《量子信息基础》2024.5.23 随堂作业：

（2024.5.28 22:00 前提交）

二阶关联函数在光子数表象下为 $g^{(2)}(\tau)=\frac{\langle n_1(t)n_2(t+\tau)\rangle}{\langle n_1(t)\rangle\langle n_2(t+\tau)\rangle}$

1. Show that, in terms of the intensity fluctuations defined by $\Delta I(t)=I(t)-\langle I(t)\rangle$, the second order correlation function $g^{(2)}(\tau)$ can be written in the form:
$$g^{(2)}(\tau)=1+\frac{\langle\Delta I(t)\Delta I(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}$$
Hence prove that $g^{(2)}(0)\geq 1$ for classical light.

$$\Delta I(t)=I(t)-\langle I(t)\rangle$$

$$I(t)=\Delta I(t)+\langle I(t)\rangle$$

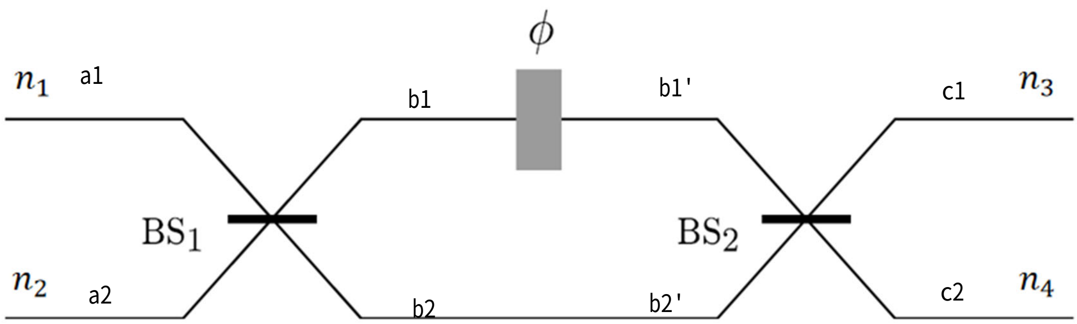
$$I(t+\tau)=\Delta I(t+\tau)+\langle I(t+\tau)\rangle$$

$$g^{(2)}(\tau)=\frac{\langle(\Delta I(t)+\langle I(t)\rangle)(\langle I(t+\tau)\rangle+\Delta I(t+\tau))\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}=1+\frac{\langle\Delta I(t)\Delta I(t+\tau)\rangle}{\langle I(t)\rangle\langle I(t+\tau)\rangle}$$

For classical light sources： $g^{(2)}(0)=1+\frac{\langle(\Delta I(t))^2\rangle}{\langle\Delta I(t)\rangle^2}\geq 1$

推导正确给 20 分，答案正确给 20 分

2. In the Mach-Zehnder interferometer shown below, drive the analytical relation between the output photon number n_3, n_4 and the input photon number n_1, n_2.



$$\begin{cases}\hat{b}_1=\frac{1}{\sqrt{2}}(\hat{a}_1+\hat{a}_2)\\ \hat{b}_2=\frac{1}{\sqrt{2}}(\hat{a}_1-\hat{a}_2)\end{cases},\quad\therefore\begin{pmatrix}\hat{b}_1\\ \hat{b}_2\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}\hat{a}_1\\ \hat{a}_2\end{pmatrix}$$

$$\begin{cases} \hat{b'}_1 = \hat{b}_1 \cdot e^{i\phi} \\ \hat{b'}_2 = \hat{b}_2 \end{cases}, \quad \therefore \begin{pmatrix} \hat{b'}_1 \\ \hat{b'}_2 \end{pmatrix} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

$$\begin{cases} \hat{c}_1 = \frac{1}{\sqrt{2}}(\hat{b'}_1 + \hat{b'}_2) \\ \hat{c}_2 = \frac{1}{\sqrt{2}}(\hat{b'}_1 - \hat{b'}_2) \end{cases}, \quad \therefore \begin{pmatrix} \hat{c}_1 \\ \hat{c}_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \hat{b'}_1 \\ \hat{b'}_2 \end{pmatrix}$$

$$\therefore \begin{pmatrix} \hat{c}_1 \\ \hat{c}_2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} e^{i\phi} + 1 & e^{i\phi} - 1 \\ e^{i\phi} - 1 & e^{i\phi} + 1 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix}$$

$$\begin{cases} n_3 = \hat{c}_1{}^{+} \hat{c}_1 \\ n_4 = \hat{c}_2{}^{+} \hat{c}_2 \end{cases}$$

$$\therefore n_3 = \frac{1}{2}[\hat{a}_1(e^{-i\phi} + 1) + \hat{a}_2(e^{-i\phi} - 1)] \cdot \frac{1}{2}[\hat{a}_1(e^{i\phi} + 1) + \hat{a}_2(e^{i\phi} - 1)]$$
$$= \frac{1}{2} n_1 (1 + cos\phi) + \frac{1}{2} n_2 (1 - cos\phi)$$

For the same reason, $n_4 = \frac{1}{2} n_1 (1 - cos\phi) + \frac{1}{2} n_2 (1 + cos\phi)$

三个中间步骤推导正确给 40 分，答案正确给 20 分

$$g^2(\tau) = \frac{\langle I(t) I(t+\tau) \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$
$$= \frac{\langle (\Delta I(t) + \langle I(t) \rangle)((\Delta I(t+\tau) + \langle I(t+\tau) \rangle)) \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$
$$= \frac{\langle \Delta I(t) \Delta I(t+\tau) + \Delta I(t) \langle I(t+\tau) \rangle + \langle I(t) \rangle \Delta I(t+\tau) + \langle I(t) \rangle \langle I(t+\tau) \rangle \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$
$$= \frac{\langle \Delta I(t) \Delta I(t+\tau) \rangle + \langle \Delta I(t) \langle I(t+\tau) \rangle \rangle + \langle \langle I(t) \rangle \Delta I(t+\tau) \rangle + \langle \langle I(t) \rangle \langle I(t+\tau) \rangle \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$
$$= \frac{\langle \Delta I(t) \Delta I(t+\tau) \rangle + \langle \Delta I(t) \rangle \langle I(t+\tau) \rangle + \langle \Delta I(t+\tau) \rangle \langle I(t) \rangle + \langle I(t) \rangle \langle I(t+\tau) \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$
$$= \frac{\langle \Delta I(t) \Delta I(t+\tau) \rangle + 0 \cdot \langle I(t+\tau) \rangle + 0 \cdot \langle I(t) \rangle + \langle I(t) \rangle \langle I(t+\tau) \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$
$$= 1 + \frac{\langle \Delta I(t) \Delta I(t+\tau) \rangle}{\langle I(t) \rangle \langle I(t+\tau) \rangle}$$