Ex. 3.19

$$\hat{\beta}^{ridge} = (X^T X + \lambda \mathbb{I})^{-1} X^T y$$

Now using SVD of X e.g. $X = UDV^T$ we obtain:

$$\hat{\beta}^{ridge} = (VDU^T UDV^T + \lambda VV^T)^{-1} VDU^T y$$

$$= (VD^2V^T + \lambda VV^T)^{-1} VDUy$$

$$= (V(D^2 + \lambda)V^T)^{-1} VDU^T y$$

and using $V^T V = \mathbb{I}$ and $V = V^{-1}$

$$= V(D^2 + \lambda)^{-1} DU^T y$$

$$= \sum_{j=1}^{p} V_j \left(\frac{d_j}{d_j^2 + \lambda} \right) u_j^T y$$

and clearly as $\lambda \to 0$, the absolute value of this equation $\|\hat{\beta}^{ridge}\|$ increases.