Ex 3.23 (c)

Part (a) Proved that the LAR algorithm Keeps the correlations tied so we need only Prove that $\lambda(\alpha)$ decreases as $\alpha$ goes from 0 to 1.

Notice:

$$(1-\alpha)^2 + \frac{\alpha(2-\alpha)}{N} RSS$$

$$= (1-\alpha)^2 + \frac{1}{N}(2\alpha - \alpha^2) RSS$$

$$= (1-\alpha)^2 + 1/N(1 - 1 + 2\alpha - \alpha^2) RSS$$

$$= (1-\alpha)^2 + 1/N(-(1-\alpha)^2 + 1) RSS$$

$$= (1-\alpha)^2 - \frac{(1-\alpha)^2}{N} RSS + \frac{1}{N} RSS$$

$$= (1-\alpha)^2 \left(1 - \frac{RSS}{N}\right) + 1/N \, RSS$$

Thus

$$\lambda(\alpha) = \frac{(1-\alpha)\lambda}{\sqrt{(1-\alpha)^2\left(1 - \frac{RSS}{N}\right) + 1/N \, RSS}}$$

Now the coefficient of determination, $R^2$, is defined as:

$$R^2 = 1 - \frac{RSS}{\sum_i (y_i - \bar{y})}$$

$$= 1 - \frac{RSS}{N} \geq 0 \quad \text{For linear regression}$$

and since this quantity is greater than or equal to zero, it is clear that as $\alpha$ increases from 0 to 1, $\lambda(\alpha)$ is monotonically decreasing.