《量子信息基础》2024.3.30 随堂作业：

（2024.4.9 晚 22 点前提交）

1. (Text book* Problem 3.16)

   Show that two noncommuting operators cannot have a complete set of common eigenfunctions. Hint: Show that if $\hat{P}$ and $\hat{Q}$ have a complete set of common eigenfunctions, then $[\hat{P},\hat{Q}]f = 0$ for any function in Hilbert space.

Assuming $\hat{P}f_n = \lambda_n f_n$ and $\hat{Q}f_n = \mu_n f_n$, and $\{f_n\}$ are a complete set of eigenfunctions

For arbitrary wavefunction

$$f = \sum_n c_n f_n$$

$$[\hat{P},\hat{Q}]f = (\hat{P}\hat{Q} - \hat{Q}\hat{P})\sum_n c_n f_n = \hat{P}\left(\sum_n c_n \mu_n f_n\right) - \hat{Q}\left(\sum_n c_n \lambda_n f_n\right)$$
$$= \sum_n c_n \mu_n \lambda_n f_n - \sum_n c_n \lambda_n \mu_n f_n = 0$$

Therefore, $\hat{P}\hat{Q} = \hat{Q}\hat{P}$ or $f = 0$. The former contradicts to $\hat{P}$ and $\hat{Q}$ are noncommuting. The latter contradicts to f is an arbitrary wavefunction.

推导和答案正确给 50 分

2. $\widehat{D}_x(a)$ is a translation operator in one dimension. When it applies to a wavefunction

   $$\widehat{D}_x(a)\psi(x) = \psi(x-a)$$

   If $\hat{f}(x)$ is commutable with $\widehat{D}_x(a)$, prove $\hat{f}(x) = \hat{f}(x-a)$.

Since $\hat{f}(x)$ is commutable with $\widehat{D}_x(a)$,

$$[\hat{f}(x), \widehat{D}(a)] = 0$$

For an arbitrary wavefunction $\psi(x)$

$$\hat{f}(x)\widehat{D}_x(a)\psi(x) = \hat{f}(x)\psi(x-a) = \widehat{D}_x(a)\hat{f}(x)\psi(x) = \hat{f}(x-a)\psi(x-a)$$

Since $\psi(x-a)$ is an arbitrary wavefunction

$$\hat{f}(x) = \hat{f}(x-a)$$

Dx(a)作用在f(x)和
\phi (x)整体上

推导和答案正确给 50 分

* David J. Griffiths, and Darrell F. Schroeter, Introduction to Quantum Mechanics (3rd Edition), Cambridge University Press (2018).