《量子信息基础》2024.4.11 随堂作业：

（2024.4.16 晚 22 点前提交）

1. (Text book* Problem 5.4)
   (a) If ψ_a and ψ_b are orthogonal, and both are normalized, what is the constant A in Equation 5.17 ?
   (b) If $\psi_a = \psi_b$ (and it is normalized), what is A? (This case, of course, occurs only for bosons.)

(a) $\psi_{\pm}(\boldsymbol{r_1}, \boldsymbol{r_2}) = A[\psi_a(\boldsymbol{r_1})\psi_b(\boldsymbol{r_2}) \pm \psi_b(\boldsymbol{r_1})\psi_a(\boldsymbol{r_2})]$

$$\int \left|\psi_{\pm}(\boldsymbol{r_1}, \boldsymbol{r_2})\right|^2 d\boldsymbol{r_1} d\boldsymbol{r_2}$$

$$= |\boldsymbol{A}|^2 \int [\psi_a(\boldsymbol{r_1})\psi_b(\boldsymbol{r_2}) \pm \psi_b(\boldsymbol{r_1})\psi_a(\boldsymbol{r_2})]^* [\psi_a(\boldsymbol{r_1})\psi_b(\boldsymbol{r_2}) \pm \psi_b(\boldsymbol{r_1})\psi_a(\boldsymbol{r_2})] d\boldsymbol{r_1} d\boldsymbol{r_2}$$

$$= |\boldsymbol{A}|^2 \left[\int |\psi_a(\boldsymbol{r_1})|^2 d\boldsymbol{r_1} \int |\psi_b(\boldsymbol{r_2})|^2 d\boldsymbol{r_2} \pm \int \psi_b^*(\boldsymbol{r_1})\psi_a(\boldsymbol{r_1}) d\boldsymbol{r_1} \int \psi_a^*(\boldsymbol{r_2})\psi_b(\boldsymbol{r_2}) d\boldsymbol{r_2}\right.$$

$$\left.\pm \int \psi_a^*(\boldsymbol{r_1})\psi_b(\boldsymbol{r_1}) d\boldsymbol{r_1} \int \psi_b^*(\boldsymbol{r_2})\psi_a(\boldsymbol{r_2}) d\boldsymbol{r_2} + \int |\psi_b(\boldsymbol{r_1})|^2 d\boldsymbol{r_1} \int |\psi_a(\boldsymbol{r_2})|^2 d\boldsymbol{r_2}\right]$$

$$= |\boldsymbol{A}|^2 (1 \pm 0 \pm 0 + 1) = 2|A|^2 = 1$$

$$\mathrm{A} = \frac{1}{\sqrt{2}}$$

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(b) If $\psi_a = \psi_b$

$$\psi_+(\boldsymbol{r_1}, \boldsymbol{r_2}) = 2A\psi_a(\boldsymbol{r_1})\psi_b(\boldsymbol{r_2})$$

$$\int |\psi_+(\boldsymbol{r_1}, \boldsymbol{r_2})|^2 d\boldsymbol{r_1} d\boldsymbol{r_2} = |\boldsymbol{A}|^2 \int [2\psi_a(\boldsymbol{r_1})\psi_b(\boldsymbol{r_2})]^* [2\psi_a(\boldsymbol{r_1})\psi_b(\boldsymbol{r_2})] d\boldsymbol{r_1} d\boldsymbol{r_2}$$

$$= 4|\boldsymbol{A}|^2 \int |\psi_a(\boldsymbol{r_1})|^2 d\boldsymbol{r_1} \int |\psi_b(\boldsymbol{r_2})|^2 d\boldsymbol{r_2} = 4|\boldsymbol{A}|^2 = \boldsymbol{1}$$

$$\mathrm{A} = \frac{1}{2}$$

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2. (Text book* Problem 5.23, 注意是教材的第二版)
   Support you had three (non-interacting) particles, in thermal equilibrium, in a one-dimensional harmonic potential, with a total energy $E = (9/2)\hbar\omega$
   (c) If they are distinguishable particles (but all with the same mass), what are the possible occupation-number configurations, and how many distinct (three particle) states are there for each one? What is the most probable configuration? If you picked a particle at random and measured its energy, what values might you get, and what is the probability of each one? What is the most probable energy?

(d) Do the same for the case of identical fermions (ignoring spin).
(e) Do the same for the case of identical bosons (ignoring spin).

The total energy of the three particles is

$$E = \left(n_1 + n_2 + n_3 + \frac{3}{2}\right)\hbar\omega = \frac{9}{2}\hbar\omega$$

$$n_1 + n_2 + n_3 = 3$$

The possible combinations of $(n_1,\ n_2,\ n_3)$ are

$$(1, 1, 1)$$

$$(0, 0, 3), (0, 3, 0), (3, 0, 0)$$

$$(0, 1, 2), (0, 2, 1), (1, 2, 0), (2, 1, 0), (1, 0, 2), (2, 0, 1)$$

(a) If particles are distinguishable

Configuration 1 is $(0, 3, 0, 0, 0, \dots)$, 1 distinct state, Q=1/10;

Configuration 2 is $(2, 0, 0, 1, 0, \dots)$, 3 distinct states, Q=3/10;

Configuration 3 is $(1, 1, 1, 0, 0, \dots)$, 6 distinct states, Q=6/10;

Configuration 3 is the most probable configuration.

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For $E_0 = \frac{1}{2}\hbar\omega$, $P_0 = \frac{3}{10} \times \frac{2}{3} + \frac{6}{10} \times \frac{1}{3} = \frac{4}{10}$;

For $E_1 = \frac{3}{2}\hbar\omega$, $P_1 = \frac{1}{10} \times 1 + \frac{6}{10} \times \frac{1}{3} = \frac{3}{10}$;

For $E_2 = \frac{5}{2}\hbar\omega$, $P_2 = \frac{6}{10} \times \frac{1}{3} = \frac{2}{10}$;

For $E_3 = \frac{7}{2}\hbar\omega$, $P_3 = \frac{3}{10} \times \frac{1}{3} = \frac{1}{10}$.

E_0 is the most probable energy, with probability of 4/10.

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(b) If particles are fermions 全同粒子，不可区分，费米子一个一态
Configuration 1 and 2 are forbidden. Only one configuration left which is

$$(1, 1, 1, 0, 0, \dots)$$

This is the most probable configuration.

For $E_0 = \frac{1}{2}\hbar\omega$, $P_0 = \frac{1}{3}$;

For $E_1 = \frac{3}{2}\hbar\omega$, $P_1 = \frac{1}{3}$;

For $E_2 = \frac{5}{2}\hbar\omega$, $P_2 = \frac{1}{3}$.

All three energies are the most probable energy.

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(c) If particles are bosons 玻色子，不可区分，可共态

Configuration 1 is $(0, 3, 0, 0, 0, \dots)$, 1 distinct state, Q=1/3;
Configuration 2 is $(2, 0, 0, 1, 0, \dots)$, 1 distinct states, Q=1/3;
Configuration 3 is $(1, 1, 1, 0, 0, \dots)$, 1 distinct states, Q=1/3;
All three configurations are the most probable configuration.

For $E_0 = \frac{1}{2}\hbar\omega$, $P_0 = \frac{1}{3} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{1}{3}$;

For $E_1 = \frac{3}{2}\hbar\omega$, $P_1 = \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{3} = \frac{4}{9}$;

For $E_2 = \frac{5}{2}\hbar\omega$, $P_2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$;

For $E_3 = \frac{7}{2}\hbar\omega$, $P_3 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

E_1 is the most probable energy, with probability of 4/9.

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3. Put 10 distinguishable particles into 4 different quantum states to let the final configuration to be $(4, 3, 2, 1)$ as the macrostate. Calculate the number of microstates in this configuration.

$$P = C_{10}^4 C_6^3 C_3^2 C_1^1 = 12600$$

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* David J. Griffiths, and Darrell F. Schroeter, Introduction to Quantum Mechanics (3rd Edition), Cambridge University Press (2018).