Ex. 3.30

$$\hat{\beta}^{lasso} = \underset{\beta}{\text{argmin}} \left\{ \tfrac{1}{2} \sum_{i=1}^{N} (y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j)^2 + \lambda \sum_{j=1}^{p} |\beta_j| \right\}$$

Now, taking a hint from exercise 3.12 we will augment the X matrix and Y vector as follows:

$$X^* = \begin{bmatrix} X \\ a\mathbb{I}_p \end{bmatrix}$$; where a is some constant and $\mathbb{I}_p$ is the identity matrix of dim $P \times P$

$$Y^* = \begin{bmatrix} Y \\ O_p \end{bmatrix}$$, where O_p is a vector of 0's of length P.

Now,

$$\hat{\beta}^{lasso} = (Y^* - X^*\beta)^T (Y^* - X^*\beta) + \lambda \sum_j |\beta_j|$$

$$= (Y - X\beta)^T (Y - X\beta) + a^2 \beta^T\beta + \lambda \sum_j |\beta_j|$$

$$= (Y - X\beta)^T (Y - X\beta) + a^2 \sum_j \beta_j^2 + \lambda \sum_j |\beta_j|$$

and setting $a^2 = \lambda^*\alpha$ and $\lambda = \lambda^*(1-\alpha)$ we obtain:

$$\hat{\beta}^{lasso} = \underset{\beta}{\text{argmin}} \left\{ \sum_{i=1}^{N} (y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j)^2 + \lambda^* \sum_{j=1}^{p} (\alpha\beta_j^2 + (1-\alpha)|\beta_j|) \right\}$$