Quantum Teleportation of Multiple Qubits Based on Quantum Fourier Transform

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***Abstract*— In this letter, we first propose quantum teleportation protocols to teleport a Greenberger–Horne–Zeilinger state and a W state via Brown state quantum channel. We find the orthogonal basis onto which Alice projects her states by quantum Fourier transforming her quantum states. According to the principle of quantum mechanics, we calculate the unitary operations Bob needs to perform to recover the teleportated states. Then, we generalize our protocols in a multi-qubit system. We show that our scheme can be easily extended to a multi-qubit system. Our method of using quantum Fourier transform to find the projective basis is universal and convenient, and it could have potential use in quantum communication.**

***Index Terms*— Quantum teleportation, entanglement, quantum Fourier transform.**

I. INTRODUCTION

QUANTUM teleportation (QT) uses the phenomenon of quantum entanglement as a means of transmission in which a quantum state is teleported without transmitting qubits. QT was first proposed by [1]. It has become one of the most important research areas in quantum information. Four years later, [2] realized QT experiment for the first time. Many different QT schemes have been put forward, for instance, the QT over non-maximally entangled channels [3], multi-level single-particle states [4], continuous variables [5] and coherent states [6]. On the other hand, QT has several applications in the quantum networking domain, especially as a routing enabler [7], [8]. It can be applied in quantum repeater for entanglement swapping.

The seminal scheme of QT involves a pair of EPR source in which the sender makes a joint Bell state measurement (BSM) on her qubits. Quantum entanglement is a very important source in this process which makes quantum mechanics so different from classical mechanics. This phenomenon was first discussed in [9]. Researchers are not only trying to find more and more interesting quantum entangled states but also maximizing the entanglement of a quantum system [10]. In [11], Brown *et al.* tried to search for highly entangled states of a multi-qubit state. By defining a cost function and using the negative partial transpose method, they found a highly entangled state for 4 and 5 qubits state via numerical search algorithm. Gao *et al.* [12] and Chen *et al.* [13] proposed protocols to teleport arbitrary 2 and 3 qubits by employing Brown state while Joy *et al.* [14] designed two deterministic secure quantum communication protocols, which were implemented by employing 3-qubit GHZ state and 5-qubit Brown state to transmit arbitrary 2 and 3 qubits.

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Motivated by the feature of Brown state, in this letter, we first propose QT protocols which teleport a Greenberger-Horne-Zeilinger (GHZ) state and a W state via Brown state channel. We find the projective basis of the sender by using quantum Fourier transform (QFT) of quantum states and we calculate the unitary operations according to the principle of quantum mechanics. Then we generalize our protocols to a multi-qubit system. We show that in order to teleport multiple qubits, the Schmidt rank of the quantum channel must be no less than the number of the coefficients of the teleported state. We show that our protocol is a generalization of the seminal protocol by Bennet. The measuring basis of the sender in our protocol is easier to calculate and the unitary operations of the receiver is simpler compared with other protocols so they are more convenient to realize experimentally.

II. QUANTUM TELEPORTATION

In a five-qubit system, the Brown state is:

$$|B_5\rangle = \frac{1}{2}(|001\rangle|\Psi^-\rangle + |010\rangle|\Phi^-\rangle + |100\rangle|\Psi^+\rangle + |111\rangle|\Phi^+\rangle),$$

where

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \quad |\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \qquad (1)$$

are Bell states. This state was verified by [15] through another algorithm and it has been used to teleport a special type of three-particle state [14], and Muralidharan and Panigrahi [16] proposed a protocol to teleport an arbitrary two-qubit state with the Brown state. Because it is the maximally entangled 5 qubit state, we first consider to use it as the quantum channel.

A. Teleporting a GHZ State With Brown State

In this section, we present our protocol of teleporting a four-qubit GHZ state with a five-qubit Brown state. The GHZ state involves at least three qubits. In GHZ state, every qubit is whether in the ground state or in the excited state. In our protocol, Alice holds the unknown four-qubit generalized GHZ state:

$$|\Omega_4\rangle = \alpha|0000\rangle + \beta|1111\rangle, \qquad (2)$$

where α and β are the complex amplitudes of the singlet states and $|\alpha|^2 + |\beta|^2 = 1$. Now, she wants to teleport it to Bob who is far away from her so she follows the following steps.

• Step 1: Alice and Bob share a five-qubit Brown state $|B_5\rangle$ and Alice also keeps the four-qubit GHZ state $|\Omega_4\rangle$. Suppose Alice holds the last qubit and Bob holds the other four qubits of $|B_5\rangle$. We can rewrite the Brown state as:

$$\begin{aligned}|B_5\rangle = &(|1110\rangle + |1001\rangle + |0100\rangle - |0011\rangle)_B|0\rangle_A \\ &+ (|1111\rangle + |1000\rangle + |0010\rangle - |0101\rangle)_B|1\rangle_A, \qquad (3)\end{aligned}$$

A and B stand for Alice and Bob, respectively. From now on, we will remove the normalized constant factor for convenience. The nine-qubit composite system is:

$$\begin{aligned}|\Theta\rangle &= |\Omega_4\rangle_A \otimes |B_5\rangle_{AB}\\ &= (\alpha|0000\rangle + \beta|1111\rangle)_A\\ &\quad \otimes [(|1110\rangle + |1001\rangle + |0100\rangle - |0011\rangle)_B|0\rangle_A\\ &\quad + (|1111\rangle + |1000\rangle + |0010\rangle - |0101\rangle)_B|1\rangle_A]\\ &= (\alpha|00000\rangle + \beta|11110\rangle)_A\\ &\quad \otimes (|1110\rangle + |1001\rangle + |0100\rangle - |0011\rangle)_B\\ &\quad + (\alpha|00001\rangle + \beta|11111\rangle)_A\\ &\quad \otimes (|1111\rangle + |1000\rangle + |0010\rangle - |0101\rangle)_B. \end{aligned} \tag{4}$$

Note that although Alice's five qubits are entangled, the first four qubits are always in the same state which means if Alice measures one of the four qubits, the other three qubits will collapse into the same state. So we can represent the four qubits as one qubit and the whole five-qubit state can be represented by a two-qubit state. If we combine the first four qubits of Alice's together and denote the subsystem as $|0_s\rangle = |0000\rangle$ and $|1_s\rangle = |1111\rangle$, then the system can be written as:

$$\begin{aligned}|\Theta\rangle &= (\alpha|0_s\rangle|0\rangle + \beta|1_s\rangle|0\rangle)_A\\ &\quad \otimes (|1110\rangle + |1001\rangle + |0100\rangle - |0011\rangle)_B\\ &\quad + (\alpha|0_s\rangle|1\rangle + \beta|1_s\rangle|1\rangle)_A\\ &\quad \otimes (|1111\rangle + |1000\rangle + |0010\rangle - |0101\rangle)_B. \end{aligned} \tag{5}$$

We define $|\Psi'^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0_s\rangle|1\rangle \pm |1_s\rangle|0\rangle)$ and $|\Phi'^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0_s\rangle|0\rangle \pm |1_s\rangle|1\rangle)$, and denote $(|1110\rangle + |1001\rangle + |0100\rangle - |0011\rangle)_B \equiv |b_0\rangle$ and $(|1111\rangle + |1000\rangle + |0010\rangle - |0101\rangle)_B \equiv |b_1\rangle$. And if we do a simple calculation, we can see that $\langle b_0|b_1\rangle = 0$. Then we have

$$\begin{aligned}|\Theta\rangle &= [\alpha(|\Phi'^{+}\rangle + |\Phi'^{-}\rangle) + \beta(|\Psi'^{+}\rangle - |\Psi'^{-}\rangle)]_A|b_0\rangle_B\\ &\quad + [\alpha(|\Psi'^{+}\rangle + |\Psi'^{-}\rangle) + \beta(|\Phi'^{+}\rangle - |\Phi'^{-}\rangle)]_A|b_1\rangle_B. \end{aligned} \tag{6}$$

- Step 2: Since $|\Psi'^{\pm}\rangle$ and $|\Phi'^{\pm}\rangle$ are the Bell basis of a two-qubit system, they form an orthogonal measurement basis of a two-qubit state. So they can be chosen as the measurement basis of Alice's projective measurement. Then Alice tells Bob her measuring result which is two-bit information via classical channel, meanwhile, Bob's qubits will collapse into the superposition of $|b_0\rangle$ and $|b_1\rangle$.
- Step 3: To recover the quantum state, Bob needs to perform unitary operation on his state. We define the unitary operations which transform $|b_0\rangle$ and $|b_1\rangle$ to $|0000\rangle$ and $|1111\rangle$ by $\mathbf{U}_{ij}$ with $\mathbf{U}_{ij}|b_i\rangle = |jjjj\rangle$, $(i,j) \in \{0,1\}$. According to the principle of quantum mechanics, $\mathbf{U}_{ij} = |jjjj\rangle\langle b_i|$, so we can easily calculate the unitary operations of Bob's side. Table I below shows Alice's measurement results and the corresponding Bob's collapsed states as well as the unitary operations Bob needs to perform on the states to recover the quantum state Eq. (2).

TABLE I

ALICE'S MEASURING RESULTS, BOB'S STATES AND THE CORRESPONDING UNITARY OPERATIONS

Alice's state	Bob's state	Unitary operation
$\lvert\Phi'^{+}\rangle$	$\frac{1}{4}(\alpha\lvert b_0\rangle + \beta\lvert b_1\rangle)$	$\mathbf{U}_{00} + \mathbf{U}_{11}$
$\lvert\Phi'^{-}\rangle$	$\frac{1}{4}(\alpha\lvert b_0\rangle - \beta\lvert b_1\rangle)$	$\mathbf{U}_{00} - \mathbf{U}_{11}$
$\lvert\Psi'^{+}\rangle$	$\frac{1}{4}(\beta\lvert b_0\rangle + \alpha\lvert b_1\rangle)$	$\mathbf{U}_{10} + \mathbf{U}_{01}$
$\lvert\Psi'^{-}\rangle$	$\frac{1}{4}(-\beta\lvert b_0\rangle + \alpha\lvert b_1\rangle)$	$\mathbf{U}_{10} - \mathbf{U}_{01}$

B. Teleporting a W State With Brown State

In this section, we introduce our protocol to teleport a three-qubit W state with a five-qubit Brown state. The W state has a special form that it is the superposition of a series of states where every qubit is in the ground state except one qubit. The unknown generalized W state is:

$$|W_3\rangle = \alpha|100\rangle + \beta|010\rangle + \gamma|001\rangle, \tag{7}$$

where $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$.

- Step 1: Suppose Alice and Bob share a Brown state $|B_5\rangle$ before and Alice holds the last two qubits, Bob holds the other three qubits. Meanwhile, Alice keeps the three-qubit W state $|W_3\rangle$. The eight-qubit composite system is:

$$\begin{aligned}|\Theta\rangle &= |W_3\rangle_A \otimes |B_5\rangle_{AB}\\ &= (\alpha|100\rangle + \beta|010\rangle + \gamma|001\rangle)_A\\ &\quad \otimes [(|001\rangle + |100\rangle)_B|01\rangle_A + (|100\rangle - |001\rangle)_B|10\rangle_A\\ &\quad + (|111\rangle + |010\rangle)_B|00\rangle_A + (|111\rangle - |010\rangle)_B|11\rangle_A]\\ &\equiv (\alpha|c_0\rangle + \beta|c_1\rangle + \gamma|c_2\rangle)_A\\ &\quad \otimes (|b_0\rangle_B|a_0\rangle_A + |b_1\rangle_B|a_1\rangle_A\\ &\quad + |b_2\rangle_B|a_2\rangle_A + |b_3\rangle_B|a_3\rangle_A), \end{aligned} \tag{8}$$

where Alice holds $|c_i\rangle$ and $|a_i\rangle$ qubits, Bob holds $|b_i\rangle$ qubits just like the above case. Note that though Alice holds five qubits, there are only three singlet states for the first three qubits which means we can choose a subspace of the three-qubit state whose basis is $\{|c_0\rangle, |c_1\rangle, |c_2\rangle\}$ to represent the first three-qubit state. And similarly, if we take the inner product of $|b_i\rangle$, we can get $\langle b_i|b_j\rangle = \delta_{ij}$.

- Step 2: Then Alice performs projective measurement on her qubits. If we define:

$$|\theta_0\rangle = |a_3c_0\rangle + |a_3c_1\rangle + |a_3c_2\rangle, \tag{9}$$
$$|\theta_1\rangle = |a_3c_0\rangle + e^{-i2\pi/3}|a_3c_1\rangle + e^{-i4\pi/3}|a_3c_2\rangle, \tag{10}$$
$$|\theta_2\rangle = |a_3c_0\rangle + e^{-i4\pi/3}|a_3c_1\rangle + e^{-i2\pi/3}|a_3c_2\rangle, \tag{11}$$

which are the QFTs [17] of states $|a_3c_0\rangle$, $|a_3c_1\rangle$ and $|a_3c_2\rangle$. Then we also define the QFTs:

$$\{\xi_{0i}\} = \mathscr{F}\{|a_0c_0\rangle, |a_1c_1\rangle, |a_2c_2\rangle, |\theta_0\rangle\}, \tag{12}$$
$$\{\xi_{1i}\} = \mathscr{F}\{|a_0c_1\rangle, |a_1c_2\rangle, |a_2c_0\rangle, |\theta_1\rangle\}, \tag{13}$$
$$\{\xi_{2i}\} = \mathscr{F}\{|a_0c_2\rangle, |a_1c_0\rangle, |a_2c_1\rangle, |\theta_2\rangle\}, \tag{14}$$

where $i \in \{0,1,2,3\}$ and $\mathscr{F}\{\cdot\}$ denotes the QFT. We have:

$$\begin{aligned}&|\Theta\rangle\\ &= (\alpha\sum_{k=0}^{k=3}|\xi_{0k}\rangle + \beta\sum_{k=0}^{k=3}e^{\frac{i\pi k}{2}}|\xi_{1k}\rangle + \gamma\sum_{k=0}^{k=3}e^{i\pi k}|\xi_{2k}\rangle)|b_0\rangle\\ &\quad + (\alpha\sum_{k=0}^{k=3}|\xi_{2k}\rangle + \beta\sum_{k=0}^{k=3}e^{\frac{i\pi k}{2}}|\xi_{0k}\rangle + \gamma\sum_{k=0}^{k=3}e^{i\pi k}|\xi_{1k}\rangle)|b_1\rangle\end{aligned}$$

TABLE II

ALICE'S MEASURING RESULTS, BOB'S STATES AND THE UNITARY OPERATIONS BOB NEEDS TO PERFORMS TO RECOVER THE QUANTUM STATES

Alice's state	Bob's stete	Unitary operation
$\|\xi_{00}\rangle$	$\alpha\|b_0\rangle+\beta\|b_1\rangle+\gamma\|b_2\rangle+(\alpha+\beta+\gamma)/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{00}+\mathbf{U}_{11}+\mathbf{U}_{22}+\mathbf{U}_{30}+\mathbf{U}_{31}+\mathbf{U}_{32}$
$\|\xi_{01}\rangle$	$\alpha\|b_0\rangle+i\beta\|b_1\rangle-\gamma\|b_2\rangle-i(\alpha+\beta+\gamma)/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{00}-i\mathbf{U}_{11}-\mathbf{U}_{22}+i(\mathbf{U}_{30}+\mathbf{U}_{31}+\mathbf{U}_{32})$
$\|\xi_{02}\rangle$	$\alpha\|b_0\rangle-\beta\|b_1\rangle+\gamma\|b_2\rangle-(\alpha+\beta+\gamma)/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{00}-\mathbf{U}_{11}+\mathbf{U}_{22}-(\mathbf{U}_{30}+\mathbf{U}_{31}+\mathbf{U}_{32})$
$\|\xi_{03}\rangle$	$\alpha\|b_0\rangle-i\beta\|b_1\rangle-\gamma\|b_2\rangle+i(\alpha+\beta+\gamma)/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{00}+i\mathbf{U}_{11}-\mathbf{U}_{22}-i(\mathbf{U}_{30}+\mathbf{U}_{31}+\mathbf{U}_{32})$
$\|\xi_{10}\rangle$	$\beta\|b_0\rangle+\gamma\|b_1\rangle+\alpha\|b_2\rangle+(\alpha+\beta e^{\frac{i2\pi}{3}}+\gamma e^{\frac{i4\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{20}+\mathbf{U}_{01}+\mathbf{U}_{12}+\mathbf{U}_{30}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{32}$
$\|\xi_{11}\rangle$	$i\beta\|b_0\rangle-\gamma\|b_1\rangle+\alpha\|b_2\rangle-i(\alpha+\beta e^{\frac{i2\pi}{3}}+\gamma e^{\frac{i4\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{20}-i\mathbf{U}_{01}-\mathbf{U}_{12}+i(\mathbf{U}_{30}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{32})$
$\|\xi_{12}\rangle$	$-\beta\|b_0\rangle+\gamma\|b_1\rangle+\alpha\|b_2\rangle-(\alpha+\beta e^{\frac{i2\pi}{3}}+\gamma e^{\frac{i4\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{20}-\mathbf{U}_{01}+\mathbf{U}_{12}-(\mathbf{U}_{30}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{32})$
$\|\xi_{13}\rangle$	$-i\beta\|b_0\rangle-\gamma\|b_1\rangle+\alpha\|b_2\rangle+i(\alpha+\beta e^{\frac{i2\pi}{3}}+\gamma e^{\frac{i4\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{20}+i\mathbf{U}_{01}-\mathbf{U}_{12}-i(\mathbf{U}_{30}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{32})$
$\|\xi_{20}\rangle$	$\alpha\|b_1\rangle+\beta\|b_2\rangle+\gamma\|b_0\rangle+(\alpha+\beta e^{\frac{i4\pi}{3}}+\gamma e^{\frac{i2\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{10}+\mathbf{U}_{21}+\mathbf{U}_{02}+\mathbf{U}_{30}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{32}$
$\|\xi_{21}\rangle$	$\alpha\|b_1\rangle+i\beta\|b_2\rangle-\gamma\|b_0\rangle-i(\alpha+\beta e^{\frac{i4\pi}{3}}+\gamma e^{\frac{i2\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{10}-i\mathbf{U}_{21}-\mathbf{U}_{02}+i(\mathbf{U}_{30}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{32})$
$\|\xi_{22}\rangle$	$\alpha\|b_1\rangle-\beta\|b_2\rangle+\gamma\|b_0\rangle-(\alpha+\beta e^{\frac{i4\pi}{3}}+\gamma e^{\frac{i2\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{10}-\mathbf{U}_{21}+\mathbf{U}_{02}-(\mathbf{U}_{30}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{32})$
$\|\xi_{23}\rangle$	$\alpha\|b_1\rangle-i\beta\|b_2\rangle-\gamma\|b_0\rangle+i(\alpha+\beta e^{\frac{i4\pi}{3}}+\gamma e^{\frac{i2\pi}{3}})/\sqrt{3}\|b_3\rangle$	$\mathbf{U}_{10}+i\mathbf{U}_{21}-\mathbf{U}_{02}-i(\mathbf{U}_{30}+e^{\frac{-i4\pi}{3}}\mathbf{U}_{31}+e^{\frac{-i2\pi}{3}}\mathbf{U}_{32})$

$$
\begin{aligned}
&+(\alpha\sum_{k=0}^{k=3}|\xi_{1k}\rangle+\beta\sum_{k=0}^{k=3}e^{\frac{i\pi k}{2}}|\xi_{2k}\rangle+\gamma\sum_{k=0}^{k=3}e^{i\pi k}|\xi_{0k}\rangle)|b_2\rangle\\
&+\frac{1}{\sqrt{3}}(\alpha\sum_{j=0}^{j=2}\sum_{k=0}^{k=3}e^{\frac{i3\pi jk}{2}}|\xi_{jk}\rangle+\beta\sum_{j=0}^{j=2}e^{\frac{i2\pi j}{3}}\sum_{k=0}^{k=3}e^{\frac{i3\pi jk}{2}}|\xi_{jk}\rangle\\
&+\gamma\sum_{j=0}^{j=2}e^{\frac{i4\pi j}{3}}\sum_{k=0}^{k=3}e^{\frac{i3\pi jk}{2}}|\xi_{jk}\rangle)|b_3\rangle. \qquad (15)
\end{aligned}
$$

According to the properties of QFT, the QFT of the computational basis is also a set of orthogonal basis of the system. In our protocol, $|c_i\rangle$ is the basis of the subspace of the three-qubit state while $|a_j\rangle$ is the basis of two-qubit state, so the composite state $|c_i\rangle|a_j\rangle$ forms a set of orthogonal complete basis of Alice's system. Meanwhile, $|\theta_k\rangle$ is orthogonal because it is the QFT of the subsystem $\{|a_3c_0\rangle, |a_3c_1\rangle, |a_3c_2\rangle\}$. So we can divide the system into three subsystems whose QFTs are $\{\xi_{0i}\}$, $\{\xi_{1i}\}$ and $\{\xi_{2i}\}$, respectively. Thus $\{\xi_{0i}\}$, $\{\xi_{1i}\}$ and $\{\xi_{2i}\}$ form a complete orthogonal set of the system. So the basis of Alice's projective measurement is chosen to be $\{\xi_{0i}, \xi_{1i}, \xi_{2i}\}$. Since there are 12 possible outcomes, she needs to transmit 4 classical bits to Bob via classical channel after the measurement.

- Step 3: To recover the quantum state, Bob needs to perform unitary operation on the collapsed state. Table II summarizes Alice's measuring results and Bob's states as well as the unitary operations Bob needs to perform to recover the quantum state Eq. (7). The unitary operations $\mathbf{U}_{ij}$ transforms $|b_i\rangle$ to $|c_j\rangle$. According to the principle of quantum mechanics, we have $\mathbf{U}_{ij}=|c_j\rangle\langle b_i|$.

C. Teleportation of Multi-Qubits GHZ and W States

The five-qubit Brown state is the maximally entangled five-qubit state which, even after tracing out one or two qubits from the state, still possesses entanglement in the final subsystem and thus, is highly robust [16]. When used in quantum teleportation, the Brown state can be helpful in understanding how quantum entanglement leads to quantum teleportation. On the other hand, the Brown state is a multi-qubit entangled state, it is experimentally hard to prepare and control. Besides, to teleport a more general multi-qubit state, a five-qubit Brown state is not sufficient. In this section, we show that in general, an entangled $(n+1)$-qubit state with Schmidt rank equals to that of the teleported state can be used as the quantum channel.

To teleport an n-qubit GHZ state

$$|\Omega_n\rangle=\alpha|0\rangle^{\otimes n}+\beta|1\rangle^{\otimes n}, \qquad (16)$$

suppose Alice and Bob share an $(n+1)$-qubit state with Schmidt rank $n_0=2$ and Alice holds the last qubit as is discussed above. Because the Schmidt rank of the quantum channel equals to 2, we can always rewrite the quantum channel as the superposition of two singlet states:

$$|\Xi\rangle=|n'\rangle_B|0\rangle_A+|n''\rangle_B|1\rangle_A, \qquad (17)$$

where $|n'\rangle$ and $|n''\rangle$ are n-qubit states in Bob's side and we suppose the two singlet states have the same weights for convenience. So the composite system is:

$$
\begin{aligned}
|\Theta\rangle &= |\Omega_n\rangle_A\otimes|\Xi\rangle_{AB}\\
&= (\alpha|0\rangle^{\otimes n}|0\rangle+\beta|1\rangle^{\otimes n}|0\rangle)_A|n'\rangle_B\\
&\quad+(\alpha|0\rangle^{\otimes n}|1\rangle+\beta|1\rangle^{\otimes n}|1\rangle)_A|n''\rangle_B, \qquad (18)
\end{aligned}
$$

which has the same form as Eq. (4). Because the first n qubits of Alice are all in the same state, by following the steps above, we can always represent the $n+1$-qubit state of Alice by a two-qubit state, arriving Eq. (6). Then Alice performs a projective measurement with the measuring basis chosen to be the Bell basis of the two qubits and then sends the result to Bob. Bob then performs the unitary operations on his qubits. The unitary operation is a combination of $\mathbf{U}'=\langle n'|i\rangle^{\otimes n}$ and $\mathbf{U}''=\langle n''|i\rangle^{\otimes n}$, where $i=\{0,1\}$, which can be calculated in the same way.

To teleport an n-qubit W state, on one hand, Bob needs to hold n qubits on his hand which are used to recover the teleported state, on the other hand, as an arbitrary n-qubit W state contains n coefficients which require at least ceil$(\log_2 n)$ qubits to represent, so Alice needs to keep ceil$(\log_2 n)$ qubits, where ceil$(\cdot)$ takes the upper integer. So to teleport an n-qubit W state, the entangled state shared by Alice and Bob must contain $n+\text{ceil}(\log_2 n)$ qubits. For convenience, suppose $n = 2^{n'}$ where n' is an integer. We show that an $(n + n')$-qubit channel shared by Alice and Bob with Schmidt rank n_0 equals to n can be used to teleport an n-qubit W state.

Let the channel be $|\Xi\rangle$, we have:

$$|\Xi\rangle = \sum_{i=0}^{i=n-1} \alpha_i |n'_i\rangle_A |n_i\rangle_B, \tag{19}$$

where $|n'_i\rangle_A$ denotes Alice holds n' qubits in state $|n'_i\rangle_A$, the same is $|n_i\rangle_B$, α_i is the coefficient and for convenience suppose $\alpha_i = \frac{1}{\sqrt{n}}$. The summation is taken from 0 to $n-1$ because the Schmidt rank is n so it can be decomposed into the superposition of n states. Suppose the n-qubit W state is:

$$\begin{aligned}|W_n\rangle &= w_1|10\cdots0\rangle + w_2|01\cdots0\rangle + \cdots + w_n|00\cdots1\rangle \\ &= w_1|w_1\rangle + w_2|w_2\rangle + \cdots + w_n|w_n\rangle,\end{aligned} \tag{20}$$

where $|w_j\rangle$ denotes all the qubits are in the 0 state except the ith qubit. The composite system is:

$$|\Theta\rangle = |W_n\rangle_A \otimes |\Xi\rangle_{AB} = \sum_{i=0}^{i=n-1} \frac{1}{\sqrt{n}} |W_n\rangle_A |n'_i\rangle_A |n_i\rangle_B. \tag{21}$$

This is the same as Eq. (8), so Alice takes measurement on her qubit with the basis chosen to be the QFT of states $\{|w_j\rangle|n'_i\rangle\}$. The dimension of the projective measurement is thus n^2. If the Schmidt rank of the channel n_0 is larger than n, then Alice first calculates the QFT of the subspace $\{|w_j\rangle|n'_{i'}\rangle\}$ with i' taken from n to $n_0 - 1$ to get Eqs. (9)-(11). Finally, Alice divides the total $n \cdot n_0$ states into n groups, calculating the QFTs of the n groups, arriving Eqs. (12)-(14). The QFTs of the n groups are chosen to be the measurement basis of Alice's projective measurement. After her measurement, she tells Bob her result. The unitary operations performed by Bob can be calculated follwing the same rule, which is, taking the tensor products of the target state and the collapsed states.

Note that our method of choosing the measurement basis of Alice by QFT can be treated as a generalization of the first QT protocol [1]. In [1], Alice and Bob share an EPR pair and they want to teleport a qubit. Alice holds one qubit of the EPR and the unknown qubit. She performs BSM on her qubits and tells Bob her result. Bob performs unitary operation on his qubit. The Bell states in Eq. (1) are the QFTs of $\{|01\rangle, |10\rangle\}$ and $\{|00\rangle, |11\rangle\}$, respectively:

$$|\Psi^{\pm}\rangle = \mathscr{F}\{|01\rangle, |10\rangle\}, \quad |\Phi^{\pm}\rangle = \mathscr{F}\{|00\rangle, |11\rangle\} \tag{22}$$

which means that Alice's measuring basis is also chosen as the QFT of her qubits.

III. Conclusion

In this article, we propose protocols to teleport a four-qubit GHZ state and a three-qubit W state. Then we generalize our protocol to a multi-qubit state system. To teleport a GHZ state, Alice always holds one qubit while Bob holds the rest qubits. Then Alice performs projective measurement on her qubits. After her measurement, she tells Bob her result and Bob takes the unitary operation according to her result. To teleport a W state, Alice needs to hold more than one qubit. The projective basis is calculated by using QFT of Alice's qubits and the unitary operations of Bob can be decided following the principle of superposition of quantum state. We show that our protocols can be easily extended to multi-qubit state. The unitary operations performed by Bob can be decomposed into a series of single-bit gate and controlled-not gate [18]. To our knowledge, using QFT in QT has not been proposed and it is a universal and convenient way to find the measuring basis of Alice's projective measurement compared with other multi-qubit QT protocols [5], [6]. Our work offers a way to multi-qubit quantum teleportation and could have potential application in quantum communication.

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