Ex 3.24

We will use the fact that the angle α between two vectors a, b is given by:

$$\alpha = \cos^{-1}\left(\frac{a \cdot b}{|a| \cdot |b|}\right) \qquad (*)$$

Noticing that u_k is an $N \times 1$ vector, we wish to ~~...~~ calculate the angle between each predictor $x_j, j \in A_k$ and the vector u_k. Using $(*)$ this is given by:

$$\alpha = \cos^{-1}\left(\frac{X_j^T \cdot u_k}{|X_j| \cdot |u_k|}\right)$$

$$= \cos^{-1}\left(\frac{X_j^T \cdot u_k}{|u_k|}\right) \qquad \text{since } |X_j| = 1 \text{ (unit norm)}$$

$\Rightarrow$ Thus if $X_j^T \cdot u_k$ is constant $\forall j \in A_k$ then the angle α is also constant.

$$X_{A_k}^T u_k = X_{A_k}^T X_{A_k} \delta_k \qquad \text{(by definition)}$$

$$= X_{A_k}^T X_{A_k} (X_{A_k}^T X_{A_k})^{-1} X_{A_k}^T r_k$$

$$= X_{A_k}^T r_k$$

And since each predictor x_j in the active set A_k has equal correlation with the current residual r_k by definition in algorithm 3.2, we can conclude that the LAR direction u_k makes an equal angle with each of the predictors in A_k.