Ex 7.5

This question is just a generalisation of Ex. 7.1 where we assume the model to be a linear smoother rather than a linear fit.

Again we will notice that $\sum_{i=1}^{N} \mathrm{cov}(\hat{y}_i, y_i) = \mathrm{trace}(\Sigma)$
where $\Sigma = \mathrm{Cov}(\hat{Y}, Y)$.

$$\Rightarrow \sum_{i=1}^{N} \mathrm{Cov}(\hat{y}_i, y_i) = \mathrm{trace}(\mathrm{Cov}(\hat{Y}, Y))$$

$$= \mathrm{trace}(\mathrm{cov}(SY, Y))$$

$$= \mathrm{trace}(S \cdot \mathrm{Cov}(Y, Y))$$

$$= \mathrm{trace}(S \cdot \mathrm{Var}(Y))$$

$$= \mathrm{trace}(S \cdot \sigma_\varepsilon^2)$$

$$= \mathrm{trace}(S) \cdot \sigma_\varepsilon^2 \qquad (\text{as } \sigma_\varepsilon^2 \text{ constant})$$