Eqn 2.24 (Ex. 2.3)

Consider 1-d case:

Want the median distance from the origin to the minimum of n $U(0,1)$ r.v.'s

(1) Need CDF for a series of $U(a,b)$ r.v.'s

for $x \sim U(a,b)$ $\quad F(x) = \mathbb{P}(X \leq x) = 1 - \mathbb{P}(X > x)$

We want $F(x) = 1 - \mathbb{P}(\min(X_1, \dots, X_n) > x)$

as they are iid $= 1 - \mathbb{P}(X_1 > x) \dots \mathbb{P}(X_n > x)$

$= 1 - \mathbb{P}(X_i > x)^n$

$$\therefore F(x) = 1 - \left(1 - \frac{x-a}{b-a}\right)^n$$

$$= 1 - \left(\frac{b-x}{b-a}\right)^n$$

~~(2) Now we want median of this dist for $b=1$, $a=0$~~

~~e.g. Find m s.t. $\int_0^m$~~

(2) Have CDF, now we wish to find the PDF

$$f(x) = \frac{d}{dx}(F(x)) = \frac{\partial}{\partial x}\left(1 - \left(\frac{b-x}{b-a}\right)^n\right)$$

$$= \frac{n}{b-a}\left(\frac{b-x}{b-a}\right)^{n-1} = f(x)$$

(3) Now lets Find the median for $b=1$, $a=0$

Find m s.t. $\int_0^m F(x)\,dx = \frac{1}{2}$

$$\Rightarrow \frac{n}{b-a}\int_0^m \left(\frac{b-x}{b-a}\right)^{n-1} = \frac{1}{2}$$

$$\Rightarrow n\int_0^m (1-x)^{n-1} = \frac{1}{2}$$

$$\Rightarrow -(1-x)^n \Big|_0^m = \frac{1}{2}$$

$$-(1-m)^n + 1 = \frac{1}{2} \Rightarrow (1-m)^n = \frac{1}{2}$$

$$\Rightarrow m = 1 - \frac{1}{2}^{1/n}$$

(4) Finally extend this to P dimensions using $e_p(r) = r^{1/P}$ and set the ratio of the area to be the median.

$$\Rightarrow e_p(M) = \left(1 - \frac{1}{2}^{1/n}\right)^{1/P} = d(P,N)$$ as required