4.2 (a)

from 4.9:

$$\log \frac{\mathbb{P}(G = \text{Class } 2 \mid X = x)}{\mathbb{P}(G = \text{Class } 1 \mid X = x)} = \log \frac{\pi_2}{\pi_1} - \frac{1}{2}(\mu_2 + \mu_1)^T \Sigma^{-1} (\mu_2 - \mu_1) + x^T \Sigma^{-1} (\mu_2 - \mu_1)$$

We will classify observation x as class 2 when:

$$\mathbb{P}(G = \text{Class } 2 \mid X = x) > \mathbb{P}(G = \text{Class } 1 \mid X = x)$$

Which is equivalent to:

$$\log \frac{\mathbb{P}(G = \text{Class } 2 \mid X = x)}{\mathbb{P}(G = \text{Class } 1 \mid X = x)} > 0$$

and estimating $\hat{\pi}_K = N_K/N$, we obtain:

Classify as class 2 when

$$\log \frac{N_2}{N_1} - \frac{1}{2}(\hat{\mu}_2 + \hat{\mu}_1)^T \hat{\Sigma}^{-1} (\hat{\mu}_2 - \hat{\mu}_1) + x^T \hat{\Sigma}^{-1} (\hat{\mu}_2 - \hat{\mu}_1) > 0$$

$$\Rightarrow x^T \hat{\Sigma}^{-1} (\hat{\mu}_2 - \hat{\mu}_1) > \frac{1}{2} (\hat{\mu}_2 + \hat{\mu}_1)^T \hat{\Sigma}^{-1} (\hat{\mu}_2 - \hat{\mu}_1) - \log \frac{N_2}{N_1}$$