



UNIVERSITY OF  
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# Simulation of metastasis base on the viscous fingering model

*Phase-field simulation computed with FiPy*

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Study Report

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# Study's goal

## 1.1 Objectives

New model of metastasis taking into account the activity of the tumour cells

## 1.2 Type of analysis

The problem is analysed as a viscous fingering problem.

## 1.3 Method

Phase-field



# Physical behaviour assumptions

## 2.1 Global description of the fluids

Mass conservation and incompressibility:  $\underline{\nabla} \cdot \underline{u} = 0$

## 2.2 Constitutive properties

### 2.2.1 Passive fluid

Darcy's law/Equation of motion:  $\underline{\nabla} p = -\beta_1 \underline{u}$

### 2.2.2 Active fluid



# Phase-field model

## 3.1 Order parameter

$\phi = 0$  for the healthy cells  
 $\phi = 1$  for the tumor cells

## 3.2 Cahn-Hilliard equation

Order parameter: conserved

Cahn-Hilliard equation:  $\frac{\partial \phi}{\partial t} + \underline{u} \cdot \underline{\nabla} \phi = \underline{\nabla} \cdot (M * \underline{\nabla} G)$

Mobility:  $M = M_C * \epsilon^2 [1]$

$$M_C = \frac{\mu_1}{\mu_2}$$

$$\beta = \beta_1 * \phi + \beta_2 * (1 - \phi)$$

## 3.3 Free energy

Free energy:  $G = \lambda * [\frac{1}{\epsilon^2} \phi(\phi - \frac{1}{2})(\phi - 1) - \nabla^2 \phi]$



# Geometric Assumptions

## 4.1 Presentation of geometry

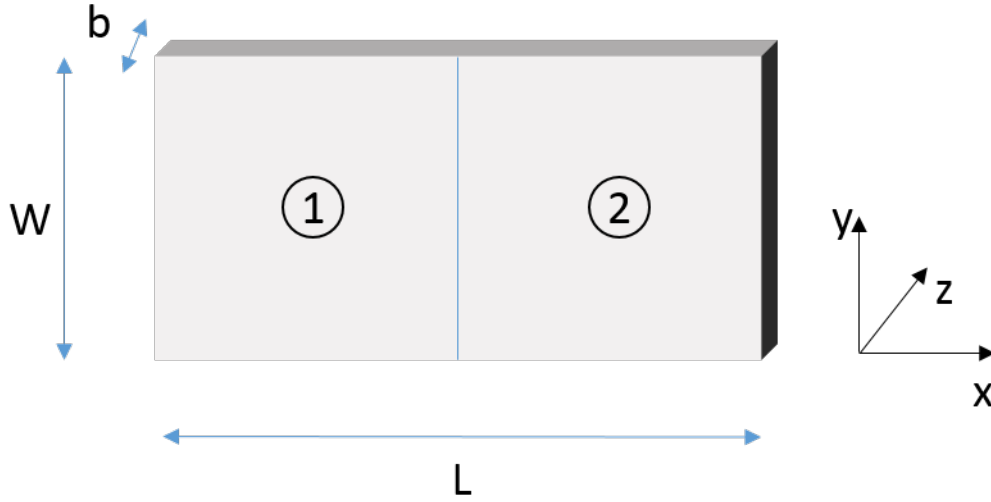


Figure 4.1: *Dispositif de simulation*

## 4.2 System of units used

## 4.3 Characteristic dimensions

Characteristic length:  $W$

Characteristic velocity:  $U_{\infty} = \frac{Q}{bW}$

## 4.4 Problem's symmetries

## 4.5 Boundary conditions

At the left: rate  $Q$ . At the right,  $U_{\infty}$ . No-slip boundaries.



# Space Discretization Assumptions

## 5.1 Discretization method

Use of FiPy: Finite Volume

## 5.2 Numerical grid

Staggered grid

## 5.3 Size and number of elements

## 5.4 Mesh convergence



# Time Discretization Assumptions

## 6.1 Numerical Scheme

Implicit by FiPy. We choose a stable time step for now.

## 6.2 Solution method



# Resolution

## **7.1 Type of problem solved**

Evolution of the two viscous fluids.

## **7.2 Initial values**

Two phases.

## **7.3 Options of resolution**

Use SIMPLE algorithm

## **7.4 Results calculated**

number of fingers, width



# Validity of the model

8.1 Convergence

8.2 Consistency

8.3 Stability

8.4 Conservation

8.5 Boundedness

8.6 Realizability

8.7 Accuracy





## Results



## Analysis and Conclusions