《量子信息基础》2024.3.7 随堂作业：

（2024.3.12 晚 22 点前提交）

1. <即教材*问题 1.5> 70 分

Consider the wave function

$$\Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$$

where A, λ, and ω are positive real constants.

(a) Normalize Ψ.

$$\int |\Psi(x,t)|^2 dx = \int \left|Ae^{-\lambda|x|}e^{-i\omega t}\right|^2 dx = |A|^2 \int_{-\infty}^{\infty} e^{-2\lambda|x|}dx$$

$$= 2|A|^2 \int_0^{\infty} e^{-2\lambda x}dx = \frac{|A|^2}{\lambda} = 1$$

$$\therefore A = \sqrt{\lambda}$$

给出正确推导和答案给 15 分，只给出答案给 5 分

(b) Determine the expectation values of x and x^2.

$$\langle x\rangle = \int_{-\infty}^{+\infty} x|\Psi(x,t)|^2 dx = \int_{-\infty}^{\infty} x\left|Ae^{-\lambda|x|}\right|^2 dx = |A|^2 \int_{-\infty}^{\infty} xe^{-2\lambda|x|}dx$$

$$= \lambda \int_{-\infty}^{\infty} xe^{-2\lambda|x|}dx$$

$\because xe^{-2\lambda|x|}$是奇函数　　$\therefore \langle x\rangle = 0$

给出正确推导和答案给 10 分，只给出推导或者答案给 5 分

$$\langle x^2\rangle = \int_{-\infty}^{+\infty} x^2|\Psi(x,t)|^2 dx = |A|^2 \int_{-\infty}^{\infty} x^2 e^{-2\lambda|x|}dx = 2\lambda \int_0^{\infty} x^2 e^{-2\lambda x}dx$$

$$= -\int_0^{\infty} x^2 de^{-2\lambda x} = -x^2 e^{-2\lambda x}\Big|_0^{\infty} + \int_0^{\infty} e^{-2\lambda x}dx^2$$

$$= \int_0^{\infty} 2xe^{-2\lambda x}dx = -\frac{1}{\lambda}\int_0^{\infty} xde^{-2\lambda x}$$

$$= -\frac{1}{\lambda}xe^{-2\lambda x}\Big|_0^{\infty} + \frac{1}{\lambda}\int_0^{\infty} e^{-2\lambda x}dx = -\frac{1}{2\lambda^2}\int_0^{\infty} de^{-2\lambda x} = \frac{1}{2\lambda^2}$$

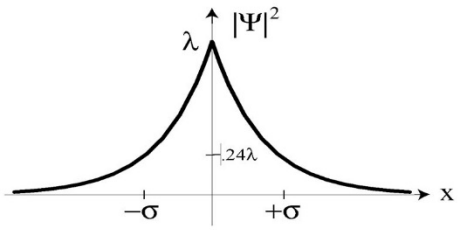
给出正确推导和答案给 10 分，只给出推导或者答案给 5 分

(c) Find the standard deviation of x. Sketch the graph of $|\Psi|^2$, as a function of x, and mark the points $(\langle x\rangle - \sigma)$ and $(\langle x\rangle + \sigma)$, to illustrate the sense in which σ represents the "spread" in x.

$$\sigma = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \frac{1}{\sqrt{2}\lambda}$$

给出正确答案给 10 分

$|\Psi|^2$的图像如下图所示：



给出正确曲线和$(\langle x\rangle \pm \sigma)$位置给 10 分，只给出一项给 5 分

(d) What is the probability that the particle would be found outside this range?

$$P = 2\int_{\sigma}^{\infty} |\Psi(x,t)|^2 dx = 2\lambda \int_{\sigma}^{\infty} e^{-2\lambda x} dx = -\int_{\sigma}^{\infty} de^{-2\lambda x} = e^{-2\lambda\sigma} = e^{-\sqrt{2}}$$
$$= 0.2431$$

给出正确推导和答案（$e^{-\sqrt{2}}$或者0.2431）给 15 分，只给出答案给 5 分

2. <PPT 最后一页> 30 分

A photon propagates in the z direction and passes a linear optical polarizer which is oriented in the x direction (see figure below). The state in Figure (a) is ψ_a while the state in Figure (b) is ψ_b.

(a) Write done the formula of ψ_c, assuming the light beam is polarized with an angle of α to the x axis.

$$\psi_{\alpha} = \cos\alpha \cdot \psi_x + \sin\alpha \cdot \psi_y$$

给出正确答案给 10 分

(b) How much is the probability that a single photon could pass the polarizer in (c)?

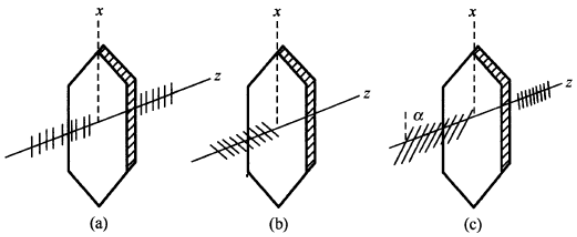
$$P = |\cos\alpha|^2$$

给出正确答案给 10 分

(c) How does the system maintain the normalization condition?

$$|\cos\alpha|^2 + |\sin\alpha|^2 = 1$$

给出正确答案给 10 分



* David J. Griffiths, and Darrell F. Schroeter, Introduction to Quantum Mechanics (3rd Edition), Cambridge University Press (2018).