# ELEC 5130/6130 RF Devices and Circuits

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Topics

# CHAPTER 2

## Basic Concepts in IC Designs

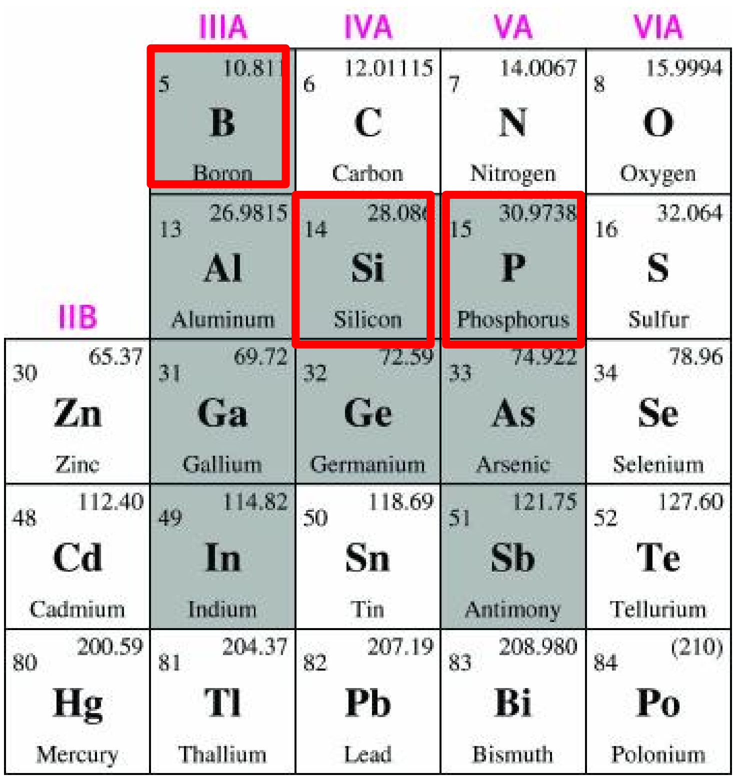
I. Device Review
II. Linearity Analysis
III. Noise Analysis
IV. System Analysis

# Semiconductor Materials

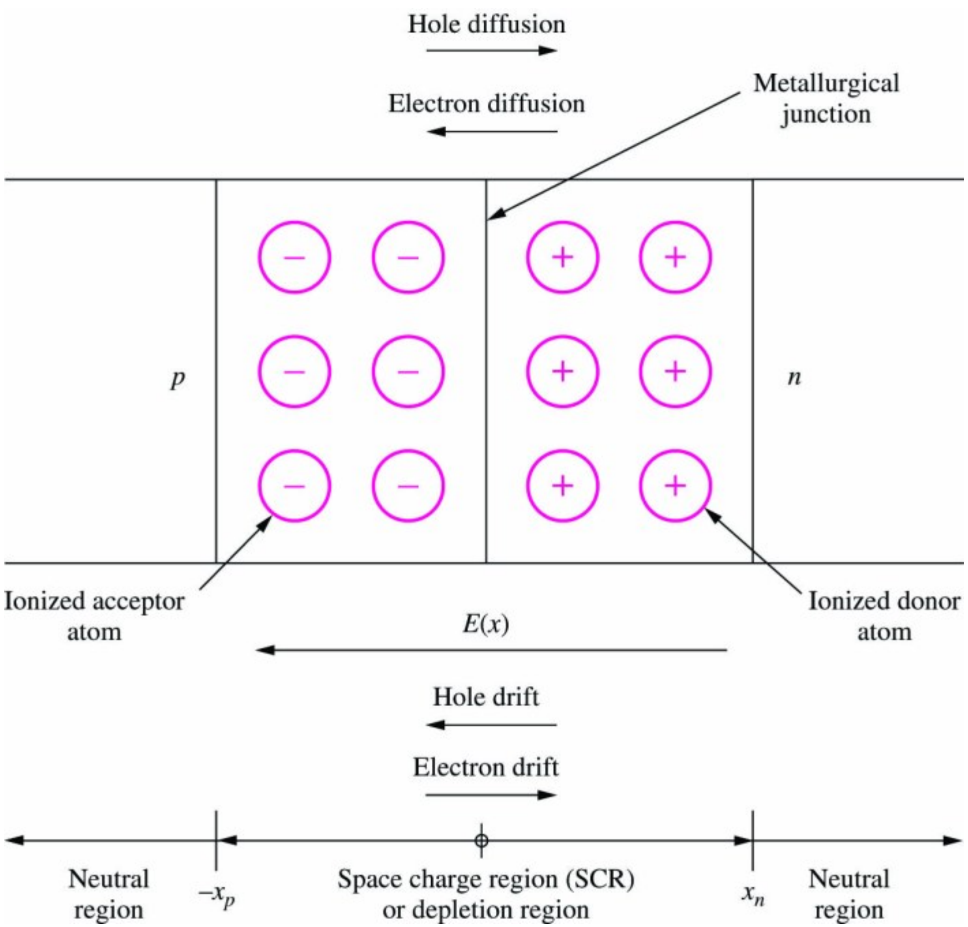
Electronic materials fall into three categories:

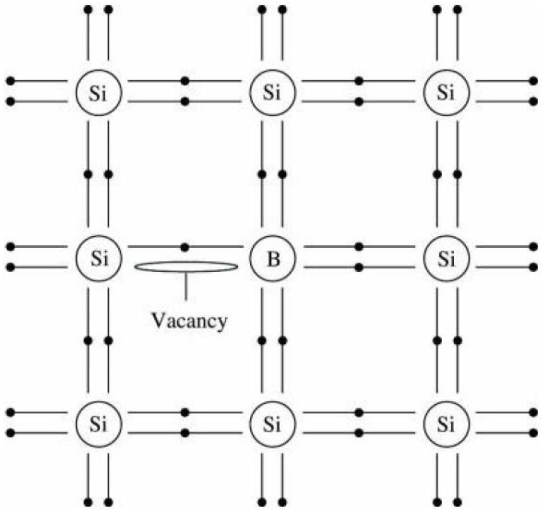
| | | |
|---|---|---|
| Insulators | Resistivity | $\rho > 10^5\ \Omega$-cm |
| Semiconductors | | $10^{-3} < \rho < 10^5\ \Omega$-cm |
| Conductors | | $\rho < 10^{-3}\ \Omega$-cm |

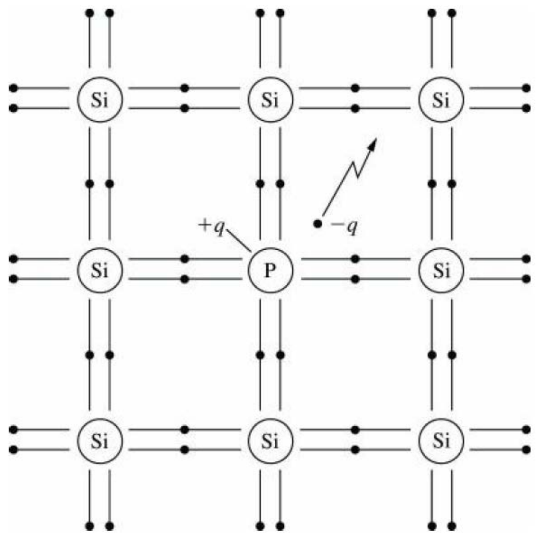
| **Semiconductor** | **Bandgap Energy $E_G$ (eV)** |
|---|---|
| Carbon (diamond) | 5.47 |
| Silicon | 1.12 |
| Germanium | 0.66 |
| Tin | 0.082 |
| Gallium arsenide | 1.42 |
| Gallium nitride | 3.49 |
| Indium phosphide | 1.35 |
| Boron nitride | 7.50 |
| Silicon carbide | 3.26 |
| Cadmium selenide | 1.70 |



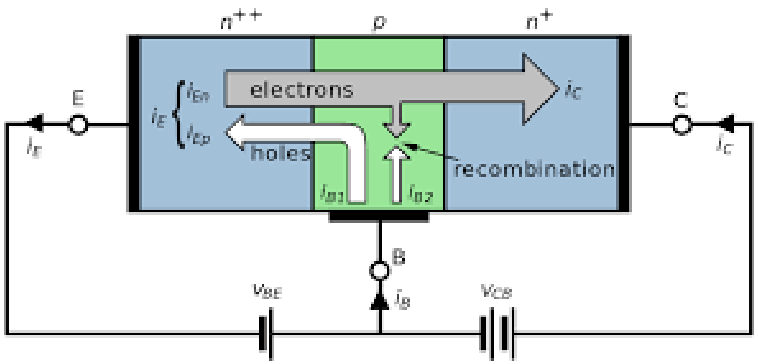
# Space-Charge Region Formation at PN Junction

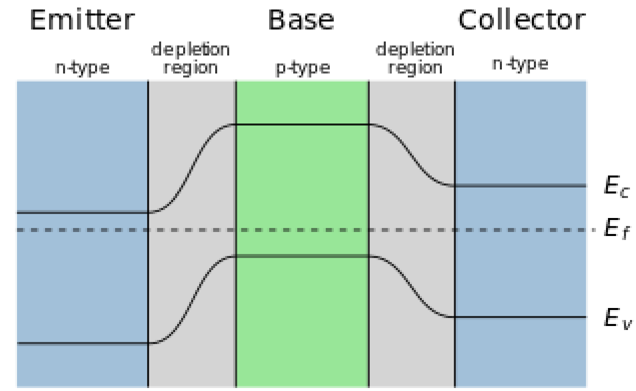


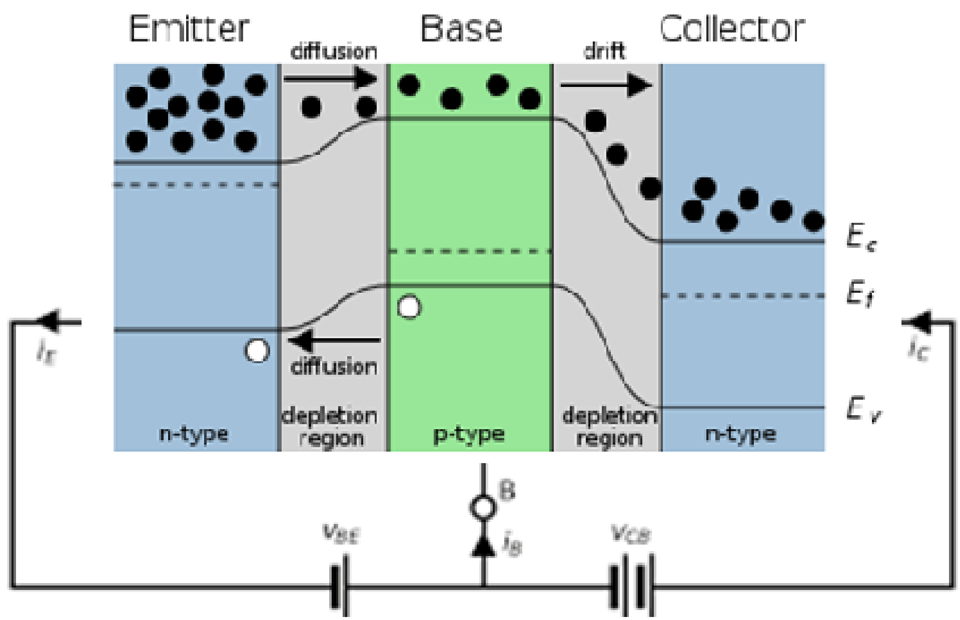



# Bipolar Transistor Basics

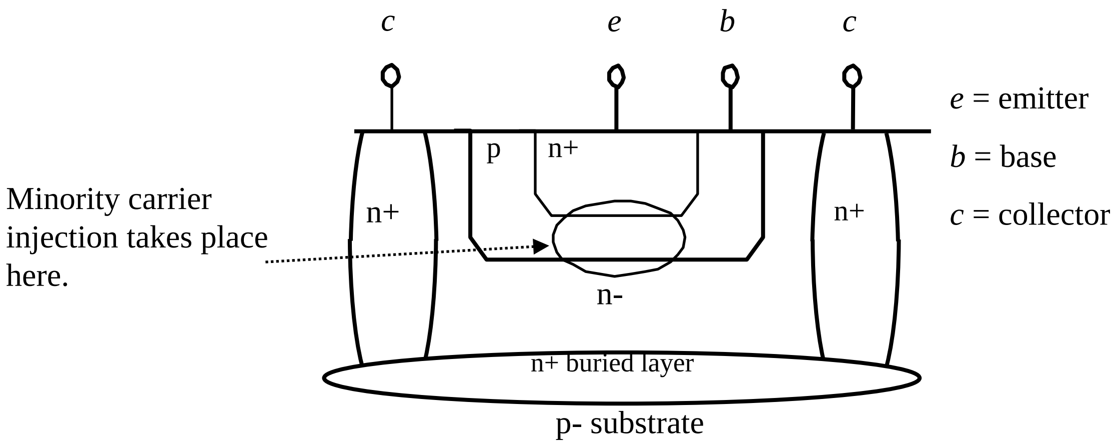


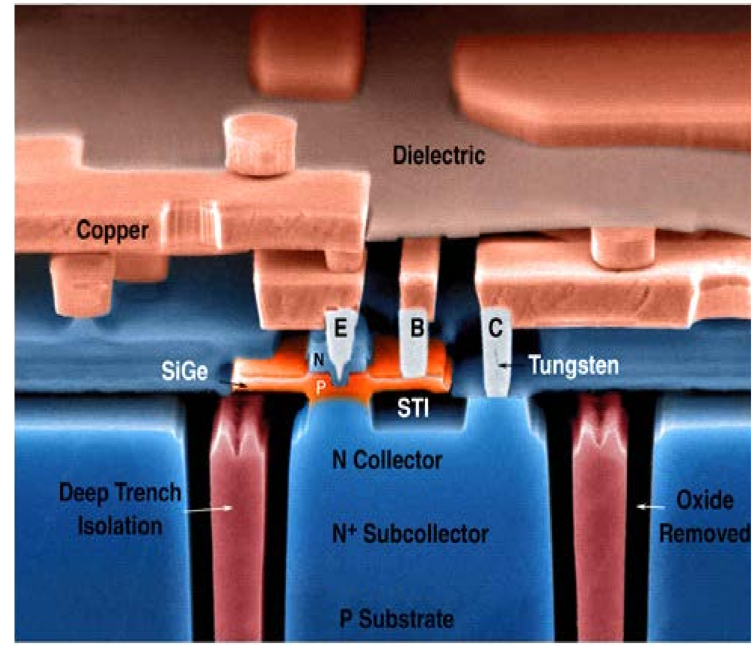



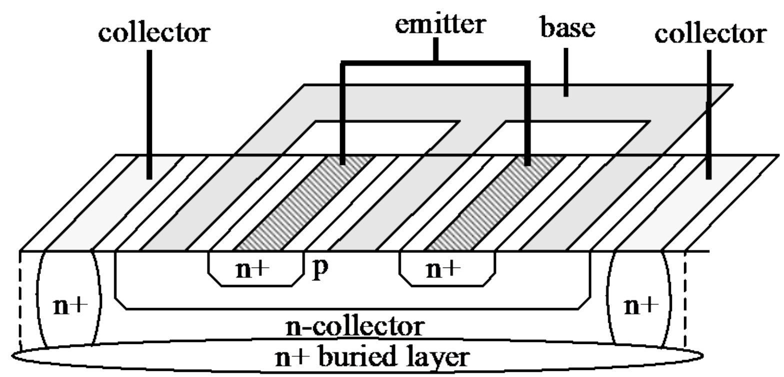

Base positive voltage pulls electrons in the emitter to higher energy level in the base. Three things can happen: (1) electrons in the base can flow out of base as $I_B$; (2) electrons can be recombined with holes in the base; (3) electrons can go across the base and b-c depletion region and flow out of collector as $I_C$. Since base layer is very thin, most of the electrons will flow to the collector.



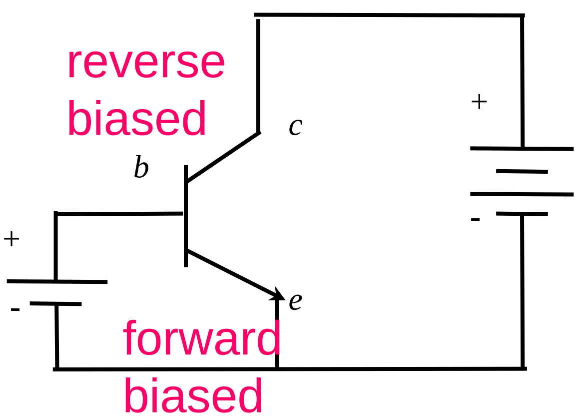
# NPN Bipolar Transistor




$$i_c + i_b = i_c$$




- Biasing:
- Normal Operation: $V_{BE} > V_{th}$, $V_{CB} > 0$
- Soft saturation: $V_{BE} > V_{th}$, $V_{CB} > -0.3V$

# BJT Electrical Characteristics

$$I_C = I_S\left(1+\frac{V_{CE}}{V_A}\right)e^{(V_{BE}/v_T)}$$
$$\approx I_S e^{(V_{BE}/v_T)}$$

Diffusion constant

Intrinsic carrier concentration

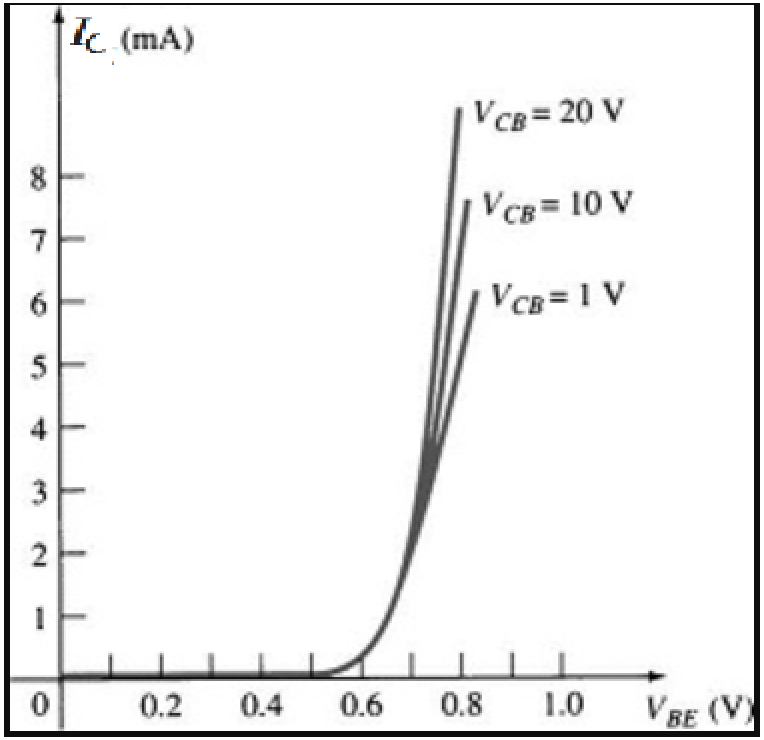
$$I_S = \frac{qD_n n_i^2}{Q_B}A_E$$

**Emitter area**

No. of dopant atoms in base per unit $A_E$

- Ratio current by scaling emitter area

$$\frac{I_{C1}}{I_{C2}} = \frac{A_{E1}}{A_{E2}}$$



# BJT Small-Signal Parameters

- Forward current gain

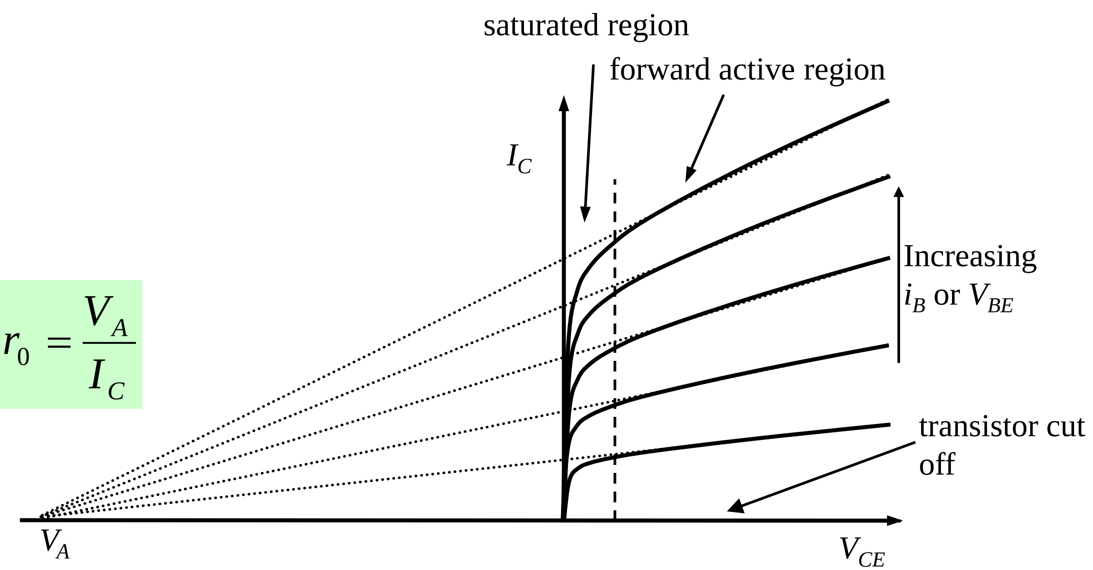
$$\beta = \underbrace{\frac{i_c}{i_b}}_{small-signal} = \underbrace{\frac{\Delta I_C}{\Delta I_B}}_{\Delta l\arg e-signal}$$

$$\beta = g_m r_\pi$$

- Trans-conductance

$$g_m = \frac{i_c}{v_\pi} = \frac{I_C}{v_T} = \frac{I_c q}{kT} \qquad v_T = \frac{kT}{q}$$

$$i_c = \beta i_b = g_m v_\pi = g_m i_b r_\pi$$

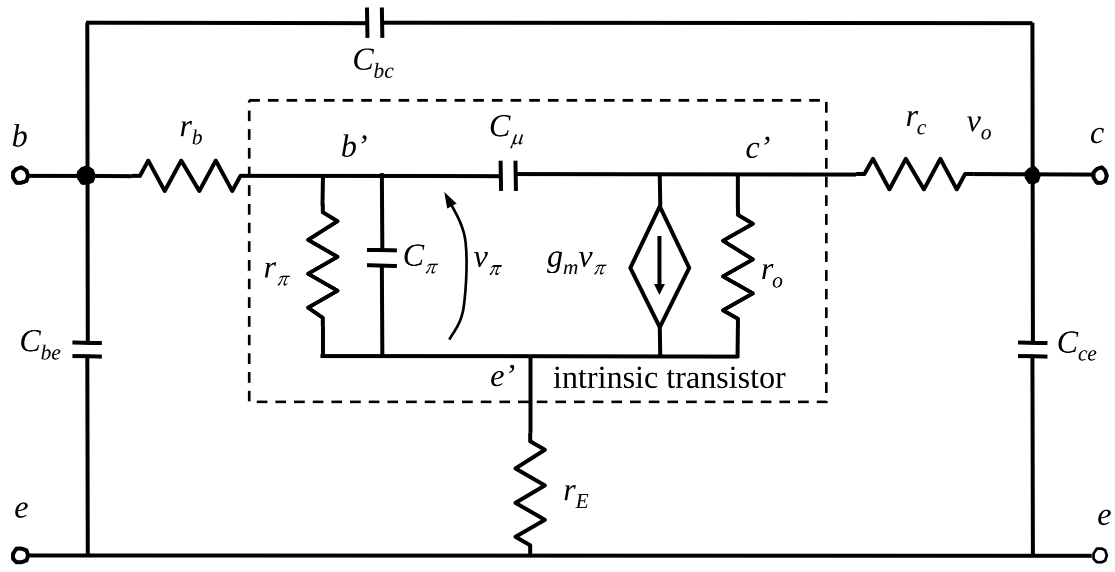



$$r_0 = \frac{V_A}{I_C}$$

$$I_C = I_S e^{(V_{BE}/v_T)}$$

$$I_C = I_S\left(1 + \frac{V_{CE}}{V_A}\right)e^{(V_{BE}/v_T)}$$

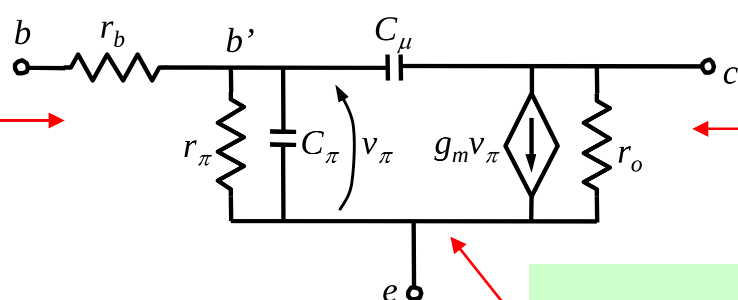
# BJT Small-Signal Model




$r_\pi, C_\pi, C_\mu, g_m, r_e, r_0$
depend on bias

## Simplified Model

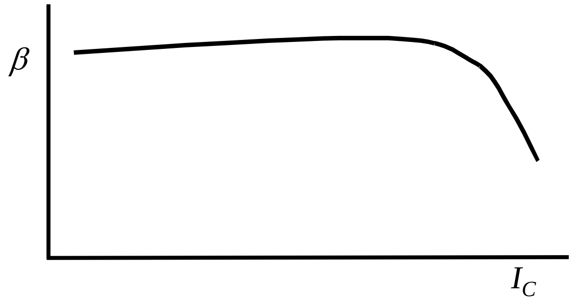
$$Z_{in} \approx r_b + r_\pi // C_\pi // C_\mu$$
$$\approx r_b + r_\pi // C_\pi$$
$$\approx r_\pi = \frac{\beta_0}{g_m} \left(\text{low freq}\right)$$

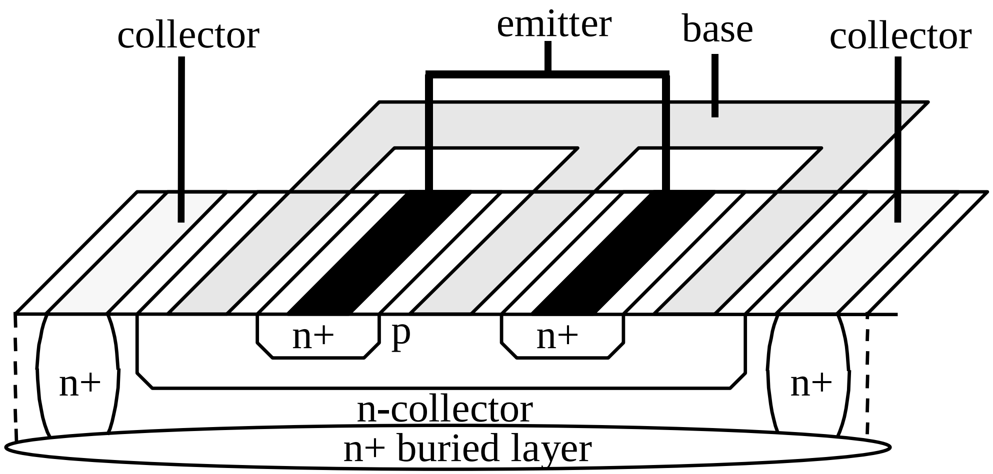



$$Z_o \approx r_0 = \frac{V_A}{I_C} = \frac{V_A}{g_m V_T}$$

$$Z_E \approx r_e = \frac{1}{g_m}$$

# BJT β Current Dependence




β drops off at high currents due to the following three effects:

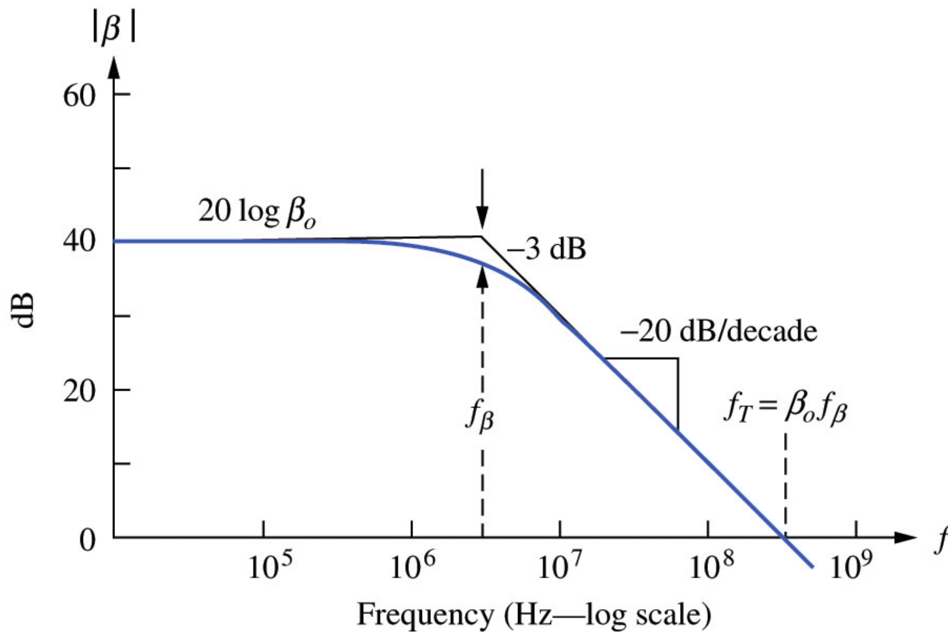
- *Decrease of emitter efficiency:* excess minority carriers in the base.
- *Kirk effect or base pushout*: the minority carrier concentration in the base-collector depletion region becomes comparable to the background donor density, leading to a dramatic increase in the effective base width.
- *Emitter crowding:* the distributed parasitic resistance at base contact causes higher current density along the edge of emitter → maximize the emitter periphery (inter-digitized layout).

# BJT 3dB Corner Frequency -- $f_\beta$

$$\beta(\omega) = \frac{\beta_o}{1 + j\,\omega/\omega_\beta}$$

3-dB corner frequency

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} = \frac{1}{2\pi r_\pi C_\pi}$$

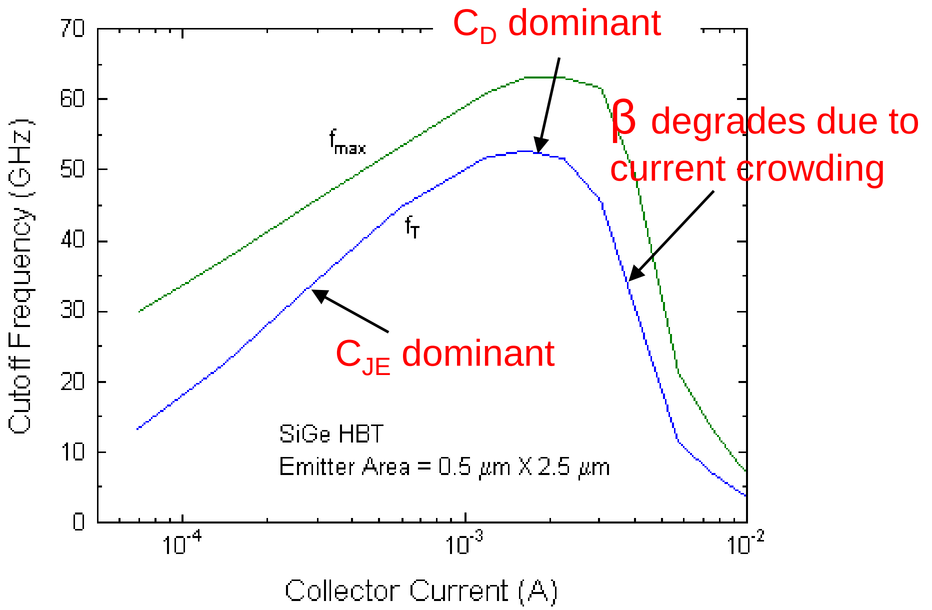



Current gain is $\beta_o = g_m r_\pi$ at low frequencies and has single pole roll-off at frequencies > $f_\beta$, crossing unity gain at $\omega_T$.

# BJT Unity-gain Frequency -- $f_T$

- $f_T$ is the frequency at which the short-circuit current gain β is equal to 1 → unity current gain-bandwidth product. Useful to specify the maximum switching frequency for CML circuits and gain/bandwidth of an amplifier.

$$f_T = \beta_0 f_\beta = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)} \approx \frac{g_m}{2\pi C_\pi} = \frac{I_C}{2\pi C_\pi V_T}$$



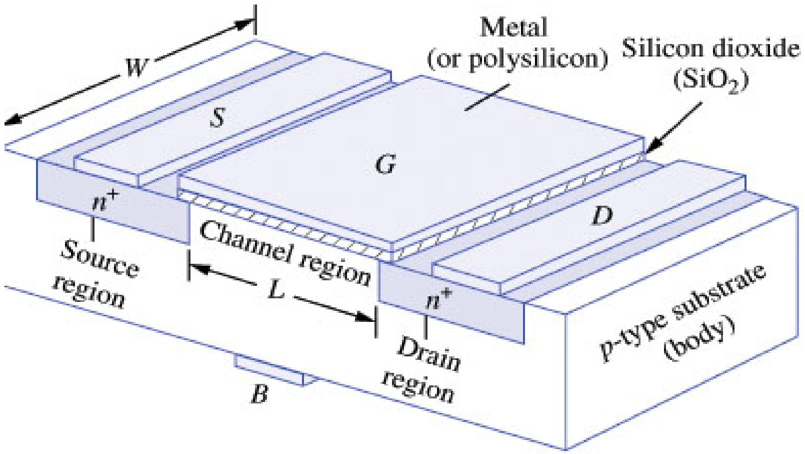

$$C_\pi = C_D + C_{JE}$$

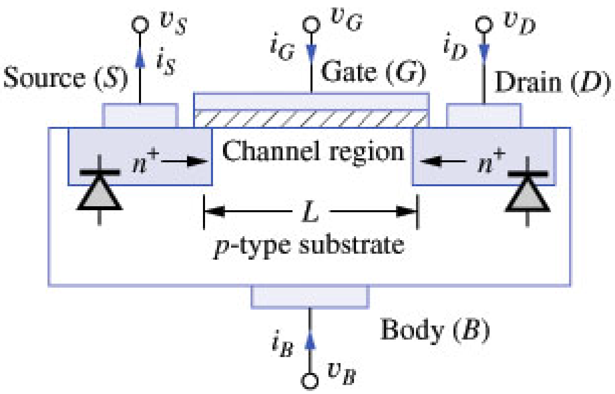
Emitter diffusion capacitance proportional to $I_C$

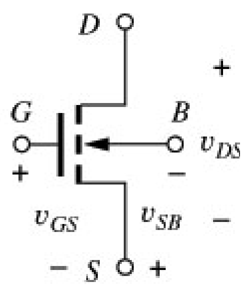
Emitter-base junction cap proportional to $A_E$

$$C_{JE} = A_E \left(\frac{q\varepsilon N_B}{V_{JE}}\right)^{\frac{1}{3}}$$

# NMOS Transistor: Structure

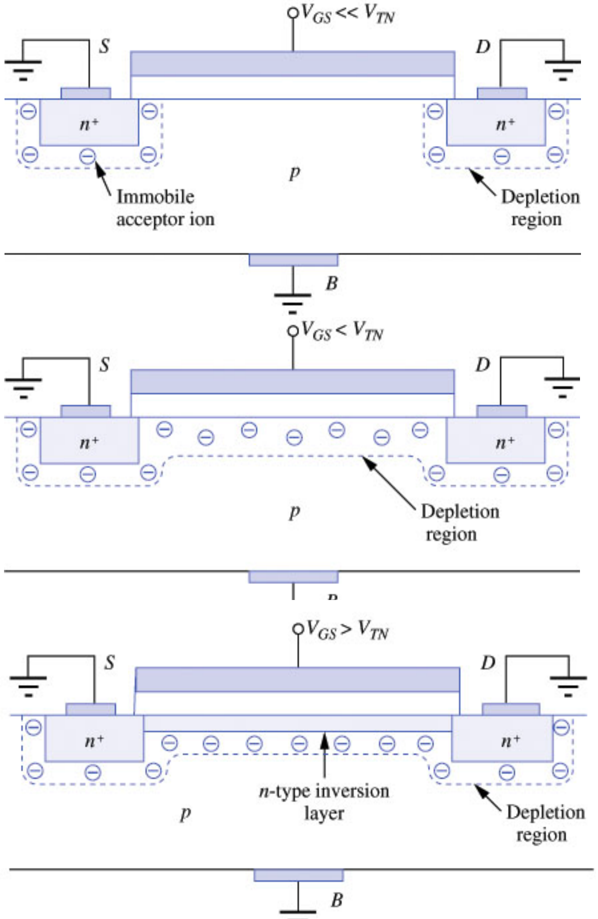


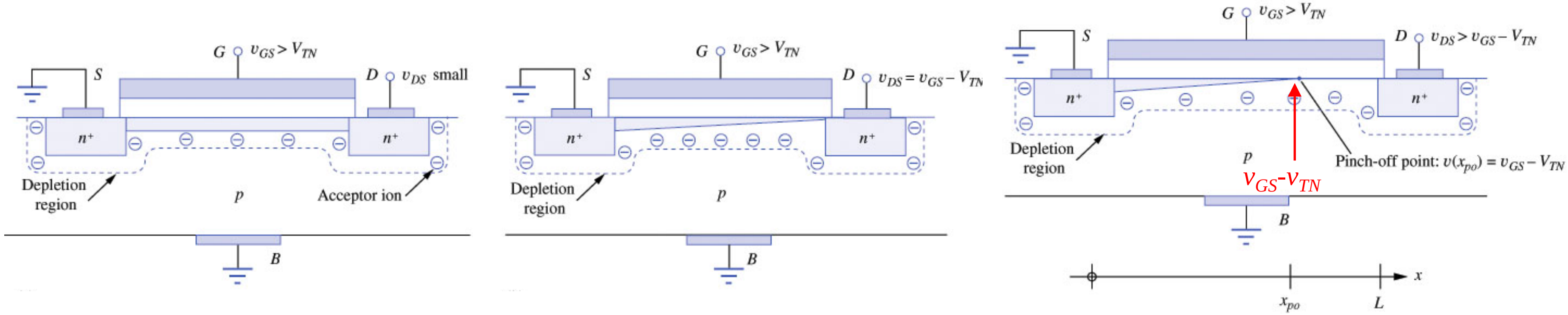

- 4 device terminals: Gate(G), Drain(D), Source(S) and Body(B).
- Source and drain regions form *pn* junctions with substrate.
- $v_{SB}$, $v_{DS}$ and $v_{GS}$ always positive during normal operation.
- $v_{SB}$ always < $v_{DS}$ and $v_{GS}$ to reverse bias *pn* junctions

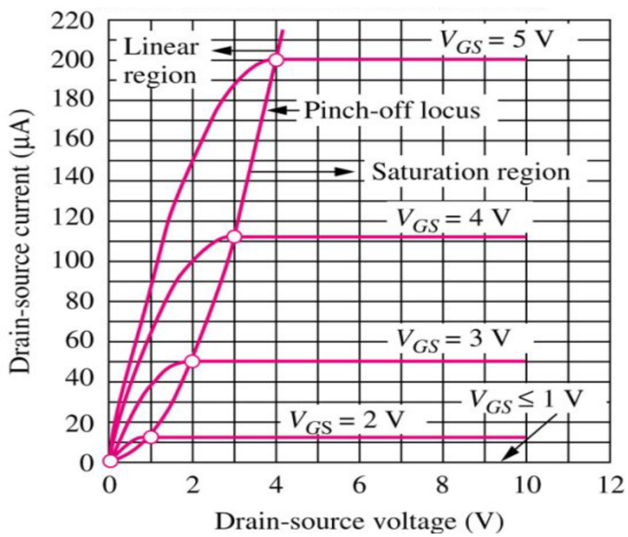
# NMOS Transistor: I-V Behavior




- $V_{GS} << V_{TN}$ : Only small leakage current flows.
- $V_{GS} < V_{TN}$: Depletion region formed under gate merges with source and drain depletion regions. No current flows between source and drain.
- $V_{GS} > V_{TN}$: Channel formed between source and drain. If $v_{DS} > 0$, finite $i_D$ flows from drain to source.
- $i_B = 0$ and $i_G = 0.$

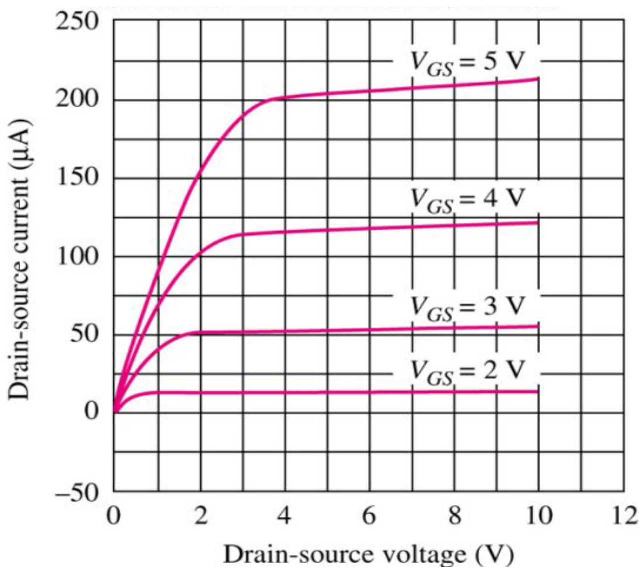
# NMOS Transistor: Saturation Region

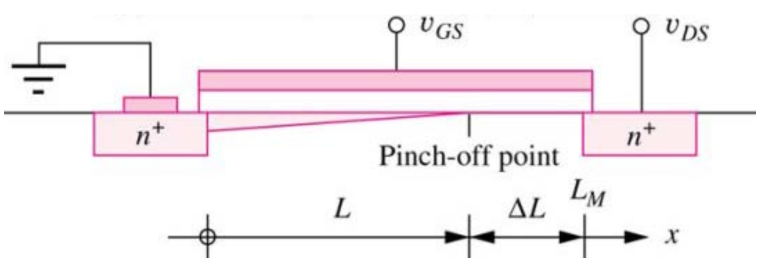



- If $v_{DS}$ increases above triode region limit, the channel region disappears and is said to be pinched-off.
- Current saturates at constant value, independent of $v_{DS}$.
- Saturation region operation mostly used for analog amplification.

# 8/24-Channel-Length Modulation



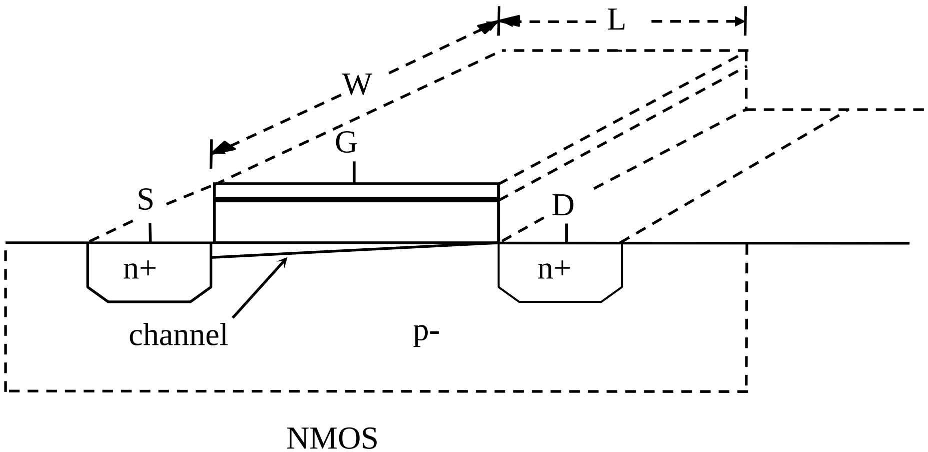

- As $v_{DS}$ increases above $v_{DSAT}$, the length of the depleted channel beyond the pinch-off point, ΔL, increases and the actual L decreases.
- $i_D$ increases slightly with $v_{DS}$ instead of being constant.

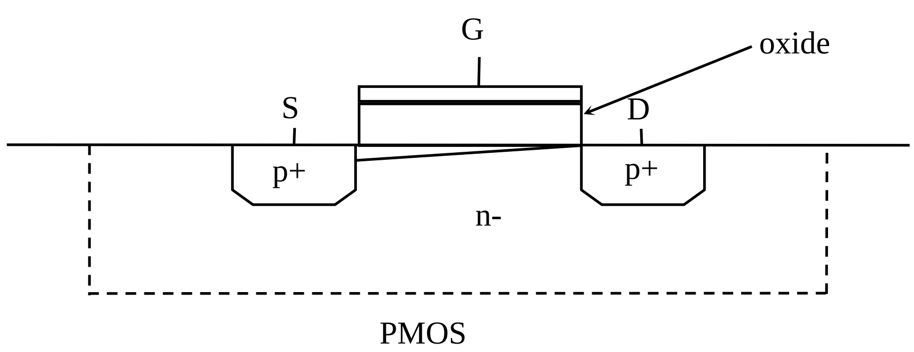
$$i_D = \frac{K_n'}{2}\frac{W}{L}\left(v_{GS} - V_{TN}\right)^2\left(1 + \lambda v_{DS}\right)$$

$\lambda$ = channel length modulation parameter

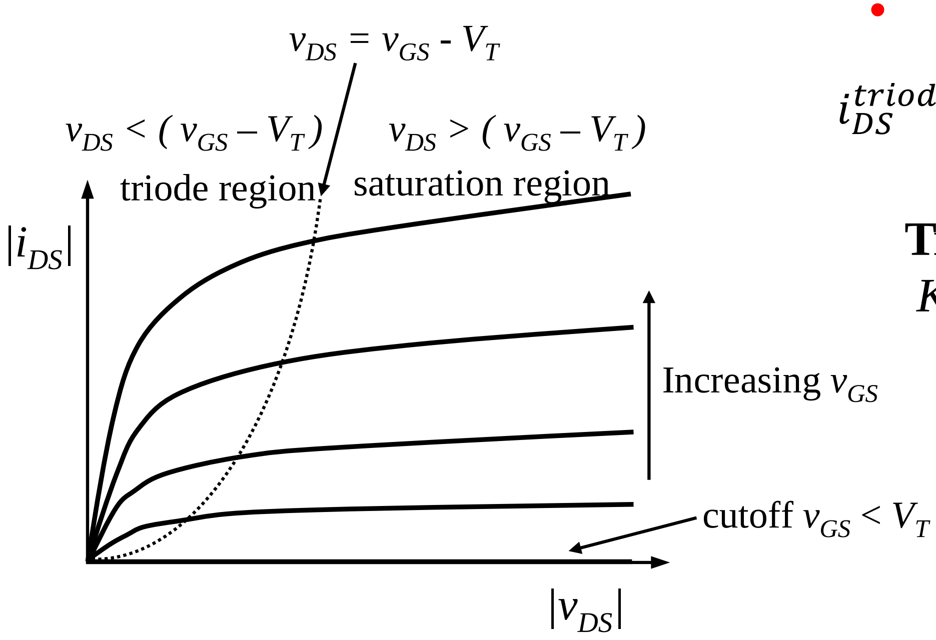
# CMOS Transistors

- CMOS is necessary to implement baseband digital or DSP functions. For low cost applications, CMOS-only process is desired to implement both digital and RF functions on the same chip – system on chip (SOC).



# CMOS Transistor Parameters




- In triode region:

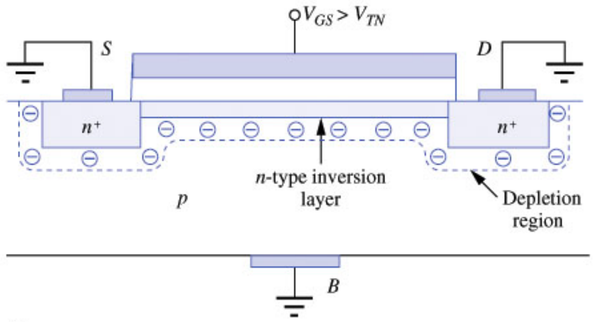
$$i_{DS}^{triode} = \mu C_{ox}\left(\frac{W}{L}\right)\left(v_{GS} - V_T - \frac{v_{DS}}{2}\right)v_{DS}$$

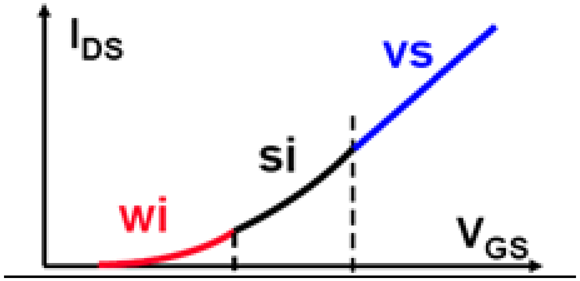
**Transconductance Parameters:**

$K_n = \mu_n C_{ox} W/L$, $K_p = \mu_p C_{ox} W/L$

**On-resistance $R_{on}$**, the resistance of the FET in the triode region near the origin:

$$R_{on} = \left[\left.\frac{\partial i_D}{\partial v_{DS}}\right|_{v_{DS}\to 0}\right]^{-1}_{Q\text{-}pt} = \frac{1}{K_n' \frac{W}{L}(V_{GS} - V_{TN})}$$



# CMOS Transistor Parameters

- In saturation region: $i_{DS}^{sat} = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{(v_{GS}-V_T)^2}{1+\alpha(v_{GS}-V_T)}(1+\lambda v_{DS}) \approx \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)(v_{GS}-V_T)^2$

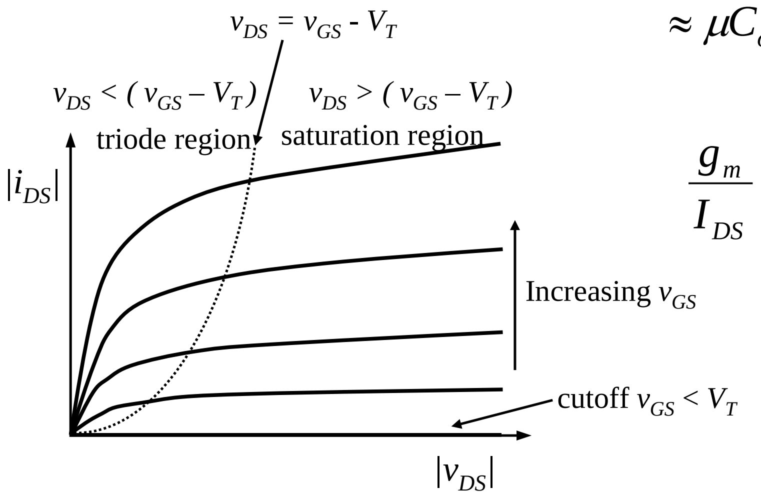
$\alpha = \theta + \frac{\mu}{2v_{scl}L}$ models the mobility degradation $\theta$ and velocity saturation effects $v_{scl}$.

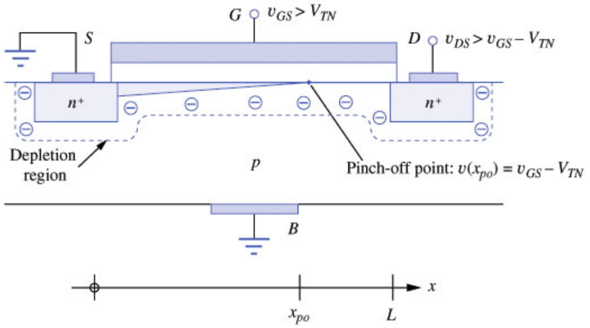
- Transconductance:

$$g_m = \frac{di_{DS}}{dv_{GS}} = \mu C_{ox}\left(\frac{W}{L}\right)(v_{GS}-V_T)(1+\lambda v_{DS})$$

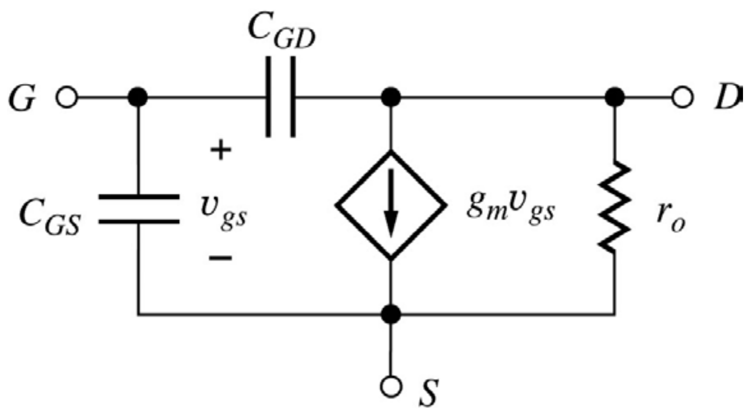
$$\approx \mu C_{ox}\left(\frac{W}{L}\right)(v_{GS}-V_T) = \sqrt{2\mu C_{ox}\left(\frac{W}{L}\right)I_{DS}} \propto \sqrt{I_{DS}}$$

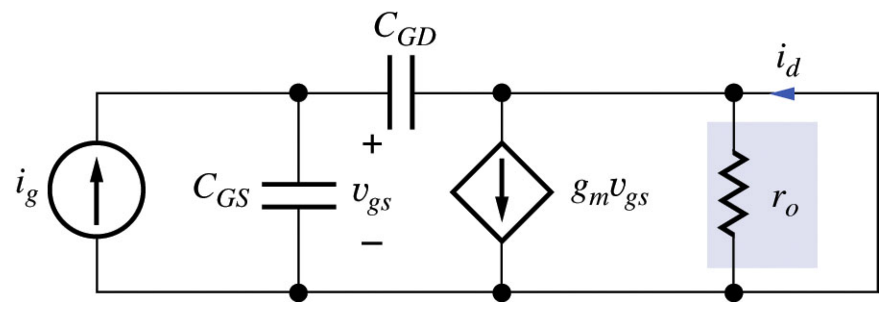
$$\frac{g_m}{I_{DS}} = \sqrt{2\mu C_{ox}\left(\frac{W}{L}\right)\frac{1}{I_{DS}}} = \frac{2}{V_{GS}-V_T} = \frac{2}{V_{ot}}$$



# High-frequency Model of MOSFET



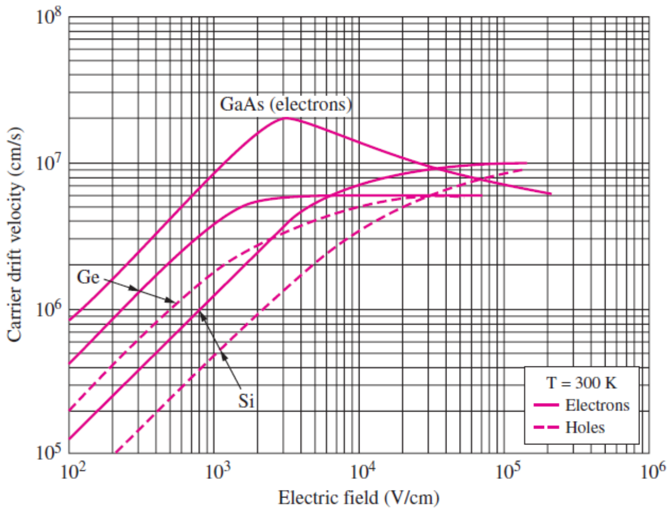



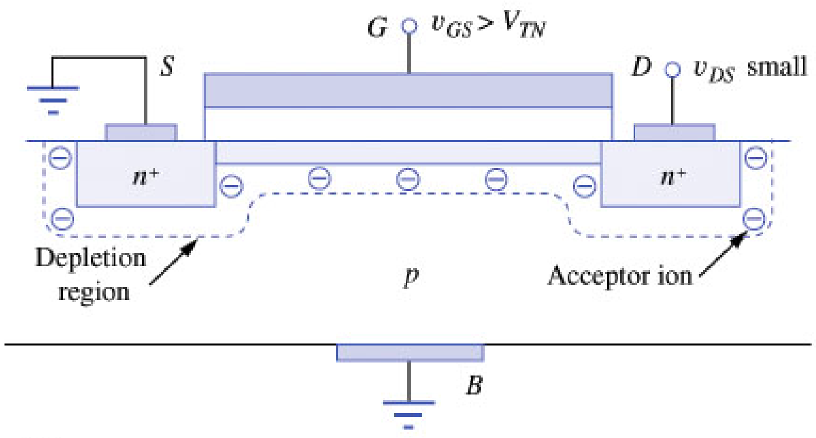

At dc, the current gain is infinite but falls at a rate of 20 dB/decade as frequency increases. The unity current gain-bandwidth product $\omega_T$ is defined as

$$\omega_T = \frac{g_m}{C_{GS} + C_{GD}}$$

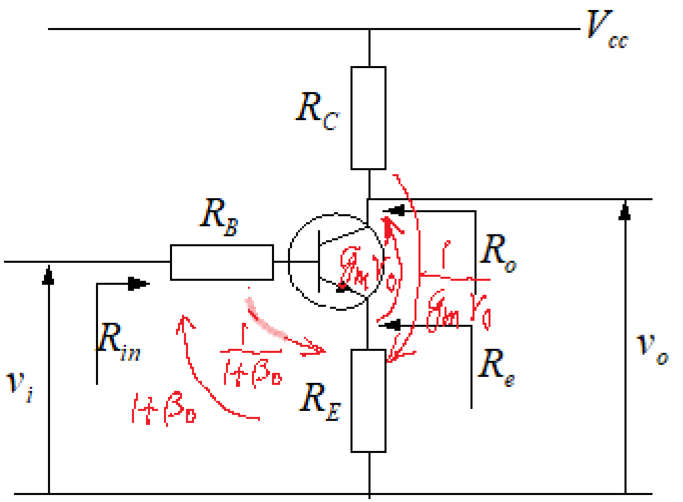
$$\omega_T = \frac{\mu_n C''_{ox} \frac{W}{L} (V_{GS} - V_{TN})}{\frac{2}{3} C''_{ox} W L} = \frac{3}{2} \frac{\mu_n (V_{GS} - V_{TN})}{L^2}$$

# Effect of velocity saturation on FET $f_T$



# Impedance transform through transistors

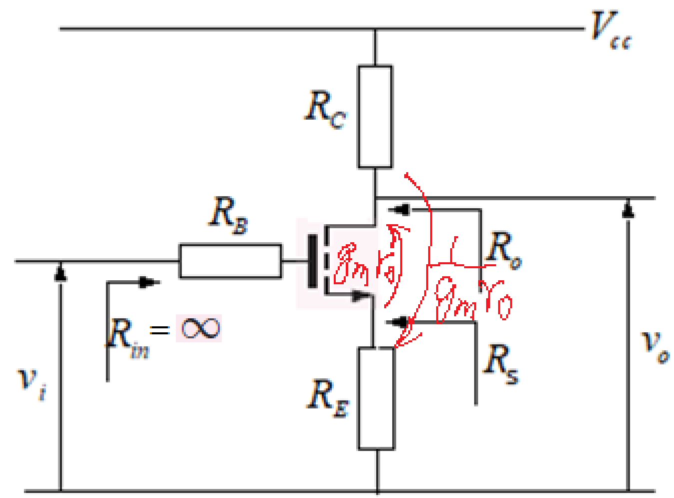



$$R_{in} = R_B + r_b + r_\pi + (1+\beta_o)R_E$$

$$R_o = R_C \| r_o[1+g_m(R_E \| r_\pi)]$$

$$\cong R_C \| r_o(1+g_m R_E) \quad \text{for } r_\pi >> R_E$$

$$R_e = R_E \left\| \left( r_e + \frac{R_B}{1+\beta_o} + \frac{R_c}{g_m r_o} \right) \right.$$



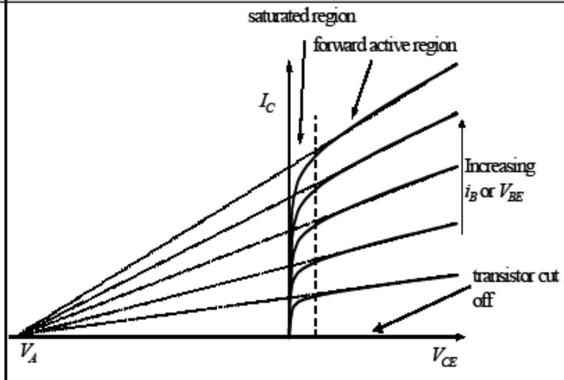
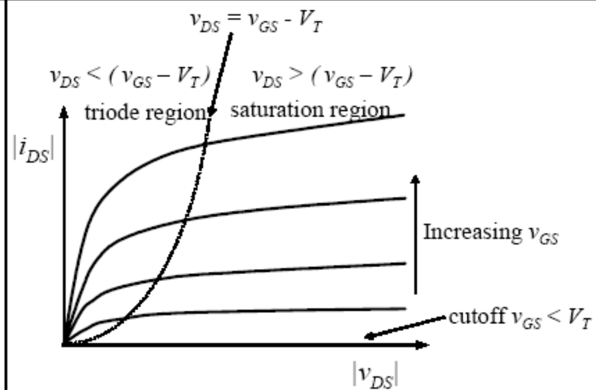

$$\beta_o = \infty$$

$$R_{in} = R_B + \infty = \infty$$

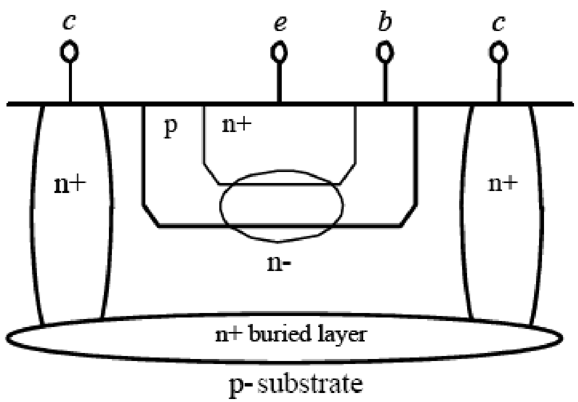
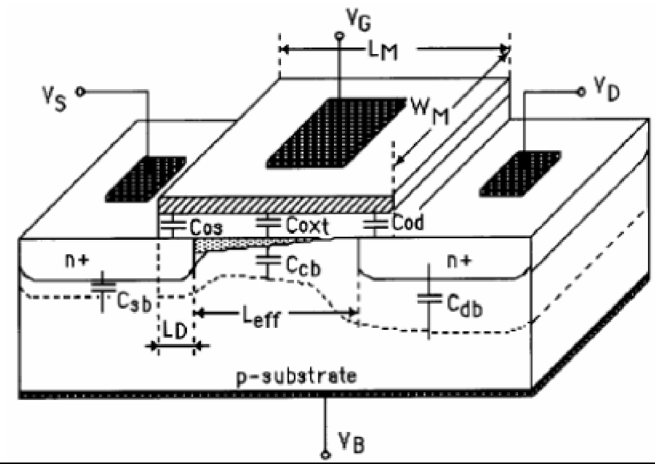
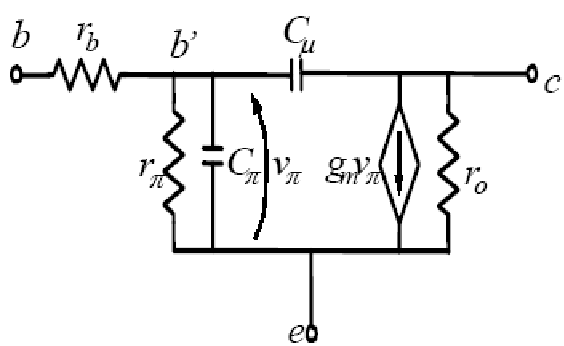
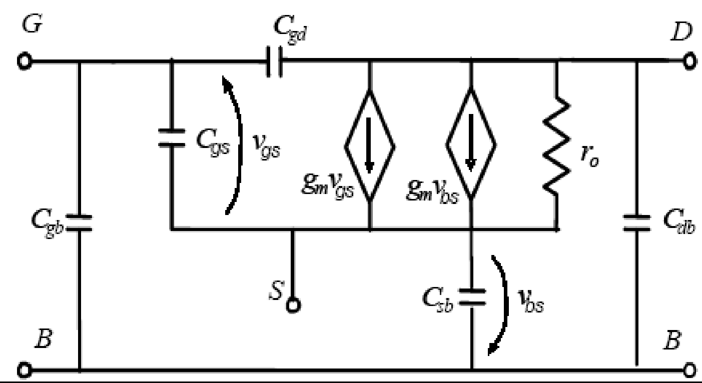
$$R_o = R_C \| r_o(1+g_m R_E)$$

$$R_S = R_E \left\| \left( \frac{1}{g_m} + \frac{R_c}{g_m r_o} \right) \right.$$

# Comparison of BJT and MOS Operational Regions

| Region | NPN | Region | NMOS |
|---|---|---|---|
| I-V curve |  | I-V curve |  |
| Cutoff | $V_{BE} < V_t$, $V_{BC} < V_t$ | Cutoff | $V_{GS} < V_t$, $V_{GD} < V_t$ |
| Forward Active | $V_{BE} > V_t$, $V_{BC} < V_t$ | Forward Active (Saturation) | $V_{GS} > V_t$, $V_{GD} < V_t$<br>$V_{DS} > V_{GS} - V_t$ |
| Reverse Active | $V_{BE} < V_t$, $V_{BC} > V_t$ | Reverse Active (Saturation) | $V_{GS} < V_t$, $V_{GD} > V_t$<br>$V_{GS} > V_{DS} - V_t$ |
| Soft Saturation | $V_{BE} > V_t$,<br>$0.8V > V_{CE} > 0.5V$<br>$0 < V_{BC} < 0.3V$ | Pinch-off | $V_{DS} = V_{GS} - V_t$ |
| Saturation | $V_{CE} < 0.5V$,<br>$V_{BC} > 0.3V$ | Triode (used as a resistor) | $V_{GS} > V_t$,<br>$V_{DS} < V_{GS} - V_t$<br>$R_{on} = \left[\mu C_{ox}\left(\frac{W}{L}\right)(v_{GS} - V_t)\right]^{-1}$ |
| | | Weak Inversion | $V_{GS} < V_t$ |

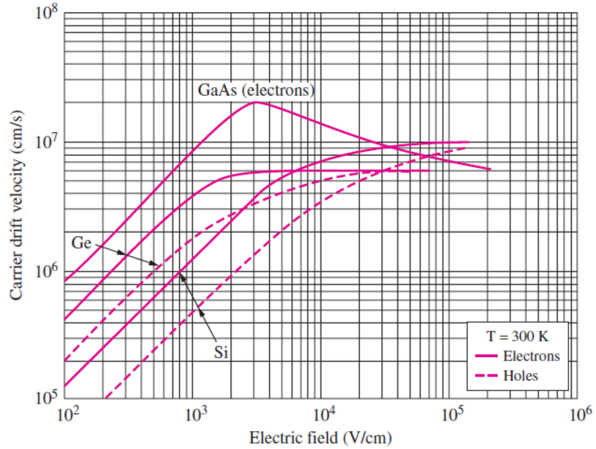
# Comparison of BJT and MOS Characteristics

| | NPN | NMOS |
|---|---|---|
| Structure | Vertical<br> | Lateral<br> |
| Small Signal Equivalent Circuit |  |  |

# Comparison of BJT and MOS Characteristics

<table>
<tr>
<td>Current</td>
<td>

$$I_C = I_S\left(1+\frac{V_{CE}}{V_A}\right)e^{(V_{BE}/v_T)}$$

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<td>

$$i_{DS}^{sat} = \frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{(v_{GS}-V_t)^2}{1+\theta(v_{GS}-V_t)}(1+\lambda v_{DS})$$

$$i_{DS}^{triode} = \mu C_{ox}\left(\frac{W}{L}\right)\left(v_{GS}-V_t-\frac{v_{DS}}{2}\right)v_{DS}(1+\lambda v_{DS})$$

$$i_{DS}^{weak} = I_t\frac{W}{L}\exp\left(\frac{v_{GS}-V_t}{V_T}\right)\left[1-\exp\left(-\frac{v_{DS}}{V_T}\right)\right]$$

wi - si transistion: $(v_{GS}-V_t)=2n\frac{kT}{q}=70mV$

si - vs transistion: $E_{scl}=5\times10^6\ \frac{V}{m}$,

$\mu=0.06\frac{m^2}{Vs}, v_{scl}=10^5\ \frac{m}{s}$

$$v_{GS}-V_t=\frac{1}{\theta}=\frac{2nLv_{scl}}{\mu}=LE_{scl}=5L[\mu m]$$

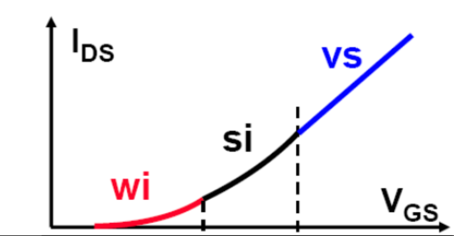
for $L=0.13\mu m, v_{GS}-V_t=0.65V$

</td>
</tr>
<tr>
<td>MOS Current under Velocity Saturation</td>
<td></td>
<td>

$$i_{DS}^{triode}=\frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{[2(v_{GS}-V_t)-v_{DS}]v_{DS}}{1+v_{DS}/E_c L}$$

$$i_{DS}^{sat}=\frac{\mu C_{ox}}{2}\left(\frac{W}{L}\right)\frac{(v_{GS}-V_t)^2(1+\lambda v_{DS})}{1+(v_{GS}-V_t)/E_c L}$$

$\xrightarrow{E_C\to 0} WC_{ox}(v_{GS}-V_t)v_{scl}$, w.v.s.

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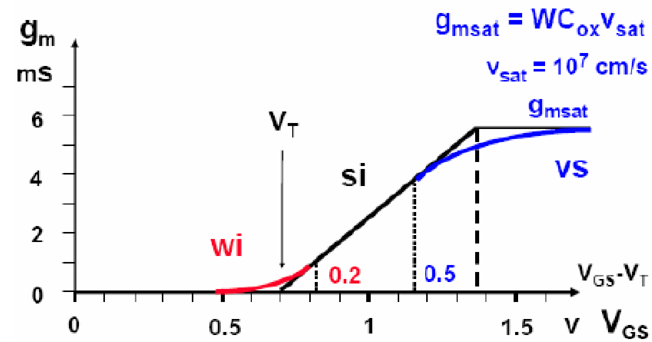
# Comparison of BJT and MOS Characteristics

<table>
<tr>
<td>Trans-<br>conductance</td>
<td>$$g_m = \frac{I_C}{v_T} = \frac{I_c q}{kT} \propto I_C$$</td>
<td>

$$g_m = \mu C_{ox}\left(\frac{W}{L}\right)(v_{GS}-V_T)(1+\lambda v_{DS})$$

$$= \frac{2I_{DS}}{v_{GS}-V_T} \approx \sqrt{2\mu C_{ox}\left(\frac{W}{L}\right)I_{DS}} \propto \sqrt{I_{DS}}$$

$$\xrightarrow{E_C \to 0} WC_{ox}v_{scl}, \text{ w.v.s.}$$



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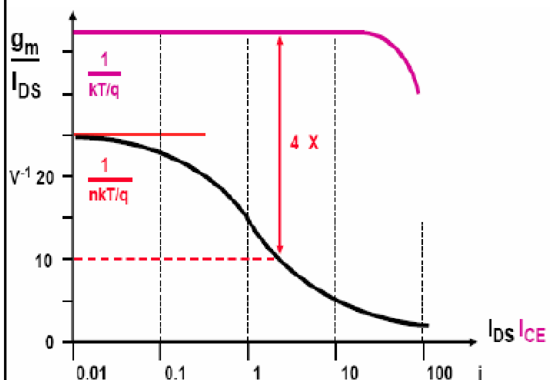
<tr>
<td>Gm to Current Ratio</td>
<td>

$$\frac{g_m}{I_C} = \frac{1}{v_T} = \frac{1}{26mV}$$



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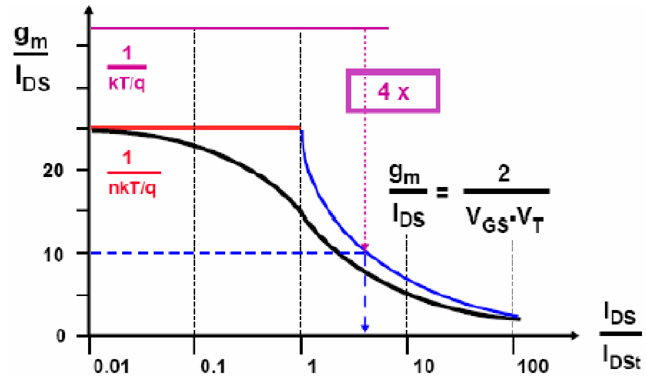
<td>

$$\frac{g_m}{I_{DS}} = \sqrt{2\mu C_{ox}\left(\frac{W}{L}\right)\frac{1}{I_{DS}}} = \frac{2}{V_{GS}-V_t}$$

$$\xrightarrow{E_C \to 0} (V_{GS}-V_t)^{-1}, \text{ w.v.s.}$$



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# Comparison of BJT and MOS Characteristics

| | | |
|---|---|---|
| Forward Current Gain | $\beta = \frac{i_c}{i_b} = g_m r_\pi$ <br> $\beta_0 \approx 100$, finite | $\beta = \frac{g_m}{S(C_{gs} + C_{gd} + C_{gb})}$ <br> $\beta_0 \to \infty$, infinite |
| Input Impedance | $Z_b \approx r_b + r_\pi // C_\pi // C_\mu$ <br> $\approx r_b(\text{high freq})$ <br> $\approx r_b + r_\pi = r_b + \frac{\beta_0}{g_m}(\text{low freq})$ | $Z_G \approx C_{gs} // C_{gd} // C_{gb}$ <br> $\approx 0(\text{high freq})$ <br> $\approx \infty(\text{low freq})$ |
| Output Impedance | $r_0 = \frac{V_A}{I_C}$ | $r_0 = \frac{V_A}{I_D} = \frac{1}{\lambda I_D}$ |
| Impedance at Emitter | $r_e = \frac{V_T}{I_C} = \frac{1}{g_m}$ | $\frac{1}{g_m}$ |
| Maximum Gain | $g_m r_o = \frac{V_A}{V_T}$ | $g_m r_o = \frac{2V_A}{V_{GS} - V_t} = \frac{1}{\lambda} \frac{2}{V_{GS} - V_t}$ |

# Comparison of BJT and MOS Characteristics

| | | |
|---|---|---|
| $C_\pi$ | $C_\pi = C_D + C_{je}$ <br> $= \tau_F g_m + C_{je}$ | $@\text{triode},\ C_{gs} = C_{gd} = \frac{WLC_{ox}}{2}$ <br> $@\text{sat},\ C_{gs} = \frac{2}{3} WLC_{ox}$ |
| Effective Transit Time | $\tau_T = \frac{1}{2\pi f_T} = \tau_F + \frac{C_{je}}{g_m} + \frac{C_\mu}{g_m}$ <br> $\approx \tau_F + C_{je}\frac{1}{g_m}$ | |
| Unity Current Gain-BW Product | $f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$ <br> $\begin{cases} = \frac{\mu}{\pi W_B^2} V_T, \text{ w.o. velocity sat} \\ \approx \frac{V_{sat}}{2\pi W_B}, \text{ with velocity sat} \end{cases}$ | $f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd} + C_{gb})}$ <br> $\begin{cases} = \frac{3\mu}{4\pi n L^2}(V_{GS} - V_t), \text{ w.o. velocity sat} \\ \approx \frac{V_{scl}}{2\pi L}, \text{with velocity sat} \end{cases}$ |
| Unity Power Gain-BW Product | $f_{\max} = \sqrt{\frac{f_T}{8\pi \cdot r_b C_\mu}}$ | $f_{\max} = \sqrt{\frac{f_T}{8\pi \cdot r_G C_{GD}}}$ |

# CHAPTER 2

## Basic Concepts in IC Designs

I. Device Review
II. Linearity Analysis
III. Noise Analysis
IV. System Analysis

# Units for Microwave and RFIC Design

Peak-to-peak voltage: $V_{pp}$

Root-mean-square voltage:

$$V_{rms} = \frac{V_{pp}}{2\sqrt{2}}$$

Power in Watt :

$$Pwatt = \frac{V^2{}_{rms}}{R} = \frac{V_{pp}^2}{8R}$$

Power in dBm :

$$P_{dBm} = 10\log_{10}\left(\frac{Pwatt\,[mW]}{1mW}\right)$$

# dBm Conversion Table

| [dBm] | [Watts] | [Volts]rms | [Volts]p | [Volts]pp |
|---|---|---|---|---|
| -30 | 0.100E-05 | 7.071 mV | 9.998 mV | 19.997 mV |
| -29 | 0.126E-05 | 7.934 mV | 11.218 mV | 22.437 mV |
| -28 | 0.158E-05 | 8.902 mV | 12.587 mV | 25.175 mV |
| -27 | 0.200E-05 | 9.988 mV | 14.123 mV | 28.246 mV |
| -26 | 0.251E-05 | 11.207 mV | 15.847 mV | 31.693 mV |
| -25 | 0.316E-05 | 12.574 mV | 17.780 mV | 35.560 mV |
| 0 | 0.100E-02 | 223.607 mV | 316.180 mV | 632.360 mV |
| 1 | 0.126E-02 | 250.891 mV | 354.760 mV | 709.520 mV |
| 2 | 0.158E-02 | 281.504 mV | 398.047 mV | 796.094 mV |
| 3 | 0.200E-02 | 315.853 mV | 446.616 mV | 893.232 mV |
| 4 | 0.251E-02 | 0.354 V | 0.501 V | 1.002 V |
| 5 | 0.316E-02 | 0.398 V | 0.562 V | 1.125 V |
| 6 | 0.398E-02 | 0.446 V | 0.631 V | 1.262 V |
| 7 | 0.501E-02 | 0.501 V | 0.708 V | 1.416 V |
| 8 | 0.631E-02 | 0.562 V | 0.794 V | 1.588 V |
| 9 | 0.794E-02 | 0.630 V | 0.891 V | 1.782 V |

# Linear vs. Nonlinear Systems

- A system is linear if for any inputs $x_1(t)$ and $x_2(t)$, $x_1(t) \rightarrow y_2(t)$, $x_2(t) \rightarrow y_2(t)$ and for all values of constants a and b, it satisfies

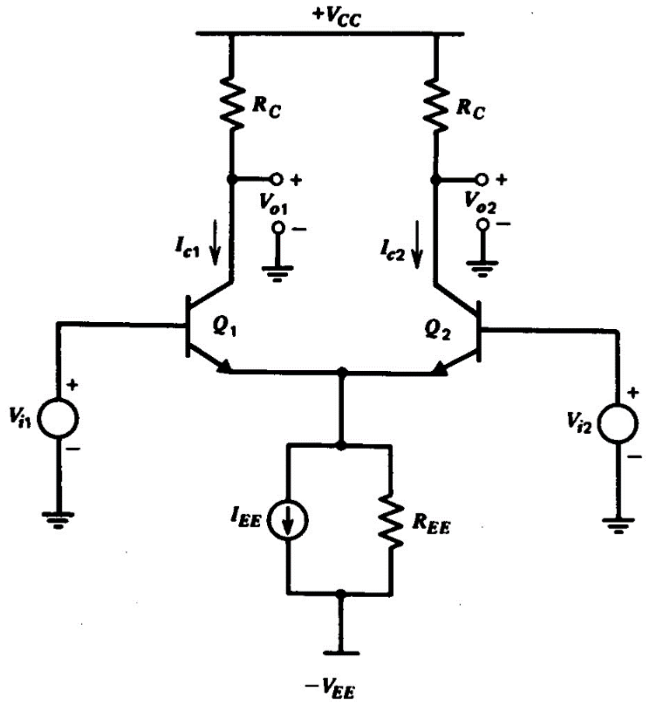
$$ax_1(t)+bx_2(t) \rightarrow ay_1(t)+by_2(t)$$

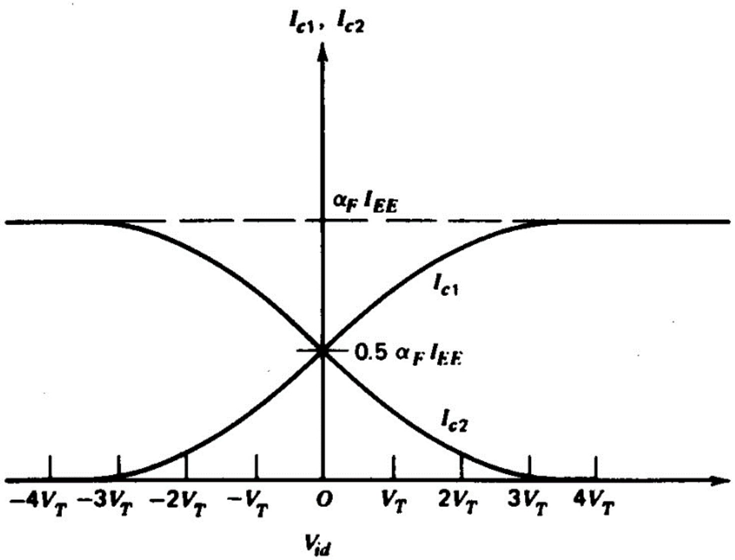
- A system is nonlinear if it does not satisfy the superposition law.

# Output of Bipolar Differential Pair

$$\frac{V_{od}}{V_{id}} = \alpha_F I_{EE} R_C \tanh\left(\frac{-V_{id}}{2V_T}\right)$$

$$\tanh(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{7}{315}x^7 + \ldots$$



# Output of MOS Differential Pair

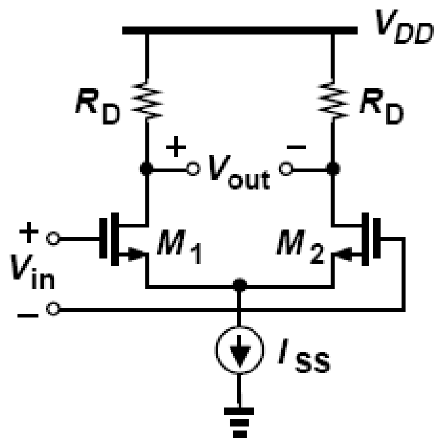
**For square-law MOS diff transistors operating in saturation, the characteristic above can be expresse**

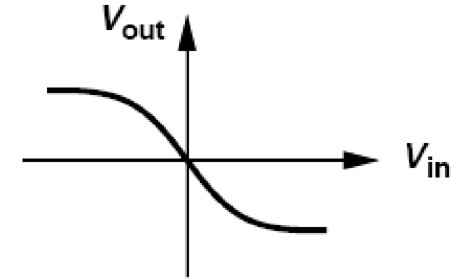
$$V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L}V_{in}\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - V_{in}^2}\,R_D$$

**If the differential input is small, approximate the characteristic by a polynomial.**

**Factoring $4I_{ss} / (\mu_n C_{ox} W/L)$ out of the square root and assuming**

$$V_{in}^2 \ll \frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}}$$

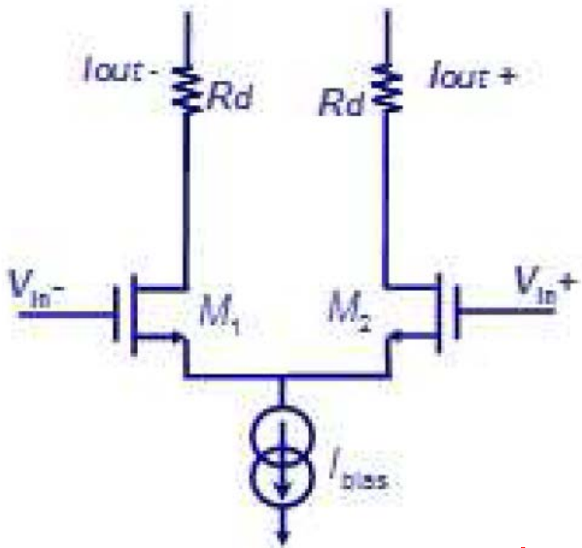



**Approximation gives us:**

$$V_{out} \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}\,V_{in}\left(1 - \frac{\mu_n C_{ox}\frac{W}{L}}{8I_{SS}}V_{in}^2\right)R_D \approx -\sqrt{\mu_n C_{ox}\frac{W}{L}I_{SS}}\,R_D V_{in} + \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{3/2}}{8\sqrt{I_{SS}}}R_D V_{in}^3$$

# Linearity of MOS Diff-Pair




$$I_d = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left(V_{gs} - V_t\right)^2$$

$$\Delta V_{in} = V_{in1} - V_{in2} = V_{gs1} - V_{gs2}$$

$$\Delta V_{out} = -\left(I_{d1} - I_{d2}\right) R_d$$

$$\Delta V_{out} = -\frac{1}{2}\mu_n C_{ox}\frac{W}{L} R_d \Delta V_{in}\sqrt{\frac{4 I_{bias}}{\mu_n C_{ox}\frac{W}{L}} - \Delta V_{in}^2}$$

$$V_{out} = a_1 V_{in} + a_2 V_{in}^2 + a_3 V_{in}^3$$

Linear gain

$$A_v = \frac{\delta V_{out}}{\delta V_{in}} = -R_d\sqrt{\mu_n C_{ox}\frac{W}{L} I_{bias}} = -R_d G_m$$

$$a_1 = \frac{\partial V_{out}}{\partial V_{in}}\Big|_{vin=0} = -Rd\sqrt{\mu_n C_{ox}\frac{W}{L} I_{bias}}$$

Symmetry due to diff circuit

$$a_2 = \frac{\partial^2 V_{out}}{\partial V_{in}^2}\Big|_{vin=0} = 0$$

Increasing $I_{bias}$ improves the linearity

$$a_3 = \frac{\partial^3 V_{out}}{\partial V_{in}^3}\Big|_{vin=0} = \frac{3}{4} R_d \frac{\left(\mu_n C_{ox}\frac{W}{L}\right)^{\frac{3}{2}}}{\sqrt{I_{bias}}}$$

# Effects of Nonlinearity

- Harmonic Distortion
- Gain Compression
- Desensitization
- Intermodulation

- For simplicity, we limit our analysis to memoryless, time invariant system. Thus,

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \ldots \qquad (3.1)$$

# Effects of Nonlinearity -- Harmonics

If a single tone signal is applied to a nonlinear system, the output generally exhibits fundamental and harmonic frequencies with respect to the input frequency. In Eq. (3.1), if $x(t) = A\cos\omega t$, then

$$
\begin{aligned}
y(t) &= \alpha_1 A\cos\omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\
&= \alpha_1 A\cos\omega t + \frac{\alpha_2 A^2}{2}(1+\cos 2\omega t) + \frac{\alpha_3 A^3}{4}(3\cos\omega t + \cos 3\omega t) \\
&= \frac{\alpha_2 A^2}{2} + (\alpha_1 A + \frac{3\alpha_3 A^3}{4})\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t
\end{aligned}
$$

Observations:

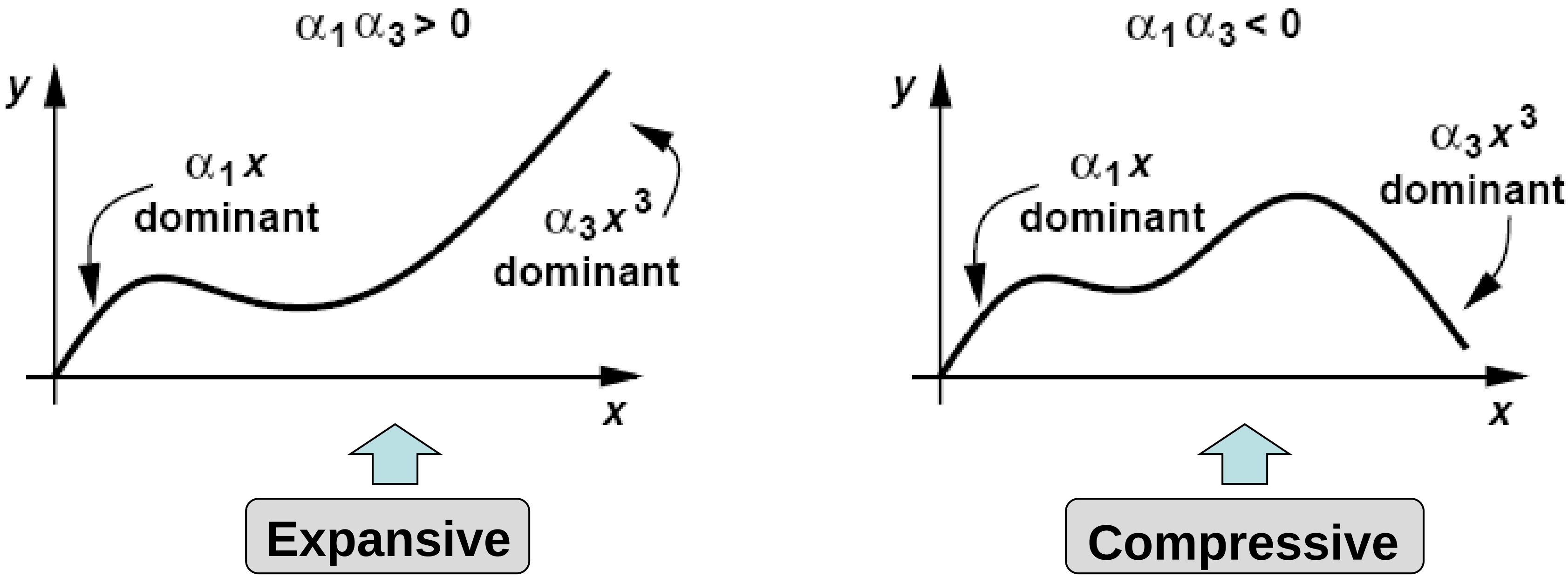
1. even order harmonics result from $\alpha_j$ with even j and vanish if the system has odd symmetry, i.e., differential circuits.
2. For large *A*, the *n*th harmonic grows approximately in proportion to $A^n$.

# Effects of Nonlinearity -- Gain Compression

$$y(t) = \frac{\alpha_2 A^2}{2} + (\alpha_1 A + \frac{3\alpha_3 A^3}{4})\cos\omega t + \frac{\alpha_2 A^2}{2}\cos 2\omega t + \frac{\alpha_3 A^3}{4}\cos 3\omega t \quad (3.2)$$

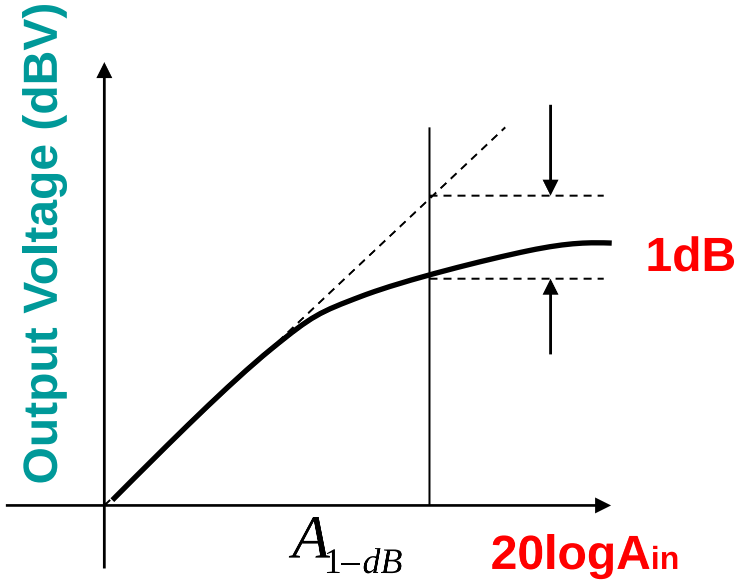
- **Under small-signal assumption, the system is normally linear and harmonics are negligible. Thus, $\alpha_1 A$ dominates → small-signal gain = $\alpha_1$.**
- **For large signal, nonlinearity becomes evident. large-signal gain = $\alpha_1 + 3\alpha_3 A^3/4$ . The gain varies when input level changes.**
- **If $\alpha_3 < 0$, the output is a "compressive" or "saturating" function of the input → the gain is compressed when A increases.**

# Gain Compression– Sign of *α₁, α₃*




- **Most RF circuit of interest are compressive, we focus on this type.**

# Effects of Nonlinearity – 1dB Compression Point




$$20\log\left|\frac{\alpha_1 A_{1-dB}}{\alpha_1 A_{1-dB} + \frac{3}{4}\alpha_3 A_{1-dB}^{\ 3}}\right| = 1dB$$

$$A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|} = 0.3808\sqrt{\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$dBm = 10\log\frac{V_{pp}^{\ 2}/8}{50\Omega \times 1mW}$$

- **1-dB compression point is defined as the input signal level that causes small-signal gain to drop 1 dB. It's a measure of the maximum input range.**

- **1-dB compression point occurs around -20 to -25 dBm (63.2 to 35.6mVpp in a 50-Ω system) in typical frond-end RF amplifiers.**

# Effects of Nonlinearity – Desensitization (Blocking)

• **Desensitization -- small signal experiences a vanishingly small gain when co-exists with a large signal, even if the small signal itself does not drive the system into nonlinear range.**

**Applying two-tone inputs *$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2 t$* to Eq.(3.1), we have**

$$y(t) = \left( \alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2 \right) \cos\omega_1 t + \cdots,$$

gain of desired signal

**For *$A_1 \ll A_2$*, it reduces to**

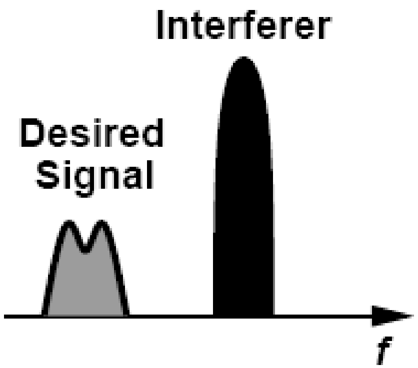
$$y(t) = \left( \alpha_1 + \frac{3}{2}\alpha_3 A_2^2 \right) A_1 \cos\omega_1 t + \cdots,$$

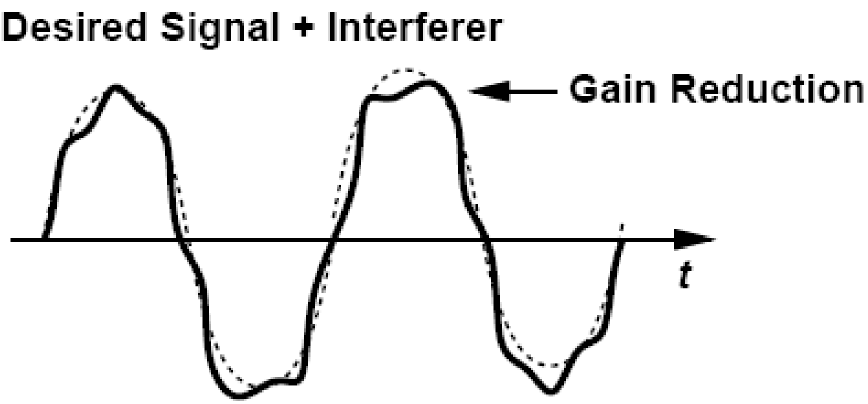
Observations:

• **Weak signal's gain decreases as a function of $A_2$ if $\alpha_3 < 0$. For sufficiently large $A_2$, the gain drops to zero → the weak signal is "blocked" by the strong signal. (the same reason that we cannot see stars during day)**

• **Many RF receivers must be able to withstand blocking signals 60 to 70 dB greater than the wanted signals.**

# Gain Compression: Desensitization




$$y(t) = \left(\alpha_1 + \frac{3}{4}\alpha_3 A_1^2 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots.$$

For $A_1 << A_2$

$$y(t) = \left(\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\right) A_1 \cos\omega_1 t + \cdots$$

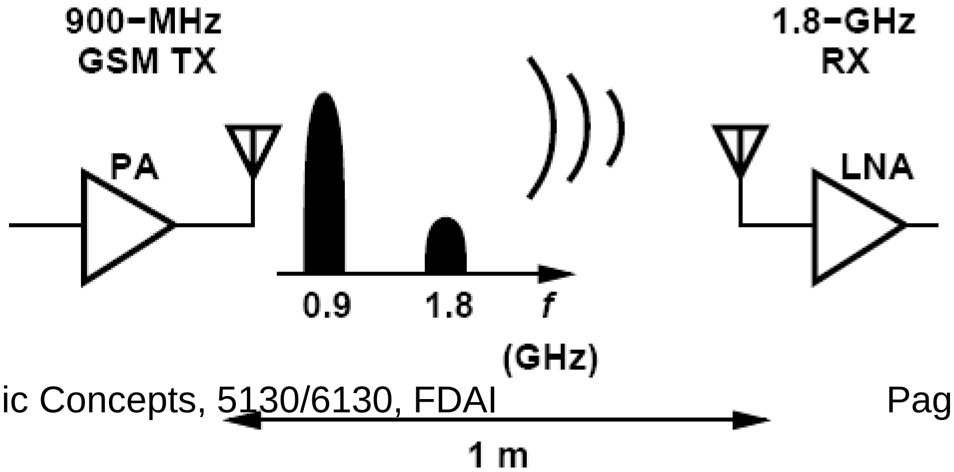
- **Desensitization: the receiver gain is reduced by the large excursions produced by the interferer even though the desired signal itself is small.**

# Example of Gain Compression

**A 900-MHz GSM transmitter delivers a power of 1 W to the antenna. By how much must the second harmonic of the signal be suppressed (filtered) so that it does not desensitize a 1.8-GHz receiver having $P_{1dB}$ = -25 dBm? Assume the receiver is 1 m away and the 1.8-GHz signal is attenuated by 10 dB as it propagates across this distance.**

***Solution:***

**The output power at 900 MHz is equal to +30 dBm. With an attenuation of 10 dB, the second harmonic must not exceed -15 dBm at the transmitter antenna so that it is below $P_{1dB}$ of the receiver. Thus, the second harmonic must remain at least 45 dB below the fundamental at the TX output. In practice, this interference must be another several dB lower to ensure the RX does not compress.**



## Effects of Nonlinearity – Intermodulation

- Harmonic distortion is due to self-mixing of a single-tone signal. It can be suppressed by low-pass filtering the higher order harmonics.

- However, there is another type of nonlinearity -- intermodulation (IM) distortion, which is normally determined by a "two tone test".

- When two signals with different frequencies applied to a nonlinear system, the output in general exhibits some components that are not harmonics of the input frequencies. This phenomenon arises from cross-mixing (multiplication) of the two signals.

# Effects of Nonlinearity – Intermodulation

- assume $x(t) = A_1\cos\omega_1 t + A_2\cos\omega_2 t$ → two tone test

$$y(t) = \alpha_1(A_1\cos\omega_1 t + A_2\cos\omega_2 t) + \alpha_2(A_1\cos\omega_1 t + A_2\cos\omega_2 t)^2$$
$$+\alpha_3(A_1\cos\omega_1 t + A_2\cos\omega_2 t)^3$$

- Expanding the right side and disregarding dc terms and harmonics, we obtain the following intermodulation products:

$$\omega = \omega_1 \pm \omega_2 : \alpha_2 A_1 A_2 \cos(\omega_1 + \omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2)t$$
$$= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t$$
$$= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_2^2 A_1}{4}\cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_2^2 A_1}{4}\cos(2\omega_2 - \omega_1)t$$

- And these fundamental components:

$$\omega = \omega_1, \omega_2 : \left(\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2\right)\cos\omega_1 t + \left(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2\right)\cos\omega_2 t$$

# Effects of Nonlinearity – Intermodulation

$$x(t) = A_1 \cos\omega_1 t + A_2 \cos\omega_2$$

$$y(t) = \frac{1}{2}\alpha_2 (A_1{}^2 + A_2{}^2) +$$

**DC Term**

$$\left[\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1 (A_1^2 + 2A_2^2)\right]\cos\omega_1 t +$$

$$\left[\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2 (2A_1^2 + A_2^2)\right]\cos\omega_2 t +$$

**1$^{st}$ Order Terms**

$$\frac{1}{2}\alpha_2 \left[A_1{}^2 \cos 2\omega_1 t + A_2{}^2 \cos 2\omega_2 t\right] +$$

$$\alpha_2 A_1 A_2 \left[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t\right] +$$

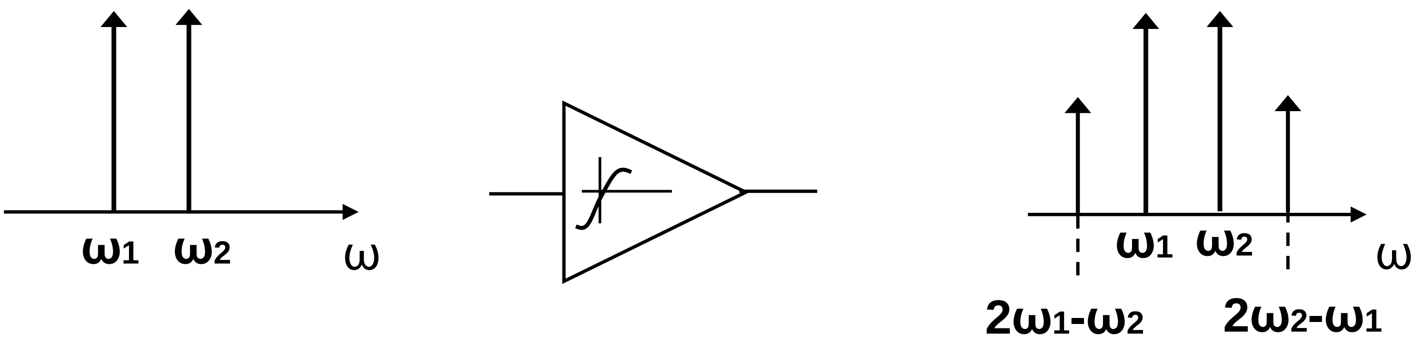
**2$^{nd}$ Order Terms**

$$\frac{1}{4}\alpha_3 \left[A_1{}^3 \cos 3\omega_1 t + A_2{}^3 \cos 3\omega_2 t\right] +$$

$$\frac{3}{4}\alpha_3 \left\{ \begin{array}{l} A_1{}^2 A_2 \left[\cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t\right] + \\ A_1 A_2{}^2 \left[\cos(2\omega_2 + \omega_1)t + \cos(2\omega_2 - \omega_1)t\right] \end{array} \right\}$$

**3$^{rd}$ Order terms**

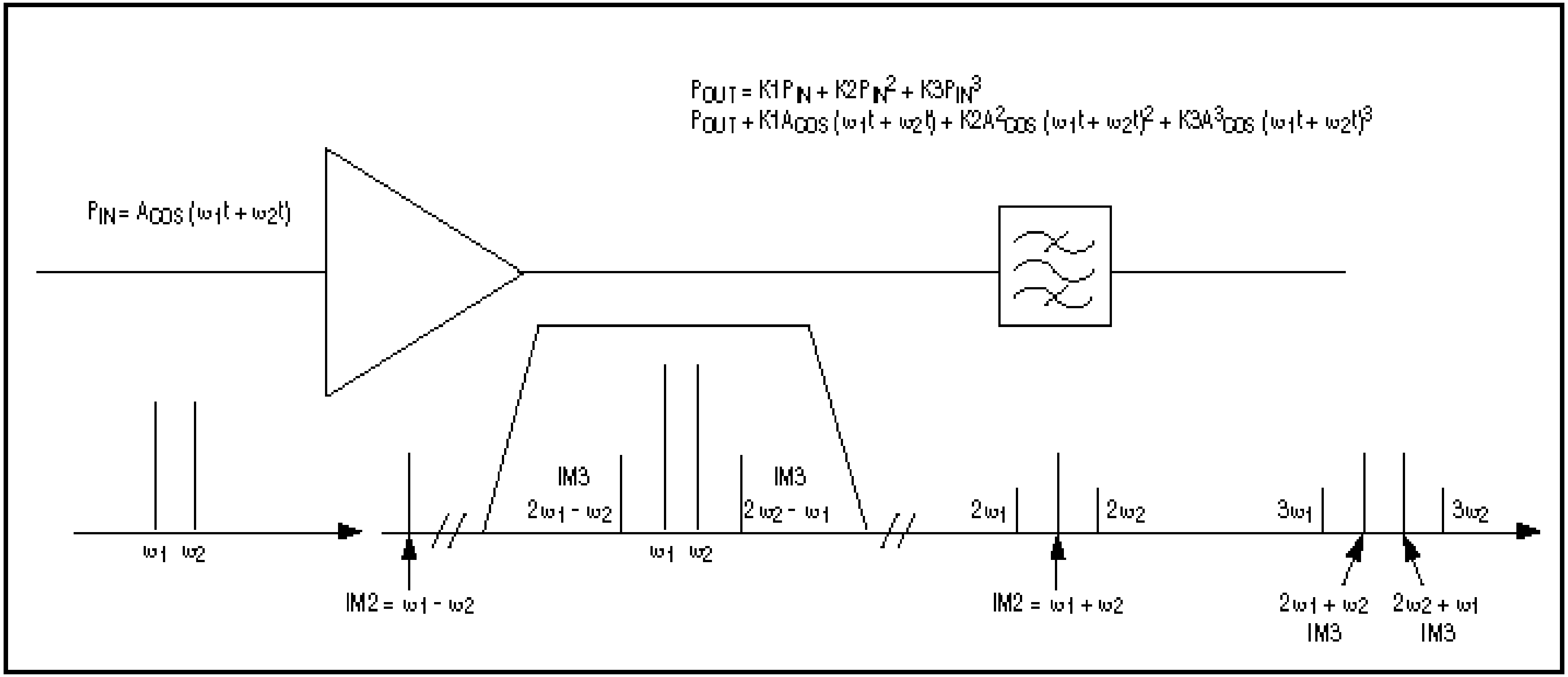
# Effects of Nonlinearity – Intermodulation



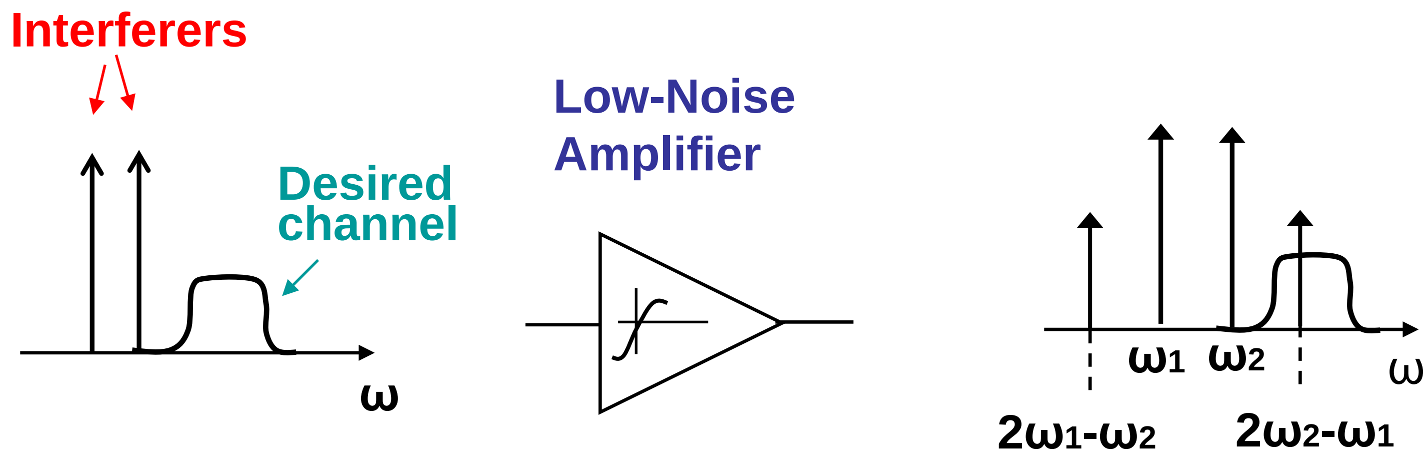

- **Of particular interest are the third-order IM products at $2\omega_1-\omega_2$ and $2\omega_2-\omega_1$. The key point here is that if the difference between $\omega_1$ and $\omega_2$ is small, $2\omega_1-\omega_2$ and $2\omega_2-\omega_1$ appear in the vicinity of $\omega_1$ and $\omega_2$.**

# IM3 falls in the vicinity of foundamentals

:In wireless communication system such as cellular handsets with narrow-band operating frequencies (i.e., a few tens of MHz), only the IM3 spurious signals (2w1 - w2) and (2w2 - w1) fall within the filter passband.



# Effects of Nonlinearity – Corruption of signal due to intermodulation




- If a weak signal accompanied by two strong interferes having third-order nonlinearity, one of the IM products falls in the band of interest, corrupting the desired component.

# Intermodulation -- Third Order Intercept Point (IP3)

- Two-tone test: $A_1=A_2=A$ and A is sufficiently small so that higher-order nonlinear terms are negligible, and the gain is relatively constant and equal to $\alpha_1$.
- As A increases, the fundamentals increases in proportion to A, whereas IM3 products increases in proportion to $A^3$.

$$x(t) = A\cos\omega_1 t + A\cos\omega_2$$

$$y(t) = \alpha_2 A^2 +$$

$$A\left[\alpha_1 + \frac{9}{4}\alpha_3 A^2\right]\cos\omega_1 t + A\left[\alpha_1 + \frac{9}{4}\alpha_3 A^2\right]\cos\omega_2 t +$$

$$\frac{1}{2}\alpha_2 A^2\left[\cos 2\omega_1 t + \cos 2\omega_2 t\right] + \alpha_2 A^2\left[\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t\right] +$$

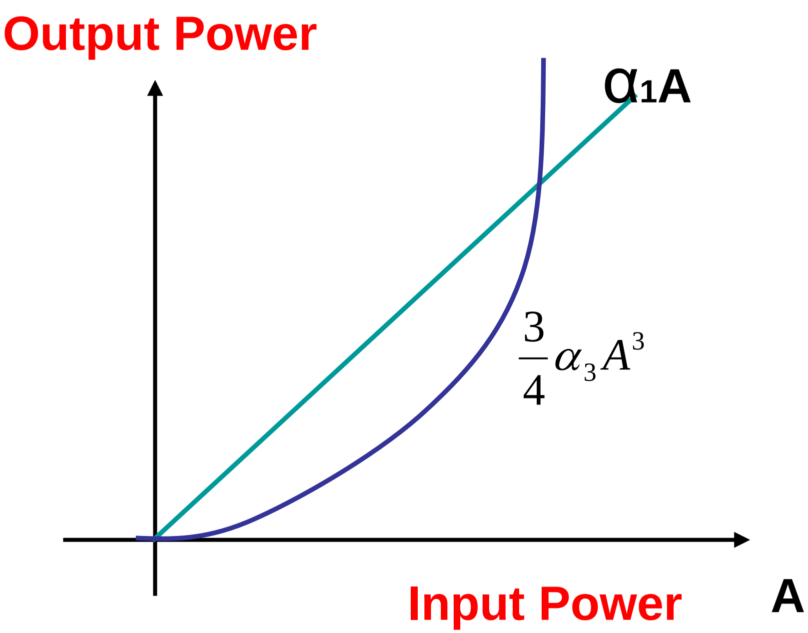
$$\frac{1}{4}\alpha_3 A^3\left[\cos 3\omega_1 t + \cos 3\omega_2 t\right] + \frac{3}{4}\alpha_3 A^3\begin{Bmatrix}\left[\cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t\right] + \\ \left[\cos(2\omega_1 + \omega_2)t + \cos(2\omega_1 - \omega_2)t\right]\end{Bmatrix}$$

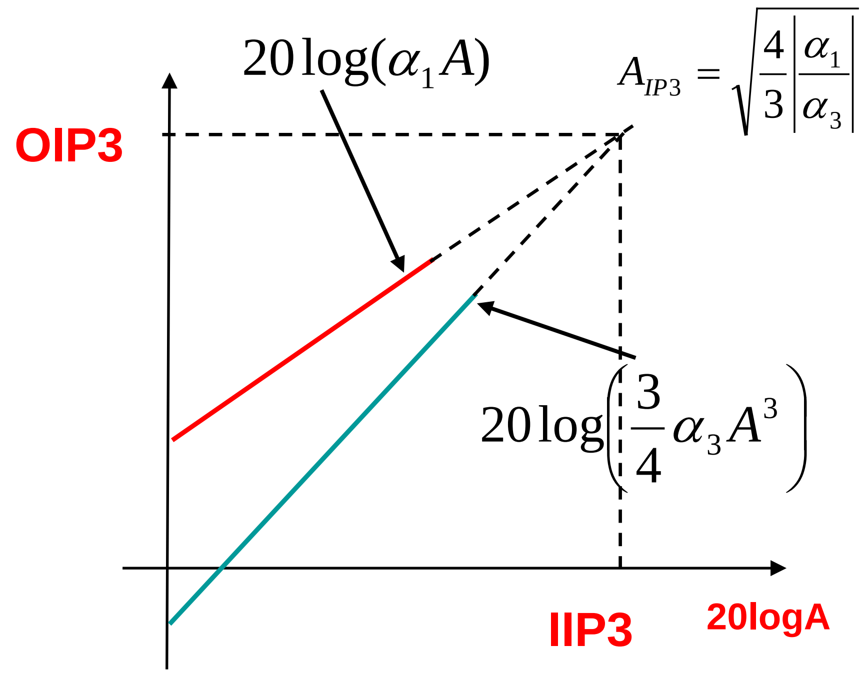
$$\alpha_1 >> \frac{9}{4}\alpha_3 A^2 \rightarrow IIP_3 = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} \text{ and } OIP_3 = \alpha_1 IIP_3$$

$$\frac{A_{1dB}}{A_{IP3}} = \sqrt{\frac{0.145}{4/3}} \approx -9.6dB$$

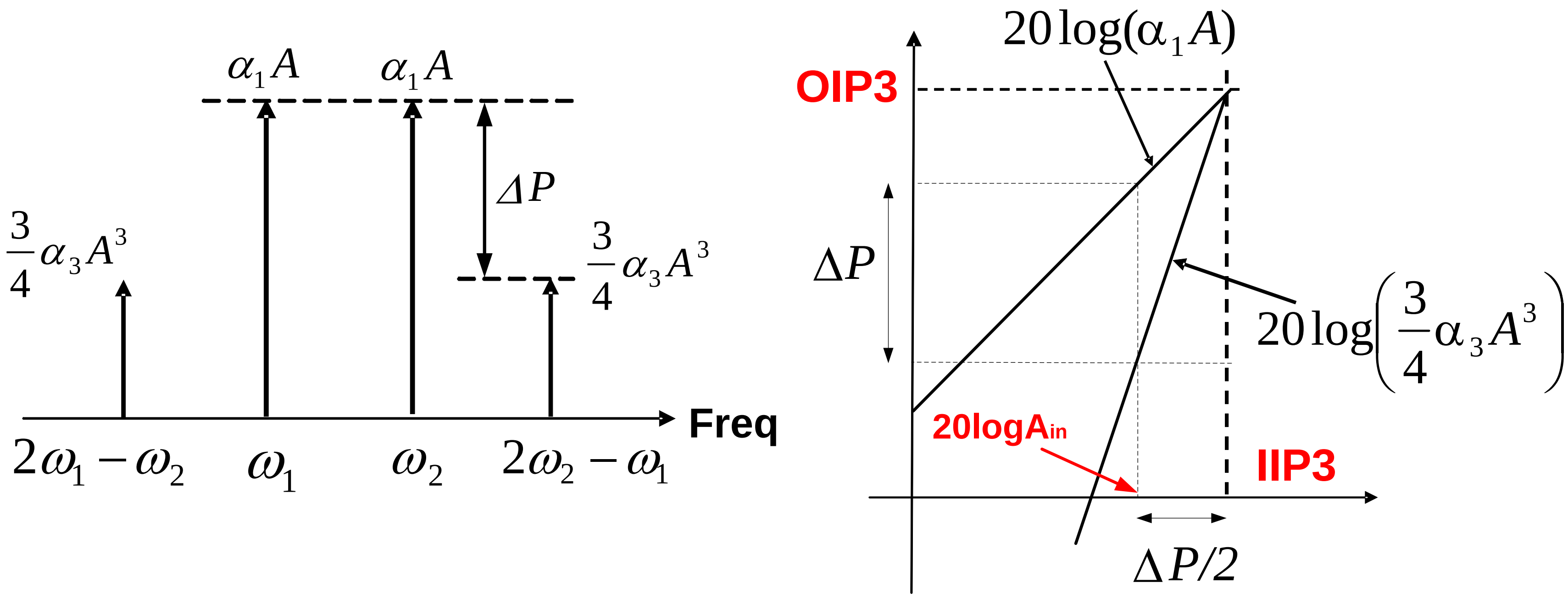
# Intermodulation -- Third Order Intercept Point (IP3)

- Plotted on a log scale, the intersection of the two lines is defined as the third order intercept point. The horizontal coordinate of this point is called the input referred $IP_3(IIP_3)$, and the vertical coordinate is called the output referred $IP_3(OIP_3)$.



# Calculate IIP3 without Extrapolation




$$IIP_3[dBm] = \frac{\Delta P[dB]}{2} + P_{in}[dBm]$$

# Relationship Between 1-dB Compression and IP3

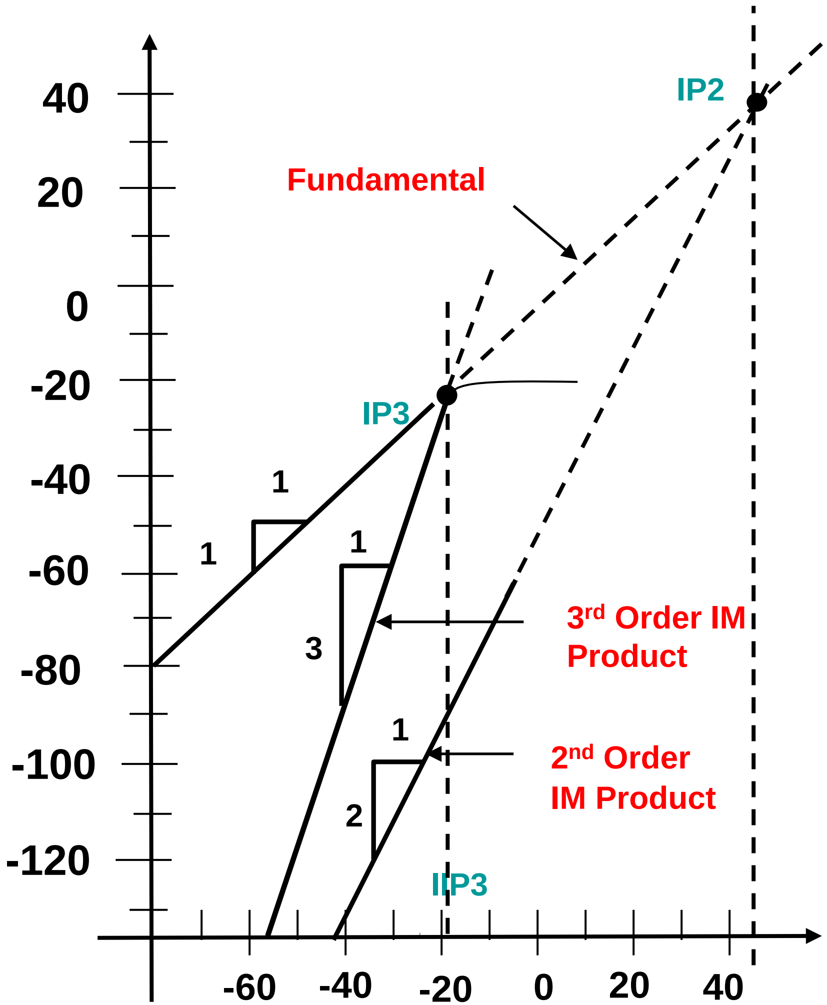
1-dB compression point with single tone applied:

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|} \qquad A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|} \qquad \frac{A_{IP3}}{A_{1dB}} = 3.04 = 9.66dB$$

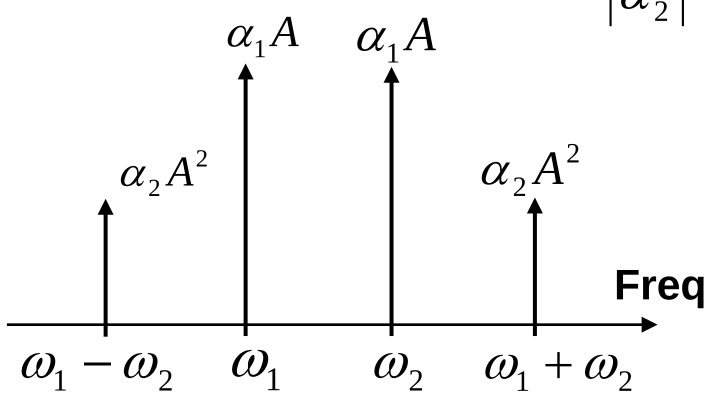
1-dB compression point with two tones applied:

$$\frac{A_{IP3}}{A_{1dB}} = \frac{2\sqrt{\frac{\alpha_1}{3\alpha_3}}}{0.22\sqrt{\frac{\alpha_1}{\alpha_3}}} = 5.25 = 14.4dB$$

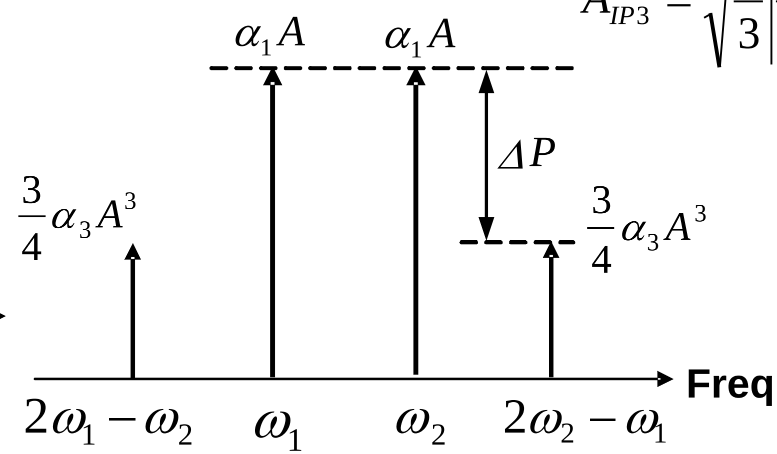
# Intermodulation – IP2 vs. IP3




- $A_1=A_2=A$
- 2ND Order IM Product: $A_{IP2} = \left|\frac{\alpha_1}{\alpha_2}\right|$




- 3RD Order IM Product $A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$

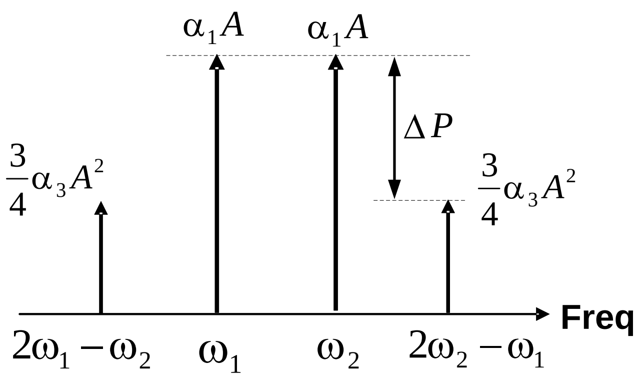


# Determine IIP3 and 1-dB Compression Point from Measurement

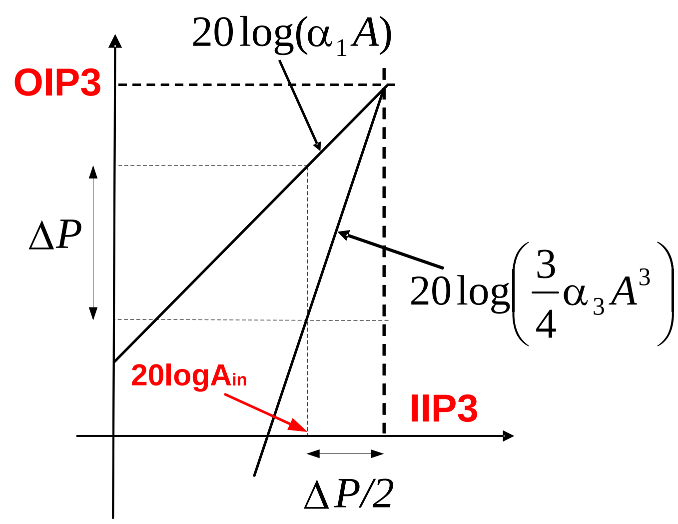
- An amplifier operates at 2 GHz with a gain of 10dB. Two-tone test with equal power applied at the input, one is at 2.01 GHz. At the output, four tones are observed at 1.99, 2.0, 2.01, and 2.02GHz. The power levels of the tones are -70,-20,-20, and -70dBm. Determine the IIP3 and 1-dB compression point for this amplifier.
- Solution: 1.99 and 2.02 GHz are the IP3 tones.

$$IIP3 = (P_1 - G) + \frac{1}{2}[P_1 - P_3] = -20 - 10 + \frac{1}{2}[-20 + 70] = -5dBm$$

$$P_{1dB} = -5 - 9.66 = -14.66dBm$$

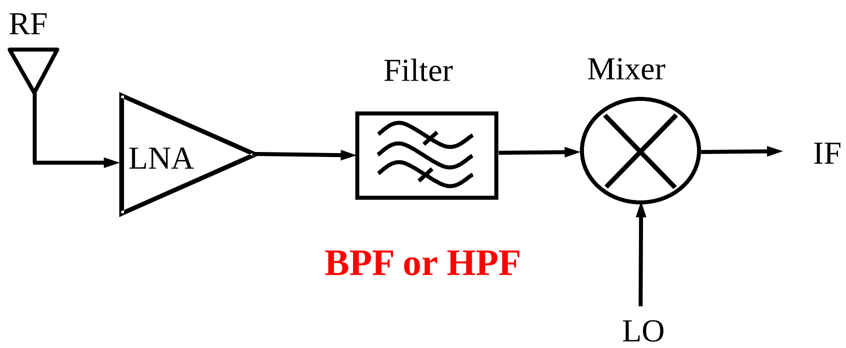


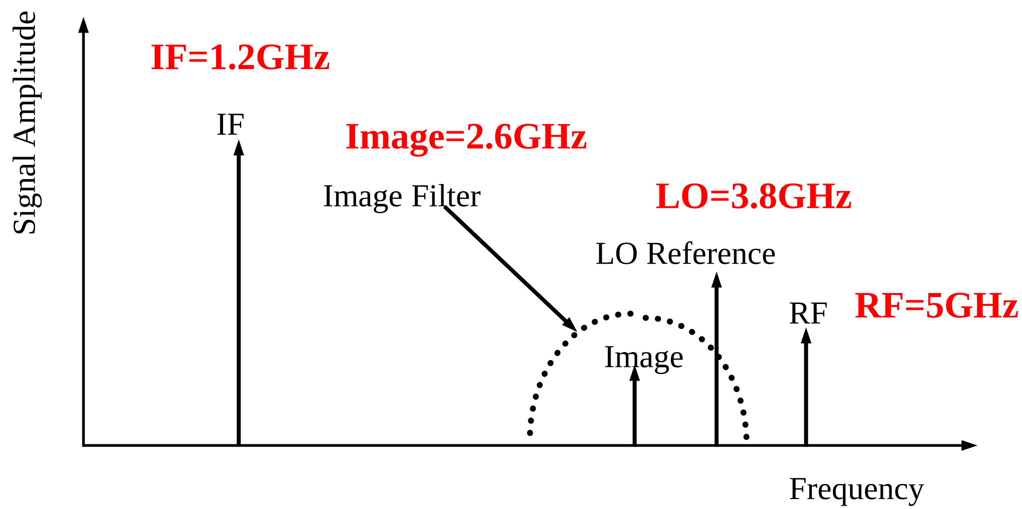

$$IIP_3[dBm] = \frac{\Delta P[dB]}{2} + P_{in}[dBm]$$



# Image Signals and Image Reject Filters

- Superheterodyne receiver front end with an LNA, an image filter, a mixer:



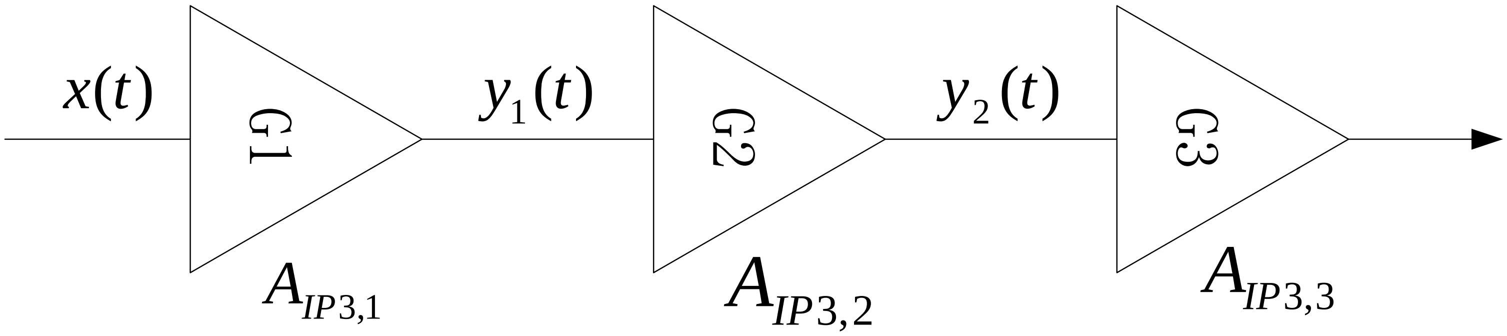



# Intermodulation of Cascade Nonlinear Stages

$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) + \ldots$$

$$y_2(t) = \beta_1 y_1(t) + \beta_2 y_1{}^2(t) + \beta_3 y_1{}^3(t) + \ldots$$

$$\ldots$$

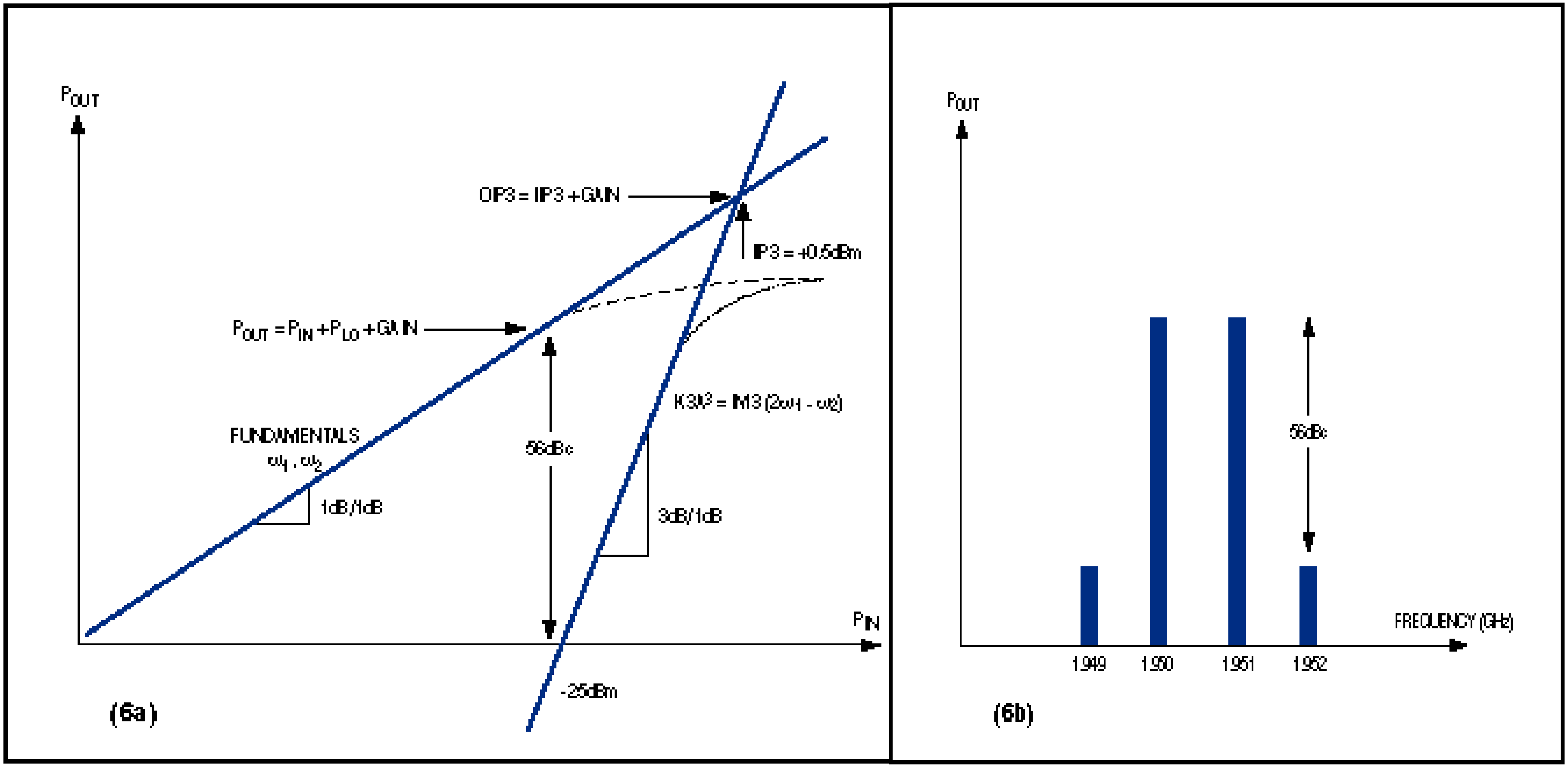



**Linearity is more important for back-end stages.**

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2 \beta_1^2}{A_{IP3,3}^2} + \ldots$$

# Design Example: SiGe Double-Balanced Downconverter

**IP3 is characterized with a -25dBm signal consisting of two tones at 1950MHz and 1951MHz. The typical operating conditions: $PRF_{IN}$ = -25dBm, IIP3 = 0.5dBm, and conversion gain = 8.4dB, current consumption = 8.7mA. P1dB=? Back off=?**



# CHAPTER 2

## Basic Concepts in IC Designs

I. Device Review
II. Linearity Analysis
III. Noise Analysis
IV. System Analysis

# Noise Figure

- **In RF design, most of the front-end receiver blocks are characterized in terms of "noise figure" rather than input referred noise.**
- **Noise factor F is defined as**

$$\mathrm{F} = \frac{SNR_{in}}{SNR_{out}} = \frac{S_i/N_i}{S_o/N_o} = \frac{N_o}{GN_i} = \frac{N_{o(total)}}{N_{o(source)}} = \frac{N_{o(source)} + N_{o(added)}}{N_{o(source)}} = 1 + \frac{N_{o(added)}}{N_{o(source)}}$$

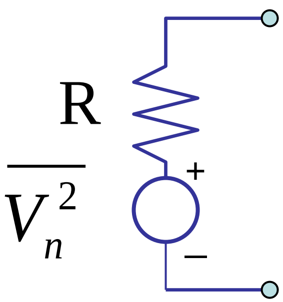
$$\text{Noise Figure, NF} = 10\log_{10} F$$

- **Noise figure measures how much the SNR degrades as the signal passes through a system.**
- **For a noiseless system, SNRin = SNRout, namely, F=1, NF=0dB, regardless of the gain. This is because both the input signal and the input noise are amplified (or attenuated) by the same factor and no additional noise is introduced.**

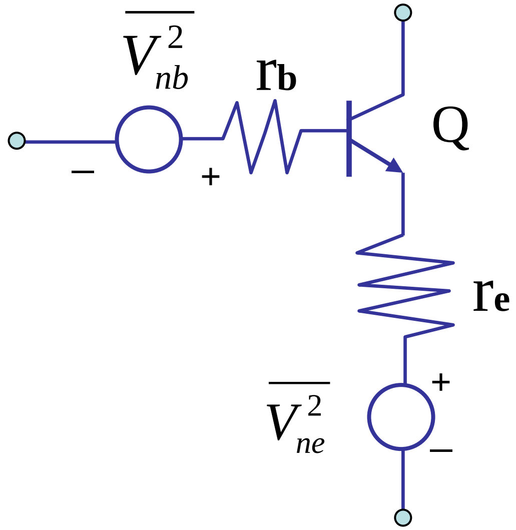
# Thermal Noise

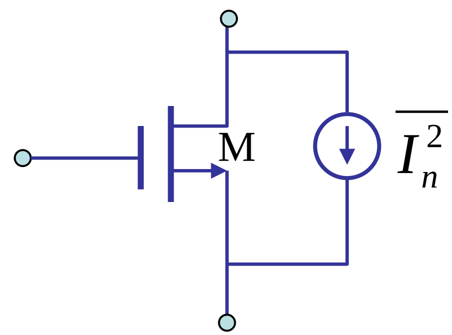
- **Thermal noise (Johnson noise) – due to random thermal motion of electrons and is generated by resistors, base and emitter resistance $r_b$, $r_E$, and $r_c$. of bipolar devices, and channel resistance of MOSFETs. Thermal noise is a white noise with Gaussian amplitude distribution.**

- **Thermal noise floor:** $$10\log\left(\frac{kT}{1mW}\right) = -174 dBm/Hz \text{ at } 290^0 K$$




$$\overline{V_n^2} = 4kTR\Delta f$$



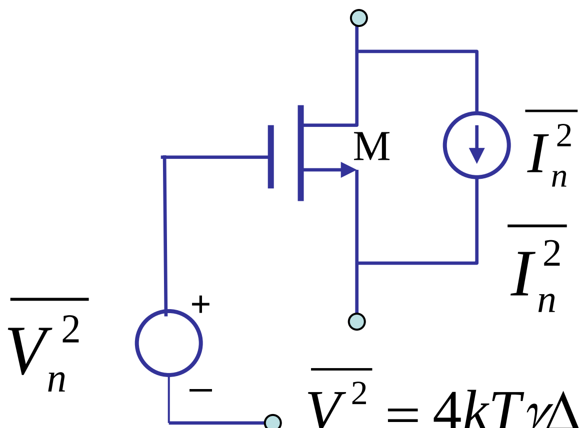




$$\overline{I_n^2} = 4kT\left(\frac{2}{3}g_m\right)\Delta f$$

>2/3 for submirocn MOS

# Thermal Noise of MOSFET

Channel thermal noise of a MOSFET can be modeled as a noise current source at the output given by 4kTγgm. This current noise source can be referred to the gate as a voltage noise source given by 4kTγ/gm.

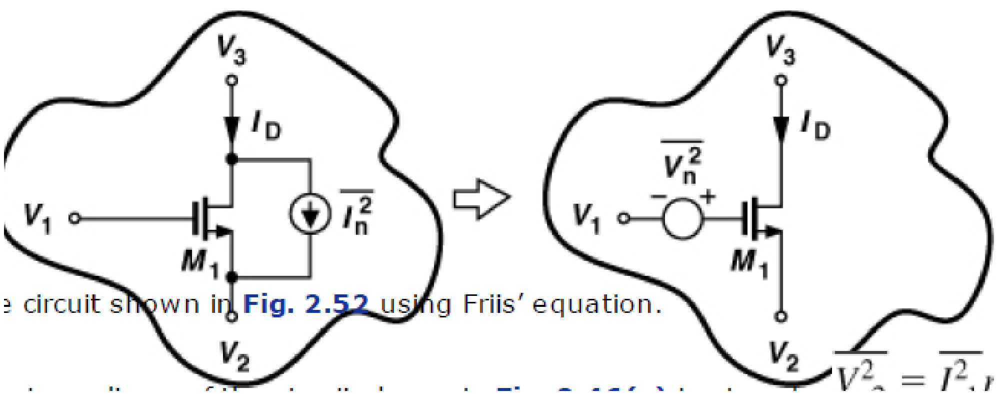



$$\overline{I_n^2} = 4kT\gamma g_m \Delta f$$

$$\overline{V_n^2} = 4kT\gamma \Delta f \, / \, g_m$$

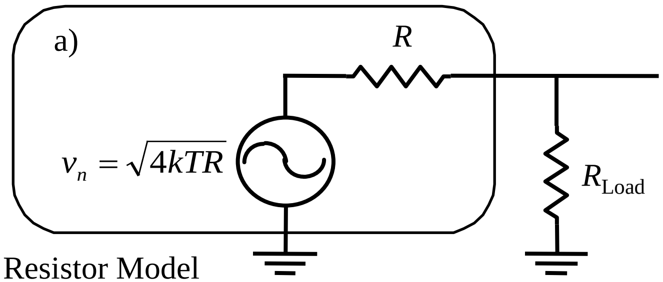
$$I_D = V_{in} \cdot g_m = \sqrt{4kT\gamma \cdot g_m}$$

$$\overline{V_n^2} = \frac{4kT\gamma}{g_m}$$

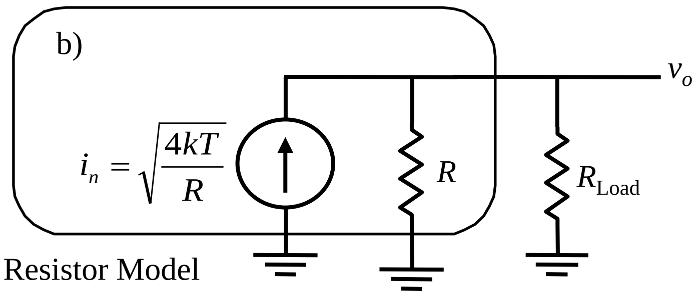


# Thermal Noise Floor

- Thermal noise power spectral density (PSD): $N_{resistor} = 4kTR$
- The rms noise voltage $v_n$ in the bandwidth $\Delta f$ : $v_n^2 = 4kTR\Delta f$
- The equivalent noise current: $i_n^2 = \dfrac{4kT\Delta f}{R}$
- For power matching $R_{LOAD} = R$, $v_0 = v_n/2$,
  Output power spectral density: $P_0 = \dfrac{v_0^2}{R} = \dfrac{v_n^2}{4R} = kT$




Resistor Model




Resistor Model

- The noise from an antenna (R) under matching condition at T=290K:

$$P_{available} = kT = 4\times10^{-21} W / Hz$$
$$= -174 dBm / Hz$$

**$k$=1.38 × 10$^{-23}$ J/K**

Independent of resistor value !!

## Thermal Noise Dominated Receiver Sensitivity

- Thermal noise floor for 200kHz BW:

$$-174dBm/Hz + 10\log_{10}(200k) = -121dBm$$

- signal-to-noise ratio:

$$SNR = \frac{S}{Noise \quad floor}$$

- For BW=200kHz and BER=$10^{-3}$, SNR and receiver sensitivity required by various modulations are given by

- QPSK, SNR=7dB, receiver sensitivity= -114dBm
- 16QAM, SNR=12dB, receiver sensitivity= -109dBm
- 64QAM, SNR=17dB, receiver sensitivity= -104dBm

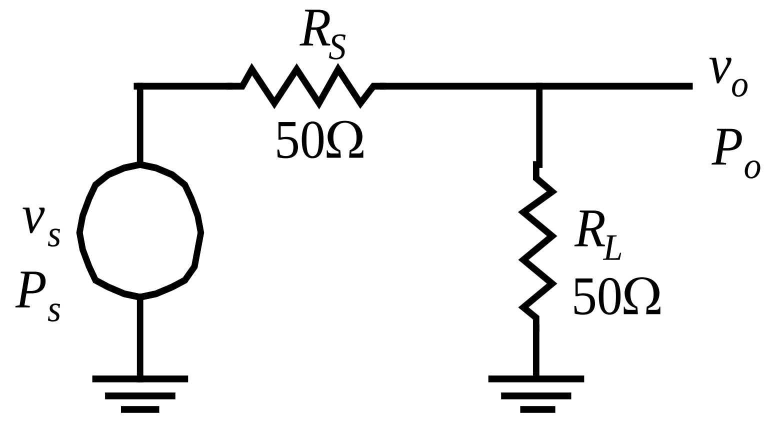
# Thermal Noise Calculation

rms thermal noise voltage from a 50Ω source is

$$\sqrt{4kTR} \approx 0.9nV/\sqrt{Hz} \qquad\qquad v_0 = 0.45nV/\sqrt{Hz}$$

For maximum power transfer, $R_L=R_S=50$ Ω,

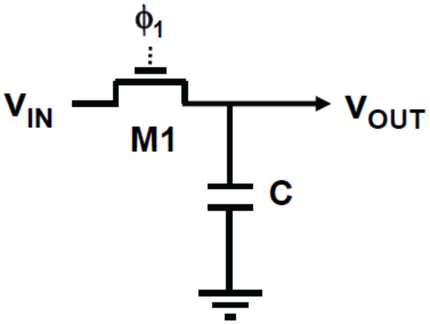
$$P_{in(available)} = \frac{v_0^2}{4R_L} = \frac{4kTR_S}{4R_L} = kT = 4\times10^{-21}$$



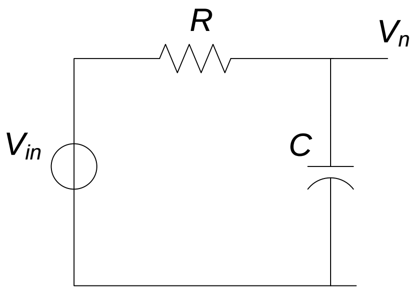

- $R_L$ is noiseless, F=1, NF=0dB
- $R_L$ is noisy, $N_{o(total)}$=2kT, F=2, NF=3dB
- Total output noise voltage (uncorrelated)

$$v_0 = \sqrt{2}0.45nV/\sqrt{Hz} = 0.636nV/\sqrt{Hz}$$

# Sample-and-Hold Thermal (kT/C) Noise




Thermal noise
PSD $S_r = 4kTR$




Sampled capacitor circuit with the switch modeled as an on-resistance R

**LPF Transfer Function:**

$$V_n(s) = V_{in}(s)\frac{1}{1 + sRC}$$

**Output Noise Power Spectral Density:**

$$S_n(f) = S_r(f)\left|\frac{1}{1 + j2\pi fRC}\right|^2$$

**Output noise power:**

$$v_n^2 = 4kTR\int_0^{\infty}\frac{1}{1 + (2\pi fRC)^2}df$$

$$= \frac{2kT}{\pi C}\int_0^{\frac{\pi}{2}}d\theta = \frac{kT}{C}$$

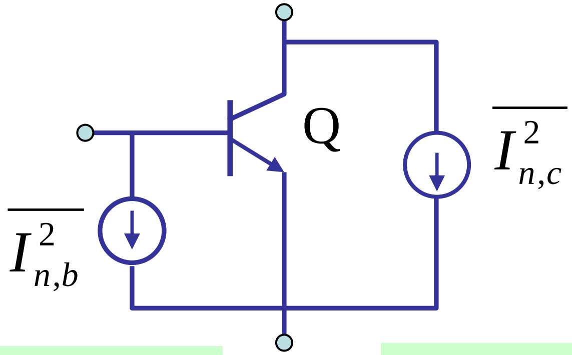
- Sampler thermal noise $kT/C$ places a fundamental limit on holding capacitor size.
- kT/C noise is independent of switch on-resistance R and is inversely proportional to C.
- To achieve fine resolution, capacitor value needs to be sufficiently large → trade-off with power and speed.

$k = 1.38 \times 10^{-23}$ J/K

# Shot Noise

- **Shot noise (Schottky noise) – due to the particle-like nature of charge carriers. Only the time-average flow of electrons and holes appears as constant current. Any fluctuation in the number of charge carriers produces a random noise current at that instant. Shot noise is a Gaussian white process associated with the transfer of charge across an energy barrier (e.g., a p-n junction). This random process is called shot noise and is expressed in amperes per root hertz.**

$$\overline{I_n^2} = 2qI\Delta f$$




$$i_{bn} = \sqrt{2qI_B}$$

$$i_{cn} = \sqrt{2qI_C}$$

# Base Shot Noise

- Base shot noise can be related to thermal noise in the resistor $r_\pi$ (but is off by a factor of 2) → the diffusion resistance is generating noise half thermally.
- Resistors under thermal equilibrium generates noise voltage of $\sqrt{4kTR}$
- Conducting PN junction is active with power added, not under thermal equilibrium.

$$v_{bn} = i_{bn} \cdot r_\pi = \sqrt{2qI_B} \cdot r_\pi = \sqrt{2q\frac{I_C}{\beta}} \cdot r_\pi = \sqrt{2q\frac{I_C}{g_m r_\pi}} \cdot r_\pi$$

$$= \sqrt{2q\frac{I_C}{\frac{I_C q}{kT} r_\pi}} \cdot r_\pi = \sqrt{2kTr_\pi}$$

# Flicker noise

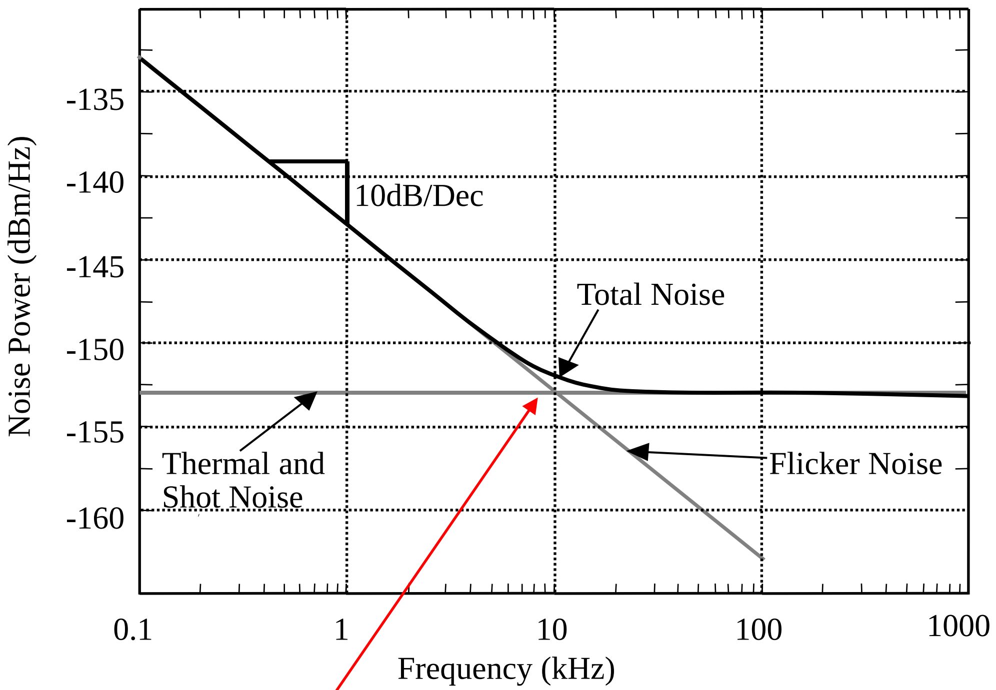
- **Flicker noise (1/f noise) – found in all active devices. In bipolar transistors, it is caused by traps associated with contamination and crystal defects in the emitter-base depletion layer. These traps capture and release carriers in a random fashion with noise energy concentrated in low frequency. K depends on processing and may vary by order of magnitude.**

$$\overline{I_n^2} = K \frac{I_C{}^a}{f} \Delta f, \ a \approx 0.5 \sim 2$$

- **In MOSFETs, 1/f noise arises from random trapping of charge at the oxide-silicon interfaces. Represented as a voltage source in series with the gate, the noise spectral density is given by**

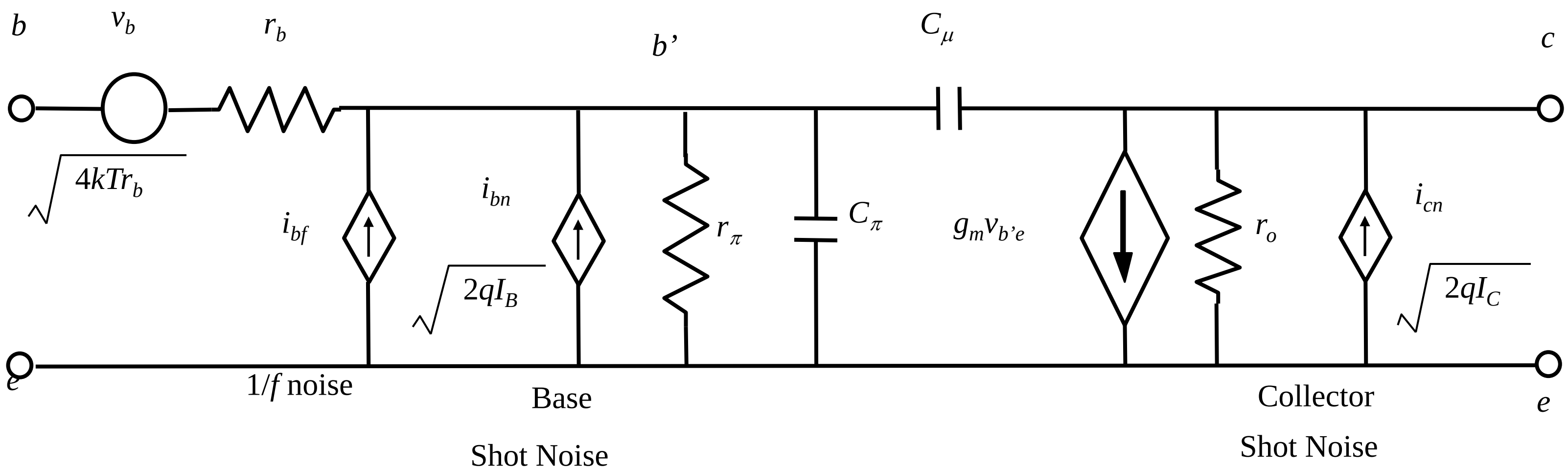
$$\overline{V_n^2} = \frac{K}{WLCox} \frac{1}{f}$$

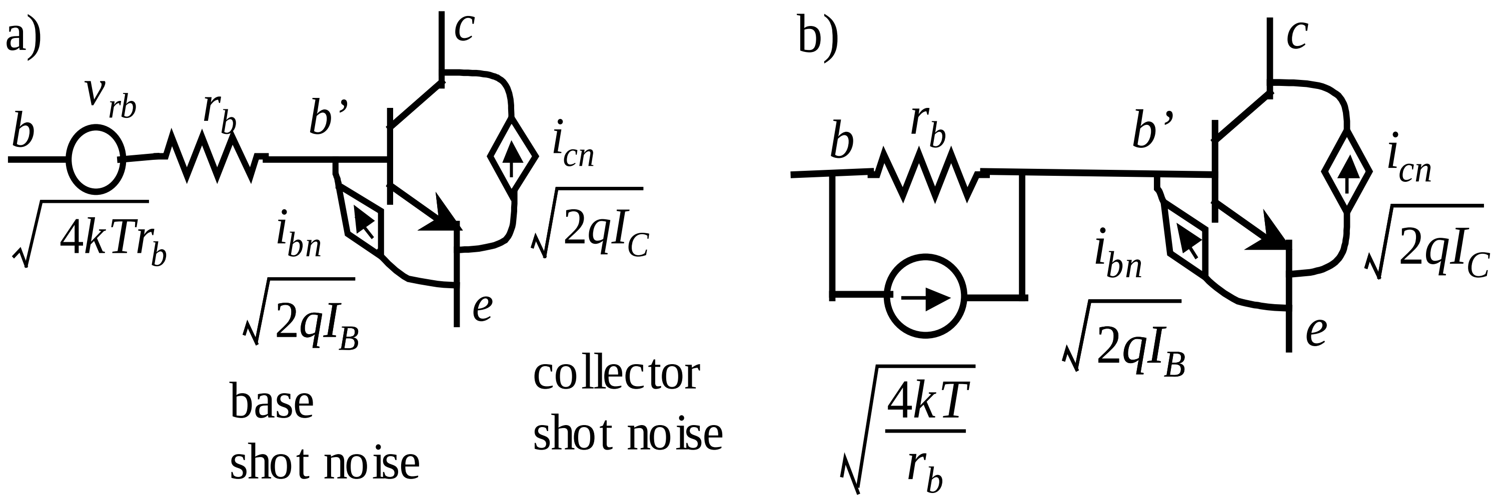
# Noise Power Spectral Density



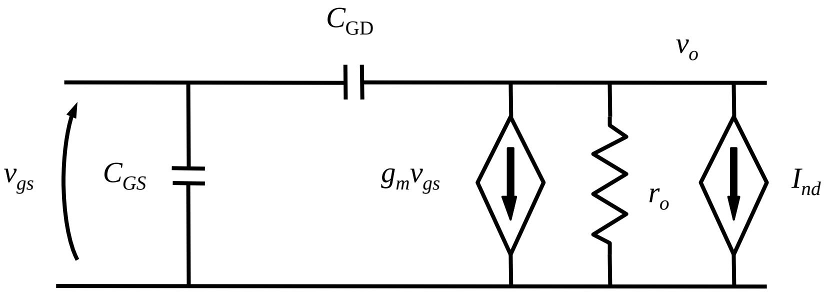

The 1/f corner frequency can be significantly higher for MOSFET

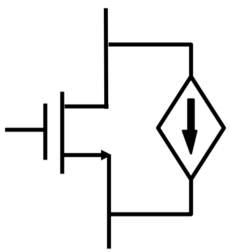
# BJT Model with Noise Sources



# CMOS Model with Noise Sources



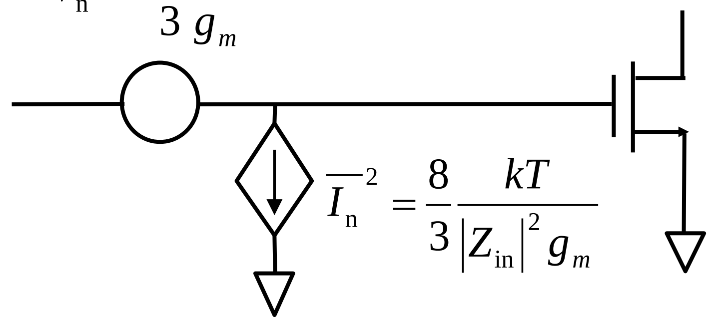
$$\overline{I_{\mathrm{nd}}}^2 = 4kT\left(\frac{2}{3}g_m\right)$$

- Gate resistance can be added to the noise model with gate resistivity ρ

$$R_{GATE} = \frac{1}{3}\rho\frac{W}{L}$$

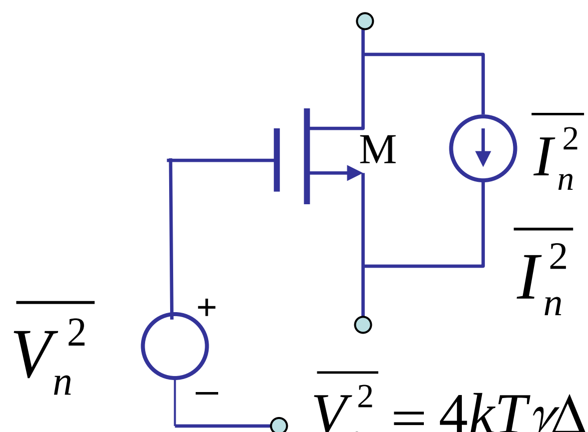
Input-referred noise

$$\overline{V_{\mathrm{n}}}^2 = \frac{8}{3}\frac{kT}{g_m}$$



$$\overline{I_{\mathrm{n}}}^2 = \frac{8}{3}\frac{kT}{\left|Z_{\mathrm{in}}\right|^2 g_m}$$

Use either $V_n$ or $I_n$ for MOS thermal noise:

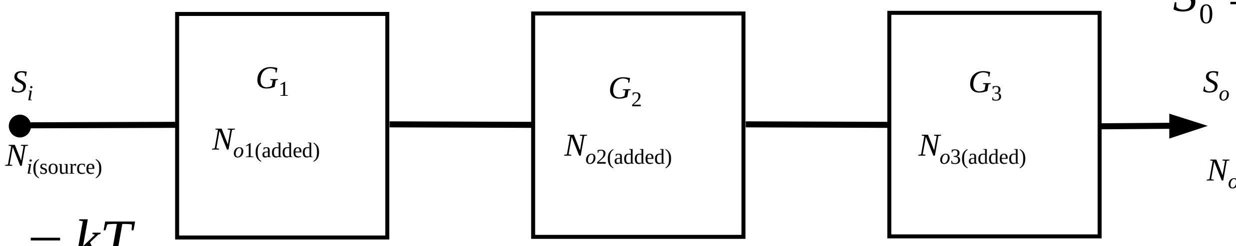



$$\overline{I_n^2} = 4kT\gamma g_m \Delta f$$

$$\overline{V_n^2} = 4kT\gamma\Delta f \,/\, g_m$$

# Noise Figure of Cascaded Stages

$$S_0 = S_i + G_i + G_2 + G_3$$




$$N_{i(source)} = kT$$

input thermal noise

output noise due to each stage

$$N_{o(total)} = N_{i(source)} G_1 G_2 G_3 + N_{o1(added)} G_2 G_3 + N_{o2(added)} G_3 + N_{o3(added)}$$

total output noise

output noise due to source

$$F = \frac{N_{o(total)}}{N_{o(source)}} = 1 + \frac{N_{o1(added)}}{N_{i(source)} G_1} + \frac{N_{o2(added)}}{N_{i(source)} G_1 G_2} + \frac{N_{o3(added)}}{N_{i(source)} G_1 G_2 G_3}$$

$$= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

Effective F is reduced by G of previous stages!

# Noise Figure of Cascade Stages

$$NF_{tot} = 1 + \left(NF_1 - 1\right) + \frac{NF_2 - 1}{G_{p1}} + \frac{NF_3 - 1}{G_{p1} G_{p1}} + \ldots.. + \frac{NF_m - 1}{G_{p1} G_{p2\ldots..} G_{p(m-1)}}$$

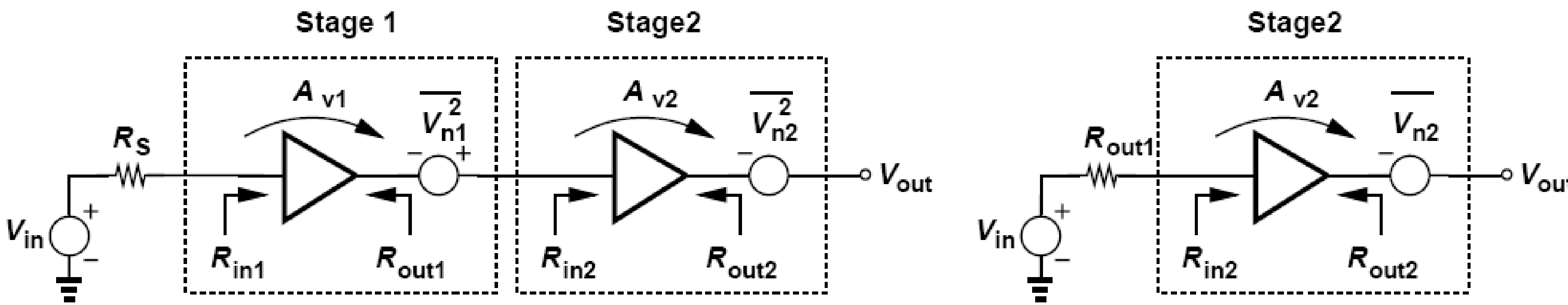
**$NF_{tot}$ – total equivalent Noise Figure**

**$NF_m$ – Noise Figure of $m^{th}$ stage**

**$G_{pm}$ – Available power gain of $m^{th}$ stage**

**Noise figure is more important for front-end stages.**

# Noise Figure of Cascaded Stages (Ⅰ)




$$A_0 = \frac{V_{out}}{V_{in}} = \frac{R_{in1}}{R_{in1}+R_S}A_{v1}\frac{R_{in2}}{R_{in2}+R_{out1}}A_{v2}$$

$$\overline{V^2_{n,out}} = \overline{V^2_{n2}} + \overline{V^2_{n1}}\frac{R^2_{in2}}{(R_{in2}+R_{out1})^2}A^2_{v2}$$

$$\mathrm{NF_{tot}} = 1 + \frac{\overline{V^2_{n,out}}}{A_0^2}\cdot\frac{1}{4kTR_S}$$

$$= 1 + \frac{\overline{V^2_{n1}}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A^2_{v1}}\cdot\frac{1}{4kTR_S}$$

$$+ \frac{\overline{V^2_{n2}}}{\left(\frac{R_{in1}}{R_{in1}+R_S}\right)^2 A^2_{v1}\left(\frac{R_{in2}}{R_{in2}+R_{out1}}\right)^2 A^2_{v2}}\cdot\frac{1}{4kTR_S}$$

$$\mathrm{F} = 1 + \frac{N_{o(added)}}{N_{o(source)}}$$

# Noise Figure of Cascaded Stages (II)

$$NF_2 = 1 + \frac{\overline{V_{n2}^2}}{\frac{R_{in2}^2}{(R_{in2}+R_{out1})^2}A_{v2}^2}\frac{1}{4kTR_{out1}} \quad NF_{tot} = NF_1 + \frac{NF_2 - 1}{\frac{R_{in1}^2}{(R_{in1}+R_S)^2}A_{v1}^2\frac{R_S}{R_{out1}}}$$

**Denominator is the "available power gain" of the 1st stage, defined as "available power" at its output, $P_{out,av}$ (the power delivered to a matched load $R_{out1}=R_{in2}$), divided by available source power, $P_{S,av}$ (the power delivered to a matched load $R_S=R_{in1}$).**

$$P_{out,av} = V_{in}^2\frac{R_{in1}^2}{(R_S+R_{in1})^2}A_{v1}^2 \cdot \frac{1}{4R_{out1}}. \qquad P_{S,av} = \frac{V_{in}^2}{4R_S}$$

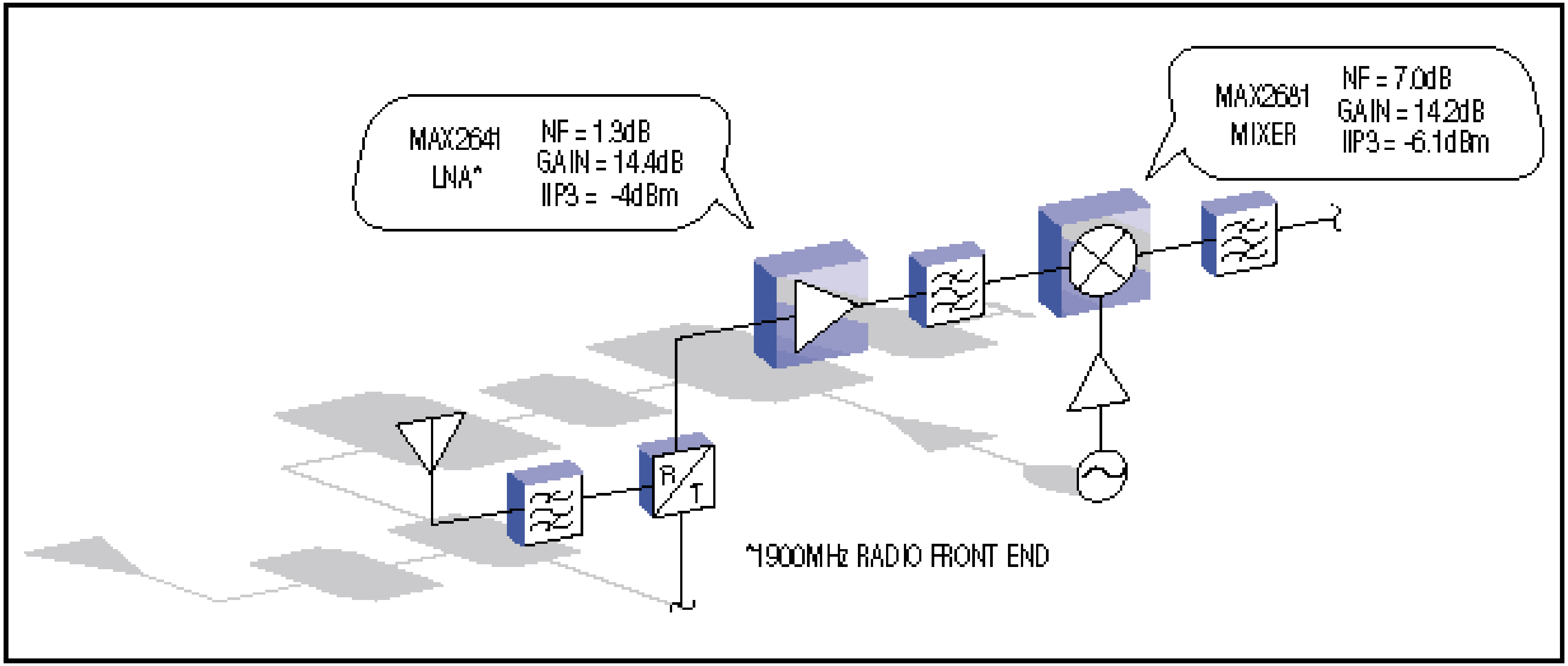
$$NF_{tot} = NF_1 + \frac{NF_2 - 1}{A_{P1}}$$

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1}\cdots A_{P(m-1)}}.$$

**Called "Friis' equation", this result suggests that the noise contributed by each stage decreases as the total available power gain preceding that stage increases, implying that the first few stages in a cascade are the most critical to the overall NF.**

# Design Example: Typical RF front end circuitry

**What is the total G, NF and IIP3 for a two-stage reiver front end with LAN and mixer?**



# CHAPTER 2

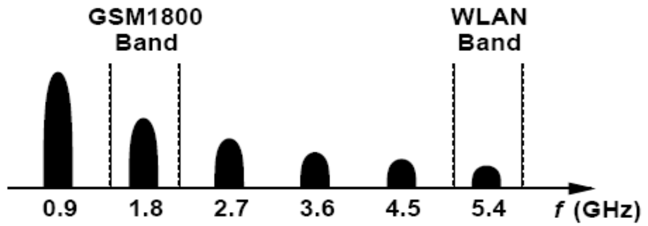
## Basic Concepts in IC Designs

I. Device Review
II. Linearity Analysis
III. Noise Analysis
IV. System Analysis

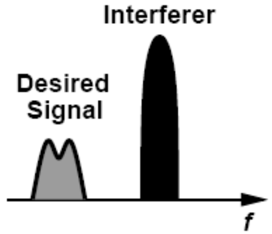
# Effects of Nonlinearity: Intermodulation—Recall Previous Discussion

**So far we have considered the case of:**

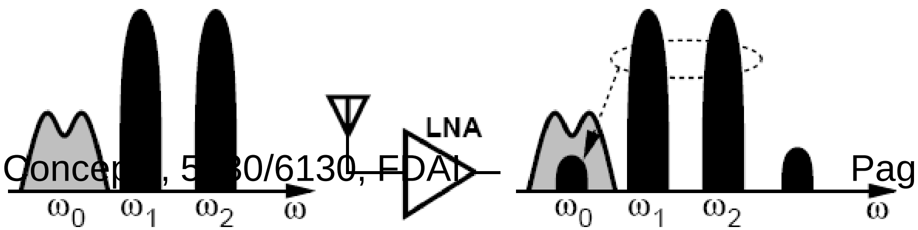
- **Single Signal** → **Harmonic distortion**




- **Signal + one large interferer** → **Desensitization**




- **Signal + two large interferers** → **Intermodulation**



# Sensitivity

- **Sensitivity -- defined as the minimum signal level that the system can detect with acceptable SNR.**

$$NF = \frac{SNR_{in}}{SNR_{OUT}} = \frac{P_{sig} / P_{RS}}{SNR_{OUT}}$$

- **The overall signal power is distributed across the channel bandwidth, B, integrating over the bandwidth to obtain total mean square power**

$$P_{sig,tot} = P_{RS} \bullet NF \bullet SNR_{OUT} \bullet B$$

$$P_{in,\min}\Big|_{dBm} = P_{RS}\Big|_{dBm/Hz} + NF\Big|_{dB} + SNR_{\min}\Big|_{dB} + 10\log B$$

**where $P_{RS}$ is the source resistance noise power.**

# Sensitivity

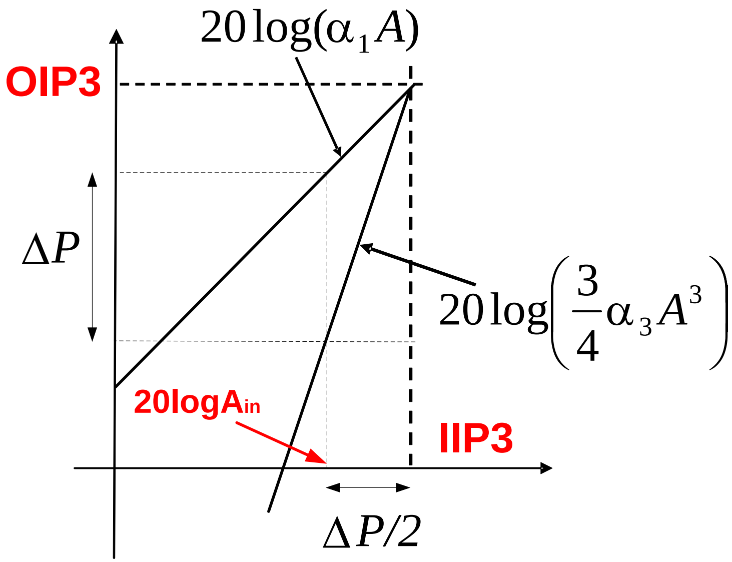
- **Assuming conjugate matching at input, we obtain $P_{RS}$ as the noise power that source resistance delivers to the receiver**

$$P_{RS} = \frac{4kTRs}{4}\frac{1}{R_{in}} = kT$$
$$= -174dBm/Hz \text{ (thermal noise floor)}$$

- **At room temperature, we obtain receiver sensitivity as**

$$P_{in,\min} = F + SNR_{\min} = \underbrace{-174dBm + NF + 10\log B}_{\text{System noise floor}} + SNR_{\min}$$

# Maximum Input Power




$$P_{IIP3} = P_{in} + \frac{P_{out} - P_{IM,out}}{2}$$
$$= P_{in} + \frac{P_{in} - P_{IM,in}}{2} = \frac{3P_{in} - P_{IM,in}}{2}$$

$$P_{in} = \frac{2P_{IIP3} + P_{IM,in}}{3}$$

**where $P_{IM,out}$ denotes output-referred power of IM$_3$ products, $P_{out}=P_{in}+G$, $P_{IM,OUT}= P_{IM,in}+G$. The input level for which the IM products become equal to the noise floor $F$ is thus given by**

$$P_{in} = \frac{2P_{IIP3} + F}{3} = \frac{2P_{IIP3} - 174dBm + NF + 10\log B}{3}$$

# Dynamic Range

- **Dynamic Range (DR) -- defined as the ratio of the maximum to minimum input levels that the circuit provides a reasonable signal quality.**
- **Spurious-Free Dynamic Range (SFDR) -- determine the upper end of dynamic range on the intermodulation behavior and the lower end on sensitivity.**
- **The upper bound of the dynamic range is defined as the maximum input power in a two-tone test for which the 3rd IM products do not exceed the noise floor F=-174dBm+NF+10logB.**
- **The SFDR is thus given by**

$$SFDR = P_{in,\max} - P_{in,mim} = \left( \frac{2P_{IIP3} + F}{3} \right) - (F + SNR_{\min}) = \frac{2(P_{IIP3} - F)}{3} - SNR_{\min}$$

- **Example: NF=9dB, $P_{IIP3}$=-15dBm, B=100kHz, $SNR_{min}$=12dB → SFDR=(-15-(-174+9+50))/1.5-12=54.7dB.**

# Homework 1

Submit your HW 1 to Canvas. Solutions to HW1 will be posted after the submission deadline.

HW 1: 2.2, 2.3, 2.7, 2.8, 2.11 (use Eq. (2.122) and (2.132)), 2.15.

2.2. Repeat Example 2.11 if one interferer has a level of −3 dBm and the other, −35 dBm.

2.3. If cascaded, stages having only *second-order* nonlinearity can yield a finite $IP_3$. For example, consider the cascade identical common-source stages shown in Fig. 2.75.

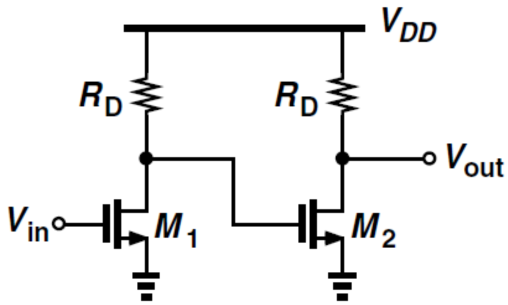



**Figure 2.75** *Cascade of CS stages.*

# Homework 1

HW 1: 2.2, 2.3, 2.7, 2.8, 2.11 (use Eq. (2.122) and (2.132)), 2.15.

2.7. A broadband circuit sensing an input $V_0 \cos \omega_0 t$ produces a third harmonic $V_3 \cos(3\omega_0 t)$. Determine the 1-dB compression point in terms of $V_0$ and $V_3$.

2.8. Prove that in Fig. 2.36, the noise power delivered by $R_1$ to $R_2$ is equal to that delivered by $R_2$ to $R_1$ if the resistors reside at the same temperature. What happens if they do not?

2.11. Determine the NF of the circuit shown in Fig. 2.52 using Friis' equation.

2.15. The input/output characteristic of a bipolar differential pair is given by $V_{out} = -2R_C I_{EE} \tanh[V_{in}/(2V_T)]$, where $R_C$ denotes the load resistance, $I_{EE}$ is the tail current, and $V_T = kT/q$. Determine the $IP_3$ of the circuit.