《量子信息基础》2024.4.17 随堂作业：

（2024.4.20 晚 22 点前提交）

1. Consider a system of two particles, they have six possible states $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6$. Calculate the number of the microstates of the system, in the following conditions:
   (1) Two particles are bosons;
   (2) Two particles are fermions;
   (3) Two particles are distinguishable.

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(1) C_6^1表示两个粒子占同一个态，C_6^2表示两个粒子占不同态

$$N_b = C_6^1 + C_6^2 = 6 + 15 = 21$$

推导和答案正确给 10 分

(2) 费米子不能同态

$$N_f = C_6^2 = 15$$

推导和答案正确给 10 分

(3) 可区分，可同态

$$N_d = 6 \times 6 = 36$$

推导和答案正确给 10 分

2. Prove that the binary entropy $S_{bin}(p)$ attains its maximum value of one at $p = 1/2$.

$$S_{bin}(p) \equiv -p \log p - (1-p) \log(1-p)$$

$$\frac{d\, S_{bin}(p)}{dp} = -\log p - \frac{1}{\ln 2} + \log(1-p) + \frac{1}{\ln 2} = -\log p + \log(1-p)$$

$$= \log\left(\frac{1-p}{p}\right) = 0$$

$$\therefore\ p = \frac{1}{2}$$

推导和答案正确给 25 分

3. Calculate the Von Neumann entropy $S(\rho)$ for the following density matrix:
   (1) $\rho = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$　　方阵A特征值：|λI-A|=0
   (2) $\rho = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
   (3) $\rho = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

（1） The eigenvalues of matrix ρ are

$\lambda_1 = 1; \lambda_2 = 0$

$$S(\rho) = -\sum_x \lambda_x \log \lambda_x = -1 \log 1 - 0 \log 0 = 0$$

推导和答案正确给 15 分

（2） The eigenvalues of matrix ρ satisfy that

$$\left(\frac{1}{2} - \lambda\right)\left(\frac{1}{2} - \lambda\right) - \frac{1}{4} = 0$$

$\lambda_1 = 1; \lambda_2 = 0$

$$S(\rho) = -\sum_x \lambda_x \log \lambda_x = -1 \log 1 - 0 \log 0 = 0$$

推导和答案正确给 15 分

（3） The eigenvalues of matrix ρ satisfy that

$$\left(\frac{2}{3} - \lambda\right)\left(\frac{1}{3} - \lambda\right) - \frac{1}{9} = 0$$

$\lambda_1 = \frac{1}{2} + \frac{\sqrt{5}}{6}; \lambda_2 = \frac{1}{2} - \frac{\sqrt{5}}{6}$

$$S(\rho) = -\sum_x \lambda_x \log \lambda_x = -\left(\frac{1}{2} + \frac{\sqrt{5}}{6}\right)\log\left(\frac{1}{2} + \frac{\sqrt{5}}{6}\right) - \left(\frac{1}{2} - \frac{\sqrt{5}}{6}\right)\log\left(\frac{1}{2} - \frac{\sqrt{5}}{6}\right)$$

$$\cong 0.55$$

推导正确给 15 分（最后答案没有给出具体数字也算正确）

* David J. Griffiths, Introduction to Quantum Mechanics (2nd Edition), Cambridge University Press (2017).