Stochastic Processes: Practice using R

1. **A Weather Model:** The weather forecast model for a certain community can be modelled by a Markov Chain with three states: Rain (R), Snow (S) Clear (C). The transition matrix is

$$\begin{array}{cc} & \begin{array}{ccc} \mathrm{R} & \mathrm{S} & \mathrm{C} \end{array} \\ \mathbf{P} = & \begin{pmatrix} \mathbf{0.2} & \mathbf{0.6} & \mathbf{0.2} \\ \mathbf{0.1} & \mathbf{0.8} & \mathbf{0.1} \\ \mathbf{0.1} & \mathbf{0.6} & \mathbf{0.3} \end{pmatrix} \end{array}$$

We want to find the probability of each state for the future. For this purpose, we will find the $n - step$ transition matrix using matrix power simulation. The code “Matrix Power 1” is in the course folder.

Note:
1. The matrix entries are entered as a vector using the c() command.
2. Modify the row and column numbers in the appropriate place.
3. For the n-step transition matrix, enter the appropriate power (i.e. the value of n) in the code.

a) Find the $2 - step$ transition matrix

b) Find the $3 - step$ transition matrix

c) a) Find the $4 - step$ transition matrix

d) Find the long-term weather distribution for each state. What is the smallest n to predict the limiting distribution?

e) For tomorrow, the meteorologist predicts a 50% chance of rain and a 50% chance of snow. Based on this information, we set the initial state distribution to be $x_{\text{initialstate}} = (0.5, 0.5, 0)$.

Hint: Modify the R-code to incorporate the initial distribution. Define the initial distribution as a 1 by n matrix for an appropriate n. Then use matrix multiplication between the initial distribution and the appropriate power of the transition matrix.

The R-code for matrix multiplication is A%*%B. Set P_n to be the transition matrix raised to the n-th power. Then do $initialstate\% * \%P1$.

Starting with the initial distribution $\pi_0 = (0.5, 0.5, 0)$,

- Find the distribution of all states one day later.
- Find the distribution of all states two days later.
- Find the distribution of all states three days later.
- Find the distribution of all states twenty days later.

2. Use the R-code ("Simulating of *Markov Chains*") to display the trajectory of the sample path of the weather model. Make sure you enter all the necessary input parameters. Read the script first and decide what you need to enter before you run the program.

Based on your simulation, answer the following questions.

- Which state is most frequent?
- Which state occurred the least?

2. Gambler's Ruin: Suppose a gambler has $20 and wants to play a game. At each game he can win or lose with equal probability. He will stop the game if he wins $60 or goes bankrupt.

- Based on what you know about this game, is the gambler more or less likely to win the game? Base your answer by finding the exact probabilities of winning and losing.
- We now want to run a simulation of the game 1000 times to estimate the probability of winning and losing the game. Use the Gambler's Ruin R-script for this. Write the probabilities from simulating the game 1000 times.
- Compare the simulated probabilities with the exact probabilities and write a comment.

3. Consider a Markov chain with four states numbered 1, 2, 3 and 4. The transition matrix is

$$P = \begin{pmatrix} 0.8 & 0.1 & 0.05 & 0.05 \\ 0.2 & 0.6 & 0.1 & 0.1 \\ 0 & 0 & 0.8 & 0.2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

By raising P to some powers n (try n = 2, …,9,10), comment on some interesting features of this Markov chain.