

Ex 3.28

$$\hat{\beta}_{\text{lasso}} = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^N (y_i - \beta_0 - \sum_{j=1}^p x_{ij} \beta_j)^2$$

$$\text{Subject to } \sum_{j=1}^p |\beta_j| \leq t$$

$$\text{Now, } \sum_{j=1}^p x_{ij} \beta_j = \sum_{j=1}^{p-1} x_{ij} \beta_j + x_{i,p} \beta_p$$

Now since the order of variables doesn't matter, I will say that this p^{th} variable has been duplicated.

$$\text{e.g. } x_{i,p} \beta_p \rightarrow x_{ip} \beta_{p_1}^* + x_{ip} \beta_{p_2}^*$$

$$= x_{ip} (\beta_{p_1}^* + \beta_{p_2}^*)$$

$$\text{Therefore if } \beta_j = a \text{ then } a = \beta_{p_1}^* + \beta_{p_2}^* \quad \textcircled{1}$$

$$\text{also, } \sum_{j=1}^p |\beta_j| \Rightarrow \sum_{j=1}^{p-1} |\beta_j| + |\beta_{p_1}^*| + |\beta_{p_2}^*| \leq t$$

$$\Rightarrow \sum_{j=1}^{p-1} |\beta_j| + |\beta_{p_1}^* + \beta_{p_2}^*| \leq t \quad (\text{triangle inequality})$$

$$\text{e.g. } |\beta_{p_1}^* + \beta_{p_2}^*| \leq t^*,$$

$$\text{where } t^* = t - \sum_{j=1}^{p-1} |\beta_j|$$