



Chapter 7: Relational Database Design

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Chapter 7: Relational Database Design

- Functional Dependency Concepts
- Normal Forms
 - Atomic Domains and First Normal Form
 - BCNF and Third Normal Form
- Functional Dependency Theory and Algorithms
- Decomposition and Testing Using Functional Dependencies
- More Normal Form
- Database-Design Process



Functional Dependencies (FD)

- Constraints on the set of legal relations.
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes.
- A functional dependency is a generalization of the notion of a *key*.
- Functional dependencies allow us to find/define dependencies whose left sides are not superkeys. It can also help us to find illegal dependencies. Consider the schema:

inst_dept (ID, name, salary, dept_name, building, budget).

We expect these functional dependencies to hold:

$\text{dept_name} \rightarrow \text{building}$ (*dept_name* is not a superkey)

and $ID \rightarrow \text{building}$ (*ID* is a superkey)

but would not expect the following to hold:

$\text{dept_name} \rightarrow \text{salary}$



Functional Dependencies -Definition

- Let R be a relation schema

$$\alpha \subseteq R \text{ and } \beta \subseteq R$$

- The **functional dependency**

$$\alpha \rightarrow \beta$$

holds on R if and only if for any **legal relations $r(R)$** , whenever any two tuples t_1 and t_2 of r agree on the attributes α , they also agree on the attributes β . That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

- Example: Consider $r(A,B)$ with the following instance of r .

1	4
1	5
3	7

- On this instance, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold.



Functional Dependencies

- Key defined with FD

- K is a superkey for relation schema R if and only if $K \rightarrow R$
- K is a candidate key for R if and only if
 - $K \rightarrow R$, and
 - for no $\alpha \subset K$, $\alpha \rightarrow R$



Functional Dependencies – Trivial FDs

- A functional dependency is **trivial** if it is satisfied by all instances of a relation
 - Example:
 - ▶ $ID, name \rightarrow ID$
 - ▶ $name \rightarrow name$
 - In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$



Closure of a Set of Functional Dependencies

- Given a set F of functional dependencies, there are certain other functional dependencies that are logically implied by F .
 - For example: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of **all** functional dependencies logically implied by F is the **closure** of F .
- We denote the *closure* of F by F^+ .
- F^+ is a superset of F .



Functional Dependencies – Categories

- Trivial and Non Trivial
- For Non Trivial FD $\alpha \rightarrow \beta$:
 - α : We care more about whether it is a superkey. Two categories for α :
 - ▶ is a superkey;
 - ▶ is not a superkey;
 - β : We care more about whether there is any part in β that is part of a candidate key, which means these parts can also determine the values of some other attributes. This will bring some complexity.
Two categories for β :
 - ▶ No part of β is part of a candidate key;
 - ▶ Some or all of β is (or is part of) a candidate key
 - Combine categories of α and β , we have four different categories for non trivial FDs
- If all α of non trivial FDs in F^+ are superkeys, in most cases, R is in good form. If some α of non trivial FDs in F^+ are not superkeys, we will do decomposition based on category of β .



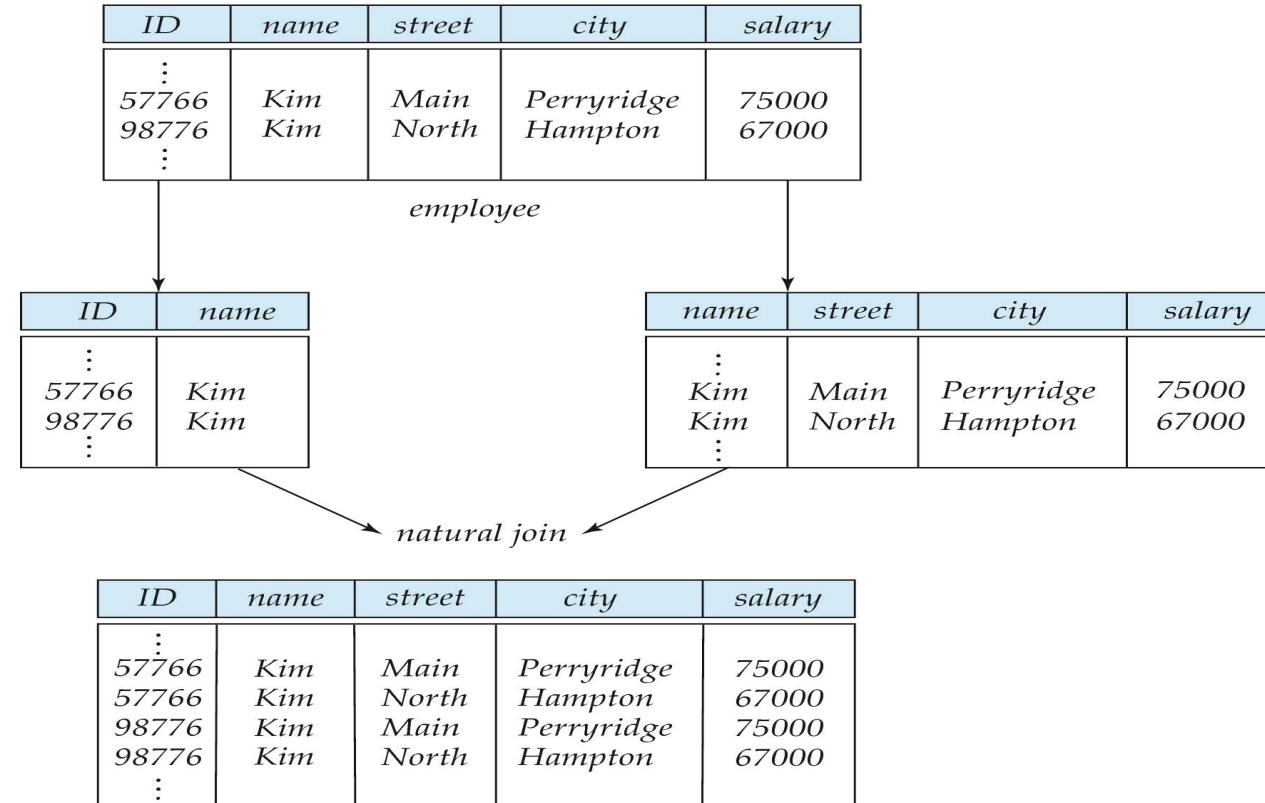
Use of Functional Dependencies

- We use functional dependencies to:
 - specify constraints on the set of legal relations
 - ▶ We say that F **holds on** R if all legal relations on R satisfy the set of functional dependencies F .
 - test relations to see if they are legal under a given set of functional dependencies.
 - ▶ If a relation r is legal under a set F of functional dependencies, we say that r **satisfies** F .
 - if R is not in good form, decompose R which satisfies F into $R_1 \dots R_n$, so that each of $R_1 \dots R_n$ is in good form
 - In some cases, combine some relations into one relation
- Note: A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not hold on all legal instances.
 - For example, a specific instance of *instructor* may, by chance, satisfy $name \rightarrow ID$.



Functional Dependency

- improper(Lossy) decomposition



- In this relation. All α in the non-trivial functional dependencies are superkeys. No need to decompose. If decompose improperly, some information will be lost. See the results of decomposition and combination above.



Functional Dependency

- proper decomposition

<i>ID</i>	<i>name</i>	<i>salary</i>	<i>dept_name</i>	<i>building</i>	<i>budget</i>
22222	Einstein	95000	Physics	Watson	70000
12121	Wu	90000	Finance	Painter	120000
32343	El Said	60000	History	Painter	50000
45565	Katz	75000	Comp. Sci.	Taylor	100000
98345	Kim	80000	Elec. Eng.	Taylor	85000
76766	Crick	72000	Biology	Watson	90000
10101	Srinivasan	65000	Comp. Sci.	Taylor	100000
58583	Califieri	62000	History	Painter	50000
83821	Brandt	92000	Comp. Sci.	Taylor	100000
15151	Mozart	40000	Music	Packard	80000
33456	Gold	87000	Physics	Watson	70000
76543	Singh	80000	Finance	Painter	120000

- *inst_dept (ID, name, salary, dept_name, building, budget)*.

with functional dependencies:

dept_name → *building, budget*

where *dept_name* is not a superkey, building or budget is not part of a candidate key

- Decompose into instructor and department



Example of Lossless-Join Decomposition

- **Lossless join decomposition**

- Decomposition of $R = (A, B, C)$

$$R_1 = (A, B) \quad R_2 = (B, C)$$

A	B	C
α	1	A
β	2	B

r

A	B
α	1
β	2

$\Pi_{A,B}(r)$

B	C
1	A
2	B

$\Pi_{B,C}(r)$

$\Pi_A(r) \bowtie \Pi_B(r)$

A	B	C
α	1	A
β	2	B



Functional Dependency

- Sometime we need to combine

- Consider combining relations
 - $\text{dept_building}(\text{dept_name}, \text{building})$ and
 - $\text{dept_budget}(\text{dept_name}, \text{budget})$
- *into one relation*
 - $\text{department}(\text{dept_name}, \text{building}, \text{budget})$
- No repetition in this case



First Normal Form

- Domain is **atomic** if its elements are considered to be indivisible units
 - Examples of non-atomic domains:
 - ▶ Set of names, composite attributes
 - ▶ Identification numbers like CS101 that can be broken up into parts
- A relational schema R is in **first normal form** if the domains of all attributes of R are atomic
- Non-atomic values complicate storage and encourage redundant (repeated) storage of data
 - Example: Set of accounts stored with each customer, and set of owners stored with each account
 - We assume all relations are in first normal form (and revisit this in Chapter 22: Object Based Databases)



First Normal Form (Cont'd)

- Atomicity is actually a property of how the elements of the domain are used.
 - Example: Strings would normally be considered indivisible
 - Suppose that students are given roll numbers which are strings of the form CS0012 or EE1127
 - If the first two characters are extracted to find the department, the domain of roll numbers is not atomic.
 - Doing so is a bad idea: leads to encoding of information in application program rather than in the database.
- First Normal Form is the basic requirements for a relation. But not the last. We need some rules to help us find whether the relation is in good form.



To generate good form relations

- Decide whether a particular relation R is in “good” form.
- In the case that a relation R is not in “good” form, decompose it into a set of relations $\{R_1, R_2, \dots, R_n\}$ such that
 - each relation is in good form
 - the decomposition is a lossless-join decomposition
- Our theory is based on:
 - functional dependencies
 - multivalued dependencies



Boyce-Codd Normal Form

A relation schema R is in BCNF with respect to a set F of functional dependencies if for all functional dependencies in F^+ of the form

$$\alpha \rightarrow \beta$$

where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:

- $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
- α is a superkey for R

Example schema *not* in BCNF:

instr_dept (ID , $name$, $salary$, $dept_name$, $building$, $budget$)

because $dept_name \rightarrow building, budget$
holds on *instr_dept*, but $dept_name$ is not a superkey



Decomposing a Schema into BCNF

- Suppose we have a schema R and a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF.

We decompose R into:

- $(\alpha U \beta)$
- $(R - (\beta - \alpha))$

- In our example,

- $\alpha = \text{dept_name}$
- $\beta = \text{building, budget}$

and inst_dept is replaced by

- $(\alpha U \beta) = (\text{dept_name, building, budget})$
- $(R - (\beta - \alpha)) = (\text{ID, name, salary, dept_name})$



Lossless-join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless join if at least one of the following dependencies is in F^+ :
 - $R_1 \cap R_2 \rightarrow R_1$
 - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition;



Decomposing Examples

- $R = (A, B, C)$
 $F = \{A \rightarrow B$
 $\quad B \rightarrow C\}$
 - $R1 = (B,C);$
 - $R2 = (A,B);$

- $R = (A, B, C)$
 $F = \{A \rightarrow B$
 $\quad C \rightarrow B\}$
 - $R1 = (C,B);$
 - $R2 = (A,C);$
 - What is the problem for the above decomposition?



BCNF and Dependency Preservation

- Constraints, including functional dependencies, are costly to check in practice unless they pertain to only one relation
- If it is sufficient to test only those dependencies on each individual relation of a decomposition in order to ensure that *all* functional dependencies hold, then that decomposition is *dependency preserving*.
- Because it is not always possible to achieve both BCNF and dependency preservation, we consider a weaker normal form, known as *third normal form*.



Third Normal Form

- A relation schema R is in **third normal form (3NF)** if for all:

$$\alpha \rightarrow \beta \text{ in } F^+$$

at least one of the following holds:

- $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \in \alpha$)
- α is a superkey for R
- Each attribute A in $\beta - \alpha$ is contained in a candidate key for R .
(NOTE: each attribute may be in a different candidate key)

- If a relation is in BCNF it is in 3NF (since in BCNF one of the first two conditions above must hold).
- Third condition is a minimal relaxation of BCNF to ensure dependency preservation.



Third Normal Form - Example

- advisor schema with application specific requirement: one student can only have one advisor in one department. This constraint can be expressed with a functional dependency, and easily checked in DBMS.

```
advisor (s_id,i_id,dept_name) //here dept_name is the department of
advisor, not student. Important FDs (How many candidate keys here?):
s_id,i_id ->dept_name;
i_id ->dept_name
s_id,dept_name->i_id //Application specific requirement. We can use a
Unique constraint while creating table "advisor" to ensure the constraint.
```

- If decompose it into two relations: advisor(s_id, i_id) and instructor(i_id, dept_name).
 - have to combine the two relations to check whether one student only has one advisor in one department.
 - duplication of information(dept_name) is avoided, but efficiency to check the constraint is decreased.
- Third normal form is a kind of tradeoff. Tolerant some duplication, but increase the efficiency of constraint checking (**dependency preservation**).



Goals of Normalization

- Let R be a relation scheme with a set F of functional dependencies.
- Decide whether a relation scheme R is in “good” form.
- In the case that a relation scheme R is not in “good” form, decompose it into a set of relation scheme $\{R_1, R_2, \dots, R_n\}$ such that
 - each relation scheme is in good form
 - the decomposition is a lossless-join decomposition
 - Preferably, the decomposition should be dependency preserving.



How good is BCNF?

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a relation
 - $inst_info (ID, child_name, phone)$
 - where an instructor may have more than one phone and can have multiple children

<i>ID</i>	<i>child_name</i>	<i>phone</i>
99999	David	512-555-1234
99999	David	512-555-4321
99999	William	512-555-1234
99999	William	512-555-4321

inst_info



How good is BCNF? (Cont.)

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies – i.e., if we add a phone 981-992-3443 to 99999, we need to add two tuples
 - (99999, David, 981-992-3443)
 - (99999, William, 981-992-3443)



How good is BCNF? (Cont.)

- Therefore, it is better to decompose *inst_info* into:

inst_child

<i>ID</i>	<i>child_name</i>
99999	David
99999	William

inst_phone

<i>ID</i>	<i>phone</i>
99999	512-555-1234
99999	512-555-4321

This suggests the need for higher normal forms, such as Fourth Normal Form (4NF), which we shall see later.



Functional-Dependency Theory

- We now consider the formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies.
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF
- We then develop algorithms to test if a decomposition is dependency-preserving



Armstrong's Axioms

- We can find F^+ , the closure of F , by repeatedly applying **Armstrong's Axioms**:
 - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ **(reflexivity)**
 - if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$ **(augmentation)**
 - if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ **(transitivity)**
- These rules are
 - **sound** (generate only functional dependencies that actually hold), and
 - **complete** (generate all functional dependencies that hold).



Example

- $R = (A, B, C, G, H, I)$
 $F = \{ A \rightarrow B$
 $\quad A \rightarrow C$
 $\quad CG \rightarrow H$
 $\quad CG \rightarrow I$
 $\quad B \rightarrow H \}$
- some members of F^+
 - $A \rightarrow H$
 - ▶ by transitivity from $A \rightarrow B$ and $B \rightarrow H$
 - $AG \rightarrow I$
 - ▶ by augmenting $A \rightarrow C$ with G , to get $AG \rightarrow CG$ and then transitivity with $CG \rightarrow I$
 - $CG \rightarrow HI$
 - ▶ by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$, and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$, and then transitivity



Procedure for Computing F^+

- To compute the closure of a set of functional dependencies F :

$F^+ = F$

repeat

for each functional dependency f in F^+

 apply reflexivity and augmentation rules on f

 add the resulting functional dependencies to F^+

for each pair of functional dependencies f_1 and f_2 in F^+

if f_1 and f_2 can be combined using transitivity

then add the resulting functional dependency to F^+

until F^+ does not change any further

NOTE: We shall see an alternative procedure for this task later



Additional rules

■ Additional rules:

- If $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds (**union**)
- If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds
(decomposition)
- If $\alpha \rightarrow \beta$ holds and $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds
(pseudotransitivity)

The above rules can be inferred from Armstrong's axioms.



Closure of Attribute Sets

- Given a set of attributes α , define the ***closure*** of α **under F** (denoted by α^+) as the set of attributes that are functionally determined by α under F
- Algorithm to compute α^+ , the closure of α under F

```
result :=  $\alpha$ ;
while (changes to result) do
    for each  $\beta \rightarrow \gamma$  in  $F$  do
        begin
            if  $\beta \subseteq result$  then  $result := result \cup \gamma$ 
        end
```



Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B, A \rightarrow C, CG \rightarrow H, CG \rightarrow I, B \rightarrow H\}$
- $(AG)^+$
 1. $result = AG$
 2. $result = ABCG$ ($A \rightarrow C$ and $A \rightarrow B$)
 3. $result = ABCGH$ ($CG \rightarrow H$ and $CG \subseteq AGBC$)
 4. $result = ABCGHI$ ($CG \rightarrow I$ and $CG \subseteq AGBCH$)
- Is AG a candidate key?
 1. Is AG a super key?
 1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$
 2. Is any subset of AG a superkey?
 1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
 2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$



Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
 - To test if α is a superkey, we compute α^+ , and check if α^+ contains all attributes of R .
- Testing functional dependencies
 - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^+), just check if $\beta \subseteq \alpha^+$.
 - That is, we compute α^+ by using attribute closure, and then check if it contains β .
 - Is a simple and cheap test, and very useful
- Computing closure of F
 - For each $\gamma \subseteq R$, we find the closure γ^+ , and for each $S \subseteq \gamma^+$, we output a functional dependency $\gamma \rightarrow S$.
 - *Example:* $R = (A, B, C)$ $F = \{A \rightarrow B, B \rightarrow C\}$



Use of Attribute Set Closure

– Example: Find all Candidate Keys

- If attribute α only appears in left side of non trivial FDs in F , it should be part of a candidate key
- If attribute α only appears in right side of non trivial FDs in F , it should not be part of a candidate key
- If attribute α does not appear in any non trivial FDs of F , it should be part of a candidate key
- Example1: $R=(A, B, C, D, E)$, $F=\{A B \rightarrow C, B \rightarrow D, AD \rightarrow E\}$
 - Candidate key: **AB**
- Example2: $R=\{A, B, C, D, E\}$, $F=\{AC \rightarrow BD, B \rightarrow A\}$
 - Candidate keys: **ACE**, **BCE**



Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
 - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
 - Parts of a functional dependency may be redundant
 - ▶ E.g.: on LHS: $F: \{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$ can be simplified to
 $F': \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$ where F' is stronger (i.e. $AC \rightarrow D$ can be derived from $A \rightarrow D$ only, do not need other FDs in F')
 - ▶ E.g.: on RHS: $F: \{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$ can be simplified to
 $F': \{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$ where F is stronger (i.e. $A \rightarrow D$ can be derived from $A \rightarrow CD$ only, do not need other FDs in F)
 - Intuitively, a canonical cover of F is a “minimal” set of functional dependencies equivalent to F , having no redundant dependencies or redundant parts of dependencies



Extraneous Attributes

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F .
 - Attribute A is **extraneous** in α (**LHS**) if $A \in \alpha$ and F logically implies $(\alpha - A) \rightarrow \beta$.
 - ▶ We can use F to calculate $(\alpha - A)^+$ to see if $(\alpha - A)^+$ includes β
 - Attribute A is **extraneous** in β (**RHS**) if $A \in \beta$ and the set of functional dependencies $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies $\alpha \rightarrow \beta$.
 - ▶ We can use F' to calculate α^+ to see if α^+ includes β
- Note: implication in the opposite direction is trivial in each of the cases above, since a “stronger” functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow B, B \rightarrow C, AB \rightarrow C\}$
 - B is extraneous in $AB \rightarrow C$ because F logically implies $A \rightarrow C$ (i.e. $A^+ = ABC$ under F , A^+ includes C).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
 - C is extraneous in $AB \rightarrow CD$ because $F' = \{A \rightarrow C, AB \rightarrow D\}$ logically implies $AB \rightarrow CD$ (i.e. $AB^+ = ABCD$ under F' , AB^+ includes CD)



Testing if an Attribute is Extraneous

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F .
- To test if attribute $A \in \alpha$ is extraneous in α
 1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
 2. check that $(\{\alpha\} - A)^+$ contains β ; if it does, A is extraneous in α
- To test if attribute $A \in \beta$ is extraneous in β
 1. compute α^+ using only the dependencies in
$$F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\},$$
 2. check that α^+ contains A ; if it does, A is extraneous in β



Canonical Cover

- A **canonical cover** for F is a set of dependencies F_c such that
 - F logically implies all dependencies in F_c , and
 - F_c logically implies all dependencies in F , and
 - No functional dependency in F_c contains an extraneous attribute, and
 - Each left side of functional dependency in F_c is unique.
- To compute a canonical cover for F :
repeat
 - Use the union rule to replace any dependencies in F
 $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
 - Find a functional dependency $\alpha \rightarrow \beta$ with an
 extraneous attribute either in α or in β
 /* Note: test for extraneous attributes done using F_c , not F^* */
 - If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$**until** F does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied



Computing a Canonical Cover

- $R = (A, B, C)$
 $F = \{A \rightarrow BC$
 $B \rightarrow C$
 $A \rightarrow B$
 $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
 - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
 - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
 - ▶ Yes: in fact, $B \rightarrow C$ is already present!
 - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
 - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
 - ▶ Yes: using transitivity on $A \rightarrow B$ and $B \rightarrow C$.
 - Can use attribute closure of A in more complex cases
- The canonical cover is:
 - $A \rightarrow B$
 - $B \rightarrow C$



Canonical Cover might not be Unique

- $F: (A \rightarrow B, B \rightarrow AC, C \rightarrow AB)$
 - one $F_C: (A \rightarrow B, B \rightarrow C, C \rightarrow A)$
 - ▶ $F'_1 = (A \rightarrow B, B \rightarrow C, C \rightarrow AB)$
 - ▶ B^+ under F'_1 is ABC, includes A, so A is extraneous in $B \rightarrow AC$ in F
 - ▶ $F'_2 = (A \rightarrow B, B \rightarrow C, C \rightarrow A)$
 - ▶ C^+ under F'_2 is ABC, includes B, so B is extraneous in $C \rightarrow AB$ in F'_1
 - ▶ $F_C = F'_2$
 - another $F_C: (A \rightarrow B, B \rightarrow AC, C \rightarrow B)$



Lossless-join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless join if at least one of the following dependencies is in F^+ :
 - $R_1 \cap R_2 \rightarrow R_1$
 - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition



Dependency Preservation Testing

- Let F_i be the set of dependencies F^+ that include only attributes in R_i .
 - ▶ A decomposition is **dependency preserving**, if
$$(F_1 \cup F_2 \cup \dots \cup F_n)^+ = F^+$$
 - ▶ If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive.



Another Way to Test Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into R_1, R_2, \dots, R_n we apply the following test (with attribute closure done with respect to F)
 - $result = \alpha$
while (changes to $result$) do
 for each R_i in the decomposition
 $t = (result \cap R_i)^+ \cap R_i$
 $result = result \cup t$
 - If $result$ contains all attributes in β , then the functional dependency
 $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \dots \cup F_n)^+$



Example

- $R = (A, B, C)$
 $F = \{A \rightarrow B, B \rightarrow C\}$
 - Can be decomposed in two different ways

- $R_1 = (A, B), R_2 = (B, C)$
 - Lossless-join decomposition:

$$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$

- $R_1 = (A, B), R_2 = (A, C)$
 - Lossless-join decomposition:

$$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$

- Not dependency preserving
(cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)



Testing for BCNF

- To check if a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF
 1. compute α^+ (the attribute closure of α), and
 2. verify that it includes all attributes of R , that is, it is a superkey of R .
- **Using F to test BCNF conformity of R :** To check if a relation schema R is in BCNF, it suffices to check only the dependencies in the given set F for violation of BCNF, rather than checking all dependencies in F^+ .
 - If none of the dependencies in F causes a violation of BCNF, then none of the dependencies in F^+ will cause a violation of BCNF either. (**For why, Think of Armstrong's Axioms**)
- **If R does not conform to BCNF, F^+ will be needed to test decomposition of R . Because if there are FDs in F that violate BCNF rules, there will probably be FDs in F^+ that violate BCNF rules for the decomposition. For why, still think of Armstrong's Axioms**
 - Consider $R = (A, B, C, D, E)$, with $F = \{ A \rightarrow B, BC \rightarrow D \}$
 - ▶ Decompose R into $R_1 = (A, B)$ and $R_2 = (A, C, D, E)$
 - ▶ Neither of the dependencies in F contain only attributes from (A, C, D, E)
 - ▶ In fact, dependency $AC \rightarrow D$ in F^+ shows R_2 is not in BCNF.



Testing BCNF for R with F

- Let F be the set of functional dependencies on R with no trivial FDs. If all functional dependencies in F of the form

$$\alpha \rightarrow \beta$$

where $\alpha \subseteq R$ and $\beta \subseteq R$, α is a superkey for R

R is in BCNF



Testing Decomposition for BCNF

- To check if a relation R_i in a decomposition of R is in BCNF,
 - Either test R_i for BCNF with respect to the **restriction of F to R_i** (that is, all FDs in F^+ that contain only attributes from R_i)
 - or use the original set of dependencies F that hold on R , but with the following test:
 - for **every set of attributes $\alpha \subseteq R_i$** , (not necessary)
check that α^+ (the attribute closure of α) either includes no attribute of $R_i - \alpha$, or includes all attributes of R_i .
 - ▶ If the condition is violated by some α in R_j , the dependency
 $\alpha \rightarrow (\alpha^+ - \alpha) \cap R_j$
can be shown to hold on R_i , and R_i violates BCNF.



BCNF Decomposition Algorithm

```
result := {R };
done := false;
compute  $F^+$ ;
while (not done) do
    if (there is a schema  $R_i$  in result that is not in BCNF)
        then begin
            let  $\alpha \rightarrow \beta$  be a nontrivial functional dependency that
                holds on  $R_i$  such that  $\alpha \rightarrow R_i$  is not in  $F^+$ ,
                and  $\alpha \cap \beta = \emptyset$ ;
            result := (result -  $R_i$ )  $\cup$  ( $R_i - \beta$ )  $\cup$  ( $\alpha, \beta$ );
        end
    else done := true;
```

Note: each R_i is in BCNF, and decomposition is lossless-join.



BCNF Decomposition – Practical Way

```
result := {R};  
compute Fc;  
while (there is change with result) do  
    For each Ri in result  
        For each α → β in Fc  
            if α ⊂ Ri  
                For each γ , where α ⊆ γ and γ ⊂ Ri  
                    compute γ+ with Fc  
                    if γ ⊂ (γ+ ∩ Ri) and (γ+ ∩ Ri) ≠ Ri // γ is not a superkey for Ri  
                        result := (result - Ri) ∪ (Ri - ((γ+ ∩ Ri) - γ)) ∪ (γ+ ∩ Ri)
```

- For $R = (A, B, C, D, E)$, with $F = \{ A \rightarrow B, BC \rightarrow D \}$
- Solution1: $R_1 = (A, B), R_2 = (A, C, D, E); AC^+ = (A, B, C, D); AC^+ \cap R_2 = (A, C, D);$ So split R2 into $(A, C, D), (A, C, E)$
- Solution2: $R_1 = (B, C, D), R_2 = (A, B, C, E); AC^+ = (A, B, C, D); AC^+ \cap R_2 = (A, B, C);$ So split R2 into $(A, B, C), (A, C, E)$



Example of BCNF Decomposition

- $R = (A, B, C, D)$
 $F = \{A \rightarrow B$
 $B \rightarrow C\}$
- Decomposition1
 - $R_1 = (B, C), R_2 = (A, B, D)$
 - $R_1 = (B, C), R_2 = (A, B), R_3 = (A, D)$ //dependency preserving
- Decomposition2
 - $R_1 = (A, B), R_2 = (A, C, D)$ //need to decompose R_2
 - $R_1 = (A, B), R_2 = (A, C), R_3 = (A, D)$ //not dependency preserving



Example of BCNF Decomposition

- *class (course_id, title, dept_name, credits, sec_id, semester, year, building, room_number, capacity, time_slot_id)*
- Functional dependencies:
 - $course_id \rightarrow title, dept_name, credits$
 - $building, room_number \rightarrow capacity$
 - $course_id, sec_id, semester, year \rightarrow building, room_number, time_slot_id$
- A candidate key $\{course_id, sec_id, semester, year\}$.
- BCNF Decomposition:
 - $course_id \rightarrow title, dept_name, credits$ holds
 - ▶ $course_id$ is not a superkey for class.
 - We replace *class* by:
 - ▶ *course(course_id, title, dept_name, credits)*
 - ▶ *class-1 (course_id, sec_id, semester, year, building, room_number, capacity, time_slot_id)*



BCNF Decomposition (Cont.)

- *course* is in BCNF
- $\text{building}, \text{room_number} \rightarrow \text{capacity}$ holds on *class-1*
 - $\{\text{building}, \text{room_number}\}$ is not a superkey for *class-1*.
 - We replace *class-1* by:
 - ▶ *classroom* (*building*, *room_number*, *capacity*)
 - ▶ *section* (*course_id*, *sec_id*, *semester*, *year*, *building*, *room_number*, *time_slot_id*)
- *classroom* and *section* are in BCNF.



BCNF and Dependency Preservation

It is not always possible to get a BCNF decomposition that is dependency preserving

- $R = (J, K, L)$

$$F = \{JK \rightarrow L$$

$$L \rightarrow K\}$$

Two candidate keys = JK and JL

- R is not in BCNF

- Any decomposition of R will fail to preserve

$$JK \rightarrow L$$

This implies that testing for $JK \rightarrow L$ requires a join



Third Normal Form: Motivation

- There are some situations where
 - BCNF is not dependency preserving, and
 - efficient checking for FD violation on updates is important
- Solution: define a weaker normal form, called Third Normal Form (3NF)
 - Allows some redundancy (with resultant problems; we will see examples later)
 - But functional dependencies can be checked on individual relations without computing a join.
 - There is always a lossless-join, dependency-preserving decomposition into 3NF.



3NF Example

■ Relation *dept_advisor*:

- $\text{dept_advisor}(\text{s_ID}, \text{i_ID}, \text{dept_name})$
 $F = \{\text{s_ID}, \text{dept_name} \rightarrow \text{i_ID}, \text{i_ID} \rightarrow \text{dept_name}\}$
- Two candidate keys: s_ID , dept_name , and i_ID , s_ID
- R is in 3NF
 - ▶ $\text{s_ID}, \text{dept_name} \rightarrow \text{i_ID}$
 - $\text{s_ID}, \text{dept_name}$ is a candidate key
 - ▶ $\text{i_ID} \rightarrow \text{dept_name}$
 - dept_name is contained in a candidate key



Redundancy in 3NF

- There is some redundancy in this schema
- Example of problems due to redundancy in 3NF
 - $R = (J, K, L)$
 $F = \{JK \rightarrow L, L \rightarrow K\}$

J	L	K
j_1	l_1	k_1
j_2	l_1	k_1
j_3	l_1	k_1
<i>null</i>	l_2	k_2

- repetition of information (e.g., the relationship l_1, k_1)
 - ($i_ID, dept_name$)
- need to use null values (e.g., to represent the relationship l_2, k_2 where there is no corresponding value for J).
 - ($i_ID, dept_name$) if there is no separate relation mapping instructors to departments



Testing for 3NF

- Optimization: Need to check only FDs in F , need not check all FDs in F^+ .
- Use attribute closure to check for each dependency $\alpha \rightarrow \beta$, if α is a superkey.
- If α is not a superkey, we have to verify if each attribute in β is contained in a candidate key of R
 - this test is rather more expensive, since it involve finding candidate keys
 - testing for 3NF has been shown to be NP-hard
 - Interestingly, decomposition into third normal form (described shortly) can be done in polynomial time



3NF Decomposition Algorithm

Let F_c be a canonical cover for F ;
 $i := 0$;

for each functional dependency $\alpha \rightarrow \beta$ in F_c **do**

if none of the schemas R_j , $1 \leq j \leq i$ contains $\alpha \beta$

then begin

$i := i + 1$;

$R_i := \alpha \beta$

end

if none of the schemas R_j , $1 \leq j \leq i$ contains a candidate key for R

then begin

$i := i + 1$;

$R_i :=$ any candidate key for R ;

end

/* Optionally, remove redundant relations */

repeat

if any schema R_j is contained in another schema R_k

then /* delete R_j */

$R_j = R_i$;

$i = i - 1$;

return (R_1, R_2, \dots, R_i)



3NF Decomposition Algorithm (Cont.)

- Above algorithm ensures:
 - each relation schema R_i is in 3NF
 - decomposition is dependency preserving and lossless-join
 - Proof of correctness is at end of this presentation



3NF Decomposition: An Example

- Relation schema:

$\text{cust_banker_branch} = (\underline{\text{customer_id}}, \underline{\text{employee_id}}, \text{branch_name}, \text{type})$

- The functional dependencies for this relation schema are:

1. $\text{customer_id}, \text{employee_id} \rightarrow \text{branch_name}, \text{type}$
2. $\text{employee_id} \rightarrow \text{branch_name}$
3. $\text{customer_id}, \text{branch_name} \rightarrow \text{employee_id}$

- We first compute a canonical cover

- branch_name is extraneous in the r.h.s. of the 1st dependency
- No other attribute is extraneous, so we get $F_C =$

$\text{customer_id}, \text{employee_id} \rightarrow \text{type}$
 $\text{employee_id} \rightarrow \text{branch_name}$
 $\text{customer_id}, \text{branch_name} \rightarrow \text{employee_id}$



3NF Decomposition Example (Cont.)

- The **for** loop generates following 3NF schema:

$(customer_id, employee_id, type)$

$(employee_id, branch_name)$

$(customer_id, branch_name, employee_id)$

- Observe that $(customer_id, employee_id, type)$ contains a candidate key of the original schema, so no further relation schema needs be added
- At end of for loop, detect and delete schemas, such as $(employee_id, branch_name)$, which are subsets of other schemas
 - result will not depend on the order in which FDs are considered
- The resultant simplified 3NF schema is:

$(customer_id, employee_id, type)$

$(customer_id, branch_name, employee_id)$



Comparison of BCNF and 3NF

- It is always possible to decompose a relation into a set of relations that are in 3NF such that:
 - the decomposition is lossless
 - the dependencies are preserved
- It is always possible to decompose a relation into a set of relations that are in BCNF such that:
 - the decomposition is lossless
 - it may not be possible to preserve dependencies.



Design Goals

- Goal for a relational database design is:
 - BCNF.
 - Lossless join.
 - Dependency preservation.
- If we cannot achieve this, we accept one of
 - Lack of dependency preservation
 - Redundancy due to use of 3NF
- Interestingly, SQL does not provide a direct way of specifying functional dependencies other than superkeys.

Can specify FDs using assertions, but they are expensive to test, (and currently not supported by any of the widely used databases!)
- Even if we had a dependency preserving decomposition, using SQL we would not be able to efficiently test a functional dependency whose left hand side is not a key.



Multivalued Dependencies

- Suppose we record names of children, and phone numbers for instructors:
 - $inst_child(ID, child_name)$
 - $inst_phone(ID, phone_number)$
- If we were to combine these schemas to get
 - $inst_info(ID, child_name, phone_number)$
 - Example data:
 - (99999, David, 512-555-1234)
 - (99999, David, 512-555-4321)
 - (99999, William, 512-555-1234)
 - (99999, William, 512-555-4321)
- This relation is in BCNF
 - Why?



Multivalued Dependencies (MVDs)

- Let R be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The **multivalued dependency**

$$\alpha \twoheadrightarrow \beta$$

holds on R if in any legal relation $r(R)$, for all pairs for tuples t_1 and t_2 in r such that $t_1[\alpha] = t_2[\alpha]$, there exist tuples t_3 and t_4 in r such that:

$$\begin{aligned}t_1[\alpha] &= t_2[\alpha] = t_3[\alpha] = t_4[\alpha] \\t_3[\beta] &= t_1[\beta] \\t_3[R - \beta] &= t_2[R - \beta] \\t_4[\beta] &= t_2[\beta] \\t_4[R - \beta] &= t_1[R - \beta]\end{aligned}$$





MVD (Cont.)

- Tabular representation of $\alpha \rightarrow\!\!\!\rightarrow \beta$

	α	β	$R - \alpha - \beta$
t_1	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$a_{j+1} \dots a_n$
t_2	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$b_{j+1} \dots b_n$
t_3	$a_1 \dots a_i$	$a_{i+1} \dots a_j$	$b_{j+1} \dots b_n$
t_4	$a_1 \dots a_i$	$b_{i+1} \dots b_j$	$a_{j+1} \dots a_n$



Example

- Let R be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.

Y, Z, W

- We say that $Y \twoheadrightarrow Z$ (Y **multidetermines** Z)
if and only if for all possible relations $r(R)$

$\langle y_1, z_1, w_1 \rangle \in r$ and $\langle y_1, z_2, w_2 \rangle \in r$

then

$\langle y_1, z_1, w_2 \rangle \in r$ and $\langle y_1, z_2, w_1 \rangle \in r$

- Note that since the behavior of Z and W are identical it follows that

$Y \twoheadrightarrow Z$ if $Y \twoheadrightarrow W$



Example (Cont.)

- In our example:

$ID \rightarrow\!\!\! \rightarrow child_name$

$ID \rightarrow\!\!\! \rightarrow phone_number$

- The above formal definition is supposed to formalize the notion that given a particular value of Y (ID) it has associated with it a set of values of Z ($child_name$) and a set of values of W ($phone_number$), and these two sets are in some sense independent of each other.
- Note:
 - If $Y \rightarrow Z$ then $Y \rightarrow\!\!\! \rightarrow Z$
 - Indeed we have (in above notation) $Z_1 = Z_2$
The claim follows.



Use of Multivalued Dependencies

- We use multivalued dependencies in two ways:
 1. To test relations to **determine** whether they are legal under a given set of functional and multivalued dependencies
 2. To specify **constraints** on the set of legal relations. We shall thus concern ourselves *only* with relations that satisfy a given set of functional and multivalued dependencies.
- If a relation r fails to satisfy a given multivalued dependency, we can construct a relations r' that does satisfy the multivalued dependency by adding tuples to r .



Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:
 - If $\alpha \rightarrow \beta$, then $\alpha \rightarrow\rightarrow \beta$

That is, every functional dependency is also a multivalued dependency
- The **closure** D^+ of D is the set of all functional and multivalued dependencies logically implied by D .
 - We can compute D^+ from D , using the formal definitions of functional dependencies and multivalued dependencies.
 - We can manage with such reasoning for very simple multivalued dependencies, which seem to be most common in practice
 - For complex dependencies, it is better to reason about sets of dependencies using a system of inference rules (see Appendix C).





Fourth Normal Form

- A relation schema R is in **4NF** with respect to a set D of functional and multivalued dependencies if for all multivalued dependencies in D^+ of the form $\alpha \twoheadrightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following hold:
 - $\alpha \twoheadrightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$ or $\alpha \cup \beta = R$)
 - α is a superkey for schema R
- If a relation is in 4NF it is in BCNF



Restriction of Multivalued Dependencies

- The restriction of D to R_i is the set D_i consisting of
 - All functional dependencies in D^+ that include only attributes of R_i
 - All multivalued dependencies of the form
$$\alpha \rightarrow\rightarrow (\beta \cap R_i)$$
where $\alpha \subseteq R_i$ and $\alpha \rightarrow\rightarrow \beta$ is in D^+



4NF Decomposition Algorithm

result: = { R };

done := false;

compute D^+ ;

Let D_i denote the restriction of D^+ to R_i

while (*not done*)

if (there is a schema R_i in *result* that is not in 4NF) **then**

begin

let $\alpha \rightarrow\!\!\!\rightarrow \beta$ be a nontrivial multivalued dependency that holds
on R_i such that $\alpha \rightarrow R_i$ is not in D_i , and $\alpha \cap \beta = \emptyset$;

result := (*result* - R_i) \cup (R_i - β) \cup (α, β);

end

else *done* := true;

Note: each R_i is in 4NF, and decomposition is lossless-join





Example

- $R = (A, B, C, G, H, I)$
 $F = \{ A \rightarrow\!\!\!\rightarrow B$
 $\quad B \rightarrow\!\!\!\rightarrow HI$
 $\quad CG \rightarrow\!\!\!\rightarrow H \}$
- R is not in 4NF since $A \rightarrow\!\!\!\rightarrow B$ and A is not a superkey for R
- Decomposition
 - a) $R_1 = (A, B)$ $(R_1$ is in 4NF)
 - b) $R_2 = (A, C, G, H, I)$ $(R_2$ is not in 4NF, decompose into R_3 and R_4)
 - c) $R_3 = (C, G, H)$ $(R_3$ is in 4NF)
 - d) $R_4 = (A, C, G, I)$ $(R_4$ is not in 4NF, decompose into R_5 and R_6)
 - $A \rightarrow\!\!\!\rightarrow B$ and $B \rightarrow\!\!\!\rightarrow HI \rightarrow A \rightarrow\!\!\!\rightarrow HI$, (MVD transitivity), and
 - and hence $A \rightarrow\!\!\!\rightarrow I$ (*MVD restriction to R_4*)
 - e) $R_5 = (A, I)$ $(R_5$ is in 4NF)
 - f) $R_6 = (A, C, G)$ $(R_6$ is in 4NF)



Further Normal Forms

- **Join dependencies** generalize multivalued dependencies
 - lead to **project-join normal form (PJNF)** (also called **fifth normal form**)
- A class of even more general constraints, leads to a normal form called **domain-key normal form**.
- Problem with these generalized constraints: are hard to reason with, and no set of sound and complete set of inference rules exists.
- Hence rarely used



Overall Database Design Process

- We have assumed schema R is given
 - R could have been generated when converting E-R diagram to a set of tables.
 - R could have been a single relation containing *all* attributes that are of interest (called **universal relation**).
 - Normalization breaks R into smaller relations.
 - R could have been the result of some ad hoc design of relations, which we then test/convert to normal form.



ER Model and Normalization

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization.
- However, in a real (imperfect) design, there can be functional dependencies from non-key attributes of an entity to other attributes of the entity
 - Example: an *employee* entity with attributes *department_name* and *building*, and a functional dependency $\text{department_name} \rightarrow \text{building}$
 - Good design would have made department an entity
- Functional dependencies from non-key attributes of a relationship set possible, but rare --- most relationships are binary



Denormalization for Performance

- May want to use non-normalized schema for performance
- For example, displaying *prereqs* along with *course_id*, and *title* requires join of *course* with *prereq*
- Alternative 1: Use denormalized relation containing attributes of *course* as well as *prereq* with all above attributes
 - faster lookup
 - extra space and extra execution time for updates
 - extra coding work for programmer and possibility of error in extra code
- Alternative 2: use a materialized view defined as
$$\text{course} \quad \text{prereq}$$
 - Benefits and drawbacks same as above, except no extra coding work for programmer and avoids possible errors



Other Design Issues

- Some aspects of database design are not caught by normalization
- Examples of bad database design, to be avoided:
 - Instead of *earnings* (*company_id*, *year*, *amount*), use
 - *earnings_2004*, *earnings_2005*, *earnings_2006*, etc., all on the schema (*company_id*, *earnings*).
 - ▶ Above are in BCNF, but make querying across years difficult and needs new table each year
 - *company_year* (*company_id*, *earnings_2004*, *earnings_2005*, *earnings_2006*)
 - ▶ Also in BCNF, but also makes querying across years difficult and requires new attribute each year.
 - ▶ Is an example of a **crosstab**, where values for one attribute become column names
 - ▶ Used in spreadsheets, and in data analysis tools



Modeling Temporal Data

- **Temporal data** have an association time interval during which the data are *valid*.
- A **snapshot** is the value of the data at a particular point in time
- Several proposals to extend ER model by adding valid time to
 - attributes, e.g., address of an instructor at different points in time
 - entities, e.g., time duration when a student entity exists
 - relationships, e.g., time during which an instructor was associated with a student as an advisor.
 t
- But no accepted standard
- Adding a temporal component results in functional dependencies like
$$ID \rightarrow street, city$$
not to hold, because the address varies over time
- A **temporal functional dependency** $X \rightarrow Y$ holds on schema R if the functional dependency $X \rightarrow Y$ holds on all snapshots for all legal instances $r(R)$.



Modeling Temporal Data (Cont.)

- In practice, database designers may add start and end time attributes to relations
 - E.g., $\text{course}(\text{course_id}, \text{course_title})$ is replaced by
$$\text{course}(\text{course_id}, \text{course_title}, \text{start}, \text{end})$$
 - ▶ Constraint: no two tuples can have overlapping valid times
 - Hard to enforce efficiently
- Foreign key references may be to current version of data, or to data at a point in time
 - E.g., student transcript should refer to course information at the time the course was taken



Example

- Consider the following relation schema and functional dependencies:
- relation schema $R=(A, B, C, D, E)$, $F= \{A B \rightarrow C, B \rightarrow D, AD \rightarrow E\}$
- Assume the above functional dependencies are the only ones that hold over R .
- 1) Find all candidate keys of R .
- 2) Identify whether R is in BCNF or 3NF or neither.
- 3) If R is not in BCNF, decompose R into a collection of BCNF relations. Please tell whether the decomposition is dependency preserving, and why?



End of Chapter

1,13,21,22,30,32

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Proof of Correctness of 3NF Decomposition Algorithm

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Correctness of 3NF Decomposition Algorithm

- 3NF decomposition algorithm is dependency preserving (since there is a relation for every FD in F_c)
- Decomposition is lossless
 - A candidate key (C) is in one of the relations R_i in decomposition
 - Closure of candidate key under F_c must contain all attributes in R .
 - Follow the steps of attribute closure algorithm to show there is only one tuple in the join result for each tuple in R_i



Correctness of 3NF Decomposition Algorithm (Cont'd.)

Claim: if a relation R_i is in the decomposition generated by the above algorithm, then R_i satisfies 3NF.

- Let R_i be generated from the dependency $\alpha \rightarrow \beta$
- Let $\gamma \rightarrow B$ be any non-trivial functional dependency on R_i . (We need only consider FDs whose right-hand side is a single attribute.)
- Now, B can be in either β or α but not in both. Consider each case separately.



Correctness of 3NF Decomposition (Cont'd.)

■ Case 1: If B in β :

- If γ is a superkey, the 2nd condition of 3NF is satisfied
- Otherwise α must contain some attribute not in γ
- Since $\gamma \rightarrow B$ is in F^+ it must be derivable from F_c , by using attribute closure on γ .
- Attribute closure not have used $\alpha \rightarrow \beta$. If it had been used, α must be contained in the attribute closure of γ , which is not possible, since we assumed γ is not a superkey.
- Now, using $\alpha \rightarrow (\beta - \{B\})$ and $\gamma \rightarrow B$, we can derive $\alpha \rightarrow B$ (since $\gamma \subseteq \alpha \beta$, and $B \notin \gamma$ since $\gamma \rightarrow B$ is non-trivial)
- Then, B is extraneous in the right-hand side of $\alpha \rightarrow \beta$; which is not possible since $\alpha \rightarrow \beta$ is in F_c .
- Thus, if B is in β then γ must be a superkey, and the second condition of 3NF must be satisfied.



Correctness of 3NF Decomposition (Cont'd.)

- Case 2: B is in α .
 - Since α is a candidate key, the third alternative in the definition of 3NF is trivially satisfied.
 - In fact, we cannot show that γ is a superkey.
 - This shows exactly why the third alternative is present in the definition of 3NF.

Q.E.D.