Ex 5.12

Setting W to be a diagonal matrix with diagonal elements W_i we can re write 5.73 as:

$$\min_{\theta} RSS(f,\lambda) = (y - N\theta)^T W (y - N\theta) + \lambda \theta^T \Omega_N \theta$$

$$\Rightarrow \hat{\theta} = (N^T W N + \lambda \Omega_N)^{-1} N^T W y$$

$$\Rightarrow S_\lambda = \underbrace{N(N^T W N + \lambda \Omega_N)^{-1} N^T}_{H} W$$

$$H^{-1} = (N^T)^{-1} (N^T W N + \lambda \Omega_N) N^{-1}$$

$$= (W + \lambda (N^T)^{-1} \Omega_N N^{-1})$$

$$= W + \lambda K \qquad \text{(same } K \text{ as in ex 5.9)}$$

$$\Rightarrow S_\lambda = HW = (W + \lambda K)^{-1} W$$

And Clearly if all weights are 1, then $W = I$ and S_λ becomes $(I + \lambda K)^{-1}$ as usual.

Now, in the case of training data with ties, we can apply the Smoothing Spline approach to the full training data set or, equivalently, we can reduce the dataset to the N unique observations where each observation has a number of repetitions in the full data M_i. Then we set the response of each observation to be the average across these repetitions e.g.

$$\bar{y}_i = \frac{\sum_{j=1}^{M_i} y_{ij}}{M_i}$$

Then the Problem can be Characterised as a Weighted Sum of Squares Problem e.g.

$$\min_{f} RSS(f,\lambda) = \sum_i M_i\{\bar{y}_i - f(x_i)\}^2 + \lambda\int f''(t)^2\,dt$$

and can be Solved as already discussed. For a more detailed Proof that tied observations can be Written ~~as~~ in ~~a~~ weighted Sum of Squares form see my solution to exercise 2.6.