Ex 3.7

Similarly to exercise 3.6 we have:

$$\mathbb{P}(\beta \mid y) \propto \mathbb{P}(y \mid \beta)\,\mathbb{P}(\beta)$$

$$\Rightarrow \prod_{i=1}^{N} \mathbb{P}(y_i \mid \beta) \prod_{j=1}^{p} \mathbb{P}(\beta_j) \cdot \mathbb{P}(\beta_0)$$ falls into proportionality

$$\propto \prod_i \exp\left(-\tfrac{1}{2}\sigma^{-2}(y_i - \beta_0 - x_i^T\beta)^2\right) \prod_j \exp\left(-\tfrac{1}{2}\tau^{-2}\beta_j^2\right)$$

$$\Rightarrow \log \mathbb{P}(\beta \mid y) \propto -\tfrac{1}{2}\left[\sigma^{-2}\sum_i (y_i - \beta_0 - x_i^T\beta)^2 + \tau^{-2}\sum_j \beta_j^2\right]$$

$$\propto \sigma^{-2}\left[\sum_i (y_i - \beta_0 - x_i^T\beta)^2 + \frac{\sigma^2}{\tau^2}\sum_j \beta_j^2\right]$$

$$\propto \sum_i (y_i - \beta_0 - x_i^T\beta)^2 + \lambda \sum_j \beta_j^2$$