

## 第一章 引言——波与矢量分析

1. 1

设  $\vec{E} = E_y \vec{y}_0 = \vec{y}_0 10^{-3} \cos(2\pi \times 10^6 t + 2\pi \times 10^{-2} x) \text{ V/m}$ , 求矢量  $\vec{E}$  的方向, 波的传播方向, 波的幅度, 频率  $f$ , 相位常数  $k$ , 相速度  $v_p$ .

解:  $\vec{E} = E_x \vec{x}_0 + E_y \vec{y}_0 + E_z \vec{z}_0 = E_y \vec{y}_0 = \vec{y}_0 10^{-3} \cos(2\pi \times 10^6 t + 2\pi \times 10^{-2} x) \text{ V/m}$

$\therefore$  矢量  $\mathbf{E}$  的方向是沿  $Y$  轴方向, 波的传播方向是  $-x$  方向;

波的幅度  $|\mathbf{E}| = E_y = 10^{-3} \text{ V/m}$

$$f = \frac{\omega}{2\pi} = \frac{2\pi \times 10^6}{2\pi} = 10^6 \text{ Hz} = 1 \text{ MHz};$$

$$k = 2\pi \times 10^{-2};$$

$$V_p = \frac{\omega}{k} = \frac{2\pi \times 10^6}{2\pi \times 10^{-2}} = 10^8 \text{ m/s}.$$

1. 2 写出下列时谐变量的复数表示 (如果可能的话)

$$(1) V(t) = 6 \cos(\omega t + \frac{\pi}{4})$$

$$(2) I(t) = -8 \sin \omega t$$

$$(3) A(t) = 3 \sin \omega t - 2 \cos \omega t$$

$$(4) C(t) = 6 \cos(120\pi t - \frac{\pi}{2})$$

$$(5) D(t) = 1 - \cos \omega t$$

$$(6) U(t) = \sin(\omega t + \frac{\pi}{3}) \sin(\omega t + \frac{\pi}{6})$$

(1) 解:  $\therefore \phi_v(z) = \pi/4$

$$\therefore V = 6e^{j\frac{\pi}{4}} = 6 \cos \frac{\pi}{4} + 6j \sin \frac{\pi}{4} = 3\sqrt{2} + 3\sqrt{2}j$$

(2) 解:  $I(t) = -8 \cos(\omega t - \frac{\pi}{2})$

$$\phi_v(z) = -\frac{\pi}{2}$$

$$\therefore I = -8e^{-j\frac{\pi}{2}} = 8j$$

(3) 解:  $A(t) = \sqrt{13}(\frac{3}{\sqrt{13}} \sin \omega t - \frac{2}{\sqrt{13}} \cos \omega t)$

令  $\cos \theta = \frac{3}{\sqrt{13}}$  则  $A(t) = \sqrt{13} \cos(\omega t + \theta - \frac{\pi}{2})$

$\therefore \varphi_v(z) = \theta - \frac{\pi}{2}$

则  $A = \sqrt{13}e^{j(\theta - \frac{\pi}{2})} = -2 - 3j$

(4) 解:  $C(t) = 6 \cos(120\pi t - \frac{\pi}{2})$

$\therefore C = 6e^{j\frac{\pi}{2}} = -6j$

(5)(6)两个分量频率不同,不可用复数表示

1. 3 由以下复数写出相应的时谐变量]

(1)  $C = 1 + j$

(2)  $C = 4 \exp(j0.8)$

(3)  $C = 3 \exp(j\frac{\pi}{2}) + 4 \exp(j0.8)$

(1) 解:

$(1 + j)e^{j\omega t} = (1 + j)(\cos \omega t + j \sin \omega t) = \cos \omega t + j \sin \omega t + j \cos \omega t - \sin \omega t$

$\therefore C(t) = \text{RE}(Ce^{j\omega t}) = \cos \omega t - \sin \omega t$

(2) 解:  $C(t) = \text{RE}(Ce^{j\omega t}) = \text{RE}(4e^{j0.8} e^{j\omega t}) = 4 \cos(\omega t + 0.8)$

(3) 解:  $Ce^{j\omega t} = (3e^{j\frac{\pi}{2}} + 4e^{j0.8})e^{j\omega t} = 3e^{j(\omega t + \frac{\pi}{2})} + 4e^{j(\omega t + 0.8)}$

得:  $C(t) = \text{RE}(Ce^{j\omega t}) = 3 \cos(\omega t + \frac{\pi}{2}) + 4 \cos(\omega t + 0.8) = 4 \cos(\omega t + 0.8) - 3 \cos(\omega t)$

1. 4

假定  $\vec{A} = \vec{x}_0 + j\vec{y}_0 + (1 + j2)\vec{z}_0, \vec{B} = -\vec{x}_0 - (1 + 2j)\vec{y}_0 + j\vec{z}_0,$

求:  $\vec{A} \cdot \vec{B}, \vec{A} \times \vec{B}, \vec{A} \cdot \vec{B}^*, \text{Re}[\vec{A} \times \vec{B}^*]$

解:  $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = -1$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (-4 + j4)\vec{x}_0 + (-1 - 3j)\vec{y}_0 + (-1 - j)\vec{z}_0$$

$$\mathbf{B}^* = -\vec{x}_0 - (1 - 2j)\vec{z}_0 \text{ 则: } \mathbf{A} \cdot \mathbf{B}^* = -1 - 2j$$

$$\mathbf{A} \times \mathbf{B}^* = \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ 1 & j & 1 + 2j \\ -1 & -(1 - 2j) & -j \end{vmatrix} \text{ 得到: } \operatorname{RE}(\mathbf{A} \times \mathbf{B}^*) = 6\vec{x}_0 - \vec{y}_0 - \vec{z}_0$$

1. 5 计算下列标量场的梯度

$$(1) u = x^2 y^2 z^2$$

$$(2) u = 2x^2 + y^2 - z^2$$

$$(3) u = xy + yz + xz$$

$$(4) u = x^2 + y^2 + 2xy$$

$$(5) u = xyz$$

$$(1) \text{ 解: } \operatorname{grad}(u) = \nabla u$$

$$= \frac{\partial x^2 y^2 z^2}{\partial x} \vec{x}_0 + \frac{\partial x^2 y^2 z^2}{\partial y} \vec{y}_0 + \frac{\partial x^2 y^2 z^2}{\partial z} \vec{z}_0$$

$$= 2xy^2 z^2 \vec{x}_0 + 2x^2 yz^2 \vec{y}_0 + 2x^2 y^2 z \vec{z}_0$$

$$(2) \text{ 解: } \operatorname{grad}(u) = \nabla u$$

$$= 4x\vec{x}_0 + 2y\vec{y}_0 - 2z\vec{z}_0$$

$$(3) \text{ 解: } \operatorname{grad}(u) = \nabla u$$

$$= (y + z)\vec{x}_0 + (x + z)\vec{y}_0 + (y + x)\vec{z}_0$$

$$(4) \text{ 解: } \operatorname{grad}(u) = \nabla u$$

$$= (2x + 2y)\vec{x}_0 + (2y + 2x)\vec{y}_0$$

$$(5) \text{ 解: } \operatorname{grad}(u) = \nabla u$$

$$= yz\vec{x}_0 + xz\vec{y}_0 + xy\vec{z}_0$$

1. 6 求曲面  $z = x^2 + y^2$  在点 (1,1,2) 处的法线方向

解：梯度的方向就是电位变化最陡的方向

$$\text{令 } T = x^2 + y^2 - z$$

则  $\nabla T = 2x\vec{x}_0 + 2y\vec{y}_0 - \vec{z}_0$  将点(1,1,2)代入得：

法线方向与  $2\vec{x}_0 + 2\vec{y}_0 - \vec{z}_0$  同向

1. 7 求下列矢量场的散度，旋度

$$(1) \vec{A} = x^2 \vec{x}_0 + y^2 \vec{y}_0 + z^2 \vec{z}_0$$

$$(2) \vec{A} = (y+z)\vec{x}_0 + (x+z)\vec{y}_0 + (x+y)\vec{z}_0$$

$$(3) \vec{A} = (x+y)\vec{x}_0 + (x^2 + y^2)\vec{y}_0$$

$$(4) \vec{A} = 5\vec{x}_0 + 6yz\vec{y}_0 + x^2 \vec{z}_0$$

$$(1) \text{ 解: } \operatorname{div}(\vec{A}) = \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$= 2x + 2y + 2z$$

$$\operatorname{curl}(\vec{A}) = \nabla \times \vec{A} = \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^2 & z^2 \end{vmatrix} = 0$$

$$(2) \text{ 解: } \operatorname{div}(\vec{A}) = 0$$

$$\operatorname{curl}(\vec{A}) = 0$$

$$(3) \text{ 解: } \operatorname{div}(\vec{A}) = 1 + 2y$$

$$\operatorname{curl}(\vec{A}) = \nabla \times \vec{A} = \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & x^2+y^2 & 0 \end{vmatrix} = (2x-1)\vec{z}_0$$

$$(4) \text{ 解: } \operatorname{div}(\vec{A}) = 6z$$

$$\operatorname{curl}(\vec{A}) = \nabla \times \vec{A} = \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 5 & 6yz & x^2 \end{vmatrix} = -6y\vec{x}_0 - 2x\vec{y}_0$$

1.11

若矢量场  $\vec{A} = x\vec{x}_0$ ，求  $\oint_S \vec{A} \cdot d\vec{S}$ ，其中  $\vec{S}$  是由  $x^2 + y^2 = r^2, z=0, z=h$  组成的闭合曲面

解：由散度定理可得：

$$\begin{aligned}\oint_S \vec{A} \cdot d\vec{S} &= \int_V (\nabla \cdot \vec{A}) dV \quad (V \text{ 为 } x^2 + y^2 = r^2, z = h \text{ 围成的体积}) \\ &= \int_V [\nabla \cdot (x\vec{x}_0)] dV \\ &= \int_V dV \\ &= \pi r^2 h\end{aligned}$$

1. 12

假定  $\vec{A} = A_x \vec{x}_0 + A_y \vec{y}_0 + A_z \vec{z}_0$ ,  $\vec{B} = B_x \vec{x}_0 + B_y \vec{y}_0 + B_z \vec{z}_0$ , 试证明：

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

证明：  $\nabla \cdot (\vec{A} \times \vec{B})$

$$\begin{aligned}&= \nabla \cdot \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \\ &= \nabla \cdot [\vec{x}_0 (A_y B_z - A_z B_y) + \vec{y}_0 (A_z B_x - A_x B_z) + \vec{z}_0 (A_x B_y - A_y B_x)] \\ &= \frac{\partial (A_y B_z - A_z B_y)}{\partial x} + \frac{\partial (A_z B_x - A_x B_z)}{\partial y} + \frac{\partial (A_x B_y - A_y B_x)}{\partial z} \\ &= \frac{B_z \partial A_y - B_y \partial A_z + A_y \partial B_z - A_z \partial B_y}{\partial x} + \frac{B_x \partial A_z - B_z \partial A_x + A_z \partial B_y - A_y \partial B_z}{\partial y} \\ &\quad + \frac{B_y \partial A_x - B_x \partial A_y + A_x \partial B_y - A_y \partial B_x}{\partial z} \\ &= B_x \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + B_y \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + B_z \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \\ &\quad - A_x \left( \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) - A_y \left( \frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) - A_z \left( \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \\ &= \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})\end{aligned}$$

1. 13

证明：

$$(1) \nabla \cdot (\Phi \vec{A}) = \vec{A} \cdot \nabla \Phi + \Phi \nabla \cdot \vec{A}$$

$$(2) \nabla \times (\Phi \vec{A}) = \nabla \Phi \times \vec{A} + \Phi \nabla \times \vec{A}$$

(1) 证明：

$$\begin{aligned}
\text{左边} &= \nabla \cdot (\Phi A_x \vec{x}_0 + \Phi A_y \vec{y}_0 + \Phi A_z \vec{z}_0) \\
&= \frac{\partial \Phi A_x}{\partial x} + \frac{\partial \Phi A_y}{\partial y} + \frac{\partial \Phi A_z}{\partial z} \\
\text{右边} &= (A_x \vec{x}_0 + A_y \vec{y}_0 + A_z \vec{z}_0) \cdot \left( \frac{\partial \Phi}{\partial x} \vec{x}_0 + \frac{\partial \Phi}{\partial y} \vec{y}_0 + \frac{\partial \Phi}{\partial z} \vec{z}_0 \right) + \Phi \left( \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) \\
&= \frac{\Phi \partial A_x + A_x \partial \Phi}{\partial x} + \frac{\Phi \partial A_y + A_y \partial \Phi}{\partial y} + \frac{\Phi \partial A_z + A_z \partial \Phi}{\partial z} \\
&= \frac{\partial \Phi A_x}{\partial x} + \frac{\partial \Phi A_y}{\partial y} + \frac{\partial \Phi A_z}{\partial z} \\
&\therefore \text{左边} = \text{右边} \\
&\text{证毕}
\end{aligned}$$

(2) 证明:

$$\begin{aligned}
\text{左边} &= \nabla \times (\Phi \vec{A}) = \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \Phi A_x & \Phi A_y & \Phi A_z \end{vmatrix} \\
\text{右边} &= \nabla \Phi \times \vec{A} + \Phi \nabla \times \vec{A} \\
&= \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial \Phi}{\partial x} & \frac{\partial \Phi}{\partial y} & \frac{\partial \Phi}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} + \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \Phi \frac{\partial}{\partial x} & \Phi \frac{\partial}{\partial y} & \Phi \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \text{左边} \\
&\text{证毕}
\end{aligned}$$

1. 14

证明:

$$(1) \nabla \cdot (\nabla \times \vec{A}) = 0$$

$$(2) \nabla \times (\nabla \Phi) = 0$$

(1) 证明:

$$\begin{aligned}
\nabla \cdot (\nabla \times \vec{A}) &= \nabla \cdot \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \\
&= \nabla \cdot [\vec{x}_0 (\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}) + \vec{y}_0 (\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}) + \vec{z}_0 (\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y})] \\
&= \frac{\partial^2 A_z}{\partial x \partial y} - \frac{\partial^2 A_y}{\partial x \partial z} + \frac{\partial^2 A_x}{\partial z \partial y} - \frac{\partial^2 A_z}{\partial x \partial y} + \frac{\partial^2 A_y}{\partial x \partial z} - \frac{\partial^2 A_x}{\partial z \partial y} \\
&= 0
\end{aligned}$$

证毕

(2) 证明:

$$\nabla \times (\nabla \Phi) = \nabla \times (\frac{\partial \Phi}{\partial x} \vec{x}_0 + \frac{\partial \Phi}{\partial y} \vec{y}_0 + \frac{\partial \Phi}{\partial z} \vec{z}_0)$$

$$= \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \Phi}{\partial x} & \frac{\partial \Phi}{\partial y} & \frac{\partial \Phi}{\partial z} \end{vmatrix} = 0$$

证毕

2. 1

市话用的平行双导线，测得其分布电路参数为：

$$R' = 0.042 \Omega / m$$

$$L' = 5 \times 10^{-7} H / m$$

$$G' = 5 \times 10^{-10} S / m$$

$$C' = 30.5 pF / m$$

求传播常数 $k$ 与特征阻抗 $Z_c$ 。

解： 
$$jk = \sqrt{(R' + j\omega L')(G' + j\omega C')}$$

$$Z_c = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$$

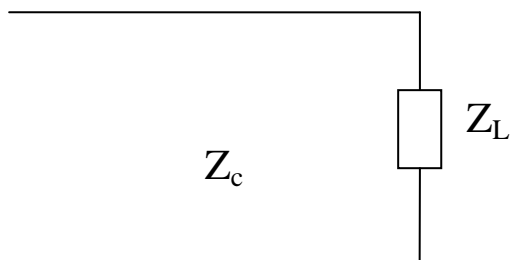
将数据代入解得（以 50Hz 代入，不是很正确）：

$$k = (16.8 - j19.6) \times 10^{-8}$$

$$Z_c = (1.5 - j1.44) \times 10^{-3} \Omega$$

2. 2

参看图， $Z_L = 80, Z_c = 50$ ，负载 $Z_L$ 电压 $5V$ ，求驻波系数 $\rho$ ，驻波最小点位置 $d_{\min} / \lambda$ ，传输线长度 $l = \lambda/4, \lambda/2, 3\lambda/8$ 处的输入阻抗以及 $V_{\max}, V_{\min}, I_{\max}, I_{\min}$ 。



解：（1）由题意可得：

$$\Gamma_v(0) = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{80 - 50}{80 + 50} = \frac{3}{13}$$

$$\rho = \frac{1 + |\Gamma_v(0)|}{1 - |\Gamma_v(0)|} = \frac{1 + \frac{3}{13}}{1 - \frac{3}{13}} = 1.6$$



$$(2) \Gamma_v(0) = \frac{3}{13} \text{ 即 } \psi(0) = 0$$

$$\therefore \frac{d_{\min}}{\lambda} = \frac{d_{\max}}{\lambda} + \frac{1}{4} = \frac{1}{4}$$

$$(3) l = \frac{\lambda}{4} \text{ 时 } kl = \frac{2\pi}{\lambda} l = \frac{\pi}{2}$$

$$Z_{in} = Z_C \frac{Z_L + jZ_C \tan kl}{Z_C + jZ_L \tan kl} = 50 \frac{80 + j50 \tan \frac{\pi}{2}}{50 + j80 \tan \frac{\pi}{2}} = 31.25\Omega$$

$$l = \frac{\lambda}{2} \text{ 时 } kl = \frac{2\pi}{\lambda} l = \pi$$

$$Z_{in} = Z_C \frac{Z_L + jZ_C \tan kl}{Z_C + jZ_L \tan kl} = 80\Omega$$

$$l = \frac{3\lambda}{8} \text{ 时 } kl = \frac{2\pi}{\lambda} l = \frac{3}{4}\pi$$

$$Z_{in} = Z_C \frac{Z_L + jZ_C \tan kl}{Z_C + jZ_L \tan kl} = -50j\Omega$$

$$(4) \left| \frac{V(0)}{V_i} \right| = |1 + \Gamma_v(0)| = \left| 1 + \frac{3}{13} \right| = \frac{16}{13} = \frac{5}{|V_i|}$$

$$\text{可得: } |V_i| = \frac{65}{16}$$

$$V_{\max} = |V_i| [1 + \Gamma_v(0)] = \frac{65}{16} \left( 1 + \frac{3}{13} \right) = 5V$$

$$V_{\min} = |V_i| [1 - \Gamma_v(0)] = \frac{65}{16} \left( 1 - \frac{3}{13} \right) = 3.125V$$

$$I_{\max} = \frac{V_{\max}}{Z_C} = 0.1A$$

$$I_{\min} = \frac{V_{\min}}{Z_C} = 0.0625A$$

2. 3

无线传输线特征阻抗  $Z_C = 50$ , 负载阻抗  $Z_L = 5\sqrt{25.99^\circ}$ ,  
求传输线长度  $l = \lambda/8, \lambda/4, 3\lambda/8$  处输入阻抗

$$\text{解: 当 } l = \frac{\lambda}{8}, \frac{\lambda}{4}, \frac{3\lambda}{8} \text{ 处 } kl = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$$

$$Z_{in} = Z_c \frac{Z_L + jZ_c \tan kl}{Z_c + jZ_L \tan kl} = 50 \frac{5\sqrt{25.99^\circ} + j50 \tan \frac{\pi}{4}}{50 + j5\sqrt{25.99^\circ} \tan \frac{\pi}{4}} = (9.1 + j53.26)\Omega$$

$$Z_{in} = Z_c \frac{Z_L + jZ_c \tan kl}{Z_c + jZ_L \tan kl} = 50 \frac{5\sqrt{25.99^\circ} + j50 \tan \frac{\pi}{2}}{50 + j5\sqrt{25.99^\circ} \tan \frac{\pi}{2}} = 50\Omega$$

$$Z_{in} = Z_c \frac{Z_L + jZ_c \tan kl}{Z_c + jZ_L \tan kl} = 50 \frac{5\sqrt{25.99^\circ} + j50 \tan \frac{3\pi}{4}}{50 + j5\sqrt{25.99^\circ} \tan \frac{3\pi}{4}} = (8.26 - j45.44)\Omega$$

2. 4

传输线终端归一化阻抗  $z_L = 0.8 + j1.0$ , 求:

- (1) 驻波系数  $\rho$
- (2) 离开驻波第一个最小点位置  $d_{\min}$
- (3) 负载反射功率与入射功率之比

解:  $\Gamma_v(0) = \frac{Z_L - Z_c}{Z_L + Z_c} = \frac{z_L - 1}{z_L + 1} = \frac{-0.2 + j1.0}{1.8 + j1.0} = |\Gamma_v(0)|e^{j\psi(0)}$

$$(1) \rho = \frac{1 + |\Gamma_v(0)|}{1 - |\Gamma_v(0)|} = 2.96$$

$$(2) d_{\min} = \frac{\psi(0)}{4\pi} \lambda + \frac{\lambda}{4} = 0.35\lambda$$

$$(3) \frac{P_r}{P_i} = |\Gamma_v(0)|^2 = \frac{1}{4}$$

2. 6

传输线特征阻抗为  $50\Omega$ , 终端开路, 测得始端输入阻抗为  $j33\Omega$ , 求传输线以波长计的电长度  $l/\lambda$

解: 终端开路时:

$$Z_{in} = -jZ_C \cot kl = 33j$$

$$\therefore \tan kl = -\frac{50}{33}$$

$$kl = \pi - \arctan \frac{50}{33}$$

$$\text{得: } \frac{l}{\lambda} = \frac{1}{2} - \frac{\arctan(\frac{50}{33})}{2\pi} = 0.343$$

2. 8

在无耗线上测得:  $Z_{in}^{sc}$  为  $j100$ ,  $Z_{in}^{oc}$  为  $-j25$ ,  $d_{\min}$  为  $0.1\lambda, 0.6\lambda, \dots$ , 驻波系数  $\rho = 3$ , 求负载阻抗

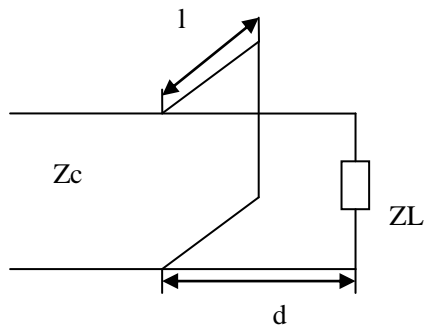
$$\text{解: } |\Gamma_v| = \frac{\rho - 1}{\rho + 1} = \frac{3 - 1}{3 + 1} = 0.5$$

$$d_{\min} = 0.1\lambda = \frac{\psi(0)\lambda}{4\pi} + \frac{\lambda}{4} \Rightarrow \psi(0) = -0.6\pi$$

$$Z_C = \sqrt{Z_{in}^{oc} \cdot Z_{in}^{sc}} = \sqrt{j100 \cdot j(-25)} = 50\Omega$$

$$\text{即 } Z_L = Z_C \frac{1 + \Gamma_v(0)}{1 - \Gamma_v(0)} = 50 \frac{1 + 0.5e^{-0.6\pi j}}{1 - 0.5e^{-0.6\pi j}}$$

2. 9



如图,  $Z_L = (30 + j60)\Omega$ ,  $Z_C = 50\Omega$ , 用可移动单可变电纳匹配器进行匹配, 用圆图决定可变电纳匹配器到负载  $Z_L$  的距离  $d$ , 以及并联短路支线长度。

解: 归一化阻抗:

$$z_L = \frac{Z_L}{Z_C} = 0.6 + j1.2$$

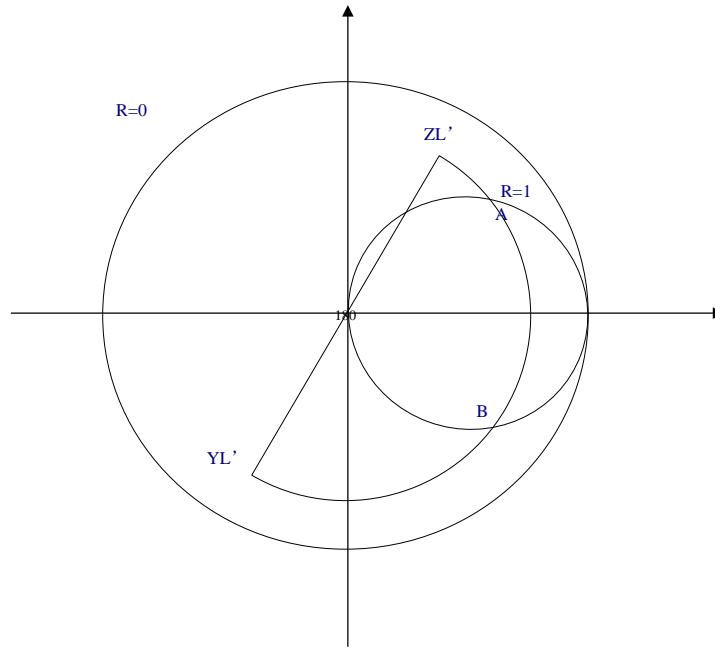
$\therefore y_L = 0.33 - j0.66$ , 由阻抗圆图可读出:

$$d_A = 0.28\lambda, \quad d_B = 0.42\lambda$$

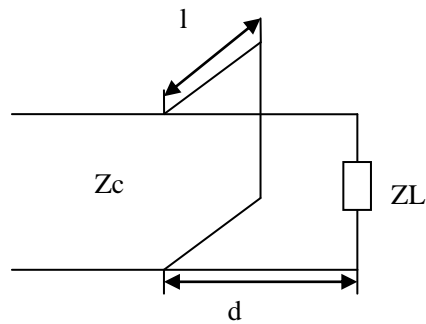
旋转后:  $y_A' = 1 + b_A j, \quad y_B' = 1 + b_B j$

在短路图中:  $y_{2A}' = -b_A j, \quad y_{2B}' = -b_B j$

由图可得:  $l_A = 0.087\lambda, \quad l_B = 0.432\lambda$



2.10



特征阻抗 $Z_C = 50\Omega$ 传输线, 终端接负载 $Z_L = (60 + 60j)\Omega$ , 并联短路支线离负载距离 $d = 0.22\lambda$ 。调节并联短路支线长度 $l$ , 求最小驻波系数 $\rho_{\min}$ 。

解: 归一化阻抗:

$$z_L = 1.2 + j1.2$$

通过 $0.22\lambda$ 后其输入阻抗见图 $z_L'$ 其导纳为 $y_L'$

由图可以知道 $y_L' = g + jb = 0.53 + jb_1$

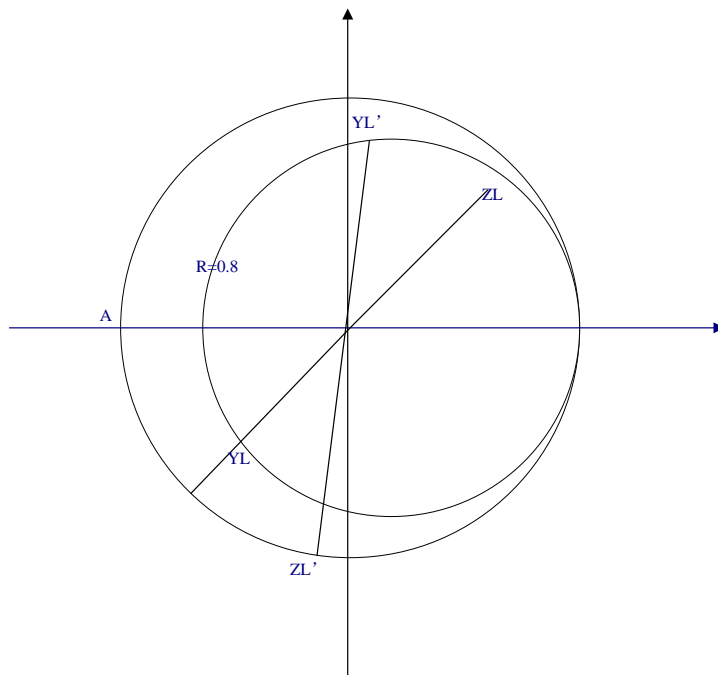
当并联短路支路 $y_2 = jb_2$ 后:

$$y_{in} = 0.53 + j(b_1 + b_2)$$

由圆图可以知道 $y_{in}$ 随着 $y_2$ 的变化围绕 $y = 0.53$ 的等 $g$ 圆旋转当转到A点时 $g$ 最小。

由图可以知道此时:

$$\rho_{\min} = 1.218$$



2. 13

有一空气介质的同轴线需装入介质支撑薄片，薄片材料为聚苯乙烯，其相对介电常数 $\epsilon_r = 2.55$ ,为使介质不引起反射，介质中心 $\phi$ 孔径应该是多少？

解：为了不引起介质反射  $Z_{c1} = Z_{c2}$

$$\text{可得: } \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0}} \ln \frac{7}{2} = \frac{1}{2\pi} \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}} \ln \frac{7}{\phi}$$

$$\text{解得: } \phi = 0.287m$$

### 第三章 麦克斯韦方程

3. 1 求以下几个量的量纲

$$(1) \vec{E} \cdot \vec{D}$$

$$(2) \vec{H} \cdot \vec{B}$$

$$(3) \vec{S}$$

$$\text{解: (1) } \frac{V}{m} \cdot \frac{C}{m^2} = \frac{V \cdot C}{m^3} = \frac{J}{m^3}$$

$$(2) \frac{A}{m} \cdot \frac{Wb}{m^2} = \frac{A \cdot Wb}{m^3} = \frac{J}{m^3}$$

$$(3) \text{ 解: } \frac{V}{m} \cdot \frac{A}{m} = \frac{V \cdot A}{m^2} = \frac{W}{m^2}$$

3. 2 写出以下时谐矢量的复矢量表示

$$(1) \vec{V}(t) = 3 \cos \omega t \vec{x}_0 + 4 \sin \omega t \vec{y}_0 + \cos(\omega t + \frac{\pi}{2}) \vec{z}_0$$

$$(2) \vec{E}(t) = (3 \cos \omega t + 4 \sin \omega t) \vec{x}_0 + 8(\cos \omega t - \sin \omega t) \vec{z}_0$$

$$(3) \vec{H}_t = 0.5 \cos(kz - \omega t) \vec{x}_0$$

$$\text{解: (1) } \vec{V}(\vec{r}) = 3\vec{x}_0 - 4j\vec{y}_0 + j\vec{z}_0$$

$$(2) \vec{E}(t) = 5 \cos(\omega t + \varphi) \vec{x}_0 - 8\sqrt{2} \cos(\omega t - \frac{3}{4}\pi) \vec{z}_0$$

$$\text{其中 } \varphi = \arcsin \frac{4}{5}$$

$$\vec{V}(\vec{r}) = 5e^{\varphi} \vec{x}_0 - 8\sqrt{2}e^{-\frac{3}{4}\pi} \vec{z}_0 = (3 + 4j)\vec{x}_0 + (8 + 8j)\vec{z}_0$$

$$(3) \vec{H}(\vec{r}) = 0.5e^{-kz} \vec{x}_0 = 0.5[\cos(kz) - j \sin(kz)] \vec{x}_0$$

3. 3 从下面的复矢量写出相应的时谐矢量

$$(1) \vec{C} = \vec{x}_0 - j\vec{y}_0$$

$$(2) \vec{C} = j(\vec{x}_0 - j\vec{y}_0)$$

$$(3) \vec{C} = \exp(-jkz) \vec{x}_0 + j \exp(jkz) \vec{y}_0$$

$$\text{解: (1) } \vec{C} = \vec{x}_0 - j\vec{y}_0 = \vec{x}_0 + \vec{y}_0 e^{j(\frac{\pi}{2})}$$

$$\vec{C}(\vec{r}, t) = \vec{x}_0 \cos \omega t + \vec{y}_0 \cos(\omega t - \frac{\pi}{2})$$

$$(2) \vec{C} = j(\vec{x}_0 - j\vec{y}_0) = \vec{x}_0 e^{j\frac{\pi}{2}} + \vec{y}_0$$

$$\vec{C}(\vec{r}, t) = \vec{x}_0 \cos(\omega t + \frac{\pi}{2}) + \vec{y}_0 \cos \omega t$$

$$(3) \quad \vec{C} = \exp(-jkz)\vec{x}_0 + j \exp(jkz)\vec{y}_0$$

$$\vec{C}(\vec{r}, t) = \vec{x}_0 \cos(\omega t - kz) + \vec{y}_0 \cos(\omega t + kz + \frac{\pi}{2})$$

3. 4 无源空间  $\vec{H} = z\vec{y}_0 + y\vec{z}_0$ ,  $\vec{D}$  随时间变化吗?

解:

$$\frac{\partial \vec{D}}{\partial t} = \nabla \times \vec{H} = \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & z & y \end{vmatrix} = \vec{x}_0 (\frac{\partial}{\partial y} y - \frac{\partial}{\partial z} z) = 0$$

$\therefore \vec{D}$  不随时间变化

3. 5 假定  $(\vec{E}_1, \vec{B}_1, \vec{H}_1, \vec{D}_1), (\vec{E}_2, \vec{B}_2, \vec{H}_2, \vec{D}_2)$  分别为源  $(\vec{J}_1, \rho_{V1}), (\vec{J}_2, \rho_{V2})$  激发的 满足麦克斯韦

方程的解。求源为  $(\vec{J}_t = \vec{J}_1 + \vec{J}_2, \rho_{Vt} = \rho_{V1} + \rho_{V2})$  时麦克斯韦方程的解。

解: 由题意可得:

$$\begin{cases} \nabla \times \vec{E}_1 = -j\omega \vec{B}_1 \dots\dots\dots(1) \\ \nabla \times \vec{H}_1 = \vec{J}_1 + j\omega \vec{D}_1 \dots\dots(2) \\ \nabla \cdot \vec{D}_1 = \rho_{V1} \dots\dots\dots(3) \\ \nabla \cdot \vec{B}_1 = 0 \dots\dots\dots(4) \end{cases} \quad \begin{cases} \nabla \times \vec{E}_2 = -j\omega \vec{B}_2 \dots\dots\dots(5) \\ \nabla \times \vec{H}_2 = \vec{J}_2 + j\omega \vec{D}_2 \dots\dots(6) \\ \nabla \cdot \vec{D}_2 = \rho_{V2} \dots\dots\dots(7) \\ \nabla \cdot \vec{B}_2 = 0 \dots\dots\dots(8) \end{cases}$$

分别将 (1) + (5), (2) + (6), (3) + (7), (4) + (8) 可以得到:

$$\begin{cases} \nabla \times \vec{E}_1 + \nabla \times \vec{E}_2 = \nabla \times (\vec{E}_1 + \vec{E}_2) = -j\omega (\vec{B}_1 + \vec{B}_2) \\ \nabla \times (\vec{H}_1 + \vec{H}_2) = \vec{J}_1 + \vec{J}_2 + j\omega (\vec{D}_1 + \vec{D}_2) = \vec{J}_t + j\omega (\vec{D}_1 + \vec{D}_2) \\ \nabla \cdot (\vec{D}_1 + \vec{D}_2) = \rho_{V1} + \rho_{V2} = \rho_{Vt} \\ \nabla \cdot (\vec{B}_1 + \vec{B}_2) = 0 \end{cases}$$

$\therefore$  当源为  $(\vec{J}_t, \rho_{Vt})$  时麦克斯韦方程的解为  $(\vec{E}_1 + \vec{E}_2, \vec{B}_1 + \vec{B}_2, \vec{H}_1 + \vec{H}_2, \vec{D}_1 + \vec{D}_2)$

3.6

如果在某一表面  $\vec{E} = 0$ , 是否就可得出在该表面  $\frac{\partial \vec{B}}{\partial t} = 0$ ? 为什么?

解: 由斯托克斯定理, 在此表面上

$$\oint_l \vec{E} \cdot d\vec{l} = \int_S (\nabla \times \vec{E}) \cdot d\vec{S} (\vec{l} \text{ 为 } \vec{S} \text{ 所包围的一条闭合曲线})$$

又  $\therefore$  在此表面上  $\vec{E} = 0$

$$\therefore \int_S (\nabla \times \vec{E}) \cdot d\vec{S} = 0$$

$$\therefore \text{在此表面上 } (\nabla \times \vec{E}) = \frac{\partial \vec{B}}{\partial t} = C (C \text{ 为常数}) \text{ 并不能得出 } \frac{\partial \vec{B}}{\partial t} = 0$$

3.7

如果在一条直线上  $\vec{E} = 0$ , 是否就可得出  $\nabla \times \vec{E} = 0$ ?

解: 同 3.6 证明方法也不能得出  $\nabla \times \vec{E} = 0$

3.8

对于调幅广播, 频率  $f$  从  $500\text{kHz}$  到  $1\text{MHz}$ , 假定电离层电子浓度  $N = 10^{12} / \text{m}^3$ , 确定电离层有效介电系数  $\epsilon_e$  的变化范围。

解: 由题可得:

$$W_p = \sqrt{\frac{Ne_2}{m\epsilon_0}} = \sqrt{\frac{10^{12} \times (1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times 8.85 \times 10^{-12}}} = 5.64 \times 10^7 (\text{rad} / \text{s})$$

$$f_p = \frac{W_p}{2\pi} = 9\text{MHz}$$

$$\epsilon_e = \epsilon_0 \left(1 - \frac{W_p^2}{W^2}\right) = \epsilon_0 \left(1 - \frac{f_p^2}{f^2}\right)$$

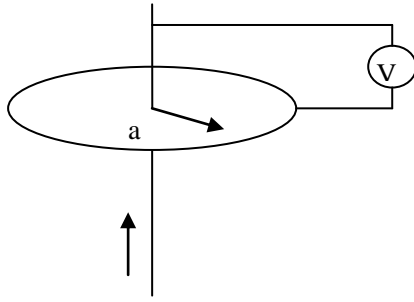
$$\epsilon_{e\max} = 8.85 \times 10^{-12} \left(1 - \left(\frac{9 \times 10^6}{5 \times 10^3}\right)^2\right) = -2.86 \times 10^{-9} (\text{F} / \text{m})$$

$$\epsilon_{e\min} = 8.85 \times 10^{-12} \left(1 - \left(\frac{9 \times 10^6}{1 \times 10^6}\right)^2\right) = -7.08 \times 10^{-10} (\text{F} / \text{m})$$

3. 9

有一半径为  $a$  的导体圆盘以角速度  $\omega$  在均匀磁场中做等速旋转, 设圆盘与磁场互相垂直, 试求该圆盘中心与它边缘之间的感应电动势。





解：由题意可得：

穿过圆盘的磁通量不发生变化

由法拉第电磁感应定律可得整个圆盘是一个等势体

∴圆盘中心与边缘的感应电动势是0

3. 10

一点电荷（电量为 $10^{-5}C$ ）做圆周运动，其角速度 $\omega=1000rad/s$ ，圆周半径 $r=1cm$ ，试求圆心处位移电流密度

解：设 $t=0$ 时 $\varphi=0$

$$\begin{aligned}\vec{D} &= \frac{q}{4\pi r^2}(-\vec{r}_0) = \frac{q}{4\pi r}(-\vec{x}_0 \cos \varphi - \vec{y}_0 \sin \varphi) \\ J_d &= \frac{dD}{dt} = \frac{q\omega}{4\pi r^2}(\vec{x}_0 \sin \omega t - \vec{y}_0 \cos \omega t) \\ &= \frac{10^{-5} \times 10^3}{4\pi(10^{-2})^2}(\vec{x}_0 \sin 100t - \vec{y}_0 \cos 100t) \\ &= 7.96(\vec{x}_0 \sin 100t - \vec{y}_0 \cos 100t)\end{aligned}$$

3. 11

假定 $\vec{E}=(\vec{x}_0 + j\vec{y}_0)e^{-jz}$ ， $H=(\vec{y}_0 - j\vec{x}_0)e^{-jz}$ ，求用 $z, \omega t$ 表示的 $\vec{S}$ 以及 $\langle \vec{S} \rangle$ 。

解： 
$$\vec{E} = (\vec{x}_0 + j\vec{y}_0)e^{-jz} = \vec{x}_0 e^{-jz} + \vec{y}_0 e^{-jz + j\frac{\pi}{2}}$$

$$\vec{E}(\vec{r}, t) = \vec{x}_0 \cos(\omega t - z) + \vec{y}_0 \cos(\omega t - z + \frac{\pi}{2}) = \vec{x}_0 \cos(\omega t - z) - \vec{y}_0 \sin(\omega t - z)$$

$$\vec{H} = (\vec{y}_0 - j\vec{x}_0)e^{-jz} = \vec{y}_0 e^{-jz} + \vec{x}_0 e^{-jz - j\frac{\pi}{2}}$$

$$\vec{H}(\vec{r}, t) = \vec{x}_0 \sin(\omega t - z) + \vec{y}_0 \cos(\omega t - z)$$

$$\begin{aligned}
\vec{S}(\vec{r}, t) &= \vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) \\
&= [\bar{x}_0 \cos(\omega t - z) - \bar{y}_0 \sin(\omega t - z)] \times [\bar{x}_0 \sin(\omega t - z) + \bar{y}_0 \cos(\omega t - z)] \\
&= \bar{z}_0 \cos^2(\omega t - z) \bar{z}_0 + \bar{z}_0 \sin^2(\omega t - z) = \bar{z}_0
\end{aligned}$$

$$\langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{T} \int_0^T \vec{S}(\vec{r}, t) dt = \frac{1}{T} \int_0^T \bar{z}_0 dt = \bar{z}_0$$

$$\begin{aligned}
\langle \vec{S}(\vec{r}, t) \rangle &= \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] \\
&= \frac{1}{2} \text{Re}[(\bar{x}_0 e^{-jz} + \bar{y}_0 e^{-jz + j\frac{\pi}{2}}) \times (\bar{x}_0 e^{jz + j\frac{\pi}{2}} + \bar{y}_0 e^{jz})] \\
&= \frac{1}{2} \text{Re}[\bar{z}_0 - \bar{z}_0 e^{-j\pi}] = \frac{1}{2} \text{Re}[2\bar{z}_0] = \bar{z}_0
\end{aligned}$$

3. 12

$$\text{证明: } \vec{S} \neq \text{Re}[\vec{E} \times \vec{H} e^{j\omega t}]$$

证明:

$$\begin{aligned}
\text{Re}[\vec{E} \times \vec{H} e^{j\omega t}] &= \text{Re}[e^{j\omega t} \begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ E_x & E_y & E_z \\ H_x & H_y & H_z \end{vmatrix}] \\
&= \text{Re}\{[\bar{x}_0(E_y H_z - E_z H_y) + \bar{y}_0(E_z H_x - E_x H_z) + \bar{z}_0(E_x H_y - E_y H_x)]e^{j\omega t}\} \\
&\neq \vec{E}(t) \times \vec{H}(t) = \vec{S}(t)
\end{aligned}$$

3. 13

$$\text{证明: } \vec{S} \neq \text{Re}[\vec{E} e^{j\omega t} \times \vec{H} e^{j\omega t}]$$

证明:

$$\begin{aligned}
&\text{Re}[\vec{E}(\vec{r}) e^{j\omega t} \times \vec{H}(\vec{r}) e^{j\omega t}] \\
&= \text{Re}\left[\begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ E_x(r) e^{j\omega t} & E_y(r) e^{j\omega t} & E_z(r) e^{j\omega t} \\ H_x(r) e^{j\omega t} & H_y(r) e^{j\omega t} & H_z(r) e^{j\omega t} \end{vmatrix}\right] \\
&\neq \begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ \text{Re}[E_x(r) e^{j\omega t}] & \text{Re}[E_y(r) e^{j\omega t}] & \text{Re}[E_z(r) e^{j\omega t}] \\ \text{Re}[H_x(r) e^{j\omega t}] & \text{Re}[H_y(r) e^{j\omega t}] & \text{Re}[H_z(r) e^{j\omega t}] \end{vmatrix} \\
&= \vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) = \vec{S}(\vec{r}, t)
\end{aligned}$$

4. 1

写出 $\omega, k, f, T, \lambda$ 的单位

解:  $\omega: \text{rad/s}, k: \text{rad/m}, f: \text{Hz}, T: \text{s}, \lambda: \text{m}$

4.2

氦氖激光器输出波长为 $6.328 \times 10^{-7} \text{m}$ , 计算它的 $f, T, k$ 。

解:  $\lambda = 6.328 \times 10^{-7} \text{m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6.328 \times 10^{-7}} = 9.92 \times 10^6 \text{ (rad/m)}$$

$$T = \frac{\lambda}{c} = \frac{6.328 \times 10^{-7}}{3 \times 10^8} = 2.11 \times 10^{-15} \text{ (s)}$$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6.328 \times 10^{-7}} = 474 \text{ (GHz)}$$

4. 3

已知均匀平面电磁波在均匀媒质中传播, 其电场强度的表示式为 $\vec{E} = \vec{y}_0 E_y = \vec{y}_0 10 \cos(\omega t - kz + 30^\circ) \text{mV/m}$ , 工作频率 $f = 150 \text{MHz}$ , 媒质的参数为 $\mu_r = 1, \varepsilon_r = 4, \sigma = 0$ , 试求:

(1) 相位常数 $k$ , 相速 $v_p$ 和波阻抗 $\eta$ ;

(2)  $t = 0, z = 1.5$ 处,  $\vec{E}, \vec{H}, \vec{S}(t), \langle \vec{S} \rangle$ 各为多少?

(3) 在 $z = 0$ 处,  $E$ 第一次出现最大值(绝对值)的时刻 $t$ 等于多少?

$$k = \omega \sqrt{\mu \varepsilon} = 2\pi f \sqrt{\mu_r \varepsilon_r} \sqrt{\mu_0 \varepsilon_0} = 2\pi f \frac{\sqrt{\mu_r \varepsilon_r}}{c}$$

$$= \frac{2\pi \times 150 \times 10^6 \sqrt{4}}{2 \times 10^8} = 2\pi \text{ (rad/s)}$$

解: (1)

$$\lambda = \frac{2\pi}{k} = 1 \text{ (m)}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_r}{\varepsilon_r}} \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{1}{4}} \eta_0 = 188 \text{ } (\Omega)$$

$$\vec{E}_{(z=1.5, t=0)} = \vec{y}_0 10 \cos(-2\pi \times 1.5 + \frac{\pi}{6}) = 5\sqrt{3} \vec{y}_0 (mV/m)$$

$$\begin{aligned} \vec{H}(\vec{r}) &= \frac{-1}{j\omega\mu} \nabla \times \vec{E}(\vec{r}) \\ (2) \quad &= \vec{x}_0 \frac{1}{j\omega\mu} \frac{\partial E_y(\vec{r})}{\partial z} - \vec{z}_0 \frac{1}{j\omega\mu} \frac{\partial E_y(\vec{r})}{\partial x} \\ &= \vec{x}_0 \frac{-10k}{\omega\mu} e^{-jkz + j\frac{\pi}{6}} \end{aligned}$$

$$\vec{H}(\vec{r}, t) = -\vec{x}_0 \frac{10k}{\omega\mu} \cos(\omega t - kz + \frac{\pi}{6})$$

$$\vec{H}_{(z=1.5, t=0)} = \frac{\sqrt{3}}{12\pi} \vec{x}_0$$

$$\vec{S}(t)_{(z=1.5, t=0)} = \vec{E}(\vec{r}, t) \times \vec{H}(\vec{r}, t) = \frac{5}{4\pi} \vec{z}_0$$

$$\langle \vec{S}(\vec{r}, t) \rangle = \frac{1}{2} \text{Re}[\vec{E}(\vec{r}) \times \vec{H}^*(\vec{r})] = \frac{5}{6\pi} \vec{z}_0$$

$$z=0$$

$$\vec{E} = \vec{y}_0 \cos(\omega t + \frac{\pi}{6})$$

$$(3) \quad \text{当 } 2\pi f + \frac{\pi}{6} = \pi \text{ 时最大}$$

$$2\pi f t = \frac{5}{6} \pi$$

$$t = \frac{5\pi}{6 \times 2\pi \times 150} = 2.78 \mu s$$

4. 4

已知自由空间电磁波的 $f_0, \lambda_0, k_0, v_0$ , 当它进入介质, 其介电常数为 $4\epsilon_0$ ,  $\mu = \mu_0$ , 求介质中电磁波的 $f, \lambda, k$ 以及 $v$ 。

$$\text{解: } f = f_0, \quad v = \frac{1}{2} v_0, \quad \lambda = \frac{v}{f} = \frac{v_0/2}{f_0}, \quad k = 2k_0$$

4. 5

$\vec{E} = E_0 e^{jkz} \vec{x}_0, \vec{H} = H_0 e^{jkz} \vec{y}_0$  满足自由空间麦克斯韦方程, 问题如下:

(1) 用 $E_0, \epsilon_0, \mu_0$ 表示 $H_0$ 和 $k$ ;

(2) 这个解是不是均匀平面波? 波沿什么方向传播? 并求出波速和 $\langle \vec{S} \rangle$ 。

$$\text{解: (1) } \nabla \times \vec{E} = -j\omega\mu\vec{H}$$

$$\begin{aligned}
\vec{H} &= \frac{-1}{j\omega\mu} \nabla \times \vec{E} \\
&= \frac{-1}{j\omega\mu} \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0 e^{jkz} & 0 & 0 \end{vmatrix} \\
&= -\vec{y}_0 \frac{k}{\omega\mu_0} E_0 e^{jkz} \\
&= H_0 e^{jkz} \vec{y}_0 \\
\therefore H_0 &= -\frac{k}{\omega\mu_0} E_0 = -\frac{\omega\sqrt{\mu_0\epsilon_0}}{\omega\mu_0} E_0 = \frac{-E_0}{\eta_0}
\end{aligned}$$

(2)  $Z = C$  时等相位面幅度相同

$\therefore$  它是均匀平面波

传播方向  $\vec{S} = \vec{E} \times \vec{H}$  方向, 为  $-z$  方向

$$v_p = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3 \times 10^8 \text{ m/s}$$

$$\langle \vec{S} \rangle = \frac{1}{2} \text{Re}[\vec{E} \times \vec{H}^*] = \frac{1}{2} \text{Re}[\vec{z}_0 E_0 H_0] = -\frac{1}{2} \frac{E_0^2}{\eta_0} \vec{z}_0$$

4. 6

商用调幅广播电台覆盖地域最低信号场强为  $25\text{Mv/m}$ , 问与之相联系的最小功率密度是多少? 最小磁场是多大?

$$\text{解: } \langle \vec{S} \rangle = \frac{1}{2} \frac{E_0^2}{\eta} = \frac{1}{2} \frac{(25 \times 10^{-3})^2}{\eta_0} = \frac{3.125 \times 10^{-4}}{\eta_0} \text{ W}$$

$$H_{\min} = \frac{25 \times 10^{-3}}{\eta_0} \text{ A/m}$$

4. 7

在无耗自由空间中, 平面电磁波的平均能流密度为  $0.26\mu\text{W/m}^2$ , 平面波沿  $z$  方向传播, 其工作频率  $f = 150\text{MHz}$ , 电场强度的表达式为  $E = E_m \cos(\omega t - kz + 60^\circ)$ . 试求在  $z = 10\text{m}$  处,  $t = 0.1\text{s}$  时的  $E, H, S$ .

$$\text{解: } \langle \vec{S} \rangle = \vec{z}_0 \frac{1}{2} \frac{E_m^2}{\eta_0}$$

$$\frac{1}{2} \frac{E_m^2}{\eta_0} = 0.26 \times 10^{-6}$$

$$\therefore E_m = \sqrt{0.26 \times 10^{-6} \times 2 \times 377} = 14 \text{ mV/m}$$

$$H = H_m \cos(\omega t - kz + \frac{\pi}{3}) = \frac{E_m}{\eta_0} \cos(\omega t - kz + \frac{\pi}{3})$$

$$z = 10 \text{ m}, t = 0.1 \mu\text{s} \text{ 处}$$

$$E = E_m \cos(2\pi f t - \frac{2\pi f}{c} z + \frac{\pi}{3}) = 14 \times 10^{-3} \cos(3 \times 10^8 \pi t - \pi z + \frac{\pi}{3})$$

$$E_{(z=10\text{m}, t=0.1\mu\text{s})} = 1.4 \times 10^{-3} \cos(3 \times 10^8 \pi \times 10^{-7} - \pi \times 10 + \frac{\pi}{3}) = 7 \text{ mV/m}$$

$$H_{(z=10\text{m}, t=0.1\mu\text{s})} = \frac{7 \times 10^{-3}}{377} = 1.86 \times 10^{-5} \text{ A/m}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$S = 1.86 \times 10^{-5} \times 7 \times 10^{-3} = 0.13 \mu\text{W/m}^2$$

4. 8 求下列场的极化性质

$$(1) \vec{E} = (j\vec{x}_0 + \vec{y}_0)e^{-jkz}$$

$$(2) \vec{E} = [(2+j)\vec{x}_0 + (3-j)\vec{z}_0]e^{-jky}$$

$$(3) \vec{E} = [(1+j)\vec{y}_0 + (1-j)\vec{z}_0]e^{-jkx}$$

$$(4) \vec{E} = [j\vec{x}_0 + j2\vec{y}_0]e^{jkx}$$

解: (1)  $\vec{E} = (j\vec{x}_0 + \vec{y}_0)e^{-jkz} = \vec{x}_0 e^{-(jkz - \frac{\pi}{2})} + \vec{y}_0 e^{-jkz}$

$$\varphi_b = 0, \varphi_a = \frac{\pi}{2}$$

$$\therefore \varphi = \varphi_b - \varphi_a = -\frac{\pi}{2}$$

$$\frac{E_y}{E_x} = A e^{j\varphi} = e^{-j\frac{\pi}{2}}$$

$\therefore$  是右手圆极化

$$(2) A e^{j\varphi} = \frac{2+j}{3-j} = \frac{1}{\sqrt{2}} e^{j\frac{\pi}{4}}$$

是左手椭圆极化

$$(3) A e^{j\varphi} = \frac{1-j}{1+j} = e^{j(-\frac{\pi}{2})}$$

是右手圆极化

$$(4) \quad A e^{j\varphi} = \frac{j}{j2} = \frac{1}{2} e^{j0}$$

是线极化

4.10

一线极化波电场的两个分量  $E_x = 6 \cos(\omega t - kz - 30^\circ)$ ,  $E_y = 8 \cos(\omega t - kz - 30^\circ)$ , 试求它分解成振幅相等旋向相反的两个圆极化波。

解：讨论  $z=0$  的情况：

$$E_x = 6 \cos(\omega t - \frac{\pi}{6})$$

$$E_y = 8 \cos(\omega t - \frac{\pi}{6})$$

$$E = \sqrt{E_x^2 + E_y^2} = 10 \cos(\omega t + \frac{\pi}{6})$$

$$\tan \theta = \frac{E_y}{E_x} = \frac{4}{3} \Rightarrow \theta = 53^\circ$$

$$E_x = 10 \cos 53^\circ \cos(\omega t - 30^\circ)$$

$$E_y = 10 \sin 53^\circ \cos(\omega t - 30^\circ)$$

$$E_x = 5[\cos(\omega t + 23^\circ) + \cos(\omega t - 83^\circ)]$$

$$E_y = 5[\sin(\omega t + 23^\circ) - \sin(\omega t - 83^\circ)]$$

$$\therefore \vec{E}(\vec{r}, t) = \vec{x}_0 E_x + \vec{y}_0 E_y$$

$$= 5[\vec{x}_0 \cos(\omega t + 23^\circ) + \vec{y}_0 \sin(\omega t - 23^\circ)] (\text{右手圆极化})$$

$$+ 5[\vec{x}_0 \cos(\omega t - 83^\circ) - \vec{y}_0 \sin(\omega t - 83^\circ)] (\text{左手圆极化})$$

$$= \vec{E}(t)_{\text{右旋}} + \vec{E}(t)_{\text{左旋}}$$

4.11

自由空间沿着  $z$  方向传播的平面波  $\vec{E} = \vec{E}_0 e^{-jkz}$ , 式中  $\vec{E}_0 = \vec{E}_r + j\vec{E}_i$ , 且  $\vec{E}_r = 2\vec{E}_i = b$ ,  $b$  为常数,  $\vec{E}_r$  在  $x$  方向,  $\vec{E}_i$  与  $x$  轴夹角为  $60^\circ$ , 试求电场强度与磁场强度的瞬时值, 并说明波的极化。

$$\text{解: } \vec{E} = \vec{E}_0 e^{-jkz} = (\vec{E}_r + j\vec{E}_i) e^{-jkz} = (b\vec{x}_0 + \frac{\sqrt{3}}{4}jb\vec{y}_0) e^{-jkz}$$

$$\therefore \vec{E}(\vec{r}, t) = \vec{x}_0 b \cos(\omega t - kz) + \vec{y}_0 \frac{\sqrt{3}}{4} b \cos(\omega t + \frac{\pi}{2} - kz)$$

$$= \bar{x}_0 b \cos(\omega t - kz) - \bar{y}_0 \frac{\sqrt{3}}{4} b \sin(\omega t - kz)$$

$$\vec{H} = \frac{\nabla \times \vec{E}}{-j\omega\mu}$$

$$= \frac{-\bar{x}_0 \frac{\sqrt{3}}{4} jkb}{\omega\mu} e^{-jkz} + \bar{y}_0 \frac{kb}{\omega\mu} e^{-jkz}$$

$$= \bar{x}_0 \frac{\sqrt{3}}{4\eta} b e^{-j(kz + \frac{\pi}{2})} + \bar{y}_0 \frac{b}{\eta} e^{-jkz}$$

$$\therefore \vec{H}(\vec{r}, t) = \bar{x}_0 \frac{\sqrt{3}}{4\eta} \cos(\omega t - kz - \frac{\pi}{2}) + \bar{y}_0 \frac{b}{\eta} \cos(\omega t - kz)$$

$$= \bar{x}_0 \frac{\sqrt{3}}{4\eta} \sin(\omega t - kz) + \bar{y}_0 \frac{b}{\eta} \cos(\omega t - kz)$$

由题意可得：

$$\begin{cases} E_x = b \cos(\omega t - kz) \\ E_y = -\frac{\sqrt{3}}{4} b \sin(\omega t - kz) \end{cases}$$

是左手椭圆极化波

4.12

均匀平面波的频率为10MHz，设地球的 $\mu = \mu_0$ ， $\varepsilon = \varepsilon_0$ ， $\sigma = 10^{-4} S/m$ ，求地球的衰减常数与趋肤深度。

$$\text{解：} \frac{\sigma}{\omega\varepsilon} = \frac{10^{-4}}{2\pi \times 10 \times 10^6 \times 8.85 \times 10^{-12} \times 4} = 0.045 \ll 1$$

$$\therefore k = \omega \sqrt{\mu\varepsilon(1 - j\frac{\sigma}{\omega\varepsilon})} \approx \omega \sqrt{\mu\varepsilon}(1 - j\frac{\sigma}{2\omega\varepsilon})$$

$$k_i = \frac{\sigma}{2} \sqrt{\frac{\mu}{\varepsilon}} = \frac{10^{-4}}{2} \sqrt{\frac{4\pi \times 10^{-7}}{4 \times 8.85 \times 10^{-12}}} = 9.4 \times 10^{-3} N/m$$

$$\sigma = \frac{1}{k_i} = 106m$$

4.13

用上例数据，设地球表面电场强度为1V/m，求地球表面功率密度。

$$\text{解：} \langle S \rangle = \frac{1}{2} \frac{|E|^2}{\eta_0} = \frac{1}{2\eta_0}$$



一平面电磁波从空气垂直地向海面传播, 已知某海域的海水参数为  $\xi_r = 80$ ,  $\sigma = 1\text{S/m}$ ,  $\mu_r = 1$ , 平面电磁波在海平面处的场强表示式为:

$$\vec{E} = \vec{x}_0 1000 e^{-k_i z} e^{j(t-k_r z)} \text{V/m}$$

工作波长为  $300\text{m}$ , 试求电场强度的振幅为  $1\text{V/m}$  时离海面的距离。

并写出这个位置上的  $\vec{E}$ ,  $\vec{H}$  的表达式

$$\text{解: } \frac{\sigma}{\omega\epsilon} = \frac{\sigma}{2\pi \frac{C}{\lambda} \epsilon_r \epsilon_0} = \frac{1}{2\pi \times \frac{2 \times 10^8}{300} \times 8.85 \times 10^{-12} \times 80} \approx 225 \gg 1 \text{ 良导体}$$

$$k = \omega \sqrt{\mu\epsilon} (1 - j \frac{\sigma}{\omega\epsilon})^{\frac{1}{2}} \approx \sqrt{\omega\mu \frac{\sigma}{2}} (1 - j)$$

$$k_r = \sqrt{\frac{\omega\mu}{\sigma}} = \sqrt{\frac{2\pi \frac{C}{\lambda} \mu\sigma}{2}} = \sqrt{\frac{2\pi \times \frac{3 \times 10^8}{300} \times 4\pi \times 10^{-7}}{2}} = 1.986 (1/\text{m})$$

$$k_i = \sqrt{\frac{\omega\mu\sigma}{2}} = 1.986 \text{ N/m}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu}{-j \frac{\sigma}{\omega}}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{j 4\pi \times 10^{-7} \times 2\pi \times 10^6}{1}} = 2.8 e^{j\frac{\pi}{4}}$$

$$1000 e^{-k_i z} = 10^{-6}$$

$$-1.986 z = \ln 10^{-9}$$

$$z = 10.43 \text{ m}$$

$$\vec{E}_{(z=10.43)} = \vec{x}_0 e^{j(2\pi \times 10^{-6} t - 1.986 z)} \mu\text{V/m}$$

$$\vec{H}_{(z=10.43)} = \vec{y}_0 \frac{E}{\eta} = \vec{y}_0 \frac{1}{2.8} e^{j(2\pi \times 10^{-6} t - 1.986 z - \frac{\pi}{4})}$$

$$= \vec{y}_0 0.357 e^{j(2\pi \times 10^{-6} t - 1.986 z - \frac{\pi}{4})} \mu\text{A/m}$$

4.15 巡航导弹用雷达测量其飞行高度, 雷达工作频率为  $3\text{GHz}$ 。假定地面为  $1\text{m}$

厚的干雪覆盖, 试求:

(1) 雷达测得的巡航导弹飞行高度, 与巡航导弹实际离开地面的高度是否有差别□

(2) 不计空气而只考虑地面覆盖的雪对传播电磁波损耗以及空气与雪交界面的反射损耗的影响, 计算由于  $1\text{m}$  厚的雪引起的雷达信号的衰减 (用  $\text{dB}$  表示), 假定  $3\text{GHz}$  时雪的  $\epsilon = 1.5\epsilon_0$ ,  $\tan \delta = 9 \times 10^{-4}$ 。

解：（1）由于雪中电磁波有损耗，所以雷达测得的高度与实际有差别

$$(2) \tan \sigma = 9 \times 10^4 \Rightarrow \sigma = \arctg 9 \times 10^4$$

$$\therefore k_i = \frac{1}{d_p} = \frac{1}{\sigma} = \frac{1}{\arctg 9 \times 10^4}$$

$$\text{设 } \vec{E} = \vec{E}_0 e^{-\frac{1}{\arctg 9 \times 10^4} z} e^{-jk_r z}$$

$$\therefore \text{通过 } 1m \text{ 后衰减为: } 20 \lg e^{-\frac{1}{\arctg 9 \times 10^4}}$$

4.16

设某海域海水低频时可用  $\varepsilon = 81\varepsilon_0$ ,  $\mu = \mu_0$ ,  $\sigma = 4S/m$  介质表示，平面波波矢与  $x$  轴夹角为  $30^\circ$ 。给出波沿  $x$  方向传播的传输线模型（给出等效传输线的特征参数）。

解：低频时海水是良导体

$$k_{zm} = \sqrt{\frac{\omega \mu_0 \sigma}{2}} (1 - j)$$

$$k_r = \sqrt{\frac{\omega \mu_0 \sigma}{2}}$$

$$Z_{TE} = \frac{\omega \mu}{k_z} = \frac{\omega \mu}{\sqrt{\frac{\omega \mu_0 \sigma}{2}}} = \sqrt{\frac{\omega \mu}{2\sigma}} (1 + j)$$

$$Z_{TM} = \frac{k_z}{\omega \varepsilon} = \frac{k_z}{\omega(\varepsilon' - j \frac{\sigma}{\omega})} \approx \frac{\sqrt{\frac{\omega \mu \sigma}{2}} (1 - j)}{-j\sigma} = \sqrt{\frac{\omega \mu}{2\sigma}} (1 + j)$$

$$Z_{TM} = Z_{TE} = Z_m = R(1 + j) = \frac{\omega \mu_0 \sigma}{2} (1 + j) = \sqrt{\frac{\omega \mu_0}{2\sigma}} (1 + j)$$

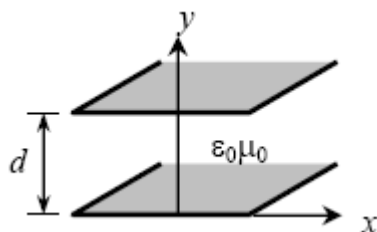
5.1 完纯导体表面  $\vec{H}_t = 3\vec{x}_0 + 4\vec{z}_0 A/m$ , 求表面电流  $\vec{J}_s$

解:  $\vec{J}_s = \vec{n}_0 \times \vec{H} = \vec{H}_t = 3\vec{x}_0 + 4\vec{z}_0$

5.2

两无限大平板间有电场  $\vec{E} = \vec{x}_0 A \sin(\frac{\pi}{d} y) e^{j(\omega t - kz)}$ , 式中 A 为常数, 平行板外空间电磁场为零, 坐标如图所示。试求:

- (1) 求  $\nabla \cdot \vec{E}, \nabla \times \vec{E}$ ;
- (2) E 能否用一位置的标量函数的负梯度表示, 为什么?
- (3) 求与 E 相联系的 H;
- (4) 确定两板上面电流密度和面电荷密度。



解: (1)  $\nabla \cdot \vec{E} = 0$

$$\begin{aligned} \nabla \times \vec{E} &= \vec{x}_0 \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \vec{y}_0 \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_z}{\partial x} \right) + \vec{z}_0 \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \\ &= -\vec{y}_0 jk A \sin\left(\frac{\pi}{d} y\right) e^{j(\omega t - kz)} - \vec{z}_0 \frac{\pi}{d} A \cos\left(\frac{\pi}{d} y\right) e^{j(\omega t - kz)} \end{aligned}$$

(2)  $\nabla \times \vec{E} \neq 0$  是有旋场, 不能用标量函数的负梯度表示

(3)

$$\begin{aligned} \vec{H} &= -\frac{1}{j\omega\mu_0} \nabla \times \vec{E} \\ &= \vec{y}_0 \frac{kA}{\omega\mu_0} \sin\left(\frac{\pi}{d} y\right) e^{j(\omega t - kz)} + \vec{z}_0 \frac{\pi}{d} \frac{1}{j\omega\mu_0} A \cos\left(\frac{\pi}{d} y\right) e^{j(\omega t - kz)} \end{aligned}$$

(4)  $\vec{J}_s = \vec{n} \times \vec{A}$

$$\vec{J}_{s(y=0)} = \vec{y}_0 \times [\vec{y}_0 \frac{kA}{\omega\mu_0} \sin(\frac{\pi}{d}y)e^{j(\omega t - kz)} + \vec{z}_0 \frac{\pi}{d} \frac{1}{j\omega\mu_0} A \cos(\frac{\pi}{d}y)e^{j(\omega t - kz)}]$$

$$= \vec{x}_0 \frac{\pi}{d} \frac{1}{j\omega\mu_0} A e^{j(\omega t - kz)}$$

$$\text{同理 } \vec{J}_{s(y=d)}$$

$$= \vec{x}_0 \frac{\pi}{d} \frac{1}{j\omega\mu_0} A e^{j(\omega t - kz)}$$

$$\rho_s = \vec{D} \cdot \vec{n}$$

$$\rho_{s(y=0)} = \vec{D} \cdot \vec{n} = 0$$

$$\text{同理 } \rho_{s(y=d)} = 0$$

5.3

有一均匀平面波垂直入射到  $z=0$  处的理想导电平面，其电场强度  $\vec{E} = E_0(\vec{x}_0 - j\vec{y}_0)e^{-jkz}$

，试确定：

- (1) 入射波和反射波的极化方式；
- (2) 导电平面上面电流密度；
- (3) 写出  $z \leq 0$  区域合成电场强度的瞬时值。

解：(1)  $\vec{E} = \vec{E}_0(\vec{x}_0 - j\vec{y}_0)e^{-jkz}$ ，所以入射波是右手圆极化

反射波为满足导体表面边界条件， $E_x^r, E_y^r$  与  $E_x^i, E_y^i$  都有  $180^\circ$  相移，且波的传播方向相反。

$\therefore \vec{E}^r = E_0(-\vec{x}_0 + j\vec{y}_0)e^{jkz}$ ，所以是左手圆极化。

$$(2) \vec{E}_{all} = \vec{E}_0(\vec{x}_0 - j\vec{y}_0)(e^{-jkz} - e^{jkz})$$

$$\vec{H} = -\frac{\nabla \times \vec{E}_{all}}{j\omega\mu}$$

$$= -\frac{1}{j\omega\mu} \begin{vmatrix} \vec{x}_0 & \vec{y}_0 & \vec{z}_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_0(e^{-jkz} - e^{jkz}) & -jE_0(e^{-jkz} - e^{jkz}) & 0 \end{vmatrix}$$

$$= (j\vec{x}_0 + \vec{y}_0)E_0 \frac{k}{\omega\mu} (e^{-jkz} + e^{jkz})$$

$$\vec{J}_s|_{z=0} = \vec{n} \times \vec{H}|_{z=0} = -\vec{z}_0 \times (j\vec{x}_0 + \vec{y}_0)E_0 \frac{k}{\omega\mu} (e^{-jkz} + e^{jkz})|_{z=0} = 2(-j\vec{y}_0 + \vec{x}_0) \frac{k}{\omega\mu}$$

(3) 此入射波可看成是两个平面波的叠加。

$$\vec{E}_1 = \vec{x}_0 E_0 e^{-jkz}, \vec{E}_2 = -j\vec{y}_0 E_0 e^{-jkz}$$

在这个坐标系下两个均为 TEM 波，

对平面波 1，在  $z \leq 0$  区域合成电场强度

$$E_x(z) = E_0(e^{-jkz} - e^{jkz}) = -2jE_0 \sin kz$$

对平面波 2，在  $z \leq 0$  区域合成电场强度

$$E_y(z) = -jE_0(e^{-jkz} - e^{jkz}) = -2E_0 \sin kz$$

所以  $z \leq 0$  区域合成电场强度的瞬时值

$$\vec{E} = \vec{x}_0 2E_0 \sin kz \sin \omega t - \vec{y}_0 2E_0 \sin kz \cos \omega t$$

5.4 计算从下列各种介质斜入射到它与空气的平面分界面时的临界角：

(1) 蒸馏水， $\xi_r = 81.1$

(2) 酒精， $\xi_r = 25.8$

(3) 玻璃， $\xi_r = 9$

(4) 云母， $\xi_r = 6$

解：(1)  $\theta_c = \arcsin \sqrt{\frac{\xi_{r2}}{\xi_{r1}}} = \arcsin \sqrt{\xi_r} = \arcsin \sqrt{81.1}$

(2)  $\theta_c = \arcsin \sqrt{\frac{\xi_{r2}}{\xi_{r1}}} = \arcsin \sqrt{\xi_r} = \arcsin \sqrt{25.8}$

(3)  $\theta_c = \arcsin \sqrt{\frac{\xi_{r2}}{\xi_{r1}}} = \arcsin \sqrt{\xi_r} = \arcsin 3$

(4)  $\theta_c = \arcsin \sqrt{\frac{\xi_{r2}}{\xi_{r1}}} = \arcsin \sqrt{\xi_r} = \arcsin 6$

5.5

一圆极化均匀平面波自空气投射到非磁性媒质表面  $z=0$ ，入射角  $\theta_i = 60^\circ$ ，入射面为  $x-z$

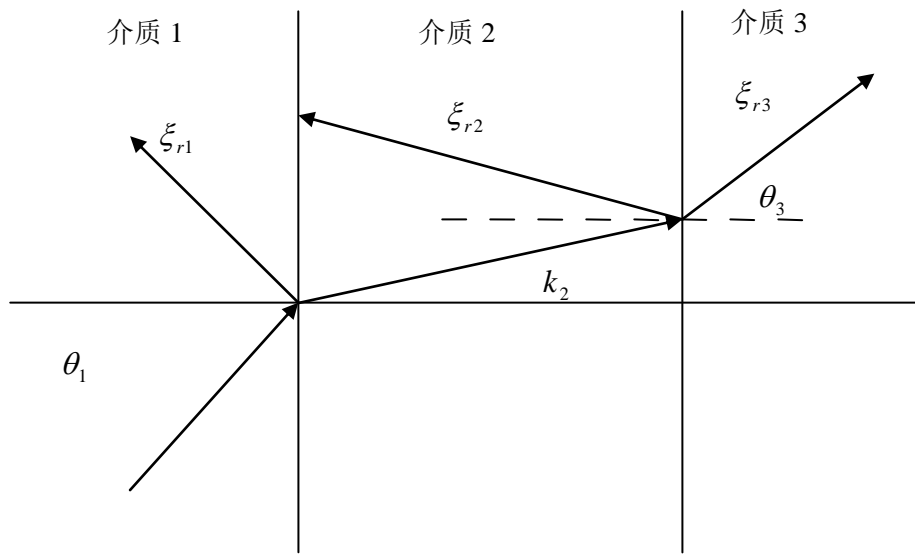
面。要求反射波电场在  $y$  方向，求媒质的相对介电系数  $\epsilon_r$ 。

解：将该圆极化波分解为 TE，TM，如果  $\theta_b = 60^\circ$  则反射波只有 TE

由  $\theta_b = 60^\circ$  得到：

$$\theta_b = \tan^{-1} \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = 60^\circ, \epsilon_{r2} = \sqrt{3}$$

5.7



如题图所示三介质系统， $\vec{k}_1, \vec{k}_2, \vec{k}_3$  分别为介质1,2,3中波矢，求用 $\theta_1$ 表示的 $\theta_3$ 。

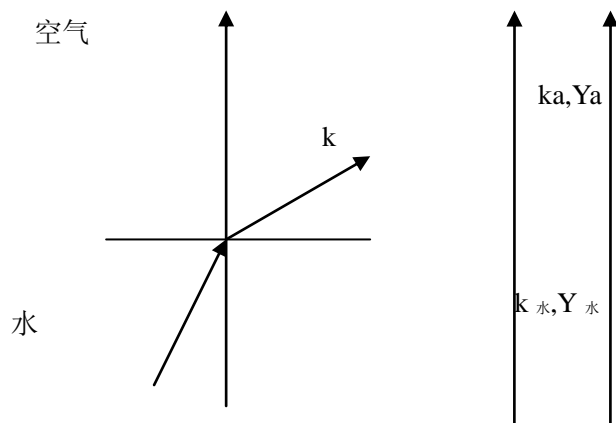
解：由 SNELL 定理可得：

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 = n_3 \sin \theta_3$$

同时 $n_1 = \sqrt{\xi_{r1}}, n_2 = \sqrt{\xi_{r2}}, n_3 = \sqrt{\xi_{r3}}$ 代入得到：

$$\theta_3 = \arcsin\left(\sqrt{\frac{\xi_{r1}}{\xi_{r3}}} \sin \theta_1\right)$$

5.8



光从水以 $\theta = 30^\circ$ 投射到与空气交界面，设光频是水介电常数 $\xi_a = 1.7\xi_0$ ， $\mu_a = \mu_0$ ，给出x方向传输线模型并用传输线模型求反射系数和折射系数。

解：以 TE 模为例：

$$k_{\text{水}} = k_{x2} = k_{\text{水}}' \cos 30^\circ = \frac{\sqrt{3}}{2} \sqrt{1.7} k_0 = \frac{\sqrt{5.1}}{2} k_0 = 1.729 k_0$$

$$k_a = \sqrt{k_1^2 - k_{x2}^2} = \sqrt{k_1^2 - \frac{1}{4} \times 1.7 \times k_0^2} = \frac{\sqrt{2.3}}{2} k_0 = 0.758 k_0$$

$$Y_{\text{水}} = k_{\text{水}} / \omega \mu = \frac{\sqrt{5.1}}{2} k_0 / \omega \mu = \frac{\sqrt{5.1}}{2\eta} = \frac{1.729}{\eta_0}$$

$$Y_a = k_a / \omega \mu = \frac{\sqrt{2.3}}{2} k_0 / \omega \mu = \frac{\sqrt{2.3}}{2\eta} = \frac{0.758}{\eta_0}$$

$$\Gamma(x=0) = \frac{Y_{\text{水}} - Y_a}{Y_{\text{水}} + Y_a} = \frac{\frac{\sqrt{5.1}}{2} - \frac{\sqrt{2.3}}{2}}{\frac{\sqrt{5.1}}{2} + \frac{\sqrt{2.3}}{2}} = 0.196$$

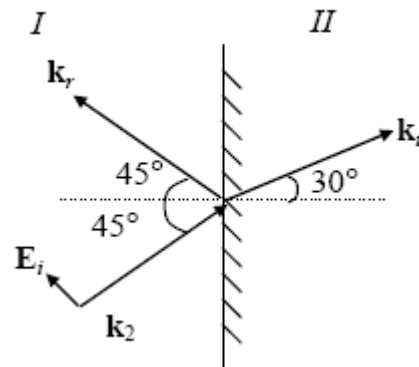
$$T(x=0) = \frac{2Y_{\text{水}}}{Y_{\text{水}} + Y_a} = \frac{\sqrt{5.1}}{\frac{\sqrt{5.1}}{2} + \frac{\sqrt{2.3}}{2}} = 1.196$$

5.9

均匀平面波由介质 I (空气) 以  $45^\circ$  角投射到无损介质 II, 已知折射角为  $30^\circ$ , 如图频率为  $300\text{MHz}$ 。求：

(1)  $\epsilon_{r2}$

(2) 反射系数  $\Gamma$



解：(1)

$$\sqrt{\epsilon_{r1}} \sin 45^\circ = \sqrt{\epsilon_{r2}} \sin 30^\circ$$

$$\sqrt{\epsilon_{r2}} = \sqrt{2}$$

$$\therefore \epsilon_{r2} = 2$$

$$\begin{aligned}
\Gamma &= \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\varepsilon_{r1} k_{z2} - \varepsilon_{r2} k_{z1}}{\varepsilon_{r1} k_{z2} + \varepsilon_{r2} k_{z1}} \\
(2) \quad &= \frac{\sqrt{6}k_0/2 - \sqrt{2}k_0}{\sqrt{6}k_0/2 + \sqrt{2}k_0} = 4\sqrt{3} - 7
\end{aligned}$$

5.10

两个各向同性介质组成的交界面， $\mu_1 \neq \mu_2, \xi_1 \neq \xi_2$  求入射波平行极化、垂直极化两种情形下的布儒斯特角。

解：对于 TE 模

$$\Gamma_{TE} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\frac{\omega\mu_2}{k_{z2}} - \frac{\omega\mu_1}{k_{z1}}}{\frac{\omega\mu_2}{k_{z2}} + \frac{\omega\mu_1}{k_{z1}}} = \frac{\mu_2 k_{z1} - \mu_1 k_{z2}}{\mu_2 k_{z1} + \mu_1 k_{z2}}$$

要使  $\Gamma_{TE} = 0$ ,  $\mu_2 k_{z1} - \mu_1 k_{z2} = 0$

即  $\mu_2 k_1 \cos \theta_B = \mu_1 k_2 \cos \theta_2$  (1)

由相位匹配条件： $k_1 \sin \theta_B = k_2 \sin \theta_2$  (2)

$$\text{由 (1) 得 } \cos \theta_2 = \frac{\mu_2 k_1}{\mu_1 k_2} \cos \theta_B, \sin \theta_2 = \sqrt{1 - \cos^2 \theta_2} = \sqrt{1 - \frac{\mu_2^2 k_1^2}{\mu_1^2 k_2^2} \cos^2 \theta_B} \quad (3)$$

(3)代入(2)得：

$$\begin{aligned}
k_1 \sin \theta_B &= k_2 \sqrt{1 - \frac{\mu_2^2 k_1^2}{\mu_1^2 k_2^2} \cos^2 \theta_B} \\
\frac{k_1}{k_2} \sqrt{1 - \cos^2 \theta_B} &= \sqrt{1 - \frac{\mu_2^2 k_1^2}{\mu_1^2 k_2^2} \cos^2 \theta_B}
\end{aligned}$$

两边平方，均整理后得到：

$$\cos^2 \theta_B = \frac{\mu_1}{\xi_1} \frac{\mu_1 \xi_1 - \mu_2 \xi_2}{\mu_1^2 - \mu_2^2}$$

$$\therefore \theta_B = \arccos \sqrt{\frac{\mu_1}{\xi_1} \frac{\mu_1 \xi_1 - \mu_2 \xi_2}{\mu_1^2 - \mu_2^2}}$$

$$\text{当 } \xi_1 = \xi_2, \theta_B = \arccos \sqrt{\frac{\mu_1}{\mu_1 + \mu_2}}$$



对于 $TM$ 模

$$\Gamma_{TM} = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{\frac{k_{z2}}{\omega\xi_2} - \frac{k_{z1}}{\omega\xi_1}}{\frac{k_{z2}}{\omega\xi_2} + \frac{k_{z1}}{\omega\xi_1}} = \frac{\xi_1 k_{z2} - \xi_2 k_{z1}}{\xi_1 k_{z2} + \xi_2 k_{z1}}$$

要使 $\Gamma_{TM} = 0$ ,  $\xi_1 k_{z2} - \xi_2 k_{z1} = 0$

即 $\xi_2 k_1 \cos \theta_B = \xi_1 k_2 \cos \theta_2$  (1)

由相位匹配条件:  $k_1 \sin \theta_B = k_2 \sin \theta_2$  (2)

$$\text{由 (1) 得 } \cos \theta_2 = \frac{\xi_2 k_1}{\xi_1 k_2} \cos \theta_B, \sin \theta_2 = \sqrt{1 - \cos^2 \theta_2} = \sqrt{1 - \frac{\xi_2^2 k_1^2}{\xi_1^2 k_2^2} \cos^2 \theta_B} \quad (3)$$

(3)代入(2)得:

$$k_1 \sin \theta_B = k_2 \sqrt{1 - \frac{\xi_2^2 k_1^2}{\xi_1^2 k_2^2} \cos^2 \theta_B}$$

$$\frac{k_1}{k_2} \sqrt{1 - \cos^2 \theta_B} = \sqrt{1 - \frac{\xi_2^2 k_1^2}{\xi_1^2 k_2^2} \cos^2 \theta_B}$$

两边平方, 均整理后得到:

$$\cos^2 \theta_B = \frac{\varepsilon_1}{\mu_1} \frac{\mu_1 \varepsilon_1 - \mu_2 \varepsilon_2}{\varepsilon_1^2 - \varepsilon_2^2}$$

$$\therefore \theta_B = \arccos \sqrt{\frac{\varepsilon_1}{\mu_1} \frac{\mu_1 \varepsilon_1 - \mu_2 \varepsilon_2}{\varepsilon_1^2 - \varepsilon_2^2}}$$

$$\text{当 } \mu_1 = \mu_2, \theta_B = \arccos \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}}$$

5.11 垂直极化平面波由媒质 I 倾斜投射到媒质 II, 如图,  $\varepsilon_1 = 4\varepsilon_0, \varepsilon_2 = \varepsilon_0$  求:

(1) 产生全反射时的临界角;

(2) 当  $\theta = 60^\circ$  时, 求  $k_x, k_{z1}$  (用  $k_0 = \omega \sqrt{\mu_0 \varepsilon_0}$  表示);

(3) 求  $k_{z2}$  (用  $k_0$  表示)

(4) 在媒质 II, 求场衰减到  $1/e$  时离开交界面的距离;

(4) 求反射系数  $\Gamma$ 。

解: (1)

$$\varepsilon_1 = 4\varepsilon_0, \varepsilon_2 = \varepsilon_0$$

$$\therefore \theta_c = \arcsin\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right) = \arcsin\sqrt{\frac{1}{2}} = 30^\circ$$

(2)

$$\theta = 60^\circ, k_x = k_1 \sin \theta_1 = k_0 \sqrt{4} \sin 60^\circ = \sqrt{3}k_0$$

$$k_{z1} = k_1 \cos 60^\circ = k_0 4 \cos 60^\circ = k_0$$

$$(3) \quad k_{z2} = \sqrt{k_0^2 - k_x^2} = \sqrt{k_0^2 - 3k_0^2} = -\sqrt{2}jk_0 = -ja_2$$

$$(4) \quad a_2 = \sqrt{2}k_0, a_2 d = 1, d = \frac{1}{a_2} = \frac{1}{\sqrt{2}k_0}$$

$$(5) \quad \Gamma = \frac{k_{z1} - k_{z2}}{k_{z1} + k_{z2}} = \frac{k_0 - j\sqrt{2}k_0}{k_0 + j\sqrt{2}k_0} = e^{j109.5^\circ}$$

5.14 一均匀平面电磁波由空气向理想介质  $\mu = \mu_0, \varepsilon = 9\varepsilon_0$  垂直入射。已知  $z=5$  米处

$$H_y = H_2 = 10e^{-jk_2 z} = 10e^{-j\frac{\pi}{4}} \text{ 毫安/米 (设介质分界面处为 } z=0, \text{ 初相 } \varphi = 0^\circ) \text{。试求:}$$

(1) 此平面电磁波的工作频率;

(2) 写出介质区域及空气区域的  $E_2, H_2, E_1, H_1$  的表示式;

(3) 在介质区域中再求:

a. 由复数振幅写成复数或瞬时的表示式;

b. 坡印廷矢量瞬时表示式  $S$  及  $S_{av}$ ;

c. 电场与磁场能量密度的瞬时表示式  $w_e, w_m$  及其最大的能量密度的大小  $w_{e\max}, w_{m\max}$

d. 能量密度的平均值  $w_{eav}, w_{mav}$ 。

$$\text{解: (1) 由题意 } k_2 z = \frac{\pi}{4}, k_2 = \frac{\pi}{4z} = \frac{\pi}{4 \times 5} = \frac{\pi}{20} (\text{rad/m})$$

$$k_2 = \omega \sqrt{\mu_0 \varepsilon} = \omega \sqrt{9\mu_0 \varepsilon_0} = \frac{\pi}{20}$$

$$\text{故 } f = \frac{k_2}{6\pi \sqrt{\mu_0 \varepsilon_0}} = \frac{\frac{\pi}{20}}{6\pi \times \frac{1}{3 \times 10^8}} 2.5 \text{ MHz}$$

$$(2) \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{9\varepsilon_0}} = \frac{1}{3}\eta_0 = 40\pi (\Omega)$$

$$\text{反射系数 } T = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{\frac{2\eta_0}{3}}{\frac{4\eta_0}{3}} = \frac{1}{2}$$

$$\text{在介质区域中 } E_2 = \eta_2 H_z = 40\pi \times 10 e^{-j\frac{\pi}{4}} = 400\pi e^{-j\frac{\pi}{4}} = E_{i0} e^{-j\frac{\pi}{4}}$$

$$\text{从而得到 } E_{i0} = 400\pi, E_{i0} = \frac{1}{\Gamma} E_{t0} = 800\pi (mV/m)$$

式中  $E_{i0}, E_{t0}$  表示入射波与入射波场强在  $z=0$  处的振幅。

在空气区域中的场强的入射波与反射波的合成，以  $E_1, H_1$  表示

$$E_1 = E_{i0}(e^{-jk_1 z} + \Gamma e^{jk_1 z}) = 800\pi(e^{-jk_1 z} - \frac{1}{2}e^{jk_1 z})$$

$$H_1 = H_t - H_r = \frac{E_{i0}}{\eta_0}(e^{-jk_1 z} - \Gamma e^{jk_1 z}) = \frac{20}{3}(e^{-jk_1 z} + \frac{1}{2}e^{jk_1 z})(mA/m)$$

$$k_1 = \frac{2\pi}{\lambda} = \frac{2\pi}{120} = \frac{\pi}{60} (rad/m)$$

在介质2中  $E_2, H_2$  为

$$E_2 = TE_{i0} e^{-jk_1 z} = 400\pi e^{-j\frac{\pi}{20_1} z}$$

$$H_2 = \frac{E_2}{\eta_2} = \frac{400\pi}{40\pi} e^{-j\frac{\pi}{20_1} z} = 10 e^{-j\frac{\pi}{20_1} z}$$

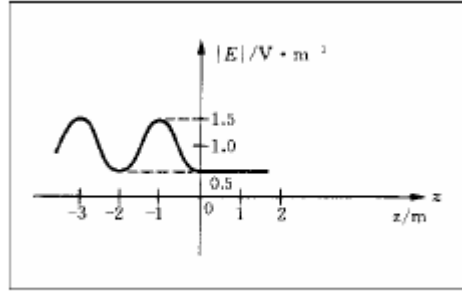
(3) 由  $E_2, H_2$  的复矢量表示  $\Rightarrow$  瞬态表示，在求坡印廷  $S(t), \langle S(t) \rangle, W_e, W_m$  等

(注意：TEM 波即可以用 TE 波的公式，也可以用 TM 波的公式)

$$\begin{aligned}
H_2 &= 10e^{-j\pi/20}\vec{y}_0 \\
E_2 &= Z_2|H_2|e^{-j\pi/20}\vec{x}_0 = 400\pi e^{-j\pi/20}\vec{x}_0 \\
Z_1 &= 120\pi, Z_2 = 40\pi, k_1 = \frac{\pi}{60} \\
\Gamma_{TM} &= \frac{Z_2 - Z_1}{Z_2 + Z_1} = -\frac{1}{2} \\
T_{TM} &= 1 - \Gamma_{TM} = \frac{3}{2} \\
H_y^i &= 10e^{-j\pi/60}/T_{TM}\vec{y}_0 = \frac{20}{3}e^{-j\pi/60}\vec{y}_0 \\
H_y^r &= \frac{20}{3}e^{-j\pi/60} \times (-\Gamma_{TM})\vec{y}_0 = \frac{10}{3}e^{-j\pi/60}\vec{y}_0 \\
H_y^1 &= H_y^i + H_y^r = \frac{1}{3}(10e^{-j\pi/60} + 10e^{j\pi/60})\vec{y}_0 \\
E_x^i &= Z_1 \times \frac{20}{3}e^{-j\pi/60}\vec{x}_0 = 800\pi e^{-j\pi/60}\vec{x}_0 \\
E_x^r &= 800\pi e^{-j\pi/60} \times \Gamma_{TM}\vec{x}_0 = -400\pi e^{j\pi/60}\vec{x}_0 \\
E_1 &= E_\lambda + E_{\bar{\lambda}} = 800\pi e^{-j\pi/60}\vec{x}_0 - 400\pi e^{j\pi/60}\vec{x}_0 \\
T_{TE} &= 1 + \Gamma_{TE} = \frac{1}{2} \\
E_x^i &= \frac{400\pi}{T_{TE}}e^{-j\pi/60}\vec{x}_0 = 800\pi e^{-j\pi/60}\vec{x}_0 \\
E_x^r &= 800\pi e^{-j\pi/60} \times \Gamma_{TM}\vec{x}_0 = -400\pi e^{j\pi/60}\vec{x}_0 \\
E_1 &= E_\lambda + E_{\bar{\lambda}} = 800\pi e^{-j\pi/60}\vec{x}_0 - 400\pi e^{j\pi/60}\vec{x}_0 \\
H_y^i &= \frac{800\pi}{Z_1}e^{-j\pi/60}\vec{y}_0 = \frac{20}{3}e^{-j\pi/60}\vec{y}_0 \\
H_y^r &= \frac{400\pi}{Z_1}e^{-j\pi/60}\vec{y}_0 = \frac{10}{3}e^{-j\pi/60}\vec{y}_0 \\
H_y^1 &= H_y^i + H_y^r = \frac{1}{3}(20e^{-j\pi/60} + 10e^{j\pi/60})\vec{y}_0
\end{aligned}$$

5.15 均匀平面波垂直投射到介质板，介质板前电场的大小示于下图，求

- (1) 介质板的介电常数  $\epsilon$
- (2) 入射波的工作频率。



解:  $\rho = \frac{1.5}{0.5} = 3$

$$|\Gamma| = \frac{\rho - 1}{\rho + 1} = \frac{3 - 1}{3 + 1} = 0.5$$

$$\psi(0) = -\pi, \Gamma = -\frac{1}{2}$$

垂直投射时,  $k_z = k$

$$\Gamma = \frac{k_{z1} - k_{z2}}{k_{z1} + k_{z2}} = \frac{k_1 - k_2}{k_1 + k_2} = \frac{k_1 - \sqrt{\epsilon_{r2}} k_2}{k_1 + \sqrt{\epsilon_{r2}} k_2} = -\frac{1}{2}$$

$$\sqrt{\epsilon_{r2}} = 3, \epsilon_{r2} = 9$$

$$\lambda = 4m, f = \frac{3 \times 10^8}{4} = 7.5 \times 10^7 \text{ m}$$

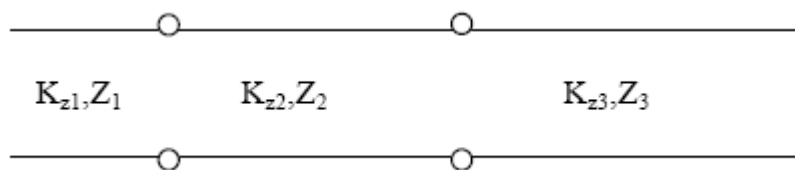
$$\omega = 2\pi f = 4.71 \times 10^8 \text{ rad/s}$$

5.16 在介电系数分别为  $\epsilon_1$  与  $\epsilon_3$  的介质中间放置一块厚度为  $d$  的介质板, 其介电常数为  $\epsilon_2$ , 三种介质的磁导率均为  $\mu_0$ ,

若均匀平面波从介质1 以  $\theta^i = 0^\circ$  垂直投射到介质板上,

(1) 试证明: 当  $\epsilon_2 = \sqrt{\epsilon_1 \epsilon_3}$ , 且  $d = \frac{\lambda_0}{4\sqrt{\epsilon_{r2}}}$  时, 没有反射。

(2) 如果  $\theta^i \neq 0^\circ$ , 导出没有反射时的  $d$  的表达式。



解: 每一层介质可等效为传输线, 如果均匀平面波从介质1 以  $\theta^i = 0^\circ$  垂直投射到介质板上, 对TE波, 传输线的特征参数为:

$$k_{z1} = \omega\sqrt{\mu_0\epsilon_1} = \sqrt{\epsilon_{r1}}k_0, k_0 = \omega\sqrt{\mu_0\epsilon_0}, Z_1 = \frac{\omega\mu_0}{k_{z1}} = \frac{\eta_0}{\sqrt{\epsilon_{r1}}}, \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$k_{z2} = \omega\sqrt{\mu_0\epsilon_2} = \sqrt{\epsilon_{r2}}k_0, Z_2 = \frac{\omega\mu_0}{k_{z2}} = \frac{\eta_0}{\sqrt{\epsilon_{r2}}}$$

$$k_{z3} = \omega\sqrt{\mu_0\epsilon_3} = \sqrt{\epsilon_{r3}}k_0, Z_3 = \frac{\omega\mu_0}{k_{z3}} = \frac{\eta_0}{\sqrt{\epsilon_{r3}}}$$

$$\text{当 } d = \frac{\lambda_0}{4\sqrt{\epsilon_{r2}}}, k_{z2}d = \frac{\pi}{2}, \text{ 即介质板相当于 } \frac{\lambda}{4} \text{ 传输线, 当 } Z_2^2 = Z_1Z_3 \text{ 时, 传输线匹配, 即没有反射,}$$

把波阻抗公式代入即可得  $\epsilon_2 = \sqrt{\epsilon_1\epsilon_3}$  , 所以得证。

$$\text{当 } \theta^i \neq 0^\circ, k_{z1} = \sqrt{\epsilon_{r1}}k_0 \cos \theta, k_0 = \omega\sqrt{\mu_0\epsilon_0}, Z_1 = \frac{\omega\mu_0}{k_{z1}}$$

$$k_{z2} = k_0\sqrt{\epsilon_{r2} - \epsilon_{r1}\sin^2 \theta}, Z_2 = \frac{\omega\mu_0}{k_{z2}}$$

$$k_{z3} = k_0\sqrt{\epsilon_{r3} - \epsilon_{r1}\sin^2 \theta}, Z_3 = \frac{\omega\mu_0}{k_{z3}}$$

$$\text{若要求没有反射波则 } Z_{in} = \frac{Z_3 + jZ_2 \tan k_{z2}d}{Z_2 + jZ_3 \tan k_{z2}d} = Z_1$$

此时为无反射时  $d$  所要满足的方程

5.19 在玻璃基片上涂复多层介质膜，试从原理上说明，只要适当选择每层膜的厚度及膜材的介电系数，该多层膜系统即可制作成增透膜系统（ $\Gamma \rightarrow 0$ ），也可做成全反射膜（ $\Gamma \rightarrow 1$ ）。

解：如果作增透膜，选择每一层介电系数、厚度使

$$\Gamma = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} \rightarrow 0$$

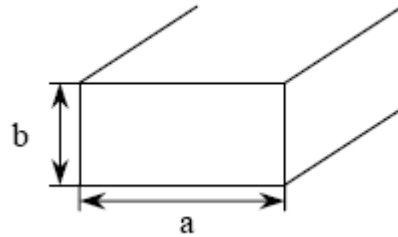
如果做全反射膜使

$$\Gamma = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} \rightarrow 1$$

## 第六章 波导

6.3 矩形波导BJ-100的宽边尺寸为 $a=22.86\text{mm}$ ,窄边尺寸为 $b=10.16\text{mm}$ ,传输频率为 $10\text{GHz}$ 的信号。求截止波长 $\lambda_c$ ,导波波长 $\lambda_g$ ,

相速 $v_p$ 和特征阻抗 $Z_c$ 。当频率 $f$ 稍微上升时,上述各参量如何变化?当宽边 $a$ 稍微变化时,上述各参量如何变化?当窄边 $b$ 稍微增大时,它们又怎么变化?



解:  $a = 22.86\text{mm}, b = 10.16\text{mm}, f = 10\text{GHz}$

$$\lambda_c = 2a, \lambda_g = \frac{\lambda}{\sqrt{1 - (\frac{\lambda}{\lambda_c})^2}}, kz = \frac{2\pi}{\lambda_g}, Z_c = \frac{\omega\mu}{kz}$$

$$\lambda = 3\text{cm}, \lambda_g = 39.76\text{mm}, v_p = 3.976 \times 10^8 \text{ m/s}, Z_{CTE10} = 499.65\Omega$$

$f \uparrow \rightarrow \lambda \downarrow$ , 分析各个式子的影响。

$f \uparrow, \lambda \downarrow, \lambda_{CTE10}$  不变,  $\lambda_{gTE10} \downarrow, v_{pTE10} \downarrow, Z_{CTE10} \downarrow$

$a \uparrow \rightarrow \lambda_c \uparrow$ , 分析各个式子的影响

$a \uparrow, \lambda_{CTE10} \uparrow, \lambda_{gTE10} \downarrow, v_{pTE10} \downarrow, Z_{CTE10} \downarrow$

$b$ 不出现在公式中, 没有影响。

6.4 上题中信号频率由 $10\text{GHz}$ 逐步增大到 $30\text{GHz}$ , 写出在波导中依次可能出现的高次模式。

解: 当 $f$ 由 $10\text{GHz} \rightarrow 30\text{GHz}$

$$\begin{aligned}\lambda_{CTE01} &= 2b = 20.32mm, \lambda_{CTE21} = \lambda_{CTM11} = 18.57mm, \lambda_{CTE02} = b = 10.16mm, \\ \lambda_{CTE20} &= 22.86mm \\ \lambda_{CTE21} &= \lambda_{CTM21} = 15.19mm, \lambda_{CTE21} = \lambda_{CTM12} = 9.9mm \\ \lambda_{CTE30} &= 15.24mm, \lambda_{CTE03} = 6.77mm, \lambda_{CTE40} = 11.43mm, \lambda_{CTE50} = 9.144mm \\ TE_{10}, TE_{20}, TE_{01}, TE_{11}, TM_{11}, TE_{30}, TE_{21}, TM_{21}, TE_{40}, TE_{02}, TE_{31}, TM_{31}\end{aligned}$$

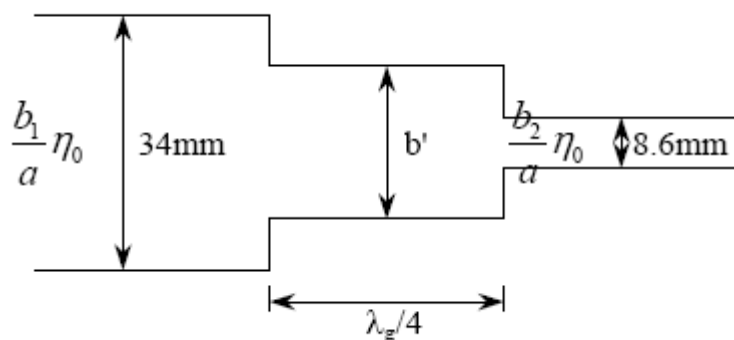
$$\text{由 } k_z = \sqrt{\omega^2 \mu_0 \varepsilon_0 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$$

改变 $(m, n)$ 的组合, 能使 $k_z$ 为实数的所有 $(m, n)$ 组合就是可能出现的高次模式

6.8

如果要把BB-32波导 ( $a = 72.14mm, b = 8.6mm$ ) 和BJ-32波导 ( $a = 72.14mm, b = 30.04mm$ ) 连接起来, 并使之反射最小, 中间应当加入一段什么样的波导? (用波导的传输线模型来解。)

解:

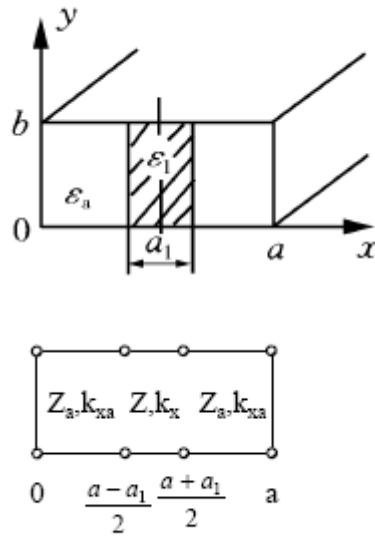


$$b' = \sqrt{b_1 b_2} = 17.11mm$$

两波导中间接入一段长为 $\frac{\lambda_g}{4}$ 的波导, 截面 $a' = 72.14mm, b'$ 的选择使得其等效阻抗平方为左右两个矩形波导等效阻抗的乘积

6.9





见图，矩形波导（截面为 $a \times b$ ）宽边中间被厚度为 $a_1$ 的介质填充（介电常数为 $\xi_1$ ）求该部分介质填充矩形波导的色散关系。

解：

$$k_y = \frac{n\pi}{b}$$

$$k_{xa} = \sqrt{k_a^2 - \left(\frac{n\pi}{b}\right)^2 - k_z^2}$$

$$k_{x1} = \sqrt{k_1^2 - \left(\frac{n\pi}{b}\right)^2 - k_z^2}$$

$$Z_{xa} = \frac{\omega\mu}{k_{xa}}$$

$$Z_{x1} = \frac{\omega\mu}{k_{x1}}$$

选择 $x = \frac{a-a_1}{2}$ 的对称面

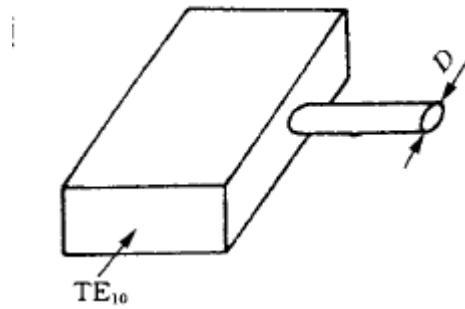
计算 $\vec{Z}$ ,  $\vec{\bar{Z}}$

代入横向谐振条件，得到 $\vec{Z} + \vec{\bar{Z}} = 0$  (1)，即 $f(kz, \omega) = 0$

利用对称性，分对称面开路，短路两种情况，结果要简单。但 (1) 式包含了对称面开路，短路两种情况。

6.11

如图，矩形波导（ $47.55 \times 22.15 \text{ mm}$ ）侧壁打一个直径 $D$ 的圆孔，并接上一段直径为 $D$ 的金属圆管，为了防止微波能量通过金属圆管向外辐射，求直径 $D$ 应小于何值。



解:

圆波导直径 $D$ 的选择,

对于矩形波导 $TE_{10}$ 模工作频率, 圆波导处于截止状态

矩形波导 $\lambda_{cTE10} = 2a = 2 \times 4.755 = 9.510cm$

$\lambda_{cTE01} = 4.43cm$

$\lambda_{cTE20} = 4.755cm$

单模工作 $9.510 \sim 4.755cm$

圆波导截止波长比 $4.755cm$ 小的多, 圆波导 $TE_{11}$ 截止波长 $\lambda_{cTE11} = 3.41a$

$\lambda_{cTE11} = 3.41a \ll 4.755cm, a \ll 1.394cm$

$(D = 2a)$

6.17 脉冲光信号沿着多模和单模光纤传输时所引起的色散效应有什么不同? 以什么因素为主?

答: 多模光纤以模间色散为主。单模光纤以模内色散为主, 即以波导色散为主。

模间色散 $\gg$ 模内色散

6.20

介质圆波导半径 $a = 500\mu m$ , 折射率 $n_1 = 2.8$ , 包层折射率 $n_2 = 2.7$ , 用图6-57所示归一化传播常数和归一化频率关系曲线计算 $LP_{01}$ 模,  $LP_{11}$ 模 $u = k_{c1}a$

解:  $a = 500\mu m, n_1 = 2.8, n_2 = 2.7$

$\lambda = 200\mu m$

$V = ak_0 \sqrt{n_1^2 - n_2^2} = 11.65$

查曲线得 $b_{LP01} = 0.97, b_{LP11} = 0.9$

由 $b = 1 - \frac{u^2}{V^2}$ , 得到 $u = V\sqrt{1-b}$

以半径为 $a$ 内充空气的金属圆波导和芯半径为 $a$ ,芯区与包层折射率为 $n_1, n_2$ 的介质圆波导为例, 比较金属波导和介质波导的导波特性

答: 圆波导, 场全部限制在波导内传播。 介质圆波导, 包层中, 在横向没有波的传播, 但包层中接近界面有高频能量储存。 金属圆波导截止条件,  $k_z$  为虚数即截止,  $k_z = 0$  是截止与非截止的临界点。介质圆波导, 包层中横向有能量传播就截止,  $k_{t2} = 0$ , 是截止与非截止的临界点, 但此时  $k_z$  可以是实数。 金属圆波导有高通滤波特性, 介质圆波导对于  $LP_{01}$  模到 DC 也能传播。

## 第八章 天线

8.1

假定波束宽度分别为 $1.5^\circ, 3^\circ$ 。问以分贝表示的天线增益是多少？

$$\text{解: } G = \frac{4\pi}{\Omega} = \frac{4\pi}{\frac{\pi}{4}\theta_B^2} = \frac{16}{\theta_B^2}$$

$$\theta_B = 1.5^\circ = \frac{\pi}{180} \times 1.5 \text{ rad}, \theta_B = 3^\circ = \frac{\pi}{180} \times 3 \text{ rad}$$

$$G_{(\theta_B=1.5^\circ)} = \frac{16}{\left(\frac{\pi}{180} \times 1.5\right)^2} = 2.335 \times 10^4$$

$$G_{(\theta_B=3^\circ)} = \frac{G_{(\theta_B=1.5^\circ)}}{4} = \frac{2.335}{4} \times 10^4 \approx 5837$$

8.3

天线发射功率为 $5\text{kW}$ ，方向性为 $36\text{dB}$ ，问距离天线 $25\text{km}$ 处，功率密度是多少？

$$\text{解: } G_D = 10^{\frac{36}{10}} = 3.98 \times 10^3$$

$$\langle S_r \rangle = G_D \frac{P}{4\pi r^2} = 3.98 \times 10^3 \frac{5 \times 10^3}{4\pi \times 25 \times 10^3} = 2.54 \times 10^{-3} \text{ W/m}^2$$

8.7

两个半波偶极子天线平行放置相距 $500\text{km}$ ，一个作发射，一个作接收，两天线之间连线与偶极子垂直，即 $\theta = 90^\circ$ ，发射天线发射功率 $1\text{kW}$ ，频率 $200\text{MHz}$ ，接收天线能接收到多少功率？

$$\text{解: 因为 } \theta = 90^\circ, G_D = \frac{3}{2} \sin^2 \theta = 1.5$$

$$f = 200 \times 10^6 \text{ Hz}, \lambda = \frac{3 \times 10^8}{200 \times 10^6} = 1.5 \text{ m}, A_\theta = \frac{G_D \lambda^2}{4\pi} = 0.296 \text{ m}^2$$

$$\langle S_r \rangle = G_D \frac{P}{4\pi r^2} = 4.777 \times 10^{-10} \text{ W/m}^2$$

8.14

一线天线阵共有40个单元每单元相距 $\frac{\lambda}{2}$ , 各单位激励电流大小相等且同轴, 计算波束宽度及第一个零点位置

解:  $N = 40, d = \lambda/2, kd = \pi, \psi = 0$

$$|F(\varphi)| = \frac{1}{N} \left| \frac{\sin \frac{Nkd \sin \varphi}{2}}{\sin \frac{kd \sin \varphi}{2}} \right| = \frac{1}{40} \left| \frac{\sin(20\pi \sin \varphi)}{\sin \frac{\pi \sin \varphi}{2}} \right|$$

令  $20\pi \sin \varphi = \pi$ ,  $\varphi = \arcsin \frac{1}{20} = 2.87^\circ$ , 即第一个零点位置

$$(ws')_{B/2} = \frac{w}{\lambda} \sin \frac{\varphi_B}{2} = \frac{Nd}{\lambda} \sin \frac{\varphi_B}{2} = 20 \sin \frac{\varphi_B}{2}$$

$$\text{令 } 20 \sin \frac{\varphi_B}{2} = 0.443$$

$$\therefore \varphi_B = 2 \arcsin \frac{0.442}{20} = 2.54^\circ$$

8.16

假定由于口面场均匀分布, 抛物面天线有效面积只是实际面积的60%, 如要得到45dB增益, 计算以下两种情况的抛物面天线直径:

(1) 500MHz

(2) 40GHz

$$\text{解: } G_D = 10^{\frac{45}{10}} = 10^{4.5} = 3.16 \times 10^4$$

$$(1) f = 500\text{MHz}, A_\theta = \frac{G_D \lambda^2}{4\pi} = 905.73\text{m}^2$$

$$A' = \frac{A_\theta}{60\%} = \frac{5}{3} A_\theta = 1509.55\text{m}^2$$

$$d = \sqrt{\frac{4A'}{\pi}} = 43.9\text{m}$$

$$(2) f = 40\text{GHz}, A_\theta = \frac{G_D \lambda^2}{4\pi} = 0.142\text{m}^2$$

$$A' = \frac{A_{\theta}}{60\%} = 0.236m^2$$

$$d = \sqrt{\frac{4A'}{\pi}} = 0.548m$$

8.20

如果雷达的基本参数是:  $P_T = 1MW, f = 5GHz, G = 45dB, P_{R\min} = -115dBm$  ( $dBm$ 是相对于 $1mW$ 是 $dB$ 数), 目标雷达截面 $\sigma = 1m^2$ , 计算雷达最大作用距离

$$\text{解: } G_D = 10^{\frac{45}{10}} = 3.6 \times 10^4$$

$$P_{R\min} = 10^{\frac{-115}{10}} mW = 3.16 \times 10^{-12} mW$$

$$R_{\max} = \left[ \frac{P_T G^2 \lambda^2 \sigma}{(4\pi)^3 P_{R\min}} \right]^{1/4} = 8.7 \times 10^5 m = 870km$$