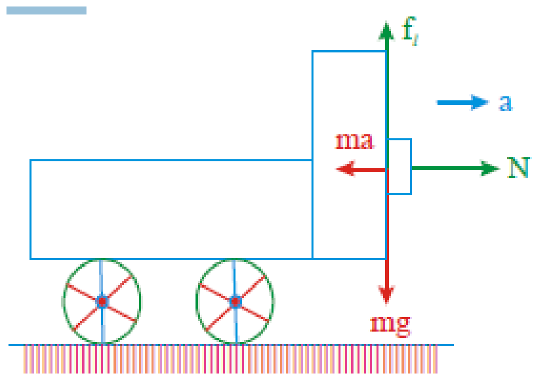
$$T = \sqrt{\frac{2\ell}{a - \mu_k g}}$$

Acceleration of the block relative to ground is

$$a'' = \mu_k g$$

Body placed in contact with the front surface of accelerated truck

When a block of mass 'm' is placed in contact with the front face of the vehicle moving with acceleration 'a' then a pseudo force 'F_{pf}' acts on the block in a direction opposite to the direction of motion of the vehicle.



Under equilibrium, f_1 = mg;

$$N = ma$$

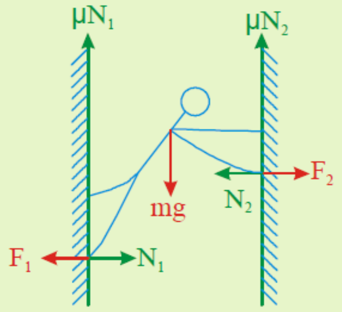
$$\mu_s N = mg$$

$$\therefore \mu_s ma = mg$$

$$\therefore a_{min} = \frac{g}{\mu_s}$$

Problem No 49 - A man of mass 40 kg is at rest between the walls as shown in the figure. If 'm ' between the man and the walls is 0.8, find the normal reactions exerted by the walls on the man.

Sol. Since man is at rest,



$N_1 - N_2 = 0$ (horizontal equilibrium)

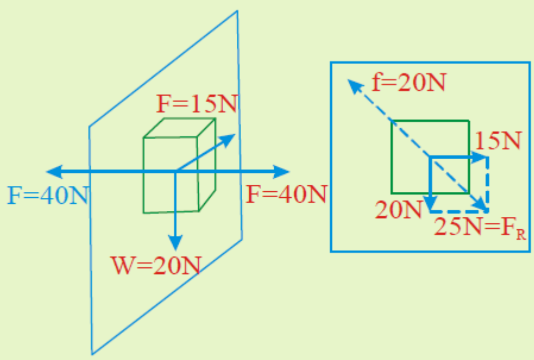
$\therefore N_1 = N_2 = N,\ F_1 = F_2 = F$ (say)

$\therefore 2\ N = mg$ (vertical equilibrium)

$= 2 \times 0.8 \times N = 400 \quad \therefore N = 250N$

Problem No 50 - A 2 kg block is in contact with a vertical wall having coefficient of friction 0.5 between the surfaces. A horizontal force of 40N is applied on the block at right angles to the wall. Another force of 15N is applied, on the plane of the wall and at right angles to 40N force. Find the acceleration of the block.

Sol.



Resultant of W=20N and 15N

$F_R = \sqrt{20^2 + 15^2} \quad = 25N$

frictional force $f = \ N = 0.5 \text{ x } 40 = 20N$

This acts in a direction, opposite to 25N force.

$\therefore$Net force acting on the block, $F_{net} = 25 - 20 = 5$

$\therefore$acceleration of the block $a = \frac{5}{2} = 2.5ms^{-2}$

Problem No 51 - A block of mass 4 kg is placed on a rough horizontal plane. A time dependent horizontal force F = kt acts on the block (k = 2 N/s). Find the frictional force between the block and the plane at t = 2 seconds and t = 5 seconds (μ = 0.2)

Sol. Given $F = kt$

When t = 2sec ; F = 2(2)=4N ..... case (i)

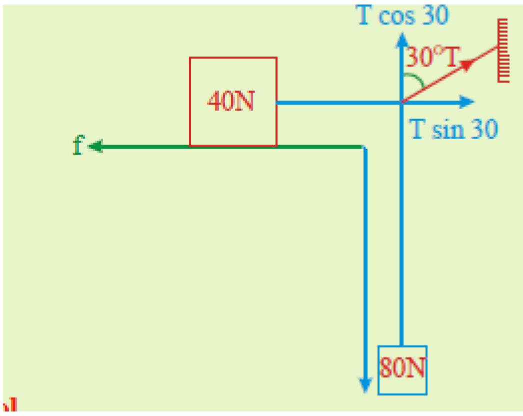
$f_{ms} = \ {}_s mg = 0.2 \times 4 \times 10 = 8N$

Here $F < f_{ms}$ $\therefore$ friction = applied force=4N

When t = 5 sec ; F = 2 (5) = 10N......case(ii)

F > f $\therefore$ frictional force < 8N

Problem No 52 - A block on table shown in figure is just on the edge of slipping. Find the coefficient of static friction between the block and table



Sol.

$f_l = T\sin\theta$

$\mu mg = T\sin\theta$ ....(1)

$80 = T\cos\theta$ ....(2)

$\frac{T\sin 30^0}{T\cos 30^0} = \frac{\mu mg}{80}$;

$\text{Tan}\,30^0 = \frac{\mu 40}{80}$; $\frac{1}{\sqrt{3}} = \frac{\mu}{2}$ $\Rightarrow \mu = \frac{2}{\sqrt{3}} = 1.15$

Probem No 53 - When a car of mass 1000 kg is moving with a velocity of 20ms^{-1} on a rough horizontal road, its engine is switched off. How far does the car move before it comes to rest if the coefficient of kinetic friction between the road and tyres of the car is 0.75?

Sol. Here $v = 20ms^{-1}$, $\mu_k = 0.75$, $g = 10ms^{-2}$

Stopping distance $S = \frac{v^2}{2\mu_k g} = 26.67m$

Problem No 54 - A horizontal conveyor belt moves with a constant velocity V. A small block is projected with a velocity of 6 m/s on it in a direction opposite to the direction of motion of the belt. The block comes to rest relative to the belt in a time 4s. μ = 0.3, g = 10 m/s^2. Find V

Sol. $\left|\vec{V}_{b,c}\right| = V_b + V_c = 6 + V$

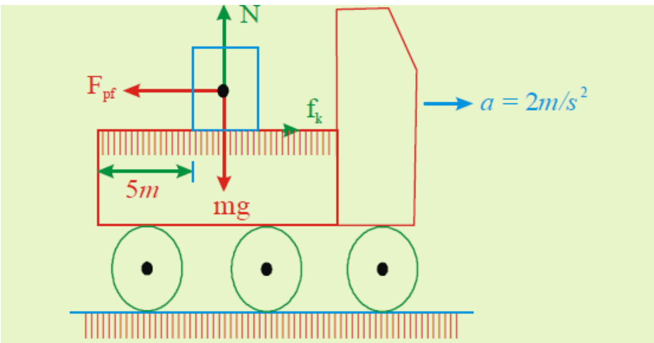
$f = \mu mg = 0.3 \times m \times 10 = 3m$

Retardation $a = \dfrac{F}{m} = \dfrac{3m}{m} = 3m/s^2$

$u_r = 6 + V, V_r = 0, t = 4\sec \;,\; a_r = -3ms^{-2}$

$V_r = u_r + a_r t,\; 0 = (6+V) - 3 \times 4, V = 6$ m/s

Problem No 55 - The rear side of a truck is open. A box of 40 kg mass is placed 5m away from the open end as shown in figure. The coefficient of friction between the box and the surface is 0.15. On a straight road, the truck starts from rest and accelerating with 2 m/s². At what distance from the starting point does the box fall off the truck? (Ignore the size of the box).



Sol : Because of the acceleration of the truck the pseudo force on the box = m x a = 40 x 2 = 80N.
This force acts opposite to the acceleration of the truck. The frictional force on the truck which acts in the forward direction $f_k = \mu N = 0.15 \times 40g$ = 58.8N Since pseudo force is greater than frictional force, the block will accelerate in backward direction relative to truck with a magnitude

$$a = \frac{80 - 58.8}{40} = 0.53\, m/s^2$$

The time taken by box to cover the distance 5m is given by

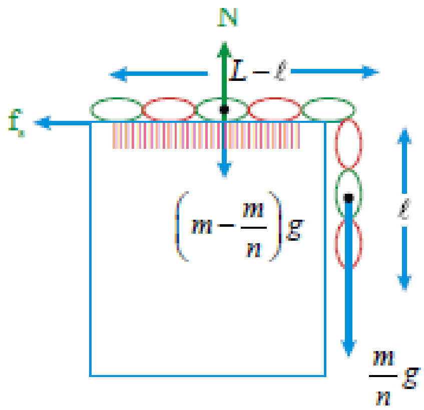
$$s = 0 + \frac{1}{2}at^2 \Rightarrow t = \sqrt{\frac{2s}{a}} = 4.34\sec$$

The distance travelled by truck in this time is , $a' = 2ms^{-2}$

$$s' = \frac{1}{2}a't^2 = \frac{1}{2} \times 2 \times (4.34)^2 = 18.87m$$

Sliding of a chain on a horizontal table

consider a uniform chain of mass "m" and length "L" lying on a horizontal table of coefficient of friction "μ_s". When 1/nᵗʰ of its length is hanging from the edge of the table, the chain is found to be about to slide from the table. Weight of the hanging part of the chain = $\frac{mg}{n}$



Weight of the chain lying on the table

$$= mg - \frac{mg}{n} = mg\left(1 - \frac{1}{n}\right)$$

When the chain is about to slide from edge of the table,

$$\left(\begin{matrix}\textit{The weight of the} \\ \textit{hanging part of the chain}\end{matrix}\right) = \left(\begin{matrix}\textit{frictional force between the} \\ \textit{chain and the table surface}\end{matrix}\right)$$

$$\frac{mg}{n} = \mu_s mg\left(1 - \frac{1}{n}\right)$$

$$\frac{mg}{n} = \mu_s mg\left(\frac{n-1}{n}\right)$$

$$\therefore \mu_s = \left(\frac{1}{n-1}\right)$$

If l is the length of the hanging part, then n = $\frac{L}{l}$

Substituting this in the above expression we get,

$$\mu_s = \frac{l}{L-l} \text{ (or) } n = \frac{L}{l} = \frac{\mu_s + 1}{\mu_s}$$

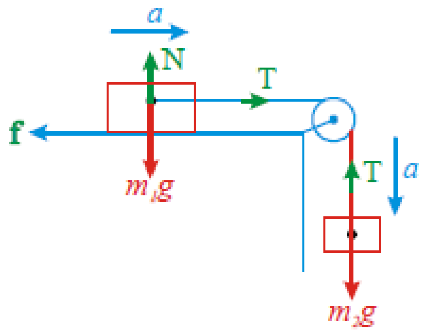
∴ The maximum fractional length of chain hanging from the edge of the table in equilibrium is $\frac{\ell}{L} = \frac{\mu_s}{\mu_s + 1}$

Fractional length of chain on the table

$$\frac{L - \ell}{L} = \frac{1}{\mu_s + 1}$$

Connected Bodies

A block of mass m_1 placed on a rough horizontal surface, is connected to a block of mass m_2 by a string which passes over a smooth pulley. The coefficient of friction between m_1 and the table is μ.



For body of mass m_2

$$m_2g - T = m_2a \quad \dots (i)$$

For body of mass m_1

$$T - f_k = m_1a$$

$$T - \mu_k N = m_1a \quad \dots (ii)$$

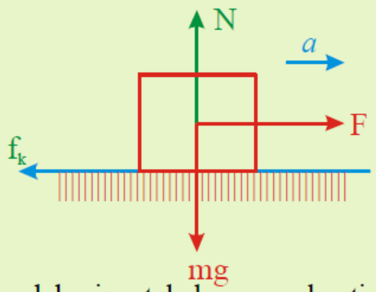
Solving Eqs (i) and (ii), we get

$$a = \left(\frac{m_2 - \mu_k m_1}{m_1 + m_2}\right) g$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2}(1 + \mu)$$

Problem No 56 - A block of mass 10kg is pushed by a force F on a horizontal rough plane is moving with acceleration $5ms^{-2}$. When force is doubled, its acceleration becomes $18ms^{-2}$. Find the coefficient of friction between the block and rough horizontal plane. ($g = 10\ ms^{-2}$).

Sol :



On a rough horizontal plane, acceleration of a block of mass 'm' is given by $a = \frac{F}{m} - \mu_k g$ ....(i)

Initially, $a = 5ms^{-2}$

$5 = \frac{F}{10} - \mu_k(10)$............(ii) $(\because m = 10kg)$

When force is doubled $a = 18\,ms^{-2}$.

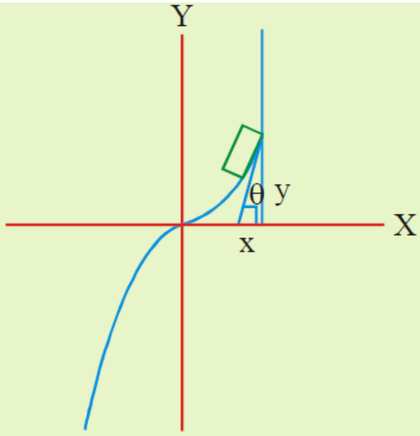
$18 = \frac{2F}{10} - \mu_k(10)$..........(iii)

Multiplying Eq(ii) with 2 and subtracting from Eq.(iii)

$8 = \mu_k(10) \Rightarrow \mu_k = \frac{8}{10} = 0.8$

Problem No 57 - A block of mass 'm' is placed on a rough surface with a vertical cross section of y = $\frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is,

Sol:



$$Tan\theta = \frac{dy}{dx} = \frac{d}{dx}\left(\frac{x^3}{6}\right) \Rightarrow Tan\theta = \frac{x^2}{2}$$

At limiting equilibrium, we get $\mu = Tan\theta \Rightarrow 0.5 = \frac{x^2}{2}$

$x^2 = 1 \Rightarrow x = \pm 1$

Now putting the values of 'x' in $y = \frac{x^3}{6}$, we get

When $x = 1 \Rightarrow y = \frac{1}{6}$; $x = -1 \Rightarrow y = -\frac{1}{6}$

So the maximum height above the ground at which the block can be placed without slipping is $y = \frac{1}{6}m$

Motion of a body on an inclined plane

Case (i): Body sliding down on a smooth inclined plane:

Let us consider a body of mass 'm' kept on a smooth inclined plane.

Normal reaction $N = mg\cos\theta$

Acceleration of sliding block $a = g\sin\theta$

If l is the length of the inclined plane and h is the height. The time taken to slide down starting from rest from the top is $t = \sqrt{\frac{2l}{g\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}}$ $\left(\because l = \frac{h}{\sin\theta}\right)$

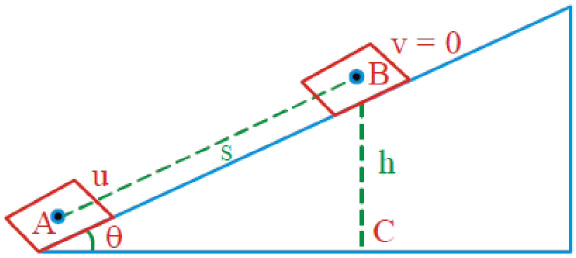
Sliding block takes more time to reach

the bottom than to fall freely in air from the top of the inclined plane to the ground.

Velocity of the block at the bottom of the inclined plane is same as the speed attained if block falls freely from the top of the inclined plane.

$$V = \sqrt{2\,gl\sin\theta} = \sqrt{2gh}$$

Case(ii): Body projected up on a smooth inclined plane:



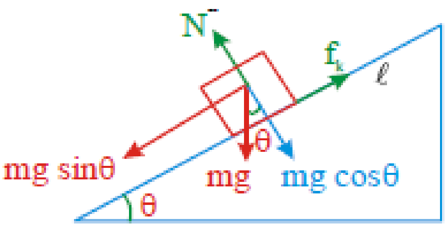
If a block is projected up the plane with a velocity u, the acceleration of the block is $a = -g\sin\theta$

Distance travelled by the block up the plane before its velocity becomes zero is $S = \frac{u^2}{2g\sin\theta}$

Time of ascent t = $\frac{u}{g\sin\theta}$

Case (iii) Motion of a body down the rough inclined plane:

Let a body of mass 'm' be sliding down a rough inclined plane of angle of inclination θ and coefficient of kinetic friction μ_k.



Let θ be the angle of inclination of a rough inclined plane, α be the angle of repose, m be the mass of the body and m be the coefficient of friction. At limiting equilibrium (about to slide)

$$mg\sin\alpha = \mu_s mg\cos\alpha$$

$$\tan\alpha = \mu_s \Rightarrow \alpha = \tan^{-1}(\mu_s)$$

1) When $\theta_1 < \alpha$; the block remains at rest on the inclined plane. Frictional force $mg\ \sin\theta_1$ (self adjusting) and acceleration a=0

2) When $\theta_2 = \alpha$; the block remains at rest on inclined plane or impending state of motion is achieved.

Mg sin θ_2 = μ_s mg cos θ_2 (at time t = 0)

Here $\theta_2 > \theta_1$ and $f_s = f_l$

acceleration a = 0

3) When θ_2 = α and (t > 0) the same inclination is continued the block moves downwards with acceleration a.

Mg sin θ_2 > μ_k mg cos θ_2 acceleration

$$a = \frac{mg\sin\theta_2 - \mu_k mg\cos\theta_2}{m}$$

$$a = \frac{\mu_s\cos\theta_2 - \mu_k mg\cos\theta_2}{m} = g\cos\theta_2\,(\mu_s - \mu_k)$$

4) When θ > α, the body slides.

$$f_k = \mu_k mg\cos\theta$$

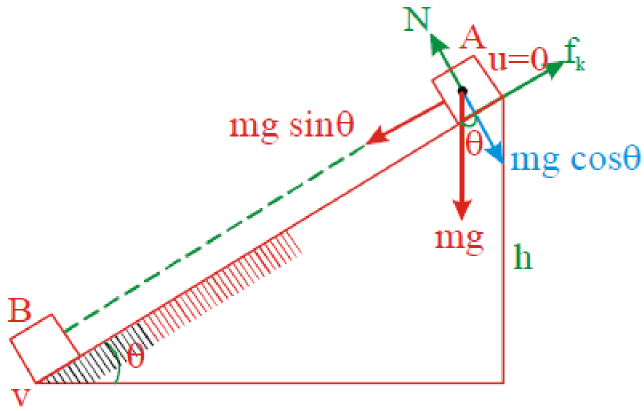
The resultant force acting on the body down the plane is

$$F_R = = mg\sin\theta = f_K$$

$$F_R = mg(\sin\theta - \mu_k\cos\theta)$$

The acceleration of the body

$$a = g(\sin\theta - \mu_k\cos\theta)$$



Velocity of the body at the bottom of the plane

$$v = \sqrt{2g(\sin\theta - \mu_k\cos\theta)\,l}$$

If 't' is the time taken to travel the distance 'l' with initial velocity u = 0 at the top of the plane,

$$t = \sqrt{\frac{2l}{g(\sin\theta - \mu_k\cos\theta)}}$$

The time taken by a body to slide down

on a rough inclined plane is 'n' times the time taken by it to slide down on a smooth inclined plane of same inclination and length, then coefficient of friction is,

$$n = \frac{t_{rough}}{t_{smooth}} = \frac{\sqrt{\frac{2l}{g(\sin\theta - \mu_k \cos\theta)}}}{\sqrt{\frac{2l}{g\sin\theta}}}$$

$$n^2 = \frac{\sin\theta}{\sin\theta - \mu_k \cos\theta}$$

$$n^2 \sin\mu - n^2 \mu_k \cos\theta = \sin\theta$$

$$\mu_k = \tan\theta \left[1 - \frac{1}{n^2}\right]$$

Case (iv): Body projected up a rough inclined plane:

If a body is projected with an initial velocity 'u' to slide up the plane, the kinetic frictional force acts down the plane and the body suffers retardation due to a resultant force

$$F_R = (mg\sin\theta + f_k)$$

Acceleration $a = -g(\sin\theta + \mu_k \cos\theta)$

Time taken to stop after travelling a distance l along the plane,

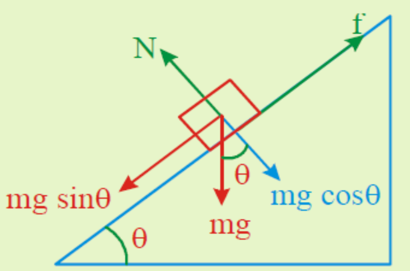
$$t = \sqrt{\frac{2l}{g(\sin\theta + \mu_k \cos\theta)}}$$

Force required to drag with an acceleration 'a' is

$$F = (\mu_k mg\cos\theta + mg\sin\theta + ma)$$

Problem No 58 - A body is moving down a long inclined plane of angle of inclination 'θ' for which the coefficient of friction varies with distance x as μ (x) = kx, where k is a constant. Here x is the distance moved by the body down the plane. The net force on the body will be zero at a distance x_0 is given by

Sol :



$$F = mg\sin\theta - f$$

$$N = mg\cos\theta \; ; \quad f = \mu N = \mu mg\cos\theta$$

$$F = mg\sin\theta - \mu mg\cos\theta$$

$$F = mg(\sin\theta - kx\cos\theta)$$

$$\text{If } F = 0\,; \quad \sin\theta - kx_0\cos\theta = 0 \Rightarrow x_0 = \frac{\tan\theta}{k}$$

Problem No 59 - A body of mass 'm' slides down a smooth inclined plane having an inclination of 45° with the horizontal. It takes 2s to reach the bottom. If the body is placed on a similar place having coefficient of friction 0.5 Then what is the time taken for it to reach the bottom?

Sol : Mass = m, $\theta = 45^0$, $\mu = 0.5$ Time taken by the body to reach the bottom without friction is

$$T_1 = \sqrt{\frac{2l}{g\sin\theta}} = 2\text{sec}$$

Time taken with friction is

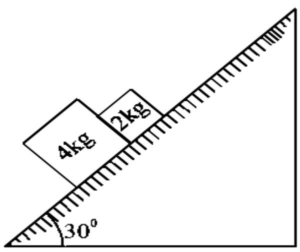
$$T_2 = \sqrt{\frac{2l}{g(\sin\theta - \mu\cos\theta)}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{\sin\theta - \mu\cos\theta}{\sin\theta}}$$

$$T_2 = T_1\sqrt{\frac{\sin\theta}{\sin\theta - \mu\cos\theta}}$$

$$= 2\sqrt{\frac{\sin 45^0}{\sin 45^0 - (0.5)\cos 45^0}}$$

$$= 2\sqrt{\frac{(1/\sqrt{2})}{(1/\sqrt{2}) - (0.5)(1/\sqrt{2})}} = 2\times\sqrt{2} = 2.828s$$

Problem No 60 - Two blocks of masses 4 kg and 2 kg are in contact with each other on an inclined plane of inclination 30° as shown in the figure. The coefficient of friction between 4 kg mass and the inclined plane is 0.3, where as between 2 kg mass and the plane is 0.2. Find the contact force between the blocks.



Sol : The acceleration of 4 kg mass,

If $\theta = 30^0, \mu_k = 0.3$

$$a_4 = g(\sin\theta - \mu_k\cos\theta) = 10\left[\frac{1}{2} - 0.3\times\frac{\sqrt{3}}{2}\right] = 2.6ms^{-2}$$

The acceleration of 2 kg mas

$$a_2 = 10\left[\frac{1}{2} - 0.2\times\frac{\sqrt{3}}{2}\right] = 3.27ms^{-2}$$

$\therefore a_2 > a_4$

Thus, there will be contact force between the blocks and they move together. If 'a' is the common acceleration,

$(m_1 + m_2)a =$

$(m_1 + m_2)g\sin\theta - (\mu_1 m_1 + \mu_2 m_2)g\cos\theta$

$$6a = 6\times10\times\frac{1}{2} - (0.3\times4 + 0.2\times2)\times10\times\frac{\sqrt{3}}{2}$$

$6a = 30 - 13.856 \Rightarrow a = 2.7ms^{-2}$

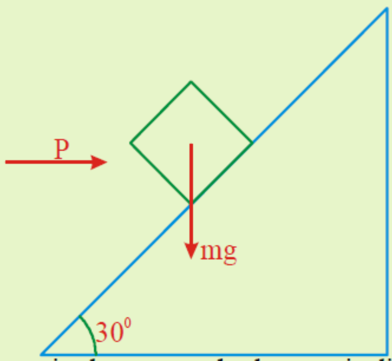
For, 4 kg mass; $mg\sin\theta + f_{contact} - f_{friction} = ma$

$$4\times10\times\frac{1}{2} + f_c - 0.3\times10\times10\times\frac{\sqrt{3}}{2} = 4 \text{ x } 2.7$$

$f_c = 10.8 + 10.4 - 20 \Rightarrow f_c = 1.2N$

Problem No 61 - A 30 kg block is to be moved up an inclined plane at an angle 30° to the horizontal with a velocity of $5ms^{-1}$. If the frictional force retarding the motion is 150 N, find the horizontal force required to move the block up the plane. ($g = 10\ ms^{-2}$)

Sol.



The force required to move a body up an inclined plane is

$F = mg\sin\ \ + f_k$

$f_k = \mu_k(mg\cos\theta + P\sin\theta) = 150N$

$= 30(10)\sin 30^0 + 150\ = 300N.$

If P is the horizontal force, $F = P\cos$

$$P = \frac{F}{\cos} = \frac{300}{\cos30^0} = \frac{300\times2}{\sqrt{3}} = 200\sqrt{3} = 346N$$

Problem No 62 - A body is sliding down an inclined plane having coefficient of friction 0.5. If the normal reaction is twice that of resultant downward force along the inclined plane, then find the angle between the inclined plane and the horizontal

Sol : $\mu = 0.5$, $N = mg\cos\theta$

$N = 2F, F = mg(\sin\theta - \mu\cos\theta)$

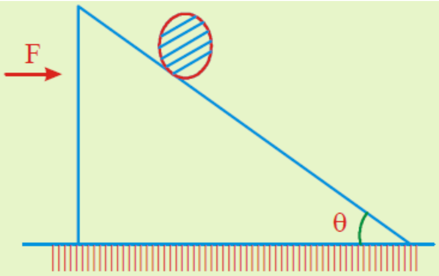
$N = 2mg(\sin\theta - \mu\cos\theta)$

$mg\cos\theta = 2mg(\sin\theta - \mu\cos\theta)$

$\cos\theta = 2\cos\theta(\tan\theta - \mu)$

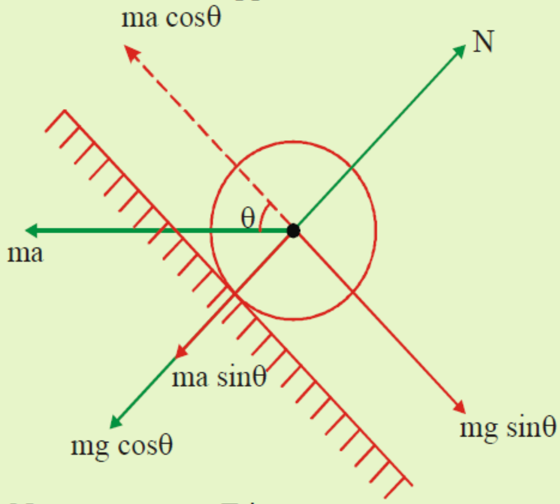
$\frac{1}{2} = \tan\theta - \frac{1}{2} \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45^0$

Problem No 63 - In the given figure, the wedge is acted upon by a constant horizontal force 'F'. The wedge is moving on a smooth horizontal surface. A ball of mass 'm' is at rest relative to the wedge. The ratio of force exerted on 'm' by the wedge when 'F' is acting and 'F' is withdrawn assuming no friction between the edge and the ball, is equal to,



Sol.

When Force F is applied



$N_1 = mg\cos\theta + F\sin\theta$

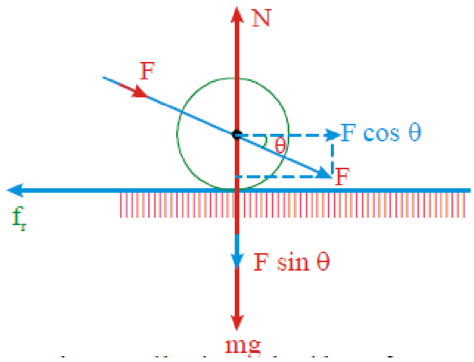
$(F\cos\theta = mg\sin\theta \Rightarrow F = mg\tan\theta)$

$$\text{If } F=0;\ N_2 = mg\cos\theta,\ \frac{N_1}{N_2} = 1 + \frac{F\sin\theta}{mg\cos\theta}$$

$$\frac{N_1}{N_2} = 1 + \frac{mg\tan\theta\sin\theta}{mg\cos\theta} = 1 + \tan^2 = \sec^2$$

Pushing and Pulling

i) A lawn roller on horizontal surface pushed by an inclined force:



When a lawn roller is pushed by a force 'F', which makes an angle θ with the horizontal, then

Normal reaction

$$N = mg + F\sin\theta$$

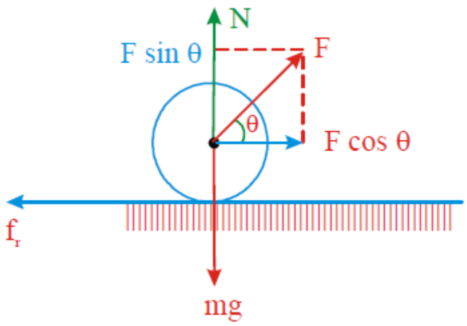
Frictional force,

$$f_r = \mu_r N = \mu_r (mg + F\sin\theta)$$

∴ The net horizontal pushing force is given by,

$$F_1 = F(\cos\theta - \sin\theta) - \mu_r mg$$

ii) A Lawn Roller on a Horizontal Surface Pulled by an inclined force



Let a lawn roller be pulled on a horizontal road by a force 'F', which makes an angle θ with the horizontal.

Normal reaction

$$N = mg - F\sin\theta$$

Frictional force

$$f_r = \mu_r N = \mu_r (mg - F\sin\theta)$$

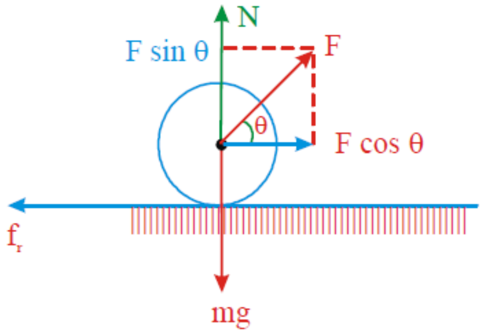
The net horizontal pulling force is

$$F_2 = (\cos\theta + \sin\theta) - \mu_r mg$$

Pulling is easier than Pushing.

Applying an Inclined Pulling Force:

Let an inclined force F be applied on the body so as to pull it on the horizontal surface as shown in the figure.



The body is in contact with the surface, and just ready to move

$$N + F\sin\theta = mg$$

$$N = mg - F\sin\theta$$

Frictional force $f_r = F\cos\theta$

$$F\cos\theta = \mu_r N$$

$$F\cos\theta = \mu_r (mg - F\sin\theta)$$

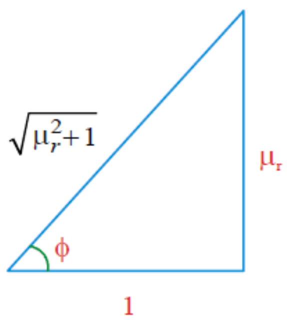
$$F = \frac{\mu_r mg}{(\cos\theta + \mu_r \sin\theta)}$$

$$F = \frac{mg\sin\emptyset}{\cos(\theta - \emptyset)} \qquad (\therefore \tan\emptyset = \mu_r)$$

For F to be minimum cos (θ-∅) should be Maximum ⇒ cos (θ-∅) = 1

⇒ θ-∅ = 0,

θ = ∅ = angle of friction



$$\therefore F_{min} = mg\sin\theta = mg\sin\emptyset$$

From the figure,

$$\sin\emptyset = \frac{\mu_r}{\sqrt{\mu_r^2+1}}$$

$$F_{min} = \frac{\mu_r mg}{\sqrt{\mu_r^2+1}}$$

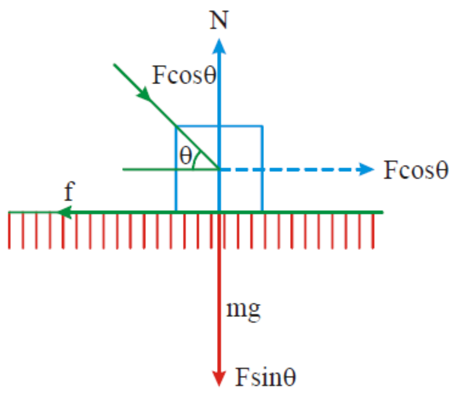
Minimum horizontal pulling force, when θ = 0

$$\cos(0 - \emptyset) = \cos\emptyset$$

$$F = \frac{mg\sin\emptyset}{\cos\emptyset} = mg\tan\emptyset$$

Applying an Inclined Pushing force:

Let an inclined force F is applied on the body so as to push it on the horizontal surface as shown in the figure.



The body is in contact with the surface, and just ready to move,

$$N = mg + F\sin\theta$$

Frictional force $\quad f_1 = F\cos\theta$

$$F\cos\theta = \mu_s N$$

$$F\cos\theta = \mu_s(mg + F\sin\theta)$$

$$F = \frac{\mu_s mg}{(\cos\theta - \mu_s\sin\theta)}$$

$$F = \frac{mg\sin\emptyset}{\cos(\theta+\emptyset)} \quad (\because \tan\emptyset = \mu_s)$$

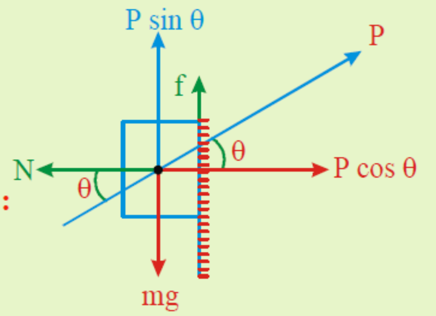
For F to be minimum θ = 0

$$\therefore F_{min}\frac{mg\sin\emptyset}{\cos\emptyset} = mg\tan\emptyset = \mu_s mg$$

(since $\mu_s = \tan\emptyset$)

Problem No 64 -A block of mass m kg is pushed up against a wall by a force P that makes an angle 'θ ' with the horizontal as shown in figure. The coefficient of static friction between the block and the wall is m. The minimum value of P that allows the block to remain stationary is

Sol :



At equilibrium, $P\sin\theta + f = mg \quad N = P\cos\theta$,

$N = P\cos\theta \quad f = (mg - P\sin\theta)$

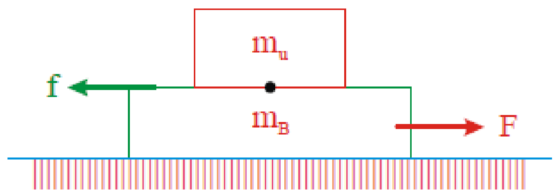
$\mu N = (mg - P\sin\theta)$

$\mu P\cos\theta = mg - P\sin\theta;\ P[\sin\theta + \mu\cos\theta] = mg$

$$\boxed{P = \frac{mg}{(\sin\theta + \mu\cos\theta)}}$$

Block on Block

Case I: Bottom block is pulled and there is no friction between bottom block and the horizontal surface.



When the bottom block is pulled, upper block is accelerated by the force of friction acting upon it.

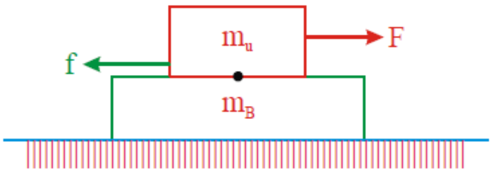
The maximum acceleration of the system of two blocks to move together without slipping is,

$a_{max} = \mu_s g$, where μ_s is the coefficient of static friction between the two blocks. The maximum applied force for which both the blocks move together is $F_{max} = \mu_s g(m_u + m_B)$

If $a < \mu_s g$ blocks move together and applied force is $F = (m_B + m_u)a$ In this case frictional force between the two block $f = m_u a$.

If $a > \mu_s g$, blocks slip relative to each other and have different accelerations. The acceleration of the upper block is $a_u = \mu_k g$ and that for the bottom block is $a_B = \frac{F - \mu_k m_u g}{m_B}$

Case - II: Upper block pulled and there is no friction between bottom block and the horizontal surface.



When the upper block is pulled, bottom block is accelerated by the force of friction acting on it. The maximum acceleration of the system of two blocks to move together without slipping is

$a_{max} = \mu_s \frac{m_u}{m_B} g$ where , μ_s =coefficient of static friction between the two blocks The maximum force for which both blocks move together is

$$F_{max} = \mu_s \frac{m_u}{m_B} g(m_u + m_B)$$

If $a < a_{max}$, blocks move together and frictional force between the two blocks is $f = m_B a$ The applied force on the upper block is $F = (m_B + m_u)a$

If $a > a_{max}$ blocks slide relative to each other and hence they have different accelerations.The acceleration of the bottom block is $a_B = \mu_K \frac{m_u}{m_B} g$ and the acceleration of the upper block is

$$a_u = \frac{F - \mu_k m_u g}{m_u}$$

A number of blocks of identical masses m each are placed one above the other. Force required to pull out Nth block from the top is
F = (2N–1) μ mg

Problem No 65 - A block of mass 4kg is placed on another block of mass 5kg, and the block B rests on a smooth horizontal table, for sliding the block A on B, a horizontal force 12N is required to be applied on it .How much maximum horizontal force can be applied on 'B' so that both A and B move together? Also find out the acceleration produced by this force.

Sol: Here M_1=4kg and M_2=5kg

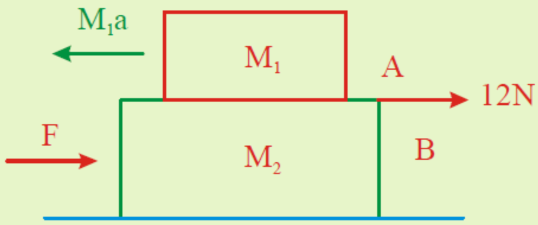
Limiting friction between the blocks is f_{lim}

Acceleration of system is

$$a = \frac{F}{M_1 + M_2} = \frac{F}{4+5} = \frac{F}{9} m/s^2$$

Because of this acceleration the block A experiences a pseudo force of magnitude

$$F_{pseudo} = M_1 a = 4 \times \frac{F}{9}$$

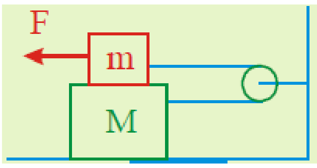


As block A move together with B, $F_{pseudo} \le f_{lim}$ For maximum value of applied force

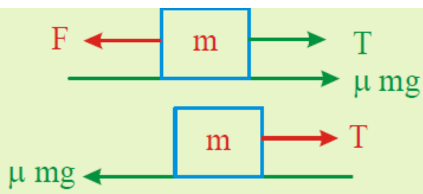
$$F_{pseudo} = f_{lim}; \quad \frac{4F}{9} = 12 \Rightarrow F = 27N$$

The acceleration of blocks $= \frac{27}{9} = 3m/s^2$

Problem No 66 - Two blocks of masses 'm' and 'M' are arranged as shown in the figure. The coefficient of friction between the two blocks is 'm', whereas between the lower block and the horizontal surface is zero. Find the force 'F' to be applied on the upper block, for the system to be under equilibrium?



Sol :



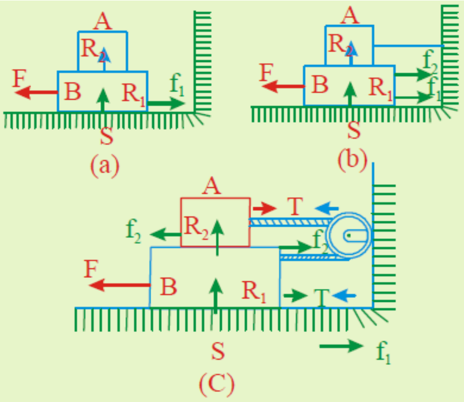
On the upper block,

F = T + f = T + μ N ; F = T + μ mg ......(1)

On the lower block T = μ mg .... (2)

from (1) and (2), we get, F = 2 μ mg

Problem No 67 - Block A weighs 4N and block B weighs 8N.The coefficient of kinetic friction is 0.25 for all surfaces. Find the force F to slide B at a constant speed when (a) A rests on B and moves with it (b) A is held at rest and (c) A and B are connected by a light cord passing over a smooth pulley as shown in fig (a),(b) and (c) respectively



Sol : (a) When A moves with B the force opposing the motion is the only force of friction between B and S the horizontal and as velocity of system is constant,

$$F = f_1 = \mu R_1 = 0.2(4+8) = 3N$$

(b) When A is held stationary, the friction opposing the motion is between A and B. So

$$F = \mu R_1 + \mu R_2 = 3 + 0.25(4) = 4N$$

(c) In this situation for dynamic equilibrium of B

$$F = \mu R_1 + \mu R_2 + T \text{ ....(i)}$$

While for the uniform motion of A,

$$T = \mu R_2 \text{ ....(ii)}$$

Substituting T from eqn (ii) in (i) we

get $F = \mu R_1 + 2\mu R_2 = 3 + 2 \times 1 = 5N$

MCQ

(Classwork)

LEVEL 1

NEWTON'S LAWS OF MOTION

1. A horizontal force "F" produces an acceleration of $6m/\ s^2$ on a block resting on a smooth horizontal surface. The same force produces an acceleration of $3m/\ s^2$ on a second block resting on a smooth horizontal surface. If the two blocks are tied together and the same force acts, the acceleration produced will be
   1) 9 $m/\ s^2$ 2) 2 $m/\ s^2$
   3) 4 $m/\ s^2$ 4) 1/ 2 $m/\ s^2$
2. A 0.2 kg object at rest is subjected to a force $(0.3\hat{\imath}$ - $0.4\ \hat{\jmath})$N . What is its velocity vector after 6 sec
   1) $(9\hat{\imath}$ -12 $\hat{\jmath})$ 2) $(8\hat{\imath}$ -16 $\hat{\jmath})$
   3) $(12\hat{\imath}$ - $9\hat{\jmath})$ 4) $(16\hat{\imath}$ -8 $\hat{\jmath})$
3. A body of mass 2 kg is moving with a velocity of $u = 3\hat{\imath} + 4\ \hat{\jmath}\ m/s$. A steady force $F = \hat{\imath} - 2\ \hat{\jmath}\ N$ begins to act on it. After four seconds, the body will be moving along.
   1) X-axis with a velocity of 2 m/s
   2) Y-axis with a velocity of 5 m/s
   3) X-axis with a velocity of 5 m/s
   4) Y-axis with a velocity of 2 m/s
4. Three forces $\overline{F_1}$, $\overline{F_2}$ and $\overline{F_3}$ are simultaneously acting on a particle of mass 'm' and keep it in equilibrium. If $F1$ force is reversed in direction only, the acceleration of the particle will be.
   1) $\overline{F}_1/\ m$ 2) $2\overline{F}_1/m$
   3) $-\overline{F}_1/m$ 4) $-2\overline{F}_1/m$
5. A block of metal weighing 2kg is resting on a frictionless plane. It is struck by a jet releasing water at a rate of 1 kg/s and at a speed of 5 m/s. The initial acceleration of the block will be
   1) 2.5 m/s^2 2) 5 m/s^2
   3) 10 m/s^2 4)20 m/s^2
6. A body of mass 2kg moving on a horizontal surface with an initial velocity of 4 ms^{-1}, comes to rest after 2 second. If one wants to keep this body moving on the same surface with a velocity of 4ms^{-1}, the force required is
   1) zero 2) 2 N 3) 4 N 4) 8 N
7. Ten coins are placed on top of each other on a horizontal table. If the mass of each coin is 10g and acceleration due to gravity is 10 ms^{-2}, what is the magnitude and direction of the force on the 7th coin (counting from the bottom) due to all the coins above it?
   1) 0.3 N downwards 2) 0.3 N upwards
   3) 0.7 N downwards 4) 0.7 N upwards
8. A ball of mass 'm' moves normal to a wall with a velocity 'u' and rebounds with the same speed. The change in momentum of the ball during the rebounding is
   1) 2mu towards the wall
   2) 2mu away from the wall
   3) zero
   4) mu away from the wall
9. Bullets of 0.03 kg mass each hit a plate at the rate of 200 bullets per second with a velocity of 30 m/s. The average force acting on the plate in newton is
   1) 120 2) 180 3) 300 4) 480
10. A vehicle of mass 10kg is moving with a velocity of 5 ms^{-1} .To stop it in 1/10 sec the required force in opposite direction is
    1) 5000N 2)500N
    3)50N 4)1000N

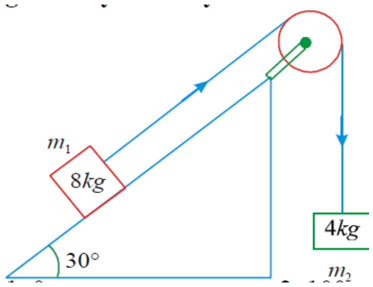
IMPULSE

11. An impulse is supplied to a moving object with the force at an angle1200 with the velocity vector. The angle between the impulse vector and the change in momentum vector is
    1) 1200 2) 00 3) 600 4) 2400
12. A 20 kg body is pushed with a force of 7N for 1.5 sec then with a force of 5N for 1.7 sec and finally with a force of 10N for 3 sec, the total impulse applied

to the body and change in velocity will be

1) $49Ns, 12.5ms^{-1}$
2) $49Ns,\ 2.45ms^{-1}$
3) $98Ns,\ 4.9ms^{-1}$
4) $4.9Ns, 2.45ms^{-1}$

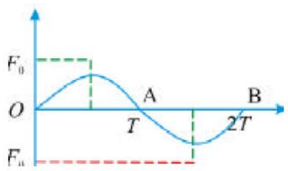
13. A body of mass 5 kg is acted upon by a net force F which varies with time t as shown in graph, then the net momentum in SI units gained by the body at the end of 10 seconds is



1) 0 2) 100
3) 140 4) 200

14. A body is acted on by a force given by $F = (10 + 2t)\ N$. The impulse received by the body during the first four seconds is
1) 40 N s 2) 56 N s
3) 72 N s 4) 32 N s

15. A unidirectional force F varying with time t as shown in the Fig. acts on a body initially at rest for a short duration 2T. Then the velocity acquired by the body is



1) $\frac{\pi F_0 T}{4m}$ 2) $\frac{\pi F_0 T}{2m}$ 3) $\frac{F_0 T}{4m}$ 4) zero

16. If the average velocity of a body moving with uniform acceleration under the action of a force is "v" and the impulse it receives during a displacement of "s" is "I", the constant force acting on the body is given by

1) $\frac{I \times v}{2s}$ 2) $\frac{2I \times v}{s}$ 3) $\frac{I \times v}{s}$ 4) $\frac{I \times s}{v}$

OBJECTS SUSPENDED BY STRINGS AND APPARENT WEIGHT

17. A 6.0kg object is suspended by a vertical string from the ceiling of an elevator which is accelerating upward at a rate of $2.2ms^{-2}$ .the tension in the string is
1) 11N 2) 72N 3) 48N 4)59N

18. A young man of mass 60 kg stands on the floor of a lift which is accelerating downwards at $1m/\ s^2$ then the reaction of the floor of the lift on the man is (Take $g = 9.8m/\ s^2$)
1) 528 N 2) 540 N 3) 546N 4) none

19. Three masses of 16 kg, 8 kg and 4kg are placed in contact as shown in Figure. If a force of 140 N is applied on 4kg mass, then the force on 16kg will be

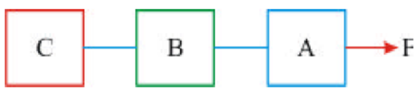
1) 140 N 2) 120 N
3) 100 N 4) 80 N

20. A body of mass M is being pulled by a string of mass m with a force P applied at one end. The force exerted by the string on the body is

1) $\frac{Pm}{(M+m)}$ 2) $\frac{PM}{(M+m)}$

3) $Pm(M+m)$ 4) $\frac{P}{(M-m)}$

21. Three equal masses A, B and C are pulled with a constant force F. They are connected to each other with strings. The ratio of the tension between AB and BC is



1) 1 : 2 2) 2 : 1 3) 3 : 1 4) 1 : 1

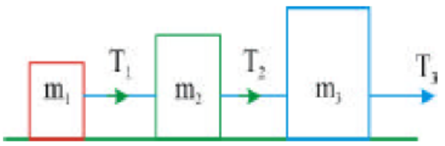
22. A coin is dropped in a lift. It takes time t_1 to reach the floor when lift is stationary. It takes time t_2 when lift is moving up with constant acceleration. Then
1) $t_1 > t_2$ 2) $t_2 > t_1$

3) $t_1 = t_2$ 4) $t_1 \geq t_2$

23. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is g/8, then the ratio of masses is
1) 8:1 2) 4:3 3) 5:3 4) 9:7

24. A pendulum bob is hanging from the roof of an elevator with the help of a light string. When the elevator moves up with uniform acceleration 'a' the tension in the string is T_1.When the elevator moves down with the same acceleration, the tension in the string is T_2. If the elevator were stationary, the tension in the string would be
1) $\frac{T_1+T_2}{2}$ 2) $\sqrt{T_1+T_2}$ 3) $\frac{T_1T_2}{T_1+T_2}$ 4) $\frac{2T_1T_2}{T_1+T_2}$

25. Three bodies are lying on a frictionless horizontal table and these are connected as shown in the figure. They are pulled towards right with a force 3 $T = 60N$ If m_1, m_2 and m_3 are equal to 10 kg, 20kg and 30kg respectively, then the values of T_1 and T_2 will be



1) 10N,10N 2) 30N,10N
3) 10N,30N 4) 30N,30N

LAW OF CONSERVATION OF MOMENTUM

26. A bullet of mass 20gm is fired from a riffle of mass 8 kg with a velocity of 100m/s. The velocity of recoil of the rifle is
1) 0.25 m/s 2) 25 m/s
3) 2.5 m/s 4) 250 m/s

27. A space craft of mass 2000 kg moving with a velocity of 600m/s suddenly explodes into two pieces. One piece of mass 500 kg is left stationary. The velocity of the other part must be (in m/s)
1) 600 2) 800 3)1500 4) 1000

28. A person weighing 60 kg in a small boat of mass 140 kg which is at rest, throws a 5 kg stone in the horizontal direction with a velocity of 14 ms^{-1}. The velocity of the boat immediately after the throw is (in m/s)
1) 1.2 2) 0.5 3) 0.35 4) 0.65

LAWS OF FRICTION

29. A body of mass 60kg is pushed with just enough force to start it moving on a rough surface with $\mu s = 0.5$ and $\mu k = 0.4$ and the force continues to act afterwards. The acceleration of the body is (in m/sec^2)
1) 0.98 2) 3.92
3) 4.90 4) Zero

30. If the coefficient of friction is 3, the angle of friction is
1) 300 2) 600 3) 450 4) 370

MOTION ON A HORIZONTAL ROUGH SURFACE

31. The coefficient of friction between a car wheels and a roadway is 0.5 The least distance in which the car can accelerate from rest to a speed of 72 kmph is (g=10 ms^{-2})
1)0m 2)20m 3)30m 4)40m

32. An eraser weighing 2N is pressed against the black board with a force of 5N. The coefficient of friction is 0.4. How much force parallel to the black board is required to slide the eraser upwards
1) 2N 2)2.8N 3)4N 4) 4.8N

33. A marble block of mass 2 kg lying on ice when given a velocity of 6ms^{-1} is stopped by friction in 10 s. Then the coefficient of friction is ($g = 10ms^{-2}$)
1) 0.02 2)0.03 3)0.06 4)0.01

34. A block of weight 100N is pushed by a force F on a horizontal rough plane moving with an acceleration 1 m/s^2, when force is doubled its acceleration becomes 10m/s^2. The coefficient of friction is (g=10ms^{-2})
1) 0.4 2) 0.6 3) 0.5 4) 0.8

35. A block of mass 5kg is lying on a rough horizontal surface. The coefficient of static and kinetic friction are 0.3 and 0.1 and $g = 10ms^{-2}$. If a horizontal force of 50N is applied on the block, the frictional force is

1)25N 2)5N 3)10N 4)Zero

36. A heavy uniform chain lies on horizontal table top. If the coefficient of friction between the chain and the table surface is 0.5, the maximum percentage of the length of the chain that can hang over one edge of the table is

1) 20% 2) 33.3% 3)76% 4) 50%

MOTION OFA BODY ON THE INCLINED PLANE

37. A body is sliding down an inclined plane forming an angle 300 with the horizontal. If the coefficient of friction is0.3 then acceleration of the body is

1)1.25.ms^{-2} 2) 2.35ms^{-2}
3) 3.4ms^{-2} 4) 4.9ms^{-2}

38. In the above problem its velocity after 3 seconds in ms^{-1} is

1)7.05 2)14.7 3)29.4 4) zero

39. In the above problem its displacement after 3 seconds is

1)78.4m 2)44.15m 3)10.57m 4)Zero

40. A block sliding down on a rough 450 inclined plane has half the velocity it would have been, the inclined plane is smooth. The coefficient of sliding friction between the block and the inclined plane is

1) $\frac{1}{4}$ 2) $\frac{3}{4}$ 3) $\frac{1}{2\sqrt{2}}$ 4) $\frac{1}{\sqrt{2}}$

41. A cube of weight 10N rests on a rough inclined plane of slope 3 in 5. The coefficient of friction is 0.6. The minimum force necessary to start the cube moving up the plane is

1)5.4N 2)10.8N 3) 2.7N 4) 18N

42. A body moves along a circular path of radius 5 m. The coefficient of friction between the surface of the path and the body is 0.5.The angular velocity in rad/s with which the body should move so that it does not leave the path is (g=10 ms^{-2})

1) 4 2) 3 3) 2 4) 1

43. A van is moving with a speed of 72 Kmph on a level road, where the coefficient of friction between tyres and road is 0.5.The minimum radius of curvature, the road must have, for safe driving of van is

1) 80 m 2) 40 m 3) 20 m 4) 4 m

44. What is the smallest radius of a circle at which a bicyclist can travel if his speed is 7 m/s and the coefficient of static friction the between tyres and road is 0.25

1) 10 m 2)20 m 3)5 m 4)15 m

LEVEL – 1 KEY

1) 2 2) 1 3) 3 4) 4 5) 1 6) 3
7) 1 8) 2 9) 2 10) 1 11) 2 12) 2
13) 3 14) 2 15) 4 16) 3 17) 2 18) 1
19)4 20)2 21)2 22) 1 23) 4 24) 1
25) 3 26) 1 27) 2 28) 3 29) 1 30) 2
31) 4 32) 3 33) 3 34) 4 35) 2 36) 2
37) 2 38) 1 39) 3 40) 2 41) 2 42) 4
43) 1 44) 2

LEVEL – 2

NEWTON'S LAWS OF MOTION

1) A ball of mass 2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2m while applying the force and the ball goes up to 2m height further, find the magnitude of the force. ($g = 10 ms^{-2}$)
1) 20N 2) 22N 3) 4N 4) 16N

2) A body of mass 3 kg is moving along a straight line with a velocity of $24 ms^{-1}$. When it is at a point 'P' a force of 9 N acts on the body in a direction opposite to its motion. The time after which it will be at 'P' again is,
1) 8s 2) 16s 3) 12s 4) 24s

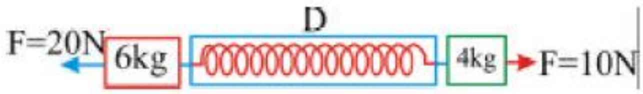
3) A ball of mass 10 gm dropped from a height of 5m hits the floor and rebounds to a height of 1.25m. If the ball is in contact with the ground for 0.1s, the force exerted by the ground on the ball is (g=10 m/s^2)
1) 0.5 N 2) 1.5 N 3) 0.15N 4) 2.5 N

4) A stream of water flowing horizontally with a speed of 15 ms^{-1} pushes out of a tube of cross sectional area 10^{-2}m^2 and hits a vertical wall near by what is the force exerted on the wall by the impact of water assuming. that it does not rebound? (Density of water = 1000 kg m^{-3})
1)1250N 2)2250N 3)4500N 4) 2550N

5) What is the magnitude of the total force on a driver by the racing car he operates as it accelerates horizontally along a straight line from rest to 60m/s in 8.0s (mass of the driver = 80 kg)
1) 0.06KN 2)0.78KN 3)1.4KN 4) 1.0KN

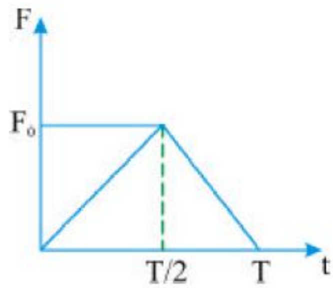
6) A base ball of mass 150 gm travelling at speed of 20 m/s is caught by a fielder and brought to rest in 0.04 s. The force applied to the ball and the distance over which this force acts are respectively
1) 75 N, 0.8 m 2) 37.5 N, 0.4 m
3) 75 N, 0.4 m 4) 37.5 N, 0.8m

7) A dynamometer D is attached to two blocks of masses 6 kg and 4 kg. Forces of 20 N and 10N are applied on the blocks as shown in Fig.
The dynamometer reads



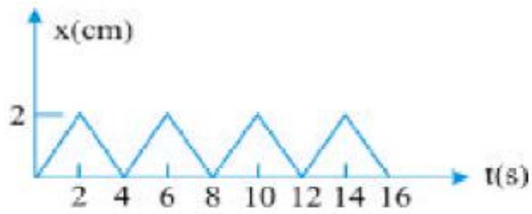
1) 10 N 2) 20 N 3) 6 N 4) 14 N

8) A particle of mass m moving with velocity u makes an elastic one-dimensional collision with a stationary particle of mass m. They are in contact for a very brief time T. Their force of interaction increases from zero to F_0 linearly in time $\frac{1}{2}$ T. The magnitude of F_0 is



1) $\frac{mu}{T}$ 2) $\frac{2mu}{T}$ 3) $\frac{mu}{2T}$ 4) $\frac{3mu}{2T}$

9) The position-time graph of a body of mass 0.04kg is shown in the figure. The time between two consecutive impulses received by the body and the magnitude of each impulse is

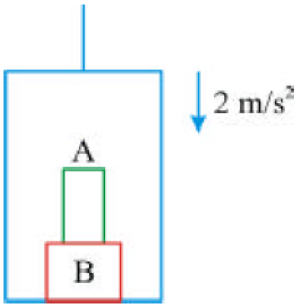


1) $4\,sec, 4 \times 10^{-4} kgm/s$
2) $2\,sec, 8 \times 10^{-4} kgm/s$
3) $6\,sec, 4 \times 10^{-4} kgm/s$
4) $8\,sec, 8 \times 10^{-4} kgm/s$

OBJECTS SUSPENDED BY STRINGS AND APPARENTWEIGHT

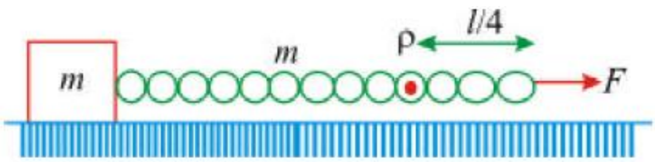
10) The elevator shown in figure is descending with an acceleration of

2m/s². The mass of the block A = 0.5 kg. The force exerted by the block A on block B is



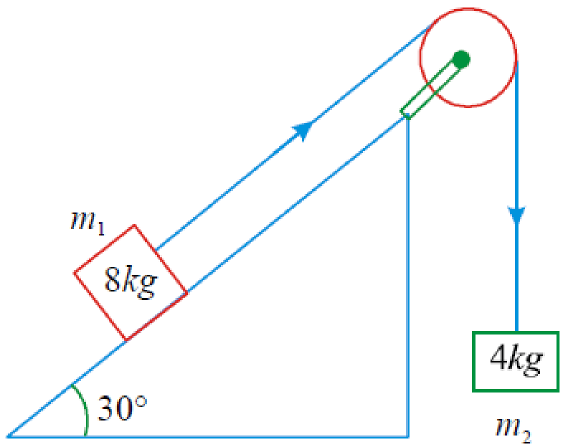
1) 2 N 2) 4 N 3) 6 N 4) 8 N

11) A block of mass m is pulled by a uniform chain of mass m tied to it by applying a force F at the other end of the chain. The tension at a point P which is at a distance of quarter of the length of the chain from the free end, will be



1) $\frac{3F}{4}$ 2) $\frac{7F}{8}$ 3) $\frac{6F}{7}$ 4) $\frac{4F}{5}$

12) Two masses of 8 kg and 4 kg are connected by a string as shown in figure over a frictionless pulley. The acceleration of the system is

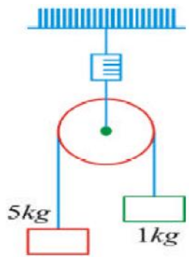


1) 4m/ s² 2) 2m/ s² 3) zero 4) 9.8m/ s²

13) Consider the system shown in figure. The pulley and the string are light and all the surface are frictionless. The tension in the string is (Take g = 10 m/s²)

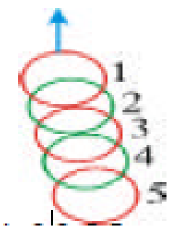
1) 0 N 2) 1 N 3) 2 N 4) 5 N

14) In the figure, a smooth pulley of negligible weight is suspended by a spring balance. Weights of 1 kg and 5kg are attached to the opposite ends of a string passing over the pulley and move with an acceleration due of gravity. During their motion, the spring balance reads a weight of



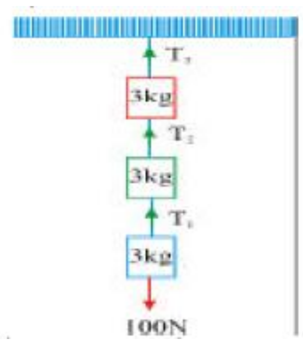
1) 6 kg
2) less than 6 kg
3)more than 6 kg
4)may be more or less than 6kg

15) A chain consisting of 5 links each of mass 0.1 kg is lifted vertically up with a constant acceleration of 2.5m/s². The force of interaction between 1st and 2nd links as shown



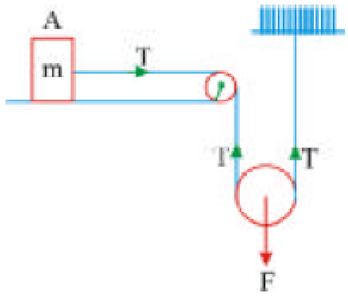
1) 6.15 N 2) 4.92 N 3) 9.84N 4) 2.46N

16) Three blocks of equal masses (each 3kg) are suspended by weightless strings as shown. If applied force is 100N, then T_1 is equal to (g =10m/s²)



1)130N 2)190N 3)100N
4)160N

17) Pulleys and strings are massless. The horizontal surface is smooth. What is the acceleration of the block

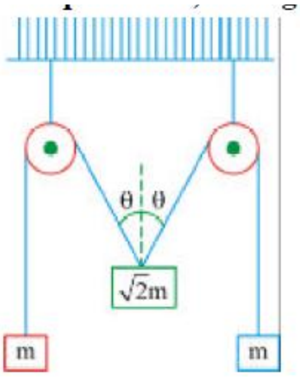


1) $\frac{F}{2m}$ 2) $\frac{F}{m}$ 3) $\frac{2F}{m}$ 4) $\frac{m}{2F}$

18) When a train starting from rest is uniformly accelerating, a plumb bob hanging from the roof of a compartment is found to be inclined at an angle of 45^0 with the vertical . The time taken by the train to travel a distance of ½km will be nearly

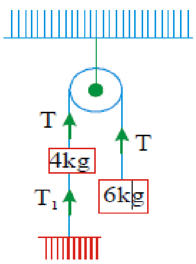
1) 7s 2)10s 3)15s 4) 25s

19) The pulley and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle 'θ' should be



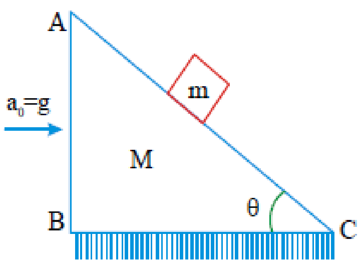
1)0^0 2) 30^0 3) 45^0 4) 60^0

20) Two bodies of masses 4kg and 6kg are attached to the ends of a string which passes over a pulley, the 4kg mass is attached to the table top by another string. The tension in this string T_1 is equal to



1) 10N 2) 10.6N 3) 25N 4) 20N

21) Acceleration of block m is $(\theta < 45^0)$



1) $g \sin\theta$ 2) $g\cos\theta$

3) $g(\cos\theta + \sin\theta)$ 4) $g(\cos\theta - \sin\theta)$

22) A stationary shell breaks into three fragments. The momentum of two of the fragments is P each and m ove at 60^0 to each other. The momentum of the third fragment is

1) P 2) 2P 3) $\frac{P}{\sqrt{3}}$ 4) $\sqrt{3}P$

23) An object initially at rest explodes, disintegrating into 3 parts of equal mass. Parts 1 and 2 have the same initial speed 'v', the velocity vectors being perpendicular to each other. Part 3 will have an initial speed of

1) $\sqrt{2}v$ 2) v / 2 3) v /$\sqrt{2}$ 4) $\sqrt{2v}$

24) A man of 50 kg is standing at one end on a boat of length 25m and mass 200 kg. If he starts running and when he reaches the other end, has a velocity $2ms^{-1}$ with respect to the boat. The final velocity of the boat is

1) $\frac{2}{3}$ ms^{-1} 2) $\frac{2}{5}$ ms^{-1}

3) $\frac{8}{5}$ ms^{-1} 4) $\frac{8}{3}$ ms^{-1}

25) A stationary body of mass 3 kg explodes into three equal pieces. Two of the pieces fly off at right angles to each other, one with a velocity $2\hat{i}$ m/s and the other with a velocity $3\hat{j}$ m/s. If the explosion takes place in 10^{-5} sec, the average force acting on the third piece in Newtons is:

1) $(2\hat{i} + 3\hat{j})10^{-5}$ 2) $-(2\hat{i} + 3\hat{j})10^{+5}$

3) $(3\hat{i} - 2\hat{j})10^{-5}$ 4) $(2\hat{i} - 2\hat{j})10^{-5}$

MOTION ON A HORIZONTAL ROUGH SURFACE

26) A particle is placed at rest inside a hollow hemisphere of radius R. The coefficient of friction between the particle and the hemisphere is $\mu = \frac{1}{\sqrt{3}}$. The maximum height up to which the particle can remain stationary is

1) $\frac{R}{2}$ 2) $\left(1 - \frac{\sqrt{3}}{2}\right)R$ 3) $\frac{\sqrt{3}}{2}R$ 4) $\frac{3R}{8}$

27) A horizontal force is applied on a body on a rough horizontal surface produces

an acceleration 'a'. If coefficient of friction between the body and surface which is μ is reduced to μ /3, the acceleration increases by 2 units. The value of 'μ' is

1) 2/3g 2) 3/2g 3) 3/g 4) 1/g

28) A block of mass 4kg is placed in contact with the front vertical surface of a lorry. The co efficient of friction between the vertical surface and block is 0.8.The lorry is moving with an acceleration of 15m/ s^2 .The force of friction between lorry and block is (g =10ms^{-2})

1) 48N 2) 24N 3) 40N 4) Zero

29) A person of mass 72kg sitting on ice pushes a block of mass of 30kg on ice horizontally with a speed of 12ms^{-1}. The coefficient of friction between the man and ice and between block and ice is 0.02. If g =10ms^{-1}, the distance between man and the block, when they come to rest is

1) 360m 2) 10m 3) 350 4) 422.5m

30) Consider a 14- tyre truck, whose only rear 8 wheels are power driven (means only these 8 wheels can produce an acceleration). These 8 wheels are supporting approximately half of the load. If coefficient of friction between road and each tyre is 0.6, then what could be the maximum attainable acceleration by this truck is

1) 6ms^{-2} 2) 24ms^{-2} 3) 3ms^{-2} 4)10ms^{-2}

31) A block is sliding on a rough horizontal surface. If the contact force on the block is $\sqrt{2}$ times the frictional force, the coefficient of friction is

1) 0.25 2) $\frac{1}{\sqrt{2}}$ 3) $\sqrt{2}$ 4) 1

32) A block is in limiting equilibrium on a rough horizontal surface. If the net contact force is $\sqrt{3}$ times the normal force, the coefficient of static friction is

1) $\sqrt{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) 0.5 4) $\frac{1}{\sqrt{3}}$

33) A block of mass 2kg is placed on the surface of trolley of mass 20kg which is on a smooth surface. The coefficient of friction between the block and the surface of the trolley is 0.25. If a horizontal force of 2 N acts on the block, the acceleration of the system in ms^{-2} is (g =10ms^{-2})

1) 1.8 2) 1.0 3) 0.9 4) 0.09

34) A man slides down on a telegraphic pole with an acceleration equal to one-fourth of acceleration due to gravity. The frictional force between man and pole is equal to (in terms of man's weight W)

1) $\frac{W}{4}$ 2) $\frac{3W}{4}$ 3) $\frac{W}{2}$ 4) W

35) A box is placed on the floor of a truck moving with an acceleration of 7 ms^{-2}. If the coefficient of kinetic friction between the box and surface of the truck is 0.5, find the acceleration of the box relative to the truck

1)1.7ms^{-2} 2)2.1ms^{-2}
3)3.5ms^{-2} 4)4.5ms^{-2}

36) A block is placed at a distance of 2m from the rear on the floor of a truck (g=10ms^{-2}). When the truck moves with an acceleration of 8ms^{-2}, the block takes 2 sec to fall off from the rear of the truck. The coefficient of sliding friction between truck and the block is

1) 0.5 2) 0.1 3) 0.8 4) 0.7

37) Sand is piled up on a horizontal ground in the form of a regular cone of a fixed base of radius R. The coefficient of static friction between sand layers is μ. The maximum volume of sand that can be piled up, without the sand slipping on the surface is

1) $\frac{\mu R^3}{3\pi}$ 2) $\frac{\mu R^3}{3}$ 3) $\frac{\pi R^3}{3\mu}$ 4) $\frac{\mu\pi R^3}{3}$

MOTION OF A BODY ON THE INCLINED PLANE

38) A body is allowed to slide from the top along a smooth inclined plane of length 5m at an angle of inclination 30^0. If

$g=10ms^{-2}$, time taken by the body to reach the bottom of the plane is

1) $\frac{\sqrt{3}}{2}s$ 2) 1.414s 3) $\frac{1}{\sqrt{2}}s$ 4) 2s

39) A body slides down a smooth inclined plane of height h and angle of inclination 30° reaching the bottom with a velocity v. Without changing the height, if the angle of inclination is doubled, the velocity with which it reaches the bottom of the plane is

1) v 2) v/2 3) 2v 4) $\sqrt{2}$ v

40) A body is projected up along an inclined plane from the bottom with speed is 2v. If it reaches the bottom of the plane with a velocity v, if θ is the angle of inclination with the horizontal and μ be the coefficient of friction.

1) $\frac{5}{3}\tan\theta$ 2) $\frac{3}{5}\tan\theta$

3) $\frac{1}{5}\tan\theta$ 4) $\frac{2}{5}\tan\theta$

41) The minimum force required to move a body up on an inclined plane is three times the minimum force required to prevent it from sliding down the plane. If the coefficient of friction between the body and the inclined plane is $\frac{1}{2\sqrt{3}}$, the angle of the inclined plane is

1) 60° 2) 45° 3) 30° 4) 15°

42) Starting from rest, the time taken by a body sliding down on a rough inclined plane at 45^0 with the horizontal is, twice the time taken to travel on a smooth plane of same inclination and same distance. Then the coefficient of kinetic friction is

1) 0.25 2) 0.33 3) 0.50 4) 0.75

43) A body is sliding down a rough inclined plane. The coefficient of friction between the body and the plane is 0.5. The ratio of the net force required for the body to slide down and the normal reaction on the body is 1 : 2. Then the angle of the inclined plane is

1) 15^0 2) 30^0 3) 45^0 4) 60^0

44) A body takes $1\frac{1}{3}$ times as much time to slide down a rough inclined plane as it takes to slide down an identical but smooth inclined plane. If the angle of inclination is 45^0, find the coefficient of friction.

1) $\frac{1}{16}$ 2) $\frac{3}{16}$ 3) $\frac{5}{16}$ 4) $\frac{7}{16}$

45) A body is sliding down an inclined plane having coefficient of friction 1/3. If the normal reaction is three times that of the resultant downward force along the inclined plane, the angle between the inclined plane and the horizontal is

1) $\tan^{-1}\left[\frac{1}{2}\right]$ 2) $\tan^{-1}(2)$

3) $\tan^{-1}\left(\frac{2}{3}\right)$ 4) $\tan^{-1}\left(\frac{3}{2}\right)$

46) A box of mass 4 kg is placed on a rough inclined plane of inclination 60^0. Its downward motion can be prevented by applying an upward pull is F and it can be made to slide upwards by applying a force 3F. The coefficient of friction between the box and inclined plane is

1) $\frac{2}{\sqrt{3}}$ 2) $\frac{\sqrt{3}}{2}$ 3) $\frac{2}{\sqrt{2}}$ 4) $\frac{1}{2}$

PULLING / PUSHING A BODY

47) A block of weight 100N is lying on a rough horizontal surface. If coefficient of friction $\frac{1}{\sqrt{3}}$. The least possible force that can move the block is

1) $\frac{100}{\sqrt{3}}N$ 2) $100\sqrt{3}$N 3) $50\sqrt{3}$N 4) 50N

48) A weight W rests on a rough horizontal plane. If the angle of friction is θ, the least force that can move the body along the plane will be

1) W $\cos\theta$ 2) W $\tan\theta$

3) W $\cot\theta$ 4) W $\sin\theta$

LEVEL – 2 KEY

1) 2	2)2	3) 2	4) 2	5)3	6)3
7)4	8) 2	9) 2	10) 2	11) 2	12)3
13) 4	14) 2	15) 2	16) 1	17) 1	18)2
19) 3	20) 4	21) 4	22) 4	23) 1	24)2
25) 2	26)2	27) 3	28) 3	29) 4	30)3
31) 4	32) 1	33) 4	34) 2	35)2	36)4
37) 4	38) 2	39) 1	40) 2	41) 3	42)4
43) 3	44) 4	45) 3	46) 2	47) 4	48)4

EXERCISE

(Homework)

LEVEL – I (A)

1. The behaviour of a body under zero resultant force is given by
   1) first law of motion
   2) second law of motion
   3) third law of motion
   4) law of gravitation
2. Which law of Newton defines an 'inertial frame of reference'?
   1) First law of motion
   2) Second law of motion
   3) Third law of motion
   4) Law of gravitation
3. The statement "acceleration is *zero* if and only if the net force is *zero*" is valid in
   1) non-inertial frames
   2) inertial frames
   3) both in inertial frames and non-inertial frames
   4) neither inertial frames nor non-inertial frames
4. You move forward when your car suddenly comes to a halt and you are thrown backward when your car rapidly accelerates. Which law of Newtons is involved in these?
   1) third law 2) second law
   3) first law 4) law of gravitation
5. You are thrown outer side when your car suddenly takes a turn. Which law of Newton is involved in this?
   1) Third law 2) Second law
   3) First law 4) Law of gravitation
6. An object is thrown vertically upward with some velocity. If gravity is turned off at the instant the object reaches the maximum height, what happens?
   1) The object continues to move in a straight line
   2) The object will be at rest
   3) The object falls back with uniform velocity

4) The object falls back with uniform acceleration

7. Which of the following is the most significant law of motion given by Newton?
   1) First law of motion
   2) Second law of motion
   3) Third law of motion
   4) Zeroth law of motion

8. The quantity of motion of a body is best represented by
   1) its mass 2) its speed
   3) its velocity 4) its linear momentum

9. A certain particle undergoes erratic motion. At every point in its motion, the direction of the particle's momentum is always
   1) the same as the direction of its velocity
   2) the same as the direction of its acceleration
   3) the same as the direction of its net force
   4) the same as the direction of its kinetic energy

10. Inside a railway car a plumb bob is suspended from the roof and a helium filled balloon is tied by a string to the floor of the car. When the railway car accelerates to the right, then
   1) both the plumb bob and balloon move to the left
   2) both the plumb bob and balloon move to the right
   3) plumb bob moves to the left and the balloon moves to the right
   4) plumb bob moves to the right and the balloon moves to the left

11. A constant force (F) is applied on a stationary particle of mass 'm'. The velocity attained by the particle in a certain displacement will be proportional to
   1) m 2) 1/m 3) $\sqrt{m}$ 4) $\frac{1}{\sqrt{m}}$

12. A constant force (F) is applied on a stationary particle of mass 'm'. The velocity attained by the particle in a certain interval of time will be proportional to
   1) m 2) 1/m 3) $\sqrt{m}$ 4) $\frac{1}{\sqrt{m}}$

13. A force produces an acceleration of *1 a* in a body and the same force produces an acceleration of a_2 in another body. If the two bodies are combined and the same force is applied on the combination, the acceleration produced in it is
   1) $a_1 + a_2$ 2) $\frac{a_1 + a_2}{a_1 a_2}$ 3) $\frac{a_1 a_2}{a_1 + a_2}$ 4) $\sqrt{a_1 a_2}$

14. To keep a particle moving with constant velocity on a frictionless surface, an external force
   1) should act continuously
   2) should be a variable force
   3) not necessary
   4) should act opposite to the direction of motion

15. If action force acting on a body is gravitational in nature, then reaction force
   1) may be a contact force
   2) may be gravitational force
   3) may be a gravitational or contact force
   4) may be a force of any origin

16. Action and reaction can never balance out because
   1) they are equal but not opposite always
   2) they are unequal in magnitude even though opposite in direction
   3) though they are equal in magnitude and opposite in direction they act on different bodies
   4) they are unequal in magnitudes

17. The propulsion of a rocket is based on the principle of conservation of
   1) linear momentum 2) energy
   3) angular momentum 4) mass

18. An automobile that is towing a trailer is accelerating on a level road. The force

that the automobile exerts on the trailer is
1) equal to the force the trailer exerts on the automobile
2) greater than the force the trailer exerts on the automobile
3) equal to the force the trailer exerts on the road
4) equal to the force the road exerts on the trailer

19. A man is standing in the middle of a perfectly smooth 'island of ice' where there is no friction between the ground and his feet. Under these circumstances
1) he can reach the desired corner by throwing any object in the same direction
2) he can reach the desired corner by throwing any object in the opposite direction
3) he has no chance of reaching any corner of the island
4) he can reach the desired corner by pursuing on the ground in that direction

20. Which law of Newton reveals the underlying symmetry in the forces that occur in nature?
1) First law
2) Second law
3) Third law
4) Law of gravitation

21. You hold a rubber ball in your hand. The Newton's third law companion force to the force of gravity on the ball is the force exerted by the
1) ball on the earth
2) ball on the hand
3) hand on the ball
4) earth on the ball

22. A lift is going up with uniform velocity. When brakes are applied, it slows down. A person in that lift, experiences
1) more weight 2) less weight
3) normal weight 4) zero weight

23. While we catch a cricket ball, we catch it at the front and make the hands move with the ball backwards. Why is that?
1) To reduce the impulse
2) To increase the time of contact, there by increase the force
3) To increase the impulse
4) To increase the time of contact, there by decrease the force

24. The change in momentum per unit time of a body represents
1) impulse 2) force
3) kinetic energy 4) resultant force

25. A father and his seven years old son are facing each other on ice skates. With their hands, they push off against one another. Regarding the forces that act on them as a result of this and the acceleration they experience, which of the following is correct?
1) Father exerts more force on the son and experiences less acceleration
2) Son exerts less force on the father and experiences more acceleration
3) Father exerts as much force on the son as the son exerts on the father, ut the father experiences less acceleration
4) Father exerts as much force on the son as the son exerts on the father, but the father experiences more acceleration

26. A student initially at rest on a frictionless frozen pond throws a 2 kg hammer in one direction. After the throw, the hammer moves off in one direction while the student moves off in the other direction. Which of the following correctly describes the above situation?
1) The hammer will have the momentum with greater magnitude
2) The student will have the momentum with greater magnitude

3) The hammer will have the greater kinetic energy
4) The student will have the greater kinetic energy

27. A ball falls towards the earth. Which of the following is correct?
1) If the system contains ball, the momentum is conserved
2) If the system contains earth, the momentum is conserved
3) If the system contains the ball and the earth, the momentum is conserved
4) If the system contains the ball and the earth and the sun, the momentum is conserved

28. A block moving in air breaks into two parts and the parts separate
1) the total momentum must be conserved
2) the total kinetic energy must be conserved
3) the total momentum must change
4) the potential energy must be conserved

29. Regarding linear momentum of a body
a) It is a measure of quantity of motion contained by the body
b) Change in momentum is the measure of impulse
c) Impulse and acceleration act in opposite direction to the change in momentum
d) In the case of uniform circular motion the linear momentum is conserved.
1) a& b are true
2) b & c are true
3) c & d are true
4) a,b & c are true

30. Compare the impulses exerted on a wall by the two objects, a golf ball and a lump of mud, both having the same mass and the velocity.
1) the golf ball imparts greater impulse
2) the lump of mud imparts the greater impulse
3) both impart equal impulse
4) nothing can be said

31. Two objects X and Y are thrown upwards simultaneously with the same speed. The mass of X is greater than that of Y. The air exerts equal resistive force on two objects, then
1) X reaches maximum height than Y
2) Y reaches maximum height than X
3) the two objects will reach the same height
4) cannot say

32. A man drops an apple in the lift. He finds that the apple remains stationary and does not fall. The lift is
1) going down with constant speed
2) going up with constant speed
3) going down with constant acceleration
4) going up with constant acceleration

33. Internal force can change
1) linear momentum as well as kinetic energy
2) linear momentum but not the Kinetic energy
3) the kinetic energy but not linear momentum
4) Neither the linear momentum nor the kinetic energy

34. A man is standing on a spring platform. Reading of spring balance is 60 kg wt. If man jumps outside the platform, then the reading of the spring balance
1) remains same
2) decreases
3) increases
4) first increases and then decreases to zero

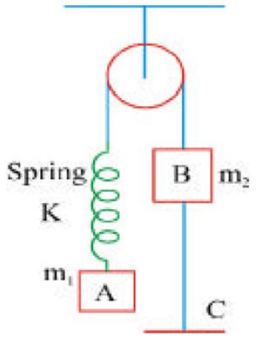
35. A stretching force of 10N is applied at one end of a spring balance and an equal force is applied at the other end at the same time. The reading of the balance is

1) 5 N 2) 10 N 3) 20 N 4) 0

36. A ball is dropped from a spacecraft revolving around the earth at a height of 1200 km. What will happen to the ball?
 1) It will continue to move with velocity V along the original orbit of spacecraft
 2) It will move with the same speed tangential to the space craft
 3) It will fall down to the earth gradually
 4) It will go far in space

37. A body is under the action of three forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$. In which case the body cannot undergo angular acceleration?
 1) $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ are concurrent
 2) $\vec{F_1}+\vec{F_2}+\vec{F_3}=0$
 3) $\vec{F_1}$,$\vec{F_2}$is parallel to $\vec{F_3}$ but the three forces are not concurrent
 4) $\vec{F_1}$ *and* $\vec{F_3}$ act at the same point but $\vec{F_3}$ acts at different point.

38. In the system shown in figure *m1* > *m2*. System is held at rest by thread BC. Just after the thread BC is burnt.



 1) acceleration of m1 will be equal to zero
 2) acceleration of m2 will be downwards
 3) magnitude of acceleration of two blocks will be non–zero and unequal
 4) magnitude of acceleration of both the blocks will be $\left(\frac{m_1-m_2}{m_1+m_2}\right)g$

39. A lift is ascending with a constant speed "V". A passenger in the lift drops a coin. The acceleration of the coin towards the floor will be
 1) Zero 2) g 3) <g 4) >g

40. A reference frame attached to the earth with respective to an observer in space
 1) is an inertial frame because Newton's laws of motion are applicable in it
 2) is an inertial frame by definition
 3) cannot be an inertial frame because earth is rotating about its axis
 4) can be an inertial frame because earth is revolving around the sun.

41. A Stationary railway platform on earth is
 1) an inertial frame of reference for an observer on earth.
 2) a Non inertial frame of reference for an observer on moon
 3) both are true
 4) both are false

42. A rotating platform for a stationary observer outside it is
 1) inertial frame of reference
 2) non inertial frame of reference
 3) both
 4) some times inertial (or) some times non inertial

43. The acceleration of a particle is found to be non zero when no force acts on the particle. This is possible if the measurement is made from
 1) inertial frame
 2) non inertial frame
 3) both
 4) some times inertial (or) some times non inertial

44 Frictional force between two bodies
 1) increases the motion between the bodies
 2) destroys the relative motion between the bodies
 3) sometimes helps and sometimes opposes the motion

4) increases the relative velocity between the bodies

45. Maximum value of static friction is
1) limiting friction
2) rolling friction
3) static friction
4) normal reaction

46. A good lubricant should be highly
1) viscous 2) non-volatile
3) both (1and 2) 4) transparent

47. Theoretically which of the following are best lubricants?
1) Solids 2) Liquids
3) Gases 4) Both 2 and 3

48. A block 'B' rests on 'A'. A rests on a horizontal surface 'C' which is frictionless. There is friction between A and B. If 'B' is pulled to the right
1) B moves forward and A to the left
2) 'B' only moves to the left
3) 'B' does not move
4) 'A' and 'B' move together to the right

49. Sand is dusted to the railway tracks during rainy season to
1) make it always wet
2) increase friction
3) to reduce consumption of fuel
4) make it always dry

50. With increase of temperature, the frictional force acting between two surfaces
1) increases
2) decreases
3) remains same
4) may increase or decrease

51. If we imagine ideally smooth surfaces and if they are kept in contact, the frictional force acting between them is
1) zero
2) a finite value but not zero
3) very large
4) we can't predict

52. If man is walking, direction of friction is
1) opposite to the direction of motion
2) same as that of direction of motion
3) perpendicular to that of direction of motion
4) 45° to the direction of motion

53. Aeroplanes are streamlined to reduce
1) fluid friction
2) sliding friction
3) kinetic friction
4) limiting friction

54. The limiting friction between two surfaces does not depend
1) on the nature of two surfaces
2) on normal reaction
3) on the weight of the body
4) on volume of the body

55. While walking on ice one should take small steps to avoid slipping. This is because smaller steps ensure
1) larger friction
2) smaller friction
3) larger normal force
4)smaller normal force

56. In order to stop a car in shortest distance on a horizontal road, one should
1) apply the brakes very hard so that the wheels stop rotating
2) apply the brakes hard enough to just prevent slipping
3) pump the brakes (press and release)
4) shut the engine off and not apply brakes

57. A body rests on a rough horizontal plane. A force is applied to the body directed towards the plane at an angle θ with the vertical. The body can be moved along the plane
1) only if θ is greater than the angle of friction
2) only if θ is lesser than the angle of friction
3) only if θ is equal to the angle of friction
4) for all values of θ

58. A lift is moving down with an acceleration equal to the acceleration due to gravity. A body of mass M kept

on the floor of the lift is pulled horizontally. If the coeficient of friction is μ, then the frictional resistance offered by the body is

1) μ_k Mg
2) Mg
3) Zero
4) $\mu_k Mg^2$

59. A body is struck to the front part of the truck. The coefficient of friction between the body and truck is m . The minimum acceleration with which the truck should travel so that the body does not fall down is

1) μ/g
2) μg
3) μ/m
4) $\mu^2 g$

60. When a bicycle is in motion, the force of friction exerted by the ground on the two wheels is such that it acts

1) in the backward direction on the front wheel and in the forward direction on the rear wheel
2) in the forward direction on the front wheel and in the backward direction on the rear wheel
3) in the backward direction on both the front and rear wheels
4) in the forward direction on both the front and rear wheels

61. A boy of mass M is applying a horizontal force to slide a box of mass M1 on a rough horizontal surface. The coefficient of friction between the shoe of the boy and the floor is m and that between the box and the floor is 'μ_1' In which of the following cases is it certainly not possible to slide the box?

1) $\mu < \mu_1$; $M < M_1$
2) $\mu > \mu_1$; $M > M_1$
3) $\mu < \mu_1$; $M > M_1$
4) $\mu > \mu_1$; $M < M_1$

62. When a person walks on a rough surface

1) the frictional force exerted by the surface keeps him moving
2) reaction of the force applied by the man on the surface keeps him moving
3) the force applied by the man keep him moving
4) weight of the man keeps him moving

63. The maximum speed of a car on a curved path of radius 'r' and the coefficient of friction μ_k is

1) $v = \sqrt{\frac{\mu_k}{gr}}$
2) $v = \sqrt{\mu_k gr}$
3) $v = \sqrt{\frac{gr}{\mu_k}}$
4) $v = \sqrt{\frac{1}{\mu_k gr}}$

64. The angle which the rough inclined plane makes with the horizontal when the body placed on it just starts sliding down is called

1) angle of Friction
2) angle of repose
3) critical angle
4) brewster's angle

65. A body of mass M is placed on a rough inclined plane of inclination θ and coefficient of friction μk. A force of (mg sin θ + μk mg cos θ) is applied in the upward direction, the acceleration of the body is

1) g sin θ
2) g (sin θ + $\mu_k$cos θ)
3) g (sin θ – $\mu_k$cos θ)
4) Zero

66. It is easier to pull a lawn roller than to push it because pulling

1) involves sliding friction
2) involves dry friction
3) increases the effective weight
4) decreases normal reaction

67. A block of mass m and surface area A just begins to slide down an inclined plane when the angle of inclination is p /5. Keeping the mass of the block same, if the surface area is doubled, the inclination of the plane at which the block starts sliding will be

1) p /5
2) p /10
3) 2 p /5
4) p /5 2

68. A block X kept on an inclined surface just begins to slide if the inclination is θ_1. The block is replaced by another

block Y and it is found that it just begins to slide if the inclination is θ_2 ($\theta_2 > \theta_1$). Then

1) Mass of X = mass of Y
2) Mass of X < mass of Y
3) Mass of X > mass of Y
4) All the three are possible

LEVEL – 1 (A) - KEY

01) 1 02) 1 03) 2 04) 3 05) 3 06) 2
07) 2 08) 4 09) 1 10) 3 11) 4 12) 2
13) 3 14) 3 15) 2 16) 3 17) 1 18) 1
19) 2 20) 3 21) 1 22) 2 23) 4 24) 4
25) 3 26) 3 27) 4 28) 1 29) 1 30) 1
31) 1 32) 3 33) 3 34) 4 35) 2 36) 1
37) 1 38) 1 39) 2 40) 3 41) 3 42) 2
43) 2 44) 3 45) 1 46) 3 47) 3 48) 4
49) 2 50) 2 51) 3 52) 2 53) 1 54) 4
55) 3 56) 2 57) 1 58) 3 59) 3 60) 1
61) 1 62) 1 63) 2 64) 2 65) 4 66) 4
67) 1 68) 4

LEVEL – 1 (B)
NEWTON'S LAWS OF MOTION

1. n balls each of mass m impinge elastically in each second on a surface with velocity u. The average force experienced by the surface will be
   1) mnu 2) 2mnu
   3) 4mnu 4) mnu/2
2. A ball reaches a racket at 60 m/s along + X direction, and leaves the racket in the opposite direction with the same speed. Assuming that the mass of the ball as 50gm and the contact time is 0.02 second, the force exerted by the racket on the ball is
   1) 300 N along + X direction
   2) 300 N along - X direction
   3) 3,00,000 N along + X direction
   4) 3,00,000 N along - X direction
3. 'P' and 'Q' horizontally push in the same direction a 1200 kg crate. 'P' pushes with a force of 500 newton and 'Q' pushes with a force of 300 newton. If a frictional force provides 200 newton of resistance, what is the acceleration of the crate?
   1) 1.3 m/s^2 2) 1.0 m/s^2
   3) 0.75 m/s^2 4) 0.5 m/s^2
4. A ball of mass 'm' moves normal to a wall with a velocity 'u' and rebounds with a velocity 'v'. The change in momentum of the ball during the rebounding is
   1) m (u+v) towards the wall
   2) m (u–v) towards the wall
   3) m(u+v) away from the wall
   4) m(u–v) away from the wall.
5. If a force of 250N acts on a body,the momentum required is 125$kgms^{-1}$ .The period for which the force acts on the body is
   1) 0.1 s 2) 0.3 s
   3) 0.5 s 4) 0.2 s
6. A machine gun fires a bullet of mass 40g with a velocity 1200 ms^{-1}. The man holding it can exert a maximum force

of 144 N on the gun. How many bullets can he fire per second at the most?

1) One
2) Three
3) Two
4) Four

7. A truck of mass 500kg is moving with constant speed $10 ms^{-1}$ .If sand is dropped into the truck at the constant rate 10kg/min, the force required to maintain the motion with constant velocity is

1) $\frac{3}{2}N$
2) $\frac{5}{4}$N
3) $\frac{7}{5}N$
4) $\frac{5}{3}N$

8. A 5000 kg rocket is set for vertical firing. The exhaust speed is $800 ms^{-1}$. To give an upward acceleration of $20 ms^{-2}$, the amount of gas ejected per second to supply the needed thrust is ($g \times 10 ms^{-2}$)

1) $127.5 kg\ s^{-1}$
2) $137.5 kg\ s^{-1}$
3) $187.5 kg\ s^{-1}$
4) $185.5 kg\ s^{-1}$

IMPULSE

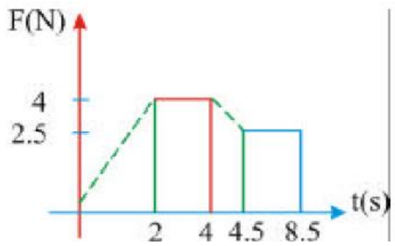
9. A small sphere of mass $m^2 kg$ moving with a velocity $\bar{u} = 4\hat{\imath} - 7\hat{\jmath} m/\ s$ collides with a smooth wall and returns with a velocity $\bar{u} = -\hat{\imath} + 3\hat{\jmath} m/\ s$. The magnitude of the impulse received by the ball is

1) $5 kg\ ms^{-1}$
2) $10\ 5\ kgms^{-1}$
3) $20\ kg\ ms^{-1}$
4) $15 kg\ ms^{-1}$

10. A ball of mass'm' is thrown at an angle is 'q' with the horizontal with an initial velocity 'u'.The change in its momentum during its flight in a time interval of 't' is

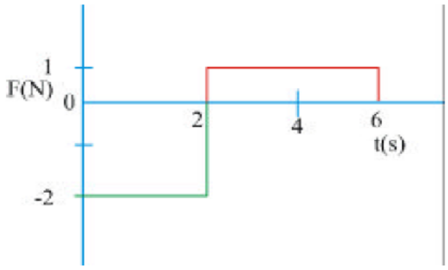
1) mgt
2) mgt cos$q$
3) mgt sin$q$
4) ½ mgt.

11. A body of mass 2kg has an initial speed $5 ms^{-1}$. A force acts on it for 4 seconds in the direction of motion. The force time graph is shown in figure. The final speed of the body is



1) $8.5\ ms^{-1}$
2) $11\ ms^{-1}$
3) $14.31\ ms^{-1}$
4) $4.31\ ms^{-1}$

12. A force time graph for the motion of a body is as shown in figure. Change in linear momentum between 0 and 6s is



1) zero
2) 8 Ns
3) 4 Ns
4) 2 Ns

13. An object of mass 3 kg is at rest. Now a force $F = 6t2\hat{\imath} + 2t\hat{\jmath}$ is applied on the object. Find the velocity of the object at t = 3 sec.

1) $18\hat{\imath} + 3\hat{\jmath}$
2) $18\hat{\imath} - 3\hat{\jmath}$
3) $3\hat{\imath} - 18\hat{\jmath}$
4) $3\hat{\imath} + 18\hat{\jmath}$

14. An impulse "I" given to a body changes its velocity from "v_1 to v_1 ". The increase in the kinetic energy of the body is given by

1) $I(v_1+v_2)$
2) $I(v_1+v_2)/2$
3) $I(v_1-v_2)$
4) $I(v_1-v_2)/2$ --

OBJECTS SUSPENDED BY STRINGS & APPARENT WEIGHT

15. A 60kg man is inside a lift which is moving up with an acceleration of $2.45 ms^{-2}$. The apparent percentage change in his weight is,

1) 20%
2) 25%
3) 50%
4) 75%

16. The apparent weight of a person inside a lift is W1 when lift moves up with certain acceleration and is W2 when lift moves down with same acceleration. The weight of person when lift moves up with constant speed is

1) $\frac{W_1+W_2}{2}$
2) $\frac{W_1-W_2}{2}$
3) $2W_1$
4) $2W_2$

17. A person of mass 60 kg is in a lift. The change in the apparent weight of the person, when the lift moves up with an acceleration of $2 ms^{-2}$ and then down with an acceleration of $2 ms^{-2}$, is (take g = 10 m/sec2)

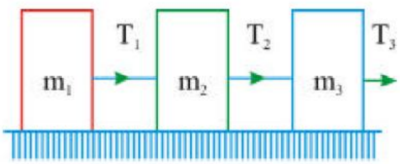
1) 120 N
2) 240 N

3) 480 N　　4) 720 N

18. A rope of length 10m and linear density 0.5kg/ m is lying length wise on a smooth horizontal floor. It is pulled by a force of 25 N. The tension in the rope at a point 6m away from the point of application is

1) 20 N　2) 15 N　3) 10 N　4) 5 N

19. Three blocks of masses m_1, m_2 and m_3 are connected by a massless string as shown in figure on a frictionless table. They are pulled with a force T_3 = 40N. If $m_1 = 10kg$, $m_2=6kg$ and $m_3 = 4kg$, then tension T_2 will be

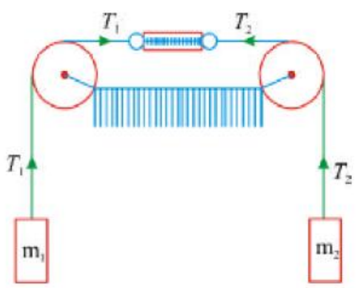


1) 10 N　　2) 20 N
3) 32 N　　4) 40 N

20. A horizontal force F pushes a 4 kg block (A) which pushes against a 2 kg block (B) as shown. The blocks have an acceleration of 3m/s^2 to the right. There is no friction between the blocks and the surfaces on which they slide. What is the net force B exerts on A?

1) 6 N to the right
2) 12 N to the right
3) 6 N to the left
4) 12 N to the left

21. Two masses m_1 and m_2 are attached to a spring balance S as shown in Figure. If $m_1 > m_2$ then the reading of spring balance will be

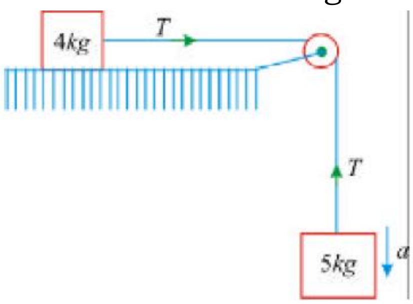


1) $(m_1 - m_2)$　　2) $(m_1 + m_2)$
3) $\frac{2m_1m_2}{m_1m_2}$　　4) $\frac{m_1m_2}{m_1+m_2}$

22. Two masses (M+m) and (M-m) are attached to the ends of a light inextensible string and the string is made to pass over the surface of a smooth fixed pulley. When the masses are released from rest, the acceleration of the system is

1) gm/M　　2) 2gM/m
3) gm/2M　　4) g $(M^2 - m^2)$/2M

23. Two bodies of masses 5kg and 4kg are tied to a string as shown. If the table and pulley are smooth, then acceleration of 5kg mass will be



1) 19.5m/s^2　　2) 0.55 m/s^2
3) 2.72 m/s^2　　4) 5.45m/s^2

LAW OF CONSERVATION OF MOMENTUM

24. The object at rest suddenly explodes into three parts with the mass ratio 2:1:1.The parts of equal masses move at right angles to each other with equal speed 'v '.the speed of the third part after explosion will be

1) v　2) $\sqrt{2}$ v　3) $\frac{v}{2}$　4) $\frac{v}{\sqrt{2}}$

25. A man and a cart move towards each other. The man weighs 64 kg and the cart weighs 32kg. The velocity of the man is 5.4 km/hr and that of the cart is 1.8 km/hr. When the man approaches the cart, he jumps on to it. The velocity of the cart carrying the man will be

1) 3 km/hr　　2) 30 km/hr
3) 1.8 km/ hr　　4) zero

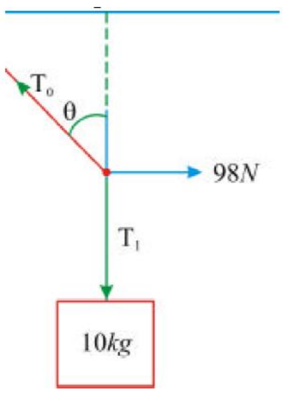
26. A bomb of mass 6 kg initially at rest explodes in to three identical

fragments. One of the fragments moves with a velocity of $10\sqrt{3}\,\hat{\imath}\ m/s$, another fragment moves with a velocity of 10 $\sqrt{3}\hat{j}\ m/s$, then the third fragment moves with a velocity of magnitude.

1) 30 m/s 2) 20 m/s
3) 15 m/s 4) 5 m/s

EQUILIBRIUM OF A PARTICLE

27. A mass of 10 kg is suspended by a rope of length 2.8m from a ceiling. A force of 98 N is applied at the midpoint of the rope as shown in figure. The angle which the rope makes with the vertical in equilibrium is



1) 300 2) 600 3) 450 4) 900

28. A mass of $M\ kg$ is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle 450 with the initial vertical direction is

1) Mg 2) $\frac{Mg}{\sqrt{2}}$
3) $Mg(\sqrt{2}+1)$ 4) $\sqrt{2}Mg$

LAWS OF FRICTION

29. The coefficients of static and dynamic friction are 0.7 and 0.4. The minimum force required to create motion is applied on a body and if it is further continued, the acceleration attained by the body in ms^{-2} is ($g = 10m/s^2$)

1) 7 2) 4 3) 3 4) Zero

30. The coefficient of static friction between contact surfaces of two bodies is 1. The contact surfaces of one body support the other till the inclination is less than

1) 30° 2) 45° 3) 60° 4) 90°

MOTION ON A HORIZONTAL ROUGH SURFACE

31. Brakes are applied to a car moving with disengaged engine, bringing it to a halt after 2s. Its velocity at the moment when the brakes are applied if the coefficient of friction between the road and the tyres is 0.4 is

1) 3.92 ms^{-1} 2) 7.84 ms^{-1}
3) 11.2ms^{-1} 4) 19.6 ms^{-1}

32. A book of weight 20N is pressed between two hands and each hand exerts a force of 40N. If the book just starts to slide down. Coefficient of friction is

1) 0.25 2) 0.2 3) 0.5 4) 0.1

33. A car running with a velocity 72 kmph on a level road, is stopped after travelling a distance of 30m after disengaging its engine ($g = 10ms^{-2}$). The coefficient of friction between the road and the tyres is

1) 0.33 2) 4.5 3) 0.67 4) 0.8

34. In the above problem car got a stopping distance of 80m on cement road then μk is ($g = 10\ m/sec^2$)

1) 0.2 2) 0.25 3) 0.3 4) 0.35

35. A 10kg mass is resting on a horizontal surface and horizontal force of 80N is applied. If $\mu = 0.2$, the ratio of acceleration without and with friction is ($g = 10ms^{-2}$)

1) 3/4 2) 4/3 3) 1/2 4) 2

36. A block of mass 20 kg is pushed with a horizontal force of 90N. If the coefficient of static and kinetic friction are 0.4 and 0.3, the frictional force acting on the block is ($g = 10ms^{-2}$)

1) 90N 2) 80N 3) 60N 4) 30N

37. A force of 150N produces an acceleration of $2ms^{-2}$ in a body and a force of 200N produces an acceleration of $3ms^{-2}$. The mass of the body and the coefficient of kinetic friction are

1) 50kg; 0.1 2) 25kg; 0.1

3) 50kg; 0.5 4) 50kg; 0.2

38. A heavy uniform chain lies on horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, the maximum percentage of the length of the chain that can hang over one edge of the table is

1) 20% 2) 25% 3) 35% 4) 15%

MOTION OF A BODY ON THE INCLINED PLANE

39. The angle of inclination of an inclined plane is 60°. Coefficient of friction between 10kg body on it and its surface is 0.2, g = 10 ms^{-2}. The acceleration of the body down the plane in ms–2 is

1) 5.667 2) 6.66 3) 7.66 4) Zero

40. In the above problem the resultant force on the body is

1) 56.6 N 2) 66.6 N
3) 76.6 N 4) 86.6 N

41. In the above problem, the frictional force on the body is

1) Zero 2) 5 N 3) 7.5 N 4) 10 N

42. In the above problem, the minimum force required to pull the body up the inclined plane

1) 66.6 N 2) 86.6 N
3) 96.6 N 4) 76.6 N

43. When a body slides down an inclined plane with coefficient of friction as µk, then its acceleration is given by

1) g(µk sin? + cos?)
2) g(µk sin? – cos?)
3) g(sin ? + µk cos?)
4) g(sin ? – µk cos ?)

44. A brick of mass 2kg just begins to slide down on inclined plane at an angle of 45° with the horizontal. The force of friction will be

1) 19.6 sin 45° 2) 9.8 sin 45°
3) 19.6 cos 45° 4) 9.8 cos 45°

45. The lengths of smooth & rough inclined planes of inclination 450 is same. Times of sliding of a body on two surfaces is t_1, t_2 and m = 0.75, then t_1 : t_2 =

1) 2 : 1 2) 2 : 3
3) 1 : 2 4) 3 : 2

PULLING / PUSHING A BODY

46. A block of weight 200N is pulled along a rough horizontal surface at constant speed by a force of 100N acting at an angle 30° above the horizontal. The coefficient of kinetic friction between the block and the surface is

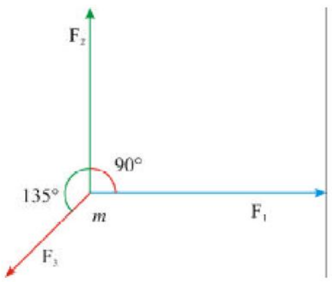
1) 0.43 2) 0.58 3) 0.75 4) 0.83

LEVEL – 1 (B) - KEY

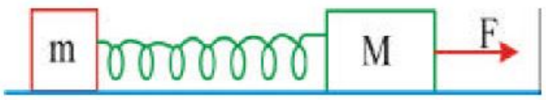
1) 2 2) 2 3) 4 4) 3 5) 3 6) 2
7) 4 8) 3 9) 2 10) 1 11) 3 12) 1
13) 1 14) 2 15) 2 16) 1 17) 2 18) 3
19) 3 20) 3 21) 3 22) 1 23) 4 24) 4
25) 1 26) 2 27) 3 28) 1 29) 3 30) 2
31) 2 32) 1 33) 3 34) 2 35) 2 36) 3
37) 1 38) 1 39) 3 40) 3 41) 4 42) 3
43) 4 44) 1 45) 3 46) 2

LEVEL – 2
NEWTON'S LAWS OF MOTION

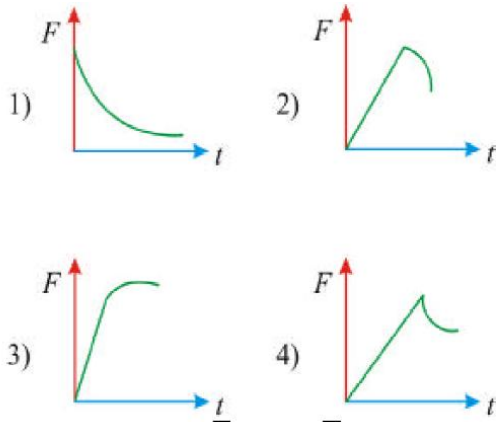
1. The momenta of a body in two perpendicular directions at any time't' are given by Px=$2t^2 + 6$ and Py = $\frac{3t^2}{2} + 3$. The force acting on the body at t = 2 sec is
   1) 5 units 2) 2 units
   3) 10 units 4) 15 units
2. When a force F acts on a body of mass m, the acceleration produced in the body is a. If three equal forces $F_1 = F_2 = F_3 = F$ act on the same body as shown in figure the acceleration produced is



   1) $(2 -1)a$ 2) $(2 +1)a$
   3) $2a$ 4) a
3. Two blocks of masses m and M are placed on a horizontal frictionless table connected by light spring as shown in the figure. Mass M is pulled to the right with a force F. If the acceleration of mass m is a, then the acceleration of mass M will be



   1) $\frac{(F-ma)}{M}$ 2) $\frac{(F+ma)}{M}$ 3) $\frac{F}{M}$ 4) $\frac{am}{M}$
4. The displacement of a body moving along a straight line is given by : $S = btn$, where 'b' is a constant and 't' is time. For what value of 'n' the body moves under the action of constant force?
   1) 3/2 2) 1 3) 2 4) 1/2
5. If F = $F_0(1 - e^{-t/\lambda})$, the F-t graph is



6. Three forces 20 2 N , 20 2 N and 40N are acting along X, Y and Z – axes respectively on a $5\sqrt{2}$ kg mass at rest at the origin. The magnitude of its displacement after 5s is,
   1) 50m 2) 25m 3) 60m 4) 100m
7. A horizontal jet of water coming out of a pipe of area of cross-section $20cm2$ hits a vertical wall with a velocity of $10ms$-1 and rebounds with the same speed. The force exerted by water on the wall is,
   1) 0.2 N 2) 10 N 3) 400 N 4) 200 N
8. A rocket of mass 40 kg has 160 kg fuel. The exhaust velocity of the fuel is 2 kms^{-1}. The rate of consumption of fuel is 4 kgs^{-1} .Calculate the ultimate vertical speed gained by the rocket.
   1) 2.82 kms^{-1} 2) 4.82 kms^{-1}
   3) 3.61 kms^{-1} 4) 5.62 kms^{-1}
9. A body of mass 5kg starts from the origin with an initial velocity $\bar{u}$ = $30\hat{\imath} + 40\hat{\jmath} ms^{-1}$. If a constant force $F = -(\hat{\imath} + 5\hat{\jmath})$ N r acts on the body, the time in which the y - component of the velocity becomes zero is
   1) 5s 2) 20s 3) 40s 4) 80s
10. A professional diver of mass 60 kg performs a dive from a platform 10 m above the water surface. Find the magnitude of the average impact force experienced by him if the impact time is 1s on collision with water surface. Assume that the velocity of the diver just after entering the water surface is $4ms^{-1}$.(g=$10ms^{-2}$)

1) $240N$ 2) $600N$ 3)$300N$ 4) $60N$

11. An open knife edge of mass 200 g is dropped from height 5m on a cardboard. If the knife edge penetrates distance 2m into the card board, the average resistance offered by the cardboard to the knife edge is ($g = 10m/s^2$)

1) 7 N 2) 25 N 3) 35 N 4) None

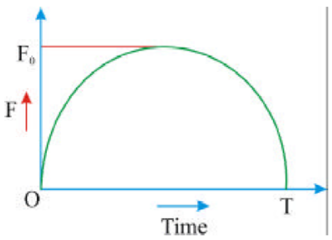
12. six forces lying in a plane and forming angles of 600 relative to one another are applied to the centre of a homogeneous sphere with a mass m=6kg. These forces are radially outward and consecutively 1N,2N,3N,4N,5N and 6N. The acceleration of the sphere is

1) 0 2) $1/2\ m/s^2$

3) $1m/s^2$ 4) $2\ m/s^2$

IMPULSE

13. A particle of mass m, initially at rest is acted upon by a variable force F for a brief interval of time T. It begins to move with a velocity u after the force stops acting. F is shown in the graph as a function of time. The curve is a semicircle. Then



1) $u = \dfrac{\pi F_0^2}{2m}$ 2) $u = \dfrac{\pi T^2}{8m}$

3) $u = \dfrac{\pi F_0 T}{4m}$ 4) $u = \dfrac{\pi F_0 T}{2m}$

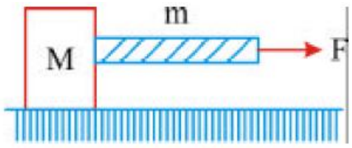
14. A ball of mass 0.2kg strikes an obstacle and moves at 600 to its original direction. If its speed also changes from 20m/s to 10m/s, the magnitude of the impulse received by the ball is

1) $2\sqrt{7}$N 2) $2\sqrt{3}$N

3) $2\sqrt{5}$N 4) $3\sqrt{2}$N

OBJECTS SUSPENDED BY STRINGS AND APPARENT WEIGHT

15. The block is placed on a frictionless surface in gravity free space. A heavy string of a mass m is connected and force F is applied on the string, then the tension at the middle of rope is



1) $\dfrac{\left(\frac{m}{2}+M\right).F}{m+M}$ 2) $\dfrac{\left(\frac{M}{2}+m\right).F}{m+M}$

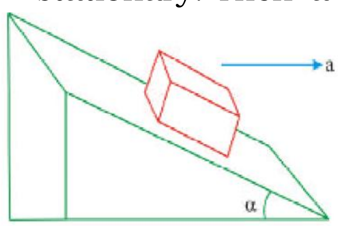
3) zero 4) $\dfrac{M.F}{m+M}$

16. A ball is suspended by a thread from the ceiling of a tram car. The brakes are applied and the speed of the car changes uniformly from 36 kmh^{-1} to zero in 5 s. The angle by which the ball deviates from the vertical is ($g = 10\ s^{-2}$)

1) $\tan^{-1}\left(\frac{1}{3}\right)$ 2) $\sin^{-1}\left(\frac{1}{5}\right)$

3) $\tan^{-1}\left(\frac{1}{5}\right)$ 4) $\cot^{-1}\left(\frac{1}{3}\right)$

17. A block is kept on a frictionless inclined surface with angle of inclination α . The incline is given an acceleration 'a' to keep the block stationary. Then 'a' is equal to



1) $\frac{g}{\tan\alpha}$ 2) g cosec α

3) g 4) g tan α

18. A man sits on a chair supported by a rope passing over a frictionless fixed pulley. The man who weighs 1,000 N exerts a force of 450 N on the chair downwards while pulling the rope on the other side. If the chair weighs 250N, then the acceleration of the chair is

1) $0.45m/s^2$ 2) 0

3) $2m/s^2$ 4) $9/25m/s^2$

19. A balloon of mass M is descending at a constant acceleration α. When a mass m is released from the balloon it starts rising with the same acceleration α. Assuming that its volume does not change, what is the value of m?

1) $\frac{\alpha}{\alpha+g}M$ 2) $\frac{2\alpha}{\alpha+g}M$

3) $\frac{\alpha+g}{\alpha}M$ 4) $\frac{\alpha+g}{2\alpha}M$

20. A monkey of mass 40 kg climbs on a massless rope of breaking strength 600 N. The rope will break if the m on key. (Take g = 10 m /s²)

1) climbs up with a uniform speed of $6m/s$
2) climbs up with an acceleration of $6m/s2$
3) climbs down with an acceleration of $4m/s^2$
4) climbs down with a uniform speed of $5m/s$

21. Two persons are holding a rope of negligible weight tightly at its ends so that it is horizontal. A 15 kg weight is attached to rope at the midpoint which now no more remains horizontal. The minimum tension required to completely straighten the rope is

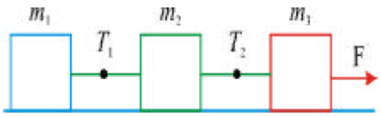
1) 150 N 2) 75 N

3) 50 N 4) Infinitely large

22. A straight rope of length 'L' is kept on a frictionless horizontal surface and a force 'F' is applied to one end of the rope in the direction of its length and away from that end. The tension in the rope at a distance 'l' from that end is

1) $\frac{F}{l}$ 2) $\frac{LF}{l}$ 3) $\left(1-\frac{l}{L}\right)F$ 4) $\left(1+\frac{l}{L}\right)F$

23. Consider three blocks of masses m_1, m_2, m_3 interconnected by strings which are pulled by a common force F on a frictionless horizontal table as in the figure. The tension T_1 and T_2 are also indicated



a) $T_2 > T_1$ if $m_2 > m_1$
b) $T_2 = T_1$ if $m_2 = m_1$
c) $T_2 > T_1$ always
d) acceleration of the system =

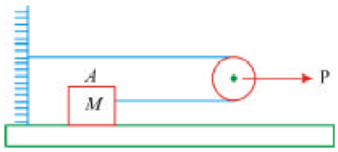
1) a, b 2) b, d 3) a, d 4) c, d

24. A railway engine of mass 50 tons is pulling a wagon of mass 40 tons with a force of 4500N. The resistance force acting is 1N per ton. The tension in the coupling between the engine and the wagon is

1) 1600 N 2) 2000 N

3) 200 N 4) 1500N

25. In the following figure, the pulley is massless and frictionless. There is no friction between the body and the floor. The acceleration produced in the body when it is displaced through a certain distance with force 'P' will be

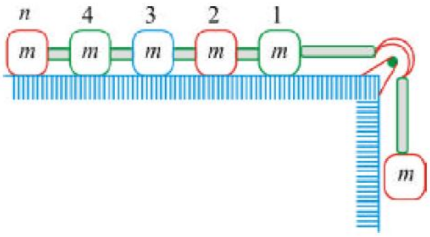


1) $\frac{P}{M}$ 2) $\frac{P}{2M}$ 3) $\frac{P}{3M}$ 4) $\frac{P}{4M}$

26. Two identical blocks each of mass "M" are tied to the ends of a string and the string is laid over a smooth fixed pulley. Initially the masses are held at rest at the same level. What fraction of mass must be removed from one block and added to the other, so that it has an acceleration of $1/5th$ of the acceleration due to gravity

1) 1/10 2) 1/5 3) 2/5 4) 1/20

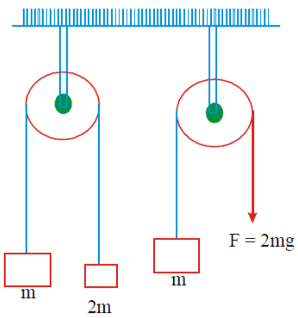
27. In the given arrangement, n number of equal masses are connected by strings of negligible masses. The tension in the string connected to nth mass is :



1) $\frac{mMg}{nm+M}$ 2) $\frac{mMg}{nmM}$
3) mg 4) mng

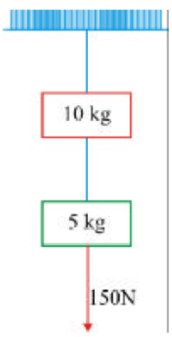
28. A 40 N block is supported by two ropes. One rope is horizontal and the other makes an angle of 300 with the ceiling. The tension in the rope attached to the ceiling is approximately
1) 80 N 2) 40 N
3) 34.6 N 4) 46.2 N

29. The pulley arrangements shown in figure are identical, the mass of the rope being negligible. In case I, the mass m is lifted by attaching a mass 2m to the other end of rope with a constant downward force F = 2mg, where g is acceleration due to gravity. The acceleration of mass m in case I is



1) zero
2) more than that in case II
3) less than that in case II
4) equal to that in case II

30. Two masses of 10 kg and 5 kg are suspended from a rigid support as shown in figure. The system is pulled down with a force of 150 N attached to the lower mass. The string attached to the support breaks and the system accelerates downwards. In case the force continues to act. what will be

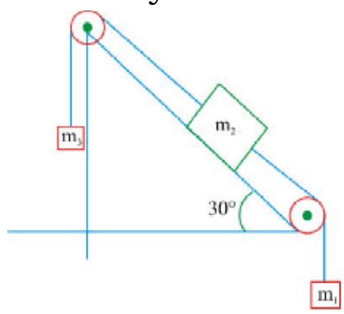


the tension acting between the two masses?
1) 300 N 2) 200 N
3) 100 N 4) zero

31. Two bodies of masses 3kg and 2kg are connected by a long string and the string is made to pass over a smooth fixed pulley. Initially the bodies are held at the same level and released from rest. The velocity of the 3kg body after one second is ($g=10m/s^2$)
1) 2m/s 2) 1m/s
3) 0.4m/s 4) 4m/s

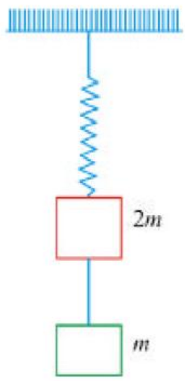
32. A block of mass 3kg which is on a smooth inclined plane making an angle of 300 to the horizontal is connected by cord passing over light frictionless pulley to second block of mass 2kg hanging vertically. What is the acceleration of each block and what is the tension of the cord?
1) 0.98 m/s^2; 17.6N
2) 1.98 m/s^2; 19.6N
3) 0.49 m/s^2; 9.8N
4) 1.47 m/s^2; 4.9N

33. If m_1 = 10kg, m_2 = 4kg, m_3 = 2kg, the acceleration of system is



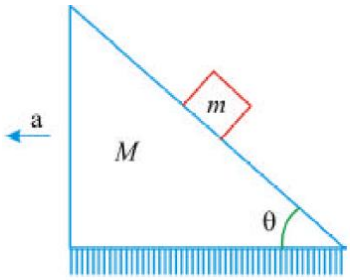
1) 5g/2 2) 5g/3
3) 5g/8 4) 5g/14

34. The string between blocks of masses 'm' and '2m' is massless and inextensible. The system is suspended by a massless spring as shown. If the string is cut, the magnitudes of accelerations of masses 2m and m (immediately after cutting)



1) g, g 2) $g, \frac{g}{2}$ 3) $\frac{g}{2}, g$ 4) $\frac{g}{2}, \frac{g}{2}$

35. All surfaces are smooth. The acceleration of mass m relative to the wedge is



1) $g \sin\theta$ 2) $g \sin\theta + a \cos\theta$
3) $g \sin\theta - a \cos\theta$ 4) $a \cos\theta$

LAW OF CONSERVATION OF MOMENTUM

36. A bullet of mass 10 gm moving with a horizontal velocity 100m/s passes through a wooden block of mass 100 gm. The block is resting on a smooth horizontal floor. After passing through the block the velocity of the bullet is 10m/s. the velocity of the emerging bullet with respect to the block is
1) 10 m/s 2) 9 m/s
3) 1 m/s. 4) 5 m/s

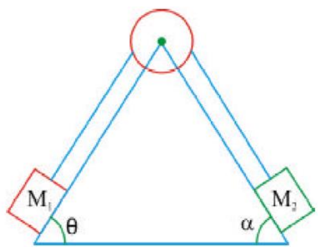
37. A shell is fired from the ground at an angle q with horizontal with a velocity 'v'. At its highest point it breaks into two equal fragments. If one fragment comes back through its initial line of motion with same speed, then the speed of the second fragment will be
1) $3v \cos\theta$ 2) $3v \cos\theta / 2$
3) $2v \cos\theta$ 4) $3v \cos\theta / 2$

38. Two trolleys of masses m and $3m$ are connected by a spring. They are compressed and released, they move off in opposite direction and come to rest after covering distances s_1 and s_2 respectively. If the frictional force between trolley and surface is same in both the cases then the ratio of distances $s_1 : s_2$ is
1) 1:9 2) 1:3 3) 3:1 4) 9:1

39. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time t = 0. They collide at time t_0 . Their velocities become $\vec{v}_1^{\,1}$ and $\vec{v}_1^{\,2}$ at time $2t_0$ while still moving in air. The value of
$\left|\left(m_1\vec{v}_1^{\,1} + m_2\vec{v}_2^{\,1}\right) - \left(m_1\vec{v}_1 + m_2\vec{v}_2\right)\right|$ **is**
1) zero 2) $(m_1 + m_2) g t_0$
3) $2(m_1 + m_2) g t_0$ 4) $\frac{1}{2}(m_1 + m_2) g t_0$

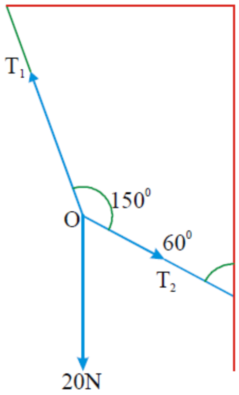
EQUILIBRIUM OF A PARTICLE

40. Two masses M_1 and M_2 connected by means of a string which is made to pass over light, smooth pulley are in equilibrium on a fixed smooth wedge as shown in figure. If q = 600 and a = 300 ,then the ratio of M_1 to M_2 is



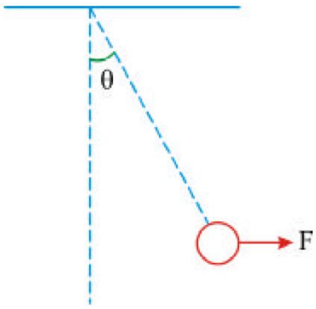
1) 1 : 2 2) 2: 3
3) 1: 3 4) 3 :1

41. If 'O' is at equilibrium then the values of the tension T_1 and T_2 respectively.



1) 20N, 30N
2) 20 $\sqrt{3}$N,20N
3) 20 $\sqrt{3}$N, 20 $\sqrt{3}$N
4) 10N, 30N

42. A 1N pendulum bob is held at an angle θ from the vertical by a 2 N horizontal force F as shown in the figure. The tension in the string supporting the pendulum bob (in newton) is



1) cos θ
2) $\frac{2}{\cos\theta}$
3) 5
4) 1

MOTION ON A HORIZONTAL ROUGH SURFACE

43. The coefficient of friction between a hemispherical bowl and an insect is 0.44 and the radius of the bowl is 0.6m. The maximum height to which an insect can crawl in the bowl will be
1) 0.4m
2) 0.2m
3) 0.3m
4) 0.1m

44. A 500 kg horse pulls a cart of mass 1500 kg along a level road with an acceleration of 1 m/s^2. If coefficient of sliding friction is 0.2, then force exerted by the earth on horse is
1)3000N
2)4000N
3)5000 N
4)6000N

45. An aeroplane requires for take off a speed of 108 kmph the run on the ground being 100m. Mass of the plane is 104kg and the coefficient of friction between the plane and the ground is 0.2. Assuming the plane accelerates uniformly the minimum force required is ($g = 10ms^{-2}$)
1) 2 x 10^4 N
2) 2.43 x 10^4 N
3) 6.5 x 10^4 N
4) 8.86 x 10^4 N

46. A duster weighs 0.5N. It is pressed against a vertical board with a horizontal force of 11N. If the co-efficient of friction is 0.5 the minimum force that must be applied on the duster parallel to the board to move it upwards is
1) 0.4 N
2) 0.7 N
3) 6 N
4) 7 N

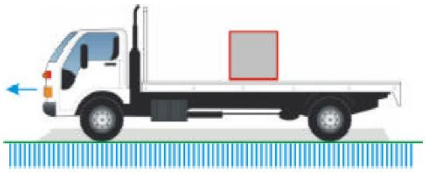
47. A man of mass 65 kg. is standing stationary with respect to a conveyor belt which is accelerating with $1m / s2$. If $s\ m$ is 0.2, the net force on the man and the maximum acceleration of the belt so that the man is stationary relative to the belt are ($g = 10m/s^2$)
1) zero, $2m/s^2$
2) $65N$, $2m/s^2$
3) zero, $1m/s^2$
4) $65N$,$1m/s^2$

48. A man of mass 60kg sitting on ice pushes a block of mass of 12kg on ice horizontally with a speed of $5ms^{-1}$ .The coefficient of friction between the man and ice and between block and ice is 0.2. If $g = 10ms^{-2}$,the distance between man and the block, when they come to rest is
1)6m 2)6.5m 3) 3m 4)7m

49. A vehicle of mass M is moving on a rough horizontal road with a momentum P. If the coefficient of friction between the tyres and the road is m , then the stopping distance is
1) $\frac{P}{2\mu \, Mg}$ 2) $\frac{P^2}{2\mu \, Mg}$ 3) $\frac{P^2}{2\mu \, M^2 g}$ 4) $\frac{P}{2\mu \, M^2 g}$

50. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in figure. The coefficient of friction between the box the surface below it is 0.15. On a straight road, the truck starts from rest

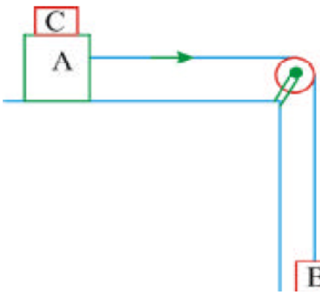
and accelerates with $2ms^{-2}$. At what distance from the starting point does the box fall from the truck? (Ignore the size of the box.)



1) $20m$ 2) $10m$ 3) $20m$ 4) $5m$

51. A grinding machine whose wheel has a radius of $\frac{1}{\pi}$ is rotating at 2.5 rev/sec. A tool to be sharpened is held against the wheel with a force of 40N. If the coefficient of friction between the tool and the wheel is 0.2, power required is
1) 40 W 2) 4 W 3) 8 W 4) 10 W

52. A block A of mass 3kg and another block B of mass 2 kg are connected by a light inextensible string as shown in figure. If the coefficient of friction between the surface of the table and A is 0.5. What maximum mass C is to be placed on A so that the system is to be in equilibrium?



1) 3kg 2) 2kg 3) 1kg 4) 4kg

Motion of a Body on the Inclined Plane

53. A block slides down a rough inclined plane of slope angle q with a constant velocity. It is then projected up the same plane with an initial velocity v. The distance travelled by the block up the plane before coming to rest is
1) $\frac{v^2}{4g\sin?}$ 2) $\frac{v^2}{2g\sin?}$ 3) $\frac{v^2}{g\sin?}$ 4) $\frac{4gv^2}{\sin?}$

54. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle q with the horizontal such that $\tan q = 2m$, then the ratio $\frac{F_1}{F_2}$ F is
1) 4 2) 1 3) 2 4) 3

55. The horizontal acceleration that should be given to a smooth inclined plane of angle $\sin^{-1}\left(\frac{1}{l}\right)$ to keep an object stationary on the plane, relative to the inclined plane is
1) $\frac{g}{\sqrt{l^2-1}}$ 2) $g\sqrt{l^2-1}$ 3) $\frac{\sqrt{l^2-1}}{g}$ 4) $\frac{g}{\sqrt{l^2+1}}$

56. A body is released from the top of a smooth inclined plane of inclination q . It reaches the bottom with velocity v . If the angle of inclination is doubled for the same length of the plane, what will be the velocity of the body on reaching the ground
1) v 2) 2v
3) $(2\cos\theta)^{\frac{1}{2}}v$ 4) $(2\sin\theta)^{\frac{1}{2}}v$

57. The force required to move a body up a rough inclined plane is double the force required to prevent the body from sliding down the plane. The coefficient of friction when the angle of inclination of the plane is 600 is (EAM - 2014)
1) $\frac{1}{\sqrt{2}}$ 2) $\frac{1}{\sqrt{3}}$ 3) $\frac{1}{2}$ 4) $\frac{1}{3}$

58. A smooth block is released from rest on a 450 inclined plane and it slides a distance ‘d’. The time taken to slide is n times that on a smooth inclined plane. The coefficient of friction (2010E)
1) $\mu_k = 1-\frac{1}{n^2}$ 2) $\mu_k = \sqrt{1-\frac{1}{n^2}}$
3) $\mu_k = \frac{1}{1-n^2}$ 4) $\mu_k = \sqrt{\frac{1}{1-n^2}}$

59. The upper half of an inclined plane of inclination ‘ q ’ is perfectly smooth while the lower half is rough. A block starting from rest at the top of the plane will again come to rest at the

bottom. The coefficient of friction between the block and the lower half of the plane is given by (2013 E)

1) $\mu = 2\tan\theta$ 2) $\mu = \dfrac{2}{\tan\theta}$

3) $\mu = \tan\theta$ 4) $\mu = \dfrac{1}{\tan\theta}$

60. A 30 kg box has to move up an inclined plane of slope 300 to the horizontal with a uniform velocity of 5 ms-1.If the frictional force retarding the motion is 150N, the horizontal force required to move the box up is($g=10ms^{-2}$)

1) $300 \times \frac{2}{\sqrt{3}}$ N 2) $300 \times \frac{\sqrt{3}}{2}$ N

3) 300N 4) 150N

PULLING / PUSHING A BODY

61. A block weighing 10kg is at rest on a horizontal table. The coefficient of static friction between the block and the table is 0.5. If a force acts downward at 60° with the horizontal, how large can it be without causing the block to move? ($g = 10ms^{-2}$)

1) 346 N 2)446 N

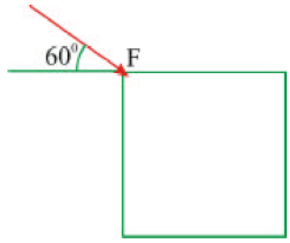
3) 746 N 4)846 N

62. A pulling force making an angle *q* with the horizontal is applied on a block of weight W placed on a horizontal table. If the angle of friction is *f* , the magnitude of the force required to move the body is equal to

1) $\dfrac{WCos\phi}{Cos(\theta-\phi)}$ 2) $\dfrac{W\sin\phi}{Cos(\theta-\phi)}$

3) $\dfrac{WTan\phi}{Sin(\theta-\phi)}$ 4) $\dfrac{WSin\phi}{Tan(\theta-\phi)}$

63. A block of mass 3 kg is kept on a frictional surface with μ = $\frac{1}{2\sqrt{3}}$ m = . The minimum force to be applied as shown to move the block is



1) 5N 2) 20 N 3) 10 N 4) 20/3 N

LEVEL – 2 KEY

01) 3 02) 1 03) 1 04) 3 05) 3 06)4
07) 3 08) 1 09) 3 10) 2 11) 1 12)3
13) 3 14) 2 15) 1 16) 3 17) 4 18)3
19) 2 20) 2 21) 4 22) 3 23) 4 24)2
25) 2 26) 2 27) 1 28) 1 29) 3 30)3
31) 1 32) 1 33) 3 34) 3 35) 2 36)3
37) 1 38) 4 39) 3 40)3 41) 2 42)3
43) 4 44) 4 45) 3 46)3 47) 1 48)2
49) 3 50) 1 51) 1 52)3 53) 1 54)4
55) 1 56) 3 57) 2 58) 1 59) 1 60)1
61) 3 62) 2 63)2

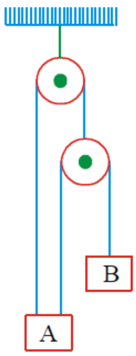
LEVEL-III

NEWTON'S LAWS OF MOTION

1) A rope is stretched between two boats at rest. A sailor in the first boat pulls the rope with a constant force of 100N. First boat with the sailor has a mass of 250kg where as the mass of second boat is double of this mass. If the initial distance between the boats was 100m, the time taken for two boats to meet each other in seconds (neglect water resistance between boats and water)
1) 13.8 2) 18.3 3) 3.18 4) 31.8

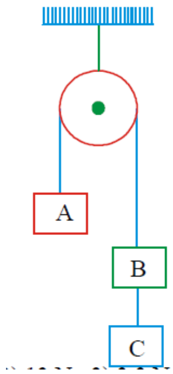
2) In order to raise a block of mass 100kg a man of mass 60kg fastens a rope to it and passes the rope over a smooth pulley. He climbs the rope with an acceleration $\frac{5g}{4}$ relative to rope. The tension in the rope is (g =10ms^{-2})
1) 1432N 2) 928 N 3) 1218N 4) 642N

3) In the pulley-block arrangement shown in figure. Find the relation between acceleration of block A and B.



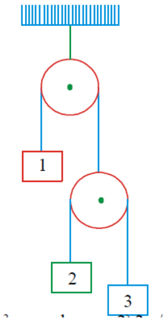
1) $a_B=-3a_A$ 2) $a_B=-a_A$
3) $a_B=-2a_A$ 4) $a_B=-4a_A$

4) Three equal weights A, B and C of mass 2 kg each are hanging on a string passing over a fixed frictionless pulley as shown in the fig. The tension in the string connecting weights B and C is



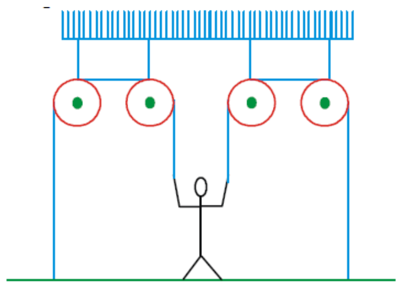
1) zero 2) 13 N 3) 3.3 N 4) 19.6 N

5) In the figure shown a_3 = 6m/s^2 (downwards) and a_2 = 4m/s^2 (upwards). Find acceleration of 1.



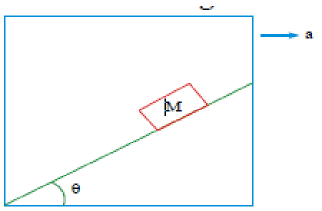
1) 1m/sec^2 upwards
2) 2m/sec^2 upwards
3) 1m/sec^2 downwards
4) 2m/sec^2 downwards

6) A man of mass m stands on a platform of equal mass m and pulls himself by two ropes passing over pulleys as shown in figure. If he pulls each rope with a force equal to half his weight, his upward acceleration would be



1) $\frac{g}{2}$ 2) $\frac{g}{4}$ 3) g 4) zero

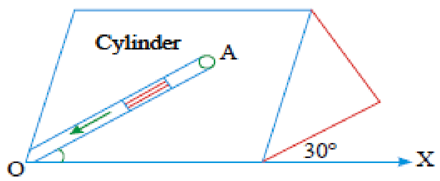
7) A block is sliding along inclined plane as shown in figure. If the acceleration of chamber is 'a' as shown in the figure. The time required to cover a distance L along inclined plane is



1) $\sqrt{\frac{2L}{g\sin\theta - a\cos\theta}}$ 2) $\sqrt{\frac{2L}{g\sin\theta + a\sin\theta}}$

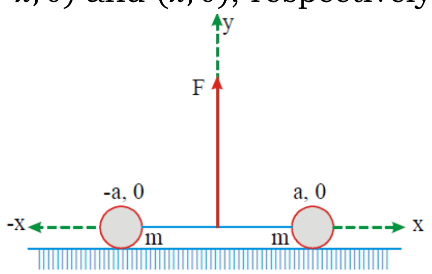
3) $\sqrt{\frac{2L}{g\sin\theta + a\cos\theta}}$ 4) $\sqrt{\frac{2L}{g\sin\theta}}$

8) An inclined plane makes an angle 30^0 with the horizontal. A groove (OA) of length 5 m cut, in the plane makes an angle 30^0 with OX. A short smooth cylinder is free to slide down under the influence of gravity. The time taken by the cylinder to reach from A to O is (g = 10 m/s^2)



1) 4 s 2) 2 s 3) 3 s 4) 1 s

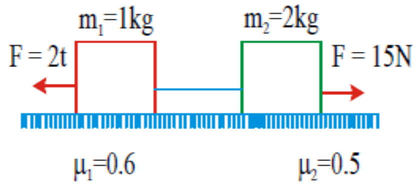
9) Two masses each equal to m are lying on X-axis at $(-a, 0)$ and $(+a, 0)$, respectively, as shown in fig. They are connected by a light string. A force F is applied at the origin along vertical direction. As a result, the masses move towards each other without loosing contact with ground. What is the acceleration of each mass? Assume the instantaneous position of the masses as $(-x, 0)$ and $(x, 0)$, respectively



1) $\frac{2F}{m}\frac{\sqrt{(a^2-X^2)}}{X}$ 2) $\frac{2F}{m}\frac{X}{\sqrt{(a^2-X^2)}}$

3) $\frac{F}{2m}\frac{X}{\sqrt{(a^2-X^2)}}$ 4) $\frac{F}{m}\frac{X}{\sqrt{(a^2-X^2)}}$

10) A lift is going up, the total mass of the lift and the passengers is 1500 kg. The variation in the speed of lift is shown in fig. Then the tension in the rope at t = 1 s will be



1)17400N 2)14700N

3)12000N 4)10000 N

11) In the above problem the tension in the rope will be least at

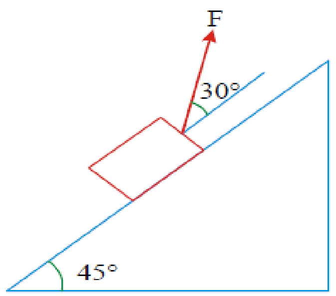
1) t = 1 s 2) t = 4 s 3) t = 9 s 4) t = 11 s

12) A piece of wire is bent in the shape of a parabola y = kx^2 (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is:

1) $\frac{a}{gk}$ 2) $\frac{a}{2gk}$ 3) $\frac{2a}{gk}$ 4) $\frac{a}{4gk}$

Friction

13) A block of mass m = 4kg is placed over a rough inclined plane having coefficient of friction μ = 0.6 as shown in fig. A force F = 10N is applied on the block at an angle 30^0. The contact force between the block and the plane is



1)10.65N 2)16.32N

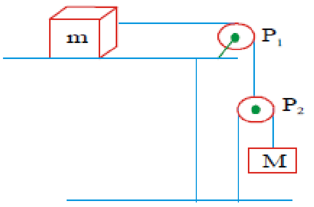
3) 27.15 N 4) 32.16 N

14) A block of mass m slides down an inclined plane of inclination θ with uniform speed. The coefficient of friction between the block and the

plane is μ. The contact force between the block and the plane is

1) $mg\sin\theta\sqrt{1+\mu^2}$
2) $\sqrt{(mg\sin\theta)^2+(\mu mg\cos\theta)^2}$
3) $mg\sin\theta$
4) mg

15) In the pulley arrangement shown, the pulley P_2 is movable. Assuming coefficient of friction between m and surface to be , the minimum value of M for which m is at rest is



1) $M=\frac{\mu m}{2}$
2) $m=\frac{\mu M}{2}$
3) $M=\frac{m}{2\mu}$
4) $m=\frac{M}{2\mu}$

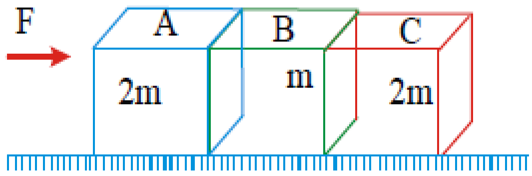
16) On an inclined plane of inclination angle 30^0, a block is placed. It is observed that the force to drag the block along the plane upwards is smaller than the force required to lift it. The maximum value of coefficient of friction is

1) $\frac{\sqrt{3}}{2}$
2) $\frac{1}{2}$
3) $\frac{1}{\sqrt{3}}$
4) $\frac{2}{3}$

17) A body slides over an inclined plane forming an angle of 45^0 with the horizontal. The distance x travelled by the body in time t is described by the equaiton $x = kt^2$, where k = 1.732. The coefficient of friction between the body and the plane has a value

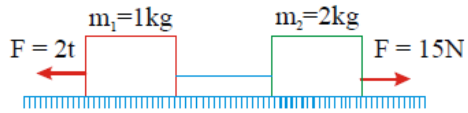
1) $\mu = 0.5$
2) $\mu = 1$
3) $\mu = 0.25$
4) $\mu = 0.75$

18) The system is pushed by a force F as shown in the figure. All surfaces are smooth except between B and C. Coefficient of friction between B and C is . Minimum value of F to prevent block B from downward slipping is



1) $\left(\frac{3}{2\mu}\right)mg$
2) $\left(\frac{5}{2\mu}\right)mg$
3) $\left(\frac{5}{2}\right)\mu mg$
4) $\left(\frac{3}{2}\right)\mu mg$

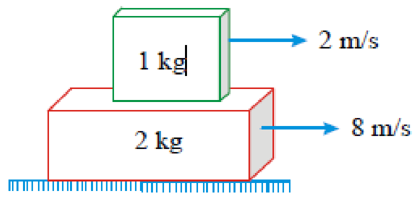
19) Two blocks A and B are separated by some distance and tied by a string as shown in the figure. The force of friction in both the blocks at t = 2s is.



1) 4N(→), 5N(←)
2) 2N(→), 5N(←)
3) 0N(→), 10N(←)
4) 1N(←), 10N(←)

Passage:

Coefficient of friction between two blocks shown in figure is $\mu = 0.4$. the blocks are given velocities of 2 m/s and 8 m/s in the directions shown in figure. Find.



20) The time when relative motion between them will stop

1) 1 sec
2) 2 sec
3) 3 sec
4) 4 sec

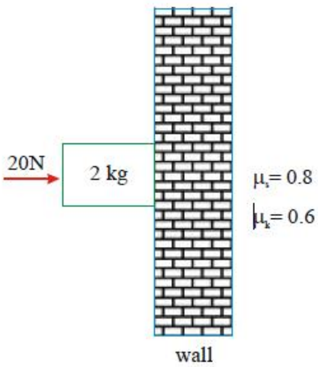
21) The common velocities of blocks upto that instant.

1) 4m/sec
2) 6m/sec
3) 8 m/sec
4) 10 m/sec

22) Displacements of 1 kg and 2 kg blocks upto that instant ($g = 10\ m/s^2$)

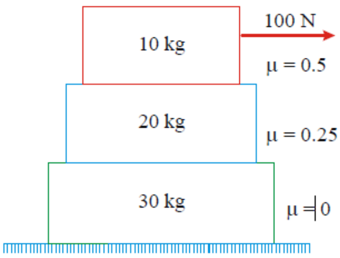
1) 4 m towards right, 7 m towards right
2) 4 m towards left, 7 m towards right
3) 4 m towards left, 7 m towards left
4) 4 m towards right, 7 m towards left

23) A 2kg block is pressed against a rough wall by a force F = 20N as shown in figure. Find acceleration of the block and force of friction acting on it. (Take $g = 10\ m/s^2$)



1) 4 m/sec^2 downward, 12N upward
2) 2 m/sec^2 downward, 6N upward
3) 12 m/sec^2 downward, 4N upward
4) 8 m/sec^2 downward, 12N upward

24) Three blocks are kept as shown in figure. Acceleration of 20 kg block with respect to ground is

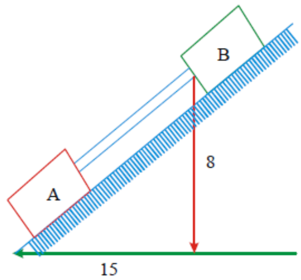


1) 5ms^{-2} 2) 2 ms^{-2} 3) 1 ms^{-2} 4) 0

25) A suitcase is gently dropped on a conveyor belt moving at a velocity of 3 m/s. If the coefficient of friction between the belt and the suitcase is 0.5, find the displacement of the suitcase relative to conveyor belt before the slipping between the two is stopped (g =10m/ s^2)

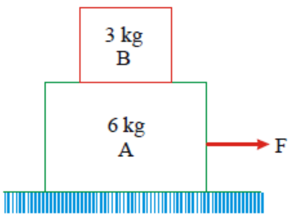
1) 2.7 m 2) 1.8 m 3) 0.9 m 4) 1.2 m

26) Blocks A and B in the fig, are connected by a bar of negligible weight. Mass of each block is 170 kg and μ_A = 0.2 and μ_A = 0.4, where μ_A and μ_B are the coefficients of limiting friction between blocks and plane. Calculate the force developed in the bar (g =10m/ sec^2).



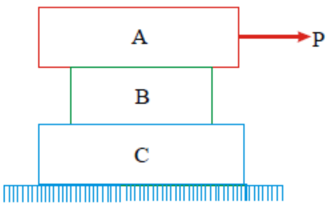
1) 150 N 2) 75 N 3) 200 N 4) 250 N

27) Two blocks A and B of masses 6 kg and 3 kg rest on a smooth horizontal surface as shown in the fig. If coefficient of friction between A and B is 0.4, the maximum horizontal force which can make them without separation is



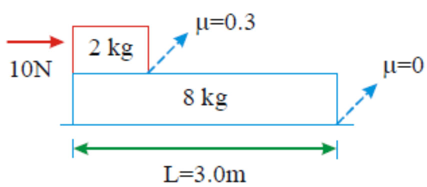
1) 72 N 2) 40 N
3) 36 N 4) 20 N

28) Find the least horizontal force P to start motion of any part of the system of the three blocks resting upon one another as shown in fig. The weights of blocks are A = 300 N, B = 100 N and C = 200 N. Between A and B, coefficient of friction is 0.3, between B and C is 0.2 and between C and the ground is 0.1.



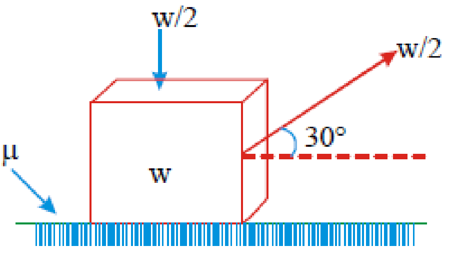
1) 60 N 2) 90 N 3) 80 N 4) 70 N

29) Determine time in which the smaller block reaches other end of bigger block as shown in the fig.



1) 4 s 2) 8 s 3) 2.19 s 4) 2.13 s

30) A block of weight W is kept on a rough horizontal surface (friction coefficient μ). Two forces W/2 each are applied as shown in the figure. Choose the correct statement.



1) For $\mu > \frac{\sqrt{3}}{5}$ block will move

2) For $\mu < \frac{\sqrt{3}}{5}$, work done by frictional force is zero (in ground frame)

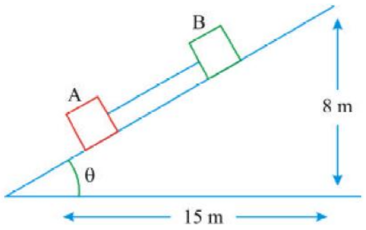
3) For $\mu > \frac{\sqrt{3}}{5}$, frictional force will do positive work (in ground frame)

4) For $\mu < \frac{\sqrt{3}}{5}$ block will move

31) A 2kg block is placed over a 4kg block and both are placed on a smooth horizontal surface. The coefficient of friction between the blocks is 0.20. The acceleration of the two blocks if a horizontal force of 12N is applied to the lower block is (g = $10 ms^{-2}$)

1) 2 ms^{-2}, 2 ms^{-2} 2) 2 ms^{-2}, 1 ms^{-2}
3) 3 ms^{-2}, 1 ms^{-2} 4) 4 ms^{-2}, 1 ms^{-2}

32) Blocks A and B shown in the figure are connected with a bar of negligible weight. A and B each has mass 170Kg, the coefficient of friction between A and the plane is 0.2 and that between B and the plane is 0.4. What is the total force of friction between the blocks and the plane (g = 10 ms^{-2})

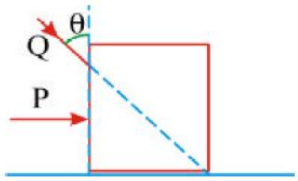


1) 900N 2) 700N
3) 600N 4) 300N

33) From the above problem what is the force acting on the connecting bar?

1) 150N 2) 100N
3) 75N 4) 125N

34) A block of mass m, lying on a rough horizontal plane is acted upon by a horizontal force P and another force Q, inclined at an angle θ with vertical. The block will remain in equilibrium, if coefficient of friction between it and surface is



1) $\frac{(P+Q\sin\theta)}{(mg+Q\cos\theta)}$ 2) $\frac{(P\cos\theta+Q)}{(mg-Q\sin\theta)}$

3) $\frac{(P+Q\cos\theta)}{(mg+Q\sin\theta)}$ 4) $\frac{(P\sin\theta-Q)}{(mg-Q\cos\theta)}$

LEVEL -3 KEY

1) 2	2) 3	3) 1	4) 2	5) 1	6) 4
7) 3	8) 2	9) 3	10) 1	11) 4	12) 2
13) 3	14) 4	15) 1	16) 3	17) 1	18) 2
19) 4	20) 1	21) 2	22) 1	23) 1	24) 3
25) 3	26) 1	27) 3	28) 1	29) 3	30) 4
31) 1	32) 1	33) 1	34) 1		

WORK

Work done by Constant Force

When a constant force $\vec{F}$ acts on a particle and the particle moves through a displacement $\vec{S}$ then the force is said to do work W on the particle.

$$\boldsymbol{W = \vec{F}.\vec{S}}$$

The scalar (dot) product of $\overrightarrow{F}$ and $\overrightarrow{S}$ can be evaluated as

$$\boldsymbol{W = \vec{F}.\vec{S} = FS\cos\theta}$$

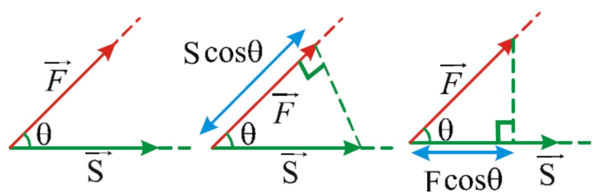
Where F is the magnitude of $\overrightarrow{F}$,

S is the magnitude of $\overrightarrow{S}$ and

θ is the angle between $\overrightarrow{F}$ and $\overrightarrow{S}$

$$W = FS\cos\theta = F(S\cos\theta)$$

$$\therefore W = \begin{pmatrix}\text{magnitude of} \\ \text{the force}\end{pmatrix} \times \begin{pmatrix}\text{component of displacement} \\ \text{in the direction of force}\end{pmatrix}$$



Work is a scalar quantity.

SI Unit is Nm or joule (J).

CGS unit is erg.

Dimensions: $[ML^2T^{-2}]$

Relation between joule and erg:

$$1\ joule = 10^7 erg$$

Other units of work:

Electron Volt (eV) $= 1.6\times10^{-19}$ J

Kilowatt hour $= 3.6\times10^{6}\ J$

Work done by multiple forces:

If a number of forces act on a body or particle then:

$$W = W_1+W_2+W_3+....$$

$$\therefore W = \overrightarrow{F_R}\cdot\vec{S}$$

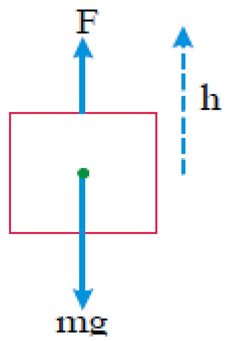
$$\left[as\ \overrightarrow{F_R} = \sum\vec{F}\right]$$

Work done in displacing a particle under the action of a number of forces is equal to the work done by the resultant force.

Nature of Work:

Work done by a force may be positive or negative or zero.

Ex : (a) If we lift a body from rest to a height h



Work done by lifting force F

$$W_1 = Fh\cos 0° = Fh\ (+v)$$

Work done by gravitational force

$$W_2 = mgh\cos 180° = -mgh\ (-ve)$$

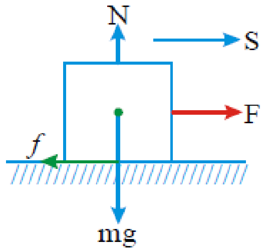
So, net work

$$W = W_1 + W_2 = Fh - mgh = (F - mg)h$$

Now, if the body is in equilibrium

$$F = mg, \qquad W = 0$$

Ex: (b) If a body is pulled on a rough horizontal road through a displacement S



Work done by normal reaction and gravity,

$W_1 = 0$ as force is ⊥ to S

Work done by pulling force F,

$W_2 = FS \cos 0° = FS(+ve)$

Work done by frictional force f,

$W_3 = fs \cos 180° = -\mu mgs$ (-ve)

Net work

$W = W_1 + W_2 + W_3 + = 0 + FS - fS$

$W = (F - f)S$

Now, if the body is in dynamic equilibrium

$f = F$, So, $W = 0$

Zero Work:

Work done is zero if,

1) Force and displacement are perpendicular.

As $W = F\, ds \cos \theta$,

so $W = 0$, if $\theta = 90°$,

i.e., if force is always perpendicular to motion, work done by the force will be zero though neither force nor displacement is zero. This is why:

(a) When a porter moves with a suitcase on his head on a horizontal level road, the work done by the lifting force or force of gravity is zero.

(b) When a body moves in a circle the work done by the centripetal force is always zero.

(c) When the bob of a simple pendulum swings, the work done by tension in the string is zero.

2) Displacement of point of application of force is zero.

As $W = \vec{F}.\vec{S}$

so, if $\vec{S} = 0$, $W = 0$

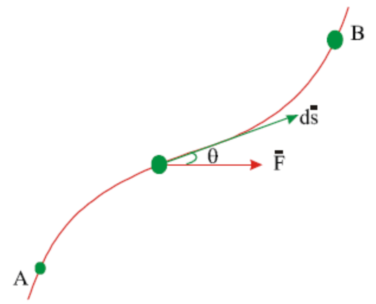
i.e., if displacement of a particle or body is zero whatever be the force, work done is zero (except nonconservative force)

(a) When a person tries to displace a wall or stone by applying a force and it (actually its centre of mass) does not move, the work done is zero.

(b) A weight lifter does work in lifting the weight from the ground but does not work in holding it up.

3) Net force acting on the body is zero.

Work done by Variable Force



When the magnitude and direction of a force varies with position, then the work done by such a force for an infinitesimal displacement ds is given by,

$$dW = \vec{F}.\vec{ds}$$

The total work done in going from A to B is

$$W_{AB} = \int_A^B \vec{F}.\vec{ds} \int_A^B (F\cos\theta) ds$$

In terms of reactangluar components

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k};$$

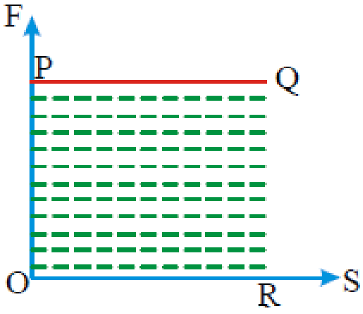
$$\vec{ds} = ds\hat{\imath} + dy\hat{\jmath} + dz\hat{k}$$

$$W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

Graphical representation of work done:

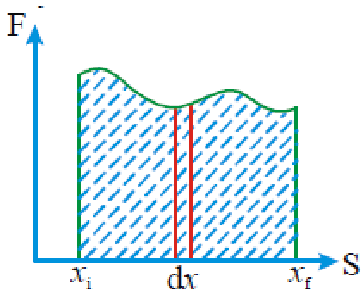
The area enclosed by the *F-S graph and displacement axis* gives the amount of work done by the force.

Graphical representation of work done by constant force:



Work = FS = Area of OPQR

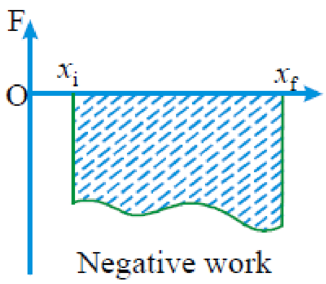
Graphical representation of work done by variable force:



For a small displacement dx the work done will be the area of the strip of width dx

$$\boldsymbol{W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F dx}$$

If area enclosed above X-axis, work done is +ve and if the area enclosed below X-axis, work done is –ve.



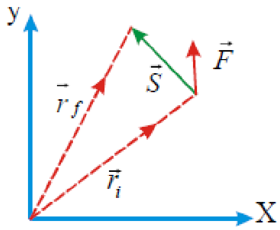
Applications

- If a force is changing linearly from F_1 to F_2 over a displacement S then work done is,

$$W = \left(\frac{F_1 + F_2}{2}\right) S$$

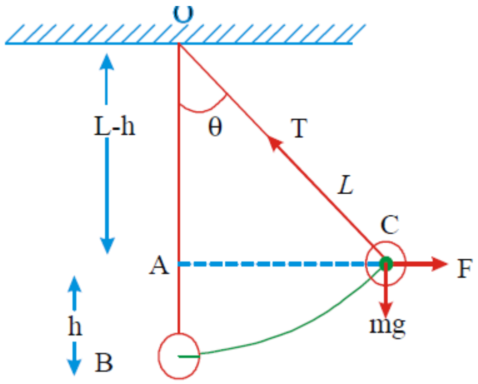
- If a force displaces the particle from its initial position $\vec{r_i}$ to final positon $\vec{r_f}$ then displacement vector is,

$$\vec{S} = \vec{r_f} - \vec{r_i}$$



$$W = \vec{F}.\vec{s} = \vec{F}.(\vec{r_f} - \vec{r_i})$$

- Work done in pulling the bob of mass m of a simple pendulum of length L through an angle θ to vertical by means of a horizontal force F.



$$\cos\theta = \frac{L-h}{L} = 1 - \frac{h}{L}$$

$$\boldsymbol{h = L(1 - \cos\theta)}$$

Work done by gravitational force,

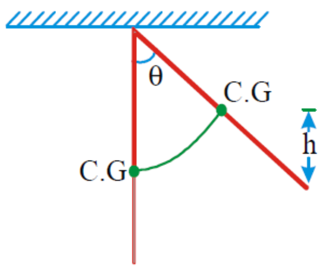
$$W = -mgh = -mgL(1 - \cos\theta)$$

Work done by horizontal force F is,

W = FL sin θ

Work done by tension T in the string is zero.

- Work done by gravitational force in pulling a uniform rod of mass m and length l through an angle θ is given by



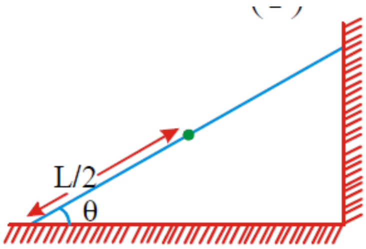
$$W = -mg\frac{l}{2}(1 - \cos\theta)$$

Where $\left(\frac{l}{2}\right)$ is the distance of centre of mass from the support.

- A ladder of mass 'm' and length 'L' resting on a level floor is lifted and held against a wall at an angle θ with the floor.

  Work done by gravitational force is

$$W_g = -mgh = -mg\left(\frac{L}{2}\right)\sin\theta$$



- A bucket full of water of total mass M is pulled by using a uniform rope of mass m and length l. Work done by pulling force

$$\boldsymbol{W = Mgl + mg\frac{l}{2}}$$

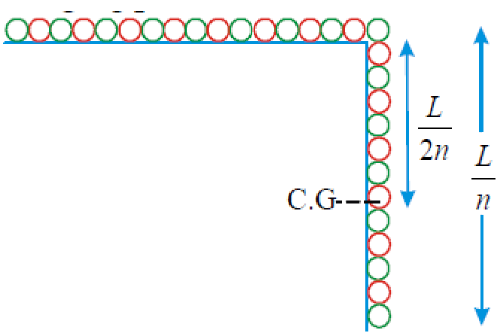
- A block of mass m is suspended vertically using a rope of negligible mass. If the rope is used to lift the block vertically up with uniform acceleration 'a', work done by tension in the rope is

$$\boldsymbol{W = -m(g+a)h} \qquad (h = \text{height})$$

  If block is lowered with acceleration 'a', then

$$\boldsymbol{W = -m(g-a)h}$$

- A uniform chain of mass M and length L is kept on a smooth horizontal table such that $\frac{1}{n^{th}}$ of its length is hanging *(l)* over the edge of the table. The work done by the pulling force to bring the hanging part (onto the table is



$$W = \left(\frac{M}{n}\right)gh = \left(\frac{M}{n}\right)g\left(\frac{L}{2n}\right)$$

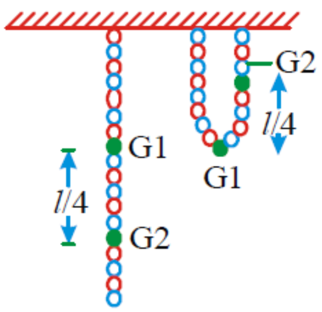
$$\therefore \boldsymbol{W = \frac{MgL}{2n^2}}$$

Mass of hanging part is $\frac{M}{n}$ and $n = \frac{L}{l}$

- A uniform chain of mass M and length L rests on a smooth horizontal table with $\frac{1}{n_1^{th}}$ part of its length is hanging from the edge of the table. Work done in pulling the chain partially such that $\frac{1}{n_2^{th}}$ part is hanging from the edge of the table is given by

$$\boldsymbol{W = \frac{MgL}{2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

- A uniform chain of mass 'M' and length L is suspended vertically. The lower end of the chain is lifted upto point of suspension.

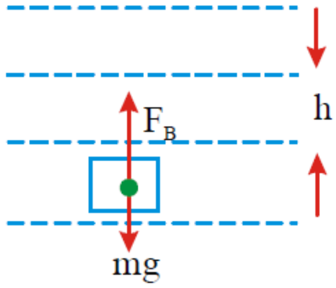


$h = \frac{L}{4} + \frac{L}{4} = \frac{L}{2}$ = rise in centre of mass of lower half of the chain.

Work done by gravitational force is

$$W_g = -\frac{M}{2}g\frac{L}{2} = -\frac{MgL}{4}$$

- The Work done in lifting a body of mass 'm' having density 'd_1' inside a liquid of density 'd_2' through a height 'h' is



$$W = mg'h = mgh\left[1 - \frac{d_2}{d_1}\right]$$

- A body of mass 'm' is placed on a frictionless horizontal surface. A force F acts on the body parallel to the surface such that it moves with an acceleration 'a', through a displacement 'S'. The work done by the force is

$$W = F\,S = ma\,S \qquad (\because \theta = 0^0)$$

- A body of mass 'm' is placed on a rough horizontal surface of coefficient of friction μ. A force F acts on the body parallel to the surface such that it moves with an acceleration 'a', through a displacement 'S'. The work done by the frictional force is,

$$f = \mu mg\cos\theta\,;\, but\ \theta = 0^0$$

$$\therefore f = \mu mg\cos\theta = \mu mg$$

$$\therefore W_f = \mu mgs$$

$$W_{net} = (f + ma)S = (\mu mg + ma)S$$

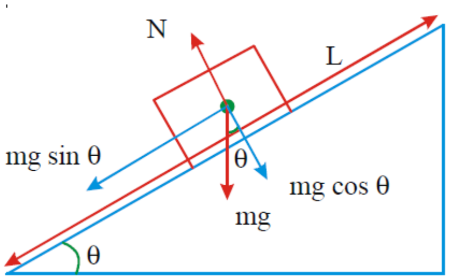
$$\therefore W = m(\mu g + a)S$$

If the body moves with uniform velocity then,

$$W = f\,S = \mu mgs$$

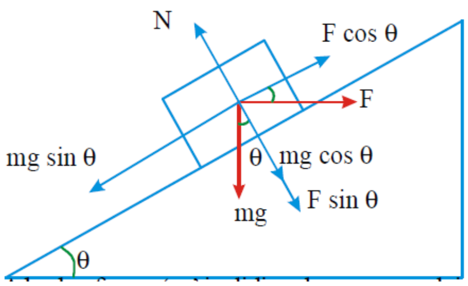
- A body of mass m is sliding down on a smooth inclined plane of inclination θ. If L is length of inclined plane, then work done by gravitational force is,

$$W = FS = mg\sin\theta\, L$$

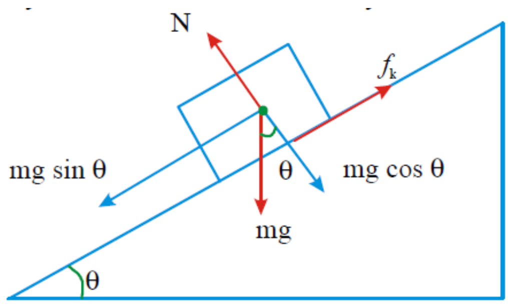


- A body of mass 'm' is moved up the smooth inclined plane of inclination θ and length L by a constant horizontal force F then work done by the resultant force is

$$W = (F\cos\theta - mg\sin\theta)L$$

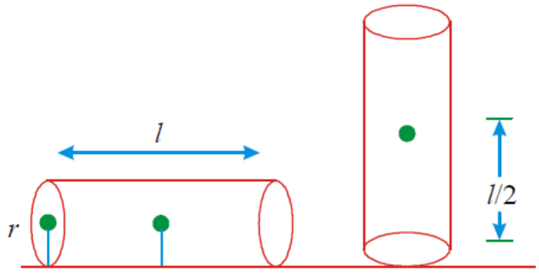


- A body of mass 'm' is sliding down on rough inclined plane of inclination θ. If L is the length of incline and μ_K is the coefficient of kinetic friction, then work done by the resultant force on the body is



$$W = (mg\sin\theta - f_k)L = (mg\sin\theta - \mu_k mg\cos\theta)L$$
$$= mgL(\sin\theta - \mu_k\cos\theta)$$

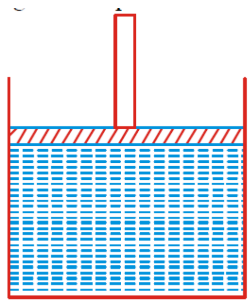
- A uniform solid cylinder of mass m, length l and radius r is lying on ground with curved surface in contact with ground. If it is turned such that its circular face is in contact with ground then work done by applied force is,



$$W = mgh = mg\left(\frac{l}{2} - r\right)$$

$$\left(\because h = \frac{l}{2} - r\right)$$

- A gas at a pressure P is enclosed in a cylinder with a movable piston. Work done by the gas in producing a small displacement dx of the piston is,

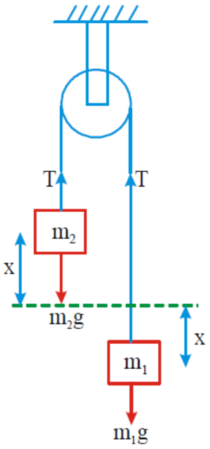


$$dW = Fdx = PA\,dx = PdV$$

Total work done by the gas during the change in its volume from V_1 to V_2 is,

$$W = \int_{V_1}^{V_2} PdV$$

- Two blocks of masses m_1 and m_2 ($m_1 > m_2$) connected by an inextensible string are passing over a smooth, massless pulley. The two blocks are released from the same level. At any instant 't', if 'x' is the displacement of each block then



Work done by gravity on block m_1 ,

$$W_1 = + m_1gx$$

Work done by gravity on block m_2 ,

$$W_2 = - m_2gx$$

Work done by gravitational force on the System,

$$W_g = m_1gx - m_2gx$$

$$W_g = (m_1 - m_2)gx = (m_1 - m_2)g\left(\frac{1}{2}at^2\right)$$

$$[\because v^2 - u^2 = 2as]$$

$$W_g = \frac{(m_1 - m_2)^2 g^2 t^2}{2(m_1 - m_2)}$$

$$\left[\because a = \frac{(m_1 - m_2)g}{m_1 + m_2}\right]$$

Note: In this case work done on the two blocks by tension is zero

$$W = T(x) + T(-x) = 0$$

Problem No 1 A body is displaced from $\vec{r}_A = (2\hat{\imath} + 4\hat{\jmath} - 6\hat{k})$ to $\vec{r}_B = (6\hat{\imath} - 4\hat{\jmath} + 2\hat{k})$ under a constant force $\vec{F} = (2\hat{\imath} + 3\hat{\jmath} - \hat{k})$. Find the work done.

Sol. Work done $W = \vec{F}.\vec{S}$; $W = \vec{F}.\left(\vec{r_B} - \vec{r_A}\right)$

$$W = \left(2\hat{i} + 3\hat{j} - \hat{k}\right).\left[\left(6\hat{i} - 4\hat{j} + 2\hat{k}\right) - \left(2\hat{i} + 4\hat{j} - 6\hat{k}\right)\right]$$

$$W = \left(2\hat{i} + 3\hat{j} - \hat{k}\right).\left(4\hat{i} - 8\hat{j} + 8\hat{k}\right)$$

W = 8 – 24 – 8 = -24 units

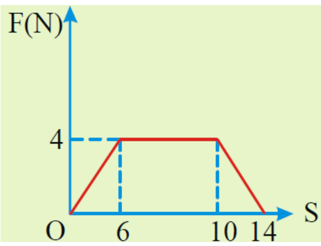
Problem No 2 A force $\vec{F} = 2x\hat{\imath} + 2\hat{\jmath} + 3z^2\hat{k}$ N is acting on a particle. Find the work done by the force in displacing the body from (1,2,3)m to (3,6,1)m.

Sol. Work done $W = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$

$$W = \int_1^3 2xdx + \int_2^6 2dy + \int_3^1 3z^2 dz$$

$$W = 2\left[\frac{x^2}{2}\right]_1^3 + 2[y]_2^6 + 3\left[\frac{z^3}{3}\right]_3^1 = -10\text{ J}$$

Problem No 3 The force acting on an object varies with the distance travelled by the object as shown in the figure. Find the work done by the force in moving the object from x = 0m to x = 14m



Sol. Work done = Area under F - S curve.

$$W = \left(\frac{1}{2} \times 6 \times 4\right) + (4 \times 4) + \left(\frac{1}{2} \times 4 \times 4\right) = 36\text{J}.$$

Problem No 4 When a rubber band is stretched by a distance 'x', it exerts a restoring force of magnitude $= ax + bx^2$, where a and b are constants. Find the work done in stretching the unstretched rubber band by 'L'.

Sol: The restoring force exerted by the rubber band when it is stretched by a distance 'x' is

$F = ax + bx^2$.

The small amount of work done on the rubber band in stretching through a small distance 'dx' is

$dW = Fdx = (ax + bx^2)dx$

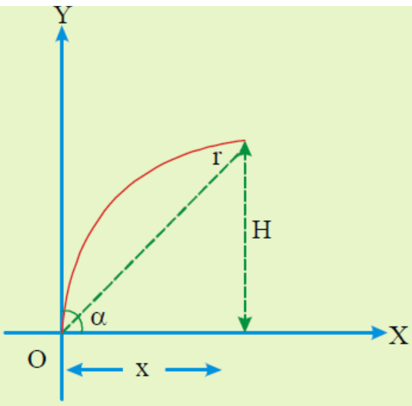
The total work done in stretching the unstretched rubber band by 'L' is

$$W = \int_0^L Fdx = \int_0^L (ax + bx^2)dx = \int_0^L axdx + \int_0^L bx^2 dx$$

$$W = a\left[\frac{x^2}{2}\right]_0^L + b\left[\frac{x^3}{3}\right]_0^L = \frac{aL^2}{2} + \frac{bL^3}{3}$$

Problem No 5 A particle of mass 'm' is projected at an angle α to the horizontal with an initial velocity u. Find the workdone by gravity during the time it reaches the highest point.

Sol:

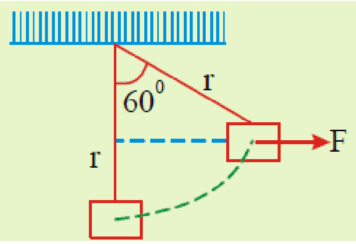


$$\vec{F}_y = -mg\,\hat{j};\ \ \vec{r}_y = \vec{H}_{max} = \left(\frac{u^2\sin^2\alpha}{2g}\right)\hat{j}$$

$$W = \vec{F}_y.\vec{r}_y = \left(-mg\hat{j}\right).\left(\frac{u^2\sin^2\alpha}{2g}\right)\hat{j}$$

$$W = -\frac{1}{2}mu^2\sin^2(\alpha)$$

Problem No 6 A 10 kg block is pulled along a frictionless surface in the form of an arc of a circle of radius 10 m. The applied force is 200 N. Find the work done by (a) applied force and (b) gravitational force in displacing through an angle 60^0



Sol: Work done by applied force $W = Fr\sin\theta$

$$W = 200\times10\times\sin60^0 = 200\times10\times\frac{\sqrt{3}}{2} = 1732\,J$$

work done by gravitational force

$$W = -mgr\left(1-\cos\theta\right)$$

$$W = -10\times9.8\times10\left(1-\cos60^0\right)$$

$$W = -98\times10\left(1-\frac{1}{2}\right) = -490\,J$$

Problem No 7 A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain back onto the table?

Sol: M = 4 kg, L = 2 m, l = 0.6 m, g = 10 m/s^2

$$\text{Work done } W = mg\frac{l}{2} = \left(\frac{M}{L}\right)l\,g\frac{l}{2}$$

$$W = \left(\frac{4}{2}\right)\times0.6\times10\times\frac{0.6}{2} = 3.6\,J$$

Problem No 8 Find the work done in lifting a body of mass 20 kg and specific gravity 3.2 to a height of 8 m in water? (g = 10 m/s^2)

Sol: Given specific gravity $\frac{\rho_b}{\rho_w} = 3.2$

$$\rho_b = 3.2\times\rho_w = 3.2\times1000 = 3200$$

$$\text{Workdone } W = mgh\left(1-\frac{\rho_w}{\rho_b}\right) = 20\times10\times8\left(1-\frac{1000}{3200}\right)$$

$$W = 20\times10\times8\left(\frac{2200}{3200}\right) = 1100\,J$$

Problem No 9 A block of mass 'm' is lowered with the help of a rope of negligible mass through a distance 'd' with an acceleration of g/3. Find the work done by the rope on the block?

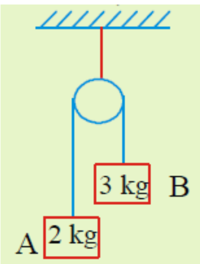
Sol: During lowering a block, tension in rope is

$T = m(g-a)$ and S = d

work done $W = -m(g-a)d$

$$W = -m\left(g-\frac{g}{3}\right)d = -\frac{2mgd}{3}$$

Problem No 10 If the system shown is released from rest. Find the net workdone by tension in first one second (g=10m/s^2)



Sol. $a = \left(\frac{m_2-m_1}{m_1+m_2}\right)g = \left(\frac{3-2}{2+3}\right)10 = 2\text{m/s}^2$

$$T = \frac{2m_1m_2g}{m_1+m_2} = \frac{2\times2\times3\times10}{2+3} = 24\text{N}$$

for each block $S = \frac{1}{2}at^2 = \frac{1}{2}\times2\times1 = 1m$

$$\therefore\ W_{net} = W_1 + W_2 = TS - TS = 0$$

Energy is the ability or capacity to do work. Greater the amount of energy possessed by the body, greater the work it will be able to do.

Energy is cause for doing work and work is effect of energy.

Energy is a scalar. Energy and work have same units and dimensions.

The different forms of energy are Mechanical energy, Light energy, Heat energy, Sound energy, Electrical energy, Nuclear energy etc.

Mechanical energy is of two types:

a) Potential Energy b) Kinetic Energy

Potential Energy (U)

Potential energy of a body is the energy possessed by a body by virtue of its position or configuration in the field.

Potential energy is defined only for conservative forces. It does not exist for non-conservative forces. In case of conservative forces,

$$\boldsymbol{F = -\left(\frac{dU}{dr}\right)}$$

$$\therefore dU = -\vec{F}.\overrightarrow{dr}$$

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}.\overrightarrow{dr}$$

$$U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}.\overrightarrow{dr} = -W$$

If $r_1 = \infty,\ U_1 = 0$

$$\therefore \boldsymbol{U = \int_{\infty}^{r} \vec{F}.\overrightarrow{dr} = -W}$$

P.E. can be +ve or -ve or can be zero.

P.E. depends on frame of reference.

Ex: Water stored in a dam, a stretched bow, a loaded spring etc., possesses P.E.

Three Dimensional formula for potential energy:

$$\therefore \vec{F} = -\left[\frac{dU}{dx}\hat{i} + \frac{dU}{dy}\hat{j} + \frac{dU}{dz}\hat{k}\right]$$

- In case of conservative force (field) potential energy is equal to negative of work done in shifting the body from some reference position to given position.

$$\therefore \boldsymbol{W = -\Delta U}$$

- A moving body may or may not have potential energy.
- Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.

Kinetic Energy

Kinetic energy is the energy possessed by a body by virtue of its motion.

Kinetic energy of a body of mass 'm' moving with a velocity 'v' is,

$$\boldsymbol{KE = \frac{1}{2}mv^2}$$

Kinetic energy is a scalar quantity.

The kinetic energy of an object is a measure of the work an object can do by the virtue of its motion.

Examples:

1) A vehicle in motion.
2) Water flowing in a river.
3) A bullet fired from a gun.

Kinetic energy depends on frame of reference.

Ex: kinetic energy of a person of mass m sitting in a train moving with speed v is zero in the frame of train but $\frac{1}{2}mv^2$ in the frame of earth.

Relation between K.E. and linear momentum

$$E = \frac{1}{2}mv^2 = \frac{P^2}{2m}$$

$$\therefore P = \sqrt{2mE}$$

- If two bodies of different masses have same momentum, then lighter body will have greater KE

  When a bullet is fired from a gun the momentum of the bullet and gun are equal and opposite.

$$\frac{KE_{bullet}}{KE_{gun}} = \frac{M_{gun}}{M_{bullet}}$$

  Hence, the KE of the bullet is greater than that of the gun.

- A body can have energy without momentum. But it cannot have momentum without energy.

- A bullet of mass 'm' moving with velocity 'v' stops in wooden block after penetrating through a distance 'x'. If F is resistance offered by the block to the bullet (Assuming F is constant inside the block),

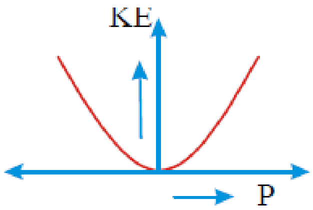
$$\frac{1}{2}mv^2 = Fx$$

$$F = \frac{mv^2}{2x}$$

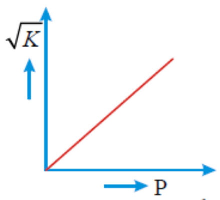
$$\therefore v^2 \propto x$$

For a given body:

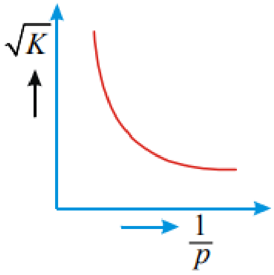
1) The graph between KE and P is a parabola.



2) The graph between $\sqrt{KE}$ and P is a straight line passing through the origin. Its slope $= \frac{1}{\sqrt{2m}}$



3) The graph between $\sqrt{KE}$ and $\frac{1}{P}$ is a rectangular hyperbola.

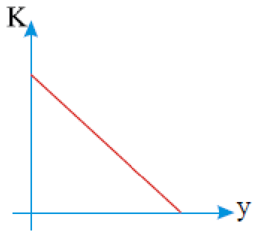


4) A particle is projected up from a point at an angle 'θ' with the horizontal. At any time 't' if 'P' is linear momentum, 'y' is vertical displacement and 'x' is horizontal displacement, then nature of the curves drawn for KE of the particle (K) against these parameters are,

i) K - y graph:

$K = K_i - mgy$;

It is a straight line

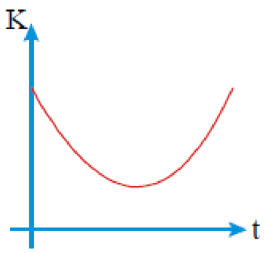


ii) K – t graph

$K = K_i - mg\left(u_y t - \frac{1}{2}gt^2\right)$

$\because y = u_y t - \frac{1}{2}gt^2$;

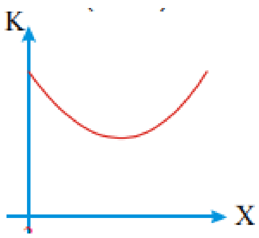
It is a parabola



iii) K – x graph

$K = K_i - mg\left(x\tan\theta - \frac{gx^2}{2u_x^2}\right) x^2$;

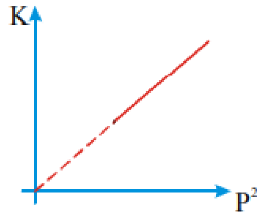
It is also a parabola



iv) K – P^2 graph

$$P^2 = 2mK$$

It is a straight line passing through origin and slope = $\frac{1}{2m}$



Conservative & Non - Conservative Forces

Conservative Force:

If work done by a force around a closed path is zero and is independent of path then the force is said to be conservative force.

Under conservative force,

$$F = -\frac{dU}{dr}$$

where U is Potential Energy.

$$U = -\int \vec{F}.\overrightarrow{dr}$$

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

and $\overrightarrow{dr} = dx\hat{\imath} + dy\,\hat{\jmath} + dz\hat{k}$

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\,\hat{\imath} + \frac{\partial U}{\partial y}\,\hat{\jmath} + \frac{\partial U}{\partial z}\,\hat{k}\right)$$

Ex: Gravitational force is a conservative force, Elastic force in a stretched spring is a conservative force

Non-Conservative Forces:

If the work done by a force around a closed path is not equal to zero and is dependent on the path, then the force is non-conservative force.

Ex:- Force of friction, Viscous force.

Work done by the non-conservative force will not be stored in the form of a Potential energy.

Potential energy is defined only for conservative forces.

Spring Force

Spring force is an example of a variable force which is a conservative.

In an ideal spring, the spring force is directly proportional to 'x'. Where x is the displacement of the block from equilibrium position. i.e.,

$$\boldsymbol{F_s = -Kx}$$

The constant K is called spring constant.

The work done on the block by the spring force as the block moves from undeformed position x = 0 to x = x_1

$$dW = \vec{F}.\overrightarrow{dx} = -Kxdx$$

$$W = \int dW = \int_0^{x_1} -Kxdx$$

$$\therefore W = -\frac{1}{2}K(x^2)_0^{x_1} = -\frac{1}{2}Kx_1^2$$

Potential energy stored in a spring:

The change in potential energy of a system corresponding to a conservative internal force is,

$$dU = -\int_0^{x_1} \vec{F}.\overrightarrow{dx}$$

$$dU = -(work\ done\ by\ the\ spring\ force)$$

$$dU = -\left(\frac{-Kx^2}{2}\right)$$

$$\therefore \Delta U = U_f - U_i = \frac{1}{2}Kx^2$$

since U_i is zero when spring is at its natural length,

$$\boldsymbol{\therefore U_f = \frac{1}{2}Kx^2}$$

Problem No 11 Two spheres whose radii are in the ratio 1:2 are moving with velocities in the ratio 3:4. If their densities are in the ratio 3:2, then find the ratio of their kinetic energies.

Sol. $\frac{r_1}{r_2} = \frac{1}{2}, \frac{v_1}{v_2} = \frac{3}{4}, \frac{\rho_1}{\rho_2} = \frac{3}{2}$

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}(V\rho)v^2 = \frac{1}{2}\left(\frac{4}{3}\pi r^3\rho\right)v^2$$

$$\frac{KE_1}{KE_2} = \frac{\rho_1}{\rho_2}\times\left(\frac{r_1}{r_2}\right)^3\times\left(\frac{v_1}{v_2}\right)^2 = \frac{3}{2}\times\left(\frac{1}{2}\right)^3\times\left(\frac{3}{4}\right)^2$$

$$\frac{KE_1}{KE_2} = \frac{3}{2}\times\frac{1}{8}\times\frac{9}{16} = \frac{27}{256}$$

Problem No 12 A particle is projected at 60^0 to the horizontal with a kinetic energy 'K'. Find the kinetic energy at the highest point?

Sol. Initial kinetic energy is $K = \frac{1}{2}mu^2$

The velocity at highest point $v_x = u\cos\theta$.

kinetic energy of a particle at highest point

$$K_H = \frac{1}{2}mv_x^2 = \frac{1}{2}mu^2\cos^2\theta = K\cos^2 60^0 = \frac{K}{4}$$

Problem No 13 An athlete in the Olympic games covers a distance of 100 m in 10s. His kinetic energy can be estimated to be in the range,

1) 200 J - 500 J

2) 2×10^5 J – 3×10^5 J

3) 20,000J - 50,000J

4) 2,000 J - 5, 000 J

Sol: Approximate mass of the athlete = 60 kg

Average velocity = 10 m/s.

Approximate $K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\times60\times10^2 = 3000J$

Range of KE = 2000 J to 5000J

Problem No 14 Kinetic energy of a particle moving along a circle of radius 'r' depends on the distance as KE = cs^2, (c is constant, s is displacement). Find the force acting on the particle.

Sol. KE = $\frac{1}{2}mv^2 = cs^2 \Rightarrow v = \left(\sqrt{\frac{2c}{m}}\right)s$

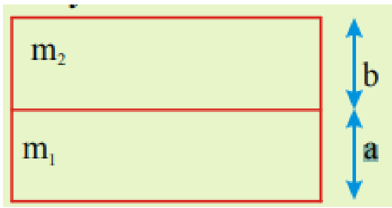
$$a_t = \frac{dv}{dt} = \sqrt{\frac{2c}{m}}\times\frac{ds}{dt} = v\sqrt{\frac{2c}{m}}$$

$$F_t = ma_t = mv\sqrt{\frac{2c}{m}} = \left[m\sqrt{\frac{2c}{m}}s\right]\sqrt{\frac{2c}{m}} = 2cs$$

Total force $F = \sqrt{F_t^2 + F_c^2} = \sqrt{(2cs)^2 + \left(\frac{mv^2}{r}\right)^2}$

$$F = 2cs\sqrt{1+\frac{s^2}{r^2}}$$

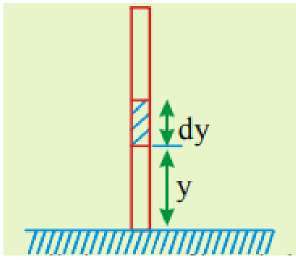
Problem No 15 A rectangular plank of mass m1 and height 'a' is on a horizontal surface. On the top of it another rectangular plank of mass m^2 and height 'b' is placed. Find the potential energy of the system?



Sol: Total potential energy of system $U = U_1 + U_2$

$$= m_1 g\frac{a}{2} + m_2 g\left(a+\frac{b}{2}\right) = \left[\left(\frac{m_1}{2}+m_2\right)a + m_2\left(\frac{b}{2}\right)\right]g$$

Problem No 16 A rod of mass m and length L is held vertical. Find its gravitational potential energy with respect to zero potential energy at the lower end?



Sol. Choose a small element of length dy, then

mass of the element $dm = \left(\frac{m}{L}\right)dy$.

The potential energy of the element dU= (dm)g(y)

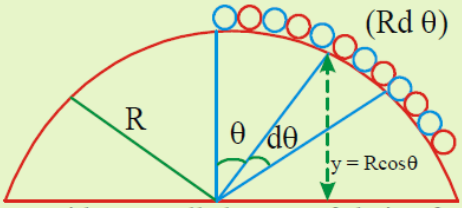
Potential energy of the entire rod

$$U = \int_0^L (dm)gy = \int_0^L \left(\frac{m}{L}\right)(dy).gy = \frac{m}{L}g\int_0^L ydy$$

$$U = \frac{m}{L}g\left(\frac{y^2}{2}\right)_0^L = \frac{mgL}{2}$$

Problem No 17 A chain of length l and mass 'm' lies on the surface of a smooth hemisphere of radius R > l with one end tied to top of the hemisphere. Find the gravitational potential energy of the chain?

Sol.



Consider a small element of chain of width $d\theta$ at an angle θ from the vertical

The mass of the element $dm = \left(\frac{m}{\ell}\right)Rd\theta$

The gravitational potential energy of the element du= (dm)gy

The gravitational potential energy of total chain

$$U = \int_0^{\frac{l}{R}} (dm)gy = \int_0^{\frac{l}{R}} \left(\frac{m}{l}Rd\theta\right)g(R\cos\theta)$$

$$U = \frac{mgR^2}{l}[\sin\theta]_0^{\frac{l}{R}} = \frac{mgR^2}{l}\sin\left(\frac{l}{R}\right)$$

Problem No 18 A spring of force constant 'k' is stretched by a small length 'x'. Find work done in stretching it further by a small length 'y'?

Sol: Initial potential energy $U_i = \frac{1}{2}kx^2$

Final potential energy $U_f = \frac{1}{2}k(x+y)^2$

Work done $W = U_f - U_i = \frac{1}{2}k(x+y)^2 - \frac{1}{2}kx^2$

$W = \frac{1}{2}ky(2x+y)$

Work – Energy Theorem

It states that, "work done by all the forces acting on a body is equal to change in its kinetic energy."

$$\boldsymbol{W = \Delta E = \frac{1}{2}m(v^2 - u^2)}$$

Work energy theorem is applicable not only for a single particle but also for a system of particles.

When it is applied to a system of two or more particles change in kinetic energy of the system is equal to work done on the system by the external as well as internal forces.

Work-energy theorem can also be applied to a system under the action of variable forces, pseudo forces, conservative as well as non-conservative forces.

Applications of work-energy theorem:

- A body of mass m starting from rest acquire a velocity 'v' due to constant force F. Neglecting air resistance,

  Work done = change in Kinetic energy

$$W = \frac{1}{2}mv^2$$

- A particle of mass 'm' is thrown vertically up with a speed 'u'. Neglecting the air friction, the work done by gravitational force, as particle reaches maximum height is,

$$W_g = \frac{1}{2}m(0)^2 - \frac{1}{2}m\times u^2 = -\frac{1}{2}mu^2$$

- A particle of mass 'm' falls freely from a height 'h' in air medium onto the ground. If 'v' is the velocity with which it reaches the ground, the work done by air friction is W_f and work done by gravitational force is W_g then,

$$W_g + W_f = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$$

- A block of mass 'm' slides down a frictionless incline of inclination 'θ' to the horizontal. If h is the height of incline, the velocity with which body reaches the bottom of incline is

$$W_g = \Delta K$$

$$mgh = \frac{1}{2}mv^2 - 0$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh}$$

- A body of mass 'm' starts from rest from the top of a rough inclined plane of inclination 'θ' and length 'l'. The velocity 'v' with which it reaches the bottom of incline if μ_k is the coefficient of kinetic friction is,

$$W_g + W_f = \Delta k$$

$$(mg\sin\theta)l + (-\mu_k mg\cos\theta)l = \frac{1}{2}mv^2 - 0$$

$$v = \sqrt{2gl(\sin\theta - \mu_k\cos\theta)}$$

- A bob of mass m suspended from a string of length l is given a speed u at its lowest position then the speed of the bob v when it makes an angle θ with the vertical is

$$W_g + W_T = \Delta K$$

$$-mgl\,(1-\cos\theta) + 0 = \frac{1}{2}m(v^2 - u^2)$$

$$v = \sqrt{u^2 - 2gl(l - \cos\ \theta)}$$

- A bullet of mass 'm' moving with velocity 'v' stops in a wooden block after penetrating through a distance x. If 'f' is the resistance offered by the block to the bullet.

$$W_f = K_f - K_i$$

$$-fx = 0 - KE_i$$

Therefore stopping distance is given by,

$$x = \frac{KE_i}{f} = \frac{mv^2}{2f} = \frac{P^2}{2mf}$$

- A block of mass 'm' attached to a spring of spring constant 'K' oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. It has a speed 'v' when the spring is at natural length. The distance it moves on a table before it comes to rest is calculated as below.

$$W_{S.F.} + W_g + W_N = \Delta K$$

(S.F. = spring force)

Let the mass be oscillating with amplitude 'x'.

On compressing the spring

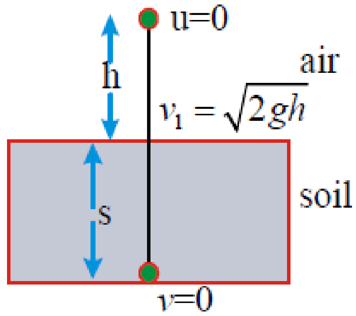
$$W_{S.F.} = -\frac{1}{2}Kx^2$$

$$W_{S.F.} = K_f - K_i$$

$$-\frac{1}{2}Kx^2 = 0 - \frac{1}{2}mv^2$$

$$\therefore x = v\sqrt{\frac{m}{K}}$$

- A pile driver of mass 'm' is dropped from a height 'h' above the ground. On reaching the ground it pierces through a distance 's' and then stops finally. If R is the average resistance offered by ground, then



$$W_g + W_R = K_f - K_i = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$Mg(h+s) + (-Rs) = 0;$$

$$R = mg\left(1 + \frac{h}{s}\right)$$

Here time of penetration is given by impulse equation

$$(R - mg)t = 0 - m\sqrt{2gh}$$

- A body of mass 'm' is initially at rest. By the application of a constant force, its velocity changes to v_0 in time t_0 the kinetic energy of the body at time 't' is

$$W = \Delta K = K_f - K_i = K_f - 0$$

$$K_f = W = mas = ma\left(\frac{1}{2}at^2\right) = ma^2t^2$$

Since a = $\frac{v_0}{t_0}$,

$$K_f = \frac{1}{2}m\left(\frac{v_0}{t_0}\right)^2 t^2$$

Problem No 19 Under the action of force 2kg body moves such that its position 'x' varies as a function of time t given by x = $\frac{t^3}{3}$, x is in metre and t in second. Calculate the workdone by the force in first two seconds.

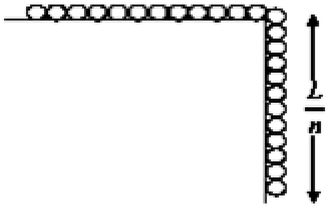
Sol. From work-energy theorem $W = \Delta KE$

$x = \frac{t^3}{3}$, Velocity $v = \frac{dx}{dt} = t^2$

At t = 0, v_1 =0, At t = 2s, v_2 = 4m/s

$$W = \frac{1}{2}m\left(v_2^2 - v_1^2\right) = \frac{1}{2}\times 2\left(4^2 - 0\right) = 16J$$

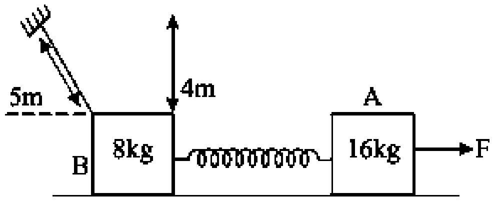
Problem No 20 A uniform chain of length 'l' and mass 'M' is on a smooth horizontal table, with (1/n)th part of its length hanging from the edge of the table. Find the kinetic energy of the chain as it completely slips off the table.



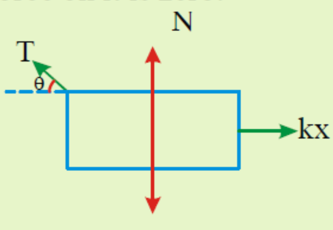
Sol: Work done $\Delta W = U_i - U_f = K_f - K_i$

$$\frac{Mgl}{2} - \frac{Mgl}{2n^2} = \frac{1}{2}Mv^2; \quad v = \sqrt{gl\left[1 - \frac{1}{n^2}\right]}$$

Problem No 21 Two blocks having masses 8 kg and 16kg are connected to the two ends of a light spring. The system is placed on a smooth horizontal floor. An inextensible string also connects B with ceiling as shown in figure at the initial moment. Initially the spring has its natural length. A constant horizontal force F is applied to the heavier block as shown. What is the maximum possible value of F so that lighter block doesn't lose contact with ground?



Sol: Draw FBD of B to get extension in spring. When block B just looses contact with ground resultant force on it is zero.



$$Kx - T\cos\theta = 0 \Rightarrow T = \frac{Kx}{\cos\theta}; \quad T\sin\theta + N - mg = 0$$

When N = 0 then $T\sin\theta = mg \Rightarrow \frac{Kx}{\cos\theta}\sin\theta = mg$

$$x = \frac{mg}{K\tan\theta} = \frac{80}{K\times(4/3)} = \frac{60}{K}$$

If spring has to just extend till this value then from work energy theorem we get

$$Fx = \frac{1}{2}Kx^2 \Rightarrow F = 30N$$

Problem No 22 A 2kg block slides on a horizontal floor with a speed of 4 m/s. It strikes an uncompressed spring and compresses it till the block is motionless. The kinetic frictional force is 15 N and spring constant is 10,000 Nm-1. Find the compression in the spring?

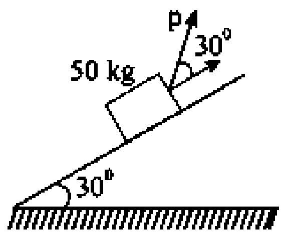
Sol: $KE = \frac{1}{2}mv^2 = W_{friction} + \frac{1}{2}Kx^2$

$$\Rightarrow \frac{1}{2}\times 2\times 4^2 = 15x + \frac{1}{2}\times 10000\times x^2$$

$$\Rightarrow 5000x^2 + 15x - 16 = 0$$

$$\Rightarrow x = 0.055\,m \text{ or } x = 5.5\,cm$$

Problem No 23 In the below figure, what constant force 'P' is required to bring the 50kg body, which starts from rest to a velocity of 10m/s in moving 7m along the plane? (Neglect friction)



Sol. Work done by force P in displacing the block by 7m, $W_1 = (F\cos\theta)(S)$

$$W_1 = \left(P\cos 30^0\right)7 = \frac{7\sqrt{3}}{2}P\,J$$

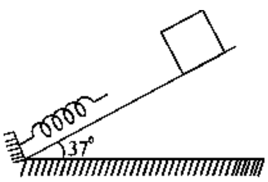
$$W_2 = -mgh = -50\times 9.8\times 7\sin 30^0 = -1715\,J$$

According to work energy theorem

$$W_1 + W_2 = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$\frac{7\sqrt{3}}{2}P - 1715 = \frac{1}{2}\times 50\times\left(10^2 - 0^2\right) \Rightarrow P = 607\,N$$

Problem No 24 Figure shows a spring fixed at the bottom end of an incline of inclination 37⁰. A small block of mass 2 kg starts slipping down the incline from a point 4.8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance 1 m up the incline. Find (i) the friction coefficient between the plane and the block and (ii) the spring constant of the spring. (g = 10 ms-2)



Sol: Applying work energy theorem for downward motion of the body $W = \Delta KE$

$$mg\sin\theta(x+d) - f \times l_1 - \frac{1}{2}Kx^2 = \Delta KE$$

$$20\sin37^0(5) - \mu \times 20\cos37^0 \times 5 - \frac{1}{2}K(0.2)^2 = 0$$

$$80\mu + 0.02K = 60 \rightarrow (1)$$

For the upward motion of the body

$$-mg\sin\theta\, l_2 + (f \times l_2) + \frac{1}{2}Kx^2 = \Delta KE$$

$$-2\times10\sin37^0 \times 1 - \mu \times 20\cos37^0 \times 1 + \frac{1}{2}K(0.2)^2 = 0$$

$$16\mu - 0.02K = -12 \rightarrow (2)$$

Adding equations (1) and (2), we get

$$96\mu = 48 \Rightarrow \mu = 0.5$$

Now, use the value of μ in equation (1), we get K = 1000 N/m.

Types of Equilibrium

A body is said to be in translatory equilibrium, if *net force* acting on the body is *zero* i.e.,

$$\vec{F}_{net} = 0$$

If the forces are conservative,

$$F = -\frac{dU}{dr}$$

and for equilibrium F = 0,

So,

$$\frac{dU}{dr} = 0$$

Therefore, at equilibrium position, slope of U-r graph is zero or the potential energy is optimum (maximum or minimum or constant).

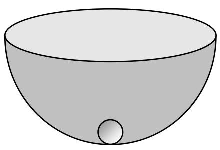
Based on this, there are three types of equilibrium

(i) Stable equilibrium

(ii) Unstable equilibrium

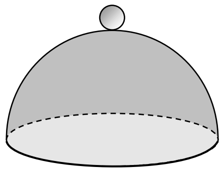
(iii) Neutral equilibrium.

Stable Equilibrium:



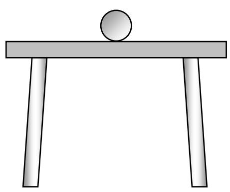
- Net force is Zero.
- $\frac{dU}{dr} = 0$ or slope of U-r graph is zero.
- When displaced from its equilibrium position, a net retarding forces start acting on the body, which has a tendency to bring the body *back to its equilibrium position.*
- PE in this equilibrium position is *minimum* as compared to its neighbouring points as $\left(\frac{d^2U}{dr^2}\right)$ is positive.
- When displaced from equilibrium position, the centre of gravity of the body comes down.

Unstable Equilibrium:



- Net force is zero
- $\frac{dU}{dr} = 0$ or slope of U-r graph is zero.
- When displaced from its equilibrium position, a net force starts acting on the body which moves the body in the direction of displacement or *away from the equilibrium position.*
- PE in equilibrium position is *maximum* as compared to other positions as $\left(\frac{d^2U}{dr^2}\right)$ is negative.
- When displaced from equilibrium position, the centre of gravity of the body goes up.

Neutral Equilibrium:

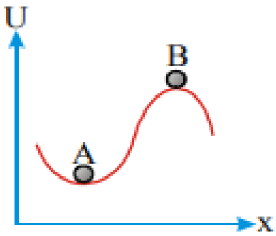


- Net force is zero.
- $\frac{dU}{dr} = 0$ or slope of U-r graph is zero.
- When displaced from its equilibrium

position the body has *neither the tendency to come back nor move away from the original position.*

- PE remains constant even if the body is moving to neighbouring points, $\frac{d^2U}{dr^2} = 0$
- When displaced from equilibrium position the centre of gravity of the body remains constant.

Potential energy and Equilibrium



In the figure, at A :

$$\frac{dU}{dr} = 0 \quad \text{and} \quad \frac{d^2U}{dr^2} > 0$$

Thus at A the particle is in stable equillibrium.

At B;

$$\frac{dU}{dr} = 0 \quad \text{and} \quad \frac{d^2U}{dr^2} < 0$$

Thus at B the particle is in unstable equilibrium.

Problem No 25 In a molecule, the potential energy between two atoms is given by U(x) = $\frac{a}{x^{12}} - \frac{b}{x^6}$. Where 'a' and 'b' are positive constants and 'x' is the distance between atoms. Find the value of 'x' at which force is zero and minimum P.E at that point.

Sol: Force is zero $\Rightarrow \frac{dU}{dx} = 0$

i.e., $a(-12)x^{-13} - b(-6)x^{-7} = 0$

$$\frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0 \Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7}$$

$$\Rightarrow x^6 = \frac{2a}{b} \quad \therefore x = \left(\frac{2a}{b}\right)^{\frac{1}{6}}$$

Substituting the value of x

$$\Rightarrow U_{\min} = a\left(\frac{b}{2a}\right)^{\frac{12}{6}} - b\left(\frac{b}{2a}\right)^{\frac{6}{6}}$$

$$U_{\min} = \left(\frac{b^2}{4a}\right) - \left(\frac{b^2}{2a}\right) \Rightarrow U_{\min} = \frac{-b^2}{4a}$$

Law of Conservation of Mechanical Energy

Total mechanical energy of a system remains constant, if only conservative forces are acting on a system of particles and the work done by all other forces is zero.

$$\therefore U_f - U_i = -W$$

From work energy theorem,

$$W = K_f = k_i$$

$$\therefore U_f - U_i = -(k_f - k_i)$$

$$\therefore U_f + k_f = U_i + k_i$$

$$\boldsymbol{U + K = constant}$$

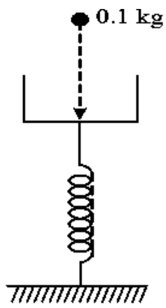
The sum of potential energy and kinetic energy remains constant in any state.

- A body is projected vertically up from the ground. When it is at height '*h*' above the ground, its potential and kinetic energies are in the ratio x : y. If H is the maximum height reached by the body, then

$$\frac{x}{y} = \frac{h}{H-h}$$

$$\therefore \frac{h}{H} = \frac{x}{x+y}$$

Problem No 26 A massless platform is kept on a light elastic spring as shown in figure. When a sand particle of 0.1kg mass is dropped on the pan from a height of 0.24m, the particle strikes the pan and the spring compresses by 0.01m. From what height should particle be dropped to cause a compression of 0.04m.



Sol. By conservation of mechanical energy

$$mg(h+y) = \frac{1}{2}Ky^2$$

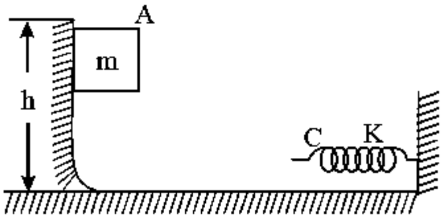
h = height of particle

y = compression of the spring

As here particle and spring remain same

$$\frac{h_1 + y_1}{h_2 + y_2} = \left(\frac{y_1}{y_2}\right)^2 ; \frac{0.24+0.01}{h_2+0.04} = \left(\frac{0.01}{0.04}\right)^2 ; \; h_2 = 3.96\,\text{m}$$

Problem No 27 A small mass 'm' is sliding down on a smooth curved incline from a height 'h' and finally moves through a horizontal smooth surface. A light spring of force constant K is fixed with a vertical rigid stand on the horizontal surface, as shown in the figure. Find the value for the maximum compression in the spring if mass 'm' is released from rest from height 'h' and hits the spring on the horizontal surface.



Sol. Conservation of energy b/w positions A and C

$$(PE_A)_{block} + KE_A = (PE_C)_{spring} + KE_C$$

$$mgh + 0 = \frac{1}{2}Kx^2 + 0 \,; mgh = \frac{1}{2}Kx^2 \,; x = \sqrt{\frac{2mgh}{K}}$$

Problem No 28 A vehicle of mass 15 quintal climbs up a hill 200m high. It then moves on a level road with a speed of 30 ms^{-1}. Calculate the potential energy gained by it and its total mechanical energy while running on the top of the hill.

Sol. m = 15 quintal = 1500kg, g = $9.8ms^{-2}$, h = 200m

P.E.gained, U = mgh = 1500 x 9.8 x 200 = 2.94 x 10^6 J

K.E. = $\frac{1}{2}mv^2 = \frac{1}{2}$ x 1500 x $(30)^2$ = 0.675 x 10^6J

Total mechanical energy

E = K + U = (0.675 + 2.94) x 10^6 = 3.615 x 10^6J

Problem No 29 A particle is released from height H. At certain height from the ground its kinetic energy is twice its gravitational potential energy. Find the height and speed of particle at that height.

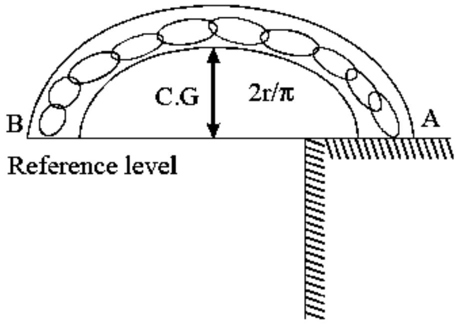
Sol. K.E = 2PE But KE = TE – PE

mg(H – h) = 2mgh; mgH = 3mgh

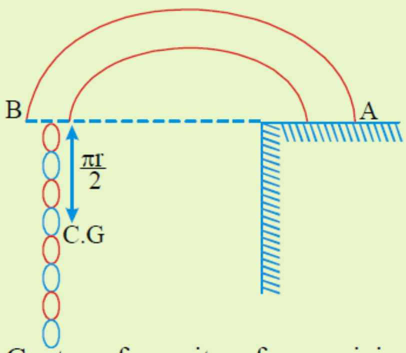
$\Rightarrow h = \frac{H}{3}$; Also K.E = 2P.E ,

$$\frac{1}{2}mv^2 = 2mg\left(\frac{H}{3}\right) \Rightarrow v = 2\sqrt{\frac{gH}{3}}$$

Problem No 30 A heavy flexible uniform chain of length π r and mass λπ r lies in a smooth semicircular tube AB of radius 'r'. Assuming a slight disturbance to start the chain in motion, find the velocity v with which it will emerge from the end of the tube?



Sol:



Centre of gravity of a semicircular arc is at a distance $\frac{2r}{\pi}$ from the centre .

Initial potential energy $U_i = (\lambda\pi r)g\left(\frac{2r}{\pi}\right)$

Final potential energy $U_f = (\lambda\pi r)g\left(\frac{-\pi r}{2}\right)$

When the chain is completely slipped off the tube, all the links of the chain have the same velocity v.

kinetic energy of chain $k = \frac{1}{2}mv^2 = \frac{1}{2}(\lambda\pi r)v^2$

From conservation of energy ,

$$\lambda\pi rg\left(\frac{2r}{\pi}\right) = (\lambda\pi r)g\left(\frac{-\pi r}{2}\right) + \frac{1}{2}(\lambda\pi r)v^2$$

On solving we get, $v = \sqrt{2rg\left(\frac{2}{\pi} + \frac{\pi}{2}\right)}$

Problem No 31 The potential energy of 1 kg particle free to move along X - axis is given by U (x) = $\left(\frac{x^4}{4} - \frac{x^2}{2}\right)$ J. The total mechanical energy of the particle is 2 J. Find the maximum speed of the particle.

Sol: For maximum value of U, $\frac{dU}{dx}=0$.

$$\therefore \frac{4x^3}{4}-\frac{2x}{2}=0 \text{ or } x=0,\ x=\pm 1.$$

$$\text{At } x=0,\ \frac{d^2U}{dx^2}=-1 \text{ and At } x=\pm 1,\ \frac{d^2U}{dx^2}=2$$

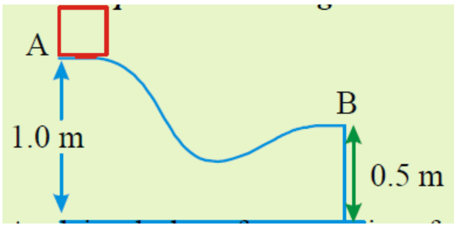
Hence U is minimum at $x=\pm 1$ with value

$$U_{\min}=\frac{1}{4}-\frac{1}{2}=-\frac{1}{4}J$$

$$K_{\max}+U_{\min}=E \text{ or } K_{\max}-\frac{1}{4}=2 \text{ or } K_{\max}=\frac{9}{4}$$

$$\Rightarrow \frac{1}{2}mv^2=\frac{9}{4} \quad \Rightarrow v_{\max}=\frac{3}{\sqrt{2}}ms^{-1}$$

Problem No 32 Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground?



Sol: Applying the law of conservation of mechanical energy for the points A and B,

$$mgH=\frac{1}{2}mv^2+mgh$$

$$g-\frac{v^2}{2}=\frac{g}{2} \text{ or } v^2=g \Rightarrow v=\sqrt{g}=3.1ms^{-1}$$

After point B the particle exhibits projectile motion with $\theta=0^0$ and $y=-0.5$m

Horizontal distance travelled by the body

$$R=u\sqrt{\frac{2h}{g}}=3.1\times\sqrt{\frac{2\times 0.5}{9.8}}=1m$$

POWER

Introduction

The rate of doing work is called power.

Power or average power is given by

$$P_{avg}=\frac{work\ done}{time}$$

$$\therefore \boldsymbol{P_{avg}=\frac{W}{t}}$$

Power is a scalar.

SI Unit: watt (W) (or) J/s,

CGS Unit : erg/sec

Other Units : kilo watt, mega watt and horse power

One horse power (hp) = 746 watt

Kilowatt-hour is the commercial unit of electric energy.

$$1\,KWh=10^3\frac{J}{sec}\times(60\times 60sec)=3.6\times 10^6\ Joule$$

Instantaneous Power

$$P=\frac{dW}{dt}$$

It is also calculated by,

$$\boldsymbol{P=\vec{F}\cdot\vec{v}=Fv\cos\theta}$$

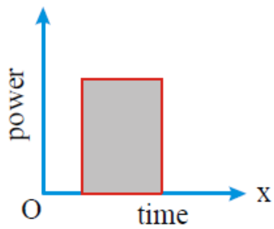
Relation between P_{avg} and P_{ins} :

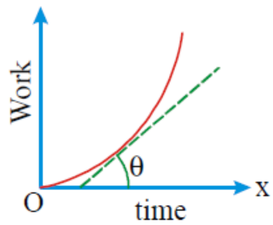
$$P_{avg}=\frac{W}{t}=\frac{mv^2}{2t}=\frac{1}{2}mv\left(\frac{v}{t}\right)=\frac{1}{2}mav$$

$$\therefore P_{avg}=\frac{1}{2}\vec{F}\cdot\vec{v}$$

$$\therefore \boldsymbol{P_{avg}=\frac{1}{2}P_{inst}}$$

The area under P – t graph gives work done.





$$P=\frac{dW}{dt}$$

$$\therefore \boldsymbol{W=\int_{t_1}^{t_2}P\,dt}$$

The slope of W-t curve gives instantaneous power

$$P = \frac{dW}{dt} = tan\theta$$

Applications on Power

- The power of a machine gun firing 'n' bullets each of mass 'm' with a velocity 'v' in a time interval 't' is given by

$$P = \frac{n\left(\frac{1}{2}mv^2\right)}{t} = \frac{nmv}{2t}$$

- A crane lifts a body of mass 'm' with a constant velocity v from the ground, its power is

$$P = Fv = mgv$$

- Power of lungs of a boy blowing a whistle is,

$$P = \frac{1}{2}\left(\begin{matrix} mass\ of\ air \\ blown\ per\ sec \end{matrix}\right)(velocity)^2$$

- Power of a heart pumping blood = (pressure) (volume of blood pumped per sec)
- A conveyor belt is moving with a constant speed 'v' horizontally and gravel is falling on it at a rate of $\frac{dm}{dt}$. Then additional force required to maintain speed v is F = v $\frac{dm}{dt}$ and additional power required to drive the belt is,

$$P = Fv = v^2 \frac{dm}{dt}$$

- When a liquid of density 'ρ' coming out of a hose pipe of area of cross section 'A' with a velocity 'v' strikes the wall normally and stops dead. Then power exerted by the liquid is

$$P = \frac{1}{2}\frac{mv^2}{t} = \frac{1}{2}Av^3$$

(mass = density × volume =$m = \rho \times A \times l$)

- A vehicle of mass 'm' is driven with constant acceleration along a straight level road against a constant external resistance 'R' when the velocity is 'v', power of engine is,

$$P = F\,v = (R + ma)\,v$$

- Efficiency (η) of a device is given by,

$$\eta = \frac{P_{in}}{P_{out}}$$

Where, P_{in} = Input power (rated power)

P_{out} = Output power (useful power)

- If a motor lifts water from a well of depth 'h' and delivers with a velocity 'v' in a time t then power of the motor is,

$$P = \frac{mgh + \frac{1}{2}mv^2}{t}$$

- Position and velocity of an automobile w. r. t. time:

An automobile of mass 'm' accelerates starting from rest, while the engine supplies constant power, its position and velocity changes w.r.t time as,

Velocity : As Fv = P = constant

i.e. $m\frac{dv}{dt} v = P$

or $\int v\, dv = \int \frac{P}{m} dt$

on integrating we get,

$$\frac{v^2}{2} = \frac{P}{m} t + C_1$$

As initially the body is at rest,

$$v = 0 \text{ at } t = 0 \Rightarrow C_1 = 0;$$

$$v = \sqrt{\frac{2Pt}{m}}$$

$$v \propto \sqrt{t}$$

Position: From the above expression

$$v = \left(\frac{2Pt}{m}\right)^{1/2}$$

(or) $\frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$

$$\int ds = \int \left(\frac{2Pt}{m}\right)^{1/2} dt$$

$$\int ds = \sqrt{\frac{2P}{m}} \int \sqrt{t}\, dt$$

Integrating on both sides we get,

$$S = \sqrt{\frac{2P}{m}}\left(\frac{2}{3}t^{3/2}\right) + C_2$$

Now at t = 0, S = 0 $\Rightarrow C_2 = 0$

$$S = \sqrt{\frac{8P}{9m}}\, t^{3/2}$$

$$\therefore S \propto t^{3/2}$$

Problem No 33 An automobile is moving at 100 kmph and is exerting attractive force of 3920 N. What horse power must the engine develop, if 20 % of the power developed is wasted?

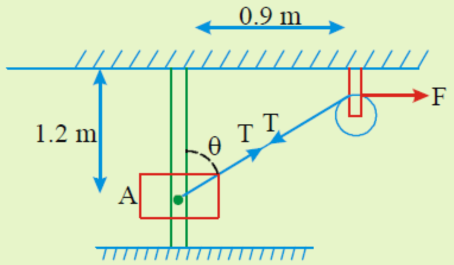
Sol : Velocity $= 100\,\text{kmph} = 100 \times \frac{5}{18}$ m/s

Force = 3920 N;;Useful power = 80%

$$\text{Power} = \frac{W}{t} = \frac{F.S}{t} = F.v \Rightarrow \frac{80}{100} P = 3920 \times 100 \times \frac{5}{18}$$

$$P = \frac{100}{80} \times 3920 \times 100 \times \frac{5}{18} = 13.16 \times 10^4 W = 182.5\text{hp}$$

Problem No 34 The 50 N collar starts from rest at A and is lifted with a constant speed of 0.6 m/s along the smooth rod. Determine the power developed by the force F at the instant shown.



Sol: Since the collar is lifted with a constant speed

$$T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg = 5 \times 10$$

Now, $P = \vec{F}.\vec{v} = T\cos\theta \times v$; Here $T = F$

$$P = 50 \times v = 50 \times 0.6 = 30W$$

Problem No 35 A machine delivers power to a body which is directly proportional to velocity of the body. If the body starts with a velocity which is almost negligible, find the distance covered by the body in attaining a velocity v.

Sol. Power $P = Fv\cos 0 = Fv = m\left(\frac{dv}{dt}\right)v \propto v$

$mv\frac{dv}{dt} = K_0 v$, Where K_0 =constant

$m\frac{dv}{dt} = K_0$; $m\left(\frac{dv}{dx}\right)\frac{dx}{dt} = K_0$

$mv\frac{dv}{dx} = K_0$; $vdv = \left(\frac{K_0}{m}\right)dx$

Integrating $\int_0^v vdv = \int_0^x \left(\frac{K_0}{m}\right)dx$;

$$\frac{v^2}{2} = \left(\frac{K_0}{m}\right)x \Rightarrow x = \frac{1}{2}\frac{mv^2}{K_0}$$

Problem No 36 Find the power of an engine which can draw a train of 400 metric ton up the inclined plane of 1 in 98 at the rate 10 ms^{-1}. The resistance due to friction acting on the train is 10 N per ton.

Sol. Given $\sin\theta = \frac{1}{98}$; $m = 400 \times 10^3$ kg

frictional force $f = 10 \times 400 = 4000$ N ;

velocity $v = 10\,\text{ms}^{-1}$

$\therefore$ Power $P = (mg\sin\theta + f)\,v$

$$\therefore P = \left[\left(400 \times 10^3 \times 9.8 \times \frac{1}{98}\right) + 4000\right] \times 10$$

= 440000W=440KW

Problem No 37 A hose pipe has a diameter of 2.5cm and is required to direct a jet of water to a height of at least 40m. Find the minimum power of the pump needed for this hose.

Sol. Volume of water ejected per sec

$$Av = \pi\left(\frac{d}{2}\right)^2 \times \sqrt{2gh}\ m^3/s;\ \therefore v = \sqrt{2gh}$$

Mass ejected per sec is $M = \frac{1}{4}\pi d^2 \times \sqrt{2gh}\,\rho$ Kg/s

Kinetic energy of water leaving hose / sec

$$K.E = \frac{1}{2}mv^2 = \frac{1}{8}\pi d^2 \times (2gh)^{\frac{3}{2}} \times \rho$$

$$= \frac{1}{8} \times 3.14 \times \left(2.5 \times 10^{-2}\right)^2 \times (2 \times 9.8 \times 40)^{\frac{3}{2}} \times 1000 = 21.5\text{KJ}$$

Problem No 38 A body of mass m accelerates uniformly from rest to velocity v0 in time t0 , find the instantaneous power delivered to body when velocity is $\frac{v_0}{2}$.

Sol. Acceleration $a = \frac{v_0}{t_0}$; Force $F = \frac{mv_0}{t_0}$

Instantaneous power $P = F.\frac{v_0}{2} = \left(\frac{mv_0}{t_0}\right)\frac{v_0}{2} = \frac{mv_0^2}{2t_0}$

COLLISION

Introduction

The strong interaction among bodies involving exchange of momentum in a short interval of time is called collision.

During collision bodies may or may not come into physical contact.

Ex: In the collision of a particle with nucleus, due to coulombic repulsive forces a particle is scattered away without any physical contact.

Based on the direction of motion of colliding bodies, collisions are classified into:

(i) Head on or one dimensional collision

(ii) oblique collision

Head on (or) one dimensional collision:

It is the collision in which the velocities of the colliding bodies are confined to same straight line before and after collision.

Oblique Collision:

It is the collision in which the velocities of the colliding bodies are not confined to same straight line before and after collision.

Oblique collision may be two dimensional or three dimensional.

Types of Collision

Based on conservation of kinetic energy collisions are classified into:

(i) Elastic Collision

(ii) Inelastic collision

Elastic Collision:

It is the collision in which both momentum and kinetic energy are conserved.

Forces involved during collision are conservative in nature.

Ex. 1. Collision between atomic particles.

2. Collision between two smooth billiard balls.

3. Collision of a particle with nucleus.

Inelastic collision:

It is the collision in which momentum is conserved but not kinetic energy. Some or all the forces involved during collision are non-conservative.

Ex: Collision between two vehicles.

Perfectly inelastic collision:

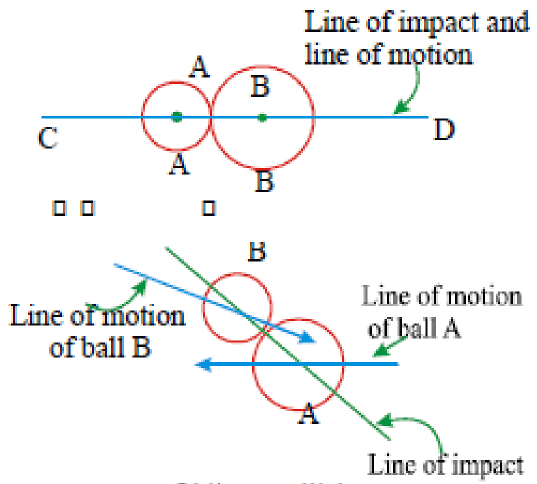
It is the collision in which the colliding bodies stick together and move as a single body after collision.

In perfectly inelastic collision the momentum remains conserved but the loss of kinetic energy is maximum.

Ex: A bullet is fired into a wooden block and remains embedded in it.

Line of impact:

The line which is perpendicular to common tangent to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both bodies.

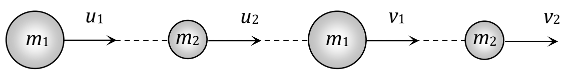


Oblique collision

Head on *Elastic* Collision

Let two bodies of masses m_1 and m_2 moving with initial velocities u_1 and u_2 in the same direction and they collide such that after collision their final velocities are v_1 and v_2 respectively.

Then,



Before collision After collision

Initial momentum = Final momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Their final velocities are given by,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \frac{2m_2u_2}{m_1 + m_2}$$

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2 + \frac{2m_1u_1}{m_1 + m_2}$$

Special cases:

1) If colliding particles have equal masses

$$m_1 = m_2 = m;$$

$$\vec{V_1} = \vec{u_2}\ ,\ \vec{V_2} = \vec{u_1}$$

2) If two bodies are of equal masses and the second body is at rest,

$$m_1 = m_2 = m \quad \text{and} \quad \vec{u_2} = \vec{0}$$

Then $\vec{v_1} = \vec{0}$; $\vec{v_2} = \vec{u_1}$

3) A lighter particle collides with heavier particle which is at rest.

$$m_1 << m_2, \quad \vec{v_2} = 0$$

$$\vec{v_1} = -\vec{u_1}, \qquad \vec{v_2} = 0$$

4) A heavier body collides with lighter body at rest $m_1 >> m_2$, $\vec{u_2} = \vec{0}$

$$\vec{v_1} = \vec{u_1} \quad , \quad \vec{v_2} = 2\vec{u_1}$$

Applications

- A body of mass m_1 moving with a velocity v_1 collides elastically with a stationary mass m_2,

1) Velocity of first body after collision

$$\vec{v_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{u_1}$$

2) Velocity of second body after collision

$$\vec{v_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{u_1}$$

3) KE of first body after collision (or) KE retained by first body,

$$\text{K.}E_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 u_1^2$$

$$\text{K.}E_{ret} = \frac{1}{2}m_1u_1^2\left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2$$

$$\boldsymbol{KE_{ret} = KE_i\left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2}$$

4) Fraction of KE retained by 1st body

$$\boldsymbol{\frac{KE_{ret}}{KE_i} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right]^2}$$

5) KE of second body after collision (or) KE transferred to the second body

$$KE_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2\left(\frac{2m_1}{m_1 + m_2}\right)^2 u_1^2$$

$$KE_2 = \left(\frac{4m_1m_2}{m_1 + m_2}\right)\left(\frac{1}{2}m_1u_1^2\right)$$

$$\boldsymbol{KE_{tra} = \left[\frac{4m_1m_2}{(m_1 + m_2)^2}\right]KE_i}$$

6) Fraction of KE transferred from 1st body to second body (or) Fraction of KE lost by 1st body is

$$\boldsymbol{\frac{KE_{tra}}{KE_i} = \frac{4m_1m_2}{(m_1 + m_2)^2}}$$

Problem No 42 A bullet of mass 'm' moving at a speed 'v' hits a ball of mass 'M' kept at rest. A small part having mass 1 m breaks from the ball and sticks to the bullet. The remaining ball is found to move at a speed 2 v in the direction of the bullet. Find the velocity of the bullet after the collision.

Sol: Mass of bullet = m and speed = v.

Mass of the ball M and fractional mass of the ball m_1

According to law of conservation of linear momentum

$$mv + 0 = (m + m_1)v_1 + (M - m_1)v_2$$

Where v_1 = final velocity of the (bullet + fractional mass)

$$v_1 = \frac{mv - (M - m_1)v_2}{(m + m_1)}$$

Problem No 43 Two bodies of masses m_1 and m_2 are moving with velocities 1ms^{-1} and 3ms^{-1} respectively in opposite directions. If the bodies undergo one dimensional elastic collision, the body of mass m_1 comes to rest. Find the ratio of m_1 and m_2

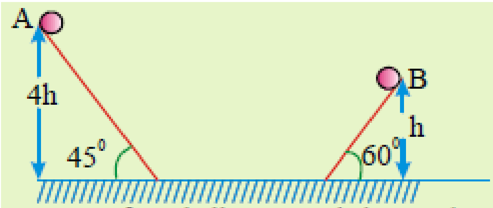
Sol. $u_1 = 1m/s, u_2 = -3m/s, v_1 = 0$

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$0 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)1 + \left(\frac{2m_2}{m_1 + m_2}\right)(-3)$$

$$m_1 - m_2 = 6m_2;\ m_1 = 7m_2;\ \frac{m_1}{m_2} = \frac{7}{1}$$

Problem No 44 Two identical balls A and B are released from the positions as shown in the figure. They collide elastically on the horizontal portion. The ratio of heights attained by A and B after collision (neglect friction)



Sol. As mass of two balls are equal, they exchange their velocities after collision.

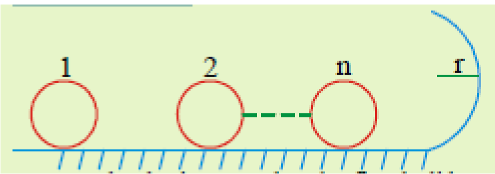
$$u_A = \sqrt{2gh}, u_B = \sqrt{2g(4h)} = \sqrt{8gh};\ h_A = \frac{u_A^2}{2g} = h;$$

$$h_B = h + \frac{v_B^2 \sin^2 60^0}{2g} = h + \frac{9h}{4} = \frac{13h}{4}$$

$$(\because (v_B)^2 - u_B^2 = -2gh \Rightarrow v_B^2 = u_B^2 - 2gh \Rightarrow v_B^2 = 6gh)$$

$$\frac{h_A}{h_B} = \frac{4}{13}$$

Problem No 45 n elastic balls are placed at rest on a smooth horizontal plane which is circular at the end with radius 'r' as shown in the figure. The masses of the balls are m, $\frac{m}{2}$, $\frac{m}{2^2}$, …… $\frac{m}{2^{n-1}}$ respectively. Find the minimum velocity that should be imparted to the first ball of mass 'm' such that the 'n^{th}' ball will complete the vertical circle.



Sol: Let speed to be imparted to the first ball be v_0. Consider the impact between the first two balls and v_1 and v_2 be the velocities of balls 1 and 2 after the impact respectively.

According to law of conservation of linear momentum $mv_0 = mv_1 + \frac{m}{2}v_2 \rightarrow (1)$

According to law of conservation of kinetic energy

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{m}{2}\right)v_2^2 \rightarrow (2)$$

Solving equations (1) and (2), we get $v_2 = \frac{4}{3}v_0$

Similarly, for n^{th} ball $v_n = \left(\frac{4}{3}\right)^{n-1} v_0 \rightarrow (3)$

For the n^{th} ball to complete the vertical circular motion $v_n = \sqrt{5gr} \rightarrow (4)$

From equations (3) and (4), we have

$$\left(\frac{4}{3}\right)^{n-1} v_0 = \sqrt{5gr};\ v_0 = \left(\frac{3}{4}\right)^{n-1}\sqrt{5gr}$$

Coefficient of Restitution (e)

Newton introduced a dimensionless parameter called the coefficient of restitution (e) to measure the elasticity of collision. It is defined as the ratio of the relative velocity of separation to the relative velocity of approach of the two colliding bodies

$$e = \frac{Relative\ velocity\ of\ separation}{Relative\ velocity\ of\ approach}$$

$$\therefore e = \frac{|\vec{V_2} - \vec{V_1}|}{\vec{u_1} - \vec{u_2}}$$

This formula is applied along the line of impact. Here the velocities mentioned in the expression should be taken along the line of impact.

For a perfectly elastic collision $e = 1$

For an inelastic collision $o < e < 1$

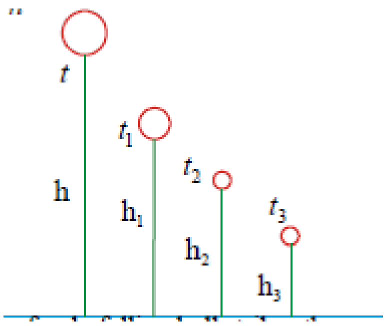
For completely inelastic collision $e = 0$

- A body dropped freely from a height 'h' strikes the floor and rebounds to a height h_1

$$e = \sqrt{\frac{h_1}{h}}$$

and after n^{th} rebound

$$h_n = e^{2n} h$$



- When a freely falling ball strikes the ground with a velocity 'v' and rebounds with a velocity v_1 then

$$e = \frac{v_1}{v}$$

and after n^{th} rebound

$$v_n = e^n v$$

- Time during n^{th} bounce to $(n+1)^{th}$ bounce is,

$$t_n = 2e^n t$$

- Total distance travelled by the ball before it stops bouncing

$$d = h + 2h_1 + 2h_2 + 2h_3 + \ldots\ldots\ldots$$
$$= h + 2e^2 h + 2e^4 h + 2e^6 h + \ldots\ldots$$
$$= h + 2e^2 h [1 + e^2 + e^4 + \ldots\ldots]$$
$$d = h \left[\frac{1+e^2}{1-e^2}\right]$$

- Total time taken by the ball to stop,

$$T = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2h}{g}}$$

- Average speed of the ball during its entire journey is given by

$$v_{avg} = \frac{Total\ distance\ travelled}{Total\ time\ taken}$$

$$v_{avg} = \frac{h\left[\frac{1+e^2}{1-e^2}\right]}{\sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]}$$

$$v_{avg} = \sqrt{\frac{gh}{2}}\frac{(1+e^2)}{(1+e)^2}$$

- Average velocity of the ball during its entire journey is given by

$$v_{avg} = \frac{Net\ displaccement}{Tptal\ time\ taken}$$

$$v_{avg} = \frac{h}{\sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]} = \sqrt{\frac{gh}{2}\frac{(1-e)}{(1+e)}}$$

Head on *Inelastic* Collision

Two bodies of masses m_1 and m_2 moving with initial velocities $\vec{u_1}$ and $\vec{u_2}$ $(u_1 > u_2)$ collide. After collision two bodies will move with velocities $\vec{v_1}$ and $\vec{v_2}$.

From Law of conservation of linear momentum,

$$m_1 (\vec{u}_1 - \vec{v}_1) = m_2 (\vec{v}_2 - \vec{u}_2)$$

By the definition of coefficient of restitution,

$$\vec{v}_2 - \vec{v}_1 = e (\vec{u}_1 - \vec{u}_2)$$

$$\vec{v}_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)\vec{u}_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)\vec{u}_2$$

$$\vec{v}_2 = \left(\frac{(1+e)m_1}{m_1 + m_2}\right)\vec{u}_1 + \left(\frac{m_2 - em_1}{m_1 + m_2}\right)\vec{u}_2$$

- Ratio of velocities after collision if $m_1 = m_2$, and $u_2 = 0$,

$$v_1 = (1 - e)\frac{u_1}{2}$$

$$v_2 = (1 + e)\frac{u_1}{2}$$

$$\frac{v_1}{v_2} = \frac{1-e}{1+e}$$

Loss of kinetic energy of the system:

$$\Delta KE = KE_i - KE_F$$

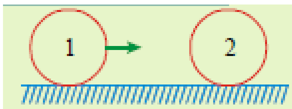
$$\Delta KE = \frac{1}{2}\left(\frac{m_1 m_2}{m_1+m_2}\right)(\vec{u}_1 - \vec{u}_2)^2(1 - e^2)$$

In case of perfectly in-elastic collision,

$e = 0$ ∴ loss in KE of system is

$$\Delta KE = \frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)(\vec{u}_1 - \vec{u}_2)^2$$

Problem No 46 Ball 1 collides with an another identical ball 2 at rest as shown in the figure. For what value of coefficient of restitution e, the velocity of second ball becomes two times that of first ball after collision?



Sol. Here $m_1 = m_2$ and $u_2 = 0$

After collision, $v_2 = \left(\frac{1+e}{2}\right)u$ & $v_1 = \left(\frac{1-e}{2}\right)u$

Given $v_2 = 2v_1$; $\left(\frac{1+e}{2}\right)u = 2\left(\frac{1-e}{2}\right)u$

$1+e = 2 - 2e$; $3e = 1$; $e = \frac{1}{3}$

Problem No 47 A body 'A' with a momentum 'P' collides with another identical stationary body 'B' one dimensionally. During the collision, 'B' gives an impulse 'J' to the body 'A'. Then the coefficient of restitution is

Sol : From the law of conservation of linear momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$mu + m(0) = mv_1 + mv_2$

$\Rightarrow P - P_1 = P_2$ where $P_2 = J$, (given)

$\therefore e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{mv_2 - mv_1}{mu - 0} = \frac{P_2 - P_1}{P}$

$= \frac{P_2 - (P - P_2)}{P} = \frac{2P_2 - P}{P} = \frac{2J - P}{P} = \frac{2J}{P} - 1$

Problem No 48 A ball of mass m collides with the ground at an angle a with the vertical. If the collision lasts for time t, the average force exerted by the ground on the ball is: (e =coefficient of restitution between the ball and the ground)

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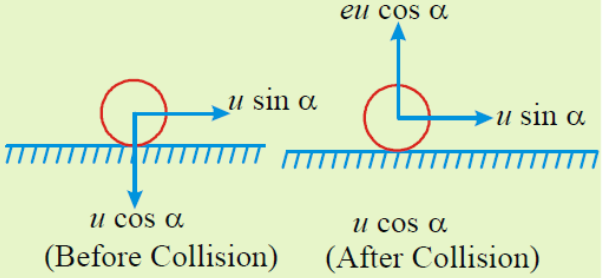


$\therefore Ft = m(eu\cos\alpha + u\cos\alpha)$ □□ $F = \frac{mu\cos\alpha(1+e)}{t}$

Problem No 49 A ball strikes a horizontal floor at an angle θ = 45□ with the normal to floor. The coefficient of restitution between the ball and the floor is e = 1/2. The fraction of its kinetic energy lost in the collision is

Sol : Let 'u' be the velocity of ball before collision. Speed of the ball after collision will become

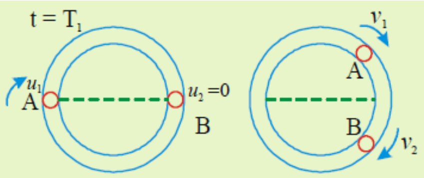
$v = \sqrt{u^2\sin^2\theta + e^2u^2\cos^2\theta}$

$= \sqrt{\left(\frac{u}{\sqrt{2}}\right)^2 + \left(\frac{u}{2\sqrt{2}}\right)^2} = \sqrt{\frac{5}{8}}u$

$\therefore$ Fraction of KE lost in collision

$= \frac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = 1 - \left(\frac{v}{u}\right)^2 = 1 - \frac{5}{8} = \frac{3}{8}$

Problem No 50 Two equal spheres A and B lie on a smooth horizontal circular groove at opposite ends of a diameter. At time t = 0, A is projected along the groove and it first impinges on B at time t = T_1 and again at time t = T_2 . If 'e' is the coefficient of restitution, find the ratio of $\frac{T_2}{T_1}$



□□□ $T_1 = \frac{\pi R}{u_1}$

$\frac{v_2 - v_1}{u_1} = e \Rightarrow v_2 - v_1 = eu_1$

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$T_2 - T_1 = \frac{2\pi R}{v_2 - v_1} \Rightarrow T_2 - T_1 = \frac{2\pi R}{eu_1}$

□□□□ □□□ $\frac{T_2}{T_1} = \frac{2+e}{e}$

Problem No 51 After perfectly inelastic collision between two identical particles moving with same speed in different directions, the speed of the combined particle becomes half the initial speed of either particle . The angle between the velocities of the two before collision is

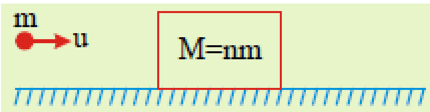
□□□ In perfectly inelastic collision between two particles, linear momentum is conserved . Let θ be the angle between the velocities of the two particles before collision. Then

$$P^2 = P_1^2 + P_2^2 + 2P_1P_2\cos\theta \quad □□$$

$$\left(2m\frac{v}{2}\right)^2 = (mv)^2 + (mv)^2 + 2(mv)(mv)\cos\theta$$

□□ cos □□ $\cos\theta = -\frac{1}{2}$ □□ $\theta = 120^0$

Problem No 52 A bullet of mass 'm' moving with velocity 'u' passes through a wooden block of mass M = nm as shown in figure. The block is resting on a smooth horizontal floor. After passing through the block, velocity of the bullet becomes 'v'. Its velocity relative to the block is



□□□ Let v' be the velocity of block. Then from conservation of linear momentum.

$$mu = mv + mnv' \text{ (or) } v' = \left(\frac{u-v}{n}\right)$$

$\therefore$ velocity of bullet relative to block will be

$$v_r = v - v' = v - \left(\frac{u-v}{n}\right) = \frac{(1+n)v - u}{n}$$

Problem No 53 A block of mass 0.50Kg is moving with a speed of 2.00 m/s on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. Find the energy loss during the collision

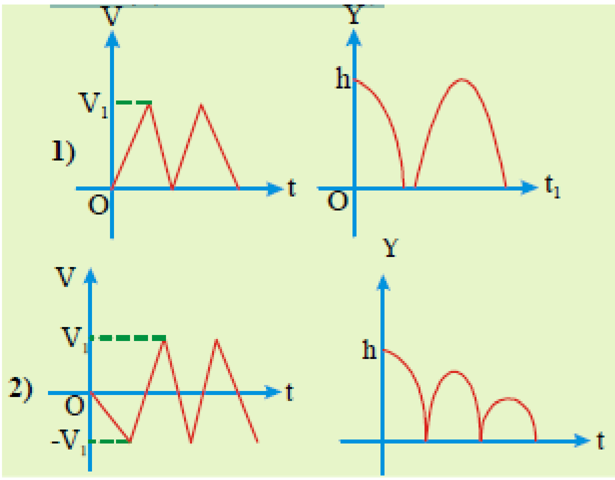
Sol: From LCLM, $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

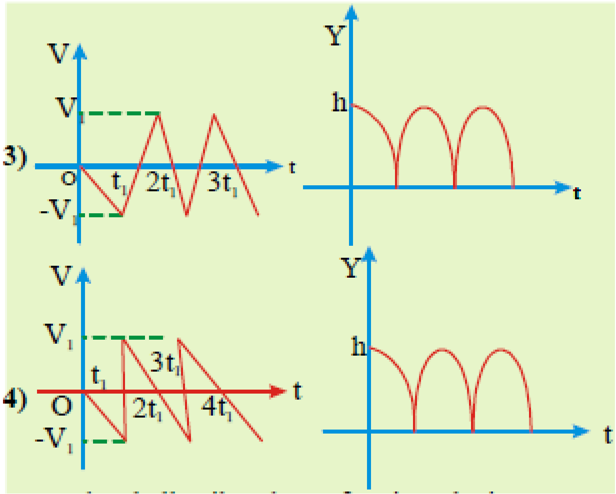
$$0.50 \times 2 + 1 \times 0 = (0.5 + 1)v \Rightarrow v = \frac{2}{3}\ ms^{-1}$$

$$\therefore \text{Energy loss } \Delta KE = \frac{1}{2}m_1u_1^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$\Delta KE = \frac{1}{2}(0.5)(2)^2 - \frac{1}{2}(1.5)\left(\frac{2}{3}\right)^2 = 0.67\ J$$

Problem No 54 Consider a rubber ball freely falling from a height h = 4.9 m on a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time and the height as a function of time will be;



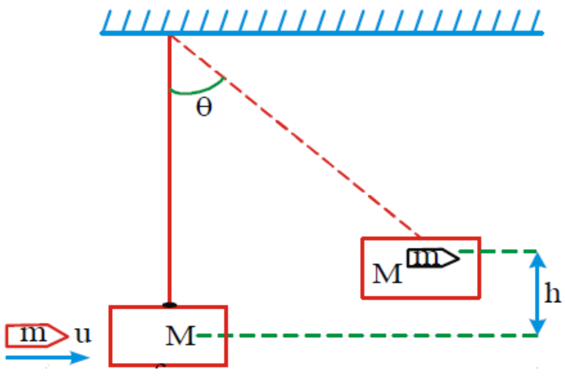


Sol: When ball strikes the surface its velocity will be reversed so correct option is (3).

Ballistic Pendulum

It is an arrangement used to determine the velocities of bullets. A log of wood of mass 'M' is suspended by a string of length '*l*' as shown in the figure. A bullet of mass 'm' is fired horizontally into the wooden block with a velocity 'u'

Case I : Let the bullet gets embedded in the block and system rises to a height 'h' as shown in the figure.



From the law of conservation of linear momentum,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)\,v$$

$$Mu + 0 = (m + M)\,v$$

$$v = \frac{mu}{m+M} \quad .... (1)$$

KE of the system after collision is given by

$$KE = \frac{1}{2}(m + M)\,v^2$$

PE at highest point = (m + M) gh

From LCE, $\frac{1}{2}(m + M)\,v^2 = (m + M)\,gh$

$$v^2 = 2gh$$

$$v = \sqrt{2gh} \quad ....(2)$$

From (1) and (2) velocity of the bullet

$$u = \frac{M+m}{m}\sqrt{2gh} = \frac{M+m}{m}\sqrt{2gl(1-\cos\theta)}$$

Loss in KE of the system = KE_1 - KE_2

$$\Delta KE = \frac{1}{2}mu^2 - \frac{1}{2}(m + M)\,v^2$$

$$\Delta KE = \frac{1}{2}\left[mu^2 - (m+M)\frac{m^2u^2}{(m+M)^2}\right]$$

$$\Delta KE = \frac{1}{2}\left[\frac{mM}{m+M}\right]u^2$$

Case II : If the bullet emerges out of the block with velocity 'v' then

$$Mu = mv + MV$$

Where $V = \sqrt{2gh}$

Problem No 55 A pendulum consists of a wooden bob of mass 'm' and of length *l*. A bullet of mass m1 is fired towards the pendulum with a speed v_1 and it emerges out of the bob with a speed $\frac{v_1}{3}$. Find the initial speed of the bullet if the bob just completes the vertical circle.

Sol: From the Law of conservation of momentum

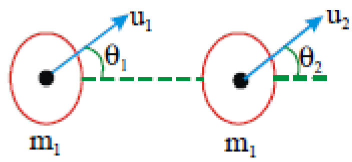
$$mv = m_1\left(v_1 - \frac{v_1}{3}\right) \text{ or } v = \frac{m_1}{m} \times \frac{2v_1}{3}$$

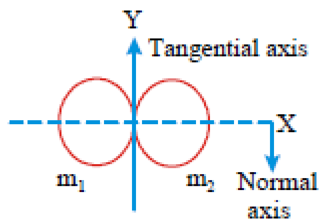
To describe a vertical circle $v = \sqrt{5gl}$

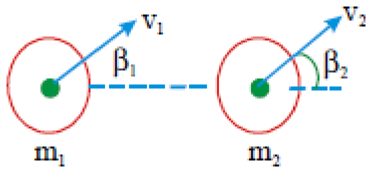
hence $\sqrt{5gl} = \frac{m_1}{m} \times \frac{2v_1}{3} \Rightarrow v_1 = \frac{m}{m_1} \times \frac{3\sqrt{5gl}}{2}$

Collision in two Dimension (Oblique Collision)

- A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles changes along common normal direction.
- No component of impulse acts along common tangent direction. Hence, linear momentum (or) linear velocity of individual particles remains unchanged along this direction.
- Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remain conserved before and after collision in any direction.
- When a particle hits elastically and obliquely another stationary particle of same mass, then they move perpendicular to each other after collision.
- Definition of coefficient of restitution can be applied along common normal direction.







From law of conservation of linear momentum along X-axis:

$m_1u_1\cos\theta_1 + m_2u_2\cos\theta_2$

$= m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$

Along Y-axis:

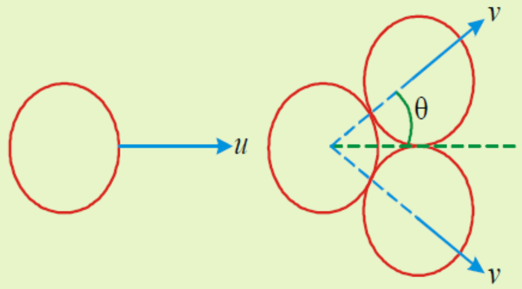
$m_1u_1\sin\theta_1 + m_2u_2\sin\theta_2$

$= m_1 v_1 \cos \beta_1 + m_2 v_2 \cos \beta_2$

Coefficient of restitution

$$e = -\frac{v_1 \cos \beta_1 - v_2 \cos \beta_2}{u_1 \cos \theta_1 - u_2 \cos \theta_2}$$

Problem No 56 Two billiard balls of same size (radius r) and same mass are in contact on a billiard table. A third ball also of the same size and mass strikes them symmetrically and remains at rest after the impact. The coefficient of restitution between the balls is.



$\sin\theta = \frac{r}{2r} = \frac{1}{2}; \quad \therefore \theta = 30^0$

From conservation of linear momentum

$mu = 2mv\cos 30^0 \quad$ or $\quad v = \frac{u}{\sqrt{3}}$

Now $e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$

in common normal direction

Hence, $e = \frac{v}{u\cos 30^0} = \frac{u/\sqrt{3}}{u\sqrt{3}/2} = \frac{2}{3}$

MCQ

(Classwork)

LEVEL – 1

WORK DONE BY CONSTANT FORCE

1. If a force $\vec{F} = (\vec{i} + 2\vec{j} + \vec{k})$ N acts on a body produces a displacement of $\vec{S} = (4\vec{i} + \vec{j} + 7\vec{k})$ m, then the work done is
   1) 9J 2) 13J 3) 5J 4) 1J
2. Work done by the gravitational force on a body of mass "m" moving on a smooth horizontal surface through a distance 's' is
   1) mgs 2) -mgs
   3) 0 4) 2mgs
3. A body of mass 1 kg is made to travel with a uniform acceleration of 30 cm/s over a distance of 2m, then work to be done is
   1) 6J 2) 60J 3) 0.6J 4) 0.3J
4. A uniform cylinder of radius 'r' length 'L' and mass 'm' is lying on the ground with the curved surface touching the ground. If it is to be oriented on the ground with the flat circular end in contact with the ground, the work to be done is
   1) mg[(L/2)-r] 2) mL[(g/2)-r]
   3) mr(gL-1) 4)mgLr
5. A meter scale of mass 400 gm is lying horizontally on the floor. If it is to be held vertically with one end touching the floor, the work to be done is
   1) 6 J 2) 4 J 3) 40J 4) 2 J
6. A force F is applied on a lawn mover at an angle of 600 with the horizontal. If it moves through a distance x, the work done by the force is
   1) Fx/2 2) F/2x
   3) 2Fx 4) 2x/F
7. A weight lifter jerks 220 kg vertically through 1.5 metre and holds still at that height for two minutes. The work done by him in lifting and in holding it still are respectively
   1) 220J, 330J 2) 3234 J, 0 J
   3) 2334 J, 10 J 4) 0 J, 3234 J
8. A tennis ball has a mass of 56.7 gm and is served by a player with a speed of 180 kmph. The work done in serving the ball is nearly

1) 710J 2) 71J 3) 918J 4) 91.8J

9. A body of mass 2kg is projected vertically up with velocity 5ms^{-1}.The work done on the body by gravitational force before it is brought to rest momentarily is
1) 250J 2) 25J 3) 0J 4) -25J

WORK DONE BY VARIABLE FORCE

10. A force F = (2+x) N acts on a particle in x - direction where 'x' is in metre. The work done by this force during a displacement from x = 1 m to x = 2 m is
1) 2 J 2) 3.5 J 3) 4.5 J 4) 5 J

KINETIC ENERGY

11. On increasing the speed of a body to 2 ms-1, its kinetic energy is quadrupled. Then its original speed must be
1) 0.25ms^{-1} 2) 1ms^{-1}
3) 4ms^{-1} 4) 2ms^{-1}

12. A bullet of mass 10 gm strikes a target at 400 m/s velocity and loses half of its initial velocity. The loss of kinetic energy in joules is
1) 800 2) 200 3) 400 4) 600

13. An object is acted on by a retarding force of 10 N and at a particular instant its kinetic energy is 6J. The object will come to rest after it has travelled a distance of
1) 3/5m 2) 5/3m
3) 4m 4) 16m

14. A man standing on the edge of the roof of a 20 m tall building projects a ball of mass 100 gm vertically up with a speed of 10ms^{-1}. The kinetic energy of the ball when it reaches the ground will be [g=10 ms^{-2}]
1) 5J 2) 20J 3) 25J 4) Zero

15. A river of salty water is flowing with a velocity 2 m/sec. If the density of water is 1.2 gm/cc, the kinetic energy of each of cubic metre of water is
1) 2.4 J 2) 24 J
3) 4.8 KJ 4) 2.4 KJ

16. If the kinetic energy of a body increases by 125% , the percentage increase in its momentum is
1) 50% 2) 62.5%
3) 250% 4) 200%

17. The kinetic energy of a body is 'K'. If onefourth of its mass is removed and velocity is doubled, its new kinetic energy is
1) K 2) 3K 3) 4K 4) 9K/4

POTENTIAL ENENGY

18. An inelastic ball falls from a height of 100 meters. It loses 20% of its total energy due to impact. The ball will now rise to a height of
1) 80 m 2) 120m 3) 60m 4) 9.8m

19. A woman weighing 63 kg eats plum cake whose energy content is 9800 calories. If all this energy could be utilized by her, she can ascend a height of
1) 1m 2) 67m 3) 100m 4) 42m

POTENTIAL ENERGY OF A SPRING

20. A spring of spring constant 5x103 N/m is stretched initially by 5cm from the unstretched position. Then the work required to stretch it further by another 5cm is.
1) 6.25Nm 2) 12.50Nm
3) 18.75Nm 4) 25Nm

21. A spring with spring constant K when stretched through 1cm, the potential energy is U. If it is stretched by 4cm, the potential energy will be
1) 4U 2) 8U 3) 16U 4) 2U

WORK ENERGY THEOREM BY CONSTANT FORCE

22. A body moving with a kinetic energy of 6J comes to rest at a distance of 1m due to a retarding force of
1) 4 N 2) 6 N 3) 5 N 4) 8 N

23. A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance of 3 meters. Assuming that the resistance due to water is negligible, the speed of the ship is
1) 0.1m/s 2) 1.5 m/s
3) 5m/s 4) 60 m/s

24. A vehicle of mass 1000 kg is moving with a velocity of 15 ms^{-1} .It is brought to rest by applying brakes and locking the wheels. If the sliding friction between the tyres and the road is 6000N, then the distance moved by the vehicle before coming to rest is

1) 37.5 m	2) 18.75 m
3) 75 m	4) 15 m

25. The workdone to accelerate a body from 30 ms-1 to 60 ms^{-1} is three times the work done to accelerate it from 10 ms-1 to 'v' . The value of 'v' in ms^{-1} is
1) 30	2) $20\sqrt{2}$	3) $30\sqrt{3}$	4) $10\sqrt{10}$

WORK ENERGY THEOREM FOR VARIABLE FORCE

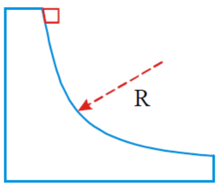
26. A block of mass 4 kg is initially at rest on a horizontal frictionless surface. A horizontal force $\vec{F} = (3+ x)\hat{I}$ newtons acts on it, when the block is at x=0. The maximum kinetic energy of the block between x=0 and x=2m is
1) 6J	2) 8J	3) 9J	4) 10J

CONSERVATION OF MECHANICAL ENERGY

27. A block of mass 4 kg slides on a horizontal frictionless surface with a speed of 2m/s. It is brought to rest in compressing a spring in its path. If the force constant of the spring is 400 N/m, by how much the spring will be compressed
1) 2×10^{-2}m	2) 0.2 m
3) 20 m	4) 200 m

28. At what height above the ground must a mass of 5 kg be to have its PE equal in value to the KE possessed by it when it moves with a velocity of 10 m/s? (g=10 m/s^2)
1) 1m	2) 5m
3) 10m	4) 50m

29. A body slides down a fixed curved track that is one quadrant of a circle of radius R, as in the figure. If there is no friction and the body starts from rest, its speed at the bottom of the track is



1) 5gR	2) 5gR
3) 2gR	4) gR

POWER

30. An electric motor in a crane while lifting a load produces a tension of 4000 N in the cable attached to the load. If the motor is winding the cable at the rate of 3ms-1, the power of the motor expressed in kilo watt units must be
1) 4	2) 3	3) 12	4) 6

31. An electric motor operates with an efficiency of 90%. A pump operated by the motor has an efficiency of 80%. The overall efficiency of the system is
1) 85%	2) 100%
3) 72%	4) 60%

32. A machine gun fires 420 bullets per minute. The velocity of each bullet is 300ms^{-1} and the mass of each bullet is 1gm. The power of the machine gun is
1) 315W	2) 315000W
3) 630W	4) 3150W

33. A 1 kg mass at rest is subjected to an acceleration of 5 m/s2 and travels 40m. The average power during the motion is
1) 40W	2) 8W
3) 50W	4) 200W

34. If the power of the motor of a water pump is 3kW, then the volume of water in litres that can be lifted to a height of 10m in one minute by the pump is (g=10 ms-2)
1) 1800	2) 180
3) 18000	4) 18

35. A particle moves with a velocity $(5\hat{i}+ 3\hat{j}+ 6\hat{j})$ m/s under the influence of a constant force $(5\hat{i}+ 3\hat{j}+ 6\hat{k})$ N. The instantaneous power applied to the particle is
1) 100W	2) 40W
3) 140W	4) 170W

ELASTIC AND INELASTIC COLLISIONS

36. A ball of 4 kg mass moving with a speed of 1 3ms- has a head on elastic collision with a 6 kg mass initially at rest. The speeds of both the bodies after collision are respectively
1) 0.6 ms^{-1}, 2.4 ms^{-1}
2) 0.6 ms^{-1}, 2.4 ms^{-1}
3) 0.6 ms^{-1}, 2.4 ms^{-1}
4) 0.6 ms^{-1}, 2.4 ms^{-1}

37. A ping - pong ball strikes a wall with a
velocity of 10 ms^{-1}. If the collision is perfectly elastic, find the velocity of ball after impact

1) 20 ms^{-1} 2) 5 ms^{-1}
3) 1.0 ms^{-1} 4) 10 ms^{-1}

38. Two identical balls collide head on. The initial velocity of one is 0.75 ms-1, while that of the other is -0.43ms-1 . If the collision is perfectly elastic, then their respective final velocities are
1) 0.75ms^{-1}; -0.43 ms^{-1}
2) -0.43ms^{-1}; 0.75 ms^{-1}
3) -0.75ms^{-1}; 0.43 ms^{-1}
4) 0.43ms^{-1}; 0.75 ms^{-1}

39. A truck of mass 15 tons moving with 1 ms^{-1} collides with a stationary truck of mass 10 tons and automatically connected to move together. The common velocity is
1) 1ms^{-1} 2) 0ms^{-1}
3) 0.4ms^{-1} 4) 0.6ms^{-1}

40. In the above problem the total KE before collision is
1) 4500J 2) 7500J
3) 3000J 4) 0J

41. In the above problem loss of KE during collision is
1) 4500J 2) 7500J
3) 3000J 4) 0J

42. A bullet of mass 'x' moves with a velocity y, hits a wooden block of mass z at rest and gets embedded in it. After collision, the wooden block and bullet moves with a velocity
1) $\frac{x}{x+z}y$ 2) $\frac{x+y}{x}y$ 3) $\frac{z}{x+y}y$ 4) $\frac{x+y}{z}y$

43. A railway truck of mass 16000kg moving with a velocity of 5ms-1 strikes another truck of mass 4000kg at rest. If they move together after impact, their common velocity is
1) 2ms^{-1} 2) 4ms^{-1}
3) 6ms^{-1} 4) 8ms^{-1}

COEFFICIENT OF RESTITUTION

44. A ball falls from a height of 10m on to a horizontal plane. If the coefficient of restitution is 0.4, then the velocity with which it rebounds from the plane after second collision is
1) 2.24ms^{-1} 2) 5.6ms^{-1}
3) 2.8ms^{-1} 4) 0.9ms^{-1}

45. A ball is dropped from a height of 3m. If coefficient of restitution between the surface and ball is 0.5, then the total distance covered by the ball before it comes to rest is
1) 3m 2) 4m 3) 5m 4) 6m

46. A glass sphere of mass 5mg, falls from a height of 3 meters on to a horizontal surface. If the coefficient of restitution is 0.5, then after the impact the sphere will rise to a height of
1) 0.075m 2) 0.75m
3) 7.5m 4) 75m

47. A particle falls from a height 'h' upon a fixed horizontal plane and rebounds. If 'e' is the coefficient of restitution, then the total distance travelled before it comes to rest is
1) $h\left(\frac{1+e^2}{1-e^2}\right)$ 2) $h\left(\frac{1-e^2}{1+e^2}\right)$
3) $\frac{h}{2}\left(\frac{1-e^2}{1+e^2}\right)$ 4) $\frac{h}{2}\left(\frac{1+e^2}{1-e^2}\right)$

LEVEL – 1 - KEY

01) 2 02) 3 03) 3 04) 1 05) 4 06) 1
07) 2 08) 2 09) 4 10) 2 11) 4 12) 4
13) 1 14) 3 15) 4 16) 1 17) 2 18) 1
19) 2 20) 3 21) 3 22) 2 23) 1 24) 2
25) 4 26) 2 27) 2 28) 2 29) 3 30) 3
31) 3 32) 1 33) 3 34) 1 35) 1 36) 3
37) 4 38) 2 39) 4 40) 2 41) 3
42) 1 43) 2 44) 1 45) 3 46) 2 47) 1

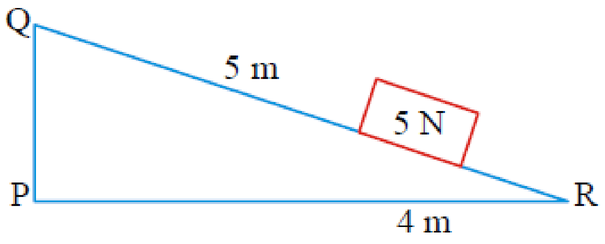
LEVEL – 2

WORK DONE BY CONSTANT FORCE

1) A bicycle chain of length 1.6 m and of mass 1 kg is lying on a horizontal floor. If g=10ms^{-2}, the work done in lifting it with one end touching the floor and the other end 1.6 m above the floor is
1) 10 J 2) 3.2 J 3) 8 J 4) 16 J

2) A bucket of mass 'm' tied to a light rope is lowered at a constant acceleration of g/4. If the bucket is lowered by a distance 'd', the work done by the rope will be (neglect the mass of the rope)
1) $\frac{1}{4}mgd$ 2) $\frac{3}{4}mgd$
3) $-\frac{3}{4}mgd$ 4) $-\frac{5}{4}mgd$

3) A weight of 5 N is moved up a frictionless inclined plane from R to Q as shown.



What is the work done in joules?
1) 15 2) 20 3) 25 4)35

4) A 5 kg stone of relative density 3 is resting at the bed of a lake. It is raised through a height of 5 m in the lake. If g = 10 m/s^2, then work done is
1) $\frac{500}{3}J$ 2) $\frac{350}{3}J$ 3) $\frac{750}{3}J$ 4) $\frac{550}{3}J$

5) Water is drawn from a well in a 5 kg drum of capacity 55 L by two ropes connected to the top of the drum. The linear density of each rope is 0.5 kgm^{-1}. The work done in lifting water to the ground from the surface of water in the well 20 m below is (g = 10 ms^{-2})
1)1.4×10^4J 2)1.5×10^4J
3)$9.8\times6\times10$ J 4) 18 J

6) A ball is dropped from the top of a tower. The ratio of work done by force of gravity in 1st, 2nd, and 3rd second of the motion of ball is
1) 1:2:3 2)1:4:16 3)1:3:5 4) 1:9:25

7) A plate of mass m, breadth 'a' and length 'b' is initially lying on a horizontal floor with length parallel to the floor and breadth perpendicular to the floor. The work done to erect it on its breadth is
1) $mg\frac{b}{2}$ 2) $mg\left(a+\frac{b}{2}\right)$
3) $mg\left(\frac{b-a}{2}\right)$ 4) $mg\left(\frac{b+a}{2}\right)$

8) A block of mass 10 kg slides down a rough slope which is inclined at 45^0 to the horizontal. The coefficient of sliding friction is 0.30. When the block has to slide 5 m, the work done on the block by the force of friction is nearly
1)115J 2)-75 2 J 3) 321.4J 4) -321.4 J

9) A uniform rope of length 'L and linear density 'μ' is on a smooth horizontal table with a length 'P' lying on the table. The work done in pulling the hanging part on to the table is
1) $\frac{\mu g(L-l)^2}{2}$ 2) $\frac{\mu g(L-l)^2}{2l^2}$
3) $\frac{\mu g(L-l)^2}{2L^2}$ 4) $\frac{\mu gL}{2(L-l)}$

10) A uniform rod of mass 2 kg and length l is lying on a horizontal surface. If the work done in raising one end of the rod through an angle 45^0 is 'W', then the work done in raising it further 45^0 is
1) W 2) $\sqrt{2}$ W
3) $\frac{W}{\sqrt{2}}$ 4) $(\sqrt{2}-1)W$

WORK DONE BY VARIABLE FORCE

11) A block is constrained to move along x-axis under a force $F = -(2x)N$. Find the work done by the force when the block is displaced from x = 2m to x = 4m
1)12J 2)8J 3)-12J 4)-8J

12) A force of (4x^2+3x)N acts on a particle which displaces it from x = 2m to x = 3m. The work done by the force is
1)32.8J 2)3.28J 3)0.328J 4)Zero

13) A body of mass 6kg is under a force which causes a displacement in it which is given by $S = \frac{t^2}{4}$ m where 't' is time. The work done by the force in 2sec is
1)12J 2)9J 3)6J 4)3J

KINETIC ENERGY

14) Two spheres of same material are moving with kinetic energies in the ratio 108:576. If the ratio of their velocities is 2:3 , then the ratio of their radii is
1)1:1 2)2:3 3) 3:4 4) 4:3

15) If the momentum of a body decreases by 30%, then kinetic energy decreases by
1) 60% 2) 51% 3) 69% 4) 90%

16) If the mass of a moving body decreased by one third of its initial mass and velocity is tripled, then the percentage change in its kinetic energy is
1)500% 2) 600% 3) 300% 4) 200%

17) The kinetic energy of a projectile at the highest point of its path is found to be 3/4th of its initial kinetic energy.

If the body is projected from the ground, the angle of projection is
1)0^0 2)30^0 3)60^0 4) 40^0

18) The kinetic energy of a moving body is given by k = $2v^2$, k being in joules and v in m/s. It's momentum when travelling with a velocity of 2 m/s will be (in kgms^{-1})
1)16 2) 4 3) 8 4) 2

POTENTIAL ENERGY

19) A simple pendulum is swinging in vertical plane. The ratio of its potential energy when it is making 45^0 and 90^0 with the vertical is
1) 1 : 1 2) $(\sqrt{2}+1)$
3) $\sqrt{2}$: 1 4) $(\sqrt{2}+1):\sqrt{2}$

POTENTIAL ENERGY OF A SPRING

20) A spring of force constant 800 Nm^{-1} is stretched initially by 5 cm. The work done in stretching from 5 cm to 15 cm is
1) 12.50 N-m 2) 18.75 N-m
3) 25.00 N-m 4) 6.25 N-m

21) When a spring is compressed by a distance 'x', the potential energy stored is U_1. It is further compressed by a distance '2x', the increase in potential energy is U_2. The ratio of U_1:U_2 is
1)1:7 2) 1:4 3)1:8 4)1:3

22) A massless spring with a force constant K = 40N/m hangs vertically from the ceiling. A 0.2kg block is attached to the end of the spring and held in such a position that the spring has its natural length and suddenly released. The maximum elastic strain energy stored in the spring is(g=10m/s^2)
1)0.1J 2)0.2J 3)0.05J 4) 0.4J

WORK ENERGY THEOREM BY CONSTANT FORCE

23) A bullet of mass'm'isfired with a velocity 'v' into a fixed log of wood and penetrates a distance 's' before coming to rest. Assuming that the path of the bullet in the log of wood is horizontal, the average resistance offered by the log of wood is
1) $\frac{mv}{2s^2}$ 2) $\frac{mv^2}{2s}$ 3) $\frac{2s}{mv^2}$ 4) $\frac{ms^2}{2v}$

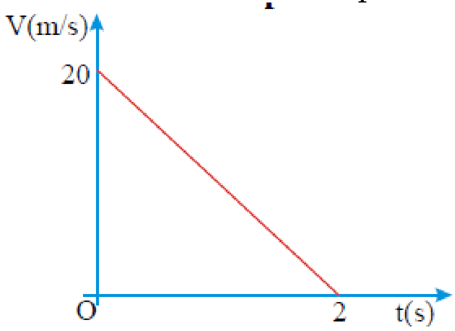
24) A ball of mass 'm' is thrown in air with speed v_1 from a height h_1 and it is caught at a height $h_2 > h_1$ when its speed becomes v_2. Find the work done on the ball by air resistance.
1) $mg(h_2-h_1)+\frac{1}{2}m(v_2^2-v_1^2)$
2) $mg(h_2-h_1)$
3) $\frac{1}{2}m(v_2^2-v_1^2)$
4) $mg(h_2-h_1)-\frac{1}{2}m(v_2^2-v_1^2)$

25) An object of mass 5 kg falls from rest through a vertical distance of 20 m and attains a velocity of 10 m/s. How much work is done by the resistance of the air on the object (g = 10 m/s^2)
1)750 J 2)– 750 J 3) 850 J 4)- 650 J

26) The velocity of a 2 kg body is changed from $(4\hat{i}+3\hat{j})$ ms^{-1} to 6 ms^{-1}. The work done on the body is
1)9 J 2)11 J 3)1 J 4) 5 J

27) An out fielder throws a cricket ball with an initial kinetic energy of 800J and an infielder catches the ball when its kinetic energy is 600J. If the path of the ball between them is assumed straight and is 20m long, the air resistance acting on the ball is
1)26.6N 2)1.33N 3)100N 4)10N

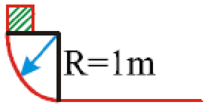
28) Velocity -time graph of a particle of mass 2kg moving in a straight line is as shown in the figure. Work done by all the forces on the particle is



1)400J 2)-400J 3)-200J 4)200J

29) A block of mass of 1kg slides down a curved track that is one -quadrant of circle of radius 1m. Its speed at the

bottom is 2m/s. The work done by the frictional force is



1)8J 2)-8J 3)4J 4)-4J

WORK ENERGY THEOREM FOR VARIABLE FORCE:

30) A block of mass 4kg is initially at rest on a horizontal frictionless surface. A force $\vec{F} = (2x + 3x^2)\hat{\imath}$ N acts horizontally on it. The maximum kinetic energy of the block between $x = 2$m and $x = 4$m in joules is
1)40 2) 36 3)68 4) 52

31) A force F=Ay^2+By+C acts on a body at rest in the Y-direction. The kinetic energy of the body during a displacement $y = -$a to y = a is
1) $\frac{2Aa^3}{3}$ 2) $\frac{2Aa^3}{3} + 2ca$
3) $\frac{2Aa^3}{3} + \frac{Ba^2}{2} + ca$ 4) $\frac{2Aa^3}{3} + \frac{Ba^2}{2}$

LAW OF CONSERVATION OF MECHANICAL ENERGY

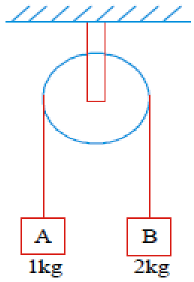
32) A 3kg model rocket is launched straight up with sufficient initial speed to reach a maximum height of 100 m, even though air resistance (a non-conservative force) performs - 900 J of work on the rocket. The height the rocket would have gone without air resistance will be
1) 70 m 2) 130 m 3) 180 m 4) 230 m

33) A body of mass 2kg is thrown up vertically with kinetic energy of 490J. If $g = 9.8m/$ s^2 , the height at which the kinetic energy of the body becomes half of the original value, is (2007 M)
1) 50m 2) 25m
3) 12.5 m 4) 19.6 m

34) A simple pendulum bob has a mass "m" and length "L". The bob is drawn aside such that the string is horizontal and then it is released. The velocity of the bob while it crosses the equilibrium position is
1) $\sqrt{gL}$ 2) $\sqrt{2gL}$ 3) $\sqrt{5gL}$ 4) $\sqrt{3gL}$

35) A 100 gm light bulb dropped from a tower reaches a velocity of 20 m/s after falling through 100 m. The energy transferred to the air due to viscous force is
1) 98 J 2) 20 J 3) 118 J 4) 78 J

36) In the arrangement shown in figure, string is light and inextensible and friction is absent every where. The speed of both blocks after the block 'A' has ascend a height of 1m will be (g=10m/s^2)



1)2m/s 2) 2.58m/s 3) 3m/s 4) 3.58 m/s

37) A car drives along a straight level frictionless road by an engine delivering constant power. Then velocity is directly proportional to
1) t 2) $\frac{1}{\sqrt{t}}$ 3) $\sqrt{t}$ 4) t^2

38) The input power to an electric motor is 200KW. Its efficiency is 80%. It operates a crane of efficiency 90%. If the crane is lifting a load of 3.6 tonnes, the velocity with which the load moves is
1) 8 ms^{-1} 2) 4 ms^{-1} 3) 2 ms^{-1} 4) 40 ms^{-1}

39) The human heart discharges 75 cm3 of blood per beat against an average pressure of 10 cm of Hg. Assuming that the pulse frequency is 75 per minute, the power of the heart is (density of Hg = 13.6 gm cm^{-3})
1) 1.25W 2) 12.5W
3) 0.125W 4)125W

40) An elevator can carry a maximum load of 1800 kg (elevator + passengers) is moving up with a constant speed of 2m/s. The frictional force opposing the motion is 400N. Determine the minimum power delivered by the motor to the elevator (in horse power).
1) 59 2)8 3)22 4)20

41) A body is initially at rest. It undergoes one - dimensional motion with constant acceleration. The power

delivered to it at time t is proportional to

1) $t^{\frac{1}{2}}$ 2) t 3) $t^{\frac{1}{3}}$ 4) t^2

42) A dam is situated at a height of 550 m above sea level and supplies water to a power house which is at a height of 50 m above sea level. 2000 kg of water passes through the turbines per second. What would be the maximum electrical power output of the power house if the whole system were 80% efficient

1)8 MW 2)10 MW
3)12.5 MW 4)16 MW

ELASTIC AND INELASTIC COLLISIONS

50) A 16 gm mass is moving in the +x direction at 30 cm/s while a 4 gm is moving in the –x direction at 50 cm/s. They collide head – on and stick together. Their common velocity after impact is

1) 0.14cm/s 2)0.14 m/s
3) 0 ms^{-1} 4) 0.3 m/s

51) A bullet of mass 50 grams going at a speed of 200 ms^{-1} strikes a wood block of mass 950 gm and gets embedded in it. The velocity of the block after the impact is

1) 5 ms^{-1} 2) 10 ms^{-1}
3) 20 ms^{-1} 4) 50 ms^{-1}

52) A block of mass 1 kg moving with a speed of 4 ms^{-1}, collides with another block of mass 2 kg which is at rest. If the lighter block comes to rest after collision, then the speed of the heavier body is

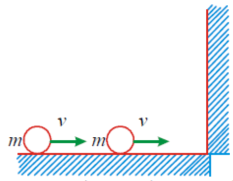
1) 2 ms^{-1} 2) 1 ms^{-1}
3) 1.5 ms^{-1} 4) 0.5 ms^{-1}

COEFFICIENT OF RESTITUTION

53) A neutron travelling with a velocity v and kinetic energy E collides perfectly elastically head on with the nucleus of an atom of mass number A at rest. The fraction of the total kinetic energy retained by the neutron is

1) $\left(\frac{A-1}{A+1}\right)^2$ 2) $\left(\frac{A+1}{A-1}\right)^2$ 3) $\left(\frac{A-1}{A}\right)^2$ 4) $\left(\frac{A+1}{A}\right)^2$

54) Two balls each of mass 'm' are moving with same velocity v on a smooth surface as shown in figure. If all collisions between the balls and balls with the wall are perfectly elastic, the possible number of collisions between the balls and wall together is



1) 1 2) 2 3) 3 4) Infinity

LEVEL – 2 - KEY

1) 3	2) 3	3) 1	4) 1	5) 1	6) 3
7) 3	8) 2	9) 1	10)4	11)3	12)1
13)4	14)3	15)2	16)1	17)2	18)3
19)4	20)3	21)3	22)3	23)2	24)1
25)2	26)2	27)4	28)2	29)2	30)3
31)2	32)2	33)3	34)2	35)4	36)2
37)3	38)2	39)1	40)1	41) 2	42)1
43)4	44)3	45)4	46)2	47) 2	48)1
49)3	50)2	51)2	52)1	53)1	54)3

EXERCISE

(Homework)

LEVEL – I (A)

WORK

1. In which of the following, the work done by the mentioned force is negative? The work done by
   1) the tension in the cable while the lift is ascending
   2) the gravitational force when a body slides down an inclined plane
   3) the applied force to maintain uniform motion of a block on a rough horizontal surface
   4) the gravitational force when a body is thrown up
2. A man pushes a wall and fails to displace it. He does
   1) negative work
   2) positive but not maximum work
   3) maximum work
   4) no work at all
3. A bucket full of water is drawn up by a person. In this case the work done by the gravitational force is
   1) negative because the force and displacement are in opposite directions
   2) positive because the force and displacement are in the same direction
   3) negative because the force and displacement are in the same direction
   4) positive because the force and displacement are in opposite directions
4. A man is rowing a boat upstream and inspite of that the boat is found to be not moving with respect to the bank. The work done by the man is
   1) zero 2) positive
   3) negative 4) may be +ve or –ve
5. A ball is thrown vertically upwards from the ground. Work done by air resistance during its time of flight is
   1) positive during ascent and negative during descent
   2) positive during ascent and descent
   3) negative during ascent and positive during descent
   4) negative during ascent and descent
6. An agent is moving a positively charged body towards another fixed positive charge. The work done by the agent is
   1) positive
   2) negative
   3) zero
   4) may be positive or negative
7. Work done by force of friction
   1) can be zero
   2) can be positive
   3) can be negative
   4) any of the above

CONSERVATIVE AND NON-CONSERVATIVE FORCES

8. Potential energy is defined for
   1) non-conservative forces only
   2) conservative forces only
   3) both conservative & non-conservative forces
   4) neither conservative nor non-conservative forces
9. Which of the following forces is called a conservative force?
   1) Frictional force 2) Air resistance
   3) Electrostatic force 4) Viscous force
10. Identify the non-conservative force in the following
    1) Weight of a body
    2) Force between two ions
    3) Magnetic force
    4) Air resistance
11. If x, F and U denote the displacement, force acting on and potential energy of a particle, then
    1) $U = F$ 2) $F = +\frac{dU}{dx}$ 3) $F = -\frac{dU}{dx}$ 4) $F = \frac{1}{x}\left(\frac{dU}{dx}\right)$
12. In the case of conservative force
    1) work done is independent of the path
    2) work done in a closed loop is zero
    3) work done against conservative force is stored is the form of potential energy
    4) all the above

KINETIC ENERGY

13. The change in kinetic energy per unit 'space' (distance) is equal to
    1) power 2) momentum
    3) force 4) pressure
14. When the momentum of a body is doubled, the kinetic energy is
    1) doubled

2) halved
3) becomes four times
4) becomes three times

15. For the same kinetic energy, the momentum shall be maximum for which of the following particle?
1) Electron 2) Proton
3) Deuteron 4) Alpha particle

16. If the momentum of a particle is plotted on X-axis and its kinetic energy on the Y-axis, the graph is a
1) straight line
2) parabola
3) rectangular hyperbola
4) circle

17. When two identical balls are moving with equal speeds in opposite direction, which of the following is true? For the system of two bodies
1) momentum is zero, kinetic energy is zero
2) momentum is not zero, kinetic energy is zero
3) momentum is zero, kinetic energy is not zero
4) momentum is not zero, kinetic energy is not zero

18. The product of linear momentum and velocity of a body represents
1) half of the kinetic energy of the body
2) kinetic energy of the body
3) twice the kinetic energy of the body
4) mass of the body

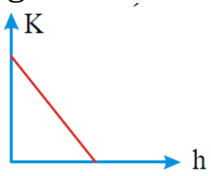
19. The KE of a freely falling body
1) is directly proportional to height of its fall
2) is inversely proportional to height of its fall
3) is directly proportional to square of time of its fall
4) 1 and 3 are true

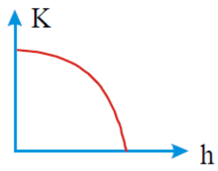
20. Consider the following statements
A) Linear momentum of a system of particles is zero
B) Kinetic energy of a system of particles is zero then
1) A does not imply B & B does not imply A
2) A implies B and B does not imply A
3) A does not imply B but B implies A
4) A implies B and B implies A

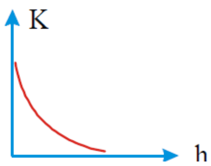
21. Internal forces can change
1) Linear momentum as well as kinetic energy
2) Linear momentum but not the kinetic energy
3) Kinetic energy but not linear momentum
4) neither the linear momentum nor the kinetic energy

22. If the force acting on a body is inversely proportional to its speed, then its kinetic energy is
1) linearly related to time
2) inversely proportional to time
3) inversely proportional to the square of time
4) a constant

23. Which of the following graphs depicts the variation of KE of a ball bouncing on a horizontal floor with height? (Neglect air resistances)

1) 

2) 

3) 

4) None of these

24. Which of the following statement is correct?
1) KE of a system cannot be changed without changing its momentum
2) KE of a system can be changed without changing its momentum
3) Momentum of a system cannot be changed without changing its KE
4) A system cannot have energy without having momentum

25. Two bodies of different masses have same linear momentum. The one having more KE is
1) lighter body 2) heavier body
3) both 4) none

26. Two bodies of masses m1 and m2 have equal KE. Their momenta is in the ratio
1) $\sqrt{m_2}:\sqrt{m_1}$ 2) $m_1:m_2$ 3) $\sqrt{m_1}:\sqrt{m_2}$ 4) $m_1^2:m_2^2$

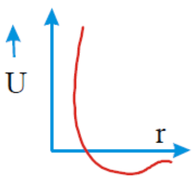
27. A body can have

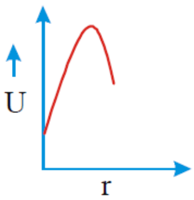
1) changing momentum and finite kinetic energy
2) zero kinetic energy and finite momentum
3) zero acceleration and increasing kinetic energy
4) finite acceleration and zero kinetic energy

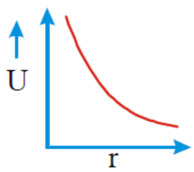
28. A rock of mass m is dropped to the ground from a height h. A second rock with mass 2m is dropped from the same height. When second rock strikes the ground, its kinetic energy will be
1) twice that of the first rock
2) four times that of the first rock
3) the same as that of the first rock
4) half that of the first rock

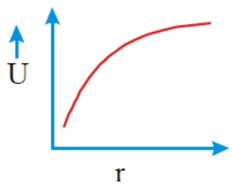
POTENTIAL ENERGY

29. These diagrams represent the potential energy U of a diatomic molecule as a function of the inter-atomic distance r. The diagram corresponds to stable molecule found in nature is

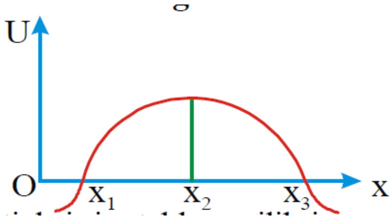
1) 

2) 

3) 

4) 

30. In the fig. the potential energy U of a particle plotted against its position x from origin. Which of the following statement is correct?



1) at x_1 particle is in stable equilibrium
2) at x_2 particle is in stable equilibrium
3) at x_3 particle is in stable equilibrium
4) at x_1 ,x_2 *and* $x_3$particle is in unstable equilibrium

POTENTIAL ENERGY OF A SPRING

31. When a spring is wound, a certain amount of PE is stored in it. If this wound spring is dissolved in acid, the stored energy
1) is completely lost
2) appears in the form of electromagnetic waves
3) appears in the form of heat raising the temperature of the acid
4) appears in the form of KE by splashing acid drops

32. Two springs have their force constants K_1 and K_2 and they are stretched to the same extension. If $K_2>K_1$ work done is
1) same in both the springs
2) more in spring K_1
3) more in spring K_2
4) independent of spring constant K

33. Two springs have their force constants K_1 and K_2 ($K_2>K_1$). When they are stretched by the same force, work done is
1) same in both the springs
2) more in spring K_1
3) more in spring K_2
4) independent of spring constant K

WORK ENERGY THEOREM

34. A lorry and a car moving with the same KE are brought to rest by applying the same retarding force. Then
1) lorry will come to rest in a shorter distance
2) car will come to rest in a shorter distance
3) both come to rest in same distance
4) any of above

CONSERVATION OF MECHANICAL ENERGY

35. A shell is fired into air at an angle q with the horizontal from the ground. On reaching the maximum height,
1) its kinetic energy is not equal to zero
2) its kinetic energy is equal to zero
3) its potential energy is equal to zero
4) both its potential and kinetic energies are zero

36. A cricket ball and a ping-pong ball are
dropped in a vacuum chamber from same height. When they have fallen half way down, they have the same
1) velocity
2) potential energy
3) kinetic energy
4) mechanical energy

37. A cyclist free-wheels from the top of a hill, gathers speed going down the hill, applies his brakes and eventually comes to rest at the bottom of the hill. Which one of the following energy changes take place.
1) Potential to kinetic and to heat energy
2) Kinetic to potential and to heat energy
3) chemical to heat and to potential energy
4) Kinetic to heat and to chemical energy

38. If 'E' represents total mechanical energy of a system while 'U' represents the potential energy, then E - U is
1) always zero
2) negative
3) either positive or negative
4) positive

39. For a body thrown vertically upwards, its direction of motion changes at the point where its total mechanical energy is
1) greater than the potential energy
2) less than the potential energy
3) equal to the potential energy
4) zero

40. Internal forces can change
1) Kinetic energy
2) mechanical energy
3) Momentum
4) 1 and 2

41. Negative of work done by the conservation forces on a system is equal to
1) the change in kinetic energy of the system
2) the change in potential energy of the system
3) the change in total mechanical energy of the system
4) the change in the momentum of the system

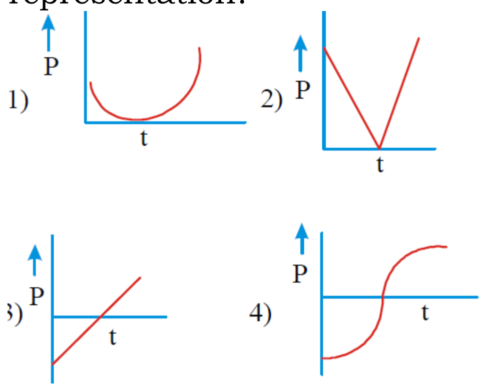
42. Which of the following statement is wrong?
1) KE of a body is independent of the direction of motion
2) In an elastic collision of two bodies, the momentum and energy of each body is conserved
3) If two protons are brought towards each other, the PE of the system increases
4) A body can have energy without momentum

43. When a body falls from an aeroplane there is increase in its:
1) acceleration
2) potential energy
3) kinetic energy
4) mass

POWER

44. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to
1) t1/2 2) t3/4 3) t3/2 4) t2

45. A particle is projected at t = 0 from a point on the ground with certain velocity at an angle with the horizontal. The power of gravitational force is plotted against time. Which of the following is the best representation?



46. A body starts from rest and acquires a
velocity V in time T. The instantaneous power delivered to the body in time 't' is proportional to
1) $\frac{V}{T}t$ 2) $\frac{V^2}{T}t^2$ 3) $\frac{V^2}{T^2}t$ 4) $\frac{V^2}{T^2}t^2$

47. A car drives along a straight level frictionless road by an engine

delivering constant power. Then velocity is directly proportional to

1) t 2) $\frac{1}{\sqrt{t}}$ 3) $\sqrt{t}$ 4) t^2

48. A particle is projected with a velocity u making an angle q with the horizontal. The instantaneous power of the gravitational force
   1) varies linearly with time
   2) is constant throughout the path
   3) is negative for complete path
   4) varies inversly with time

COLLISIONS

49. A ball with initial momentum $\vec{P}$ collides with rigid wall elastically. If $\overrightarrow{P^1}$ be its momentum after collision then
   1) $\overrightarrow{P^1} = \vec{P}$ 2) $\overrightarrow{P^1} = -\vec{P}$
   3) $\overrightarrow{P^1} = 2\vec{P}$ 4) $\overrightarrow{P^1} = -2\vec{P}$
50. Choose the false statement
   1) In a perfect elastic collision the relative velocity of approach is equal to the relative velocity of separation
   2) In an inelastic collision the relative velocity of approach is less than the relative velocity of separation
   3) In an inelastic collision the relative velocity of separation is less than the relative velocity of approach
   4) In perfect inelastic collision relative velocity of separation is zero
51. Two particles of different masses collide head on. Then for the system
   1) loss of KE is zero, if it was perfect elastic collision
   2) If it was perfect inelastic collision, the loss of KE of the bodies moving in opposite directions is more than that of the bodies moving in the same direction
   3) loss of momentum is zero for both elastic and inelastic collision
   4) 1, 2 and 3 are correct
52. A 2 kg mass moving on a smooth frictionless surface with a velocity of 1 $10ms$- hits another 2kg mass kept at rest, in a perfect inelastic collision. After collision, if they move together
   1) they travel with a velocity of $5ms^{-1}$ in the same direction
   2) they travel with a velocity of 10 ms^{-1} in the same direction
   3) they travel with a velocity of 10 ms^{-1} in opposite direction
   4) they travel with a velocity of 1 $5ms$- in opposite direction
53. A body of mass 'm' moving with a constant velocity v hits another body of the same mass moving with the same velocity v but in opposite direction and sticks to it. The velocity of the compound body after the collision is
   1) 2v 2) v 3) v/2 4) zero
54. In an inelastic collision, the kinetic energy after collision
   1) is same as before collision
   2) is always less than before collision
   3) is always greater than before collision
   4) may be less or greater than before collision
55. A ball hits the floor and rebounds after an inelastic collision. In this case
   1) the momentum of the ball just after the collision is same as that just before the collision
   2) The mechanical energy of the ball remains the same in the collision
   3) The total momentum of the ball and the earth is conserved
   4) the total kinetic energy of the ball and the earth is conserved
56. About a collision which is not correct
   1) physical contact is must
   2) colliding particles can change their direction of motion
   3) the effect of the external force is not considered
   4) linear momentum is conserved
57. In one– dimensional elastic collision, the relative velocity of approach before collision is equal to
   1) relative velocity of separation after collision
   2) 'e' times relative velocity of separation after collision
   3) '1/e' times relative velocity of separation after collision
   4) sum of the velocities after collision
58. Two identical bodies moving in opposite direction with same speed, collide with each other. If the collision is perfectly elastic then

1) after the collision both comes to rest
2) after the collision first comes to rest and second moves in the opposite direction with same speed.
3) after collision they recoil with same speed
4) both and 1 and 2

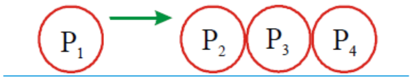
59. A body of mass 'm' moving with certain velocity collides with another identical body at rest. If the collision is perfectly elastic and after the collision both the bodies moves
1) in the same direction
2) in opposite direction
3) in perpendicular direction
4) at 45° to each other

60. Six steel balls of identical size are lined up along a straight frictionless groove. Two similar balls moving with speed v along the groove collide with this row on the extreme left end. Then
1) one ball from the right end will move on with speed v
2) two balls from the extreme right end will move on with speed v and the remaining balls will be at rest
3) all the balls will start moving to the right with speed v/8
4) all the six balls originally at rest will move on with speed v/6 and the incident balls will come to rest

61. A lighter body moving with a velocity v collides with a heavier body at rest. Then
1) the lighter body rebounced with twice the velocity of bigger body
2) the lighter body retraces its path with the same velocity in magnitude
3) the heavier body does not move practically
4) both (2) and (3)

62. A heavier body moving with certain velocity collides head on elastically with a lighter body at rest, then
1) smaller body continues to be in the same state of rest
2) smaller body starts to move in the same direction with same velocity as that of bigger body
3) the smaller body start to move with twice the velocity of the bigger body in the same direction
4) the bigger body comes to rest

63. A perfectly elastic ball P_1 of mass 'm' moving with velocity v collides elastically with three exactly similar balls P_2, P_3, P_4 lying on a smooth table. Velocity of the four balls after collision are



1) 0,0,0,0 2) v, v, v, v
3) v, v, v,0 4) 0, 0, 0, v

64. Two bodies P and Q of masses m_1 and m_1 ($m_2 > m_1$) are moving with velocity v_1 and v_2 respectively, collide with each other. Then the force exerted by P on Q during the collision is
1) greater that the force exerted by Q on P
2) less than the force exerted by Q on P
3) same as the force exerted by Q on P
4) same as the force exerted by Q on P but opposite in direction

65. The coefficient of restitution (e) for a perfectly elastic collision is
1) -1 2) 0 3) ∞ 4)1

66. A ball of mass M moving with a velocity v collides perfectly inelastically with another ball of same mass but moving with a velocity v in the opposite direction. After collision
1) both the balls come to rest
2) the velocities are exchanged between the two balls
3) both of them move at right angles to the original line of motion
4) one ball comes to rest and another ball travels back with velocity 2v

67. A ball of mass 'm' moving with speed 'u' undergoes a head-on elastic collision with a ball of mass 'nm' initially at rest. Find the fraction of the incident energy transferred to the second ball.

1) $\frac{n}{n+1}$ 2) $\frac{n}{(n+1)^2}$ 3) $\frac{2n}{(1+n)^2}$ 4) $\frac{4n}{(1+n)^2}$

68. A small bob of a simple pendulum released from 30° to the vertical hits another bob of the same mass and size lying at rest on the table vertically

below the point of suspension. After elastic collision, the angular amplitude of the pendulum will be
1) 30° 2) 60° 3) 15° 4) zero

69. Two spheres 'X' and 'Y' collide. After collision, the momentum of X is doubled. Then
1) the initial momentum of X and Y are equal
2) the initial momentum of X is greater then that of Y
3) the initial momentum of Y is double that of X
4) the loss in momentum of Y is equal to the initial momentum of X

70. A bullet is fired into a wooden block. If the bullet gets embedded in wooden block, then
1) momentum alone is conserved
2) kinetic energy alone is conserved
3) both momentum and kinetic energy are conserved
4) neither momentum nor kinetic energy are conserved

71. During collision, which of the following statement is wrong?
1) there is a change in momentum of individual bodies
2) the change in total momentum of the system of colliding particle is zero
3) the change in total energy is zero
4) law of conservation of momentum is not valid

LEVEL 1 (A) - KEY

01) 4 02) 4 03) 1 04) 1 05) 4 06) 1
07) 4 08) 2 09) 3 10) 4 11) 3 12) 4
13) 3 14) 3 15) 4 16) 2 17) 3 18) 3
19) 4 20) 3 21) 3 22) 1 23) 1 24) 1
25) 1 26) 3 27) 1 28) 1 29) 1 30) 4
31) 3 32) 3 33) 2 34) 3 35) 1 36) 1
37) 1 38) 4 39) 3 40) 4 41) 2 42) 2
43) 3 44) 3 45) 3 46) 3 47) 3 48) 1
49) 2 50) 2 51) 4 52) 1 53) 4 54) 2
55) 3 56) 1 57) 1 58) 3 59) 3 60) 2
61) 4 62) 3 63) 4 64) 4 65) 4 66) 1
67) 4 68) 4 69) 4 70) 1 71) 4

LEVEL – 1 (B)

WORK DONE BY CONSTANT FORCE

1. If $\vec{F} = 2\hat{\imath}+3\,\hat{\jmath}+ 4\,\hat{k}$ acts on a body and displaces it by $\vec{S} = 3\hat{\imath}+ 2\hat{\jmath}+ 5\hat{k}$, then the work done by the force is
1) 12 J 2) 20 J 3) 32 J 4) 64 J

2. A force of 1200 N acting on a stone by means of a rope slides the stone through a distance of 10m in a direction inclined at 600 to the force. The work done by the force is
1) 6000 3J 2) 6000J
3) 12000J 4) 8000J

3. A man weighing 80 kg climbs a staircase carrying a 20 kg load. The staircase has 40 steps, each of 25 cm height. If he takes 20 seconds to climb, the work done is
1) 9800J 2) 490 J
3) 98x105J 4) 7840J

4. The work done by a force $\vec{F}$=3$\hat{\imath}$-4$\hat{\jmath}$+5$\hat{k}$ displaces the body from a point (3,4,6) to a point (7,2,5) is
1) 15 units 2) 25 units
3) 20 units 4) 10 units

5. A force $\vec{F}$ = (6$\hat{\imath}$ -8$\hat{\jmath}$)N, acts on a particle and displaces it over 4 m along the X-axis and 6m along the Y-axis. The work done during the total displacement is
1) 72 J 2) 24 J
3) - 24 J 4) zero

6. A lawn roller is pulled along a horizontal surface through a distance of 20 m by a rope with a force of 200 N. If the rope makes an angle of 60° with the vertical while pulling, the amount of work done by pulling force is
1) 4000 J 2) 1000 J
3) 2000 3J 4) 2000 J

WORK DONE BY VARIABLE FORCE

7. An object has a displacement from position vector $\vec{r}_1 = (2\hat{\imath} + 3\hat{\jmath})$ m $\vec{r}_2 = (4\hat{\imath} + 6\hat{\jmath})$ N, then work done by the force is
1) 24J 2) 33J 3) 83J 4) 45J

KINETIC ENERGY

8. A shot is fired at 30° with the vertical from a point on the ground with kinetic energy K. If air resistance is ignored, the kinetic energy at the top of the trajectory is
1) 3K/4 2) K/2 3) K 4) K/4

9. A body starts from rest and is acted on by a constant force. The ratio of kinetic energy gained by it in the first five seconds to that gained in the next five seconds is
1) 2 : 1 2) 1 : 1 3) 3 : 1 4) 1 : 3

10. The mass of a simple pendulum bob is 100 gm. The length of the pendulum is 1 m. The bob is drawn aside from the equilibrium position so that the string makes an angle of 60° with the vertical and let go. The kinetic energy of the bob while crossing its equilibrium position will be
1) 0.49 J 2) 0.94 J
3) 1 J 4) 1.2 J

11. A body starts from rest and moves with uniform acceleration. What is the ratio of kinetic energies at the end of 1st, 2nd and 3rd seconds of its journey?
1)1 : 8 : 27 2)1 : 2 : 3
3)1 : 4 : 9 4)3 : 2 : 1

12. A liquid of specific gravity 0.8 is flowing in a pipe line with a speed of 2 m/s. The K.E. per cubic meter of it is
1) 160 J 2) 1600 J
3) 160.5 J 4) 1.6 J

13. A 60 kg boy lying on a surface of negligible friction throws horizontally a stone of mass 1 kg with a speed of 12 m/s away from him. As a result with what kinetic energy he moves back?
1) 2.4 J 2) 72 J
3) 1.2 J 4) 36 J

14. Two stones of masses m and 2 m are projected vertically upwards so as to reach the same height. The ratio of the kinetic energies of their projection is
1) 2 : 1 2) 1 : 2
3) 4 : 1 4) 1 : 4

15. A neutron, one of the constituents of a
nucleus, is found to pass two points 60 metres apart in a time interval of 1.8×10^{-4} sec. The mass of the neutron is 1.67×10^{-27} kg. Assuming that the speed is constant, its kinetic energy is
1) 9.3×10^{-17} joule 2) 9.3×10^{-14} joule
3) 9.3×10^{-21} joule 4) 9.3×10^{-11} joule

POTENTIAL ENERGY

16. A tank of size 10 m × 10 m × 10 m is full of water and built on the ground. If g = 10 ms^{-2}, the potential energy of the water in the tank is
1) 5×10^7J 2) 1×10^8 J
3) 5×10^4 J 4) 5×10^5 J

17. A bolt of mass 0.3kg falls from the ceiling of an elevator moving down with an uniform speed of 7m/s. It hits the floor of the elevator (length of the elevator = 3m) and does not rebound. What is the heat produced by impact?
1)8.82J 2)7.72J 3)6.62J 4)5.52J

POTENTIAL ENERGY OF A SPRING

18. A spring when compressed by 4 cm has 2 J energy stored in it. The force required to extend it by 8 cm will be
1) 20 N 2) 2 N
3) 200 N 4) 2000 N

19. The elastic potential energy of a stretched spring is given by E = $50x^2$. Where x is the displacement in meter and E is in joule, then the force constant of the spring is
1)50Nm 2)100N m^{-1}
3)100 N/m^2 4) 100 Nm

WORK ENERGY THEOREM BY CONSTANT FORCE

20. A body of mass 2 kg is projected with an initial velocity of 5 ms-1 along a rough horizontal table. The work done on the body by the frictional forces before it is brought to rest is
1) 250 J 2) 25 J
3) -250 J 4) -25 J

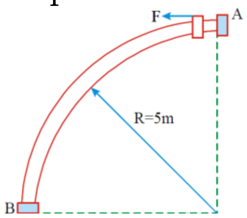
21. An object is acted on by a retarding force of 10 N and at a particular instant its kinetic energy is 6 J. The object will come to rest after it has travelled a distance of
1) 3/5 m 2) 5/3 m
3) 4 m 4) 16 m

22. By applying the brakes without causing a skid, the driver of a car is able to stop his car with in a distance of 5 m, if it is going at 36 kmph. If the car were going at 72 kmph, using the same brakes, he can stop the car over a distance of
1) 10 m 2) 2.5 m
3) 20 m 4) 40 m

23. A bullet fired into a trunk of a tree loses 1/4 of its kinetic energy in travelling a distance of 5 cm. Before stopping it travels a further distance of
1) 150 cm 2) 1.5 cm
3) 1.25 cm 4) 15 cm

WORK ENERGY THEOREM FOR VARIABLE FORCE

24. A bead of mass $\frac{1}{2}\,kg$ starts from rest from "A" to move in a vertical plane along a smooth fixed quarter ring of radius 5m, under the action of a constant horizontal force F = 5 N as shown. The speed of bead as it reaches
point "B" is

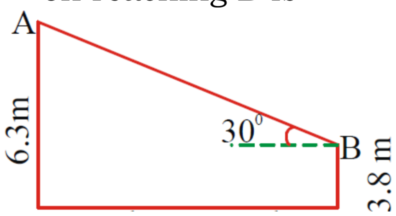


1) 14.14 m/s 2) 7.07 m/s
3) 5 m/s 4) 25 m/s

CONSERVATION OF MECHANICAL ENERGY

25. A cradle is 'h' meters above the ground at the lowest position and 'H' meters when it is at the highest point. If 'v' is the maximum speed of the swing of total mass 'm' the relation between 'h' and 'H' is
1) ½ mv² + h = H 2) (v²/2g) + h = H
3) (v²/g) + 2h = H 4) (v²/2g) + H = h

26. AB is a frictionless inclined surface making an angle of 300 with horizontal. A is 6.3 m above the ground while B is 3.8 m above the ground. A block slides down from A, initially starting from rest. Its velocity on reaching B is



1) 7 m s^{-1} 2) 14 m s^{-1}
3) 7.4 m s^{-1} 4) 4.9 m s^{-1}

27. A stone of mass "m" initially at rest and dropped from a height "h" strikes the surface of the earth with a velocity "v". If the gravitational force acting on the stone is W, then which of the following identities is correct?
1) mv - mh = 0
2) ½ mv² - Wh² = 0
3) ½ mv² - Wh = 0
4) ½ mv² - mh = 0

POWER

28. A motor boat is going in a river with a velocity $\vec{V} = (4\hat{\imath} + 2\hat{\jmath} + \hat{k})$ ms^{-1} . If the resisting force due to stream is $\vec{F} = (5\hat{\imath} - 10\hat{\jmath} + \hat{k})$ N, then the power of the motor boat is
1) 100 W 2) 50 W
3) 46 W 4) 23 W

29. Two riffles fire the same number of bullets in a given interval of time. The second fires bullets of mass twice that fired by the first and with a velocity that is half that of the first. The ratio of their powers is
1) 1 : 4 2) 4 : 1
3) 1 : 2 4) 2 : 1

30. A car weighing 1000 kg is going up an incline with a slope of 2 in 25 at a steady speed of 18 kmph. If g= 10 ms-2, the power of its engine is
1) 4 kW 2) 50 kW
3) 625 kW 4) 25 kW

31. A crane can lift up 10,000 kg of coal in 1 hour from a mine of 180 m depth. If the efficiency of the crane is 80 %, its input power must be (g = 10 ms^{-2})
1) 5 kW 2) 6.25 kW
3) 50 kW 4) 62.5 kW

32. A man carries a load of 50 kg through a height of 40 m in 25 seconds. If the power of the man is 1568 W, his mass is
1) 5 kg 2) 1000 kg
3) 200 kg 4) 50 kg

33. An electric motor creates a tension of 4500 newton in a hoisting cable and reels it at the rate of 2m/s. What is the power of the motor?
1) 15 kW 2) 9 kW
3) 225 W 4) 9000 kW

34. A juggler throws continuously balls at the rate of three in each second, each with a velocity of 10 ms^{-1}. If the mass of each ball is 0.05 kg his power is
1) 2 W 2) 50 W 3) 0.5 W 4) 7.5 W

ELASTIC AND INELASTIC COLLISIONS IN ONE DIMENSION

35. A 6 kg mass travelling at 2.5 ms^{-1} collides head on with a stationary 4 kg mass. After the collision the 6 kg mass travels in its original direction with a speed of 1ms^{-1} . The final velocity of 4 kg mass is
 1) 1 ms^{-1} 2) 2.25ms^{-1}
 3) 2 ms^{-1} 4) 0 ms^{-1}
36. A body of mass 10 kg moving with a velocity of 5 ms-1 hits a body of 1 gm at rest. The velocity of the second body after collision, assuming it to be perfectly elastic is
 1) 10 ms^{-1} 2) 5 ms^{-1}
 3) 15 ms^{-1} 4) 0.10 ms^{-1}
37. A block of mass 1 kg moving with a speed of 1 4 ms^{-1}, collides with another block of mass 2 kg which is at rest. The lighter block comes to rest after collision. The loss in KE of the system is
 1) 8 J 2) 4 × 10 J^{-7}
 3) 4 J 4) 0 J
38. A marble going at a speed of 2 ms^{-1} hits another marble of equal mass at rest. If the collision is perfectly elastic, then the velocity of the first marble after collision is
 1) 4 ms^{-1} 2) 0 ms^{-1}
 3) 2 ms^{-1} 4) 3 ms^{-1}
39. A massive ball moving with a speed v collides head on with a fine ball having mass very much smaller than the mass of the first ball at rest. The collision is elastic and then immediately after the impact, the second ball will move with a speed approximately equal to
 1) v 2) 2v 3) v/3 4) infinite
40. A 1 kg ball moving at 12 m/s collides head on with a 2 kg ball moving in the opposite direction at 24 m/s. The velocity of each ball after the impact, if the coefficient of restitution is 2/3 is
 1) -28 m/s ; -4 m/s
 2) 28 m/s ; -4 m/s
 3) 20 m/s ; 24 m/s
 4) -20 m/s ; -4 m/s
41. A 6 kg mass collides with a body at rest. After the collision, they travel together with a velocity one third the velocity of 6 kg mass. The mass of the second body is
 1) 6 kg 2) 3 kg 3) 12 kg 4) 18 kg
42. A body of mass m moving at a constant velocity v hits another body of the same mass moving with a velocity v/2 but in the opposite direction and sticks to it. The common velocity after collision is
 1) v 2) v/4 3) 2v 4) v/2
43. An 8 gm bullet is fired horizontally into a 9 kg block of wood and sticks in it. The block which is free to move, has a velocity of 40 cm/s after impact. The initial velocity of the bullet is
 1) 450 m/s 2) 450 cm/s
 3) 220 m/s 4) 220 cm/s
44. A block of wood of mass 9.8 kg is suspended by a string. A bullet of mass 200 gm strikes horizontally with a velocity of 100 ms-1 and gets embedded in it. The maximum height attained by the block is (g = 10 ms^{-2})
 1) 0.1 m 2) 0.2 m
 3) 0.3 m 4) 0 m
45. A 15 gm bullet is fired horizontally into a 3 kg block of wood suspended by a string. The bullet sticks in the block, and the impact causes the block to swing 10 cm above the initial level. The velocity of the bullet nearly is (in ms^{-1})
 1) 281 2) 326 3) 184 4) 58
46. A body of mass 20 gm is moving with a certain velocity. It collides with another body of mass 80 gm at rest. The collision is perfectly inelastic. The ratio of the kinetic energies before and after collision of the system is
 1) 2 : 1 2) 4 : 1
 3) 5 : 1 4) 3 : 2

COEFFICIENT OF RESTITUTION

47. A rubber ball drops from a height ‘h’. After rebounding twice from the ground, it rises to h/2. The co-efficient of restitution is
 1) $\frac{1}{2}$ 2) $\left(\frac{1}{2}\right)^{1/2}$ 3) $\left(\frac{1}{2}\right)^{1/4}$ 4) $\left(\frac{1}{2}\right)^{1/6}$
48. A body dropped freely from a height h onto a horizontal plane, bounces up and down and finally comes to rest. The coefficient of restitution is e. The

ratio of velocities at the beginning and after two rebounds is

1) 1 : e 2) e : 1 3) 1 : e^2 4) e^2 : 1

49. In the above problem, the ratio of times of two consecutive rebounds is

1) 1 : e 2) e : 1 3) 1 : e^2 4) e^2 : 1

50. In the above problem the ratio of distances travelled in two consecutive rebounds is

1) 1 : e 2) e : 1
3) 1 : e^2 4) e^2 : 1

51. A ball is dropped onto a horizontal floor. It reaches a height of 144 cm on the first bounce and 81 cm on the second bounce. The coefficient of restitution is

1) 0 2) 0.75 3) 81/144 4) 1

52. A ball is dropped onto a horizontal floor. It reaches a height of 144 cm on the first bounce and 81 cm on the second bounce. The height it attains on the third bounce is

1) 45.6 cm 2) 81 cm
3) 144 cm 4) 0 cm

53. A ball is dropped from height 'H' onto a horizontal surface. If the coefficient of restitution is 'e' then the total time after which it comes to rest is

1) $\sqrt{\frac{2H}{g}}\left(\frac{1-e}{1+e}\right)$ 2) $\sqrt{\frac{2H}{g}}\left(\frac{1+e}{1-e}\right)$

3) $\sqrt{\frac{2H}{g}}\left(\frac{1+e^2}{1-e^2}\right)$ 4) $\sqrt{\frac{2H}{g}}\left(\frac{1-e^2}{1+e^2}\right)$

54. A stationary body explodes into two fragments of masses m_1 and m_2. If momentum of one fragment is p, the energy of explosion is

1) $\frac{p^2}{2(m_1+m_2)}$ 2) $\frac{p^2}{2\sqrt{m_1 m_2}}$

3) $\frac{p^2(m_1+m_2)}{2m_1 m_2}$ 4) $\frac{p^2}{2(m_1-m_2)}$

LEVEL – 1 (B) - KEY

01)3 02)2 03)1 04)1 05)3 06)3
07)3 08)4 09)4 10)1 11)3 12)2
13)3 14)2 15)1 16)1 17)1 18)3
19)2 20)4 21)1 22)3 23)4 24)1
25)2 26)1 27) 3 28)3 29)4 30)1
31)2 32)4 33)2 34)4 35)2 36)1
37)3 38)2 39)2 40)1 41)3 42)2
43)1 44)2 45) 1 46) 3 47)3 48)3
49)1 50) 3 51) 2 52) 1 53)2 54) 3

LEVEL – 2

WORK DONE BY CONSTANT FORCE

1. A body of mass 5 kg is moved up over 10 m along the line of greatest slope of a smooth inclined plane of inclination 30° with the horizontal. If g = 10 m/s², the work done will be

1) 500 J 2) 2500 J
3) 250 J 4) 25 J

2. A particle of mass 0.5 kg is displaced from position $\vec{r}_1$ (2,3,1) to $\vec{r}_2$ (4,3,2) by applying a force of magnitude 30 N which is acting along $(\hat{\imath}+\hat{\jmath}+\hat{k})$. The work done by the force is

1) 10 3 J 2) 30 3 J
3) 30 J 4) 40 J

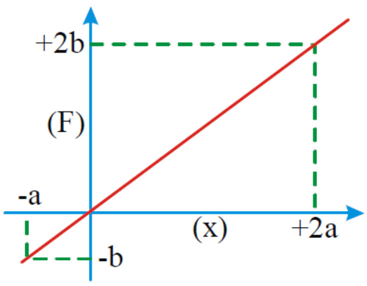
3. Kinetic energy of a particle moving in a straight line varies with time t as $K2 = 4t$. The force acting on the particle

1) is constant
2) is increasing
3) is decreasing
4) first increases and then decreases

4. A block of mass 5 kg initially at rest at the origin is acted upon by a force along the positive X - direction represented by F=(20 +5x)N. Calculate the work done by the force during the displacement of the block from x = 0 to x = 4m.

1) 100 J 2) 150 J
3) 120 J 4) 75 J

5. A force F acting on a particle varies with the position x as shown in the graph. Find the work done by the force in displacing the particle from x = -a to x=+2a



6. A force $\vec{F}$ = $(2\hat{\imath} + 3\hat{\jmath} - 4\hat{k})N$ acts on a particle which is constrained to move in the XOY plane along the line x = y. If the particle moves $5\sqrt{2}m$, the work done by force in joule is

1) $25\sqrt{2}$ 2) $5\sqrt{58}$ 3) 25 4) 10

7. Two forces each of magnitude 10 N act

simultaneously on a body with their directions inclined to each other at an angle of 120° and displaces the body over 10 m along the bisector of the angle between the two forces. Then the work done by each force is

1) 5 J 2) 1 J 3) 50 J 4) 100 J

8. 'n' identical cubes each of mass 'm' and edge 'L' are on a floor. If the cubes are to be arranged one over the other in a vertical stack, the work to be done is

1) Lmng (n-1)/2 2) Lg(n-1)/mn
3) (n-1)/Lmng 4) Lmng/2(n-1)

9. A chain of mass m and length 'L' is over
hanging from the edge of a smooth horizontal table such that 3/4th of its length is lying on the table. The work done in pulling the chain completely on to the table is

1) mgL/16 2) mgL/32
3) 3mgL/32 4) mgL/8

WORK DONE BY VARIABLE FORCE

10. A body is displaced from (0,0) to (1m, 1m) along the path x = y by a force F = $(x^2\hat{j} + y\hat{i})$ N. The work done by this force will be

1) $\frac{4}{3}J$ 2) $\frac{5}{6}J$ 3) $\frac{3}{2}J$ 4) $\frac{7}{5}J$

11. A particle moves under the effect of a force F = C x from x = 0 to x = x_1. The work done in the process is (treat C as a constant)

1) C^2/x_1^2 2) Cx_1^2
3) $1/2Cx_1^2$ 4) $1/2C^2/x_1^2$

12. Under the action of a force, a 2 kg body moves such that its position 'x' in meters as a function of time 't' in seconds given by: $x = t^2/2$. The work done by the force in the first 5 seconds is

1) 2.5 J 2) 0.25 J
3) 25 J 4) 250 J

13. A body of mass 5 kg at rest under the action of a force which gives its velocity given by v = 3×t m/s, here 't' is time in seconds. The work done by the force in two seconds will be

1) 90 J 2) 45 J
3) 180 J 4) 30 J

KINETIC ENERGY

14. A body freely falls from a certain height onto the ground in a time 't'. During the first one third of the interval it gains a kinetic energy ΔK_1 and during the last one third of the interval, it gains a kinetic energy ΔK_2 . The ratio $\Delta K_1 : \Delta K_2$ is

1) 1 : 1 2) 1 : 3
3) 1 : 4 4) 1 : 5

15. A man has twice the mass of a boy and has half the kinetic energy of the boy. The ratio of the speeds of the man and the boy must be

1) 2 : 1 2) 4 : 1
3) 1 : 4 4) 1 : 2

16. The speed of a car changes from 0 to 5 ms-1 in the first phase and from 5 ms-1 to 10 ms^{-1} in the second phase and from 10 ms^{-1} to 15 ms^{-1} during the third phase. In which phase the increase in kinetic energy is more?

1) first phase
2) second phase
3) third phase
4) same in all the three phases

POTENTIAL ENERGY

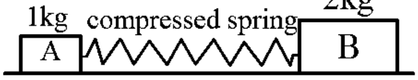
17. A rubber ball falling from a height of 5m rebounds from a hard floor to a height of 3.5m. The % loss of energy during the impact is

1) 20% 2) 30% 3) 43% 4) 50%

18. When a long spring is stretched by x cm, its P.E is U. If the spring is stretched by Nx cm, the P.E stored in it will be

1) $\frac{U}{n}$ 2) NU 3) N^2U 4) $\frac{U}{N^2}$

19. An elastic spring is compressed between two blocks of masses 1 kg and 2 kg resting on a smooth horizontal table as shown. If the spring has 12J of energy and suddenly released, the velocity with which the larger block of 2 kg moves will be



1) 2 m/s 2) 4 m/s
3) 1 m/s 4) 8 m/s

20. A block of mass 2 kg is on a smooth horizontal surface. A light spring of force constant 800 N/m has one end

rigidly attached to a vertical wall and lying on that horizontal surface. Now the block is moved towards the wall compressing the spring over a distance of 5 cm and then suddenly released. By the time the spring regains its natural length and looses contact with the block, the velocity acquired by the block will be

1) 200m/s 2) 100 m/s
3) 2m/s 4) 1m/s

WORK ENERGY THEOREM FOR CONSTANT FORCE

21. A bullet of mass 10gm is fired horizontally with a velocity $1000ms^{-1}$ from a riffle situated at a height 50 m above the ground. If the bullet reaches the ground with a velocity $500ms^{-1}$, the work done against air resistance in the trajectory of the bullet is (in joule) ($g = 10ms^{-2}$)
1) 5005 2) 3755
3) 3750 4) 17.5

22. A drop of mass 1.00 g falling from a height 1.00 km. It hits the ground with a speed of $50.0ms^{-1}$. What is the work done by the unknown resistive force?
1) -8.75J 2) 8.75J
3) -4.75J 4) 4.75J

23. A block of mass 5 kg is initially at rest on a rough horizontal surface. A force of 45 N acts on it in a horizontal direction and pushes it over a distance of 2 m. The force of friction acting on the block is 25 N. The final kinetic energy of the block is
1) 40 J 2) 90 J
3) 50J 4) 140 J

WORK ENERGY THEOREM FOR VARIABLE FORCE

24. A block of mass 2 kg is initially at rest on a horizontal frictionless surface. A horizontal force $F = (9 - x^2)\bar{\imath}$ newton acts on it, when the block is at $x = 0$. The maximum kinetic energy of the block between $x = 0$m and $x = 3$ m in joule is
1) 24 2) 20 3) 18 4) 15

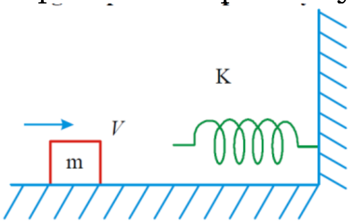
Conservation of mechanical energy

25. A freely falling body takes 4s to reach the ground. One second after release, the percentage of its potential energy, that is still retained is
1) 6.25% 2) 25%
3) 37.5% 4) 93.75%

26. A vertically projected body attains the maximum height in 6s. The ratio of kinetic energy at the end of 3rd second to decrease in kinetic energy in the next three seconds is
1) 1: 1 2) 1 : 3 3) 3 : 1 4) 9 : 1

27. Two identical blocks A and B, each of mass 'm' resting on smooth floor are connected by a light spring of natural length L and the spring constant k, with the spring at its natural length. A third identical block C (mass m) moving with a speed v along the line joining A and B collides with A. The maximum compression in the spring is:
1) $v\sqrt{\frac{m}{2k}}$ 2) $m\sqrt{\frac{v}{2k}}$ 3) $\sqrt{\frac{mv}{2k}}$ 4) $\frac{mv}{2k}$

28. A block of mass m=25kg on a smooth horizontal surface with a velocity $\vec{v} = 3ms^{-1}$ meets the spring of spring constant k = 100 N/m fixed at one end as shown in figure. The maximum compression of the spring and velocity of block as it returns to the original position respectively are



1) $1.5m$, $3ms^{-1}$ 2) $1.5m, 0ms^{-1}$
3) $1.0m, 3ms^{-1}$ 4) $0.5m$, $2ms^{-1}$

29. A body is thrown vertically up with certain initial velocity, the potential and kinetic energies of the body are equal at a point P in its path. If the same body is thrown with double the velocity upwards, the ratio of potential and kinetic energies of the body when it crosses the same point, is
1) 1:1 2) 1: 4 3) 1:7 4) 1:8

POWER

30. A machine rated as 150W, changes the velocity of a 10kg mass from $4ms^{-1}$ to 10 ms^{-1} in 4s. The efficiency of the machine is nearly
1) 70% 2) 30% 3) 50% 4) 40%

31. A pump is required to lift 600 kg of water per minute from a well 25m deep and to eject it with a speed of 50 ms^{-1}. The power required to perform the above task is
1) 10 kW 2) 15 kW
3) 20 kW 4) 25 Kw

32. A tank on the roof of a 20 m high building can hold 10 m3 of water. The tank is to be filled from a pond on the ground in 20 minutes. If the pump has an efficiency of 60 %, then the input power in kW is
1) 1.1 2) 2.74
3) 5.48 4) 7.0

33. An electric fan, with effective area of crosssection 'A', accelerates air of density ' r ' to a speed 'v'. What is the power needed for this process?
1) r Av 2) ½ r Av
3) r Av2 4) ½ r Av^3

ELASTIC AND INELASTIC COLLISIONS

34. A body of mass 5kg moving with a speed of $3ms^{-1}$ collides head on with a body of mass 3kg moving in the opposite direction at a speed of $2ms^{-1}$. The first body stops after the collision. The final velocity of the second body is
1) $3ms^{-1}$ 2) $5ms^{-1}$
3) $-9ms^{-1}$ 4)$30ms^{-1}$

35. Three identical particles moving with velocities $v_0\hat{i}$ $-3v_0\hat{j}$ *and* $5v_0\hat{k}$ collide successively with each other in such a way that they form a single particle. The velocity of resultant particle in i, j, k form is
1) $v_0\left(\hat{i}-3\hat{j}+5\hat{k}\right)$ 2) $\frac{v_0}{3}\left(\hat{i}-3\hat{j}+5\hat{k}\right)$
3) $\frac{v_o}{2}\left(\hat{i}-3\hat{j}+5\hat{k}\right)$ 4) $\frac{v_0}{3}\left(\hat{i}+3\hat{j}+5\hat{k}\right)$

36. From the top of a tower of height 100m a 10 gm block is dropped freely and a 6gm bullet is fired vertically upwards from the foot of the tower with velocity $100ms^{-1}$ simultaneously. They collide and stick together. The common velocity after collision is ($g=10ms^{-2}$)
1) $27.5ms^{-1}$ 2) $150ms^{-1}$
3) $40ms^{-1}$ 4) $100ms^{-1}$

37. A steel ball of radius 2cm is initially at rest. It is struck head on by another steel ball of radius 4cm travelling with a velocity of 81 cm/s. If the collision is elastic their respective final velocities are
1) 63 cm/s, 144 cm/s
2) 144 cm/s, 63 cm/s
3) 19 cm/s, 100 cm/s
4) 100 cm/s, 19 cm/s

38. A steel ball of radius 2cm is initially at rest. It is struck head on by another steel ball of radius 4cm travelling with a velocity of 81cm/s. The common velocity if it is perfectly inelastic collision
1) 144 cm/s 2)61 cm/s
3)81 cm/s 4) 72 cm/s

COEFFICIENT OF RESTITUTION

39. A tennis ball bounces down a flight of stairs, striking each step in turn and rebounding to half of height of the step. The coefficient of restitution is
1) 1/2 2) $\frac{1}{\sqrt{2}}$ 3) $\left(\frac{1}{\sqrt{2}}\right)^{1/2}$ 4) $\left(\frac{1}{\sqrt{2}}\right)^{1/4}$

40. A ball hits the ground and loses 20% of its momentum. Coefficient of restitution is
1) 0.2 2) 0.4 3) 0.6 4) 0.8

41. A plastic ball falling from a height 4.9m rebounds number of times. If total time for second collision is2.4 sec, then coefficient of restitution is
1) 0.3 2) 0.4 3) 0.7 4) 0.6

42. A ball is dropped from a height 'h' on to a floor of coefficient of restitution 'e'. The total distance covered by the ball just before second hit is (2008 E)
1) $h(1-2e^2)$ 2) $h(1+2e^2)$
3) $(1+e^2)$ 4) he^2

43. In two separate collisions, the coefficient of restitutions 1 *e* and 2 *e* are in the ratio 3:1. In the first collision the relative velocity of approach is twice the relative velocity of separation. Then, the ratio between relative velocity of approach and relative velocity of separation in the second collision is
(2007 E)
1) 1:6 2) 2:3 3) 3:2 4) 6:1

44. A sphere of mass m moving with constant velocity u, collides with another stationary sphere of same mass. If e is the coefficient of restitution, the ratio of the final

velocities of the first and second sphere is (2007 M)

1) $\frac{1+e}{1-e}$ 2) $\frac{1-e}{1+e}$ 3) $\frac{e}{1-e}$ 4) $\frac{1+e}{e}$

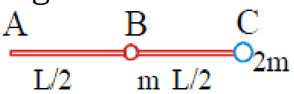
LEVEL – 2 - KEY

01) 3 02) 2 03) 1 04) 3 05) 1 06)3
07)3 08)1 09)2 10)2 11) 3 12)3
13)1 14)4 15)4 16)3 17)2 18)3
19)1 20)4 21)2 22)1 23)1 24)3
25)4 26)1 27)1 28)1 29)3 30)1
31)2 32)2 33)4 34)1 35)2 36)1
37)2 38)4 39)2 40)4 41)3 42)2
43)4 44)2

LEVEL – 3

WORK DONE BY A FORCE

1) A long rod ABC of mass "m" and length "L" has two particles of masses "m" and "2m" attached to it as shown in the figure. The system is initially in the horizontal position. The work to be done to keep it vertical with A hinged at the bottom is



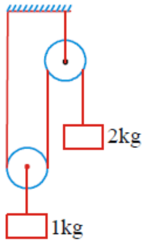
1) 2mgL 2) 3mgL/2
3) 5mgL/2 4) 3mgL

2) A particle of mass 100g is thrown vertically upwards with a speed of 5m/s. The work done by the force of gravity during the time the particle goes up is
1) -0.5J 2) -1.25J
3) 1.25J 4) 0.5J

3) A large slab of mass 5 kg lies on a smooth horizontal surface, with a block of mass 4 kg lying on the top of it. The coefficient of friction between the block and the slab is 0.25. If the block is pulled horizontally by a force of F = 6 N, then the work done by the force of friction on the slab, between the instants $t = 2s$ to $t = 3s$ is ($g = 10\ ms^2$)

1) 2.4 J 2) 5.55 J 3) 4.44J 4) 10 J

4) In the pulley – block system shown in figure, strings are light. Pulleys are massless and smooth. System is released from rest. In 0.3 seconds



a) work done on 2 kg block by gravity is 6J
b) work done on 2 kg block by string is -2J
c) work done on 1 kg block by gravity is −1.5J
d) work done on 1 kg block by string is 2 J
1) only a, d are correct
2) only b, d are correct
3) only a, b, c are correct
4) All are correct

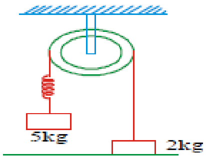
5) A body of mass 0.5 kg travels in a straight line with a velocity $v = ax^{\frac{3}{2}}$ where $a = 5m^{-\frac{1}{2}}s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$m?
1) 50 J 2) 20 J 3) 80 J 4) 45.5 J

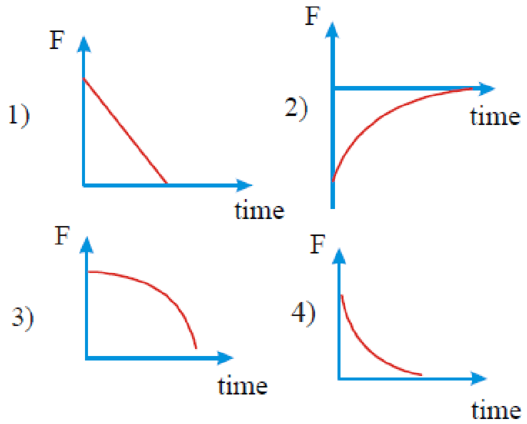
P.E AND K.E

6) A particle of mass 2 kg starts moving in a straight line with an initial velocity of 2 m/s at a constant acceleration of 2m/s^2. Then rate of change of kinetic energy
1) is four times the velocity at any moment
2) is two times the displacement at any moment
3) is four times the rate of change of velocity at any moment
4) is constant throughout

7) A running man has half the kinetic energy that a running boy of half his mass has. The man speeds up by 1.0 m/s and now has the same kinetic energy as the boy. The original speed of the man expressed in m/s units must be
1) $\sqrt{2}+1$ 2) $\sqrt{2}-1$
3) $\sqrt{2}+2$ 4) $\sqrt{2}-2$

8) The kinetic energy (KE) versus time graph for a particle moving along a straight line is shown in the figure. The force vs time graph for the particle may be



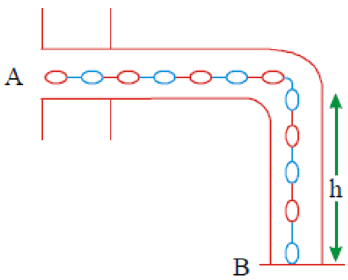


WORK - ENERGY THEOREM

9) A lifting machine, having an efficiency of 80% uses 2500 J of energy in lifting a 10 kg load over a certain height. If the load is now allowed to fall through that height freely, its velocity at the end of the fall will be (g=10 m/s^2)

1) 10 m s^{-1} 2) 15 m s^{-1}
3) 20 m s^{-1} 4) 25 m s^{-1}

10) A chain AB of length L is lying in a smooth horizontal tube so that a fraction h of its length L, hangs freely and touches the surface of the table with its end B. At a certain moment, the end A of the chain is set free. The velocity of end A of the chain, when it slips out of tube , is



1) $h\sqrt{\frac{2g}{Lh}}$ 2) $\sqrt{2gh\log_e\left(\frac{L}{h}\right)}$
3) $\sqrt{2ghl\log_e\left(\frac{L}{h}\right)}$ 4) $\frac{1}{hL}\sqrt{2g}$

11) A block of mass m = 1 kg moving on a horizontal surface with speed $v_i = 2ms^{-1}$ enters a rough patch ranging from = 0 . 10m to x = 2.01m. The retarding force $F_r = \frac{-k}{x}$ for 0.1m < x < 2.01 m, F_r = 0 for x < 0 .1m and x >2.01 m where k = 0.5 J, the final speed v_f of the block as it crosses this patch

1) 2ms^{-1} 2) 1ms^{-1} 3) 3ms^{-1} 4) 0.5ms^{-1}

12) A 1.5kg block is initially at rest on a horizontal frictionless surface. A horizontal force $\vec{F} = (4 - x^2)\hat{\imath}$ is applied on the block. Initial position of the block is at x = 0. The maximum kinetic energy of the block between x = 0 and x = 2m is

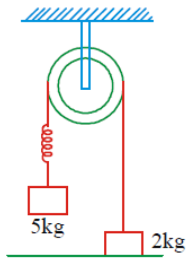
1) 2.33J 2) 8.67 J 3) 5.33 J 4) 6.67 J

CONSERVATION OF MECHANICAL ENERGY

13) The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, the speed with which the bob arrives at the lowest point, given that it dissipated 5% of its initial energy against air resistance is (m/s)

1)3.14 2) 5.28 3) 1.54 4) 8.26

14) System shown in fig is released from rest with mass 2kg in contact with the ground. Pulley and spring are massless and the friction is absent everywhere. The speed of 5kg block when 2kg block leaves the contact with the ground is (force constant of the spring k=40 N/m and g = 10m/ s^2)

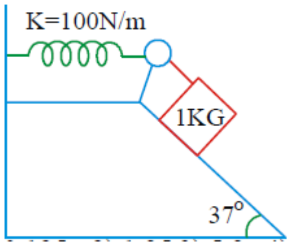


1) $\sqrt{2}$ ms 2) $2\sqrt{2}$ ms
3) 2 ms 4) $\sqrt{2}$ ms

15) The potential energy of a particle of mass m is given by $U = \frac{1}{2}kx^2$ for $x < 0$ and $U = 0$ for $x \geq 0$. If total mechanical energy of the particle is E. Then its speed at $x = \sqrt{\frac{2E}{k}}$ is

1) zero 2) $\sqrt{\frac{2E}{m}}$ 3) $\sqrt{\frac{E}{m}}$ 4) $\sqrt{\frac{E}{2m}}$

16) A 1 kg block situated on a rough inclined plane is connected to spring of a spring constant 100 Nm^{-1} as shown in fig. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless. (g = 10m/ s^2)



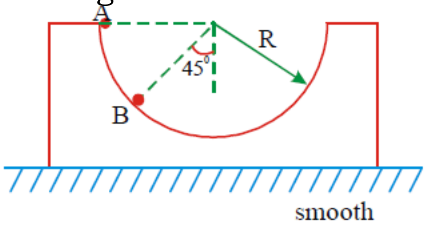
1) 0.125 2) 1.25 3) 5.2 4) 4.5

17) A light spring of force constant 'K' is held between two blocks of masses 'm' and '2m'. The two blocks and the spring system rests on a smooth horizontal floor. Now the blocks are moved towards each other compressing the spring by 'x' and suddenly released. The relative velocity between the blocks when the spring attains its natural length will be

1) $\left(\sqrt{\frac{3K}{2m}}\right)x$ 2) $\left(\sqrt{\frac{2K}{3m}}\right)x$

3) $\left(\sqrt{\frac{K}{3m}}\right)x$ 4) $\left(\sqrt{\frac{K}{2m}}\right)x$

18) A ball of mass m is released from A inside a smooth wedge of mass m as shown in fig. What is the speed of the wedge when the ball reaches point B?



1) $\left(\frac{gR}{3\sqrt{2}}\right)^{\frac{1}{2}}$ 2) $\sqrt{2gR}$

3) $\left(\frac{5gR}{2\sqrt{3}}\right)^{\frac{1}{2}}$ 4) $\sqrt{\frac{3}{2}gR}$

19) Power supplied to a particle of mass 2kg varies with time as P = $3t^2/2$W. Here t is in second. If velocity of particle at $t = 0$ is $v = 0$, the velocity of particle at time t= 2s will be

1) 1 m/s 2) 4 m/s

3) 2 m/s 4) 2 $\sqrt{2}$ m/s

20) A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particle by the forces acting on it, is

1) zero 2) $mk^2 r^2 t^2$

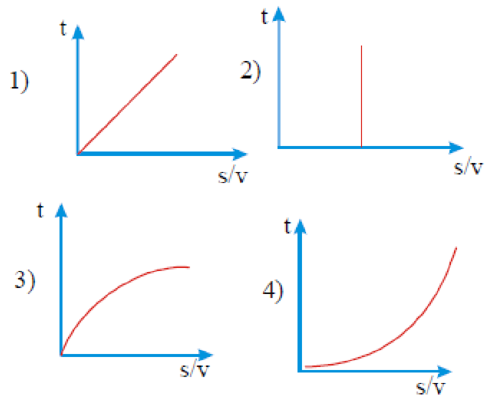
3) $mk^2 r^2 t$ 4) $mk^2 rt$

21) A constant power P is applied to a particle of mass m. The distance travelled by the particle when its velocity increases from v_1 to v_2 is (neglect friction)

1) $\frac{3P}{m}(v_2^2 - v_1^2)$ 2) $\frac{m}{3p}(v_2 - v_1)$

3) $\frac{m}{3P}(v_2^3 - v_1^3)$ 4) $\frac{m}{3P}(v_2^2 - v_1^2)$

22) A body is moved from rest along a straight line by a machine delivering constant power. The ratio of displacement and velocity (s/v) varies with time t as



23) Power applied to a particle varies with time as P = ($3t^2$-2t+1) watt, where 't' is in second. Find the change in kinetic energy between t = 2s and t = 4s

1)32J 2)46J 3)61J 4)100J

24) A car of mass M accelerates starting from rest. Velocity of the car is given by $v = \left(\frac{2Pt}{M}\right)^{\frac{1}{2}}$ where P is the constant power supplied by the engine. The position of car as a function of time is given as

1) $\left(\frac{8P}{9M}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ 2) $\left(\frac{9P}{8M}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

3) $\left(\frac{8P}{9M}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$ 4) $\left(\frac{9P}{8M}\right) t^3$

COLLISIONS

34) A block of wood of mass 3M is suspended by a string of length $\frac{10}{3}$ m. A bullet of mass M hits it with a certain velocity and gets embedded in it. The block and the bullet swing to one side till the string makes 120^0 with the initial position. The velocity of the bullet is ($g=10ms^{-2}$)

1) $\frac{40}{\sqrt{3}} ms^{-1}$ 2) $20\ ms^{-1}$
3) $30\ ms^{-1}$ 4) $40\ ms^{-1}$

35) A wooden block of mass 10gm is dropped from the top of a cliff 100m high. Simultaneously a bullet of same mass is fired from the foot of the cliff vertically upwards with a velocity of $100ms^{-1}$. If the bullet after collision gets embedded in the block, the common velocity of the bullet and the block immediately after collision is ($g=10\ ms^{-2}$)

1) 40 ms^{-1} downward
2) 40ms^{-1} upward
3) 80ms^{-1} upward
4) zero

36) A particle of mass m has a velocity $-v_0i$, while a second particle of same mass has a velocity v_0j. After the particles collide, first particle is found to have a velocity $\frac{-1}{2} v_0 \bar{i}$ then the velocity of other particle is

1) $\frac{-1}{2} v_0 \bar{i} + v_0 \bar{j}$ 2) $\frac{1}{2} v_0 \bar{i} + v_0 \bar{j}$
3) $v_0 \bar{i} + v_0 \bar{j}$ 4) $-v_0 \bar{i} + v_0 \bar{j}$

37) At high altitude a body at rest explodes into two fragments of equal masses with one fragment receiving horizontal velocity of 10 ms^{-1}. Time taken by the two radius vectors connecting point of explosion to fragment to make 90^0 is ($g=10ms^{-2}$)

1) 10 s 2) 4 s 3) 2 s 4) 1 s

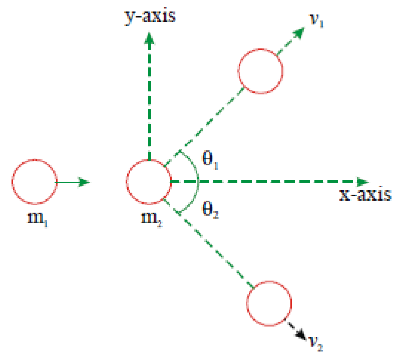
38) A test tube of mass 20 gm is filled with a gas and fitted with a stopper of 2gm. It is suspended horizontally by means of a thread of 1m length and heated. When the stopper kicks out, the tube just completes a circle in vertical plane. The velocity with which the stopper kicked out is

1) 7ms^{-1} 2)10ms^{-1} 3) 70ms^{-1} 4) 0.1ms^{-1}

39) Two bodies move towards each other and collide inelastically. The velocity of the first body is 2m/s and that of the second is 4m/sec before impact. The common velocity after collision is 1m/s in the direction of the first body. The number of times did the KE of the first body exceed that of the second body before collision.

1) 4.25 2) 3.25 3) 2.25 4) 1.25

40) Three particles A, B and C of equal masses, moving with the same speed 'v' along the medians of an equilateral triangle, collide at the centroid G of the triangle. After collision, A comes to rest and B retraces its path with a speed 'v'. The speed of C after the collision is



1) v along BG 2) $\frac{v}{2}$ along GB
3) Zero 4) v along CG

41) A moving sphere P collides another sphere Q at rest. If the collision takes place along the line joining their centers of mass such that their total kinetic energy is conserved and the fraction of K.E. transferred by the colliding particle is $\frac{8}{9}$, then the mass of P and the mass of Q bears a ratio

1) $\sqrt{8}$:3 2) 9 : 8 3) 2 : 3 4) 2 : 1

42) A particle strikes a horizontal frictionless floor with a speed 'u' at an angle 'θ' with the vertical and rebounds with a speed 'v' at an angle 'α' with the vertical. Find the value of 'v' if 'e' is the coefficient of restitution.

1) $v = u\sqrt{e^2\sin^2\theta + \cos^2\theta}$
2) $v = u\sqrt{e^2\cos^2\theta + \sin^2\theta}$
3) $v = u\sqrt{e^2\cos^2\theta + \tan^2\theta}$
4) $v = u\sqrt{\cot^2\theta + e^2\cos^2\theta}$

43) Two spheres A and B of equal masses lie on the smooth horizontal circular groove at opposite ends of diameter and at the end of time 't', 'A' impinges on 'B'. If 'e' is the coefficient of

restitution, the second impinge will occur after a time

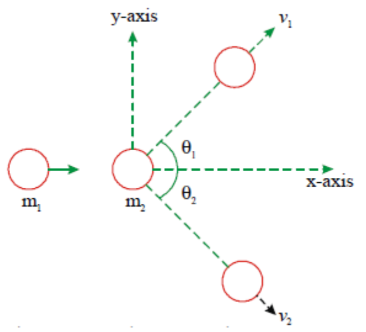
1) $\frac{2t}{e}$ 2) $\frac{t}{e}$ 3) $\frac{\pi t}{e}$ 4) $\frac{2\pi t}{e}$

44) A ball is thrown at an angle of incidence $'\theta'$ on a horizontal plane such that the incident direction and the reflected direction are at right angles to each other. If the coefficient of restitution is 'e' then $'\theta'$ is equal to

1) $\tan^{-1}(e)$ 2) $\tan^{-1}(2e)$

3) $\tan^{-1}(\sqrt{2}e)$ 4) $\tan^{-1}(\sqrt{e})$

45) Consider the collision depicted in fig to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the target. The billiard player wants to 'sink' the target ball in a corner pocket, which is at an angle $\theta_2 = 37^0$. Assume that the collision is elastic and that friction and rotational motion are not important, then θ_1 is



1) 37^0 2) 90^0 3) 45^0 4) 53^0

46) A projectile is fixed on a horizontal ground. Coefficient of restitution between the projectile and the ground is 'e'. If a, b and c be the ratio of time of flight $\left[\frac{T_1}{T_2}\right]$ maximum height $\left[\frac{H_1}{H_2}\right]$ and horizontal range $\left[\frac{R_1}{R_2}\right]$ in first two collisions with the ground, then

1) $a = \frac{1}{e}$ 2) $b = \frac{1}{e^2}$

3) $c = \frac{1}{e}$ 4) 1, 2 & 3

47) A wall moving with velocity 2cms^{-1} towards the ball and ball is moving towards the wall with a velocity 10cms^{-1} . It hits the wall normally and makes elastic collision with wall. The velocity of ball after collision with wall in cms^{-1}

1) 12 2) 8 3) 14 4) 16

48) A body A moves towards a wall with velocity V. The wall also moves towards the body A with velocity V_0. After collision the body moves in opposite direction with velocity V$^{|}$ which is $\left(1+\frac{2V_0}{V}\right)$ times the velocity V. The coefficient of restitution is

1) $\frac{1}{4}$ 2) $\frac{1}{3}$ 3) 1 4) $\frac{1}{2}$

49) A sphere A of mass m moving with certain velocity hits another stationary sphere B of different mass. If the ratio of velocities of the spheres after collision is $\frac{V_A}{V_B} = \frac{1-e}{1+e}$, where e is coefficient of restitution. The initial velocity of sphere A with which it strikes is

1) $V_A + V_B$ 2) $V_A - V_B$

3) $V_B - V_A$ 4) $\frac{(V_B - V_A)}{2}$

50) A ball A of mass 3m is placed at a distance d from the wall on a smooth horizontal surface. Another ball B of mass m moving with velocity u collides with ball A. The coefficient of restitution between the balls and the wall and between the balls is e

a) the velocity of ball B after collision is $\frac{u(3e-1)}{4}$

b) the velocity of ball B after collision is $\frac{u(2e+1)}{4}$

c) after collision, ball A will move away by distance $\frac{d(2e+1)}{(2e-1)}$ during the time ball B returns back to wall.

d) after collision, ball A will move away by distance $\frac{d(e+1)}{(3e-1)}$ during the time ball B returns back to wall

1) a,d 2) a,c 3) b,d 4) c,d

LEVEL - 3

01)4	02)2	03)2	04)4	05)1	06)1
07)1	08)4	09)3	10)2	11)2	12)3
13)2	14)2	15)2	16)1	17)1	18)1
19)3	20)3	21)3	22)1	23)2	24)1
25)3	26)4	27)2	28)3	29)3	30)2
31)4	32)4	33)1	34)4	35)2	36)1
37)4	38)3	39)4	40)1	41)4	42)2
43)1	44)4	45)4	46)4	47)3	48)3
49)1	50)1				