Ex 3.5

Begin with 3.41 :

$$\hat{\beta}^{ridge} = \underset{\beta}{\text{argmin}} \left\{ \underbrace{\sum_{i=1}^{N} (y_i - \beta_0 - \sum_{j=1}^{p} x_{ij}\beta_j)^2}_{\text{Add and subtract } \bar{x}_j \text{ here}} + \lambda \sum_{j=1}^{p} \beta_j^2 \right\}$$

$$\Rightarrow \sum_i (y_i - \beta_0 - \sum_j (x_{ij} + \bar{x}_j - \bar{x}_j) \beta_j)^2$$

$$= \sum_i (y_i - \beta_0 - \sum_j \bar{x}_j - \sum_j (x_{ij} - \bar{x}_j) \beta_j)^2$$

Giving:

$$\hat{\beta}^{ridge} = \text{argmin} \left\{ \sum_{i=1}^{N} (y_i - \beta_0^c - \sum_j (x_{ij} - \bar{x}_j) \beta_j^c)^2 + \lambda \sum_j \beta_j^{c\,2} \right\}$$

Where: $\beta_0^c = \beta_0 + \sum_j \bar{x}_j$

$\beta_j^c = \beta_j$ as required.

Therefore centering the data only effects the coefficient estimates for the intercept while all others remain the same.

Clearly the Lasso (3.52) will have the exact same result.