Ex 3.4

One pass of the Gram-Schmidt procedure (alg 3.1) is taken to obtain the z_i's and $\hat{\gamma}_{ik}$'s and in turn to write:

$$X = ZD^{-1}D\Gamma$$
$$= QR$$

Now

$$X\hat{\beta} = y$$
$$\Rightarrow QR\hat{\beta} = y$$
$$\Rightarrow R\hat{\beta} = Q^T y \quad \text{(as } Q \text{ is orthogonal)}$$

Since R is upper triangular we have:

$$\underset{(p+1)\times(p+1)}{\begin{bmatrix} r_{11} & \cdots & r_{1,p+1} \\ 0 & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ 0 \cdots & 0 & r_{p+1,p+1} \end{bmatrix}} \underset{(p+1\times 1)}{\begin{bmatrix} \hat{\beta}_1 \\ \vdots \\ \hat{\beta}_{p+1} \end{bmatrix}} = \underset{((p+1)\times 1)}{\begin{bmatrix} q_1 y \\ \vdots \\ q_{p+1} y \end{bmatrix}}$$

Which is clearly solvable $\forall \hat{\beta}_i \; i \in 1 \ldots p+1$ by solving for $\hat{\beta}_{p+1}$ and substituting to find other values sequentially.