

Derivation 4.9

$$\log \frac{P(G=k|X=x)}{P(G=2|X=x)} = \log \left(\frac{\frac{f_k(x) \pi_k}{\sum_i f_i(x) \pi_i}}{\frac{f_2(x) \pi_2}{\sum_i f_i(x) \pi_i}} \right) = \log \left(\frac{f_k(x) \pi_k}{f_2(x) \pi_2} \right)$$

①

②

$$= \log(f_k(x) \pi_k) - \log(f_2(x) \pi_2)$$

$$\textcircled{1} = \log \left(\frac{\exp(-\frac{1}{2}(x-\mu_k)^T \Sigma^{-1}(x-\mu_k))}{(2\pi)^{p/2} |\Sigma|^{1/2}} \right)$$

$$= -\frac{1}{2}(x-\mu_k)^T \Sigma^{-1}(x-\mu_k) - \log \left(\frac{(2\pi)^{p/2} |\Sigma|^{1/2}}{\pi_k} \right)$$

$$\textcircled{2} = -\frac{1}{2}(x-\mu_e)^T \Sigma^{-1}(x-\mu_e) - \log \left(\frac{(2\pi)^{p/2} |\Sigma|^{1/2}}{\pi_e} \right)$$

$$\textcircled{1} - \textcircled{2} = -\frac{1}{2}(x-\mu_k)^T \Sigma^{-1}(x-\mu_k) + \frac{1}{2}(x-\mu_e)^T \Sigma^{-1}(x-\mu_e) + \log \left(\frac{\pi_k}{\pi_e} \right)$$

$$= -\frac{1}{2}x^T \Sigma^{-1}x + \frac{1}{2}x^T \Sigma^{-1}\mu_k + \frac{1}{2}\mu_k^T \Sigma^{-1}x - \frac{1}{2}\mu_k^T \Sigma^{-1}\mu_k$$

$$+ \frac{1}{2}x^T \Sigma^{-1}x - \frac{1}{2}x^T \Sigma^{-1}\mu_e - \frac{1}{2}\mu_e^T \Sigma^{-1}x + \frac{1}{2}\mu_e^T \Sigma^{-1}\mu_e + \log \left(\frac{\pi_k}{\pi_e} \right)$$

$$= \frac{1}{2}x^T \Sigma^{-1}(\mu_k - \mu_e) + \frac{1}{2}(\mu_k - \mu_e)^T \Sigma^{-1}x - \frac{1}{2}(\mu_k + \mu_e)^T \Sigma^{-1}(\mu_k - \mu_e)$$

$$+ \log \left(\frac{\pi_k}{\pi_e} \right)$$

$$= x^T \Sigma^{-1}(\mu_k - \mu_e) - \frac{1}{2}(\mu_k + \mu_e)^T \Sigma^{-1}(\mu_k - \mu_e) + \log \left(\frac{\pi_k}{\pi_e} \right)$$