4.2 (e)

From Part (d) we have:

$$\hat{\beta}_0 = \frac{1}{N}\left[N_1 a + N_2 b - (N_1\hat{\mu}_1^T + N_2\hat{\mu}_2^T)\hat{\beta}\right]$$

$$\hat{\beta} = c^{**}\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1)$$

Now,

$$\hat{f}(x) = \hat{\beta}_0 + x^T\hat{\beta}$$

$$\Rightarrow \hat{f}(x) = 1/N(N_1 a + N_2 b) - 1/N(N_1\hat{\mu}_1^T + N_2\hat{\mu}_2^T)\hat{\beta} + x^T\hat{\beta}$$

$$= 1/N(N_1 a + N_2 b) - 1/N(N_1\hat{\mu}_1^T + N_2\hat{\mu}_2^T - Nx^T)(c^{**}\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1))$$

Now we will classify as class 2 if $\hat{f}(x) > 0$, e.g.

$$x^T c^{**}\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1) > 1/N(N_1 a + N_2 b) + 1/N(N_1\hat{\mu}_1^T + N_2\hat{\mu}_2^T)(c^{**}\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1))$$

$$\Rightarrow x^T\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1) > \frac{1/N(N_1 a + N_2 b)}{c^{**}} + 1/N(N_1\hat{\mu}_1^T + N_2\hat{\mu}_2^T)\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1)$$

Which is <u>not</u> the same as the LDA rule. However, assuming $N_1 = N_2 = N/2$ we obtain:

$$x^T\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1) > \frac{1}{2c^{**}}(a+b) + \frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_2)^T\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1)$$

And assuming $a = -b$ (targets are coded as a single value with opposite signs.)

$$x^T\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1) > 1/2(\hat{\mu}_1 + \hat{\mu}_2)^T\hat{\Sigma}^{-1}(\hat{\mu}_2 - \hat{\mu}_1)$$

Which <u>is</u> the same as the LDA rule since $\log\left(\frac{N_2}{N_1}\right) = \log(1) = 0$