量子信息基础　C2-1　Assignment Answer

Question 1、A partical of mass m has the wave function $\psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]}$, where A and a are positive real constants.

(a) Normalize ψ.

$$1 = 2|A|^2\int_0^{\infty} e^{-2amx^2/\hbar}dx = 2|A|^2\frac{1}{2}\sqrt{\frac{\pi}{(2am/\hbar)}} = |A|^2\sqrt{\frac{\pi}{2am}};\, A = (\frac{2am}{\pi\hbar})^{1/4}.$$

(b) Determine the expectation values of x, x^2 and Δx^2 （$\langle\Delta x^2\rangle = \langle x^2\rangle - \langle x\rangle^2$）.

$$\langle x\rangle = \int_{-\infty}^{\infty} x|\psi|^2 dx = 0.$$

$$\langle x\rangle^2 = 2|A|^2\int_0^{\infty} x^2 e^{-2amx^2/\hbar}dx = 2|A|^2\frac{1}{2^2(2am/\hbar)}\sqrt{\frac{\pi\hbar}{2am}} = \frac{\hbar}{4am}.$$

$$\langle\Delta x^2\rangle = \langle x^2\rangle - \langle x\rangle^2 = \frac{\hbar}{4am}.$$

(c) Determine the expectation values of p, p^2 and Δp^2 （$\langle\Delta p^2\rangle = \langle p^2\rangle - \langle p\rangle^2$）.

$$\langle p\rangle = m\frac{d\langle x\rangle}{dt} = 0.$$

$$\langle p^2\rangle = \int \psi^*\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)^2\psi dx = -\hbar^2\int \psi^*\frac{\partial^2\psi}{\partial x^2}dx = -\hbar^2\int \psi^*\left[-\frac{2am}{\hbar}\left(1-\frac{2amx^2}{\hbar}\right)\psi\right]dx$$

$$= 2am\hbar\left\{\int |\psi|^2 dx - \frac{2am}{\hbar}\int x^2|\psi|^2 dx\right\}$$

$$= 2am\hbar\left(1-\frac{2am}{\hbar}\langle x^2\rangle\right) = 2am\hbar\left(1-\frac{2am}{\hbar}\frac{\hbar}{4am}\right) = 2am\hbar\left(\frac{1}{2}\right) = am\hbar.$$

$$\langle\Delta p^2\rangle = \langle p^2\rangle - \langle p\rangle^2 = am\hbar.$$

(d) Check the Heisenberg's uncertainty principle in this special case.

$$\Delta x\Delta p = \sqrt{\frac{\hbar}{4am}}\sqrt{am\hbar} = \frac{\hbar}{2}.$$

Question 2、A photon propagates in the z direction and passes a linear optical polarizer which is oriented in the x direction (see figure below). The state in Figure (a) is ψ_a while the state in Figure (b) is ψ_b.

(a) Write done the formula of ψ_c, assuming the light beam is polarized with an angle of α to the x axis.

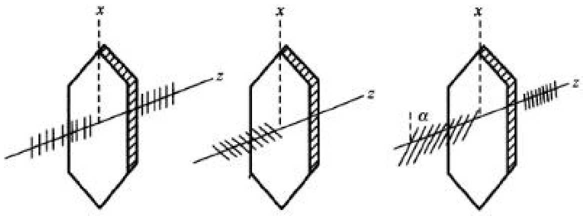
$$\psi_{\alpha} = \cos\alpha\cdot\psi_x + \sin\alpha\cdot\psi_y$$

(b) How much is the probability that a single photon could pass the polarizer in (c)?

$$P = |\cos\alpha|^2$$

(c) How does the system maintain the normalization condition?

$$|\cos\alpha|^2 + |\sin\alpha|^2 = 1$$



(a) (b) (c)

Question 3、 Show that

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = 0$$

for any two (normalizable) solutions to the Schrödinger equation (with the same $V(x)$), Ψ_1 and Ψ_2.

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = \int_{-\infty}^{\infty} \frac{\partial \Psi_1^*}{\partial t}\Psi_2 + \Psi_1^* \frac{\partial \Psi_2}{\partial t} dx$$

Schrodinger equation

$$-i\hbar\frac{\partial \Psi_1^*}{\partial t} =- \frac{\hbar^2}{2m}\frac{\partial^2 \Psi_1^*}{\partial x^2} + V\Psi_1^*$$

$$i\hbar\frac{\partial \Psi_2}{\partial t} =- \frac{\hbar^2}{2m}\frac{\partial^2 \Psi_2}{\partial x^2} + V\Psi_2$$

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = \int_{-\infty}^{\infty} \left[\frac{\hbar}{2im}\frac{\partial^2 \Psi_1^*}{\partial x^2} - \frac{V}{i\hbar}\Psi_1^*\right]\Psi_2 + \Psi_1^*\left[-\frac{\hbar}{2im}\frac{\partial^2 \Psi_2}{\partial x^2} + \frac{V}{i\hbar}\Psi_2\right] dx$$

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = \int_{-\infty}^{\infty} \frac{\hbar}{2im}\frac{\partial^2 \Psi_1^*}{\partial x^2}\Psi_2 - \Psi_1^* \frac{\hbar}{2im}\frac{\partial^2 \Psi_2}{\partial x^2} dx$$

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = \frac{\hbar}{2im}\int_{-\infty}^{\infty} \frac{\partial}{\partial x}\left(\frac{\partial \Psi_1^*}{\partial x}\Psi_2 - \Psi_1^* \frac{\partial \Psi_2}{\partial x}\right) dx$$

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = \frac{\hbar}{2im}\left(\frac{\partial \Psi_1^*}{\partial x}\Psi_2 - \Psi_1^* \frac{\partial \Psi_2}{\partial x}\right)\Bigg|_{-\infty}^{\infty}$$

$$\frac{d}{dt}\int_{-\infty}^{\infty} \Psi_1^* \Psi_2 dx = 0$$