Phase Structure in a Relational Network Under Thermodynamic Annealing

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Repository: `https://github.com/Atyzze/relational_reality/`

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Abstract

We study a relational network model in which no background geometry, embedding space, or physical laws are imposed. A graph of $N = 4444$ nodes evolves under thermodynamic annealing driven by local phase synchronization and topological stress minimization. After equilibration, distances are defined purely from graph structure and used to infer geometric organization via metric-preserving projections. Across parameter regimes, the system exhibits multiple stable and metastable geometric phases, including extended, phase-separated, and re-homogenized configurations. We report these phases and the procedures used to generate and diagnose them, without assigning physical interpretation.

1 Introduction

Many models of emergent geometry presuppose a target manifold or continuum limit. Here we take a different approach: no geometric structure is assumed, and geometry is inferred only after the system has equilibrated. The goal of this work is not interpretation, but documentation. We describe how relational dynamics alone give rise to structured distance organization and report the distinct regimes observed across runs.

2 Model and Dynamics

2.1 Relational System

The system consists of $N = 4444$ nodes connected by a dynamically evolving graph. No coordinates or embedding space are defined at any stage of the evolution. All interactions depend solely on node–node relations.

Each node carries a continuous internal phase variable ψ. The model contains no explicit geometric quantities.

2.2 Energy Function

The Hamiltonian penalizes two local sources of frustration:

1. **Phase frustration:** neighboring nodes favor phase synchronization.
2. **Topological frustration:** graph configurations with excess local stress are penalized.

No geometric target, curvature term, or gravitational law is included.

2.3 Annealing Procedure

The system evolves via a Metropolis–Hastings update scheme for 4.44×10^6 steps. Updates consist of local rewiring or adjustment moves accepted stochastically according to the energy change.

Equilibration is determined empirically by:

- stabilization of mean degree $\langle k \rangle$ and triangle count,
- vanishing rewiring acceptance rate,
- persistent high local phase correlation,
- numerical gauge-invariance checks within floating-point tolerance.

3 Distance Construction and Diagnostics

3.1 Graph Distance

After equilibration, a distance matrix D_{ij} is computed using shortest-path distances on the final graph. Distances are symmetric and satisfy metric consistency by construction. Distances are not dynamical variables and do not feed back into the evolution.

3.2 Density Proxy

A scalar diagnostic quantity ρ_i is computed for each node based on local relational observables (including phase variance and loop participation). This quantity is used only for coloring and correlation analysis and does not represent spatial density.

3.3 Metric Inference

Classical multidimensional scaling (MDS) is applied to D_{ij} to obtain a two-dimensional embedding that best preserves pairwise distances. This embedding is used solely as a diagnostic visualization of the inferred metric structure.

Force-directed (spring–mass) layouts are computed independently for comparison and are not distance preserving.

4 Results

4.1 Equilibration Diagnostics

Figure 1 shows representative equilibration telemetry. Mean degree, triangle count, and energy stabilize, while rewiring acceptance approaches zero. Local phase correlation remains high throughout, indicating that geometric freezing does not disrupt phase coherence.

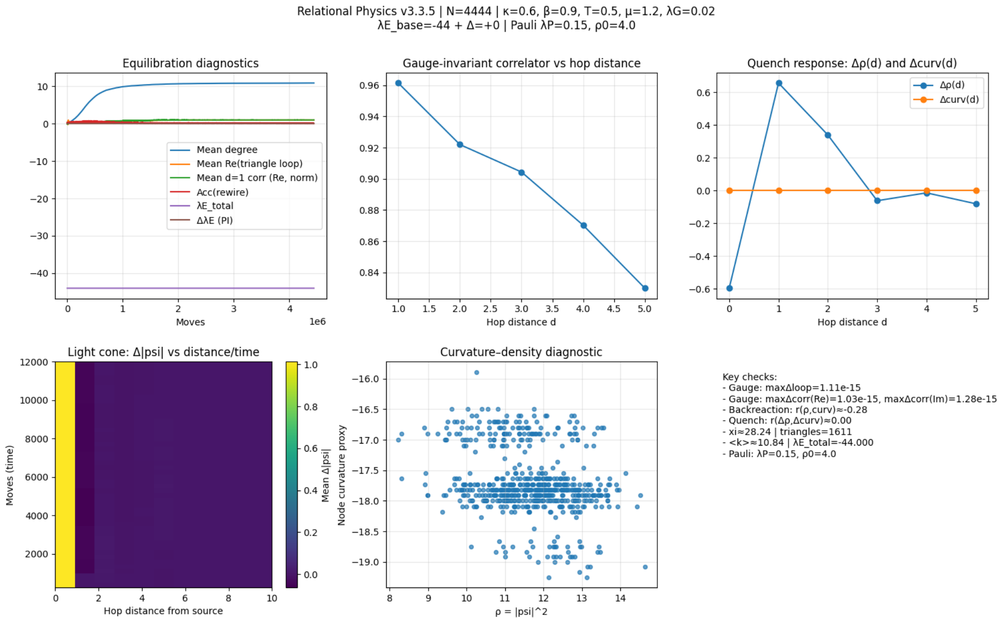

Figure 1: **Equilibration diagnostics.** Degree and triangle stabilization (top left), correlation vs hop distance (top right), light-cone propagation (bottom left), and density–curvature correlation (bottom right).

4.2 Observed Geometric Phases

Across runs and parameter sweeps, three qualitative regimes are observed.

Extended Phase

At low effective density, the inferred geometry exhibits a large effective radius, weak clustering, and broad distance distributions.

Core–Shell Phase

At intermediate density, the system develops a phase-separated structure consisting of a compact high-density core and an extended low-density shell. This configuration is metastable but reproducible across runs.

Re-homogenized Phase

At higher density, the system reorganizes into a single coherent geometric structure with smooth density gradients and reduced distance variance.

Representative metric projections are shown in Figure 2.

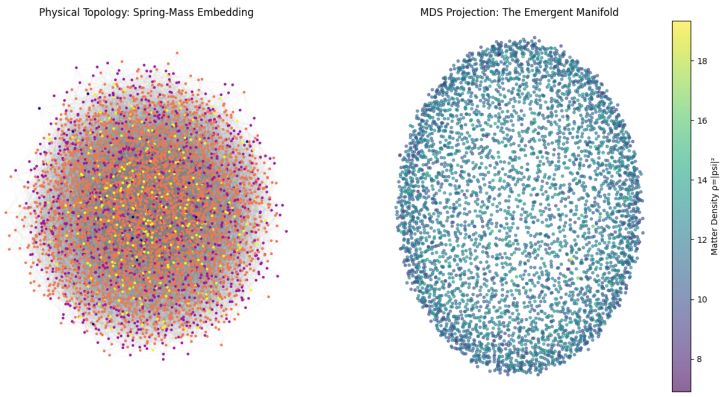

Figure 2: **Metric-preserving projection after equilibration.** Colors indicate the diagnostic density proxy ρ_i.

4.3 Density–Curvature Correlation

We compute Ollivier–Ricci curvature on the equilibrated graph and examine its correlation with the density proxy ρ. A consistent negative correlation is observed across runs, with a representative Pearson coefficient $r \approx -0.26$.

A linear fit is shown in Figure 3. The slope is reported solely as a dimensionless diagnostic of coupling strength.

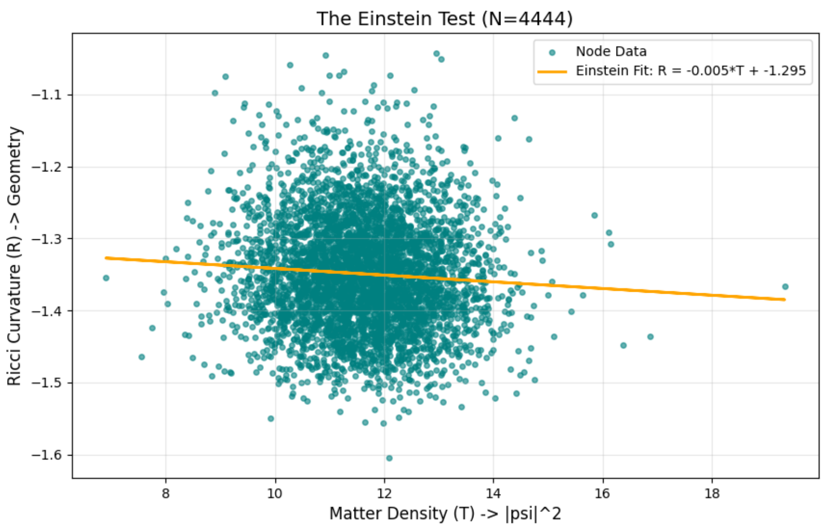

Figure 3: **Density–curvature correlation.** Scatter plot and linear fit shown for a representative run.

5 Robustness and Limitations

The observed phases persist across random initializations and parameter sweeps. Metric-preserving projections remain stable, while force-directed layouts exhibit increasing distortion at higher density.

No claim is made regarding physical spacetime, gravitational dynamics, or continuum limits. All geometric structure is inferred empirically from graph distances.

6 Conclusion

We have documented multiple density-dependent geometric phases in a purely relational network evolved under thermodynamic annealing. The results demonstrate that adaptive relational dynamics alone can generate structured metric organization without imposing background geometry or target embeddings.