《量子信息基础》2024.3.21 随堂作业：

（2024.3.26 晚 22 点前提交）

1. (Text book* Problem 3.4)
   (a) Show that the sum of two Hermitian operators is Hermitian.

Assume $\hat{Q}$ and $\hat{S}$ are Hermitian operators, and $f(x)$ and $g(x)$ are arbitrary functions.

$\langle f|\hat{Q}g\rangle = \langle \hat{Q}f|g\rangle$ and $\langle f|\hat{S}g\rangle = \langle \hat{S}f|g\rangle$. Therefore

$$\langle f|(\hat{Q}+\hat{S})g\rangle = \langle f|\hat{Q}g\rangle + \langle f|\hat{S}g\rangle = \langle \hat{Q}f|g\rangle + \langle \hat{S}f|g\rangle = \langle (\hat{Q}+\hat{S})f|g\rangle$$

So $(\hat{Q}+\hat{S})$ are Hermitian

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   (b) Suppose $\hat{Q}$ is Hermitian, and α is a complex number. Under what condition (on α) is $\alpha\hat{Q}$ Hermitian?

$$\langle f|\alpha\hat{Q}g\rangle = \alpha\langle f|\hat{Q}g\rangle$$

$$\langle \alpha\hat{Q}f|g\rangle = \alpha^*\langle f|\hat{Q}g\rangle$$

If α is a real number, $\alpha\hat{Q}$ is Hermitian.

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   (c) When is product of two Hermitian operators Hermitian?

$$\langle f|\hat{Q}\hat{S}g\rangle = \langle f|\hat{Q}(\hat{S}g)\rangle = \langle \hat{Q}f|\hat{S}g\rangle = \langle \hat{S}\hat{Q}f|g\rangle$$

If $\hat{Q}\hat{S}$ is Hermitian,

$$\langle f|\hat{Q}\hat{S}g\rangle = \langle \hat{Q}\hat{S}f|g\rangle$$

$\hat{Q}\hat{S}$ needs to be commutable.

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(d) Show that the position operator ($\hat{x}$) and the Hamiltonian operator $\left(\hat{H} = -\left(\frac{\hbar^2}{2m}\right)\frac{d^2}{dx^2} + V(x)\right)$ are hermitian.

$$\langle f|\hat{x}g\rangle = \int_{-\infty}^{+\infty} f^*(x)xg(x)dx = \int_{-\infty}^{+\infty} \left(xf(x)\right)^* g(x)dx = \langle \hat{x}f|g\rangle$$

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$$\langle f|\hat{H}g\rangle = \int_{-\infty}^{+\infty} f^*(x)\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)\right]g(x)dx$$

这一项=0　　再对这一项分部积分

$$= -\frac{\hbar^2}{2m}f^*\frac{dg}{dx}\bigg|_{-\infty}^{+\infty} + \frac{\hbar^2}{2m}\int_{-\infty}^{+\infty}\frac{df^*}{dx}\frac{dg}{dx}dx + \int_{-\infty}^{+\infty}[V(x)f(x)]^*g(x)dx$$

这一项=0

$$= \frac{\hbar^2}{2m}g\frac{df^*}{dx}\bigg|_{-\infty}^{+\infty} - \frac{\hbar^2}{2m}\int_{-\infty}^{+\infty}\frac{d^2f^*}{dx^2}gdx + \int_{-\infty}^{+\infty}[f(x)V(x)]^*g(x)dx$$

$$= \int_{-\infty}^{+\infty}\left[-\frac{\hbar^2}{2m}\frac{d^2f}{dx^2} + V(x)f\right]^* g(x)dx = \langle \hat{H}f|g\rangle$$

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2. A Hermitian operator $\hat{A}$ has a complete orthonormal set of eigenfunctions $|\psi_n\rangle$ with associated eigenvalues α_n. Show that we can always write

$$\hat{A} = \sum_i \alpha_i |\psi_i\rangle\langle\psi_i|$$

$$\hat{A}|\psi_n\rangle = \alpha_n|\psi_n\rangle$$

$$\hat{A}|\psi_n\rangle = \sum_i \alpha_i|\psi_i\rangle\langle\psi_i|\psi_n\rangle = \alpha_n|\psi_n\rangle$$

正交性，i=n时才有值

$$\therefore\ \hat{A} = \sum_i \alpha_i|\psi_i\rangle\langle\psi_i|$$

推导和答案正确给 20 分

* David J. Griffiths, and Darrell F. Schroeter, Introduction to Quantum Mechanics (3rd Edition), Cambridge University Press (2018).