UNIVERSITY OF CAMBRIDGE

Simulation of metastasis base on the viscous fingering model

Phase-field simulation computed with FiPy

2017

Study Report

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Study's goal

1.1 Objectives

New model of metastasis taking into account the activity of the tumour cells

1.2 Type of analysis

The problem is analysed as a viscous fingering problem.

1.3 Method

Phase-field

Physical behaviour assumptions

2.1 Global description of the fluids

Mass conservation and incompressibility: $\underline{\nabla} \cdot \underline{u} = 0$

2.2 Constitutive properties

2.2.1 Passive fluid

Darcy's law/Equation of motion: $\underline{\nabla} p = -\beta_1 \underline{u}$

2.2.2 Active fluid

Phase-field model

3.1 Order parameter

phi = 0 for the healthy cells
phi = 1 for the tumor cells

3.2 Cahn-Hilliard equation

Order parameter: conserved
Cahn-Hilliard equation: $\frac{\partial \phi}{\partial t} + \underline{u} \cdot \underline{\nabla}\phi = \underline{\nabla} \cdot (M * \underline{\nabla} G)$
Mobility: $M = Mc * \epsilon^2$ [1]

$Mc = \frac{\mu_1}{\mu_2}$

$\beta = \beta_1 * \phi + \beta_2 * (1 - \phi)$

3.3 Free energy

Free energy: $G = \lambda * [\frac{1}{\epsilon^2}\phi(\phi - \frac{1}{2})(\phi - 1) - \nabla^2\phi]$

Geometric Assumptions

4.1 Presentation of geometry

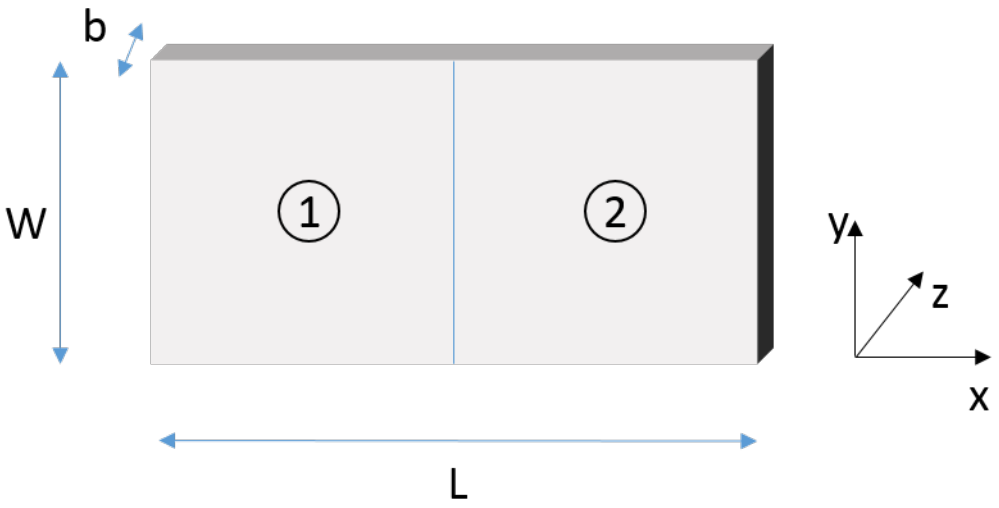

Figure 4.1: *Dispositif de simulation*

4.2 System of units used

4.3 Characteristic dimensions

Characteristic length: W
Characteristic velocity: $U_\infty = \frac{Q}{bW}$

4.4 Problem's symmetries

4.5 Boundary conditions

At the left: rate Q. At the right, U_∞. No-slip boundaries.

Space Discretization Assumptions

5.1 Discretization method

Use of FiPy: Finite Volume

5.2 Numerical grid

Staggered grid

5.3 Size and number of elements

5.4 Mesh convergence

Time Discretization Assumptions

6.1 Numerical Scheme

Implicit by FiPy. We choose a stable time step for now.

6.2 Solution method

Resolution

7.1 Type of problem solved

Evolution of the two viscous fluids.

7.2 Initial values

Two phases.

7.3 Options of resolution

Use SIMPLE algorithm

7.4 Results calculated

number of fingers, width

Validity of the model

8.1 Convergence

8.2 Consistency

8.3 Stability

8.4 Conservation

8.5 Boundedness

8.6 Realizability

8.7 Accuracy

Results

Analysis and Conclusions