表一 测量铜棒直径 d 和长度 L

实验次数	1	2	3	4	5	6	平均值
d/mm	5.932	5.936	5.945	5.940	5.938	5.938	5.938
L/mm	160.0	159.4	159.5	159.3	159.5	159.9	159.6

铜棒直径d的不确定度为$\Delta d=\sqrt{u_a^2+u_b^2}=\sqrt{\frac{\sum_{i=1}^{6}(d_i-\bar{d})^2}{6\times5}+\left(\frac{0.04}{\sqrt{3}}\right)^2}=0.023mm$

铜棒直径$d=(5.938\pm0.023)mm$

铜棒长度L的不确定度为$\Delta L=\sqrt{u_a^2+u_b^2}=\sqrt{\frac{\sum_{i=1}^{6}(L_i-\bar{L})^2}{6\times5}+\left(\frac{0.2}{\sqrt{3}}\right)^2}=0.16mm$

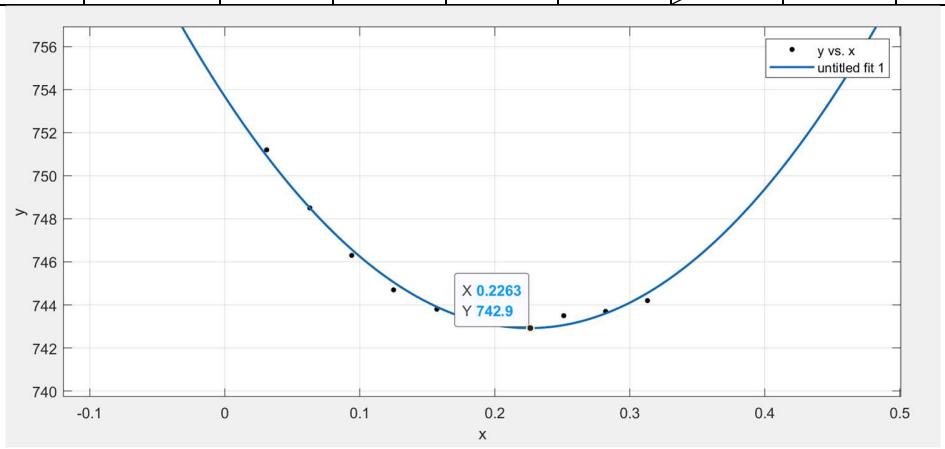
铜棒长度$L=(159.60\pm0.16)mm$

铜棒质量m的不确定度为$\Delta m=\sqrt{u_a^2+u_b^2}=\sqrt{\left(\frac{0.001}{\sqrt{3}}\right)^2}=0.0006g$

铜棒质量$m=(37.6360\pm0.0006)g$

表二 不同节点共振频率表

x/mm	5	10	15	20	25	30	35	40	45	50
x/L	0.031	0.063	0.094	0.125	0.157	0.188	0.219	0.251	0.282	0.313
f/Hz	751.2	748.5	746.3	744.7	743.8	743.5		743.5	743.7	744.2



取各个拟合点与实际测量点对比得到标准差$\sigma=\sqrt{\frac{\sum_{i=1}^{9}\left(f_{i拟合}-f_{i测量}\right)^2}{9}}=0.29Hz$

$\therefore \Delta f=3\sigma=0.9Hz$　　二次拟合得到$f_{基频共振}=742.9Hz$

$f_{基频共振}=(742.9\pm0.9)Hz$

$\bar{E}=1.6067\frac{\bar{L}^3\bar{m}}{\bar{d}^4}f_{基频共振}^2=1.0913\times10^{11}Pa$

$\frac{\Delta E}{\bar{E}}=\sqrt{\left(\frac{3\Delta}{\bar{L}}\right)^2+\left(\frac{4\Delta d}{\bar{d}}\right)^2+\left(\frac{\Delta m}{\bar{m}}\right)^2+\left(\frac{2\Delta}{\bar{f}}\right)^2}=0.016$　　$\therefore \Delta E=0.018\times10^{11}Pa$

$\therefore E=(1.091\pm0.018)\times10^{11}Pa$