DuMuX.Porous.Media

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1 Numerical simulation of immiscible two-phase flows

1.1 Test 1.1 : Buckley-Leverett equation

The goal is to approximate using DuMuX the 1-D equation for immiscible and incompressible two-phase in an 1-D homogeneous porous media of length L during a period of time]0 , T[. If the capillary pressure ($C_p = 0$) and the gravity are neglected, without source term, we obtain the Buckley-Leverett equation :

$$\phi \frac{\partial S}{\partial t} + q_T \frac{\partial F_w(S)}{\partial x} = 0, \quad x \in]0, L[, \, t \in]0, T[,$$

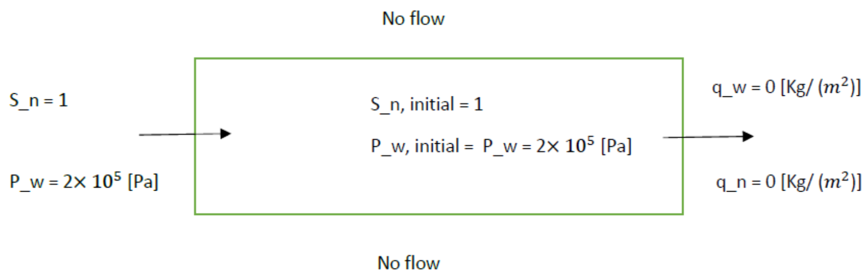
with initial and boundary conditions :

$$S(x, 0) = S_0(x), \quad x \in]0, L[,$$
$$S(0, t) = S_g(t), \quad t \in]0, T[,$$

where q_T is the total velocity of the flow, S_0, S_g (entry saturation) are given functions. $F_w \in C^1([0, 1])$ is the fractional flux and is given by :

$$F_w(S) = \frac{\frac{K_{rw}(S)}{\mu_w}}{\frac{K_{rw}(S)}{\mu_w} + \frac{K_{rn}(S)}{\mu_n}},$$

where, $K_{r\alpha}$ denotes the relative permeability of phase α and μ_α the viscosity of phase α. We use the model $2p$ of DuMuX. We consider $1 - D$ domain with $L = 1m$ and time of simulation $T = 0.2s$.The properties of porous medium are $\phi = 0.2$ and $K = 10^{-5}m^2$. $Cp = 0$ and relative permeabilities are given by $K_{rw}(S) = S^2$ and $K_{rn}(S) = (1 - S)^2$.The viscosities are $\mu_w = 1$ pa.s. The densities are $\rho_w = 1Kg.m^{-3}$ and $\rho_n = 1\,Kg.m^{-3}$.



Boundary and initial conditions.

We performed the simulation on DuMuX by plotting S_w at each time $t = 0.05$, 0.1, 0.15 and 0.2 seconds and we got the following results :

For $t = 0.05$ sec :

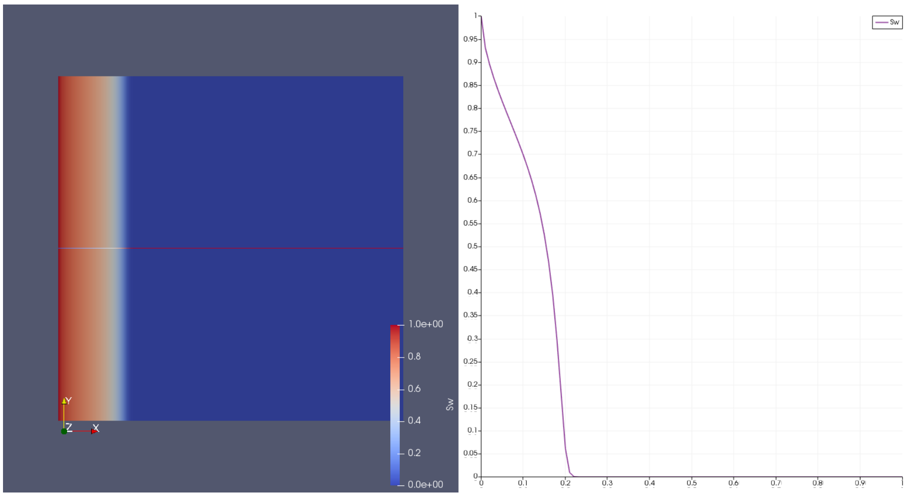


Figure 1: we noticed when we inject the wetting phase, at $L = 0$ m $S_w = 1$ (red region), after that S_w decreased progressively to achieve 0.7 at $L = 0.1$ m (orange region where there is some non-wetting phase remained) and the non-wetting phase pushed by the wetting phase, until S_w reached zero at $L = 0.2$ m (blue region where the wetting phase not achieve and not push the non-wetting phase). **We conclude that the wetting phase injected and push the non-wetting phase and it invades only the first 0.2 m of the medium in 0.05 sec.**

For $t = 0.1$ sec :

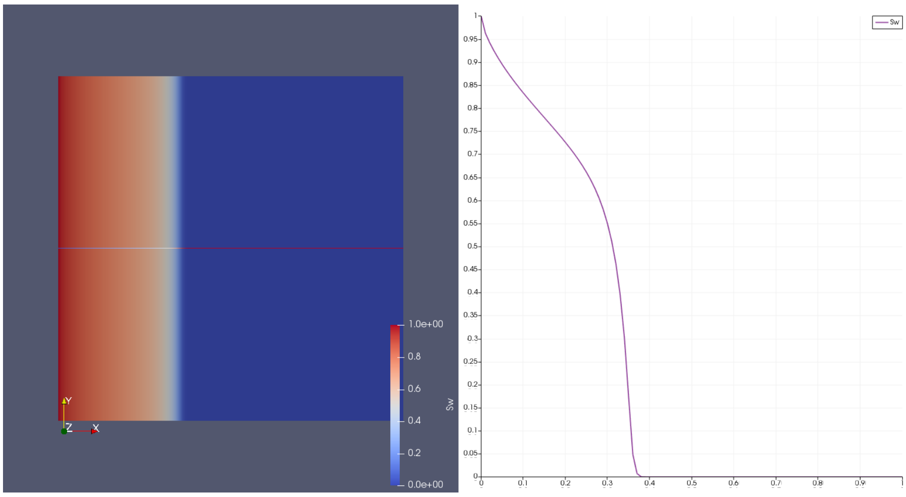


Figure 2: At $L = 0$ m, $S_w = 1$ (red region), after that S_w decreased progressively to achieve 0.83 at $L = 0.1$ m, that shows that the wetting phase invade more pores and sweep or push more non-wetting phase, similarly at $L = 0.2$ m and 0.3 m the wetting phase reached 0.73 and 0.5 respectively (we see white and orange regions) until S_w achieved zero (blue region) at $L = 0.38$ m . We conclude that the wetting phase invade only the first 0.38 m of the medium in 0.1 sec.

For $t = 0.15$ **sec** :

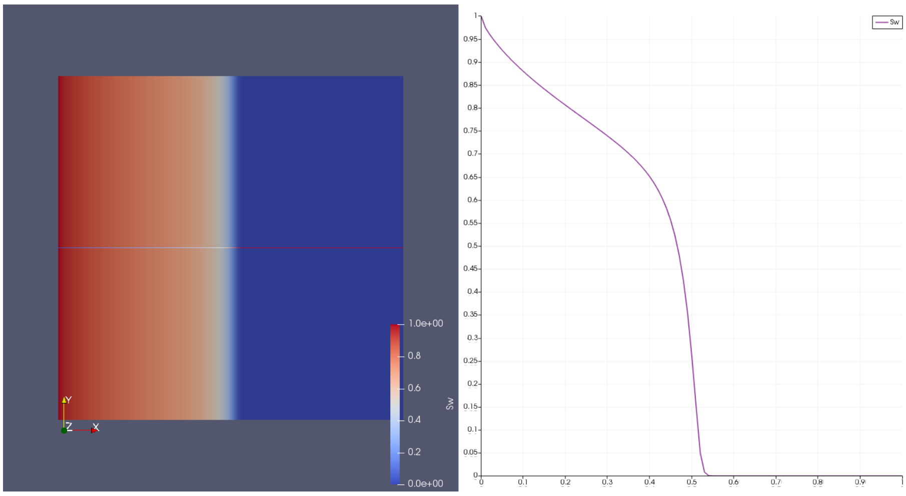


Figure 3: In 0.15 sec we notice that at $L = 0$ m, $S_w = 1$ (red region), after that S_w decreased progressively to achieve 0.85 (red region) at $L = 0.1$ m, that shows that **the wetting phase invade more and more pores and sweep more non-wetting phase,** similarly at $L = 0.2$ m, 0.3 m and 0.4 m the wetting phase reached $0.8, 0.74$ and 0.65 (orange region where the wetting phase push more the non-wetting phase and there is few non-wetting phase) respectively until S_w achieved zero (blue where there is only non-wetting phase) at $L = 0.53$ m. We conclude that the wetting phase invaded only the first 0.53 m of the medium in 0.15 sec.

For $t = 0.2$ **sec** :

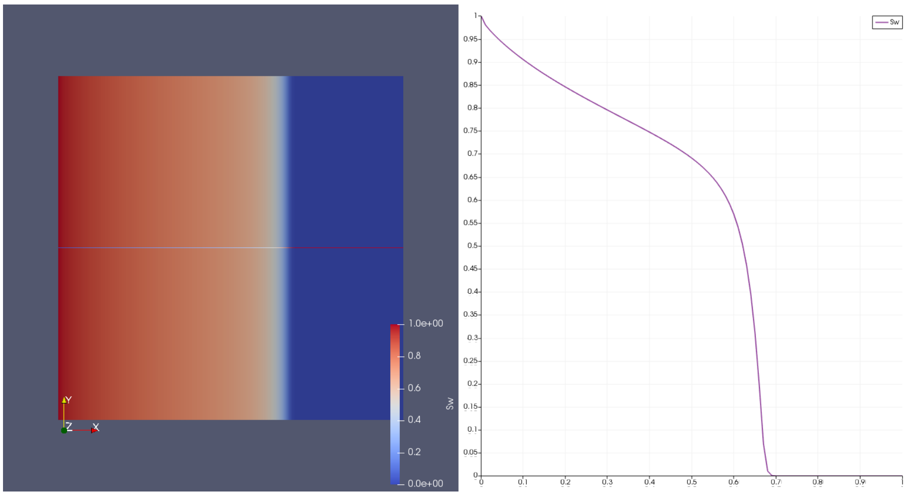


Figure 4: At the end of simulation time 0.2 sec we notice that at $L = 0$ m, $S_w = 1$ (red region), after that S_w decreased progressively to achieve 0.9 (red region)at $L = 0.1$ m, that shows that **the wetting phase invade more and more pores and sweep more non-wetting phase,** similarly at $L = 0.2$ m, 0.3 m, 0.4 m and 0.5 m the wetting phase reached $0.85, 0.8, 0.75$ and 0.7 respectively (orange and red regions) until S_w achieved zero (blue regions)at $L = 0.7$ m. We conclude that the wetting phase invade only the first 0.7 m of the medium in 0.2 sec.

To sum up, we can say that the wetting phase invade more pores and push the non-wetting phase (red,orange and white regions) and the sweep efficiency increases, so more non-wetting phase extracted from the medium as time increase, and there is still non-wetting phase not extracted from the medium (blue regions).

1.2 Test 1.2 : 2D case

We consider a domain $\Omega =]0,1[\times]0,1[$ initially saturated by a non-wetting phase ($S_n = 1$) witha velocity $\mu_n = 5Pa.s$ and density $\rho_n = 1Kg.m^{-3}$. A wetting phase is injected with viscosity $\mu_w = 1Pa.s$ and density $\rho_w = 1Kg.m^{-3}$ on the left border. The boundary is divided into three parts:

- On the left border a pressure condition $P_w = 1$ Pa and saturation condition $S_w = 1$ are imposed.
- On the right corner a pressure condition $P_w = 0$ Pa and saturation condition $S_n = 1$ are imposed.
- Top and bottom borders are impermeable : $\overrightarrow{q_n}.\overrightarrow{n} = \overrightarrow{q_w}.\overrightarrow{n} = 0\ m.s^{-1}$.
  Porisity is homogeneous and equals 0.2. The capillary pressure in null ($C_p = 0$), so $P_w = P_n$. Initial pressure is given by : $P = P_w = P_n 1 - x_1 Pa$. Relative permeabilities are given by :
  $$K_{rw}(S) = S^2, \quad K_{rn}(S) = (1-S)^2.$$

Two cases are studied. The 1st one deals with analytic permeability while the 2nd one consists in using 100 random points in the domain where the permeabiity is varying in the neighborhoods of these latters in comaparison with the rest of the domain.

1.2.1 Test 1.2.1

The permeability is given by :

$$K(x,y) = max\left(e^{-\left(\frac{2y-1-0.2sin(10x)}{0.2}\right)^2}, 0.01\right)$$

We perform a simulation with DuMuX with $T = 0.4s$ by representing the permeability and the saturation of the wetting phase (S_w) at times $t = 0.1, 0.2, 0.3$ and $0.4\,sec$. The permeability will be given as a scalar and not as a matrix.

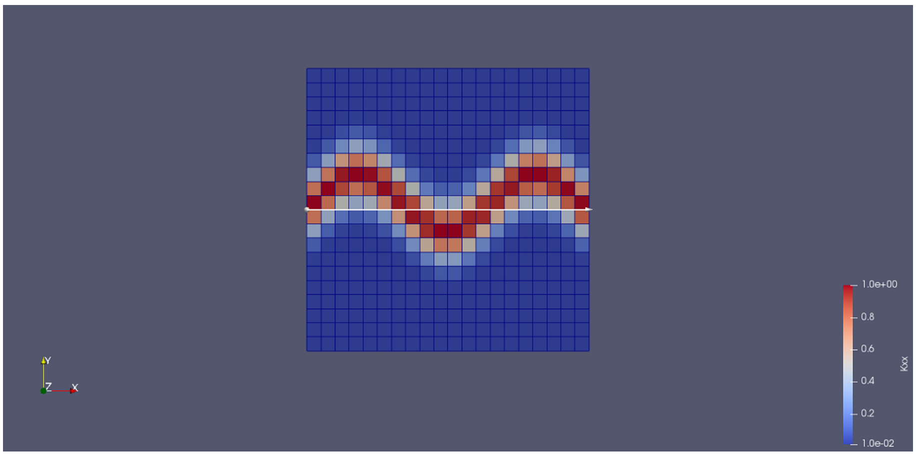


Figure 5: As the permeability is independent of time we notice that it will be the same in all the cases for $t = 0.1, 0.2, 0.3$ and 0.4. We realise that there is high permeability on the middle (red and orange regions) where the fluid can enter and invade more through this region, but the permeability is low in the extremities (white and then all blue), **we will see below in the Sw figures that the wetting phase will not pass through the blue regions (impermeable region) and it will follow the path of red, orange and white regions of permeability.**

For $t = 0.1$ **sec** :

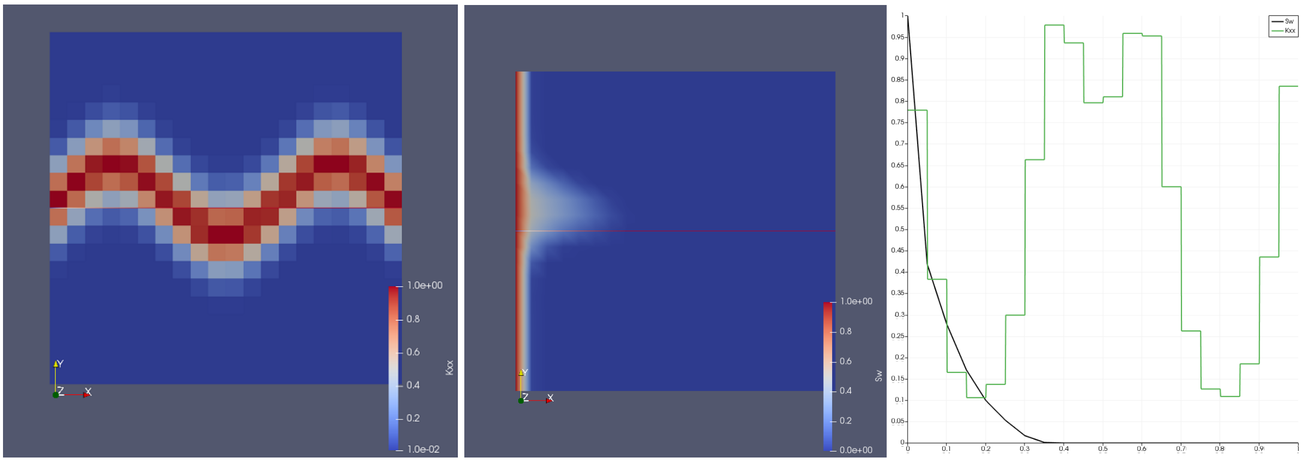


Figure 6: At the beginning $S_w = 1$ at $L = 0$ m (red region), afterwards, S_w starts to decrease to reach zero (blue region) at $L = 0.35$ m. **As the permeability is higher in the middle (red and orange regions) we notice that the wetting phase push the non-wetting phase and invade the medium from the middle following the high permeability regions (the red and orange regions in the permeability) and reach** 0.35 **m in** 0.1 **sec.**

For $t = 0.2$ **sec** :

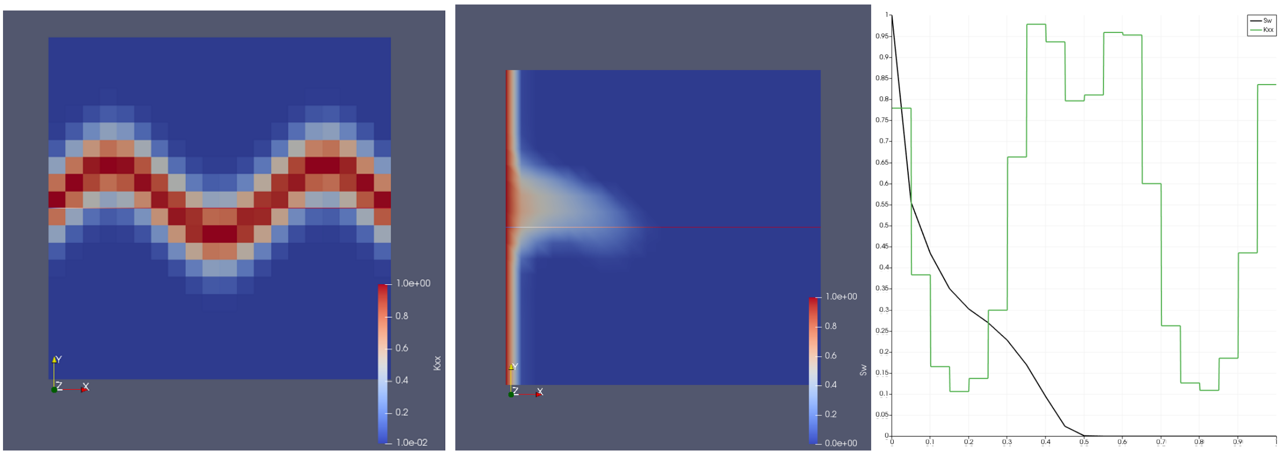


Figure 7: Similarly, $S_w = 1$ at $L = 0$ m, afterwards, S_w starts to decrease to reach zero at $L = 0.5$ m. **As the permeability is higher in the middle (red and orange regions) and low in the extremities (white and blue regions) we notice that the wetting phase invade more pores and sweep (push) more non-wetting phase following the path of red and orange regions in the permeability figure.** We also realise that the wetting phase reaches 0.5 m in 0.2 sec.

For $t = 0.3$ **sec**:

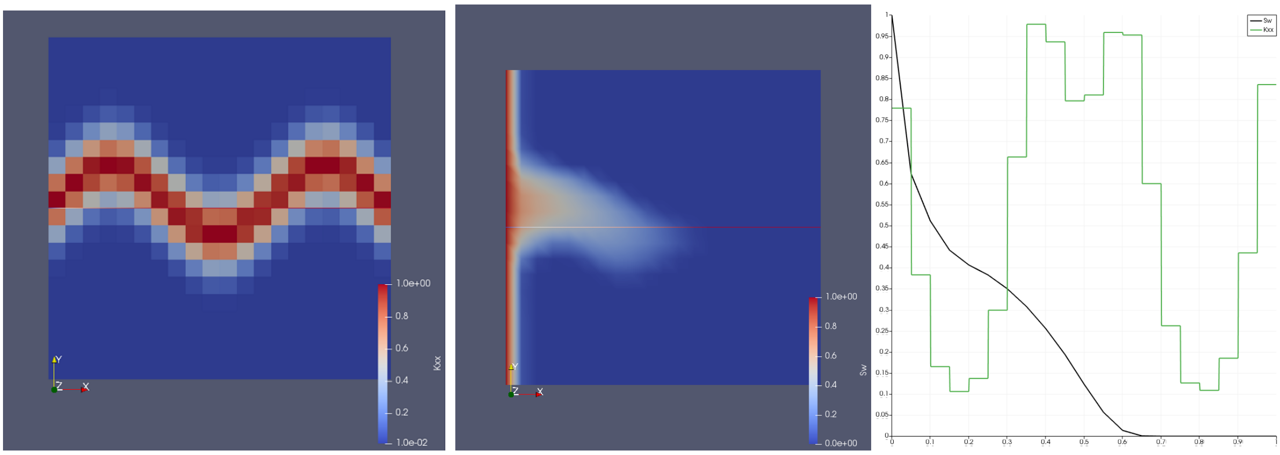


Figure 8: $S_w = 1$ at $L = 0$ m, afterwards, S_w starts to decrease to reach zero at $L = 0.65$ m. As the permeability is higher in the middle we noticed that the wetting phase invade more pores and sweep more non-wetting phase following the path where the red, orange and white regions in the figure of permeability. Also the wetting phase spread and occupy more pores in the medium as times of simulation increase (even if S_w is low and we have more white zones) and reaches 0.65 m in 0.3 sec which is good compared to the above cases.

For $t = 0.4$ **sec** :

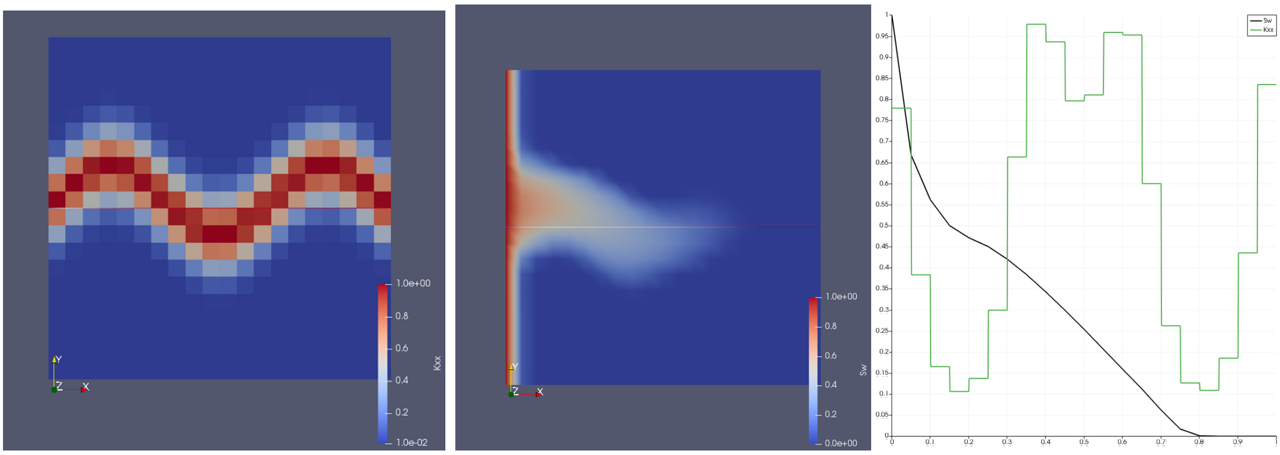


Figure 9: At the beginning $S_w = 1$ at $L = 0$ m, afterwards, S_w starts to decrease to get orange region where S_w goes from 0.8 to 0.6 (that means the wetting phase invade the non-wetting phase) until it reaches zero at $L = 0.8$ m. As the permeability is higher in the middle we noticed that the wetting phase sweep more non-wetting phase, also spread and occupy more pores in the medium to reach 0.8 m in 0.4 sec which is better than the above cases. We see also more white regions because S_w decreased from 0.5 to 0 in this region from 0.15 m to 0.8 m where the wetting phase is less than the non-wetting phase in most regions.

1.2.2 Test 1.2.2

The random permeability is now give by :

$$K(x,y) = min\Big(max\Big(\sum_{i=1}^{i=N_p} \sigma_i(x,y), 0.01\Big), 4\Big),$$

where

$$\sigma_i(x,y) = e^{-\left(\frac{\left|(x,y)-(x^i,y^i)\right|}{0.05}\right)^2},$$

$N_p = 100$ represents a number for which the coordinates (x^i, y^i) randomly distributed according to a uniform law in the domain.

We perform a simulation with DuMuX with $T = 0.6$ sec by representing the permeability and S_w at times $t = 0.15, 0.3, 0.45$ and $0.6\,sec$.

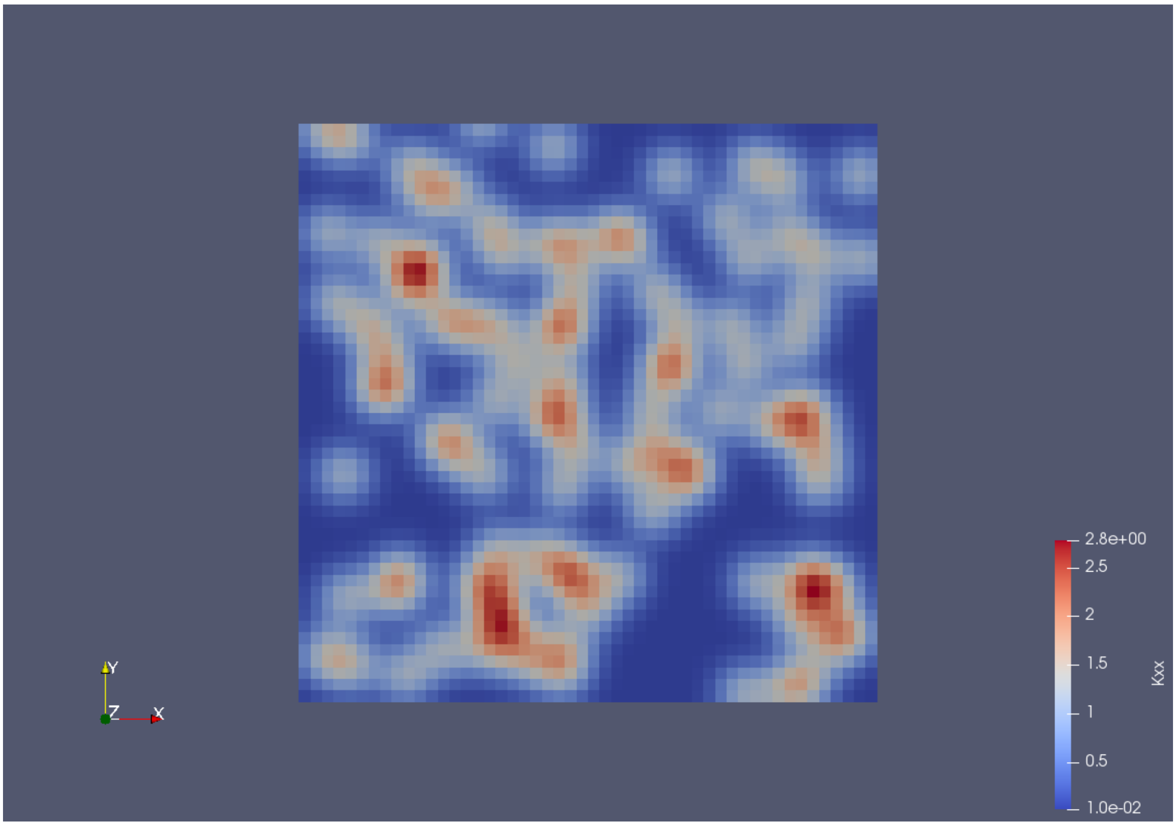


Figure 10: As the permeability is independent of time, so it will be the same during the simulation. The permeability is randomly distributed, it is high in some pores (red zones) where the fluid can go through the pores easily, where as its a little bit lower in another areas, and also there is impermeable areas (blue areas) where the wetting phase can't pass through them.

Moreover, we see in the right corner down, there is red zone (permeability high) and behind this region there is blue region (permeability very low) this means that the wetting phase will not reach that area because it will face region of low permeability that prevent the wetting phase to enter the pores and push the non-wetting phase.

For $t = 0.15$ **sec** :

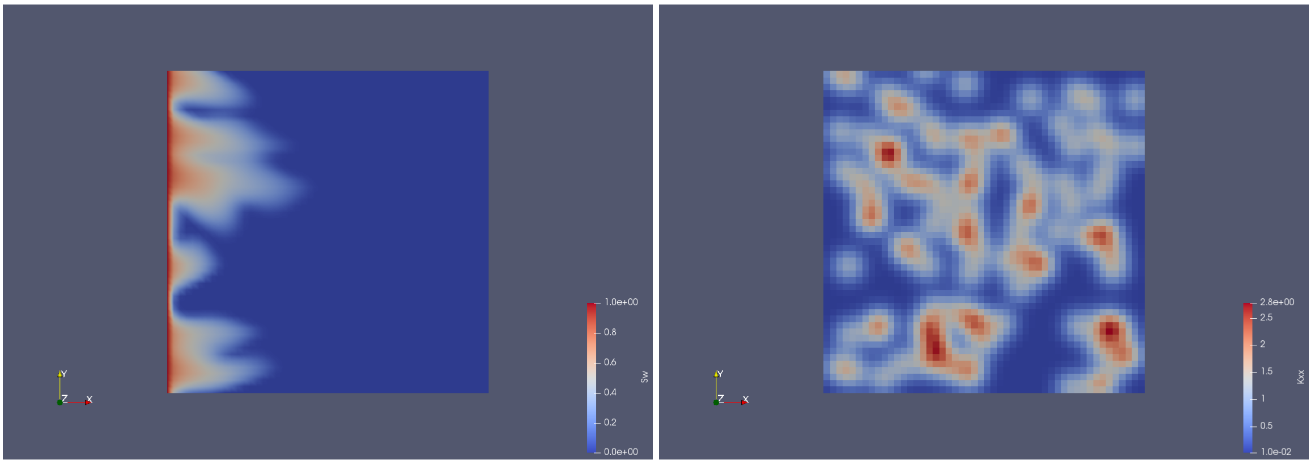


Figure 11: We notice that the wetting phase invade the pores where the permeability is not low (white, orange and red zones). Moreover, there is areas where the permeability is blue the wetting phase can't invade and enter through the pores.

For $t = 0.3$ **sec** :

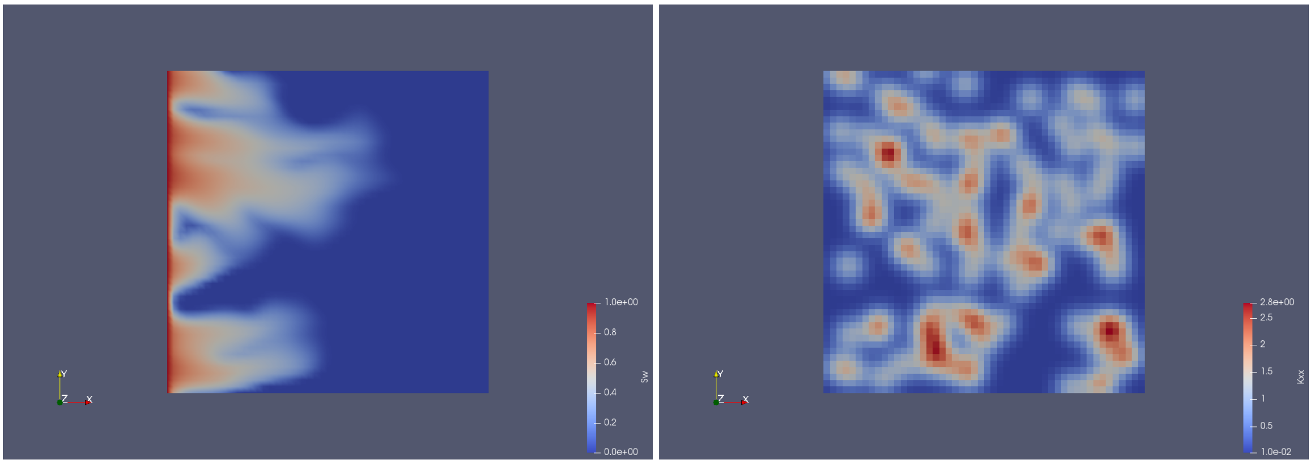


Figure 12: We notice here that the wetting phase keep going through the regions of high permeability and invade more through the porous medium and occupy more pores and it sweep more non-wetting phase. Moreover, there is some areas in the previous figure where the wetting phase did not entered through them and now the wetting phase invade it.

For $t = 0.45$ **sec** :

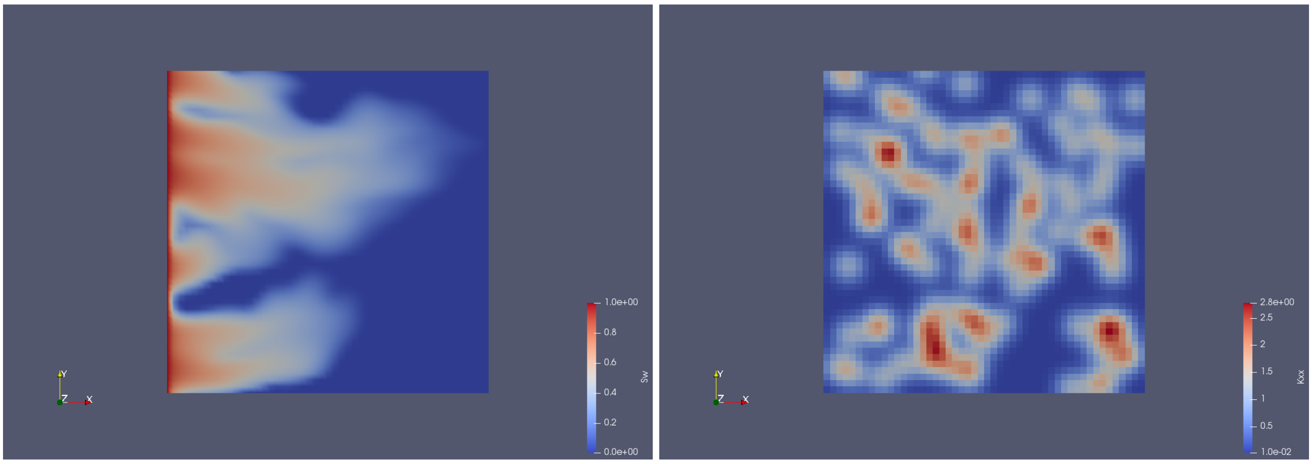


Figure 13: As the time increase we can deduce that the wetting phase occupy more pores and spread through the medium more and more. Moreover, more pores becomes more saturated in wetting phase and also the wetting phase continue to go through other pores where we see the white regions.

For $t = 0.6$ **sec** :

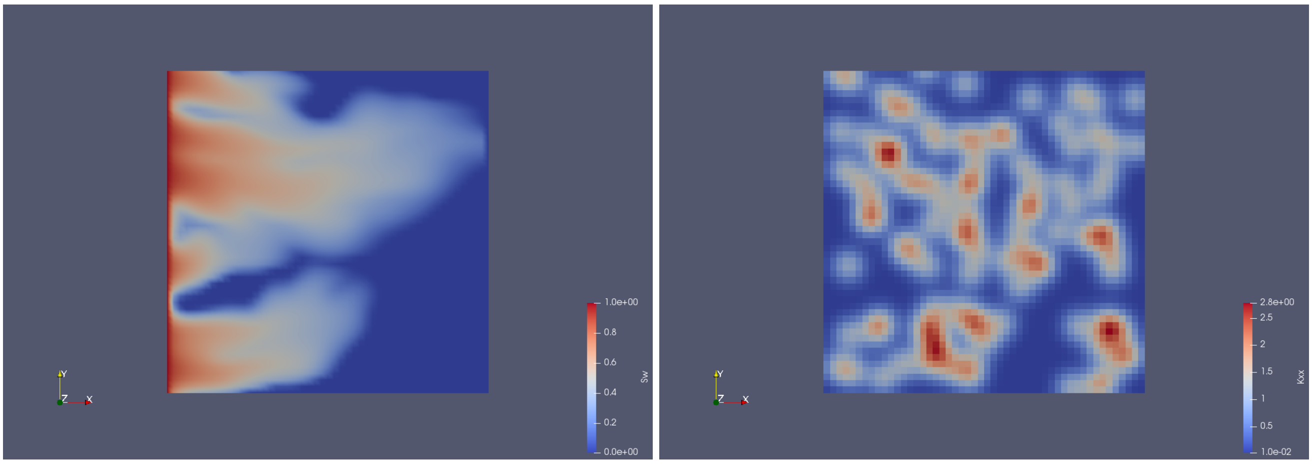


Figure 14: When the simulation complete it, we notice that the blue regions where the permeability is low the wetting phase does not pass through the medium that is why we see blue regions in Sw graph too, and white regions in permeability (low permeability) implies white regions in Sw (wetting phase reach this region but Sw will not be high compared to Sn) but in the red regions in permeability where the permeability is high the wetting phase will go easily through the pores and it will invade the non-wetting phase (red region in Sw).

Important remark, if we look on the permeability graph (right corner below) the permeability is high (red zone) but in contrary the Sw gives blue (right corner down) its so low (blue) that is because if we go back to the permeability we notice that before that region (red zone in the right corner down) there is big blue region behind it, that means the wetting phase can not pass, thus it is logical to obtain blue region in the right corner down in Sw graph (wetting phase does not reach this area).