《量子信息基础》2024.4.25 随堂作业：

（2024.4.30 晚 22 点前提交）

1. In the EPRB experiment, state whether each of the following states is entangled (i.e. can it be factored into a product of states of the individual particles, in which case it is not entangled)
   (a) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |0_1, 0_2\rangle)$
   (b) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |1_1, 0_2\rangle)$
   (c) $\frac{3}{5}|0_1, 1_2\rangle + \frac{4i}{5}|1_1, 1_2\rangle$
   (d) $\frac{1}{2}(|0_1, 0_2\rangle + |0_1, 1_2\rangle + |1_1, 0_2\rangle + |1_1, 1_2\rangle)$

(a) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |0_1, 0_2\rangle) = \frac{1}{\sqrt{2}}|0_1\rangle(|1_2\rangle - |0_2\rangle)$

Not entangled

推导和答案正确给 20 分

(b) $\frac{1}{\sqrt{2}}(|0_1, 1_2\rangle - |1_1, 0_2\rangle)$

Entangled

推导和答案正确给 20 分

(c) $\frac{3}{5}|0_1, 1_2\rangle + \frac{4i}{5}|1_1, 1_2\rangle = \left(\frac{3}{5}|0_1\rangle + \frac{4i}{5}|1_1\rangle\right)|1_2\rangle$

Not entangled

推导和答案正确给 20 分

(d) $\frac{1}{2}(|0_1, 0_2\rangle + |0_1, 1_2\rangle + |1_1, 0_2\rangle + |1_1, 1_2\rangle) = \frac{1}{2}\big(|0_1\rangle(|0_2\rangle + |1_2\rangle) + |1_1\rangle(|0_2\rangle + |1_2\rangle)\big) = \frac{1}{2}(|0_1\rangle + |1_1\rangle)(|0_2\rangle + |1_2\rangle)$

Not entangled

推导和答案正确给 20 分

2. Consider a Bell state $\frac{1}{\sqrt{2}}(|0_1, 0_2\rangle + |1_1, 1_2\rangle)$ of two photons.
   (a) Express it now on a basis of $|\theta\rangle$ and $|\theta + \pi/2\rangle$ by rotating the original basis by an angle θ. Hint: on such a basis, $|1\rangle = \sin\theta|\theta\rangle + \cos\theta\,|\theta + \pi/2\rangle$ and also a similar expression for $|0\rangle$.
   (b) In the new basis of such sate, show that the two photons will always come out of the same arm of each polarizer when two aligned polarizers are used to examine the pair of photons.

(a)

$$|1\rangle = \sin\theta|\theta\rangle + \cos\theta\,|\theta+\pi/2\rangle$$
$$|0\rangle = \cos\theta|\theta\rangle - \sin\theta\,|\theta+\pi/2\rangle$$

$$\begin{aligned}\langle 1|0\rangle &= (\langle\theta|\sin\theta + \langle\theta+\pi/2|\cos\theta)(\cos\theta|\theta\rangle - \sin\theta\,|\theta+\pi/2\rangle)\\ &= \sin\theta\cos\theta - \cos\theta\sin\theta = 0\end{aligned}$$

$$\begin{aligned}
&\frac{1}{\sqrt{2}}(|0_1,0_2\rangle + |1_1,1_2\rangle)\\
&\quad= \frac{1}{\sqrt{2}}\left(\cos\theta|\theta_1\rangle - \sin\theta\left|\theta_1+\frac{\pi}{2}\right\rangle\right)\left(\cos\theta|\theta_2\rangle - \sin\theta\left|\theta_2+\frac{\pi}{2}\right\rangle\right)\\
&\quad+ \frac{1}{\sqrt{2}}\left(\sin\theta|\theta_1\rangle + \cos\theta\left|\theta_1+\frac{\pi}{2}\right\rangle\right)\left(\sin\theta|\theta_2\rangle + \cos\theta\left|\theta_2+\frac{\pi}{2}\right\rangle\right)\\
&\quad= \frac{1}{\sqrt{2}}\cos\theta\cos\theta|\theta_1,\theta_2\rangle \cancel{- \frac{1}{\sqrt{2}}\cos\theta\sin\theta\left|\theta_1,\theta_2+\frac{\pi}{2}\right\rangle}\\
&\quad\cancel{- \frac{1}{\sqrt{2}}\sin\theta\cos\theta\left|\theta_1+\frac{\pi}{2},\theta_2\right\rangle} + \frac{1}{\sqrt{2}}\sin\theta\sin\theta\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle\\
&\quad+ \frac{1}{\sqrt{2}}\sin\theta\sin\theta|\theta_1,\theta_2\rangle \cancel{+ \frac{1}{\sqrt{2}}\sin\theta\cos\theta\left|\theta_1,\theta_2+\frac{\pi}{2}\right\rangle}\\
&\quad\cancel{+ \frac{1}{\sqrt{2}}\cos\theta\sin\theta\left|\theta_1+\frac{\pi}{2},\theta_2\right\rangle} + \frac{1}{\sqrt{2}}\cos\theta\cos\theta\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle\\
&\quad= \frac{1}{\sqrt{2}}|\theta_1,\theta_2\rangle + \frac{1}{\sqrt{2}}\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle
\end{aligned}$$

(b)

There are only the $|\theta_1,\theta_2\rangle$ or $\left|\theta_1+\frac{\pi}{2},\theta_2+\frac{\pi}{2}\right\rangle$ states. So the photons will only come out of the same arm of the polarizers.

(a)和(b)推导和答案正确给 20 分