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Computational Science and Engineering

Software Lab Project

Influence of twist angle and pre bending of wind turbine rotor blades on the 3D-eigenmodes

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List of Abbreviations

1. FAST : Fatigue, Aerodynamics, Structures, and Turbulence
2. ADAMS : Automatic Dynamic Analysis of Mechanical Systems

Abstract

The main objective of the project is to study the influence of structural twist and pre bending on 3D eigen modes of a wind turbine blade. The proposed program is based on the Finite element formulation of classical beam element to compute stiffness and mass matrices without considering the shear deformation of the beam (Euler Bernoulli beam).
The stiffness and mass matrices are computed using the blade structural data of NREL 5MW wind turbine. Required data were extracted from the space node geometry in formulating the transformation matrices for calculating global stiffness and mass matrices and are assembled into system matrices. The eigen frequencies are computed after applying the appropriate boundary conditions for three different cases. Thus obtained results are verified and compared with the proposed results from FAST and ADAMS.

Keywords: Euler-Bernoulli beam, Eigen value problem, Local coordinate system, global coordinate system, Boundary conditions.

1. Introduction

Vibration is a mechanical phenomenon whereby oscillations occur in structures about an equilibrium point [1]. These occur due to imbalance in the axis of the shaft, misalignment etc., and thus causes structural damages due to cyclic loading resulting in fatigue.
A system oscillating continuously in absence of any driving force or damping force is called the natural frequency of the system [2]. When the frequency of vibration due to applied force on the system matches the natural frequency the amplitude of vibration increases and causes resonance.

Wind turbine consists of long slender structures (blades, towers), the cross sectional forces (both static and dynamic) and moments are very important for dimensioning of components. Precise calculation of the eigenfrequencies and the eigenmodes of a Wind turbine is of great importance for the design process of a Wind Turbine, in order to avoid possible resonant states by constructive measures. In the simulating a Wind Turbine as a multi-body-system, at least the first two lowest flapwise and edgewise bending eigenmodes of the rotor blades are used. The blade is considered as a Euler Bernoulli beam and the prebending and the twist angle of the blades are usually neglected while solving the eigen value problem.

The main objectives of this project is to determine the influence of prebending and the structural twist of blades on 3-D eigen modes by Coding a procedure for element stiffness and inertia matrices for the Bernoulli-beam element; Coding the procedures for the local-global transformations of the element matrices; Assembly of the system stiffness and mass matrices; Solution of the eigenvalue problem using MUPAD system functions; and comparing the results with the proposed results by FAST and ADAMS software.

2. Euler Bernoulli Beam

The Euler-Bernoulli beam theory, sometimes called the classical beam theory is the most commonly used because it is simple and provides reasonable engineering approximations for many problems.[3]

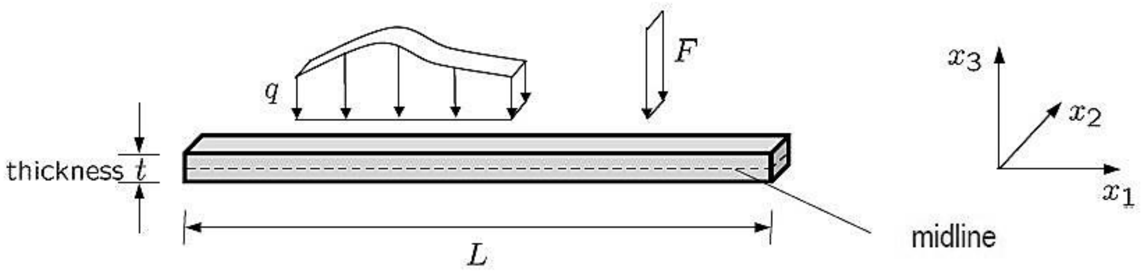

Fig 1: Bernoulli Beam [3]

The main assumptions in the Euler Bernoulli beam are

1. The beam is rigid and the axis of the beam is straight in unloaded configuration
2. Kinematic assumption: Material points on the normal to midline remain on the normal during the deformation.
3. The kinematic assumption determines the axial displacements of the material points across the thickness.

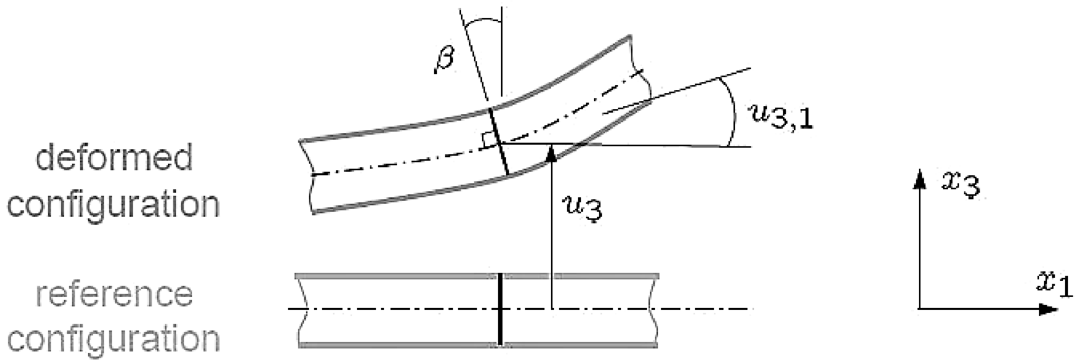

Fig 2: Deformed Bernoulli beam [3]

2.1 Mathematical formulation for Beam Vibrations

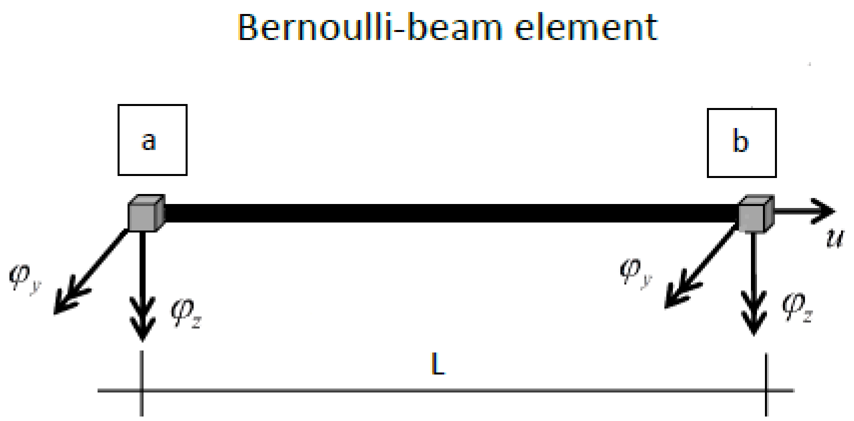

Fig 3: A beam under transverse vibration

Consider a prismatic beam with six degree of freedom and the support configuration as shown above. The properties of the beam are as mentioned in the table 1.

L	0.3 m
A	0.00334 m^2
Azz	0.0000214m^4
Ayy	0.00000117m^4
I_t	0.000000135m^4

Table 1:- Structural properties Bernoulli beam

Where:
A_{zz}: Moment of inertia along the Z-Z axis or edgewise inertia.
A_{yy}: Moment of inertia along the Y-Y axis or flapwise inertia.
I_t : Torsional moment

The general equation of transverse beam vibration is given by [4]

$$EI(x)\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A(x)\frac{\partial^2 w(x,t)}{\partial x^2} = f(x,t) \tag{1}$$

For free vibration f(x,t) is equal to zero and the general solution to beam equation is given by

$$W(x) = Acos\lambda x + Bcos\lambda x + Ccos\lambda x + Dcos\lambda x \tag{2}$$

The values of A,B,C,D are obtained by applying the appropriate boundary conditions depending upon the type of support configuration the beam posses.

2.2 Analytical approach

For the above considered beam after applying the boundary conditions the analytical solution is given by

$$\omega_{yi} = \left(\frac{i\pi}{l}\right)^2 \sqrt{\frac{E\ I_y}{\rho A}} = \left(\frac{i\pi}{l}\right)^2 \sqrt{\frac{EA_{zz}}{\rho A}} \qquad i = 1....n \tag{3}$$

$$\omega_{zi} = \left(\frac{i\pi}{l}\right)^2 \sqrt{\frac{EI_z}{\rho A}} = \left(\frac{i\pi}{l}\right)^2 \sqrt{\frac{EA_{yy}}{\rho A}} \qquad i = 1....n \tag{4}$$

$$f = \frac{\omega}{2\pi}$$

$$for\ i = 1$$

$$\omega_{yi} = \left(\frac{1\pi}{0,3}\right)^2 \sqrt{\frac{2.1*10^{11}*0.0000214}{26.3}} = 45.331\ rad$$

$$f_{yi} = \frac{\omega_{yi}}{2\pi} = 72.1466\ Hz$$

$$\omega_{zi} = \left(\frac{1\pi}{0.3}\right)^2 \sqrt{\frac{2.1 * 10^{11} * 0.00000117}{26.3}} = 105.994\ rad$$

$$f_{zi} = 16.8694\ Hz$$

3. Solution approach using Finite Element Method

In FEM approach, the beam is decomposed in small but finite lengths. The ends are considered as nodes a, and b as shown in the fig 3. In this approach a desired parameter is computed for each finite element along the definition of domain. Assembly of results of such parameters gives a smooth curve of the entity under observation. This will closely resemble the results obtained from the analytical approach.

In our case the beam is divided into ten equal finite lengths each of 0.3 metres.

To observe the Vibrations of beam, the Motion equation is given by [5]

$$M\frac{d^2x(t)}{dt^2} + C\ \frac{dx(t)}{dt^2} + K\ x(t) = f(x,t) \tag{5}$$

Since it is free vibration and the damping is neglected the above equation is reduced to

$$M\frac{d^2x(t)}{dt^2} + K\ x(t) = 0 \tag{6}$$

The steps for solution of the above equation is as shown below

$$x(t) = a\sin\omega t \tag{7}$$

$$(K - \omega_0^2 M)a = 0 \tag{8}$$

$$(KM^{-1} - \omega_0^2 I)a = 0 \tag{9}$$

$$(A - \omega_0^2 I)a = 0 \tag{10}$$

where $A = KM^{-1}$ K- Stiffness matrix

M- Mass Matrix ω_0: Eigen frequency

3.1 Stiffness Matrix : For isotropic beam with 2 nodes and considering all the six degrees of freedom the element stiffness matrix is given by

$$\underline{\underline{K}} = \left[\begin{array}{cccccc|cccccc}
\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
 & \frac{12EA_{yy}}{L^3} & 0 & 0 & 0 & \frac{6EA_{yy}}{L^2} & 0 & -\frac{12EA_{yy}}{L^3} & 0 & 0 & 0 & \frac{6EA_{yy}}{L^2} \\
 & & \frac{12EA_{zz}}{L^3} & 0 & -\frac{6EA_{zz}}{L^2} & 0 & 0 & 0 & -\frac{12EA_{zz}}{L^3} & 0 & -\frac{6EA_{zz}}{L^2} & 0 \\
 & & & \frac{GI_T}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_T}{L} & 0 & 0 \\
 & & & & \frac{4EA_{zz}}{L} & 0 & 0 & 0 & \frac{6EA_{zz}}{L^2} & 0 & \frac{2EA_{zz}}{L} & 0 \\
 & & & & & \frac{4EA_{yy}}{L} & 0 & -\frac{6EA_{yy}}{L^2} & 0 & 0 & 0 & \frac{2EA_{yy}}{L} \\
\hline
 & & & & & & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \frac{12EA_{yy}}{L^3} & 0 & 0 & 0 & -\frac{6EA_{yy}}{L^2} \\
 & & & & symm. & & & & \frac{12EA_{zz}}{L^3} & 0 & \frac{6EA_{zz}}{L^2} & 0 \\
 & & & & & & & & & \frac{GI_T}{L} & 0 & 0 \\
 & & & & & & & & & & \frac{4EA_{zz}}{L} & 0 \\
 & & & & & & & & & & & \frac{4EA_{yy}}{L}
\end{array}\right]$$

Description :

EA : Axial Stiffness

EA_{yy}: Bending stiffness along Flapwise

EA_{zz}: Bending stiffness along Edgewise

GI_t: Torsional stiffness

L: Length of the beam element

3.2 Mass matrix:

The element mass matrix for the same isotropic beam with 2 nodes and six degree of freedom is given by

$$\underline{\underline{M}} = mL\left[\begin{array}{cccccc|cccccc}
\frac{1}{3} & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\
 & \frac{13}{35}+\frac{6}{5}\frac{\Theta_z}{mL^2} & 0 & 0 & 0 & \frac{11L}{210}+\frac{1}{10}\frac{\Theta_z}{mL} & 0 & \frac{9}{70}-\frac{6}{5}\frac{\Theta_z}{mL^2} & 0 & 0 & 0 & -\frac{13L}{420}+\frac{1}{10}\frac{\Theta_z}{mL} \\
 & & \frac{13}{35}+\frac{6}{5}\frac{\Theta_y}{mL^2} & 0 & -\frac{11L}{210}-\frac{1}{10}\frac{\Theta_y}{mL} & 0 & 0 & 0 & \frac{9}{70}-\frac{6}{5}\frac{\Theta_y}{mL^2} & 0 & \frac{13L}{420}-\frac{1}{10}\frac{\Theta_y}{mL} & 0 \\
 & & & \frac{\Theta_p}{3m} & 0 & 0 & 0 & 0 & 0 & \frac{\Theta_p}{6m} & 0 & 0 \\
 & & & & \frac{L^2}{105}+\frac{2}{15}\frac{\Theta_y}{m} & 0 & 0 & 0 & -\frac{13L}{420}+\frac{1}{10}\frac{\Theta_y}{mL} & 0 & -\frac{L^2}{140}-\frac{1}{30}\frac{\Theta_y}{m} & 0 \\
 & & & & & \frac{L^2}{105}+\frac{2}{15}\frac{\Theta_z}{m} & 0 & \frac{13L}{420}-\frac{1}{10}\frac{\Theta_z}{mL} & 0 & 0 & 0 & -\frac{L^2}{140}-\frac{1}{30}\frac{\Theta_z}{m} \\
\hline
 & & & & & & \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\
 & & & & & & & \frac{13}{35}+\frac{6}{5}\frac{\Theta_z}{mL^2} & 0 & 0 & 0 & -\frac{11L}{210}-\frac{1}{10}\frac{\Theta_z}{mL} \\
 & & & & & & & & \frac{13}{35}+\frac{6}{5}\frac{\Theta_y}{mL^2} & 0 & \frac{11L}{210}+\frac{1}{10}\frac{\Theta_y}{mL} & 0 \\
 & & & symm. & & & & & & \frac{\Theta_p}{3m} & 0 & 0 \\
 & & & & & & & & & & \frac{L^2}{105}+\frac{2}{15}\frac{\Theta_y}{m} & 0 \\
 & & & & & & & & & & & \frac{L^2}{105}+\frac{2}{15}\frac{\Theta_z}{m}
\end{array}\right]$$

$$\Theta_p = m\frac{(A_{yy}+A_{zz})}{A} \quad [kg \cdot m]$$

$$\Theta_y = m\frac{A_{zz}}{A} \quad [kg \cdot m]$$

$$\Theta_z = m\frac{A_{yy}}{A} \quad [kg \cdot m]$$

Description:
θ_y: Mass moment of inertia flapwise
θ_z : Mass moment of inertia edgewise
θ_p: $\theta_y + \theta_z$
M: Distributed mass
L : Length of the beam element

3.3 Transformation matrix :

Members in Structural system are typically oriented in differing directions. In order to perform analysis, the element stiffness equations needs to be transformed to a common global coordinate system from local coordinate system. Once the element equations are expressed in global coordinate system, the equation of each elements comprising a structure can be assembled.

The obtained transformation matrix for 6 degrees of freedom is shown below

$$A = \begin{bmatrix} cos\,\alpha & -sin\,\alpha & 0 \\ \sin\alpha & cos\,\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} cos\,\beta & 0 & -sin\,\beta \\ 0 & 1 & 0 \\ sin\,\beta & 0 & cos\,\beta \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\,\gamma & -sin\,\gamma \\ 0 & sin\,\gamma & cos\,\gamma \end{bmatrix}$$

The matrix T obtained as a 3X3 matrix from A*B*G is given as follows

$$T_0 = \begin{bmatrix} cos\alpha\;\;cos\beta & -cos\gamma\;\;sin\alpha\; - cos\alpha\;\;sin\beta\;\;sin\gamma & sin\alpha\;\;sin\gamma - cos\alpha\;\;cos\gamma\;\;sin\beta \\ cos\beta\;sin\alpha & cos\alpha\;\;cos\,\gamma\; - sin\alpha\;\;sin\beta\;\;sin\gamma & -cos\alpha\;\;sin\gamma - cos\gamma\;sin\alpha\;\;sin\beta \\ sin\beta & cos\beta\;sin\,\gamma & cos\,\beta\;cos\,\gamma \end{bmatrix}$$

The transformation matrix is assembled with this T matrix to make a 12x12 matrix as shown below

$$\boldsymbol{T}(ik) = \underbrace{\begin{bmatrix} T_0 & 0 & 0 & 0 \\ 0 & T_0 & 0 & 0 \\ 0 & 0 & T_0 & 0 \\ 0 & 0 & 0 & T_0 \end{bmatrix}}_{12\,x\,12}$$

The resulting transformation matrix is used in transforming the element stiffness and mass matrices in local coordinates into global coordinate system.

$$\boldsymbol{K}_j = \boldsymbol{T}\cdot\boldsymbol{K}_e\cdot\boldsymbol{T}^T \; and \; M_J = \boldsymbol{T}\cdot\boldsymbol{M}_e\cdot\boldsymbol{T}^T \tag{11}$$

3.4 Assembly of Global Matrices

Each transformed element stiffness matrix Kj i j and mass matrix Mj i j is added to the appropriate location of the overall, or “global”stiffness matrix KG and Global Mass Matrix MG that relates to all of the beam displacements and forces. This process is called assembly and is as shown below. [6]

$$\boldsymbol{KG} = \underbrace{\begin{bmatrix} k_{11} & k_{12} & . & . \\ k_{21} & k^{(1)}{}_{22} + k^{(2)}{}_{22} & k_{23} & . \\ . & k_{32} & k^{(2)}{}_{33} + k^{(3)}{}_{33} & k_{34} \\ . & . & . & . \end{bmatrix}}_{298\,x\,298}$$

3.5 Boundary Conditions

A general mathematical principle is that the number of boundary conditions necessary to solve a differential equation has to match with the order of the differential equation. In finite element analysis boundary conditions are the specified values of the field variables on the boundaries of the field correspondingly the displacements belonging to these nodes are assigned zero. In our case, these are

1) u, v, w, ϕ_x for the first node and
2) v, w, ϕ_x for the last node of the beam

Where $v_a = [u_a \quad u_v \quad u_w \quad \phi_x \quad \phi_y \quad \phi_z\,]$ are the six displacement vectors at each node of the beam element.

3.6: Results

The following are the results of eigen frequencies obtained by taking a simple Euler Bernoulli beam with the support configuration as shown in the Fig 3. It can be observed that the results obtained from the analytical method and the results from the developed code using MUPAD are in close proximities.

n	f_{yn} Analytical [Hz]	f_{zn} Analytical [Hz]	f Mupad [Hz]	Percentage Difference [%]
1	72.1466	16.8695	16.8664	0.018
2	288.5865	67.4779	67.4331	0.066
3	649.3196	72.1466	71.8949	0.35
4	1154.3460	151.8254	151.6417	0.121

Table 2:- Eigen Frequency and Relative difference

This observation of the 10 element Bernoulli beam showcases a very negligible difference between the analytical and FEM values. This forms the basis of modelling of actual blade profile which is approximated as an Euler Bernoulli beam.

4. Eigen modes and Eigen Frequency for the Wind turbine blade

The same above used FEM approach in solving for eigen frequencies of a simple Euler Bernoulli beam is used in our case.

4.1 Blade Structural properties

The structural data of the wind turbine blade belongs to a 5-MW reference Wind turbine for offshore development. Table 4.1 shows the blade structural properties. The values in the first column defined as 'Radius' are the spanwise locations along the blade pitch axis. The values of the flapwise, edgewise inertia and the stiffness are given about the principal structural axes of each cross section as oriented by the structural twist angle "Strct Twst". "GJ Stff" represents blade torsion stiffness and "EA Stff" the blade extensional stiffness. The values in the column 2 are the distributed blade section mass per unit length values "BMassDen". [7]

Radius (m)	BMassDen (kg/m)	EdgStff (N•m2)	FlpStff (N•m2)	StrcTwst (º)	GJStff (N•m2)	EAStff (N)	FlpIner (kg•m)	EdgIner (kg•m)
1.5	678.935	1.81E+10	1.81E+10	13.308	5.56E+09	9.73E+09	972.86	973.04
1.7	678.935	1.81E+10	1.81E+10	13.308	5.56E+09	9.73E+09	972.86	973.04
2.7	773.363	1.96E+10	1.94E+10	13.308	5.43E+09	1.08E+10	1091.52	1066.38
3.7	740.55	1.95E+10	1.75E+10	13.308	4.99E+09	1.01E+10	966.09	1047.36
4.7	740.042	1.98E+10	1.53E+10	13.308	4.67E+09	9.87E+09	873.81	1099.75

5.7	592.496	1.49E+10	1.08E+10	13.308	3.47E+09	7.61E+09	648.55	873.02
6.7	450.275	1.02E+10	7.23E+09	13.308	2.32E+09	5.49E+09	456.76	641.49
7.7	424.054	9.14E+09	6.31E+09	13.308	1.91E+09	4.97E+09	400.53	593.73
8.7	400.638	8.06E+09	5.53E+09	13.308	1.57E+09	4.49E+09	351.61	547.18
9.7	382.062	6.88E+09	4.98E+09	13.308	1.16E+09	4.03E+09	316.12	490.84
10.7	399.655	7.01E+09	4.94E+09	13.308	1.00E+09	4.04E+09	303.6	503.86
11.7	426.321	7.17E+09	4.69E+09	13.308	8.56E+08	4.17E+09	289.24	544.7
12.7	416.82	7.27E+09	3.95E+09	13.181	6.72E+08	4.08E+09	246.57	569.9
13.7	406.186	7.08E+09	3.39E+09	12.848	5.47E+08	4.09E+09	215.91	601.28
14.7	381.42	6.24E+09	2.93E+09	12.192	4.49E+08	3.67E+09	187.11	546.56
15.7	352.822	5.05E+09	2.57E+09	11.561	3.36E+08	3.15E+09	160.84	468.71
16.7	349.477	4.95E+09	2.39E+09	11.072	3.11E+08	3.01E+09	148.56	453.76
17.7	346.538	4.81E+09	2.27E+09	10.792	2.92E+08	2.88E+09	140.3	436.22
19.7	339.333	4.50E+09	2.05E+09	10.232	2.61E+08	2.61E+09	124.61	398.18
21.7	330.004	4.24E+09	1.83E+09	9.672	2.29E+08	2.36E+09	109.42	362.08
23.7	321.99	4.00E+09	1.59E+09	9.11	2.01E+08	2.15E+09	94.36	335.01
25.7	313.82	3.75E+09	1.36E+09	8.534	1.74E+08	1.94E+09	80.24	308.57
27.7	294.734	3.45E+09	1.10E+09	7.932	1.44E+08	1.63E+09	62.67	263.87
29.7	287.12	3.14E+09	8.76E+08	7.321	1.20E+08	1.43E+09	49.42	237.06
31.7	263.343	2.73E+09	6.81E+08	6.711	8.12E+07	1.17E+09	37.34	196.41
33.7	253.207	2.55E+09	5.35E+08	6.122	6.91E+07	1.05E+09	29.14	180.34
35.7	241.666	2.33E+09	4.09E+08	5.546	5.75E+07	9.23E+08	22.16	162.43
37.7	220.638	1.83E+09	3.15E+08	4.971	4.59E+07	7.61E+08	17.33	134.83
39.7	200.293	1.58E+09	2.39E+08	4.401	3.60E+07	6.48E+08	13.3	116.3
41.7	179.404	1.32E+09	1.76E+08	3.834	2.74E+07	5.40E+08	9.96	97.98
43.7	165.094	1.18E+09	1.26E+08	3.332	2.09E+07	5.31E+08	7.3	98.93
45.7	154.411	1.02E+09	1.07E+08	2.89	1.85E+07	4.60E+08	6.22	85.78
47.7	138.935	7.98E+08	9.09E+07	2.503	1.63E+07	3.76E+08	5.19	69.96
49.7	129.555	7.10E+08	7.63E+07	2.116	1.45E+07	3.29E+08	4.36	61.41
51.7	107.264	5.18E+08	6.11E+07	1.73	9.07E+06	2.44E+08	3.36	45.44
53.7	98.776	4.55E+08	4.95E+07	1.342	8.06E+06	2.12E+08	2.75	39.57
55.7	90.248	3.95E+08	3.94E+07	0.954	7.08E+06	1.82E+08	2.21	34.09
56.7	83.001	3.54E+08	3.47E+07	0.76	6.09E+06	1.60E+08	1.93	30.12
57.7	72.906	3.05E+08	3.04E+07	0.574	5.75E+06	1.09E+08	1.69	20.15
58.7	68.772	2.81E+08	2.65E+07	0.404	5.33E+06	1.00E+08	1.49	18.53
59.2	66.264	2.62E+08	2.38E+07	0.319	4.94E+06	9.22E+07	1.34	17.11
59.7	59.34	1.59E+08	1.96E+07	0.253	4.24E+06	6.32E+07	1.1	11.55
60.2	55.914	1.38E+08	1.60E+07	0.216	3.66E+06	5.33E+07	0.89	9.77
60.7	52.484	1.19E+08	1.28E+07	0.178	3.13E+06	4.45E+07	0.71	8.19
61.2	49.114	1.02E+08	1.01E+07	0.14	2.64E+06	3.69E+07	0.56	6.82
61.7	45.818	8.51E+07	7.55E+06	0.101	2.17E+06	2.99E+07	0.42	5.57
62.2	41.669	6.43E+07	4.60E+06	0.062	1.58E+06	2.13E+07	0.25	4.01
62.7	11.453	6.61E+06	2.50E+05	0.023	2.50E+05	4.85E+06	0.04	0.94
63	10.319	5.01E+06	1.70E+05	0	1.90E+05	3.53E+06	0.02	0.68

Table 3: Blade structural properties

4.2. Reference system

The local reference or the local coordinate system refers to the principal structural axes oriented along the cross section at each node. The data in table 3 is given as per the local coordinates. Hence the stiffness and mass matrices calculated using the blade structural properties are the element stiffness and mass matrices of local coordinate system.

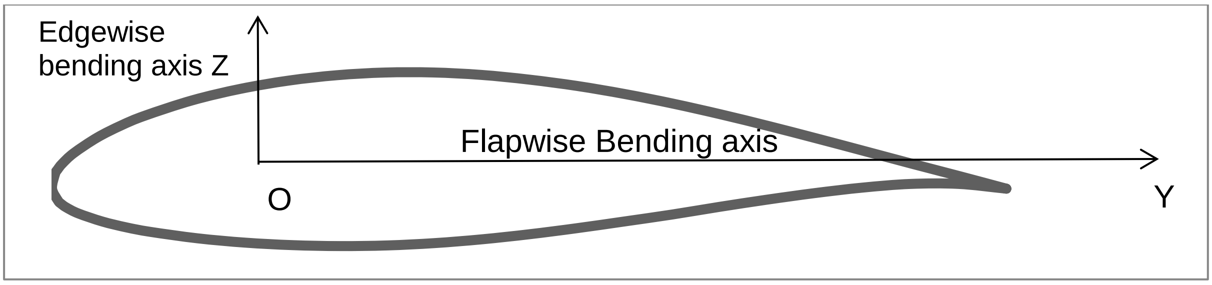

Fig 4: Sectional view of a blade with coordinates

The orientation in terms of local and global reference system for the beam element is given as follows
Local reference is {x,y,z}
Global Reference {X1, X2, X3}

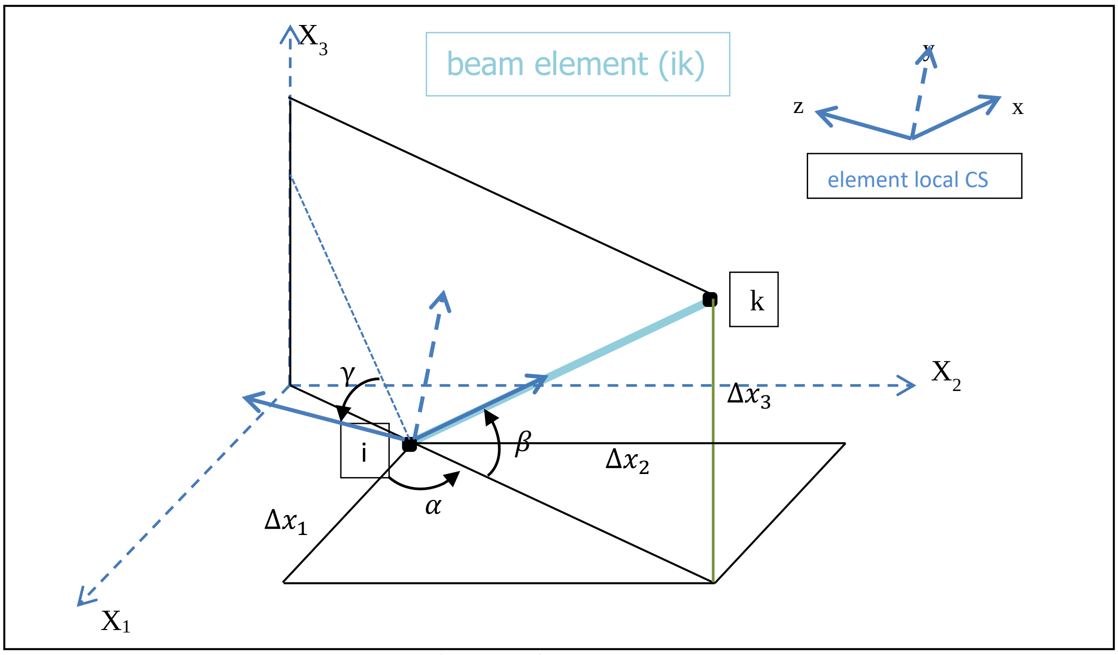

Fig 5: Global reference system

Before assembling into system stiffness and mass matrices, the element matrices should be expressed in the common global reference system with all nodal variables referred to a common point. The fig 4 and 5 illustrates the local and global reference system.

4.3 Prebending

Prebending of wind turbine blades is a simple solution directed towards the problem of tower clearance. It ensures sufficient distance between the rotor blades and the tower to avoid collision when the turbine is operating. The prebent shape of the blade must be such that when the turbine rotor is subjected to wind and inertial loads, the blades are straightened into their design configuration. The prebent blade data is taken from the graph at the same points as that of blade structural data. [10]

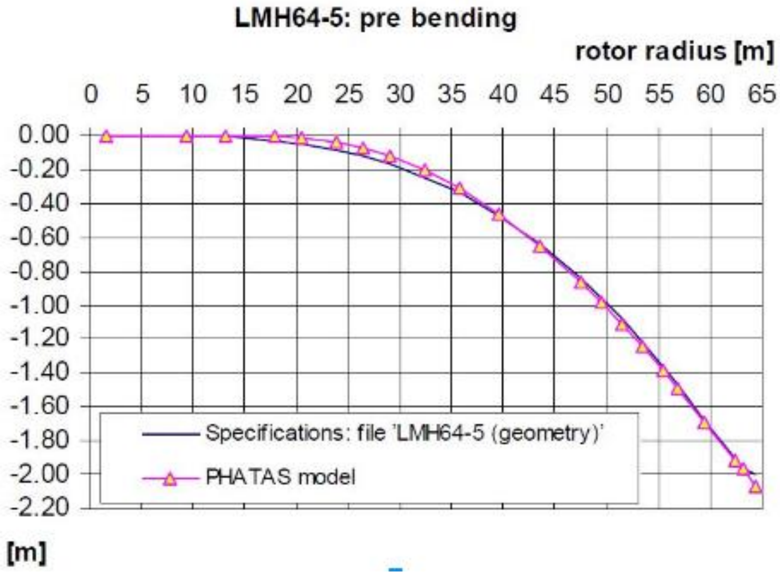

Fig 6 :- Prebending data of blade profile [8]

4.4 Structural twist:

The twist angle comes from the twist of the blade, i.e. the positioning of the different airfoils along the blade axes. So it is build into the blade and cannot be change after manufacturing of the blade, i.e. it is constant during operation of a wind turbine. The reason for that angle is to achieve optimal angle of attack at every section of the rotor blade (at optimal operation point) thereby achieving maximum power coefficient. [9]

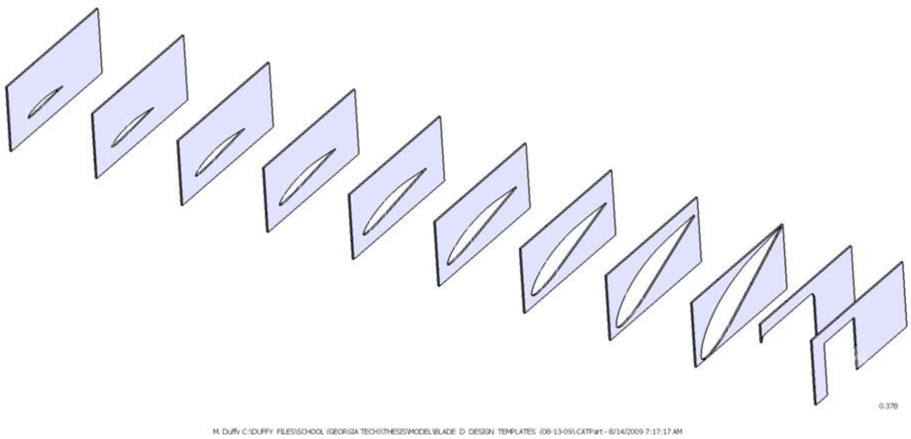

Fig 7:- Blade sections showing twist angle

4.5 Case explanation:

We form four cases using the given parameters and are as mentioned below

1. Both Structural twist and pre bending neglected (gamma = 0, pre bending = 0).
2. Neglecting Structural twist and considering only pre bending (gamma = 0, Pre bending ≠ 0).
3. Considering both structural twist and pre bending (gamma ≠ 0, Pre bending ≠ 0).
4. Considering only Structural twist and neglecting pre bending (gamma ≠ 0,Pre bending = 0).

In each case the steps followed are

1. The blade data from the table 4.1 Is chosen element wise
2. Element stiffness and mass matrices are calculated.
3. These two matrices are transformed from local to global coordinate system.
4. The transformed global stiffness and global mass matrices are assembled into global system stiffness and global system mass matrix.
5. Boundary condition is applied upon the Global system matrix (stiffness and mass matrix).
6. Thus reduced Global system stiffness and mass matrices are used to calculate Eigen frequencies and Eigen vectors using the solution approach as shown in the Eqn(10)

5. Results and discussions:

The First four eigen modes obtained from each cases are as shown below

1. Both Structural twist and prebending neglected (gamma = 0, prebending = 0).

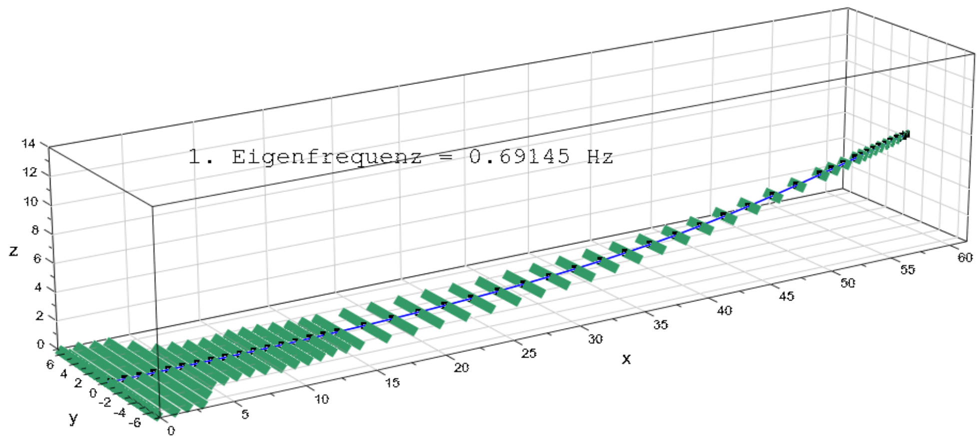

Fig 8:- 1st flapwise eigen mode

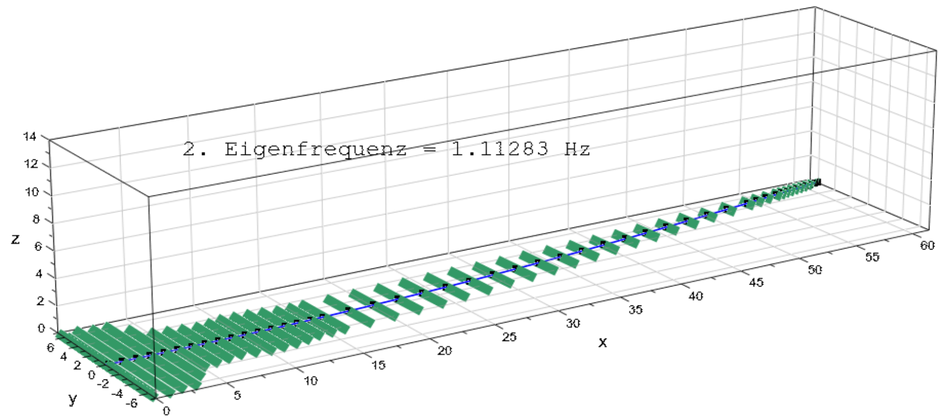

Fig 9:- 1st edgewise eigen mode

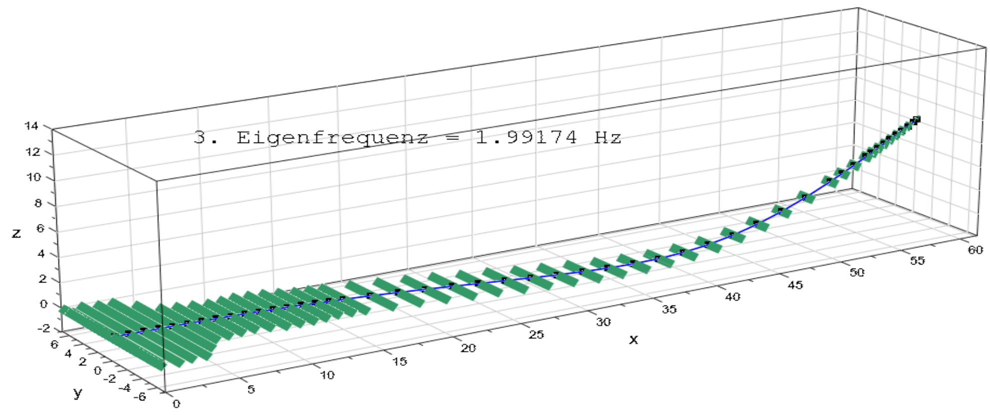

Fig 10: 2nd flapwise eigen mode

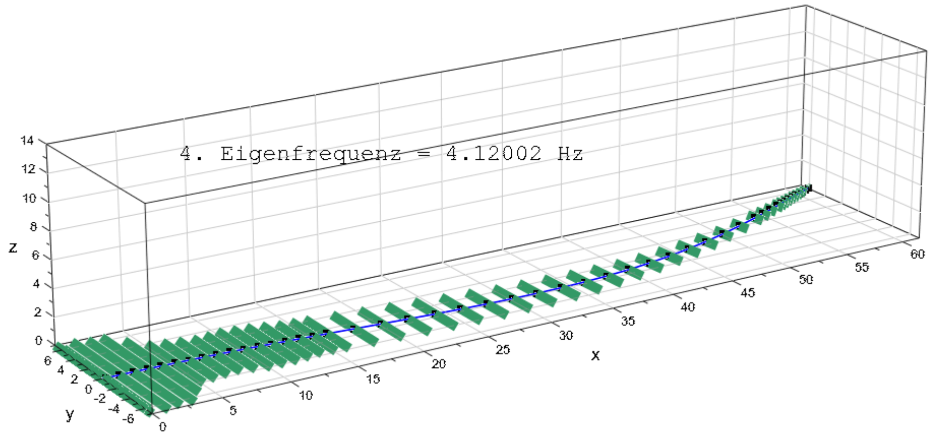

Fig 11: 2nd edgewise eigen mode

2. Neglecting Structural twist and considering only prebending (gamma = 0, Prebending ≠ 0).

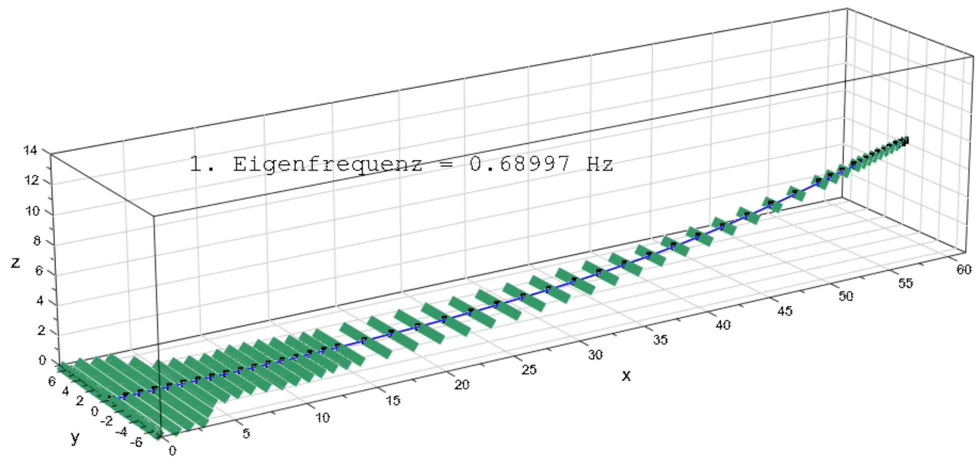

Fig 12: 1st flapwise eigen mode

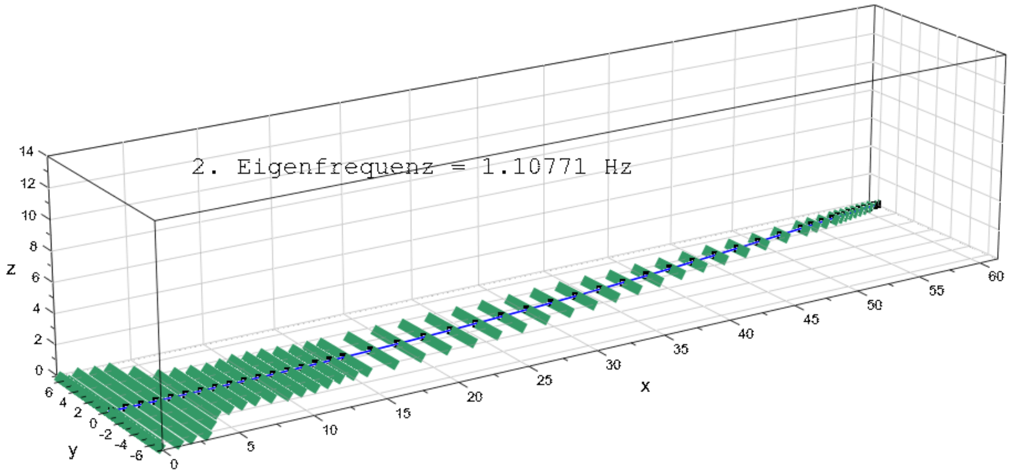

Fig 13:- 1st edgewise eigen mode

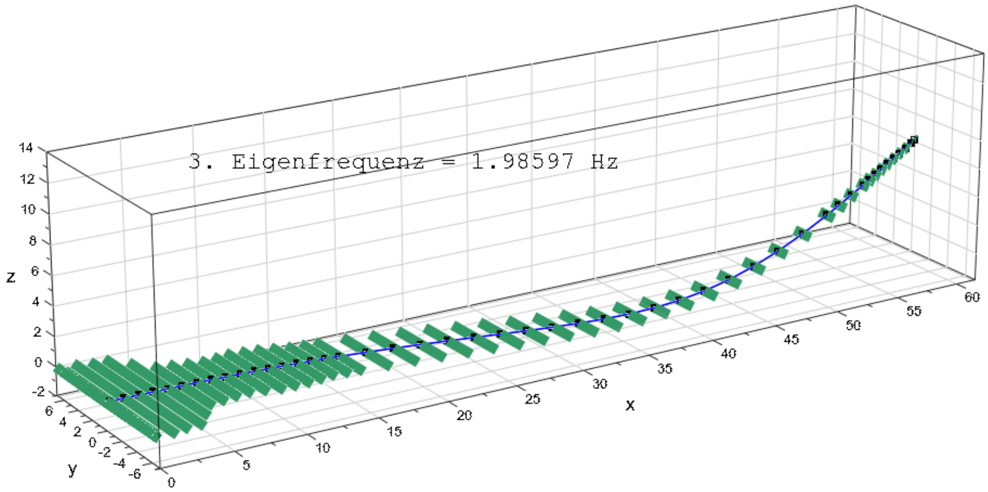

Fig 14:- 2nd flapwise eigen mode

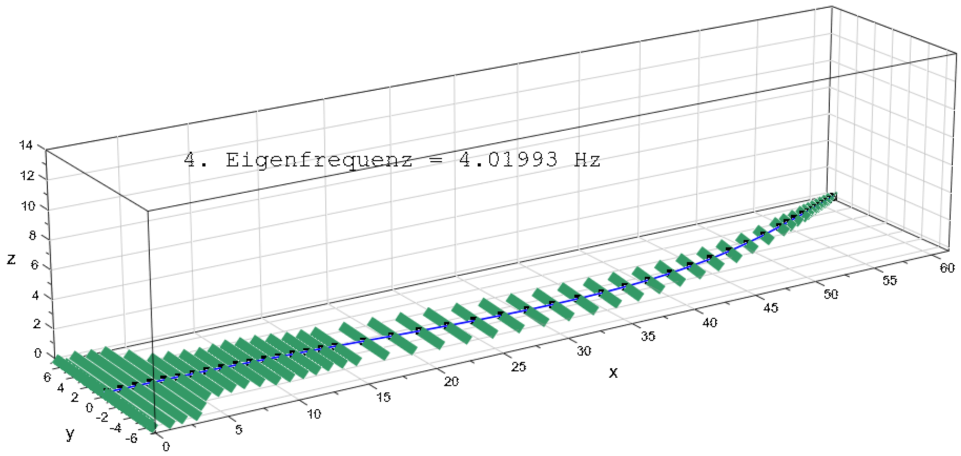

Fig 15:- 2nd edgewise eigen mode

3. Considering both structural twist and prebending (gamma ≠ 0,Prebending ≠ 0).

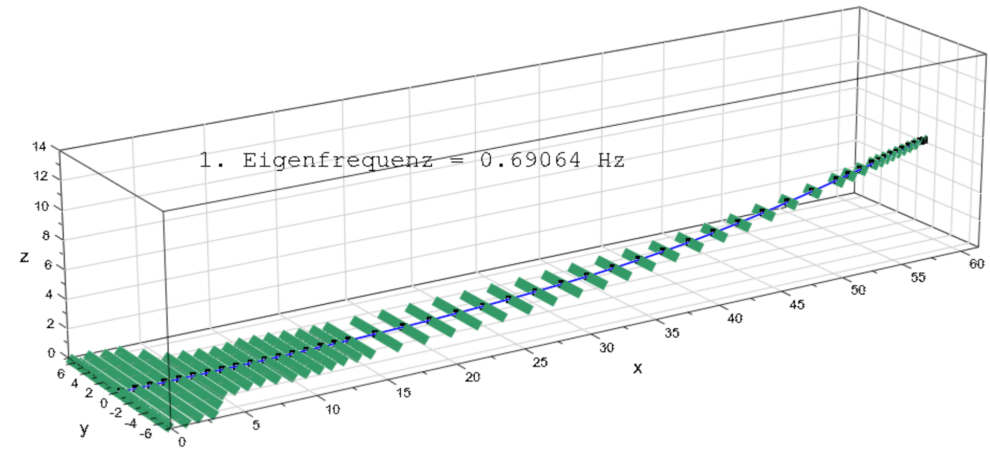

Fig 16 :- 1st flapwise eigen mode

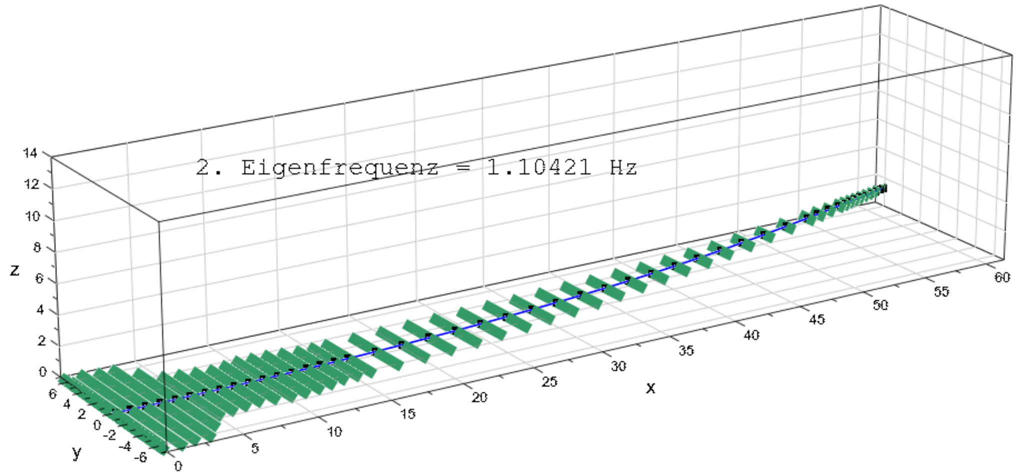

Fig 17: 1st edgewise eigen mode

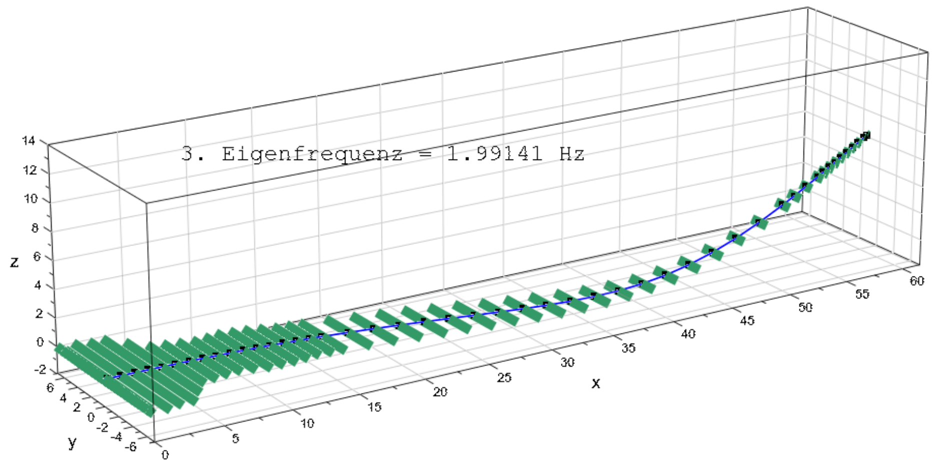

Fig 18:- 2nd flapwise eigen mode

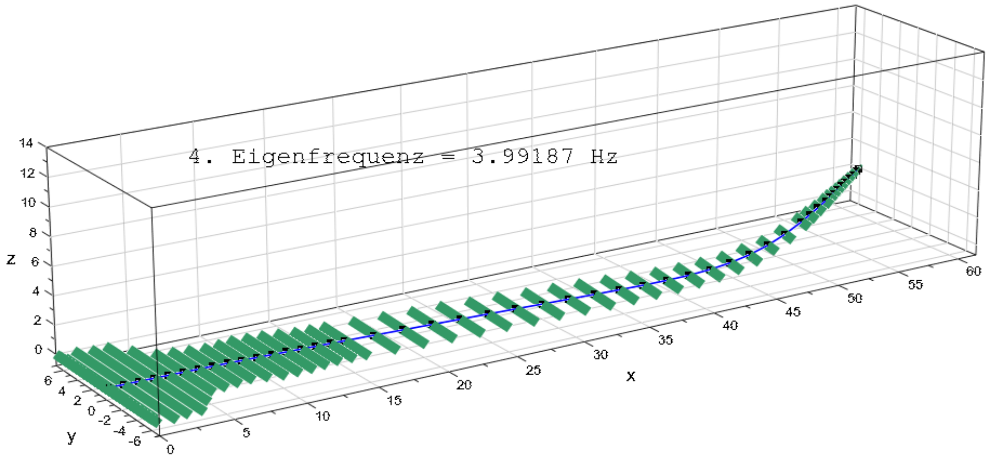

Fig 19:- 2nd edgewise eigen mode

4. Considering only Structural twist and neglecting prebending (gamma ≠ 0, Prebending = 0).

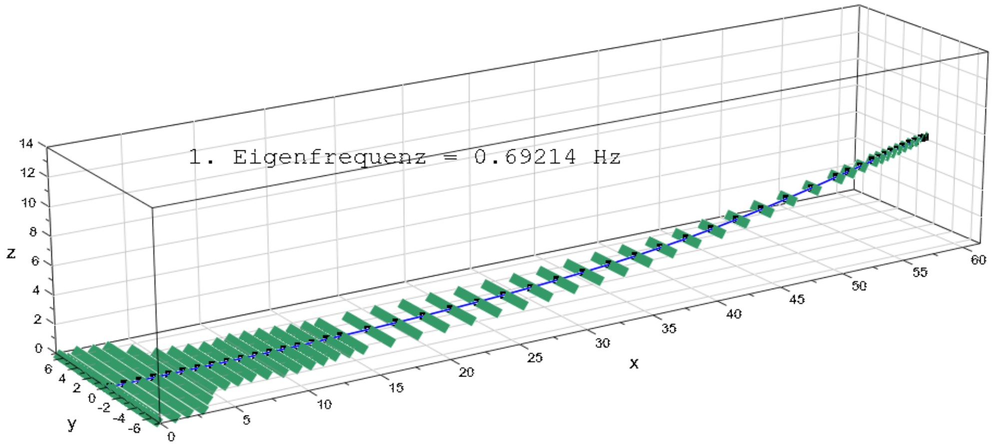

Fig 20:- 1st flapwise eigen mode

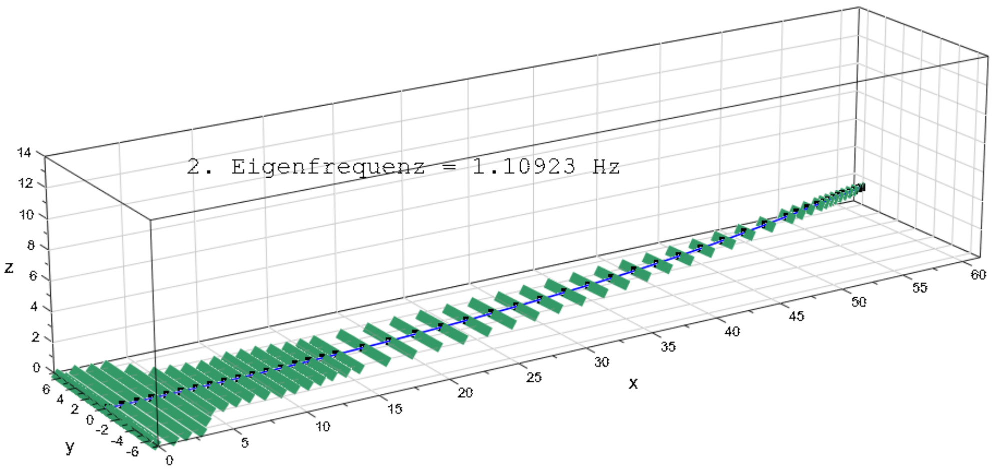

Fig 21:- 1st edgewise eigen mode

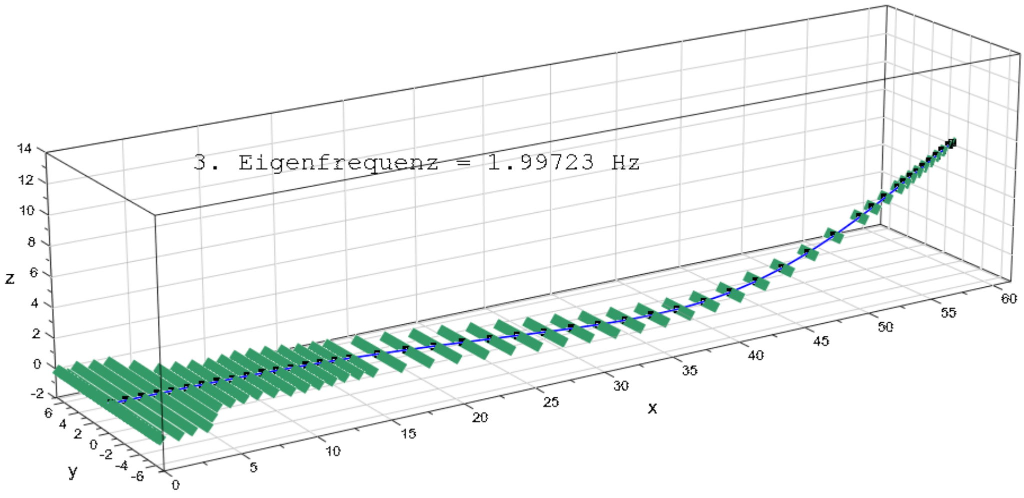

Fig 22:- 2nd flapwise eigen mode

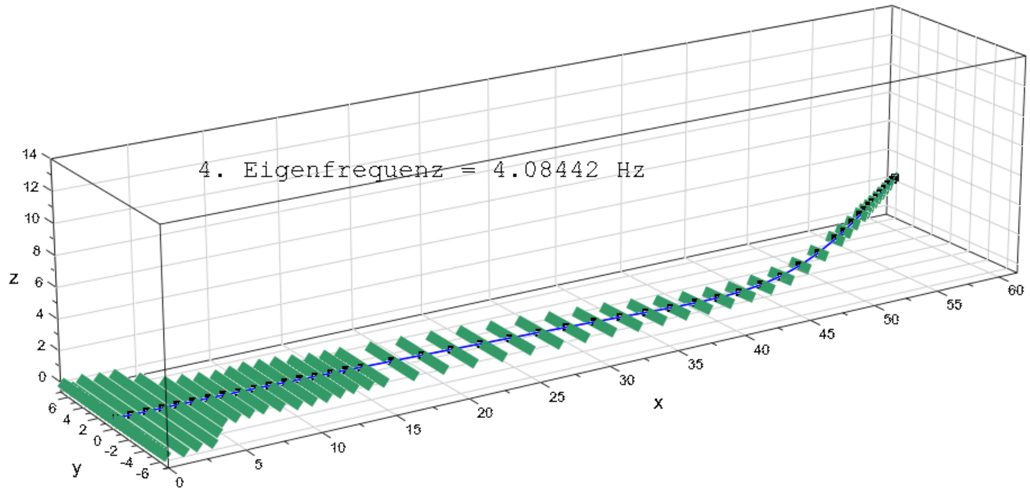

Fig 23:- 2nd edgewise eigen mode

The first two flapwise and edgewise eigenfrequencies obtained from above mentioned four cases is shown in the table 4

Description	**Gamma 0 Prebending 0 [Hz]**	**Pre bending non 0 Gamma 0 [Hz]**	**Gamma + Prebending [Hz]**	**Gamma non 0 Prebending 0 [Hz]**
1st Flapwisse Eigenfrequency	0.6914	0.6899	0.6906	0.6921
1st Edgewise eigenfrequency	1.1128	1.1077	1.1042	1.1092
2nd Flapwise Eigenfrequency	1.9917	1.9859	1.9914	1.9972
2nd Edgewise eigenfrequency	4.1200	4.0199	3.9918	4.0844

Table 4:- Edgewise and flapwise eigen frequencies

Comparing the case of considering only structural twist and neglecting prebending with the results published using FAST and ADAMS software in the report of NREL 5MW wind turbine [7] and table 4

mode	Description	FAST [Hz]	ADAMS [Hz]	Gamma non 0 prebending 0 [Hz]	Relative difference between FAST and Mupad[%]	Relative difference between ADAMS and mupad [%]
4	1st blade asymmetric flapwise yaw	0.6664	0.6296	0.6921	3.86	9.93
5	1st blade asymmetric flapwise pitch	0.6675	0.6686	0.6921	3.69	3.52
6	1st blade collective flap	0.6993	0.7019	0.6921	1.02	1.39
7	1st blade asymmetric edgewise pitch	1.0793	1.0740	1.1092	2.77	3.28
8	1st blade asymmetric edgewise yaw	1.0898	1.0877	1.1092	1.783	1.99
9	2nd blade asymmetric flapwise yaw	1.9337	1.6507	1.9972	3.29	20.993
10	2nd blade asymmetric flapwise pitch	1.9223	1.8558	1.9972	3.9	7.62
11	2nd blade collective flap	2.0205	1.9601	1.9972	1.1517	1.8943

Table 5:- Comparision of eigen frequencies with FAST and ADAMS

Percentage difference between case 1 and the case gamma ≠0 [%]	Percentage difference between case 2 and the case gamma ≠0 [%]	Percentage difference between case 3 and the case gamma ≠0 [%]
0.098	0.31	0.22
0.32	0.14	0.45
0.28	0.56	0.29
0.87	1.58	2.27

Table 6:- Percentage Difference between the cases 1,2,3 with case 4

The results obtained from Case 1, case 2, case 3 are compared with case considering only structural twist and it is as shown in the table 6.The relative differences as can be seen are very less and makes no considerable difference with considering structural twist and prebending.

6. Conclusion:

The purpose of this project was to develop an efficient MATLAB program to compute the eigen frequencies of the of a wind turbine blade using the structural properties and study the influence of the prebending and structural twist on 3D eigen modes. Eigen frequencies with different cases were evaluated and comparisons were made with the proposed results from FAST and ADAMS Software.

- The percentage difference between results obtained from the case considering only structural twist and the results proposed from the FAST is far less than the relative difference between the same cases with the results proposed from ADAMS software.

- The results obtained by considering the blade as a Euler Bernoulli beam are in close approximation to the proposed results from that of FAST and ADAMS Software.

- Comparing the results obtained from cases 1, 2 and 3 with the results obtained by considering only structural twist i.e. case 4, it can be concluded that the relative differences are very less and the influence of both structural twist and prebending deviates by 3%.

- As seen from the table 6 it can be concluded that, a 2 meter of prebending does not make a considerable difference in eigen frequency obtained by considering both structural twist and prebending.

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