Ex. 2.2

Bayes decision boundry:

Find X s.t $f(X|P_i,\Sigma_i) = f(X|q_i,\Sigma_i)$

Where P_i, q_i are the means of the 20 Gaussians and Σ_i is their associated cov.

e.g. X s.t. $\sum_{i=1}^{10} \frac{1}{2\pi\sqrt{|\Sigma_i|}} \exp(-\frac{1}{2}(X-P_i)\Sigma_i^{-1}(X-P_i))$

$= \sum_{i=1}^{10} \frac{1}{2\pi\sqrt{|\Sigma_i|}} \exp(-\frac{1}{2}(X-q_i)\Sigma_i^{-1}(X-P_i))$

Notice $\Sigma_i = \frac{1}{5}\mathbb{I} \quad \forall i$

$\Rightarrow \Sigma_i^{-1} = (\frac{1}{5}\mathbb{I})^{-1} = 5\mathbb{I}^{-1} = 5\mathbb{I}$

$\Rightarrow X$ s.t. $\sum_{i=1}^{10} \exp(-\frac{1}{2}(X-P_i)5\mathbb{I}(X-P_i)) = \sum_{i=1}^{10} \exp(-\frac{1}{2}(X-q_i)5\mathbb{I}(X-q_i))$

More formally:

$$\left\{X : \sum_{i=1}^{10} \exp(-\tfrac{5}{2}\|X-P_i\|^2) = \sum_{i=1}^{10} \exp(-\tfrac{5}{2}\|X-q_i\|^2)\right\}$$