《量子信息基础》2024.3.28 随堂作业：

（2024.4.6 晚 22 点前提交）

1. (Text book* Problem 3.27)

   Let $\hat{Q}$ be an operator with a complete set of orthonormal eigenvectors:

$$\hat{Q}|e_n\rangle = q_n|e_n\rangle$$

   (a) Show that $\hat{Q}$ can be written in terms of its spectral decomposition:

$$\hat{Q} = \sum_n q_n|e_n\rangle\langle e_n|$$

   Hint: An operator is characterized by its action on all possible vectors, so what you must show is that

$$\hat{Q}|\alpha\rangle = \left\{\sum_n q_n|e_n\rangle\langle e_n|\right\}|\alpha\rangle$$

   for any $|\alpha\rangle$.

Define an arbitrary vector $|\alpha\rangle$

$$|\alpha\rangle = \sum_n c_n\,|e_n\rangle$$

$$\hat{Q}|\alpha\rangle = \sum_n c_n\,\hat{Q}|e_n\rangle = \sum_n \langle e_n|\alpha\rangle \cdot q_n|e_n\rangle = \left\{\sum_n q_n|e_n\rangle\langle e_n|\right\}|\alpha\rangle$$

$$\hat{Q} = \sum_n q_n|e_n\rangle\langle e_n|$$

推导和答案正确给 30 分

   (b) Another way to define a function of $\hat{Q}$ is via the spectral decomposition:

$$f(\hat{Q}) = \sum_n f(q_n)|e_n\rangle\langle e_n|$$

   Show that this is equivalent to

$$e^{\hat{Q}} \equiv 1 + \hat{Q} + \frac{1}{2}\hat{Q}^2 + \frac{1}{3!}\hat{Q}^3 + \cdots$$

   when $f(\hat{Q}) = e^{\hat{Q}}$.

$$f(\hat{Q}) = e^{\hat{Q}} = \sum_n e^{q_n}|e_n\rangle\langle e_n| = \sum_n \left(1 + q_n + \frac{1}{2}q_n^2 + \frac{1}{3!}q_n^3 + \cdots\right)|e_n\rangle\langle e_n|$$
$$= \sum_n |e_n\rangle\langle e_n| + \sum_n q_n|e_n\rangle\langle e_n| + \frac{1}{2}\sum_n q_n^2|e_n\rangle\langle e_n| + \frac{1}{3!}\sum_n q_n^3|e_n\rangle\langle e_n|$$
$$= 1 + \hat{Q} + \frac{1}{2}\hat{Q}^2 + \frac{1}{3!}\hat{Q}^3 + \cdots$$

1应该是单位矩阵？

推导和答案正确给 30 分

2. An operator $\hat{Q}$ has the complete sets of Eigen wave functions $\{|a_n\rangle\}$ and $\{|b_n\rangle\}$ in the A and B representations respectively. Assuming $\{|a_n\rangle\}$ and $\{|b_n\rangle\}$ are connected by unitary transformation

$$|b_n\rangle = \hat{U}|a_n\rangle$$

prove that

$$\hat{Q}_{(B)} = \hat{U}\hat{Q}_{(A)}\hat{U}^\dagger$$

见课3-3 这里表示的是本征值/平均值

$$\langle b_n|\hat{Q}_{(B)}|b_n\rangle = \langle a_n|\hat{U}^\dagger\hat{Q}_{(B)}\hat{U}|a_n\rangle = \langle a_n|\hat{Q}_{(A)}|a_n\rangle$$

$$\hat{U}^\dagger\hat{Q}_{(B)}\hat{U} = \hat{Q}_{(A)}$$

$$\therefore \hat{Q}_{(B)} = \hat{U}\hat{Q}_{(A)}\hat{U}^\dagger$$

推导和答案正确给 40 分

* David J. Griffiths, and Darrell F. Schroeter, Introduction to Quantum Mechanics (3rd Edition), Cambridge University Press (2018).