1. Gas transport in a porous medium

The evolution of the density u of a gas flowing trough a porous medium can be described by the so called *porous medium equation*: for $m > 1$,

$$\partial_t u - \Delta u^m = 0. \tag{1}$$

For space dimension d in the domain $\mathbb{R}^d \times (0, \infty)$ the equation has a radially symmetrict exact solution, the so-called Barenblatt solution:

$$b(x,t) = \max\left(0, t^{-\alpha}\left(1 - \frac{\alpha(m-1)r^2}{2dmt^{\frac{2\alpha}{d}}}\right)^{\frac{1}{m-1}}\right) \tag{2}$$

Here, $r = |x|$ and $\alpha = \frac{1}{m-1+\frac{2}{d}}$. This solution has finite support and spreads a finite amount of mass over the space domain.

1.1. Implementation.

Choose (e.g.) $L = 1$.

- Implement a finite volume method for solving the equation, you can use the Julia package VoronoiFVM.jl.
- Choose time values $t_0 < t_1$ such that the support of $b(x, t_1)$ is contained in $(-L, L)$
- Solve the problem in a space-time domain $(-L, L) \times (t_0, t_1)$ with initial value $u(x, t_0) = b(x, t_0)$
  - Provide a space-time plot of the solution
  - Calculate the solution on several discretization grids with increasing nunber of points, calculate the error of the solution and plot it vs. grid spacing
- Repeat this for the 2D case in $(-L, L)^2 \times (t_0, t_1)$

1.2. Optional.

- Discuss ways to improve performance
- Use alternative timestepping methods from DifferentialEquations.jl (This possibility is currently under development for VoronoiFVM, please come back to me if you want to try this out – in fact the more sophisticated schemes allow for faster solution.)

1.3. Report.

- Introduce the problem and some information on the physical background
- Describe the finite volume space discretization approach
- Discuss possibilities for the time discretization. Are there any obstacles for implementing the explicit Euler method ?
- Discuss possible solution methods for the discretized problem
- Present simulation results. Suggestion: use 2D space-time plots for the 1D problems.
- Discuss the optional topics.