Wish to show:

$$\beta_1 \mathbb{I}(x < \varepsilon_1) + x\beta_4 \mathbb{I}(x < \varepsilon_1) + \beta_2 \mathbb{I}(\varepsilon_1 \leq x < \varepsilon_2)$$
$$+ x\beta_5 \mathbb{I}(\varepsilon_1 \leq x < \varepsilon_2) + \beta_3 \mathbb{I}(\varepsilon_2 \leq x) + x\beta_6 \mathbb{I}(\varepsilon_2 \leq x)$$

With constraints:

$$\beta_1 + \varepsilon_1 \beta_4 = \beta_2 + \varepsilon_1 \beta_5 \qquad (1)$$
$$\beta_2 + \varepsilon_2 \beta_5 = \beta_3 + \varepsilon_2 \beta_6 \qquad (2)$$

is equivalent to the following (unconstrained) expression:

$$\alpha_1 + \alpha_2 x + \alpha_3 (x - \varepsilon_1)_+ + \alpha_4 (x - \varepsilon_2)_+ \qquad (*)$$

Now we can reduce (*) to 3 cases:

(a) $x < \varepsilon_1$

$\Rightarrow \alpha_1 + \alpha_2 x$

(b) $\varepsilon_1 < x < \varepsilon_2$

$\Rightarrow \alpha_1 + \alpha_2 x + \alpha_3 (x - \varepsilon_1) \Rightarrow \alpha_1 - \alpha_3 \varepsilon_1 + (\alpha_2 + \alpha_3) x$

(c) $\varepsilon_2 < x$

$\Rightarrow \alpha_1 + \alpha_2 x + \alpha_3 (x - \varepsilon_1) + \alpha_4 (x - \varepsilon_2) \Rightarrow \alpha_1 - \alpha_3 \varepsilon_1 - \alpha_4 \varepsilon_2 + (\alpha_2 + \alpha_3 + \alpha_4) x$

Now if we set $\alpha_1 = \beta_1$, $\alpha_2 = \beta_4$, $\alpha_1 - \alpha_3 \varepsilon_1 = \beta_2$ and $\alpha_2 + \alpha_3 = \beta_5$ we find that:

$\beta_1 + \varepsilon_1 \beta_4 = \alpha_1 + \varepsilon_1 \alpha_2 = \alpha_1 - \alpha_3 \varepsilon_1 + \alpha_3 \varepsilon_1 + \varepsilon_1 \alpha_2$

$= \beta_2 + \varepsilon_1 \beta_5$ (satisfying constraint (1))

and setting $\alpha_1 - \alpha_3 \varepsilon_1 - \alpha_4 \varepsilon_2 = \beta_3$ and $\alpha_2 + \alpha_3 + \alpha_4 = \beta_6$ we obtain:

$\beta_2 + \varepsilon_2 \beta_5 = \alpha_1 - \alpha_3 \varepsilon_1 + \varepsilon_2 (\alpha_2 + \alpha_3) = \alpha_1 - \alpha_3 \varepsilon_1 - \alpha_4 \varepsilon_2 + \alpha_4 \varepsilon_2 + \varepsilon_2 (\alpha_2 + \alpha_3)$

$= \beta_3 + \varepsilon_2 \beta_6$ (satisfying constraint (2))

and under these transformation of coefficients we find that (*) and the original eqn. are equivalent and satisfy constraints (1) and (2).