Ex. 3.1

W.t.s. F for dropping a single coefficient from the model is equal to the square of the corresponding z-score.

First we will show that z_j is t-distributed.

$$z_j = \frac{\hat{\beta}_j}{\hat{\sigma}\sqrt{v_j}} \qquad (3.12)$$

$$= \frac{\sigma\sqrt{v_j}\, Z}{\hat{\sigma}\sqrt{v_j}} \quad \left(\begin{array}{l}\text{as } \hat{\beta}_j \sim N(\beta_j, \sigma^2 v_j) \text{ and under} \\ H_0\ \beta_j = 0 \Rightarrow \hat{\beta}_j = \sqrt{\sigma^2 v_j}\, Z \text{ where} \\ Z \sim N(0,1)\end{array}\right)$$

$$= \frac{\sigma Z}{\hat{\sigma}}$$

Now by 3.11 we have:

$$\hat{\sigma}^2 \sim \frac{\sigma^2}{N-p-1} \chi^2_{N-p-1} \qquad (*)$$

So $$\hat{\sigma} = \sigma\sqrt{\frac{Y}{\ell}} \quad \left(\begin{array}{l}\text{where } Y \sim \chi^2(\ell) \\ \text{and } \ell = N-p-1\end{array}\right)$$

Now $$z_j = \frac{Z}{\sqrt{Y/\ell}} \sim t(\ell) \quad (\text{by definition of t-dist}) \quad (**)$$

Secondly we will examine $F = \dfrac{(RSS_0 - RSS_1)/p_1 - p_0}{RSS_1/N-p_1-1}$

Notice:

$$RSS = \hat{\sigma}^2(N-p-1) \qquad (3.8)$$

$$= \sigma^2 \chi^2_{N-p-1} \qquad (\text{by } *)$$

So $$RSS_0 - RSS_1 = \sigma^2[\chi^2_{N-p_0-1} - \chi^2_{N-p_1-1}]$$

$$= \sigma^2[\chi^2_{(N-p_0-1)-(N-p_1-1)}]$$

since sum of χ^2 is single χ^2 with summed d.o.f.

$$= \sigma^2[\chi^2_{p_1-p_0}]$$

Note, since we are considering dropping just a single variable we have: $p_1 - p_0 = 1$

$$\Rightarrow \quad RSS_0 - RSS_1 = \sigma^2 \chi^2_1$$

and $RSS_1 = \sigma^2 \chi^2_\ell \qquad (\ell = N - p_1 - 1)$

So $$F = \frac{(RSS_0 - RSS_1)/p_1 - p_0}{RSS_1/N - p_1 - 1}$$

reduces to:

$$F = \frac{\sigma^2 X}{\sigma^2 Y/\ell} \qquad \text{where} \qquad X \sim \chi^2_1 \Rightarrow \overset{\overset{Z}{\shortparallel}}{\sqrt{X}} \sim N(0,1)$$

$$Y \sim \chi^2_\ell$$

By definition of χ^2 dist.

$$= \frac{X}{Y/\ell} = \frac{Z^2}{Y/\ell}$$

$$\Rightarrow Z_j^2 = \left(\frac{Z}{\sqrt{Y/\ell}}\right)^2 = \frac{Z^2}{Y/\ell} = F$$

(using **)

Thus, when dropping a single coefficient, the F-statistic is equivalent to the corresponding Z-score squared.