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# 集成电路原理与设计

## 12. 组合逻辑门

Combinational Logic Gates

宋爽

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# Syllabus



课数	内容	课数	内容
1	导论	9	差分放大器
2	器件模型	10	运算放大器
3	电容特性, 小信号模型	11	导线
4	工艺流程	12	反相器
5	模拟基本单元	13	反相器
6	电流镜与基准	14	组合逻辑
7	单级放大器	15	集成电路专题讲座一
8	课堂测验	16	集成电路专题讲座二

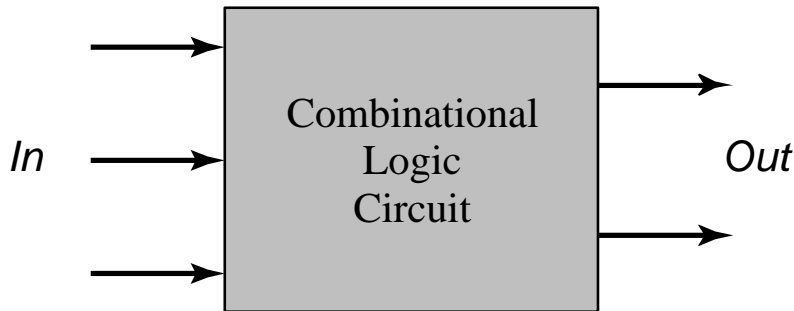
# Goals

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- In-depth discussion of logic families in CMOS
  - Static and Dynamic
  - Non-ratioed and ratioed logic
  - Pass-transistor
- Optimizing a logic gate for area, speed, energy, or robustness

# Combinational vs. Sequential Logic

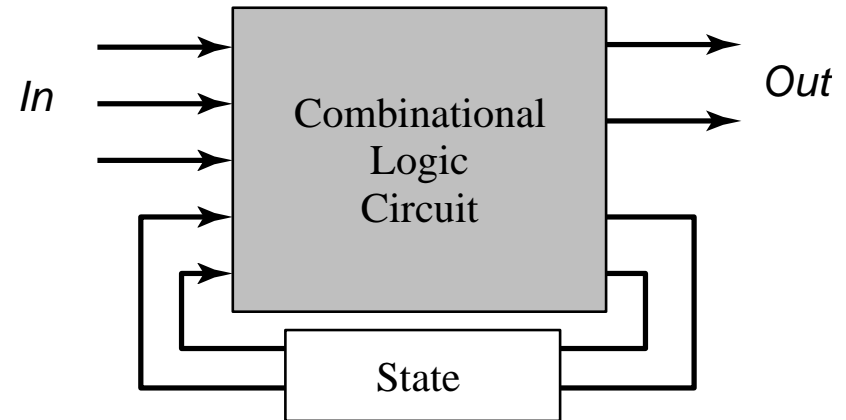
## Combinational



$$\text{Output} = f(\text{In})$$

组合逻辑：输出只与  
当前状态有关

## Sequential



$$\text{Output} = f(\text{In}, \text{Previous In})$$

时序逻辑：输出与当前状态  
及上一时钟周期状态有关

# Combinational logic expressions

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- ❑ Combinational logic: function value is a combination of **function arguments**. (布尔函数)
- ❑ A **logic gate** implements a particular logic function.
- ❑ Both specification (**logic equations**) and implementation (logic **gate networks**) are written in Boolean logic.

## Why designing gates for logic functions is non-trivial:

- ❑ may not have logic gates in the library for all logic expressions;
- ❑ a logic expression may map into gates that consume a lot of **area, delay, or power**.

# CMOS Circuit Styles

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## □ Static CMOS

*output connected to either VDD or GND via a low-resistance path*

- **high noise margins**
- **low output impedance**, high input impedance
- no steady state path between VDD and GND
- delay a function of load capacitance and transistor resistance

## □ Dynamic CMOS

*relies on temporary storage of signal values on the capacitance of **high-impedance** circuit nodes*

- simpler, faster gates
- increased **sensitivity to noise**



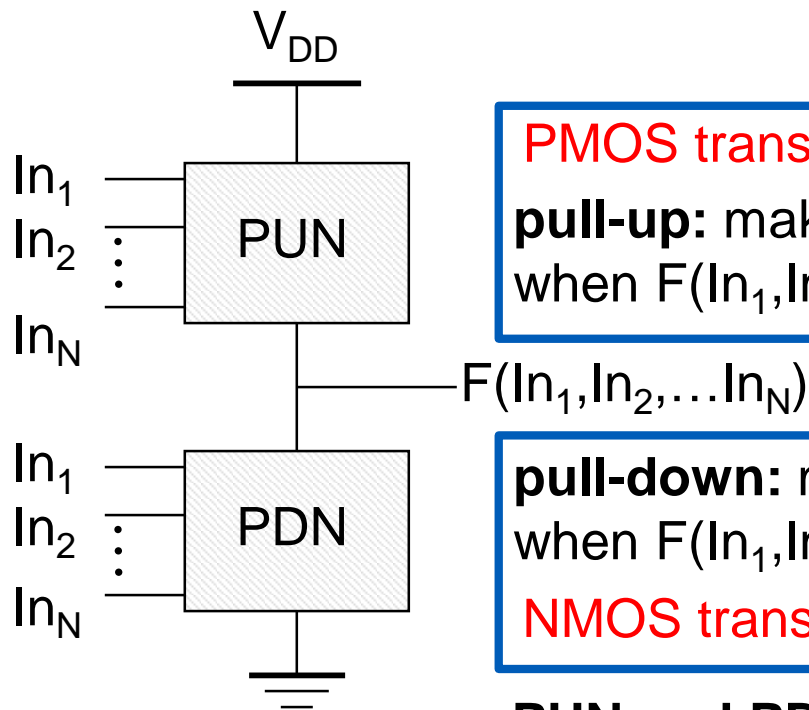
# Outline

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- **Static CMOS Design**
  - **Complementary CMOS**
  - **Ratioed Logic**
  - **Pass-Transistor Logic**
- **Dynamic CMOS Design**

# Static Complementary CMOS

- Complementary: complementary Pull-up Network (PUN, P-type) vs. Pull-down Network (PDN, N-type)
- Static: do not rely on stored charge
- Simple, effective, reliable; hence ubiquitous



**PMOS transistors only**

**pull-up:** make a connection from  $V_{DD}$  to  $F$  when  $F(In_1, In_2, \dots, In_N) = 1$

**pull-down:** make a connection from  $F$  to GND when  $F(In_1, In_2, \dots, In_N) = 0$

**NMOS transistors only**

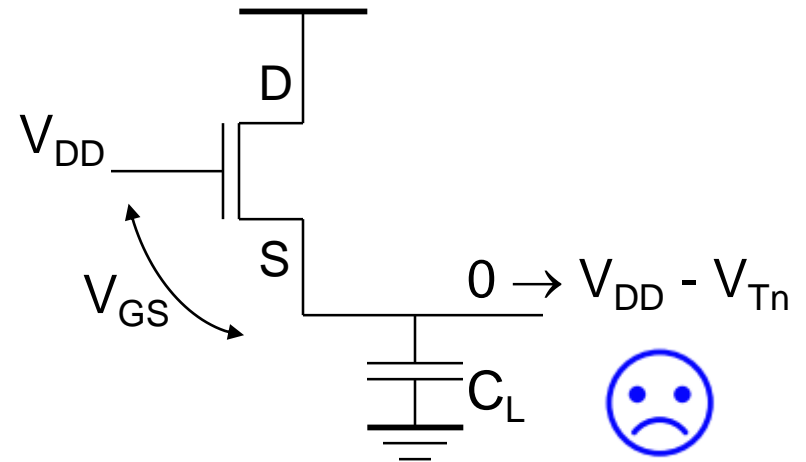
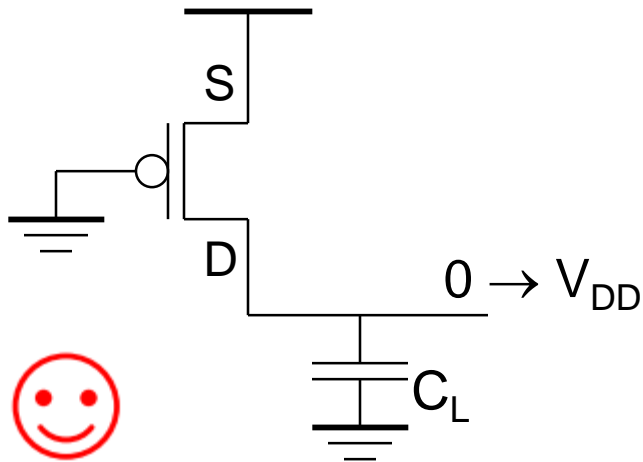
**PUN and PDN are dual logic networks**



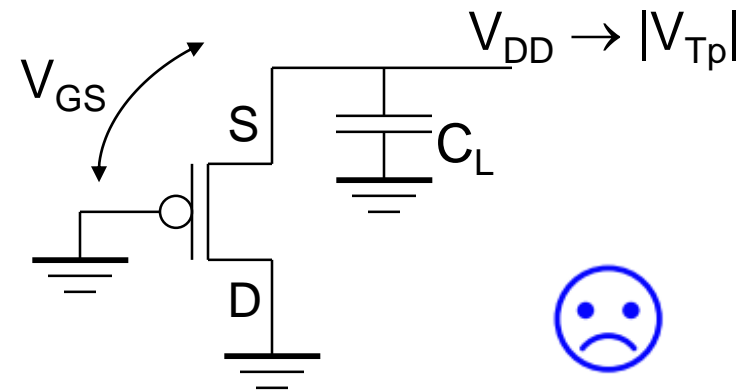
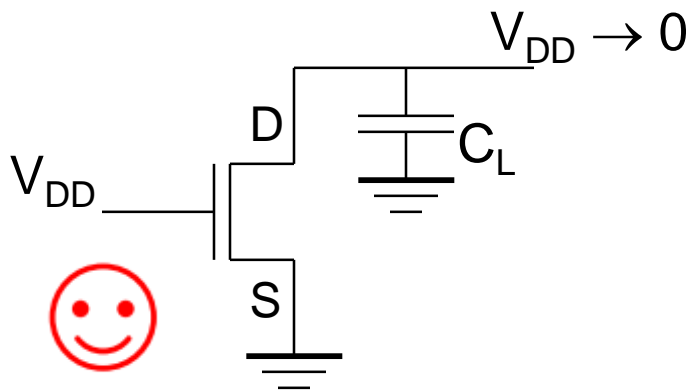
# Threshold Drops

## A. Why PMOS in PUN and NMOS in PDN?

PUN



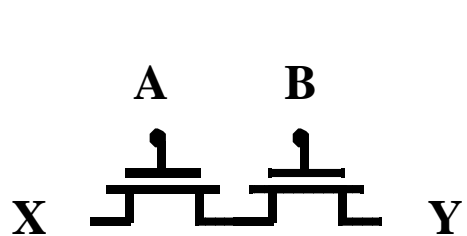
PDN



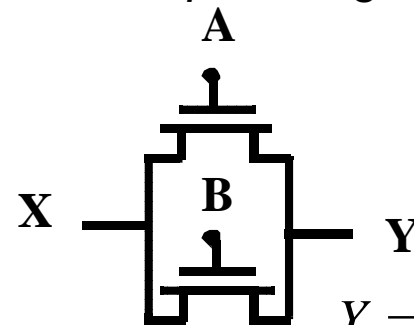
# NMOS/PMOS in Series/Parallel Connection

**B.** Transistors can be thought as a **switch** controlled by its gate signal

□ NMOS switch closes when switch control input is high

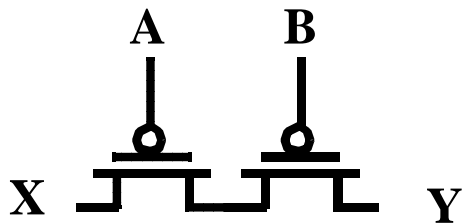


$$Y = X \text{ if } A \text{ and } B$$

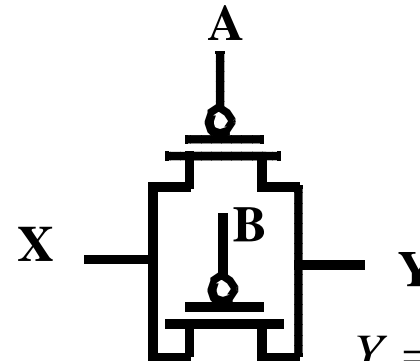


$$Y = X \text{ if } A \text{ or } B$$

□ PMOS switch closes when switch control input is low



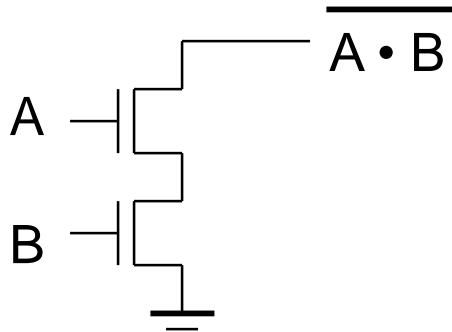
$$Y = X \text{ if } \bar{A} \text{ and } \bar{B} = \overline{A + B}$$



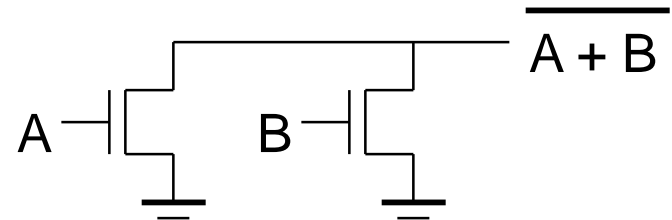
$$Y = X \text{ if } \bar{A} \text{ or } \bar{B} = \overline{AB}$$

# Construction of PDN

- NMOS devices in **series** implement a NAND function



- NMOS devices in **parallel** implement a NOR function



# Dual PUN and PDN

## C. PUN and PDN are dual networks 对偶网络

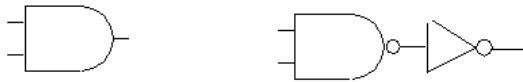
- De Morgan's theorems

$$\overline{A + B} = \overline{A} \cdot \overline{B} \quad [!(A + B) = !A \cdot !B \text{ or } !(A | B) = !A \& !B]$$

$$\overline{A \cdot B} = \overline{A} + \overline{B} \quad [!(A \cdot B) = !A + !B \text{ or } !(A \& B) = !A | !B]$$

- a **parallel** connection of transistors in the PUN corresponds to a **series** connection of the PDN

## D. Complementary gate is naturally **inverting** (NAND, NOR) *{NAND, NOR} is a complete sets; {AND, OR} is not complete.*

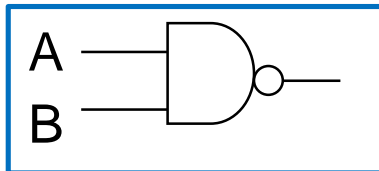
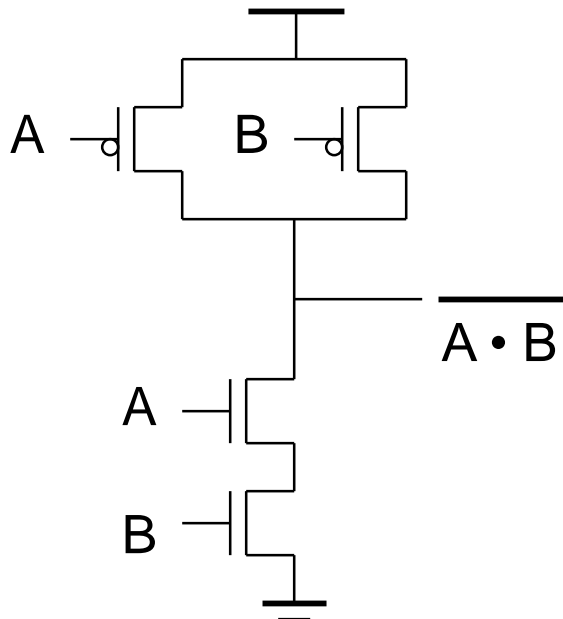


$$\text{AND} = \text{NAND} + \text{INV}$$

## E. Number of transistors for N-input logic gate: **2N**

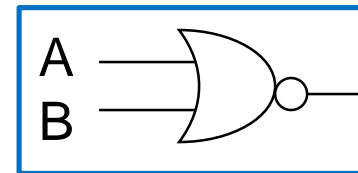
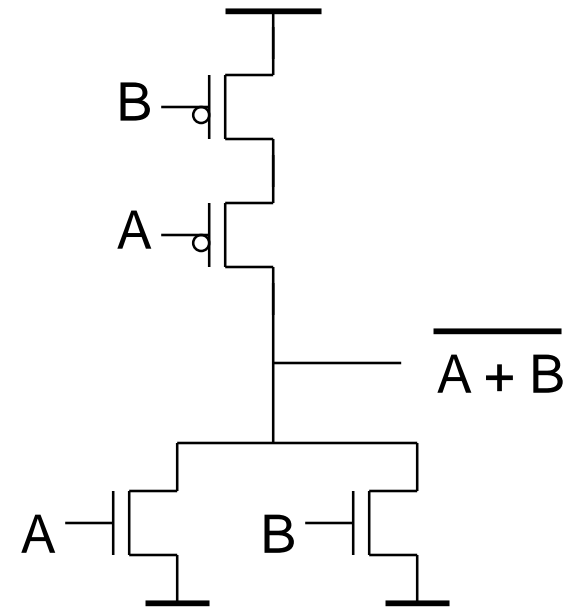
# Dual PUN and PDN: Example

## CMOS NAND



A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

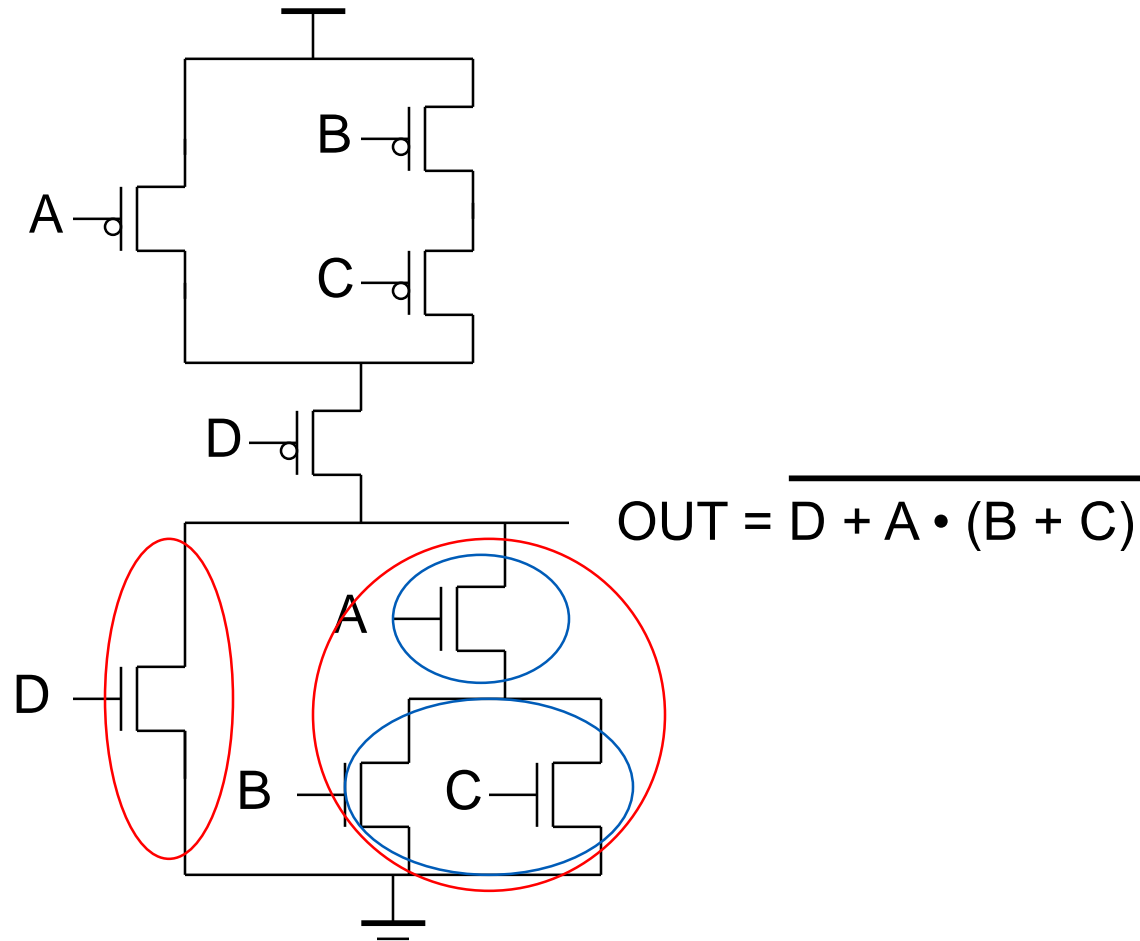
## CMOS NOR



A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

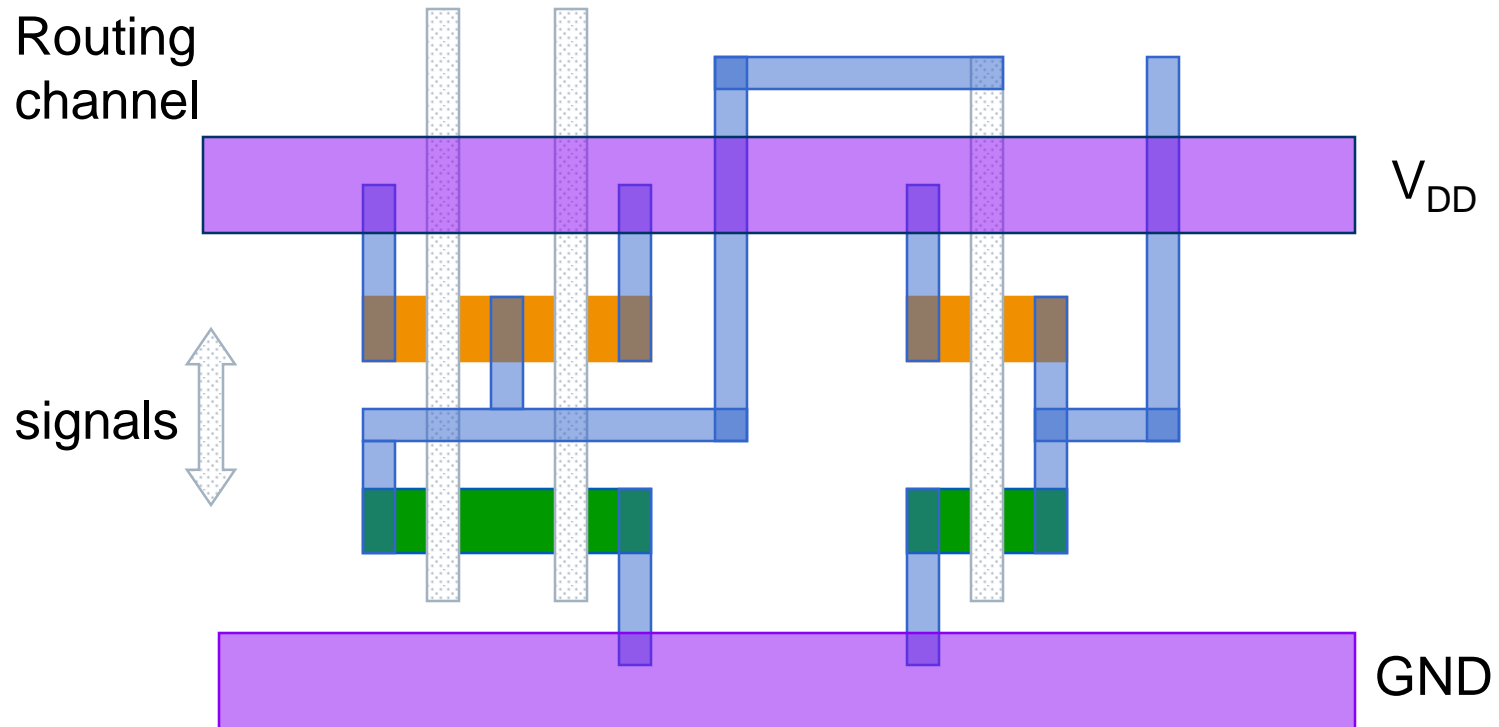
# Dual PUN and PDN: Example

## □ Complex CMOS Gate



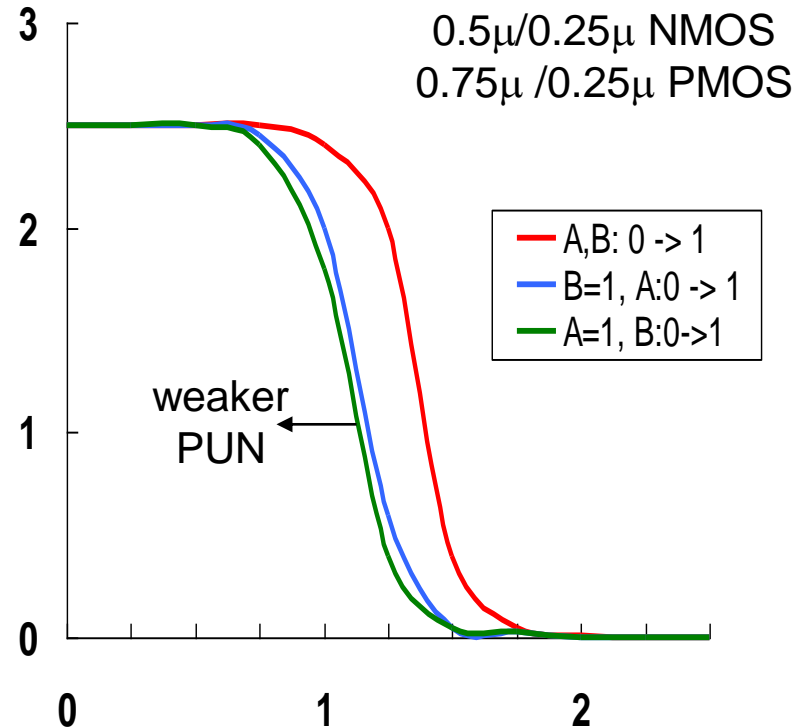
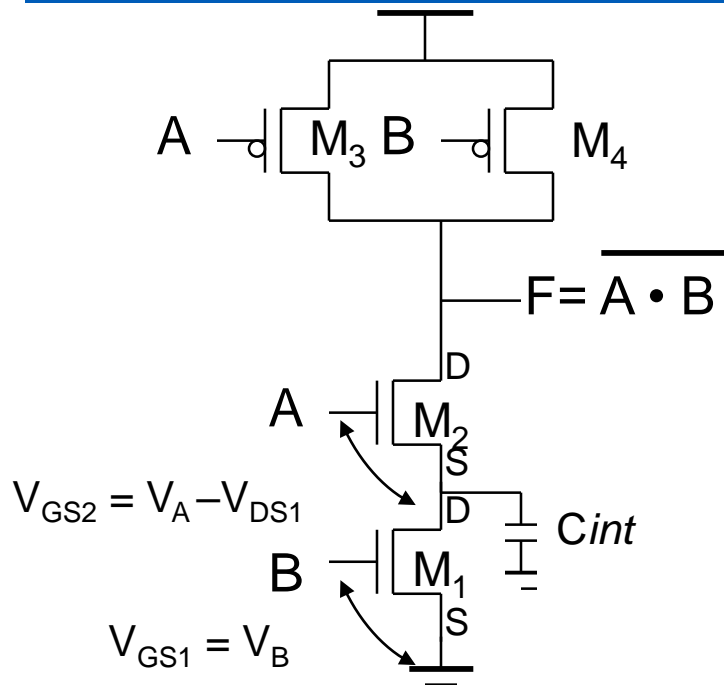
# Dual PUN and PDN: Example

## □ CMOS Gate Layout



What logic function is this?

# VTC: Data-Dependent



- $V_{TH}$  of  $M_2$  is higher than  $M_1$ : the body effect ( $\gamma$ )

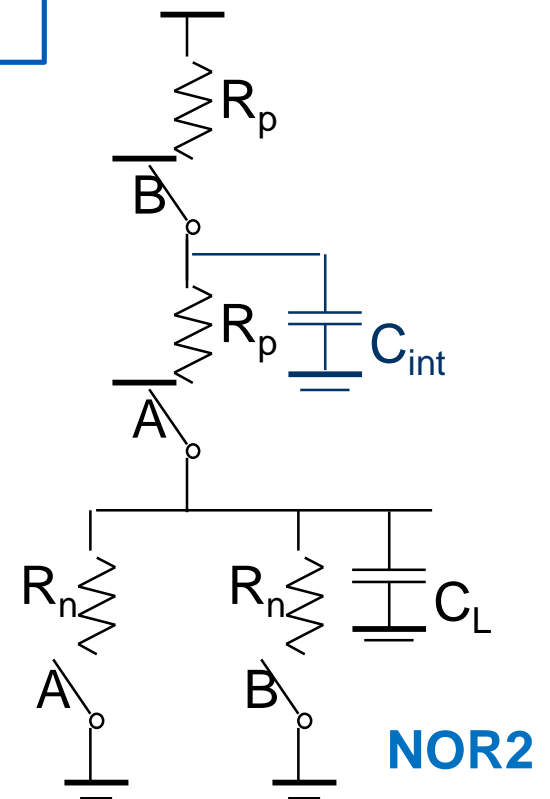
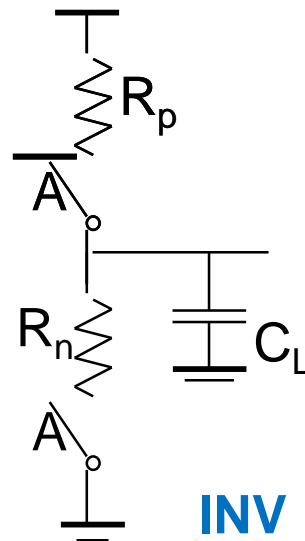
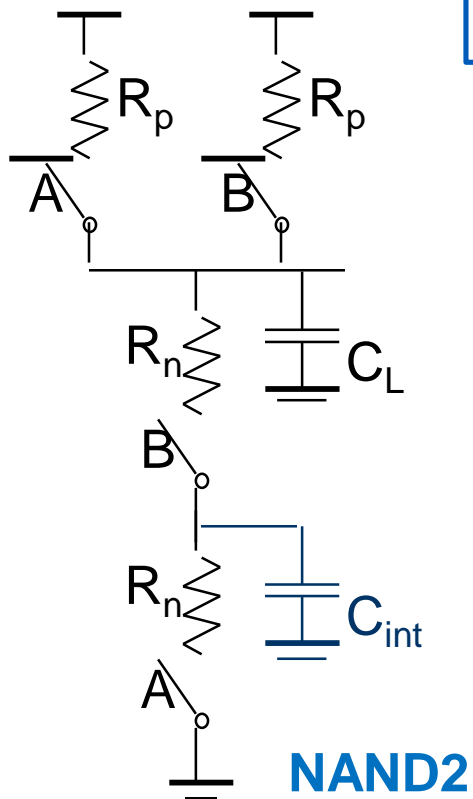
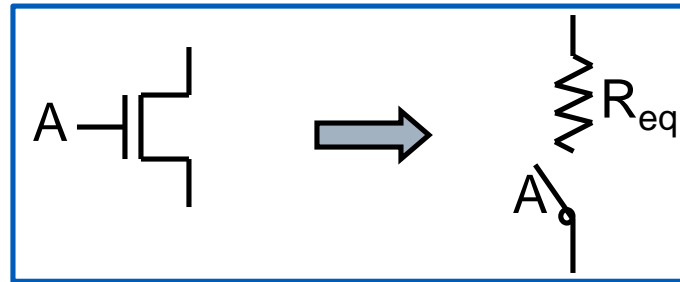
$$V_{Tn1} = V_{Tn0} \quad V_{Tn2} = V_{Tn0} + \gamma(\sqrt{|2\phi_F| + V_{int}}) - \sqrt{|2\phi_F|}$$



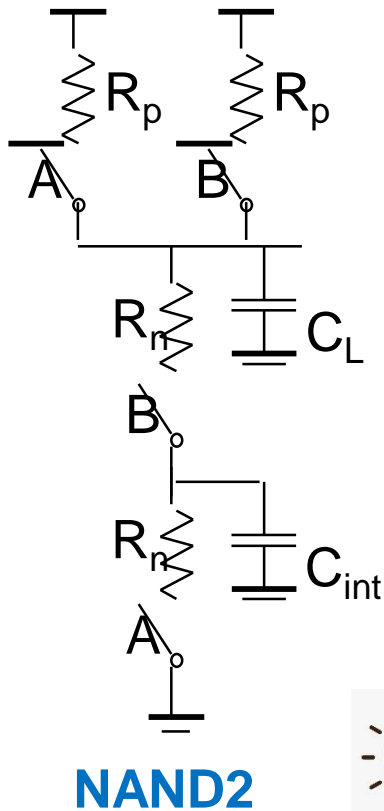
***The noise margins are input-pattern dependent.***



# Switch Delay Model



# Propagation Delay: Input Patterns



□ Delay is dependent on **the input patterns**

□ Input: high to low transition

– both inputs go low:  $0.69 \frac{R_p}{2} C_L$

– one input goes low:  $0.69 R_p C_L$

□ Input: low to high transition

– both inputs go high:  $0.69 2R_n C_L$

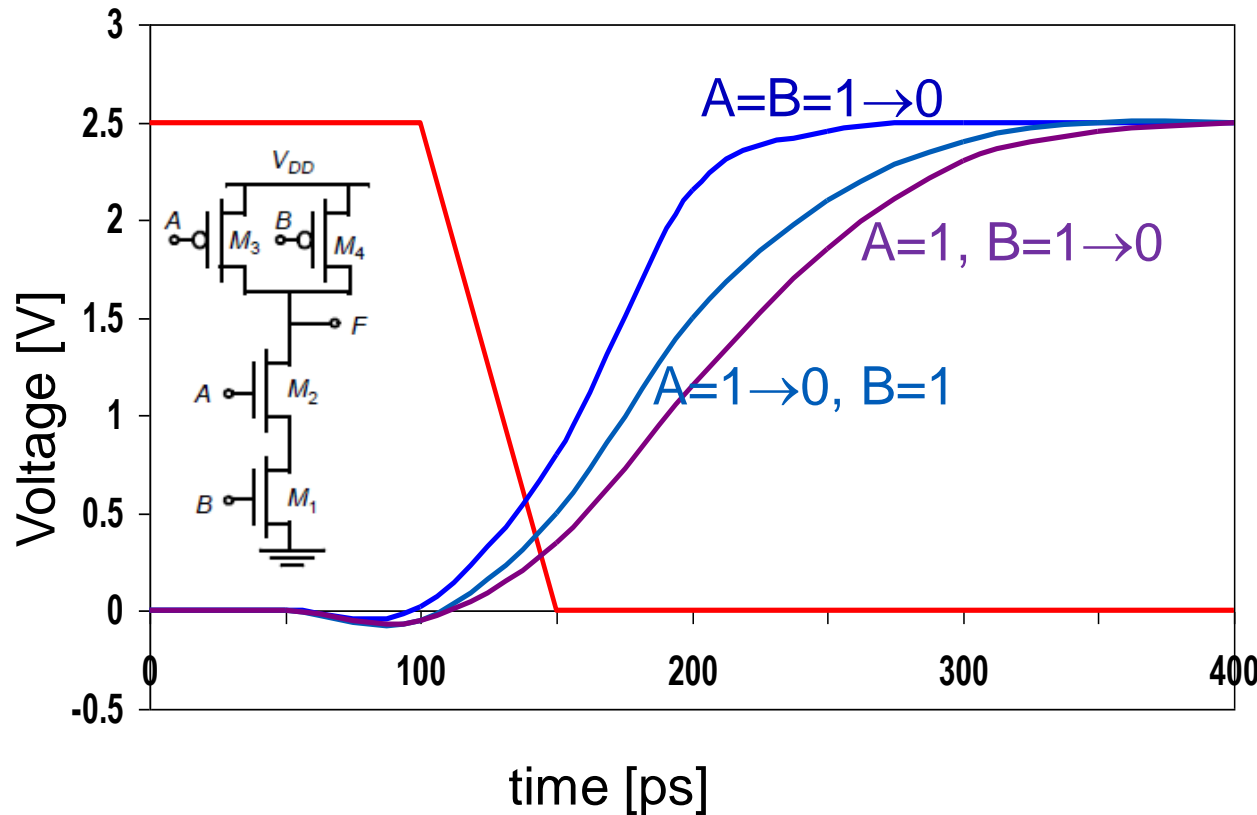
*Adding devices in series: slow down*

*Wider device: avoid a performance penalty*



**A gate with multi-in's: the combination of input that triggers **the worst-case conditions** (resistance in series)**

# Delay Dependence on Input Patterns

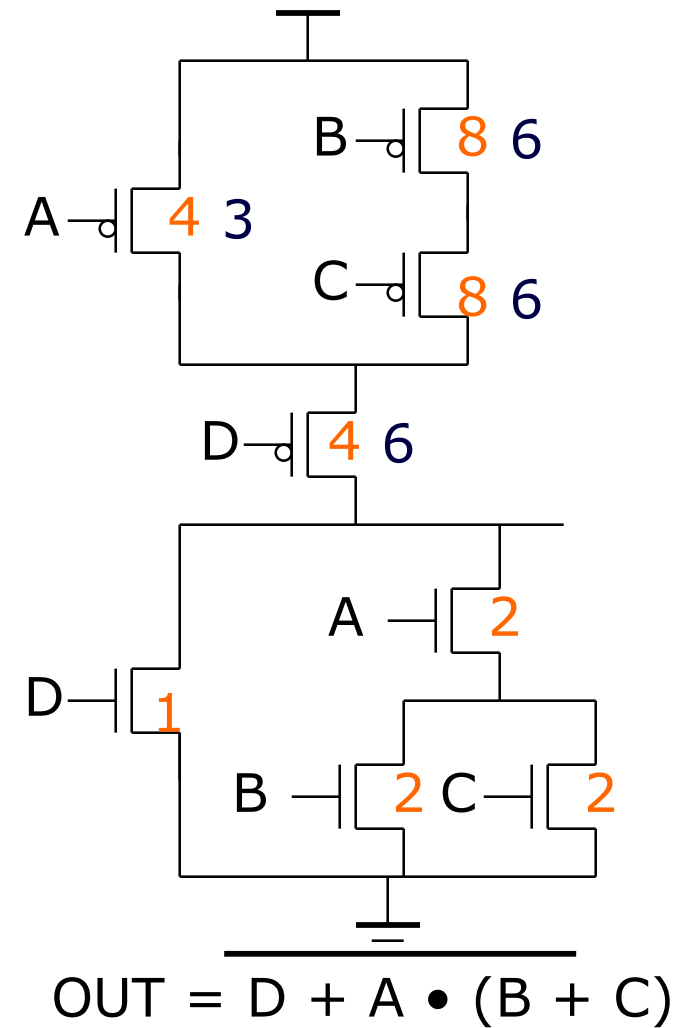
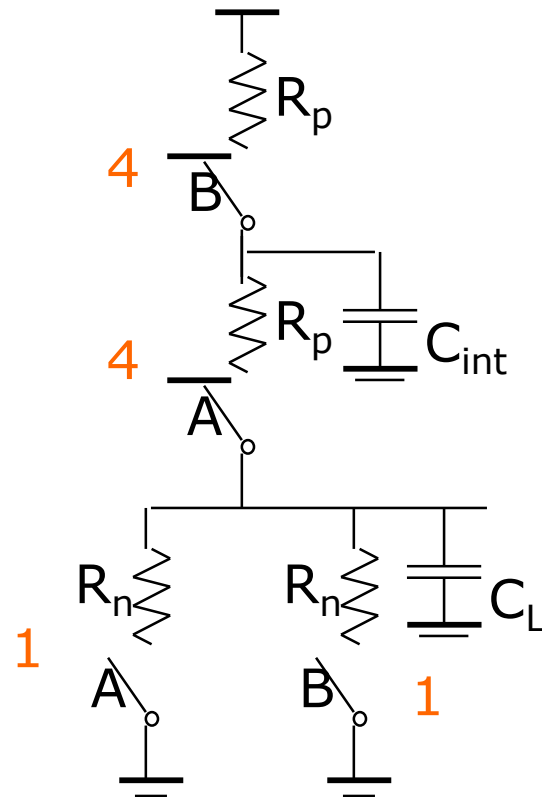
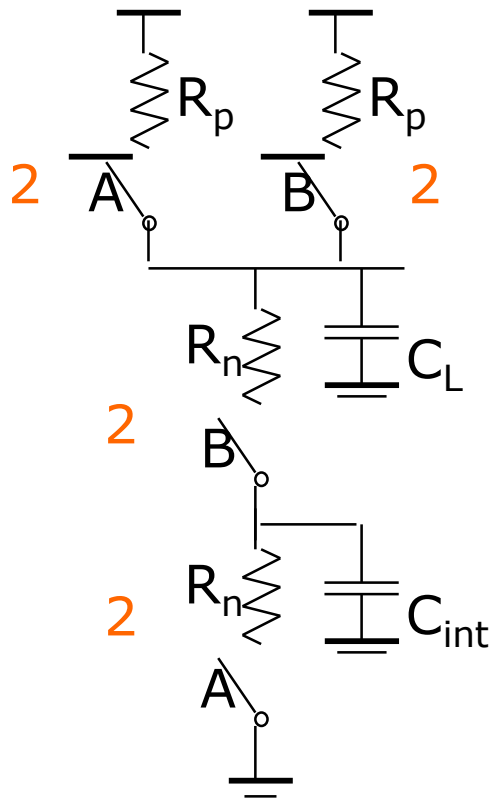


Inverter: NMOS =  $0.5\mu\text{m}/0.25\mu\text{m}$   
 PMOS =  $1.5\mu\text{m}/0.25\mu\text{m}$

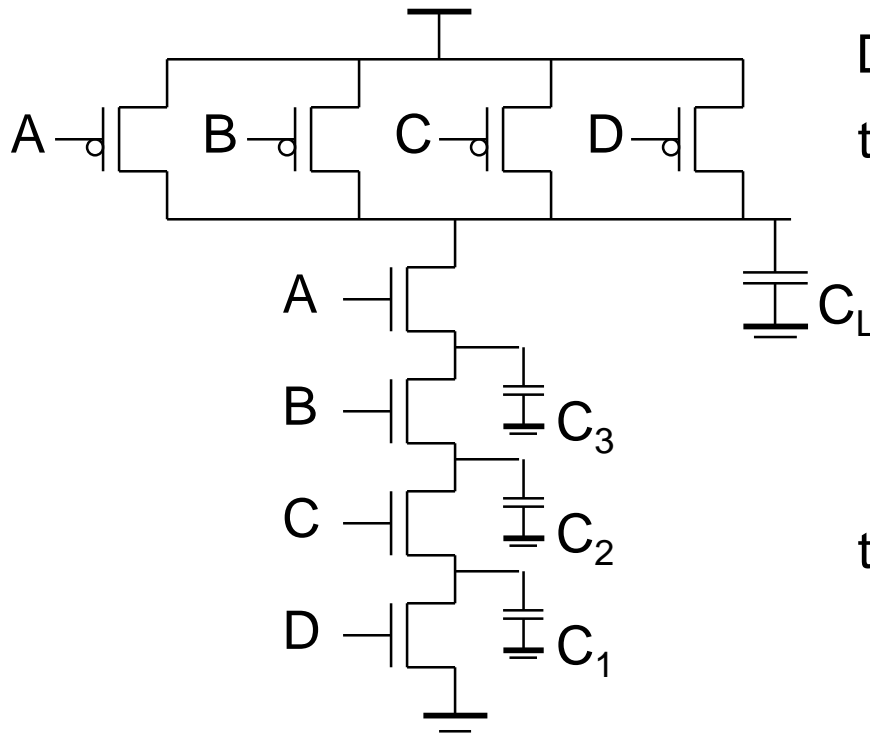


NMOS =  $1.0\mu\text{m}/0.25\mu\text{m}$   
 PMOS =  $1.5\mu\text{m}/0.25\mu\text{m}$

# Transistor sizing



# Fan-In Considerations



Distributed RC model (Elmore delay)

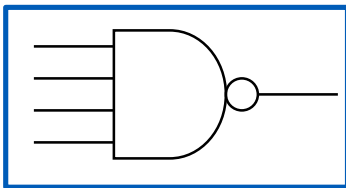
$$t_{pHL} = 0.69 [R_1 C_1 + (R_1 + R_2) C_2 + (R_1 + R_2 + R_3) C_3 + (R_1 + R_2 + R_3 + R_4) C_L]$$



NMOS: same size  $R_{eqn}$

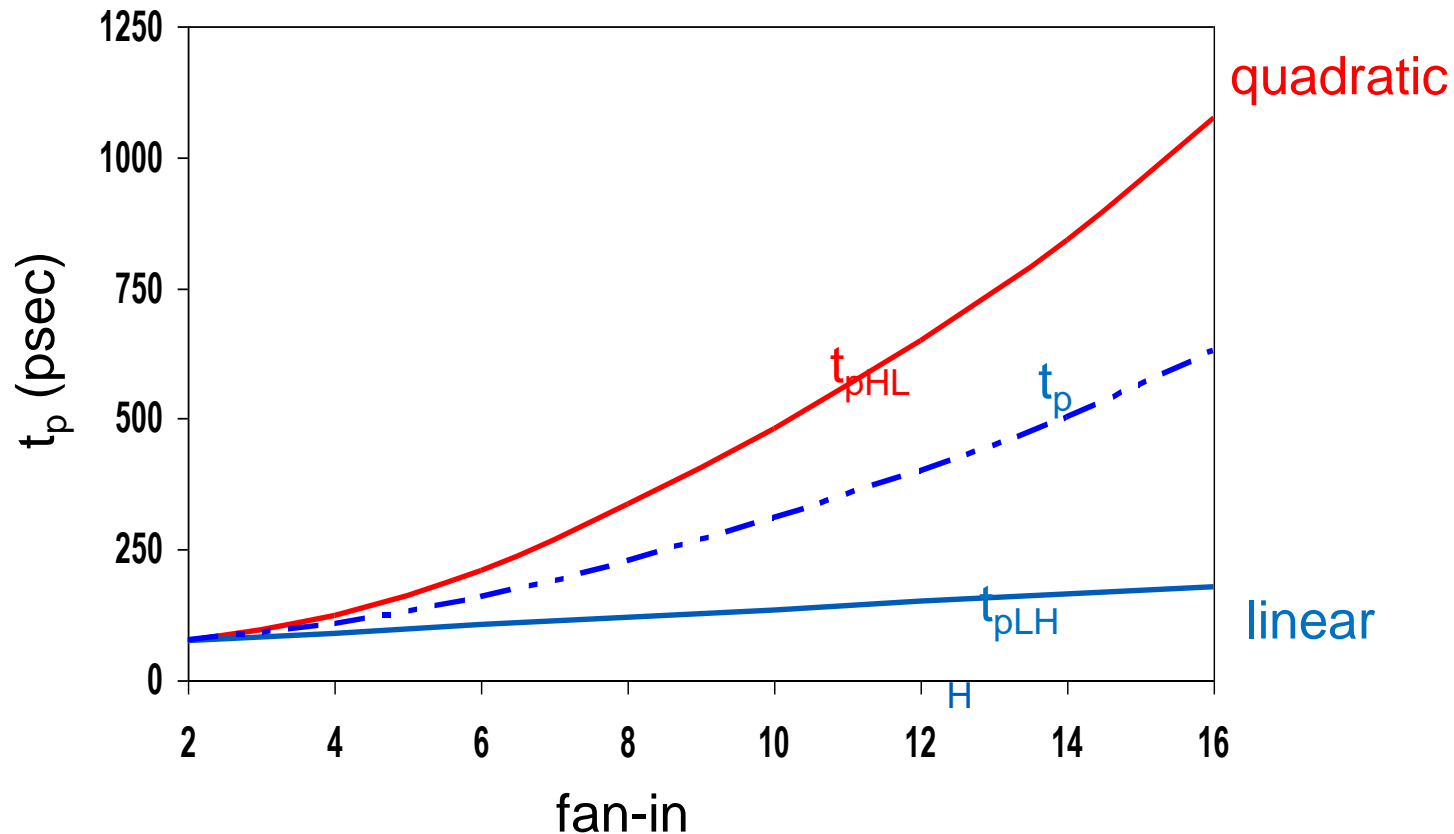
$$t_{pHL} = 0.69 R_{eqn} (C_1 + 2C_2 + 3C_3 + 4C_L)$$

***the worst-case***



- ❑ The number of transistor required to implement an N fan-in is **2N** => area, capacitance
- ❑ Propagation delay deteriorates rapidly as a function of fan-in – **quadratically** in the worst case. ->  $0.5 \cdot (1+N) \cdot N$

# $t_p$ as a Function of Fan-In



**NOTE!:** Gates with a fan-in greater than 4 should be avoided.



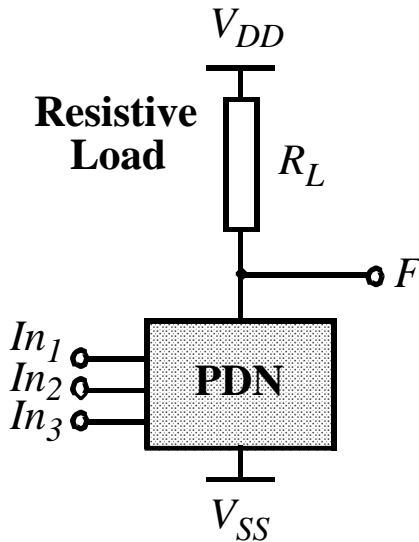
# Outline

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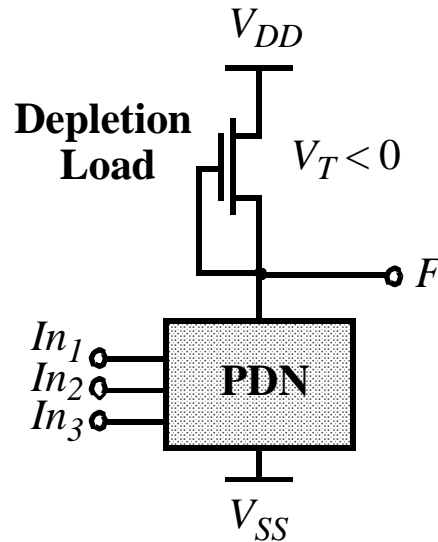
- **Static CMOS Design**
  - Complementary CMOS
  - **Ratioed Logic**
  - Pass-Transistor Logic
- **Dynamic CMOS Design**

# Ratioed Logic

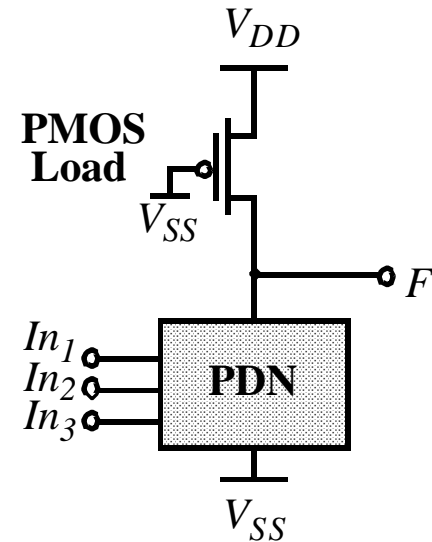
**Goal: to reduce the number of devices over complementary CMOS**



(a) resistive load



(b) depletion load NMOS



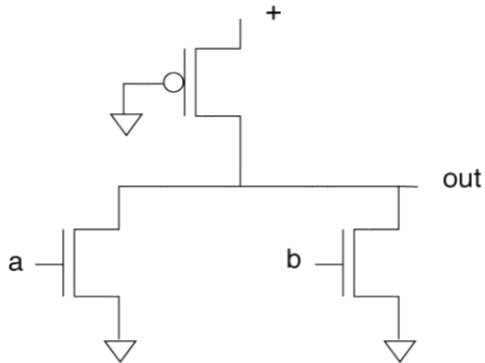
(c) pseudo-NMOS

- ❑  $2N \rightarrow N+1$
- ❑  $V_{OH}=VDD, V_{OL} \neq 0$  -» noise margin  $\downarrow$ , static power  $\uparrow$
- ❑ Tradeoff: the size of the load relative to the PDN vs. NM, delay, power
- ❑ Ratioed Logic
- ❑ Pseudo-nMOS



# Pseudo-NMOS: 2-input NOR

- Uses a p-type as a resistive pullup, n-type network for PDN.



- Consumes static **power**.
- Much **smaller** PUN network than static gate.
- Pull-up time is longer.
- Wiring is simple.

- $V_{OH}=VDD$ ,  
 $V_{OL} \neq 0$  (**Vss**):  $V_{OL} = 0.2 (V_{DD} - V_{SS})$  is one plausible choice.
- For logic 0 output, PUN and PDN form a **voltage divider**.  
Must choose N/P transistor sizes to create  $R_{eq}$  of the required ratio.
- Effective resistance of PDN must be computed in worst case:  
**series n-types** means larger transistors.

# Transistor ratio calculation

- In steady state logic 0 output:

PUN: *saturation* region  $V_{ds} > V_{gs} - V_t$ ; PDN: *linear region*  $V_{ds} < V_{gs} - V_t$ .

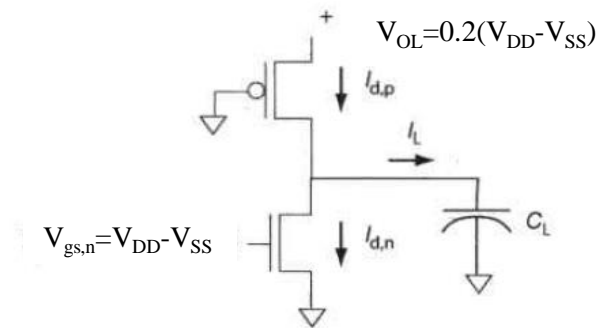
PUN and PDN:  $I_{dp} = I_{dn}$ .

- Equate two currents while generate  $V_{OL}$ :

$$I_{dp} = \frac{1}{2} k_p' (W_p / L_p) (V_{DD} - V_t)^2;$$

$$I_{dn} = \frac{1}{2} k_n' (W_n / L_n) [2(V_{DD} - V_t)V_{OL} - V_{OL}^2];$$

$$\Rightarrow V_{OL} = (V_{DD} - V_t) \left( 1 - \sqrt{1 - \frac{k_p' (W_p / L_p)}{k_n' (W_n / L_n)}} \right)$$



$V_{OL}$  is  $0.2V_{DD}$ ,  $V_t$ ,  $k_p'$  and  $k_n'$  is dependent on technology.

Using 0.18 um parameters,  $V_{DD}=1.8V$ ,  $V_t \approx 0.4 \sim 0.5V$ ,  $k_p'/k_n' \approx 0.18$

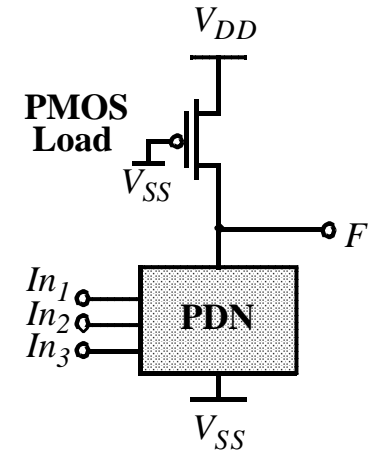
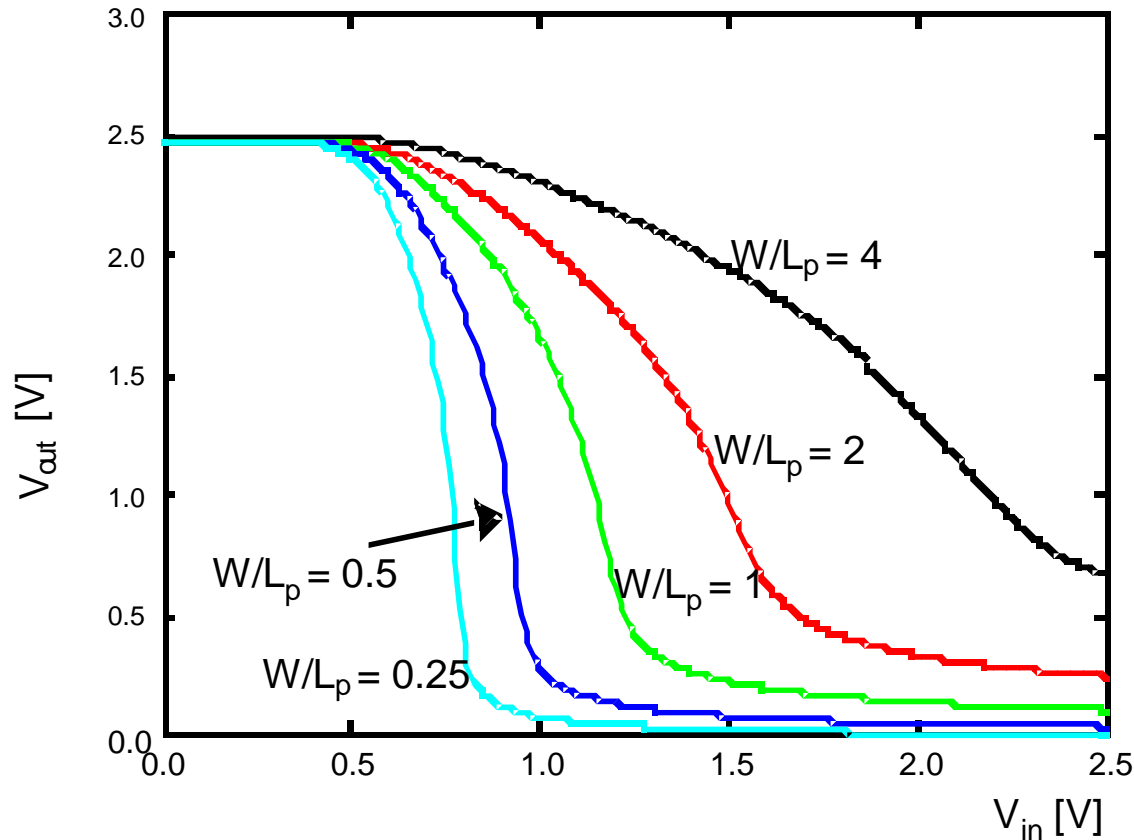
$$\Rightarrow W_p/L_p = 2.5 W_n/L_n \quad (\text{or } L_p/W_p = 0.4 L_n/W_n)$$

since equivalent resistance of MOS transistor  $R_t = (L/W)R_{N,P}$ ,  $R_N = 0.2R_P$ ,

$$\Rightarrow \text{equivalent } R_{tp} = R_P (L_p/W_p) = 5 R_N 0.4 L_n/W_n = 2 R_{tn}, \tau = R_t C_L,$$

$\Rightarrow$  Pullup time is 2 times longer than pulldown for pseudo-NMOS.

# Pseudo-NMOS VTC



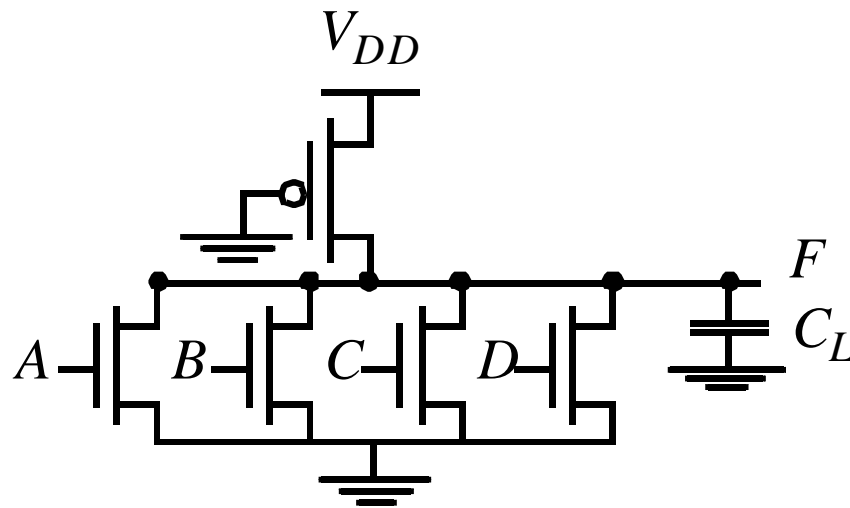
$$V_{OL} \approx \frac{\mu_p \cdot W_p}{\mu_n \cdot W_n} V_{DSATp}$$

- $V_{OL} \downarrow \rightarrow (W/L)_p < (W/L)_n$
- Static power

Size	$V_{OL}$ (V)	Power ( $\mu$ W)	$t_{plh}$ (ps)
4	0.693	564	14
2	0.273	295	56
1	0.133	160	123
0.5	0.064	80	268
0.25	0.031	41	569

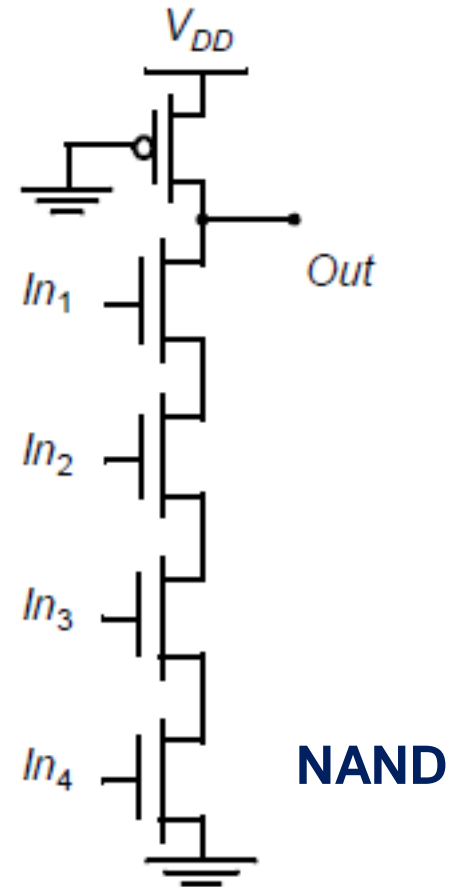
# Pseudo-NMOS: 4-input NOR and NAND

- Small area:  $2N \rightarrow N+1$



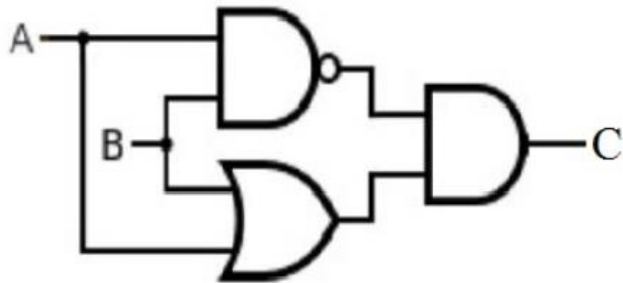
$V_{OH} = V_{DD}$  (similar to complementary CMOS)

**NOR**

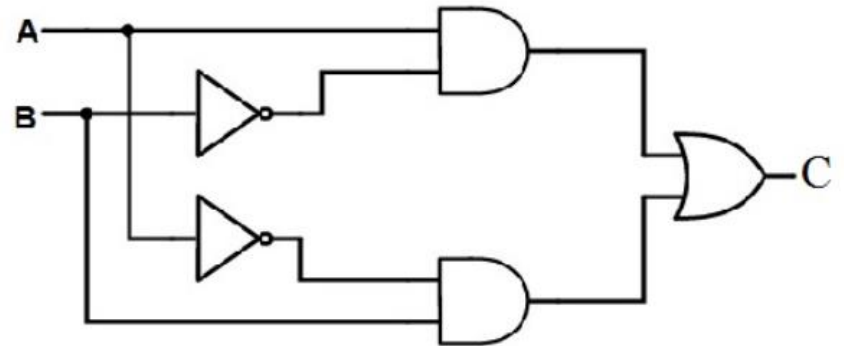


# XOR gates

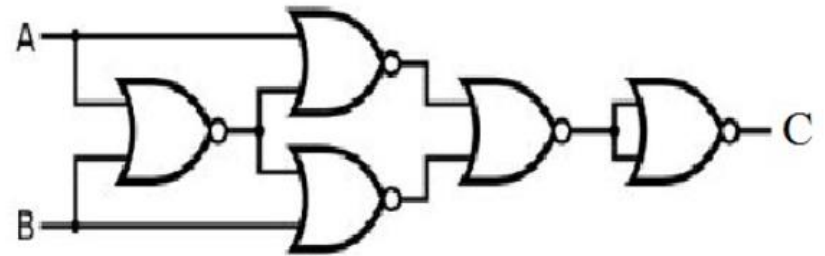
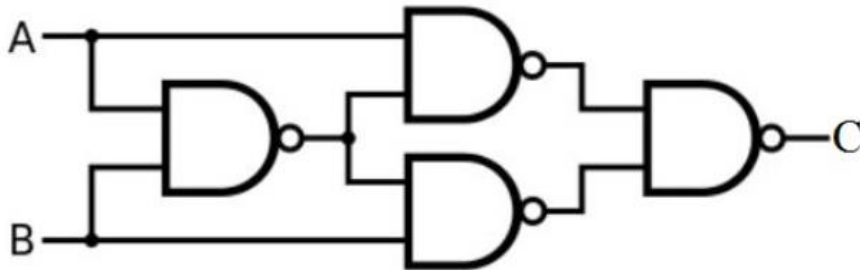
- Can be implemented with different gates



$$(A + B) \cdot (\bar{A} + \bar{B})$$



$$A \cdot \bar{B} + \bar{A} \cdot B$$



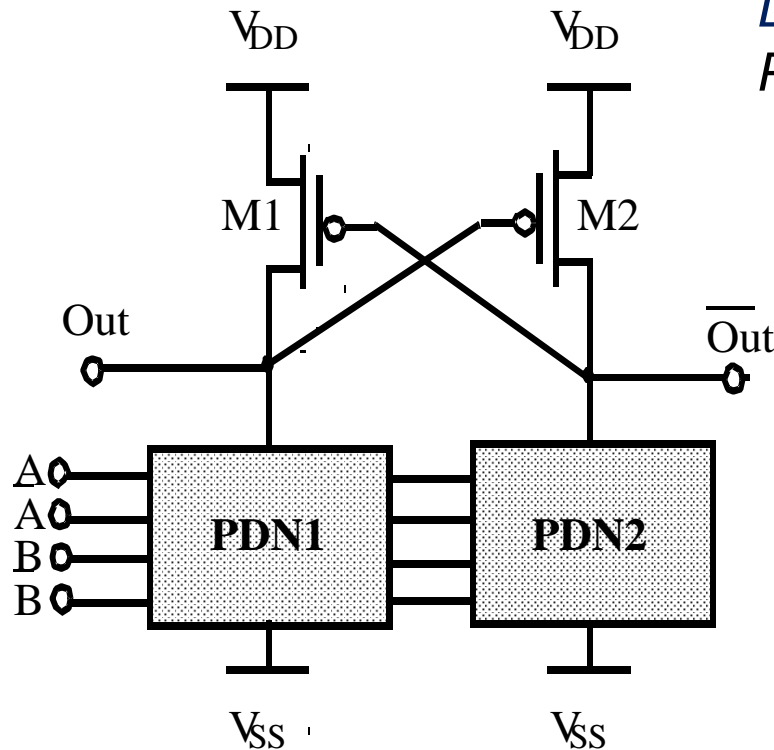
# Improved Loads - DCVSL

- Differential Cascode Voltage Switch Logic (DCVSL)

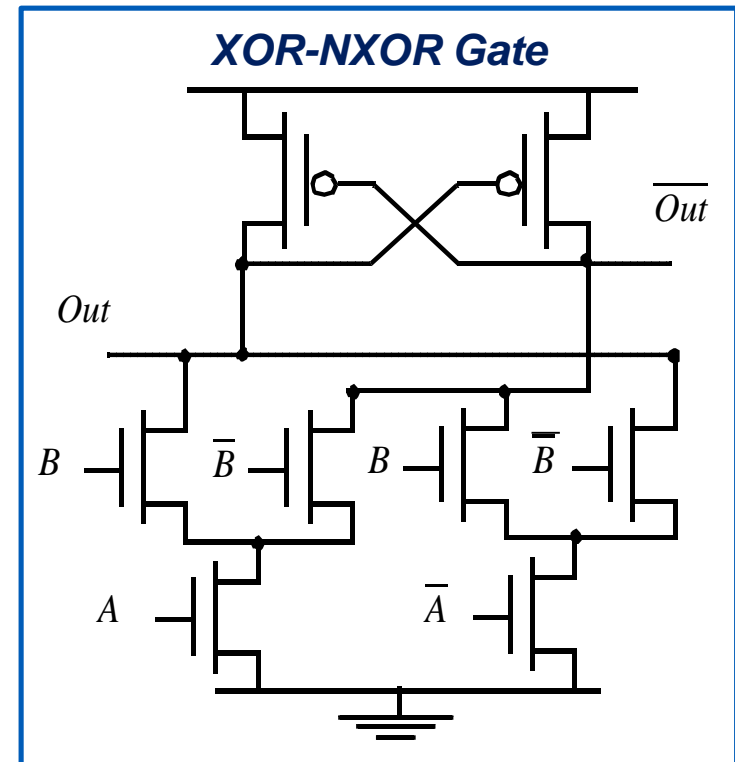
*Differential logic + Positive feedback*

*PDN1 and PDN2: 1) NMOS*

*2) exclusive*

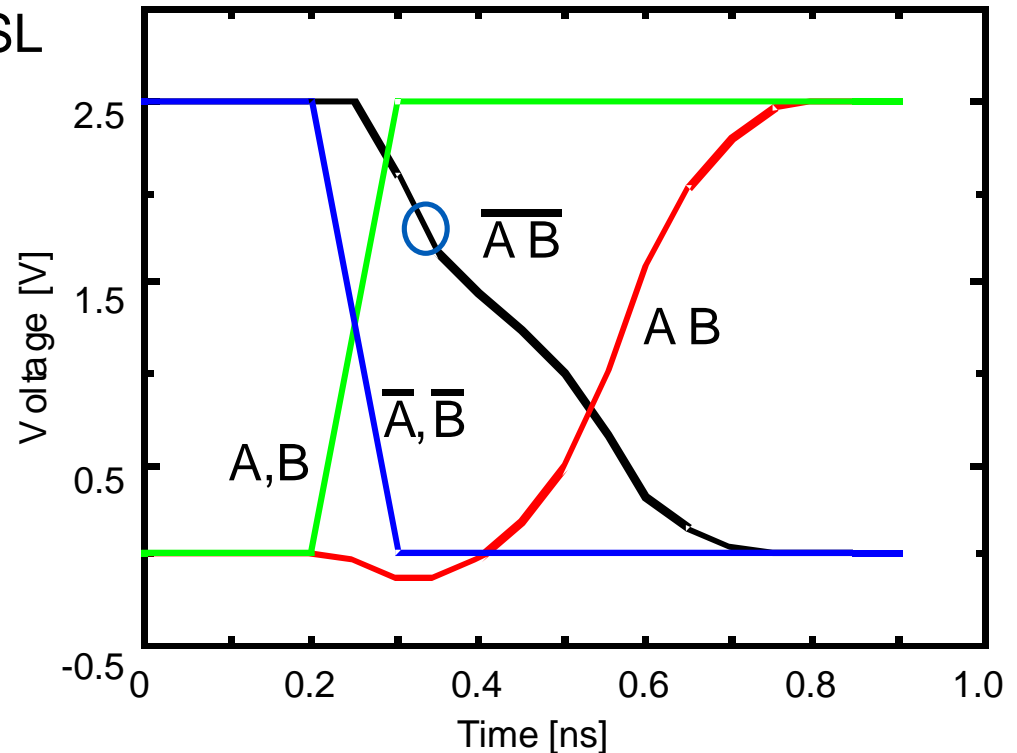
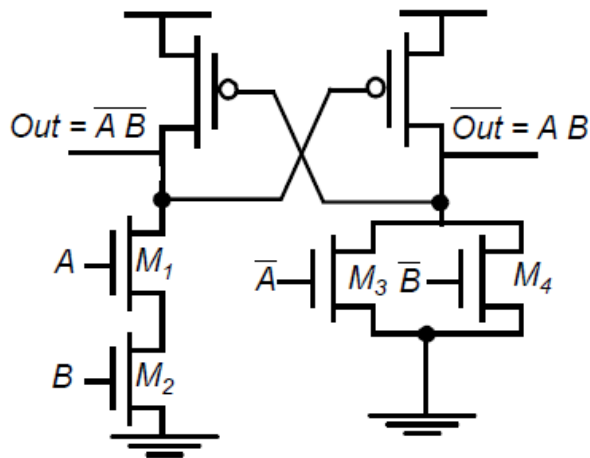


*-- Differential Output*



# DCVSL Transient Response: AND/NAND

## □ AND/NAND gate in DCVSL



### NOTE:

- As *Out* is pulled down to  $VDD - |V_{Tp}|$ , !*Out* starts to charge up to  $VDD$  quickly.
- The delay from the input to *Out* is 197 psec and to !*Out* is 321 psec.



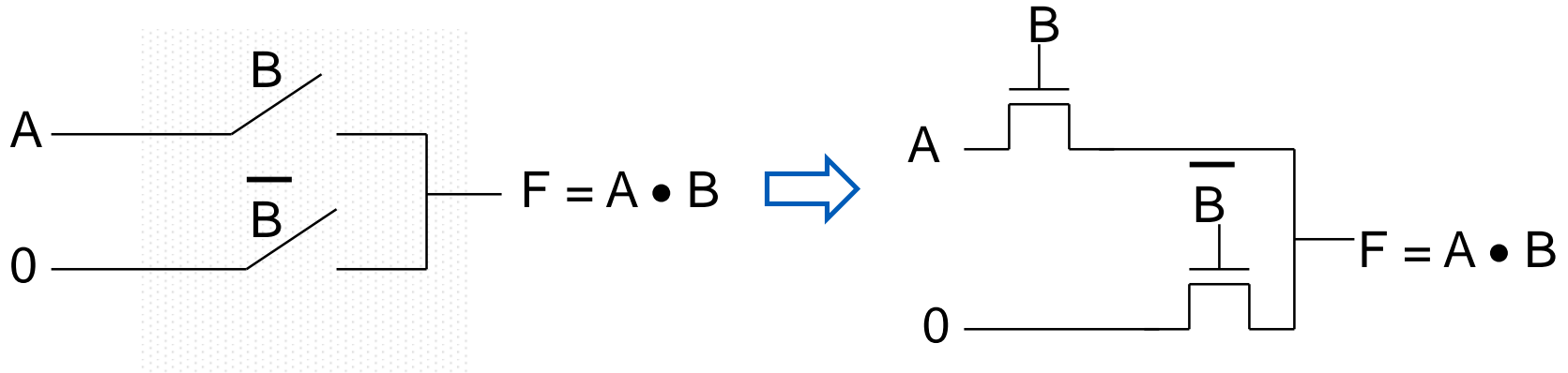
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- **Dynamic CMOS Design**

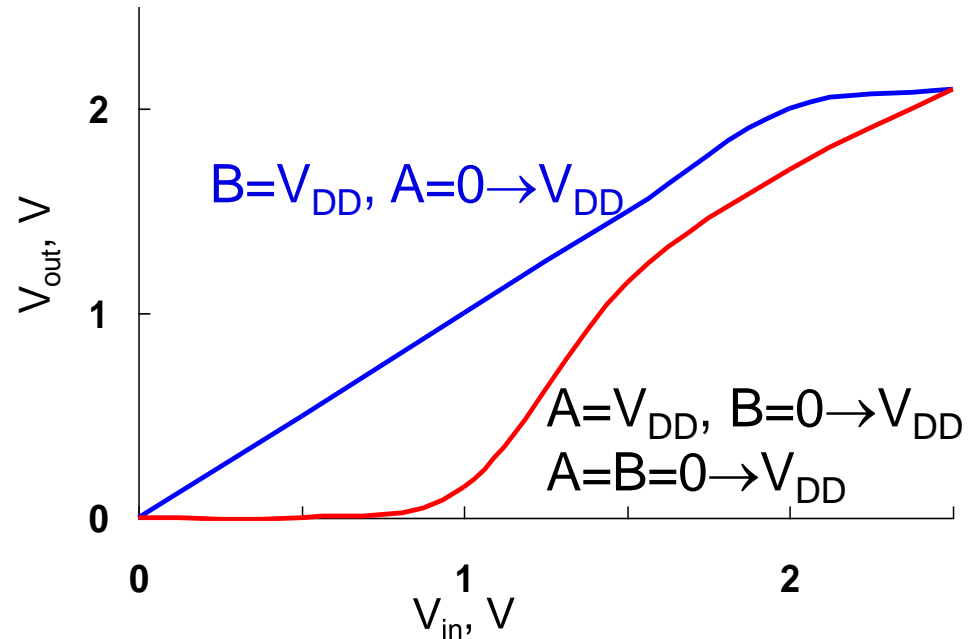
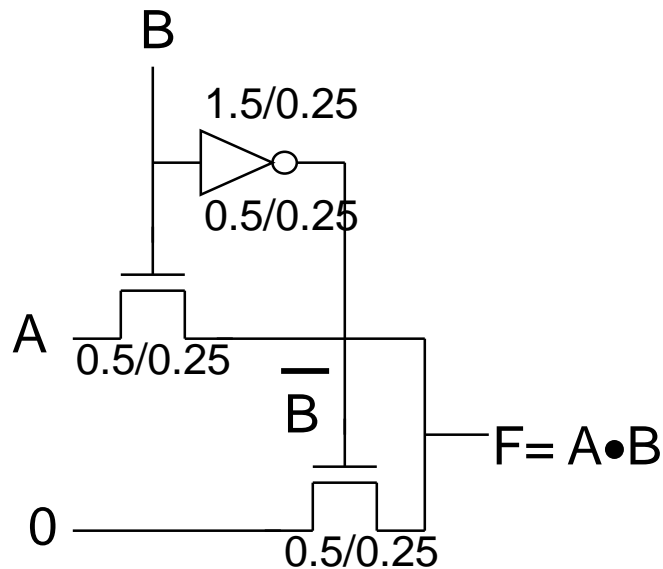


# Pass-Transistor (PT) Logic



- Gate is static - a low-impedance path exists to both supply rails under all circumstances
- N transistors instead of 2N
- No static power consumption
- Ratioless

# Pass-Transistor AND Gate: VTC



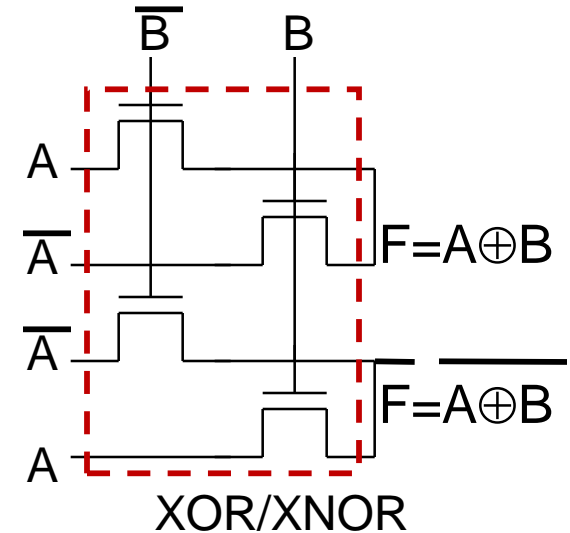
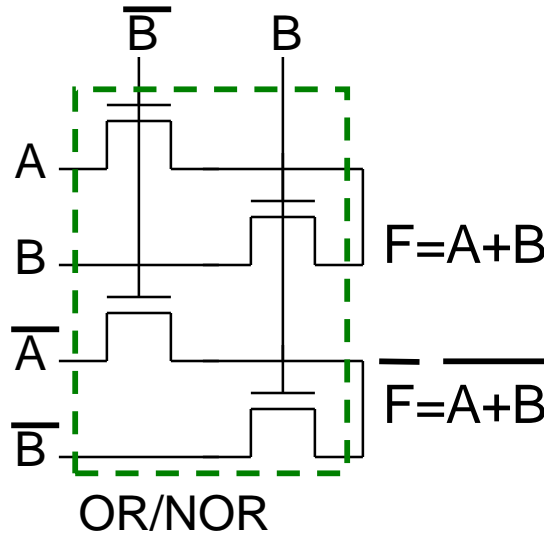
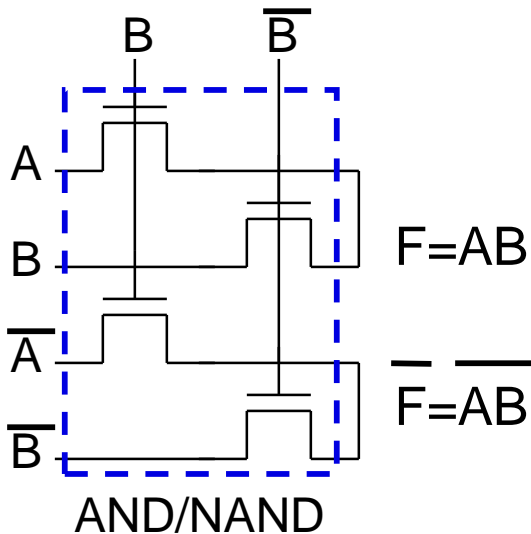
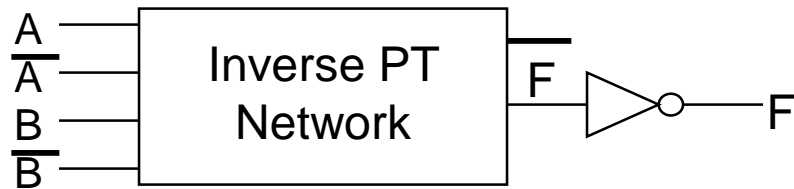
- Be data-dependent
- Pure PT logic is not regenerative - the signal gradually **degrades** after passing through a number of PTs (can fix with **static CMOS inverter** insertion)

# Differential PT Logic (CPL/DPL)

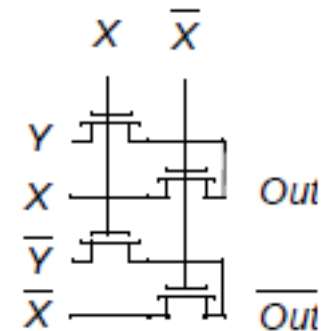
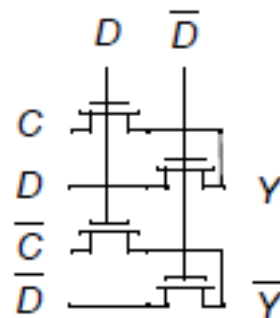
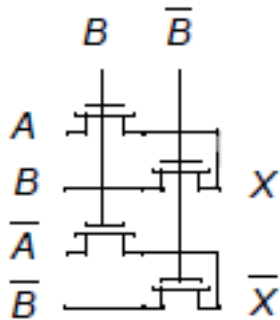
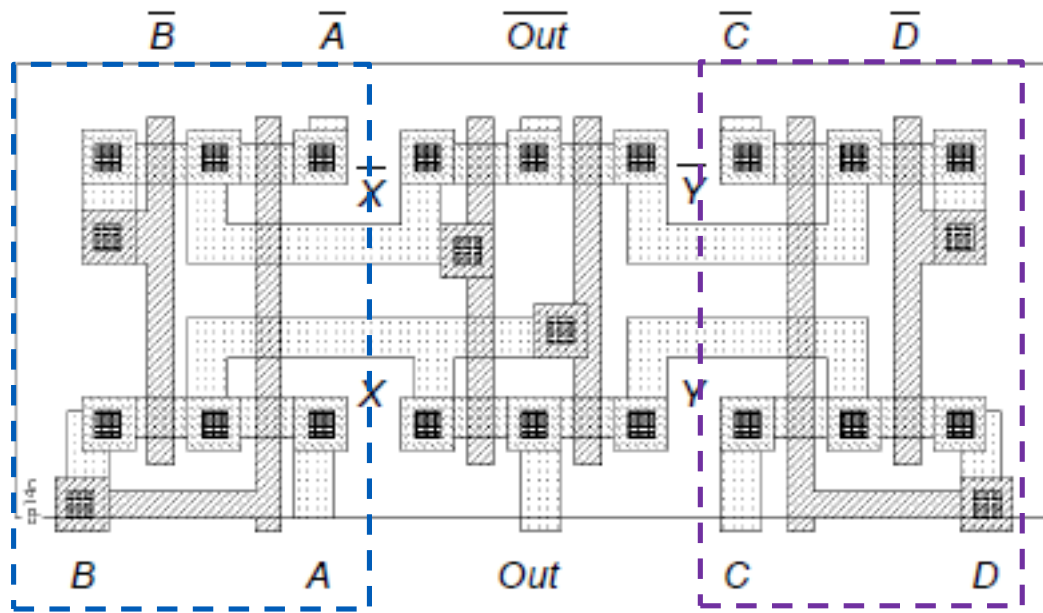


**CPL: Complementary Pass-transistor Logic**

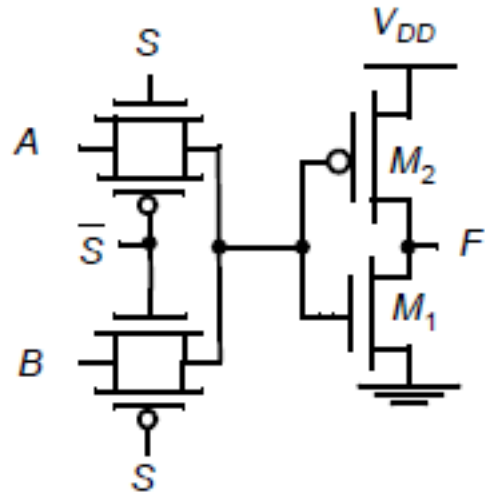
- Complementary data input/ output
- Static gates
- Modular



# Four-input NAND in CPL

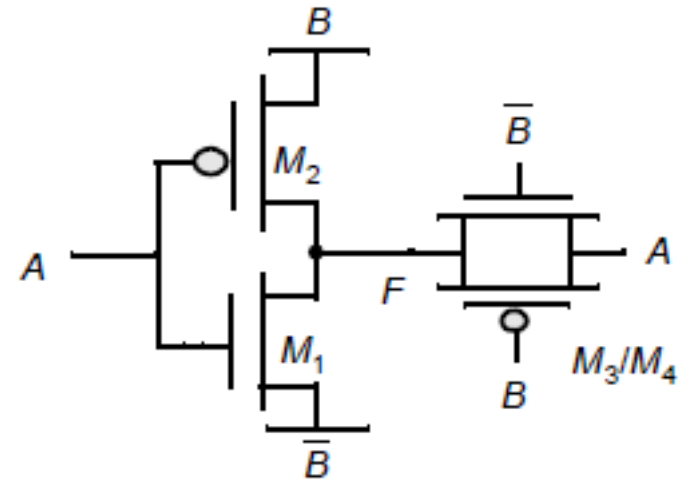


# Transmission Gate: Example



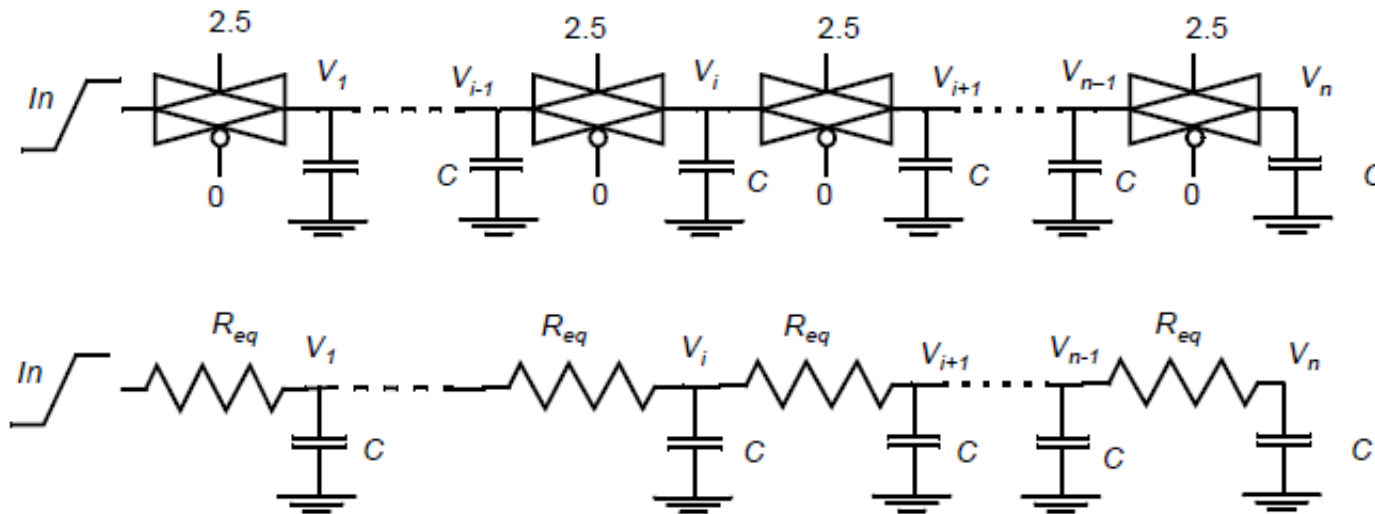
Transmission gate: Multiplexer

$$\bar{F} = (A \cdot S + B \cdot \bar{S})$$



XOR

# Delay in Transmission Gate Networks

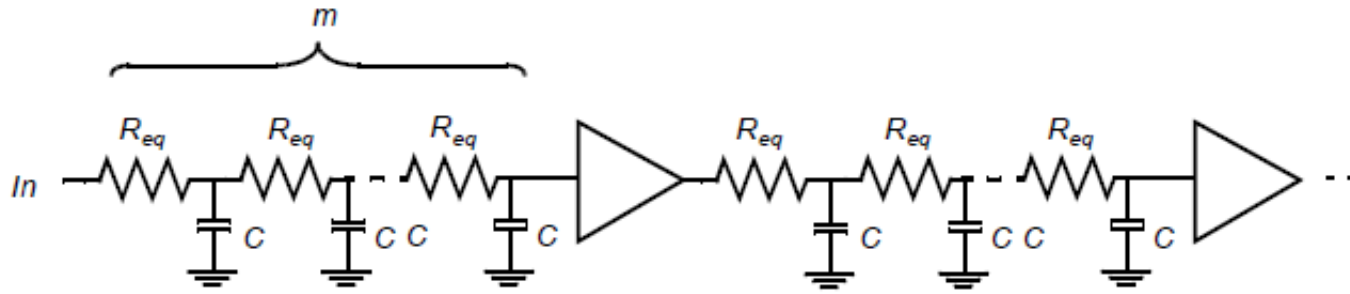


□ Delay of RC Chain 
$$t_p(V_n) = 0.69 \sum_{k=0}^n CR_{eq}k = 0.69CR_{eq}\frac{n(n+1)}{2}$$

The propagation delay is proportional to  $n^2$  and increased rapidly with the switches in the chain.

□ Delay optimization: break the chain and insert buffers every  $m$  switches

# TG Networks : Delay Optimization



## □ Delay of Buffered Chain

$$\begin{aligned}
 t_p &= 0.69 \left[ \frac{n}{m} C R_{eq} \frac{m(m+1)}{2} \right] + \left( \frac{n}{m} - 1 \right) t_{buf} \\
 &= 0.69 \left[ C R_{eq} \frac{n(m+1)}{2} \right] + \left( \frac{n}{m} - 1 \right) t_{buf} \Rightarrow m_{opt} = 1.7 \sqrt{\frac{t_{pbuf}}{C R_{eq}}} \quad \mathbf{m=3 \text{ or } 4}
 \end{aligned}$$

The number of switches per segment grows with increasing values of  $t_{buf}$ .



# Outline

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- **Static CMOS Design**
- **Dynamic CMOS Design**
  - **Dynamic Logic: Basic Principles**
  - **Speed and Power Dissipation of Dynamic Logic**
  - **Cascading Dynamic Gates**

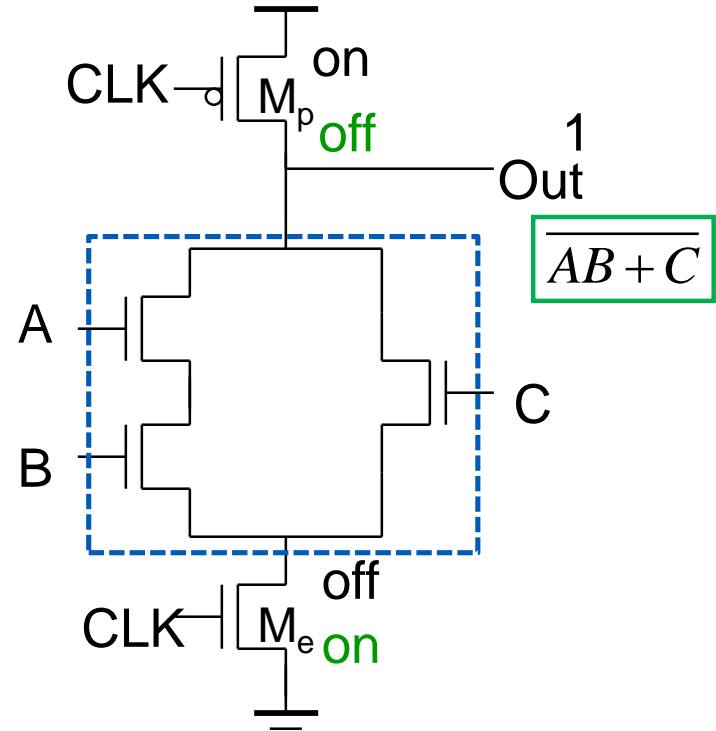
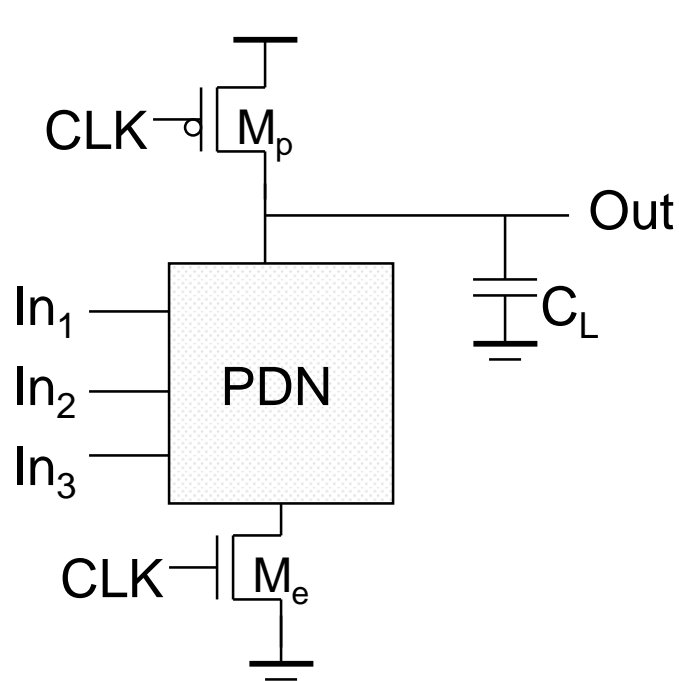


# Dynamic CMOS

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- In **static** circuits at every point in time (except when switching) the output is connected to either GND or  $V_{DD}$  via a low resistance path.
  - Complementary: fan-in of N requires 2N devices
  - Pseudo-NMOS: fan-in of N requires N+1 devices
- **Dynamic** circuits rely on the **temporary storage** of signal values on the **capacitance** of high impedance nodes.
  - requires only N+2 transistors
  - takes a sequence: **precharge** phase, conditional **evaluation** phases

# Dynamic Gate



Two phase operation:

- Precharge (CLK = 0)
- **Evaluate** (CLK = 1)

$$Out = \overline{CLK} + (\overline{AB + C}) \cdot CLK$$

# Conditions on Output

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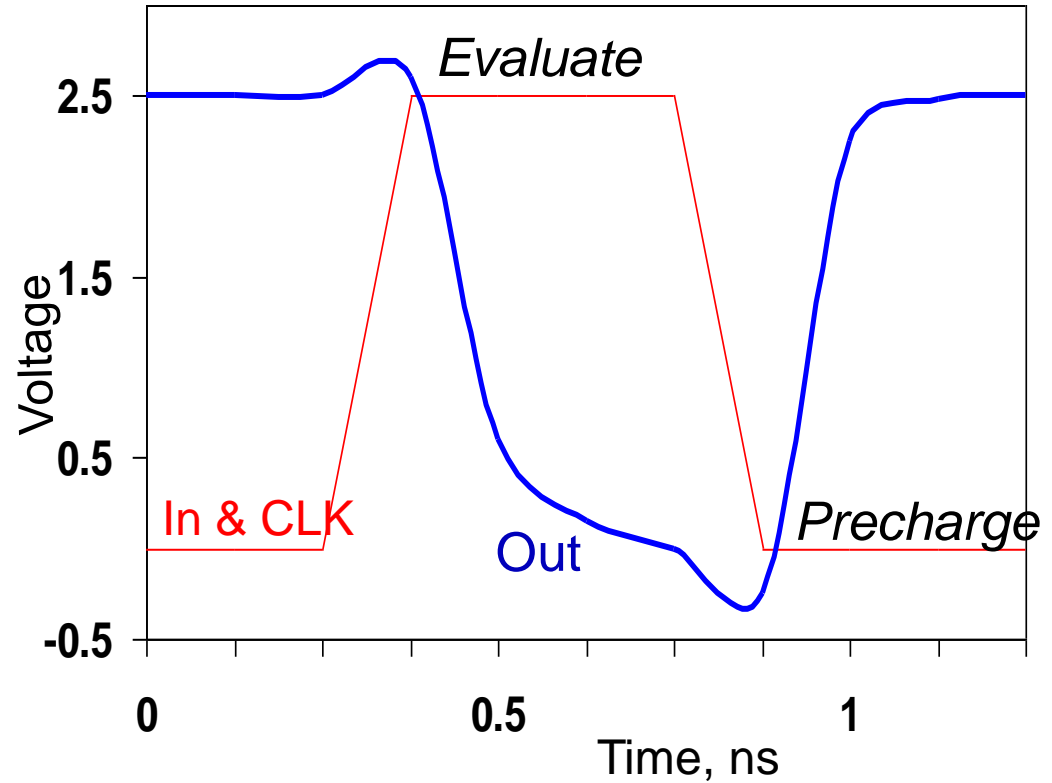
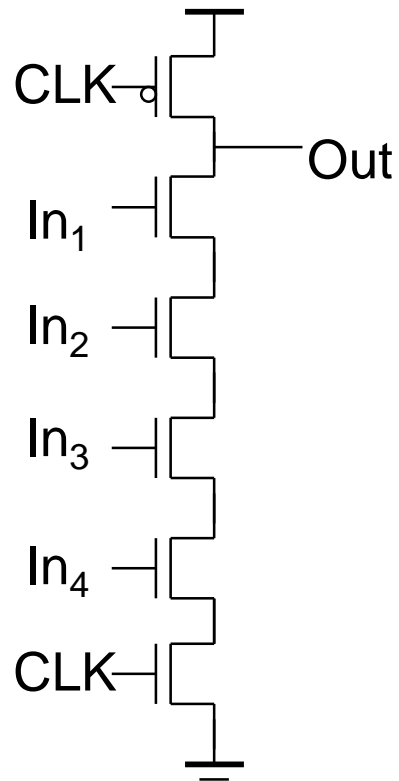
- Once the output of a dynamic gate is discharged, it **cannot be charged again** until the next precharge operation.
- Inputs to the gate can make **at most** one transition during evaluation.
- Output can be in the high impedance state during and after evaluation (PDN off), state is stored on  $C_L$

# Properties of Dynamic Gates

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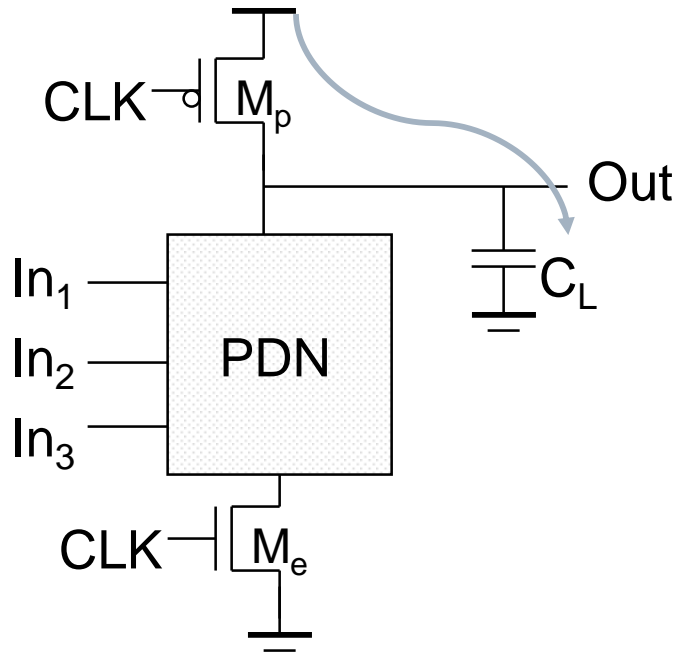
- Logic function is implemented by the PDN only
  - transistors number:  **$N+2$**  (vs. static complementary CMOS:  $2N$ )
  - be **smaller** in area
- Full swing outputs:  $V_{OL}=GND$ ,  $V_{OH}=V_{DD}$
- Nonratioed - sizing of PMOS is not important for proper functioning (only for performance)
- Faster switching speeds (*charge leakage*)
- Power dissipation should be better
  - only dynamic power – no short circuit power consumption since the pull-up path is not on when evaluating
  - lower  $C_L$ -  $C_{int}$  (fewer transistors connected to the drain output)
    - $C_{ext}$  (output load is one per connected gate, not two)
  - by construction can have at most one transition per cycle – **no glitching**
- Needs a precharge clock

# Dynamic Behavior



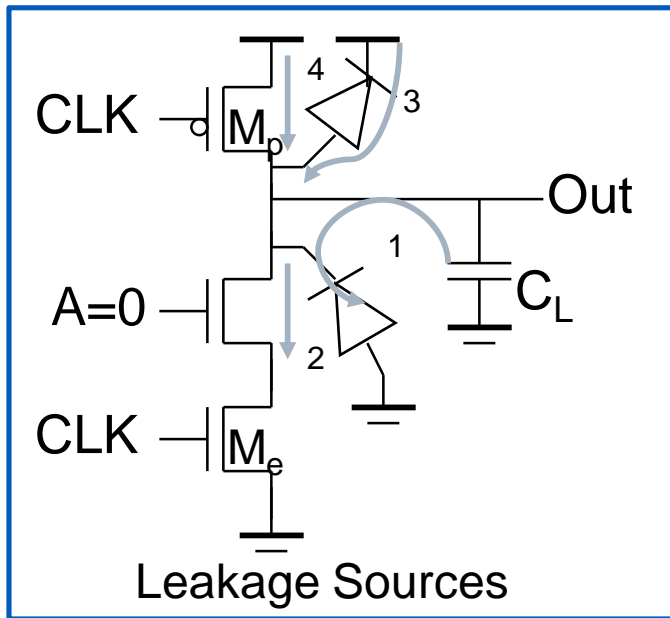
# Power Consumption

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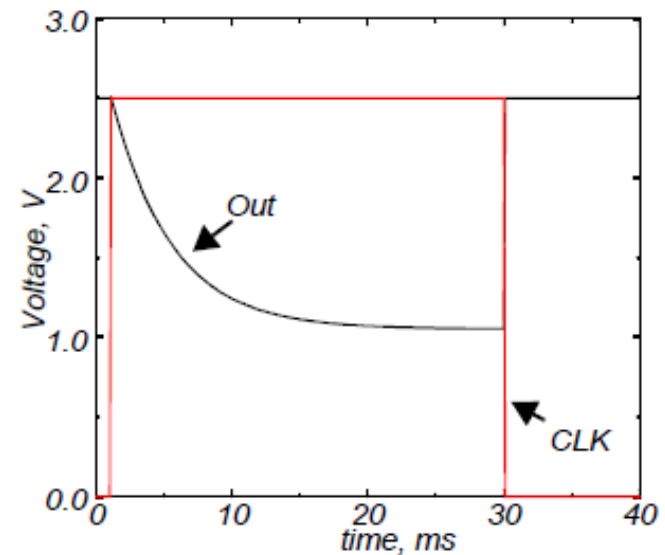
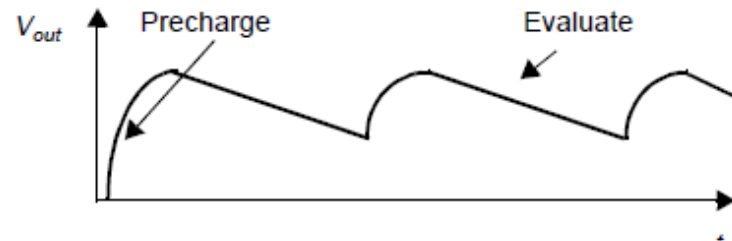
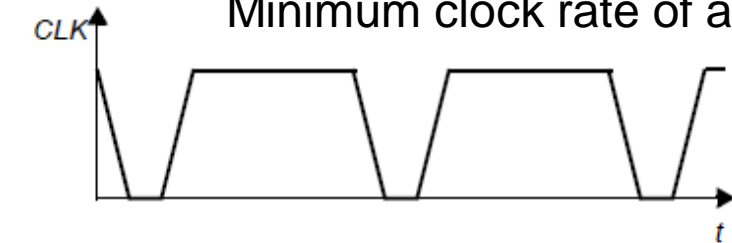
Power only dissipated  
when previous Out = 0

# Issues in Dynamic Design: Charge Leakage

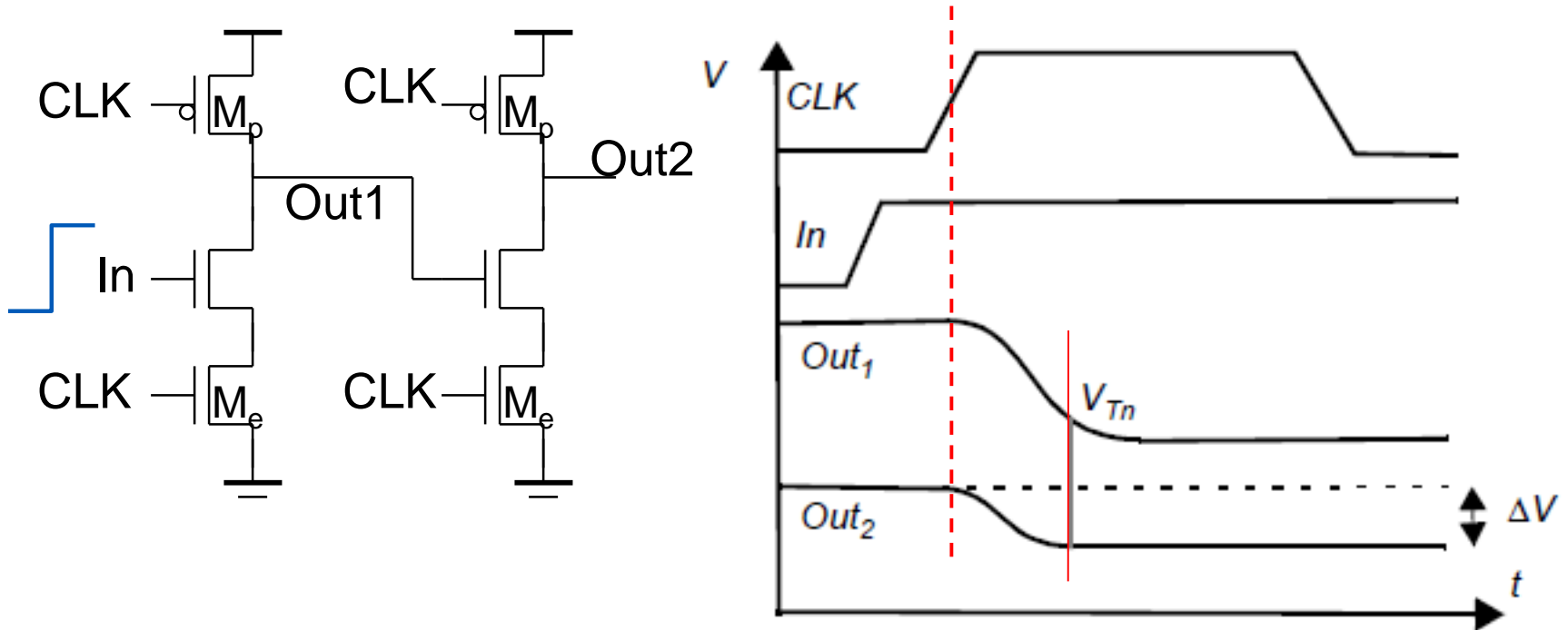


Output settles to an intermediate voltage determined by a resistive divider of the pull-up and pull-down networks (*Once the output drops below  $V_M$  of the fan-out logic gate, the output is interpreted as a low voltage*)

Minimum clock rate of a few kHz



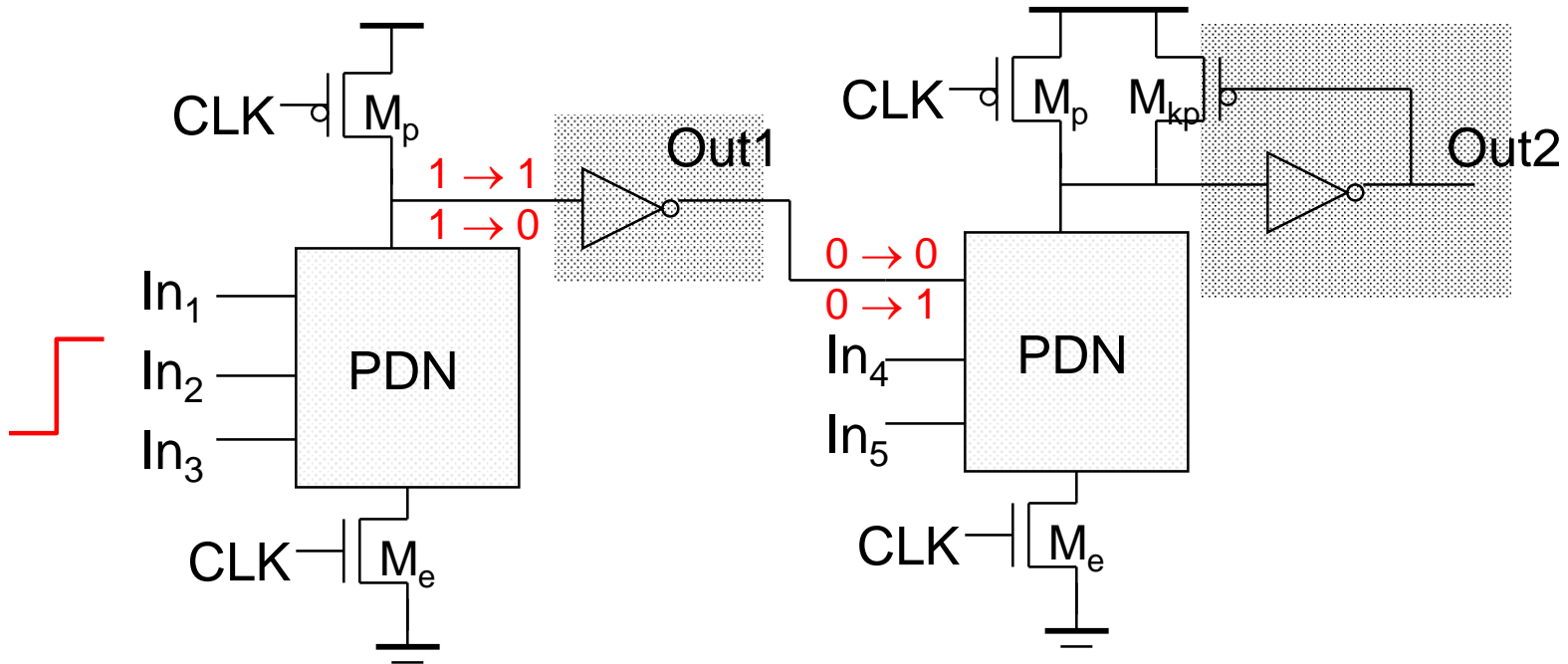
# Cascading Dynamic Gates



Only a single 0  $\rightarrow$  1 transition allowed at the inputs during the evaluation period!

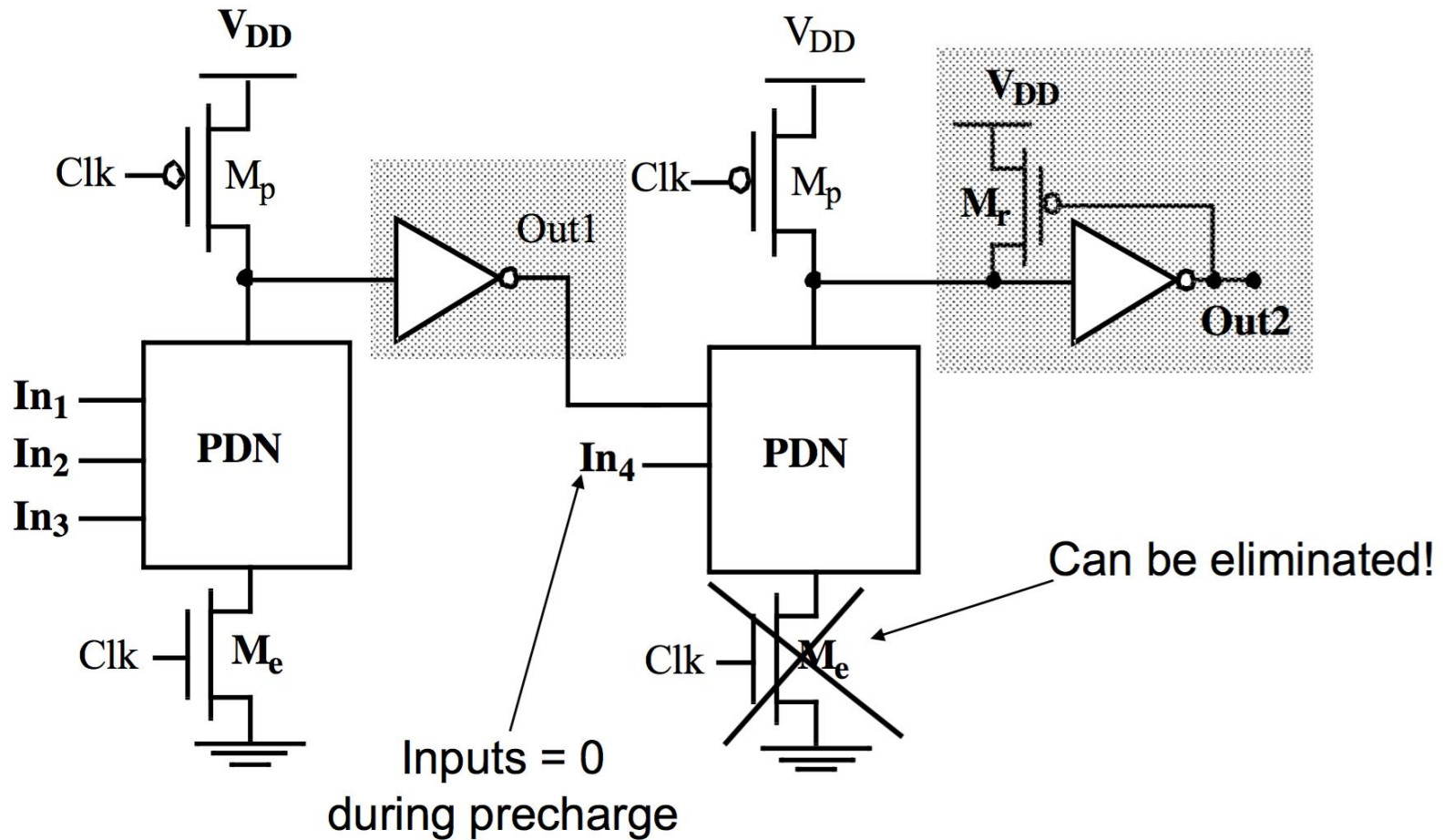


# Domino Logic

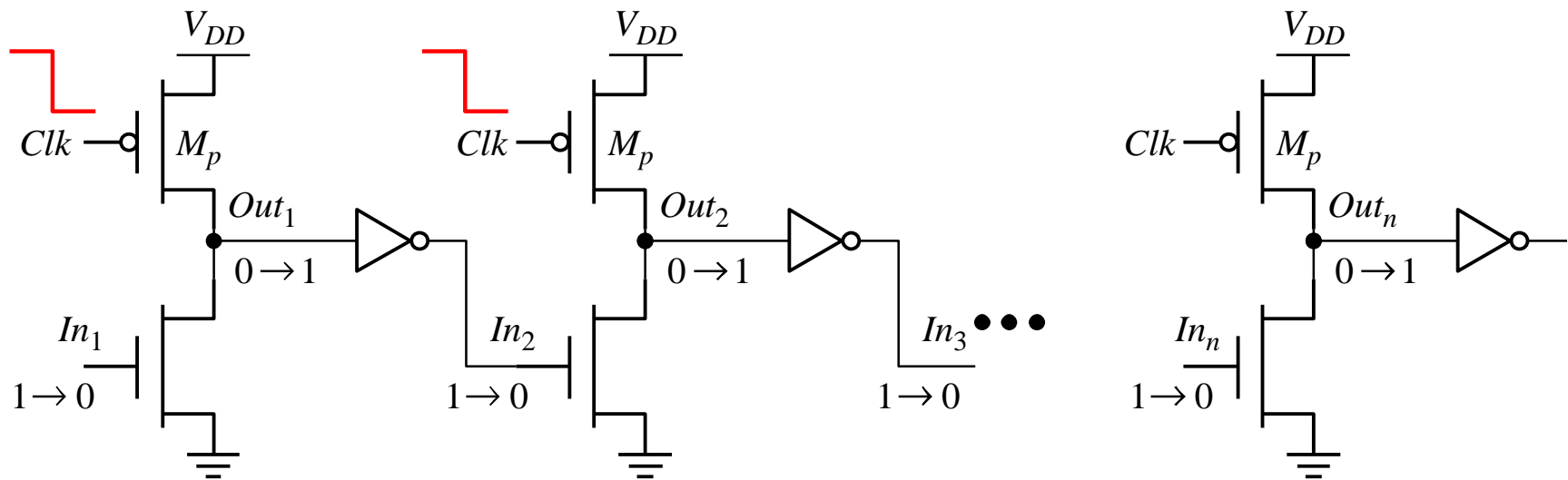


- ❑ Only non-inverting logic can be implemented
- ❑ Very high speed
  - ❑ static inverter can be skewed, only L-H transition
  - ❑ Input capacitance reduced – smaller logical effort

# Designing with Domino Logic



# Footless Domino



- ❑ The first gate in the chain needs a foot switch  
Precharge is rippling – short-circuit current
- ❑ A solution is to delay the clock for each stage

# Properties of Domino Gates

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- ❑ Logic function is implemented by the PDN only requires  $N + 2$  transistors (vs. static complementary CMOS:  $2N$ )
- ❑ Be smaller in area (vs. static complementary CMOS)
- ❑ Full swing outputs ( $V_{OL} = \text{GND}$  and  $V_{OH} = \text{VDD}$ )
- ❑ Faster switching speeds:
  - $C_{\text{int}}$  reduced (fewer transistors connected to the drain output)
  - $C_{\text{ext}}$  (output load is one per connected gate, not two)
- ❑ **Inverter is required**
- ❑ Needs a precharge clock.

# Summary

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- ❑ Static **complementary static CMOS logic** combines dual PDN and PUN networks. The performance of a CMOS gate is a strong function of the fan-in, and also is a function of the fan-out.
- ❑ The ratioed logic style consists of an active pull-down network connected to a load device.
- ❑ **Pass-transistor** logic implements a logic gate as a simple switch network.
- ❑ Dynamic logic: a two-phase scheme consisting of a pre-charge followed by an evaluation step.
- ❑ Current trend is towards an increased use of **complementary static CMOS logic**: design support through EDA tools, robust, more amenable to voltage scaling.



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# 集成电路原理与设计

## 11. 逻辑门

宋爽

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