第一次作业：第二章 3、4、7

3、

设 $X(t) = At + (1 - |A|)B, t \geqslant 0$, 这里 A 和 B 独立同分布,$P(A = 0) = P(A = 1) = P(A = -1) = \frac{1}{3}$

(1) 写出 $\{X(t)\}$ 的所有样本函数;

(2) 计算 $P(X(1) = 1), P(X(2) = 1), P(X(1) = 1, X(2) = 1)$.

解:

$$X(t) = At + (1 - |A|)B, t \geqslant 0$$

$$P(A = 0) = P(A = 1) = P(A = -1) = \frac{1}{3}, P(B = 0) = P(B = 1) = P(B = -1) = \frac{1}{3}$$

(1)

$$A = 0, B = 0, X_1(t) = 0$$

$$A = 0, B = 1, X_2(t) = 1$$

$$A = 0, B = -1, X_3(t) = -1$$

$$A = 1, X_4(t) = t$$

$$A = -1, X_5(t) = -t$$

(2)

$$P(X(1) = 1) = P(A = 0, B = 1) + P(A = 1) = \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} = \frac{4}{9}$$

$$P(X(2) = 1) = P(A = 0, B = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

$$P(X(1) = 1, X(2) = 1) = P(A = 0, B = 1) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$$

4、

设 $X(t)=AXt+1-A, t\geqslant 0$, 这里 A 和 X 相互独立,$P(A=0)=P(A=1)=\dfrac{1}{2}, X\sim N(1,1)$

(1) 计算 $P(Z(1)<1), P(Z(2)<2), P(Z(1)<1, Z(2)<2)$;

(2) 计算 $\mu_z(t), R_z(s,t)$.

解:

$$Z(t)=AXt+1-A, t\geqslant 0, P(A=0)=P(A=1)=\frac{1}{2}, X\sim N(1,1)$$

(1)

$$Z_1(t)=1, Z_2(t)=Xt$$
$$P(Z(1)<1)=P(Z_2(t)|A=1)=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$$
$$P(Z(2)<2)=P(Z_2(t)|A=1)+P(Z_1(t)|A=0)$$
$$=\frac{1}{2}\times P(2X<2)+\frac{1}{2}$$
$$=\frac{1}{2}\times\frac{1}{2}+\frac{1}{2}=\frac{3}{4}$$

(2)

$$\mu_z(t)=E(Z(t))=\frac{1}{2}+\frac{1}{2}t=\frac{t+1}{2}$$
$$R_z(s,t)=C_z(s,t)+\mu_z(t)\mu_z(s)=\frac{1}{2}+st$$

7、

甲、乙两人在玩一种游戏, 用 V_n 表示前 n 次甲赢的总次数, W_n 表示甲恰好赢 n 次的时刻。则对任何 $k,n\geqslant 1$, 事件 $\{W_k>n\},\{W_k\geqslant n\},\{W_k<n\},\{W_k\leqslant n\}$ 分别与下列哪个事件相等:

(A) $\{V_n\leqslant k\}$ (B) $\{V_n<k\}$ (C) $\{V_n>k\}$ (D) $\{V_n\geqslant k\}$

(E) $\{V_{n-1}\leqslant k\}$ (F) $\{V_{n-1}<k\}$ (G) $\{V_{n-1}>k\}$ (H) $\{V_{n-1}\geqslant k\}$

解:

$$\{W_k>n\}\leftrightarrow\{V_n<k\}$$
$$\{W_k\geqslant n\}\leftrightarrow\{V_{n-1}<k\}$$
$$\{W_k<n\}\leftrightarrow\{V_{n-1}\geqslant k\}$$
$$\{W_k\leqslant n\}\leftrightarrow\{V_n\geqslant k\}$$