Ex. 3.23

(a) Firstly we will show that:

$$\langle x_j, y-u(\alpha)\rangle = (1-\alpha)\langle x_j, y\rangle \quad (*)$$

By viewing in terms of the entire ~~the~~ Matrix X it is clear that:

$\langle x_j, y-u(\alpha)\rangle = x_j^T(y-u(\alpha))$ for $j=1,\dots,p$

is equivalent to the j^{th} element of the vector:

$$\begin{aligned}
&X^T(y-u(\alpha))\\
&= X^Ty - X^Tu(\alpha)\\
&= X^Ty - \alpha X^TX\hat{\beta}\\
&= X^Ty - \alpha X^TX(X^TX)^{-1}X^Ty\\
&= (1-\alpha)X^Ty
\end{aligned}$$

and switching back to the original notation, this is equivalent to:

$(1-\alpha)\langle x_j, y\rangle$ for $j=1,\dots,p$

Thus proving $(*)$.

Now returning to the original problem, we wish to solve:

$$\begin{aligned}
&\tfrac{1}{N}|\langle x_j, y-u(\alpha)\rangle|\\
&= \tfrac{1}{N}|(1-\alpha)\langle x_j, y\rangle| \quad \text{by } (*)\\
&= \tfrac{1}{N}(1-\alpha)|\langle x_j, y\rangle| \quad \text{since } \alpha\in[0,1]
\end{aligned}$$

and using the fact $\tfrac{1}{N}|\langle x_j, y\rangle| = \lambda$:

$$\begin{aligned}
&= \tfrac{1}{N}(1-\alpha)N\lambda\\
&= (1-\alpha)\lambda \quad \text{as required}
\end{aligned}$$