Ex 5.9

Wish to show:

$$S_\lambda = N(N^TN + \lambda\Omega_N)^{-1}N^T = (\mathbb{I} + \lambda K)^{-1}$$

where K does not depend on λ.

assuming N is invertible:

$$\begin{aligned} S_\lambda^{-1} &= (N(N^TN + \lambda\Omega_N)^{-1}N^T)^{-1} \\ &= (N^T)^{-1}(N^TN + \lambda\Omega_N)N^{-1} \\ &= \mathbb{I} + \lambda(N^T)^{-1}\Omega_N N^{-1} \\ &= \mathbb{I} + \lambda K \end{aligned}$$

$$\Rightarrow S_\lambda = (\mathbb{I} + \lambda K)^{-1}$$