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Multi-hop quantum teleportation via general distinct EPR pairs and its application in quantum network

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Abstract—**Quantum teleportation (QT) provides a useful way to securely transfer unknown quantum state between distant nodes without physically sending the states themselves. In this paper, we present a teleportation scheme of the most general hybrid multi-qudit state via EPR pairs with different dimensions in the chain network. In this structure, the EPR channels shared between neighbor nodes could be distinct, which means that the shared EPR pairs could be different in dimension and the parameters with each other, demonstrating a universality scenario in realistic transmission. In addition, we discuss the condition that can ensure the integrity of the transmitted information with available channel resource, and our scheme could provide a method to solve the resemble problem in the construction of future quantum network.**

Keywords-component; quantum information process; quantum teleportation; multi-qudit;

I. INTRODUCTION

In quantum communication, information is encoded as quantum states and transmitting this information requires establishing quantum entanglement between the sending and receiving nodes. In order to achieve this process, optical devices are used to control and manipulate the quantum states of photons. Optical fibers play a crucial role in quantum teleportation [1], [2], the characteristics of optical fibers can protect quantum states from decoherence effects, ensuring high-quality quantum state transmission. As for the quantum teleportation, proposed in [3], has received much attention in the past few decades. Currently, quantum teleportation has made great progress both in experimental [4], [5] and has played an irreplaceable role as a fundamental ingredient in the development of many quantum information processing tasks. Several experiments based on orbital angular momentum [6] and photonic systems [7]-[9] have demonstrated high-dimensional teleportation with high efficiency and fidelity. To specify the scenario where a multi-qudit state is teleported in a quantum network, Wang and Chen et al. proposed a scheme for constructing a theoretical quantum network model with a multi-level system [10], where any two communication sites can construct entangled channels via Bell states. Moreover, several protocols have been proposed for use in complicated quantum networks [11], [12]. These schemes demonstrate the prime building blocks for future quantum networks and open the door to explore teleportation applications.

In recent years, there have been various protocols proposed for the teleportation of multi-particle states [13], [14]. In 2019, Kim and Lee et al. demonstrated the teleportation of two different types of hybrid qubits, which has shown the potential for realizing more complicated hybrid entanglement states [15]. Teleportation of such hybrid entanglement states is crucial for enabling flexible interfacing between different physical systems in future quantum networks.

However, the construction of quantum network is still facing the challenge where the entangled resources shared between nodes are not proportionate to teleport some complicated states. In particular, the dimensions of the channel and the teleported qudits may be incompatible where the channels distributed among different nodes in quantum network are two-level Bell pairs and the dimension of the teleported multi-qudit states may be arbitrary instead of only two. In order to overcome the above shortcomings and utilize available channel resources in quantum nodes, our scheme aims to transfer arbitrary unknown hybrid multi-qudit states via the given channel. To solve this problem, a set of control operators are devised to implement onto different particles in quantum nodes. In this work, we are focused on the unknown hybrid multi-qudit states with arbitrary integer dimensions $D_1 D_2 \ldots D_p$. To transmit the state, each intermediate node selects a measurement basis based on its shared channel with neighboring nodes. The receiver Bob then recovers the state based on the measurement results and the channel parameters from the intermediate nodes.

The rest of the paper is organized as follows. In Sec.II, We will begin by introducing single-particle transmission with non-standard teleportation form, followed by the case of multi-hop node networks. In the latter part, we will provide a method to recover the original form. Next, To provide a general solution for multi-particle transmission, we will utilize the conclusions drawn in the previous chapter and extend the discussion to arbitrary multi-particle scenarios in Sec.III. In the end, we will discuss the practical application of our scheme in optical systems and provide a summary of our scheme in Sec.IV.

II. SINGLE QUDIT TELEPORTATION BASED ON GENERAL EPR CHANNELS

A. *The scenario Without Intermediate Node*

At the beginning of the discussion, let us first consider the case of one general qudit state, which is teleported through k-level EPR pairs. The channel is shared between the sender B and the receiver B in advance. The general unknown state can be written as follows

$$|\psi\rangle_1=\sum_{j=0}^{d-1}\alpha_j\left|j\right\rangle_1, \tag{1}$$

with α_j carrying the amplitude and the phase information of the state $\left|\psi\right\rangle_1$ and satisfying the normalization condition $\sum_{j=0}^{d-1}\left|\alpha_j\right|^2=1$. Then, we assume the dimension of each EPR pair is k. In order to ensure the integrity of the transmitted information, it should be greater than the number of the teleported state $\left|\psi\right\rangle_1$, which is $k>d$. In this term, we assume that the k-level EPR pair shared between the sender A and the receiver B can be parameterized as the set (p,q) as

$$\left|\varphi_{(p,q)}\right\rangle_{23}=\frac{1}{\sqrt{k}}\sum_{j'=0}^{k-1}e^{\frac{2\pi ip}{k}j'}\left|j',j'\oplus q\right\rangle_{23}. \tag{2}$$

It represents that the EPR pairs shared between two parties and each pair contains k superposition states. At the beginning of teleportation, we name the initial state as $\left|\Pi_0\right\rangle_{123}=\left|\psi\right\rangle_1\left|\varphi_{(p,q)}\right\rangle_{23}$, which is a direct product of single-qudit state and the k-level EPR pair. Then, the sender is supposed to start the protocol by utilizing the projective measurement on her holding particle, where the orthogonal basis $\{\left|\Lambda_{(m_1,m_2)}\right\rangle_{12}\}$ can be written as follows

$$\left|\Lambda_{(m_1,m_2)}\right\rangle_{12}=\frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}e^{-\frac{2\pi im_1}{d}j}\left|j,j\oplus m_2\right\rangle_{12} \tag{3}$$

where $j\oplus m_2$ represents a modular addition valuing from *0* to *k-1*. It may seem similar to other typical quantum teleportation protocols, but what sets our proposal apart is the non-standard Bell measurement we utilize. Then, the receiver Bob obtains

$$\left|\nu\right\rangle_3=\sum_{j=0}^{d-1}\alpha_j e^{(\frac{2\pi ip}{k}+\frac{2\pi im_1}{d})j}\left|j\oplus m_2\oplus q\right\rangle_3 \tag{4}$$

with $j\oplus m_2\oplus q$ denting the modular addition in *k*. And it can be verified that the recovery operation is $R=\sum_{j'=0}^{k-1}e^{-(\frac{2\pi ip}{k}+\frac{2\pi im_1}{d})j'}\left|j'\right\rangle\left\langle j'\oplus m_2\oplus q\right|$. It can be verified that through the aforementioned recovery operation, the receiver B is able to obtain the transmitted information represented by k-dimensional particle. In the next subsection, we will investigate the use of arbitrary general EPR pairs as channels in chain-type multi-hop networks. A further discussion on recovering the information to its original form will be provided in Sec II-C.

B. Scenario of chain structure with M hops nodes

In the scenario of multi-hop teleportation, we assume that there are M intermediate nodes between the first sender A_0 and the receiver B, where the relay nodes are denoted as $A_1\,A_2\,\ldots\,A_M$. We represent each EPR pair used as a channel as $\left|\varphi_{\{t,(p,q)\}}\right\rangle_{2t+1,2t+2}$, where $t=1,2,\ \ldots\ ,M$ is index order of the senders. Hence we can rewrite the initial state of the system as follows

$$\left|\Pi_0\right\rangle_{1\ldots 2M+2}=\left|\psi\right\rangle_1\left|\varphi_{\{0,(p,q)\}}\right\rangle_{23}\cdots\left|\varphi_{\{M,(p,q)\}}\right\rangle_{2M+1,2M+2} \tag{5}$$

The dimensions of the Bell pairs shared between each intermediate node may be different, where the dimension of the channel shared by each node needs to be greater than that of the channel used in the previous transmission process. If this condition is not met for any of the nodes, errors leading to loss of information integrity may occur. Following the same recursive approach as in the previous subsection, each node sequentially performs a non-standard Bell measurement on the two particles it possesses. For each sen $A_t(t=1,2,\ \ldots\ ,M)$, their measurement outcomes $\{\left|\Lambda_{\{t,(m_1,m_2)\}}\right\rangle\}$ will cause the final state of the system to collapse to $\left|\nu^{\sum\{t,(p,q)\}}_{\sum\{t,(m_1,m_2)\}}\right\rangle$. In this state, the superscript denotes the parameters of the multi-hop channel, while the subscript represents the measurement outcomes of each node. It can be described as:

$$\left|\nu\right\rangle_{2M+2}=\sum_{j=0}^{d-1}\alpha_j e^{(\sum_t\frac{2\pi ip^t}{k^t}+\frac{2\pi im_1^t}{d^t})j}\left|j\oplus\sum_t m_2^t\oplus q^t\right\rangle_{2M+2}, \tag{6}$$

in which $d^0=d,d^1=k^0,\ldots,d^{M-1}=k^M$. Therefore, the state possessed by B is determined by the different channel parameters and measurement outcomes at each intermediate node. Hence the recovery operation of the receiver B can be written as $R=\sum_{j'=0}^{k-1}e^{-(\sum_t\frac{2\pi ip^t}{k^t}+\frac{2\pi im_1^t}{d^t})j'}\left|j'\right\rangle\left\langle j'\oplus\sum_t m_2^t\oplus q^t\right|$ in a recursive way. It can be verified that Bob obtains results similar to those in the previous section, i.e., a superposition state containing the complete information to be transmitted but in a different form.

C. Recovery operation

In order to obtain the superposition state in its original form, in this subsection we will discuss how to achieve state transformation between different states by introducing an auxiliary particle. Moreover, the introducing ancillary state is $\left|\chi\right\rangle_b=\left|0\right\rangle_b$, where $\left|0\right\rangle_b$ can be regarded as one single qudit with dimension d in the ancillary qudit state $\left|\chi\right\rangle_b$. Therefore, we rewrite the state of system $\left|\Psi_1\right\rangle_{Bb}$ as follows

$$\left|\Psi_1\right\rangle_{Bb}=(\sum_{j'=0}^{k-1}\alpha_{j'}\left|j'\right\rangle_B)\otimes\left|\chi\right\rangle_b \tag{7}$$

where k represents the dimension of the pair shared between A_M and B. In the next step, the controlled gate U_{Bb} is supposed to be implemented by Bob to entangle these two parts together, which is $U_{Bb}=\sum_{j'=0}^{k-1}\left|j'\right\rangle\left\langle j'\right|_B\otimes X_b^{j'}$. Next, we perform a generalized X-basis measurement on the controlled party of the previous controlled operation, obtaining the measurement outcome m_B, in which the auxiliary system is collapse to

$$|\chi'\rangle_b = \sum_{j=0}^{d-1} \alpha_j e^{\frac{2\pi i m_B}{k} j} |j\rangle_b \quad (8)$$

The recovery operation determined by m_B can be written as $R_b = \sum_{j=0}^{d-1} e^{-\frac{2\pi i m_B}{k} j} |j\rangle\langle j|$. Thus, our protocol has achieved single-particle teleportation using general channels. In order to obtain more general conclusions, we can simply extend the protocol to the case of multi-particle system.

III. Multi-qudit Teleportation Based on Multi-hop Chain Network

In this section, we will extend the protocol to a more general form by considering multiparticle transmission. let us first consider the general hybrid multi-qudit state

$$|\varphi\rangle_A = \sum_{d_1,d_2,\ldots,d_p=0}^{D_1-1,D_2-1,\ldots,D_p-1} \alpha_{d_1 d_2 \ldots d_p} |d_1 d_2 \ldots d_p\rangle_A \quad (9)$$

Obviously, this multiparticle system satisfies the normalization condition $\sum_{d_1,d_2,\ldots,d_p=0}^{D_1-1,D_2-1,\ldots,D_p-1} \left|\alpha_{d_1 d_2 \ldots d_p}\right|^2 = 1$. In an intuitive way, the teleported state $|\varphi\rangle_A$ can be rewritten as

$$|\varphi\rangle_A = \sum_D \alpha_D |D\rangle_A \quad (10)$$

Here $|D\rangle$ is an abbreviation of the state $|d_1 d_2 \ldots d_p\rangle$, which has the same form as the single-particle transmission in the previous section. For the set D, its size represents the size of p entangled state products, which corresponds to the amount of information contained in a general multi-particle entangled state. In particular, we assume the total number of the dimension of each EPR pairs shared between two nodes is $K_q K_{q-1} \ldots K_1$. Here, K is the dimension of shared channel and q is its index indicating the order of the pairs. It can be verified that the dimension of each pair shared between two nodes could be distinct with each other. The number $K_q K_{q-1} \ldots K_1$ should be greater than the number $D_p D_{p-1} \ldots D_1$ of dimensions in teleported hybrid state to ensure the integrity of the transmitted information. Meanwhile, these two number should be close enough to preserve the effectiveness.

Therefore, after M rounds of teleportation, Bob obtains a general hybrid state of multi-particle, represented in the denotation D', where the concrete form depends on the practical channel. To recover the original entangled form, the auxiliary particles should be introduced by the receiver. The ancilla state is given by $|\chi\rangle_b = |00\ldots0\rangle_{p,p-1,\ldots,1}$, where $|0\rangle_p$ can be regarded as one single qudit with dimension D_p in the ancillary multi-qudit state system $|\chi\rangle_b$. Then, according to the description in the previous section, Bob performs controlled operations on multi-particle states, and performs X-basis measurements on each controlled particle separately. In which B obtains the measurement outcomes $m_B^q m_B^{q-1} \ldots m_B^1$, and the auxiliary system is collapse to

$$|\chi'\rangle_b = \sum_D \alpha_D e^{\sum_q \frac{2\pi i}{D_q} k_q m_q} |D\rangle_b \quad (11)$$

where D_q represents the q-th digit of the information D in the previous form. The representation of the information D serves as a bridge connecting the two entangled states. It can be verified that the recovery operation can be written as

$$R_b = \sum_D e^{-\sum_q \frac{2\pi i}{D_q} k_q m_q} |D\rangle\langle D|_b \quad (12)$$

Bob obtains the final state $|\chi''\rangle_b = \sum_D \alpha_D |D\rangle_b$, which can be verified that it is equal to $\sum_{d_1,d_2,\ldots,d_p=0}^{D_1-1,D_2-1,\ldots,D_p-1} \alpha_{d_1 d_2 \ldots d_p} |d_1 d_2 \ldots d_p\rangle$.

In addition, a summary of the necessary steps is given as follows

- step 1: The initiation of teleportation, including choose an path based on the teleported multi-qudit and the available channel.
- step 2: The nodes on path perform the general Bell measurement to entangle separate channels.
- step 3: The sender and the intermediate nodes send the channel parameter and the measurement outcomes via classic channel.
- step 4: The recovery operation are implemented on the receiver's qudits according to the results in classic channel.

IV. Discussion and Conclusion

A. The strategy in small word network

The small world network structure is a decentralized model where the connections between nodes are generated based on certain rules or probabilities. We illustrate the scenario where the connection probability p=0.25 in Fig.1. In this simulation, we assume that there are simultaneously n_t distinct tasks occurring in this network. The task involves teleportation from one node to another randomly, and the teleported qudits are also randomly selected in terms of length and dimension. The length of the teleported qudits can be selected from {1,2,3} and the dimension of each qudit can be chosen from {2,3,4}. In addition, the distribution rate of channels is assumed to be the same for the simplicity.

In Fig.2, we depict the comparison between the original scenario and our strategy of the total transmission time with the increasing tasks. For our strategy, we choose the most jammed edge in the graph and substitute it for other channels. For instance, we substitute dimension 2 with 3, 3 with 4, and 4 with 2,2 by valuing all potential substitutions. It can be noted that our method can decrease the delay time in Monte Carlo simulation, effectively. Moreover, our method can be improved by introducing more substitute strategy and non-shortest path to decrease the total transmission time in the future study.

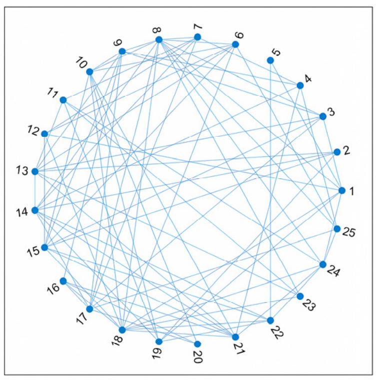

Figure 1. The graph of small world network (p=0.25)

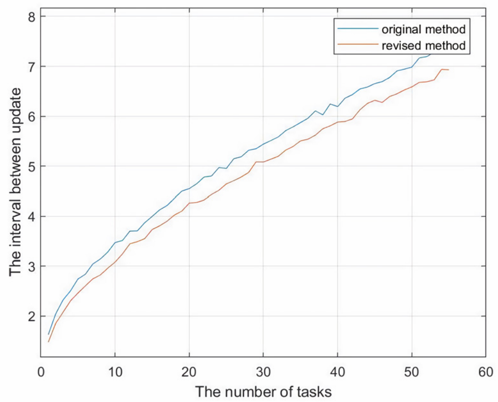

Figure 2. The comparison between two strategies

B. Conclusion

In this paper, we consider to teleport an arbitrary hybrid multi-qudit state in quantum network. The utilization of available EPR pairs is crucial for constructing a uniform quantum network when there are no appropriate quantum channels available between remote nodes. In addition, the used EPR pairs are general and distinct, hence the model we established can describe the network structure of most EPR-based quantum communication systems. In particular, we simulate the transmission scenario in decentralized network model, and it shows that our strategy can relieve the unbalanced channel resource effectively. We hope our work would be helpful for the construction of universal quantum network.

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