# Bidirectional Quantum Teleportation by Using Two GHZ-States as the Quantum Channel

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***Abstract*— Bidirectional quantum teleportation protocol enables two way quantum communication between two quantum nodes in a quantum communication network. In quantum network, the quantum nodes are linked to each other through a quantum channel. In this letter, by using the quantum channel composed of two three-qubit GHZ states, we propose an improved protocol for asymmetric bidirectional quantum teleportation which is based on local CNOT gate operations, Bell state measurements, GHZ-state measurements and unitary operations. Compared with previous protocols for bidirectional quantum teleportation, the proposed protocol requires less consumption of quantum and classical resources, possesses higher intrinsic efficiency and less operation complexity.**

***Index Terms*—. Bidirectional quantum teleportation, GHZ-state, CNOT operation.**

## I. Introduction

QUANTUM entanglement is a key ingredient in many quantum information processing (QIP) tasks like quantum teleportation (QT) [1], quantum key distribution (QKD) [2], [3] and quantum secure direct communication (QSDC) [4]–[6] etc. The generation and manipulation of entanglement lie at heart of QIP. A useful method to create entangled state is entanglement swapping [7]. Because of the ability of entangling distant qubits, entanglement swapping has a unique place in quantum networking and quantum repeater protocols [8].

QT, an astonishing feature of quantum entanglement, is a technique of transmitting quantum information, which is embedded in an unknown quantum state, from one place to a distant place with the assist of shared entangled state between sender and receiver, local operations and classical communications [1]. Since the first theoretical idea of QT was proposed by the authors [1] in 1993, it has attracted much attention of researchers both theoretically [9]–[15] and experimentally [16]–[21]. For perfect QT i.e., teleportation with unit fidelity and unit probability, the quantum channel should be pure and maximally entangled [14]. But in practical situation, decoherence reduces both purity and entanglement of the entangled state, and gives rise to quantum noise. Therefore, the minimization of decoherence effect is very essential. For this purpose, quantum entanglement distillation is used, which reduces the influence of decoherence and improves the quality of entangled states [22].

Bidirectional quantum teleportation (BQT) [23] is a two-way transmission of quantum information in which two participants can transmit their unknown quantum states to each other with or without the help of a supervisor. Due to the capability of transmitting quantum information between two participants in two directions simultaneously, BQT may be regarded as the expansion of QT protocol. BQT protocols are very important for both quantum communication and quantum computation. Recently, BQT has attracted much attention of researchers and found applications in QSDC [24], quantum remote control [25] and cryptographic switch [26].

In 2013, by utilizing five-qubit cluster state as the quantum channel, the authors in [23] proposed a protocol for BQT in which two participants Alice and Bob can teleport their unknown single-qubit quantum states to each other with the help of a supervisor Charlie. Several protocols for BQT involving three participants [27]–[32] and two participants [33]–[37] have been proposed by using various types of entangled states as the quantum channel.

In 2016, by utilizing five-qubit cluster state as the quantum channel, the authors in [33] proposed a different protocol for BQT in which Alice can teleport a two-qubit entangled state to Bob and at the same time Bob can teleport a single-qubit state to Alice without the help of supervisor. In [34], the authors proposed another protocol for BQT in which two participants Alice and Bob share two three-qubit GHZ states as the quantum channel and they can mutually transmit two-qubit entangled states to each other. The first protocol for BQT of arbitrary two-qubit states was proposed in [35] by using eight-qubit entangled state as the quantum channel.

In 2019, by utilizing a six-qubit entangled state as the quantum channel, the authors in [36] proposed a new protocol for BQT in which they claimed that Alice can transmit a special type of three-qubit entangled state to Bob and at the same time Bob can transmit an arbitrary single-qubit state to Alice by using GHZ-state measurement, single-qubit von Neumann measurement, Bell state measurement and unitary operations. The six-qubit entangled state used as the quantum channel in [36], [37] is

$$|C_6\rangle = \frac{1}{2}(|000000\rangle + |000111\rangle + |111000\rangle + |111111\rangle)_{123456} \tag{1}$$

which is just a product of two three-qubit GHZ states i.e., $|C_6\rangle = |GHZ\rangle_{123} \otimes |GHZ\rangle_{456}$, where $|GHZ\rangle = (|000\rangle + |111\rangle)/\sqrt{2}$

In [36], the authors assumed that qubits 1 & 6 are with Alice and qubits 2, 3, 4 & 5 are with Bob. They consider the simultaneous teleportation of three-qubit entangled state of the form $|\varphi\rangle_{ABC} = a_0|000\rangle + a_1|111\rangle$ with $|a_0|^2 + |a_1|^2 = 1$ from Alice to Bob and an arbitrary single-qubit state of the form $|\varphi\rangle_D = b_0|0\rangle + b_1|1\rangle$ with $|b_0|^2 + |b_1|^2 = 1$ from Bob to Alice. In their proposed scheme for BQT, Alice performs GHZ-state measurement (GSM) and single-qubit Von Neumann measurement (SM) on her qubits (A, B, 1) & C respectively

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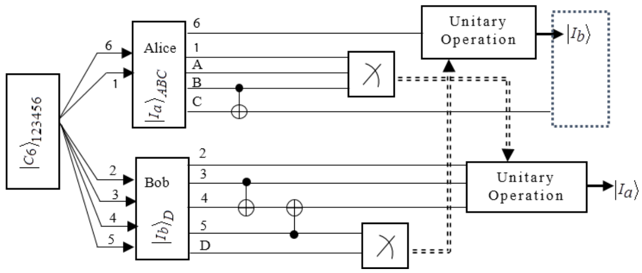


Fig. 1. Quantum circuit for BQT via six-qubit entangled state. Dashed arrows represent classical communication.

and Bob performs Bell state measurements (BSM) on his qubits (D, 5). They convey their measurement results to each other and on the basis of these measurement results, Alice and Bob perform suitable unitary operations on their own qubits (*see Table-II of Ref.* [*36*]). Finally, to reconstruct the original states, first Bob performs CNOT operation on his qubits 3 & 4 with qubit-3 as control qubit & qubit-4 as target qubit and then Alice performs CNOT operation on qubit 6 & 4 with qubit-6 as control qubit & qubit-4 as target qubit (*see Fig. 2 of Ref.* [36]). Since qubit-6 is with Alice and qubit-4 is with Bob, the CNOT operation performed by Alice on qubits 6 & 4 is *nonlocal CNOT operation* [38]–[40]. Nonlocal CNOT gate operation can be implemented by local operations, classical communications and shared entanglement. In [38], [39], the authors proved that one shared e-bit (maximally entangled pairs of qubits) and two cbits (one bit of classical communication from Alice to Bob plus one bit of classical communication from Bob to Alice) are necessary and sufficient for the nonlocal implementation of a quantum CNOT gate operation. Hence, the BQT protocol proposed in [36] requires extra resources such as additional classical communication, shared entanglement and local operations. A new scheme avoiding such nonlocal operation will give an important improvement to BQT performance.

In this letter, by utilizing the same six-qubit entangled state as the quantum channel, we propose an improved protocol for BQT to avoid nonlocal CNOT gate operation. That is, our proposed BQT protocol avoid the additional requirements of classical communications, entanglement sharing and local operations. In other words, the extra consumption of one ebit of entanglement and two cbits of classical communication arising due to nonlocal CNOT operation has been avoided.

## II. Bidirectional Quantum Teleportation (3↔1) via Six-Qubit Entangled State

The unequal number of qubits possessed by each participants in the quantum channel motivates the interest in asymmetric BQT protocol. In the present quantum channel, which is given in Eq. (1), Alice possesses two qubits and Bob possesses four qubits and therefore asymmetric BQT 3↔1 is considered here.

Let, Alice has a three-qubit entangled state of the form

$$|I_a\rangle = (a_0|000\rangle + a_1|111\rangle)_{ABC}\,, \quad \text{with } |a_0|^2 + |a_1|^2 = 1 \tag{2}$$

and Bob has an arbitrary single-qubit information state of the form

$$|I_b\rangle = (b_0|0\rangle + b_1|1\rangle)_D\,, \quad \text{with } |b_0|^2 + |b_1|^2 = 1 \tag{3}$$

Suppose Alice and Bob want to teleport these quantum states to each other simultaneously. For implementing BQT, let Alice and Bob share the six-qubit entangled state $|C_6\rangle_{123456}$, which is given in Eq. (1), as the quantum channel in which qubits 1 & 6 belong to Alice and qubits 2, 3, 4, & 5 belong to Bob.

The combined state of the whole system is given by

$$\begin{aligned}
&|\phi\rangle_{ABCD123456} \\
&= |I_a\rangle_{ABC} \otimes |I_b\rangle_D \otimes |C_6\rangle_{123456} \\
&= \frac{1}{2}\left(a_0|000\rangle + a_1|111\rangle\right)_{ABC} \otimes \left(b_0|0\rangle + b_1|1\rangle\right)_D \\
&\quad \otimes (|000000\rangle + |000111\rangle + |111000\rangle + |111111\rangle)_{123456}
\end{aligned} \tag{4}$$

To implement BQT (3↔1), the proposed scheme is executed in the following way:

*Step-1*: *Local operations*: *CNOT gate operations.*

- Alice performs CNOT gate operation on her qubit pair (B, C) with qubit-B as control qubit & qubit-C as target qubit.
- Bob performs CNOT gate operations on his two qubit pairs: first on qubit pair (3, 4) with qubit-3 as control qubit and qubit-4 as target qubit and then on qubit pair (4, 5) with qubit-5 as control qubit and qubit-4 as target qubit.

After this step, the initial state becomes

$$\begin{aligned}
|\phi'\rangle = \frac{1}{2}&[(a_0|000\rangle_{AB1}|000\rangle_{234} + a_0|001\rangle_{AB1}|111\rangle_{234} \\
&+a_1|110\rangle_{AB1}|000\rangle_{234} + a_1|111\rangle_{AB1}|111\rangle_{234}) \otimes |0\rangle_C \\
&\otimes (b_0|00\rangle_{D5}|0\rangle_6 + b_0|01\rangle_{D5}|1\rangle_6 + b_1|10\rangle_{D5}|0\rangle_6 \\
&+b_1|11\rangle_{D5}|1\rangle_6]]
\end{aligned} \tag{5}$$

The state $|\phi'\rangle$ can also be written as

$$\begin{aligned}
&|\phi'\rangle \\
&= \frac{1}{4}\left[\{|G_1\rangle_{AB1}\left(a_0|000\rangle + a_1|111\rangle\right)_{234} + |G_2\rangle_{AB1}\right. \\
&\quad \left(a_0|000\rangle - a_1|111\rangle\right)_{234} + |G_3\rangle_{AB1}\left(a_0|111\rangle + a_1|000\rangle\right)_{234} \\
&\quad + |G_4\rangle_{AB1}\left(a_0|111\rangle - a_1|000\rangle\right)_{234}\} \otimes |0\rangle_C \\
&\quad \otimes \left\{\left|\phi^+\right\rangle_{D5}\left(b_0|0\rangle + b_1|1\rangle\right)_6 + \left|\phi^-\right\rangle_{D5}\left(b_0|0\rangle - b_1|1\rangle\right)_6\right. \\
&\quad \left. + \left|\psi^+\right\rangle_{D_5}\left(b_0|1\rangle + b_1|0\rangle\right)_6 + \left|\psi^-\right\rangle_{D5}\left(b_0|1\rangle - b_1|0\rangle\right)_6\}\right]
\end{aligned} \tag{6}$$

in which $|\ \phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \& \, |\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ are Bell states and the states $|G_i\rangle\,, i = 1,2,3,4$ are GHZ states given by

$$|G_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), |G_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \tag{7.a}$$

$$|G_3\rangle = \frac{1}{\sqrt{2}}(|001\rangle + |110\rangle), |G_4\rangle = \frac{1}{\sqrt{2}}(|001\rangle - |110\rangle)\Big) \tag{7.b}$$

*Step-2*: *Measurements and classical communications:*

- Alice performs Von Neumann type measurement on her qubits (A, B, 1) by using the states $|G_i\rangle\,, i = 1, 2, 3, 4$.
- Bob performs BSM on his qubit pair (D, 5) by using Bell-basis $\{|\phi^{\pm}\rangle\,, |\psi^{\pm}\rangle\}$.

Alice and Bob convey their measurement results to each other via classical communications. The collapsed states of qubits 2, 3, 4 & 6 corresponding to the measurement results of Alice and Bob are given in Table-I.

TABLE I

COLLAPSED STATES OF QUBITS 2, 3, 4 & 6 AND ALICE'S (BOB'S) UNITARY OPERATIONS (UO) CORRESPONDING TO BOB'S (ALICE'S) MEASUREMENT RESULTS (MR)

| **Alice's MR** | **Bob's MR** | **Collapsed state of qubits 2, 3, 4 & 6** | **Alice's UO** $U_6$ | **Bob's UO** $U_2 \otimes U_3 \otimes U_4$ |
|---|---|---|---|---|
| $\|G_1\rangle_{AB1}$ | $\|\phi^+\rangle_{D5}$ | $(a_0\|000\rangle + a_1\|111\rangle)_{234} \otimes (b_0\|0\rangle + b_1\|1\rangle)_6$ | $I$ | $I \otimes I \otimes I$ |
| $\|G_1\rangle_{AB1}$ | $\|\phi^-\rangle_{D5}$ | $(a_0\|000\rangle + a_1\|111\rangle)_{234} \otimes (b_0\|0\rangle - b_1\|1\rangle)_6$ | $\sigma_z$ | $I \otimes I \otimes I$ |
| $\|G_1\rangle_{AB1}$ | $\|\psi^+\rangle_{D5}$ | $(a_0\|000\rangle + a_1\|111\rangle)_{234} \otimes (b_0\|1\rangle + b_1\|0\rangle)_6$ | $\sigma_x$ | $I \otimes I \otimes I$ |
| $\|G_1\rangle_{AB1}$ | $\|\psi^-\rangle_{D5}$ | $(a_0\|000\rangle + a_1\|111\rangle)_{234} \otimes (b_0\|1\rangle - b_1\|0\rangle)_6$ | $\sigma_z\sigma_x$ | $I \otimes I \otimes I$ |
| $\|G_2\rangle_{AB1}$ | $\|\phi^+\rangle_{D5}$ | $(a_0\|000\rangle - a_1\|111\rangle)_{234} \otimes (b_0\|0\rangle + b_1\|1\rangle)_6$ | $I$ | $\sigma_z \otimes \sigma_z \otimes \sigma_z$ |
| $\|G_2\rangle_{AB1}$ | $\|\phi^-\rangle_{D5}$ | $(a_0\|000\rangle - a_1\|111\rangle)_{234} \otimes (b_0\|0\rangle - b_1\|1\rangle)_6$ | $\sigma_z$ | $\sigma_z \otimes \sigma_z \otimes \sigma_z$ |
| $\|G_2\rangle_{AB1}$ | $\|\psi^+\rangle_{D5}$ | $(a_0\|000\rangle - a_1\|111\rangle)_{234} \otimes (b_0\|1\rangle + b_1\|0\rangle)_6$ | $\sigma_x$ | $\sigma_z \otimes \sigma_z \otimes \sigma_z$ |
| $\|G_2\rangle_{AB1}$ | $\|\psi^-\rangle_{D5}$ | $(a_0\|000\rangle - a_1\|111\rangle)_{234} \otimes (b_0\|1\rangle - b_1\|0\rangle)_6$ | $\sigma_z\sigma_x$ | $\sigma_z \otimes \sigma_z \otimes \sigma_z$ |
| $\|G_3\rangle_{AB1}$ | $\|\phi^+\rangle_{D5}$ | $(a_0\|111\rangle + a_1\|000\rangle)_{234} \otimes (b_0\|0\rangle + b_1\|1\rangle)_6$ | $I$ | $\sigma_x \otimes \sigma_x \otimes \sigma_x$ |
| $\|G_3\rangle_{AB1}$ | $\|\phi^-\rangle_{D5}$ | $(a_0\|111\rangle + a_1\|000\rangle)_{234} \otimes (b_0\|0\rangle - b_1\|1\rangle)_6$ | $\sigma_z$ | $\sigma_x \otimes \sigma_x \otimes \sigma_x$ |
| $\|G_3\rangle_{AB1}$ | $\|\psi^+\rangle_{D5}$ | $(a_0\|111\rangle + a_1\|000\rangle)_{234} \otimes (b_0\|1\rangle + b_1\|0\rangle)_6$ | $\sigma_x$ | $\sigma_x \otimes \sigma_x \otimes \sigma_x$ |
| $\|G_3\rangle_{AB1}$ | $\|\psi^-\rangle_{D5}$ | $(a_0\|111\rangle + a_1\|000\rangle)_{234} \otimes (b_0\|1\rangle - b_1\|0\rangle)_6$ | $\sigma_z\sigma_x$ | $\sigma_x \otimes \sigma_x \otimes \sigma_x$ |
| $\|G_4\rangle_{AB1}$ | $\|\phi^+\rangle_{D5}$ | $(a_0\|111\rangle - a_1\|000\rangle)_{234} \otimes (b_0\|0\rangle + b_1\|1\rangle)_6$ | $I$ | $i\sigma_y \otimes i\sigma_y \otimes i\sigma_y$ |
| $\|G_4\rangle_{AB1}$ | $\|\phi^-\rangle_{D5}$ | $(a_0\|111\rangle - a_1\|000\rangle)_{234} \otimes (b_0\|0\rangle - b_1\|1\rangle)_6$ | $\sigma_z$ | $i\sigma_y \otimes i\sigma_y \otimes i\sigma_y$ |
| $\|G_4\rangle_{AB1}$ | $\|\psi^+\rangle_{D5}$ | $(a_0\|111\rangle - a_1\|000\rangle)_{234} \otimes (b_0\|1\rangle + b_1\|0\rangle)_6$ | $\sigma_x$ | $i\sigma_y \otimes i\sigma_y \otimes i\sigma_y$ |
| $\|G_4\rangle_{AB1}$ | $\|\psi^-\rangle_{D5}$ | $(a_0\|111\rangle - a_1\|000\rangle)_{234} \otimes (b_0\|1\rangle - b_1\|0\rangle)_6$ | $\sigma_z\sigma_x$ | $i\sigma_y \otimes i\sigma_y \otimes i\sigma_y$ |

TABLE II

COMPARISON AMONG SIX BQT PROTOCOLS

| **Protocol** | **Q** | **Measurements** | **C-bit** | **BQT** | **Efficiency τ (%)** |
|---|---|---|---|---|---|
| [33] | 5 | 2 BSM, 1 SM | 5 | 2↔1 | 30 |
| [34] | 6 | 4 SM | 4 | 2↔2 | 40 |
| [35] | 8 | 8 SM | 8 | 2↔2 | 25 |
| [36] | 7 | 1 GSM, 1 SM, 1 BSM | 7 | 3↔1 | 28.5 |
| [37] | 6 | 2 GSM | 4 | 2↔2 | 40 |
| Our | 6 | 1 GSM, 1 BSM | 4 | 3↔1 | 40 |

TABLE III

COMPARISON BETWEEN PROPOSED ASYMMETRIC BQT PROTOCOL AND PREVIOUS ASYMMETRIC BQT PROTOCOL [36]

| **Protocol** (3↔1) | **CNOT Operation** | **Measurements** | **Efficiency τ** |
|---|---|---|---|
| [36] | Nonlocal | 1 GSM, 1 BSM & 1SM | 28.5% |
| Our | Local | 1GSM & 1 BSM | 40% |

*Step-3*: *Unitary Transformations*:

- Alice performs appropriate unitary transformation on her qubit-6 depending upon the measurement results received from Bob.
- Bob performs appropriate unitary transformation on his qubits 2, 3, 4 depending upon the measurement results received from Alice.

Thus, BQT is successfully realized. The unitary operations are given in Table-I and the quantum circuit implementing for asymmetric BQT is given in Fig. 1.

## III. CONCLUSION

The proposed BQT protocol claims an asymmetric quantum information transmission between two participants since the number of the qubits owned by each participants in the present quantum channel is not equal. The previous protocol for asymmetric BQT [36] needs to perform non-local CNOT gate operation that requires extra resources such as additional classical communication, entanglement sharing and local operations. Avoiding such an operation gives an important improvement to BQT performance. In this letter, we have proposed an improved protocol for BQT in which BQT is realized successfully by using local CNOT operations, GHZ-state measurements (GSM), Bell-state measurements (BSM), and unitary operations. Unlike the previous BQT protocol [36], our proposed BQT protocol avoid nonlocal CNOT operation and therefore it does not require additional local operations, 1-ebit of entanglement and 2-cbit of classical communication. Hence, our proposed asymmetric BQT protocol requires less quantum and classical resources as compared with previous BQT protocol [36].

In Table-II, the proposed BQT protocol is compared with previous BQT protocols [33]–[37]. The comparison are made from five aspects namely quantum resource consumption (Q), measurements, no. of c-bits used in classical communications, type of BQT and intrinsic efficiency. In [36], the intrinsic efficiency is defined as $\tau = q_s/(q_u + b_t)$ in which $q_s$ indicates the number of qubits consisting of the quantum information to be transmitted, $q_u$ is the number of qubits used in quantum channel and $b_t$ represents the number of c-bits used in classical communication. In [36], the authors reported that the efficiency of their proposed BQT protocol is 36.3%. But, if the resources used in the implementation of nonlocal CNOT operation are taken into account, then the efficiency of their BQT protocol [36] will be 28.5%. The further comparison between proposed BQT protocol and the previous BQT protocol [36] is given in Table-III. Table-II and Table-III show that our proposed BQT protocol possesses higher intrinsic efficiency as compared with previous BQT protocols [33], [36]. Also, our proposed asymmetric BQT protocol requires less consumption of c-bits as compared with previous asymmetric BQT protocols [33], [36]. The operation complexity in our proposed BQT protocol is less than previous BQT protocol [36].

Consequently, compared with previous BQT protocol [36], the proposed BQT protocol (i) needs to perform local CNOT operations instead of nonlocal CNOT operation (ii) reduces the operation complexity, (iii) requires less consumption of quantum and classical resources and (iv) possesses higher intrinsic efficiency. Thus, our proposed protocol has remarkable advantages and we hope that the present work is an important contribution in the development of the field of quantum information theory.

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