



BITS Pilani
Pilani Campus

Artificial & Computational Intelligence

AIML CLZG557

M3 : Game Playing

Dr. Sudheer Reddy

Course Plan



- M1 Introduction to AI
- M2 Problem Solving Agent using Search
- M3 Game Playing
- M4 Knowledge Representation using Logics
- M5 Probabilistic Representation and Reasoning
- M6 Reasoning over time
- M7 Ethics in AI



Module 3 : Searching to play games

A. Minimax Algorithm

B. Alpha-Beta Pruning

C. Making imperfect real time decisions



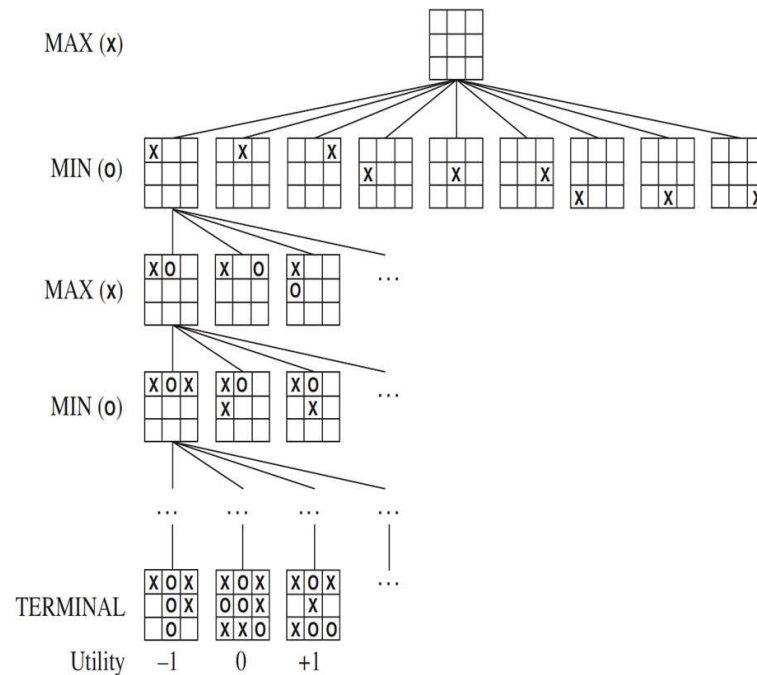
Learning Objective

At the end of this class , students should be able to:

1. Convert a given problem into adversarial search problem
2. Formulate the problem solving agent components
3. Design static evaluation function value for a problem
4. Construct a Game tree
5. Apply Min-Max
6. Apply and list nodes pruned by alpha pruning and nodes pruned by beta pruning

Task Environment

Phases of Solution Search by PSA



Assumptions – Environment :

Static

Observable

Discrete

Deterministic

Number of Agents

Game Problem

Study & design of games enables the computers to model ways in which humans think & act hence simulating human intelligence.

AI for Gaming:

- Interesting & Challenging Problem
- Larger Search Space Vs Smaller Solutions
- Explore to better the Human Computer Interaction



Characteristics of Games:

- Observability
- Stochasticity
- Time granularity
- Number of players



Adversarial Games:

Goals of agents are in conflict where one's optimized step would reduce the utility value of the other.

Games as Search Problem

PSA : Representation of Game:

INITIAL STATE: S_0

PLAYER(s)

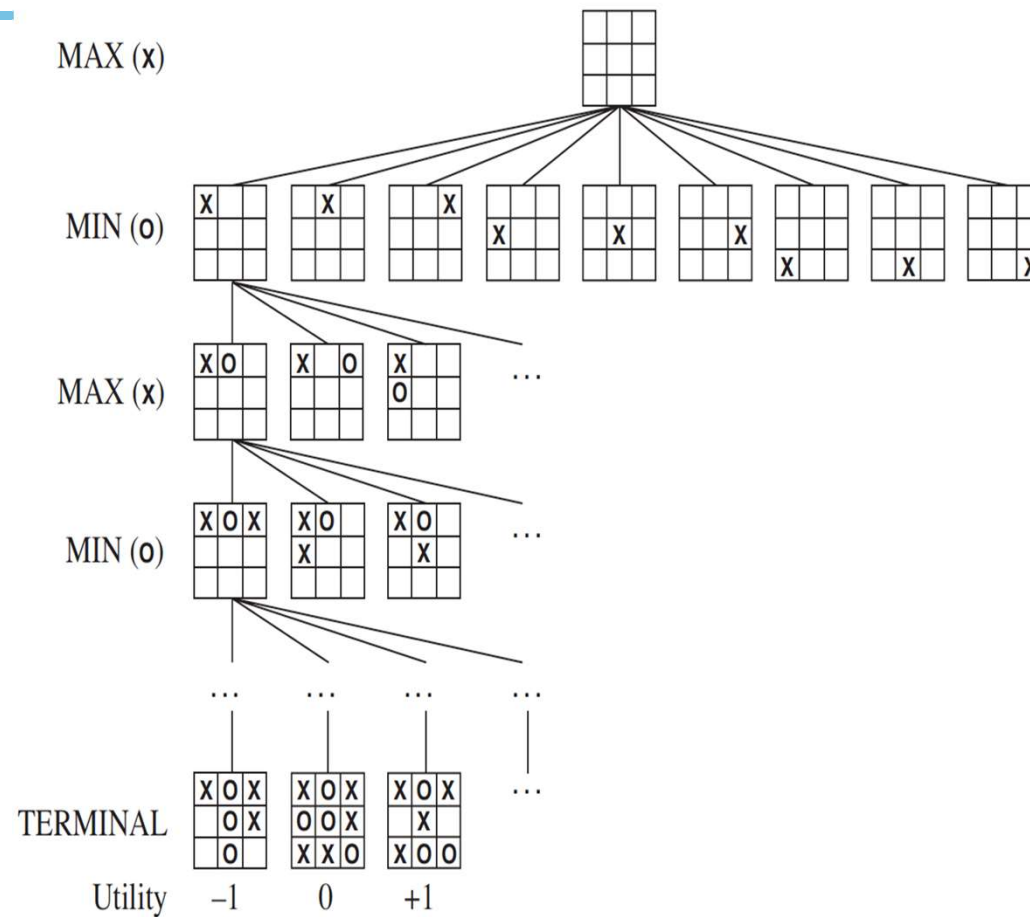
ACTIONS(s)

RESULT(s, a)

TERMINAL-TEST(s)

UTILITY(s, p)

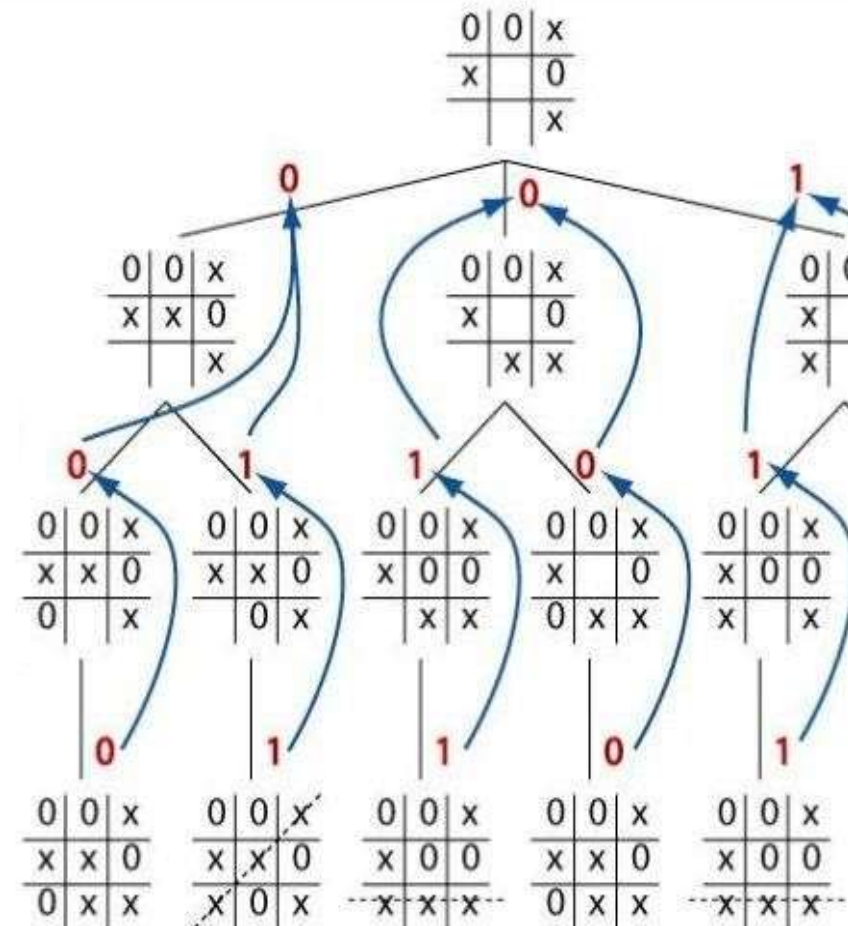
Eg., Tic Tac Toe



Min-Max Algorithm

Idea: Uses Depth – First search exploration to decide the move

Let
start Player = MAX
Depth $m = 3$





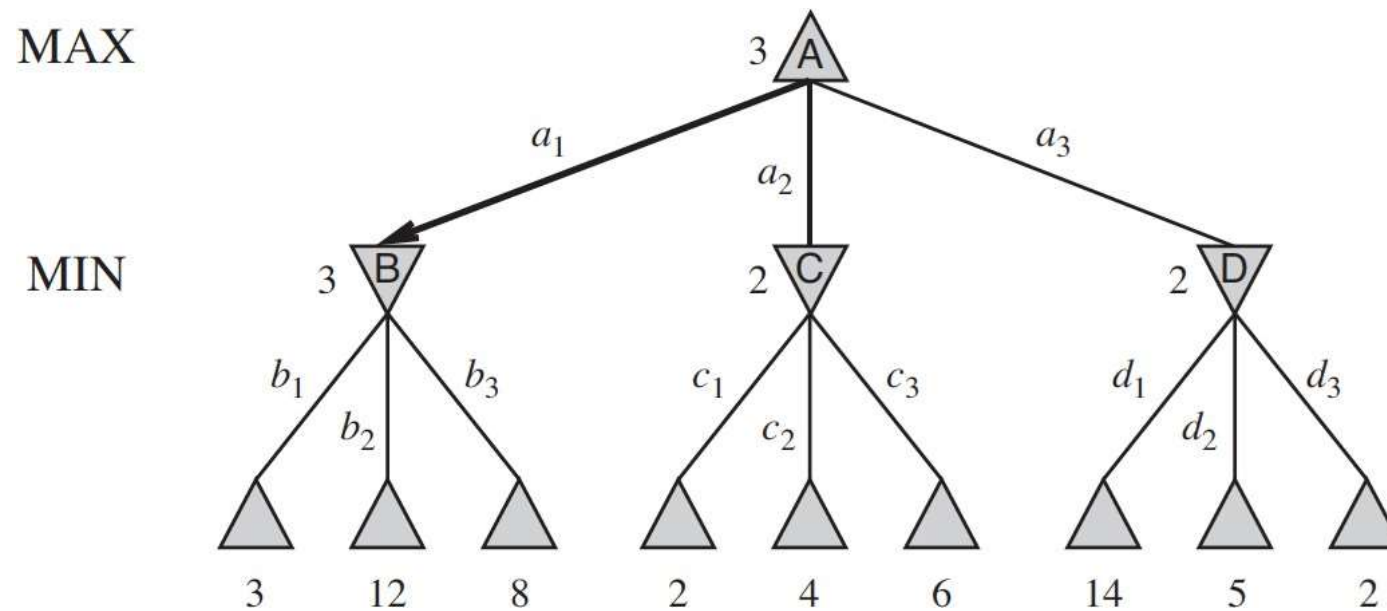
Min-Max Algorithm

function MINIMAX-DECISION(*state*) **returns** *an action*
 return $\arg \max_{a \in \text{ACTIONS}(s)} \text{MIN-VALUE}(\text{RESULT}(\text{state}, a))$

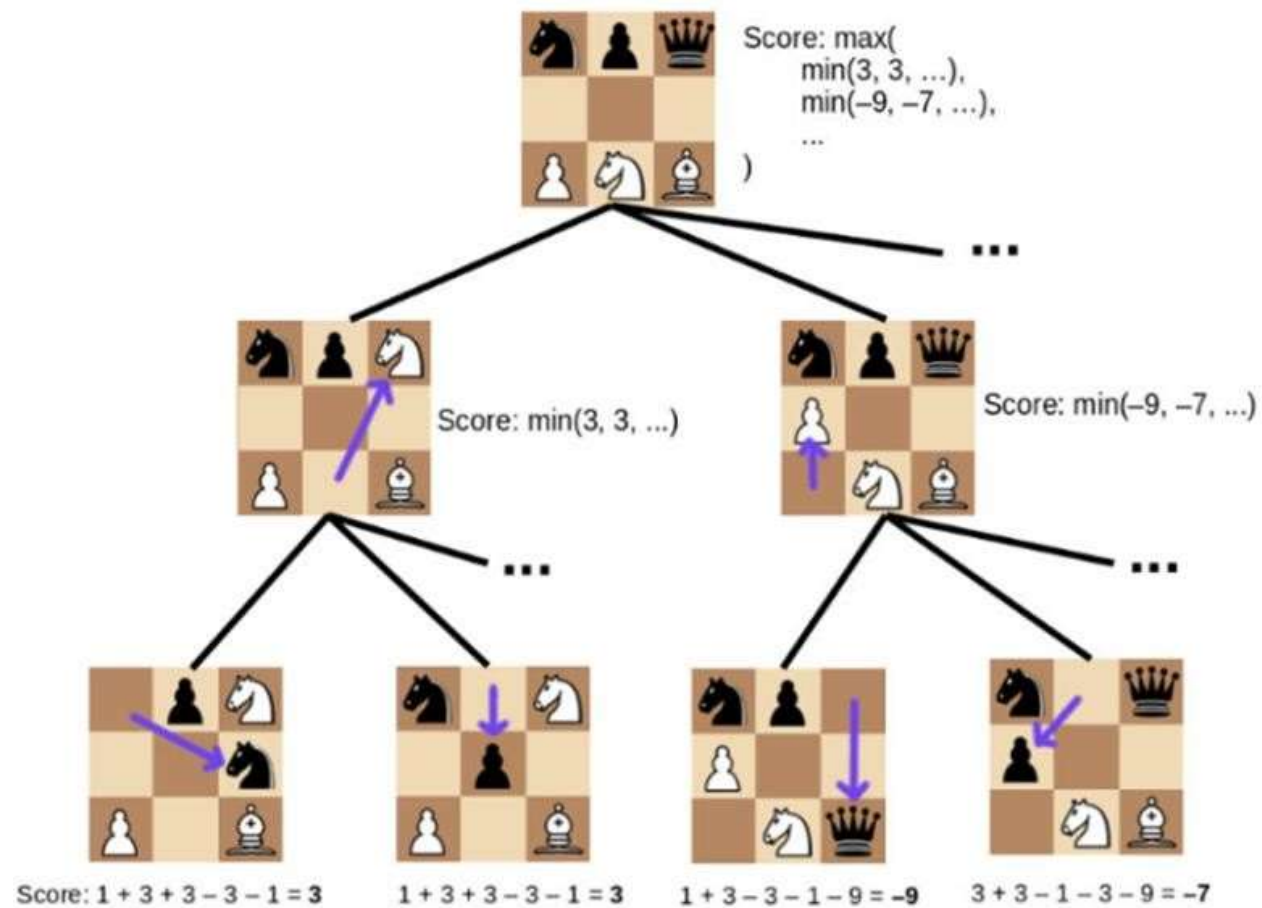
function MAX-VALUE(*state*) **returns** *a utility value*
 if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
 $v \leftarrow -\infty$
 for each *a* **in** ACTIONS(*state*) **do**
 $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s, a)))$
 return *v*

function MIN-VALUE(*state*) **returns** *a utility value*
 if TERMINAL-TEST(*state*) **then return** UTILITY(*state*)
 $v \leftarrow \infty$
 for each *a* **in** ACTIONS(*state*) **do**
 $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s, a)))$
 return *v*

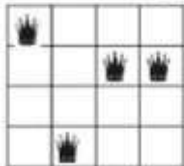
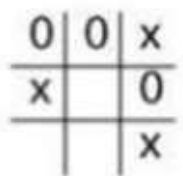
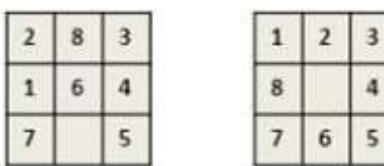
Book Example



Design of Static Evaluation Values



Design of Static Evaluation Values

N-Queens	Tic-Tac-Toe	N-Tile													
															
<table><tr><td>1</td><td>4</td><td>2</td><td>2</td><td>4</td></tr></table>	1	4	2	2	4	<table><tr><td>Max's Share</td><td>2</td></tr><tr><td>Min's Share</td><td>1</td></tr><tr><td>Board Value</td><td>1</td></tr></table>	Max's Share	2	Min's Share	1	Board Value	1	<table><tr><td>No.of.Tiles Out of Place</td><td>5</td></tr></table>	No.of.Tiles Out of Place	5
1	4	2	2	4											
Max's Share	2														
Min's Share	1														
Board Value	1														
No.of.Tiles Out of Place	5														

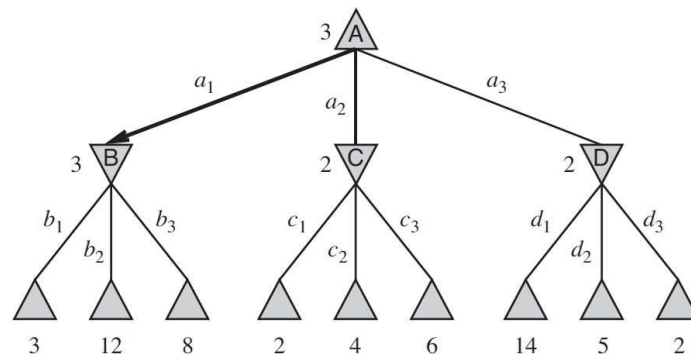
$$\text{Eval}(S) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$= 0.6 (\text{MaxChance} - \text{MinChance}) + 0.4 (\text{MaxPairs} - \text{MinPairs})$$



MAX

MIN



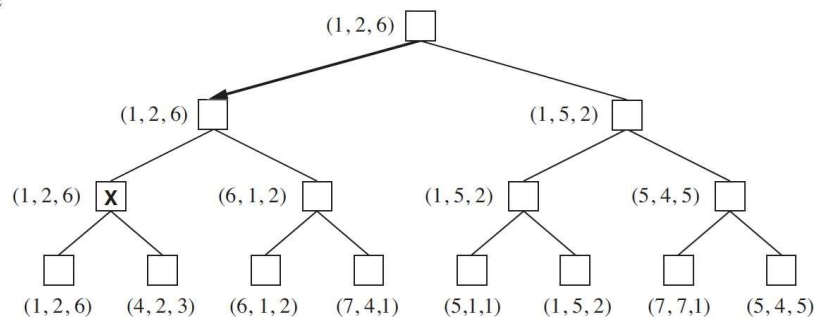
Two Player Game : 1-Ply Game

to move
A

B

C

A

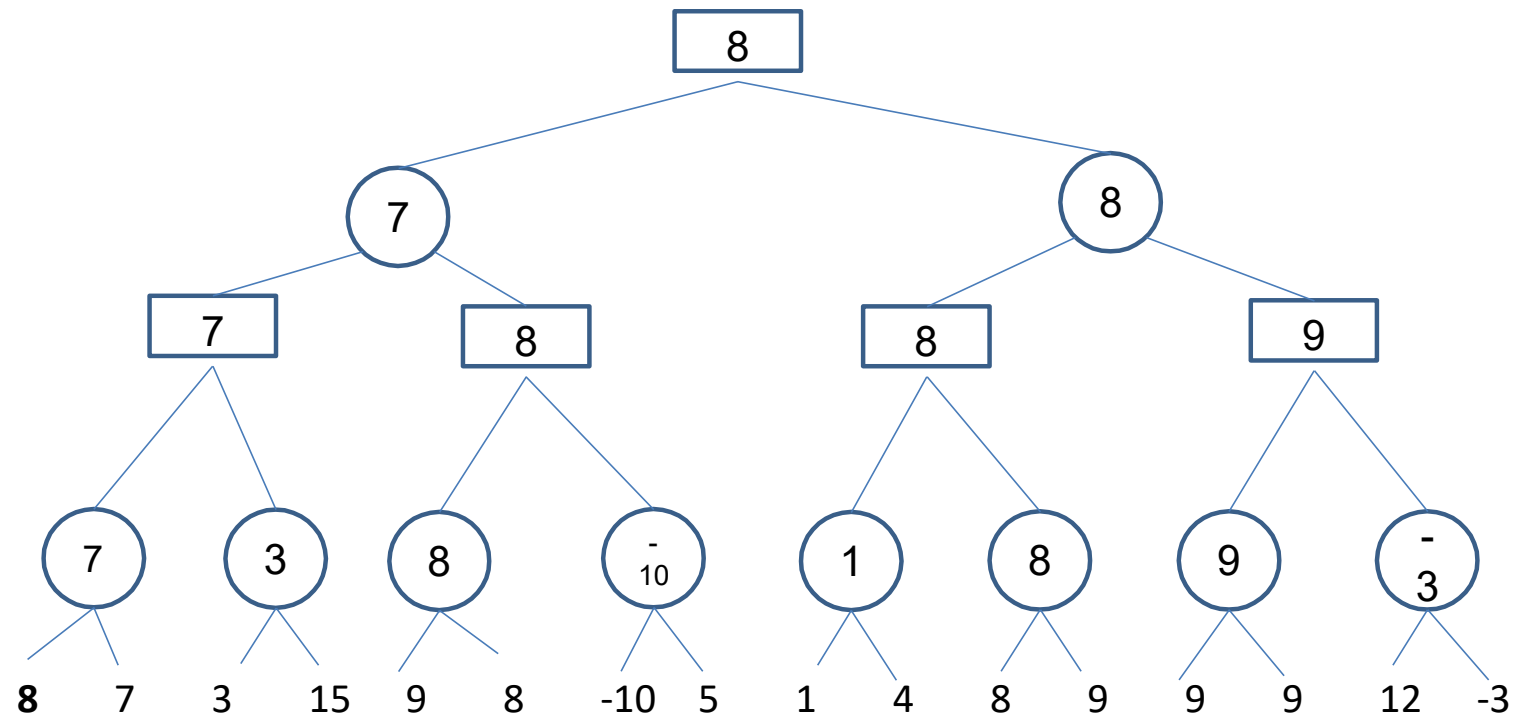


Multiplayer Game

Min-Max Algorithm – Example -1

Squares represent MAX nodes

Circles represent MIN nodes

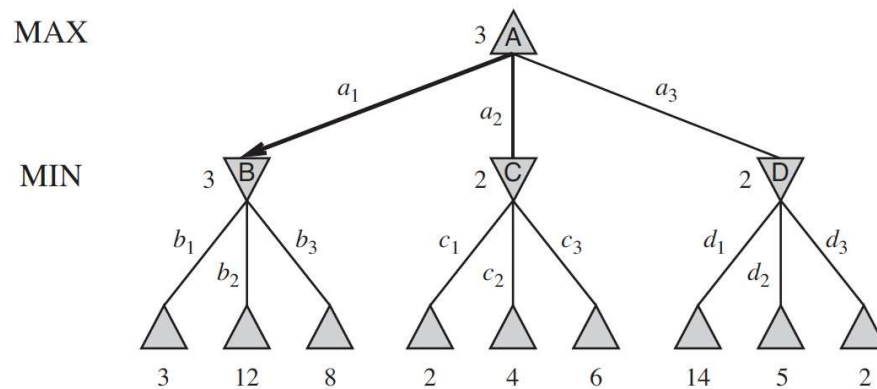
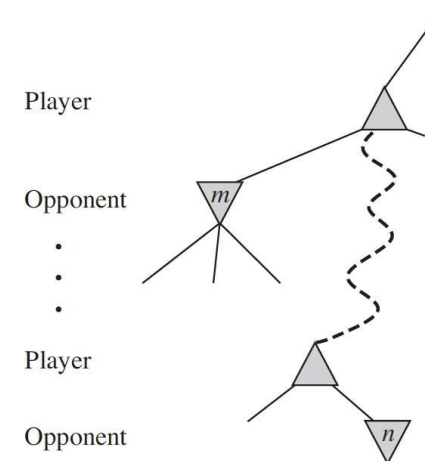




Alpha – beta Pruning

General Principle:

At a node n if a player has better option at the parent of n or further up, then n node will never be reached .Hence the entire subtree pruned

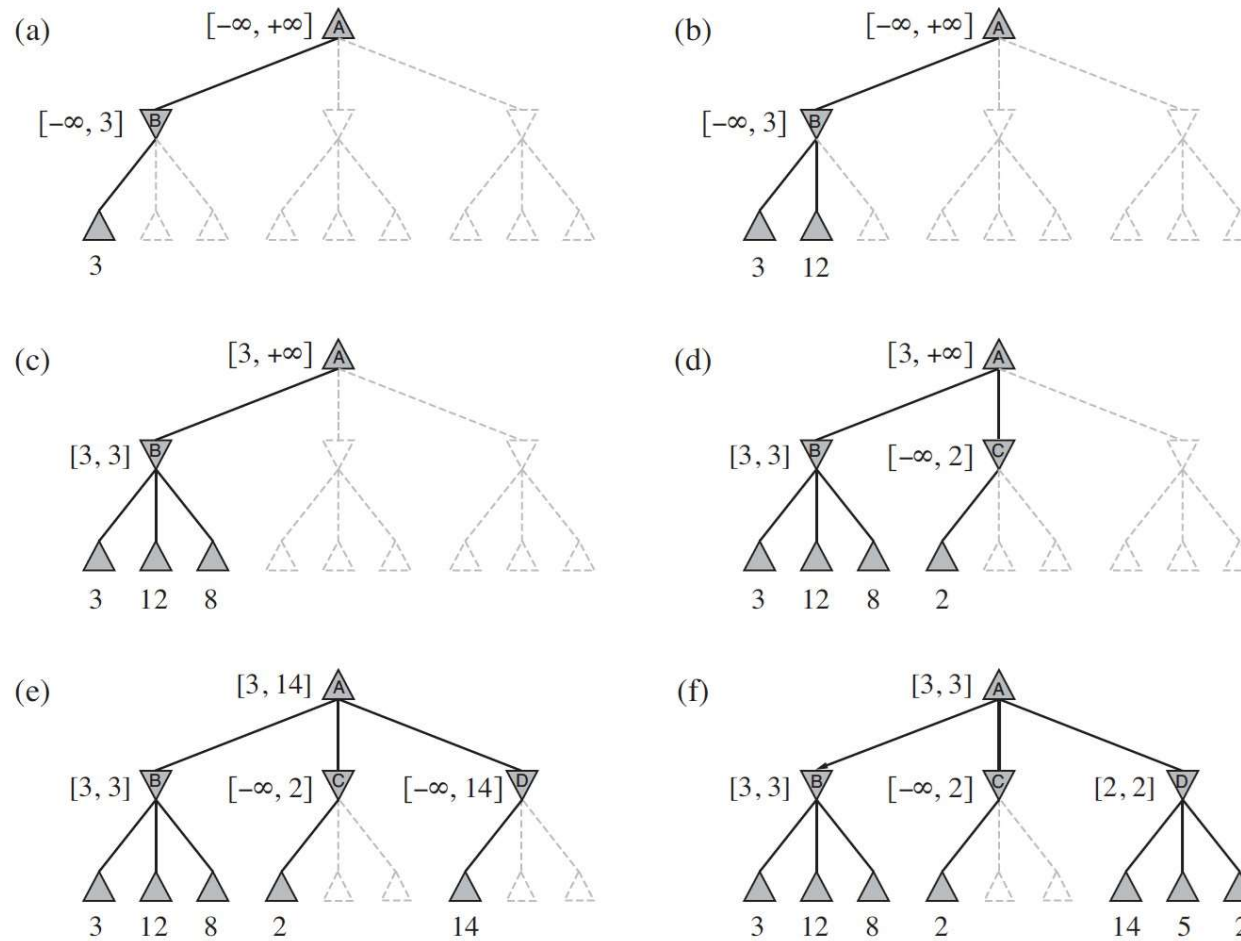


$$\begin{aligned}
 \text{MINIMAX}(\text{root}) &= \max(\min(3, 12, 8), \min(2, x, y), \min(14, 5, 2)) \\
 &= \max(3, \min(2, x, y), 2) \\
 &= \max(3, z, 2) \quad \text{where } z = \min(2, x, y) \leq 2 \\
 &= 3.
 \end{aligned}$$

Alpha Beta Pruning



Book Example



Min-Max Algorithm



Alpha beta Modifications

```
function ALPHA-BETA-SEARCH(state) returns an action
   $v \leftarrow \text{MAX-VALUE}(\text{state}, -\infty, +\infty)$ 
  return the action in ACTIONS(state) with value v
```

```
function MAX-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
   $v \leftarrow -\infty$ 
  for each a in ACTIONS(state) do
     $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$ 
    if  $v \geq \beta$  then return v
     $\alpha \leftarrow \text{MAX}(\alpha, v)$ 
  return v
```

```
function MIN-VALUE(state,  $\alpha$ ,  $\beta$ ) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
   $v \leftarrow +\infty$ 
  for each a in ACTIONS(state) do
     $v \leftarrow \text{MIN}(v, \text{MAX-VALUE}(\text{RESULT}(s, a), \alpha, \beta))$ 
    if  $v \leq \alpha$  then return v
     $\beta \leftarrow \text{MIN}(\beta, v)$ 
  return v
```

Is it possible to compute the minimax decision for a node without looking at every successor node?



Alpha – beta Pruning

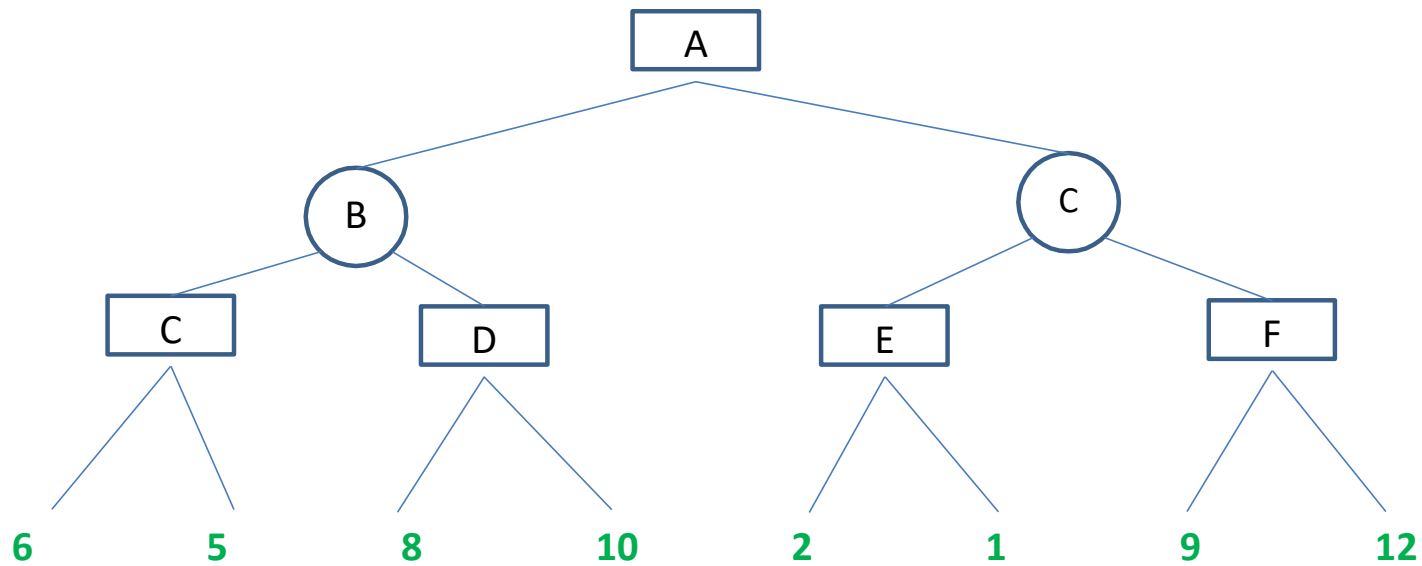
Steps in Alpha – Beta Pruning:

1. At root initialize $\alpha = -\infty$ and $\beta = +\infty$. This is to set the worst case boundary to start the algorithm which aims to increase α and decrease β as much as optimally possible
2. Navigate till the depth / limit specified and get the static evaluated numeric value.
3. For every value VAL being analyzed : Loop till all the leaf/terminal/specified state level nodes are analyzed & accounted for OR until **$\beta \leq \alpha$** .
 1. If the player is MAX :
 1. If $VAL > \alpha$
 2. then reset $\alpha = VAL$
 3. also check **if** $\beta \leq \alpha$ **then** tag the path as unpromising (TO BE AVOIDED) **and** prune the branch from game tree. Rest of their siblings are not considered for analysis
 2. Else if the player is MIN:
 1. If $VAL < \beta$
 2. then reset $\beta = VAL$
 3. also check **if** $\beta \leq \alpha$ **then** tag the path as unpromising (TO BE AVOIDED) **and** prune the branch from game tree. Rest of their siblings are not considered for analysis

Alpha Beta Pruning - Another Example



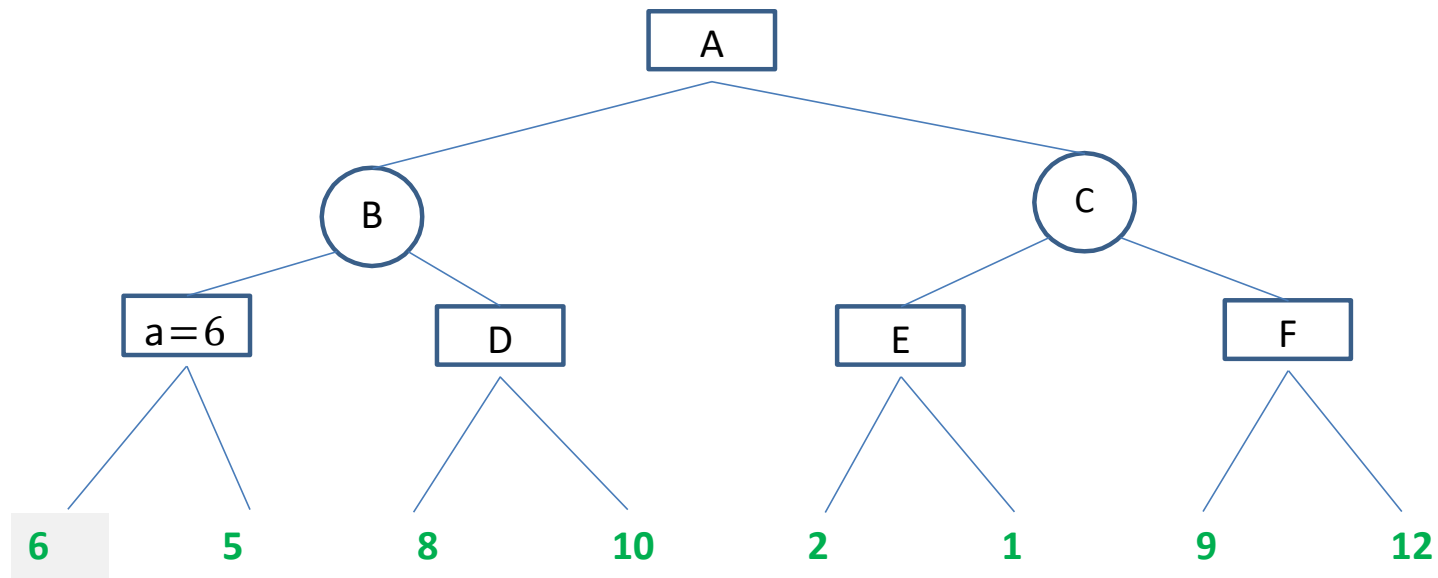
Idea –Pruning



Alpha Beta Pruning



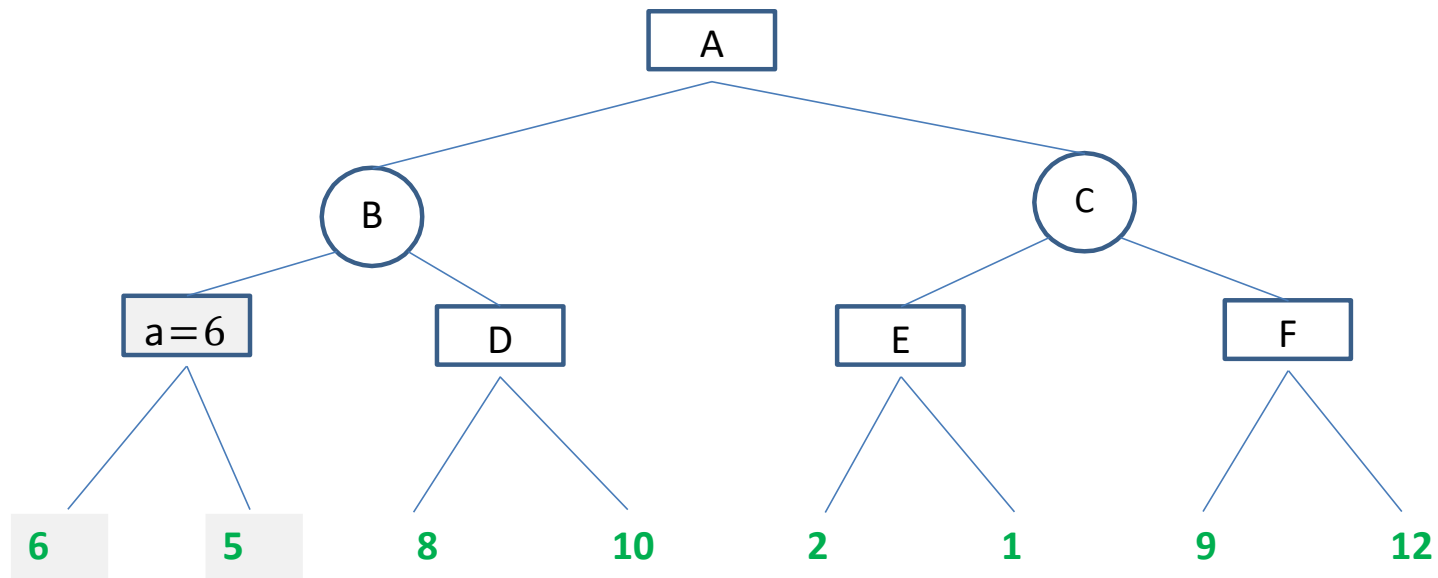
Idea –Pruning



Alpha Beta Pruning



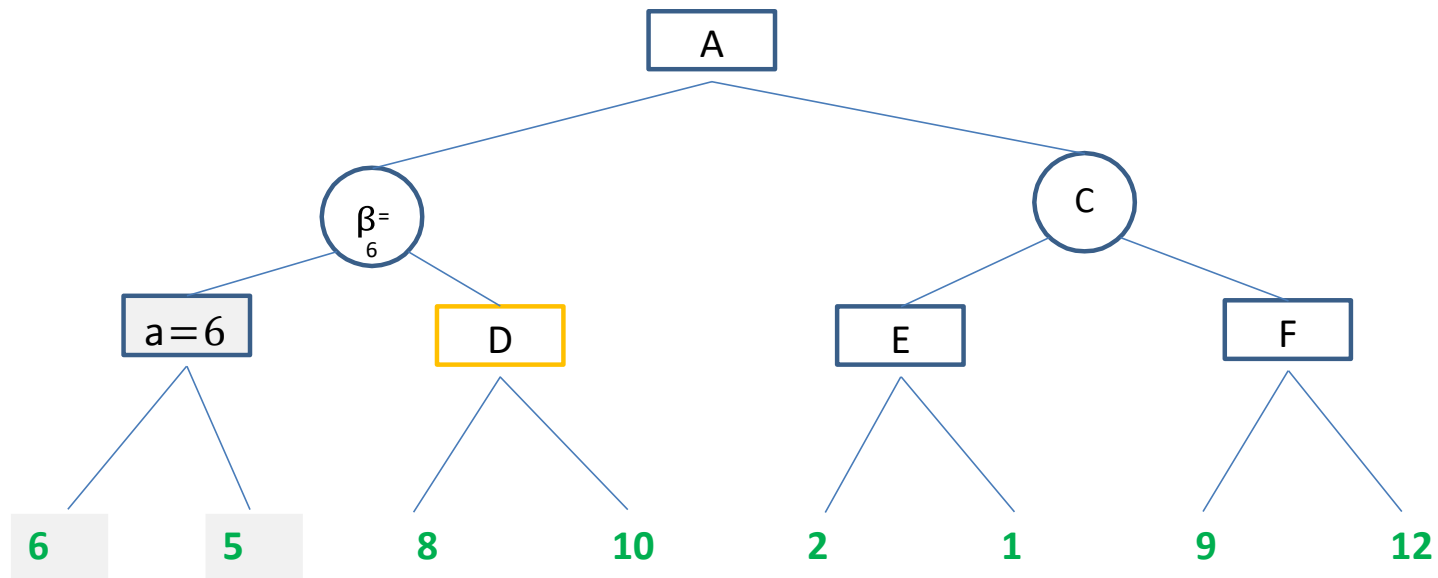
Idea –Pruning



Alpha Beta Pruning



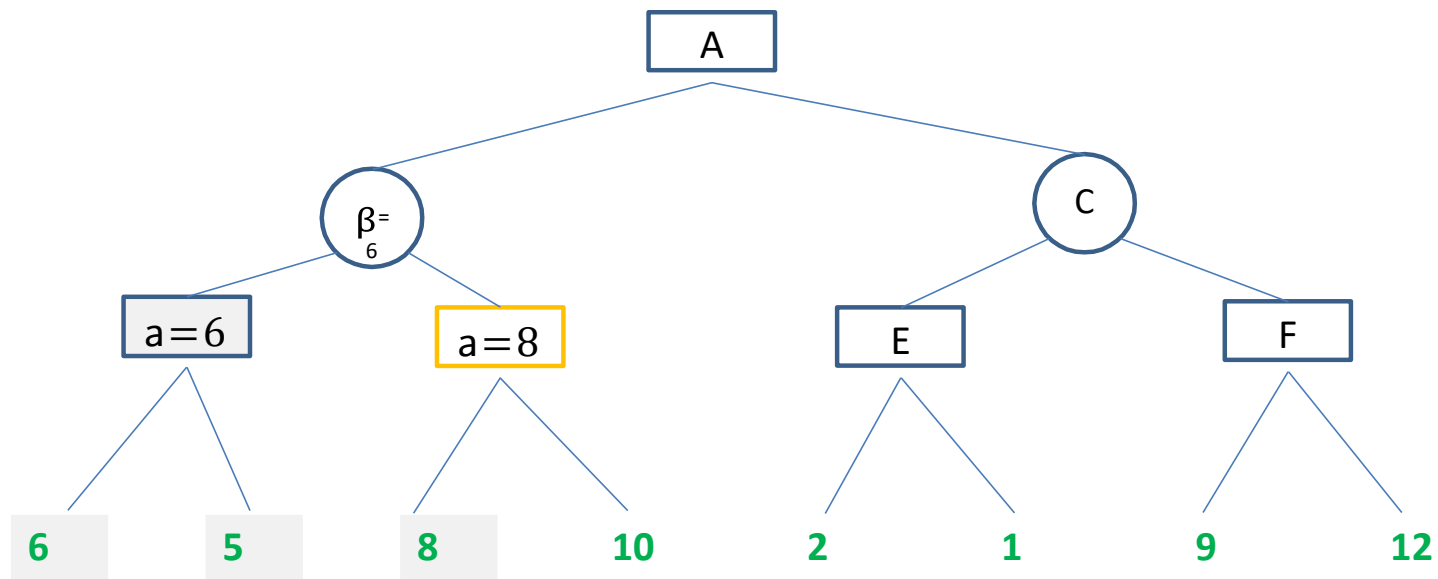
Idea –Pruning

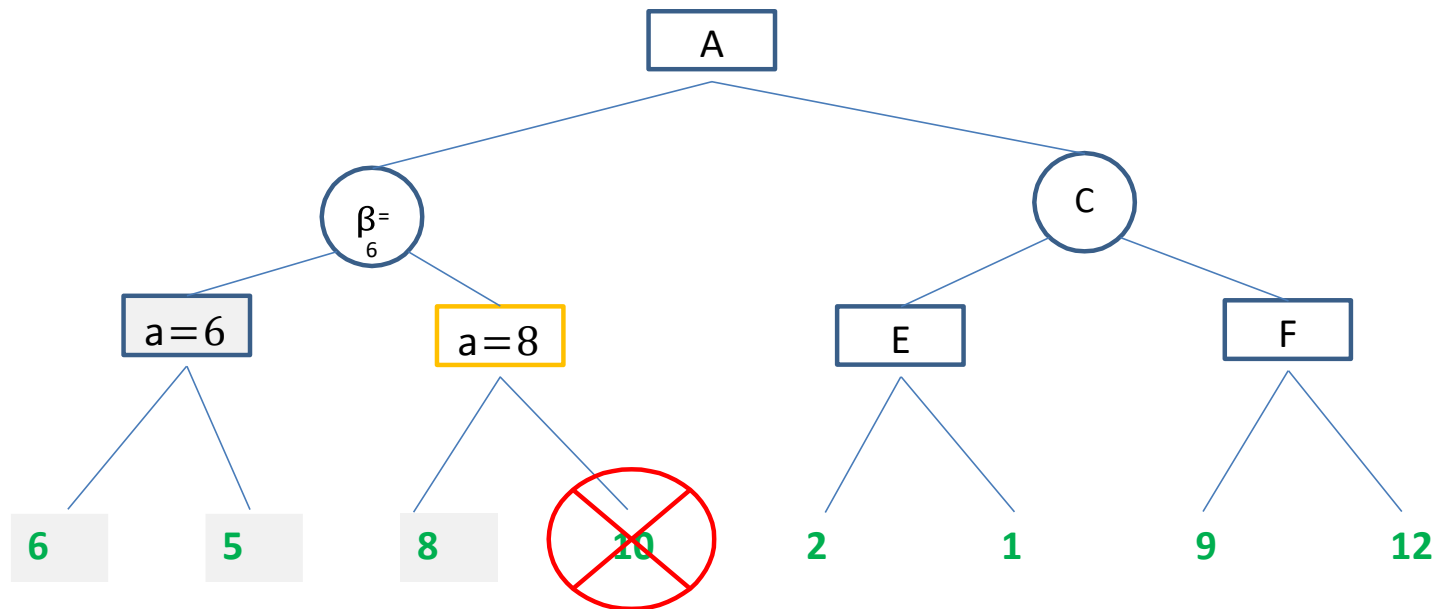


Alpha Beta Pruning



Idea – Alpha Pruning

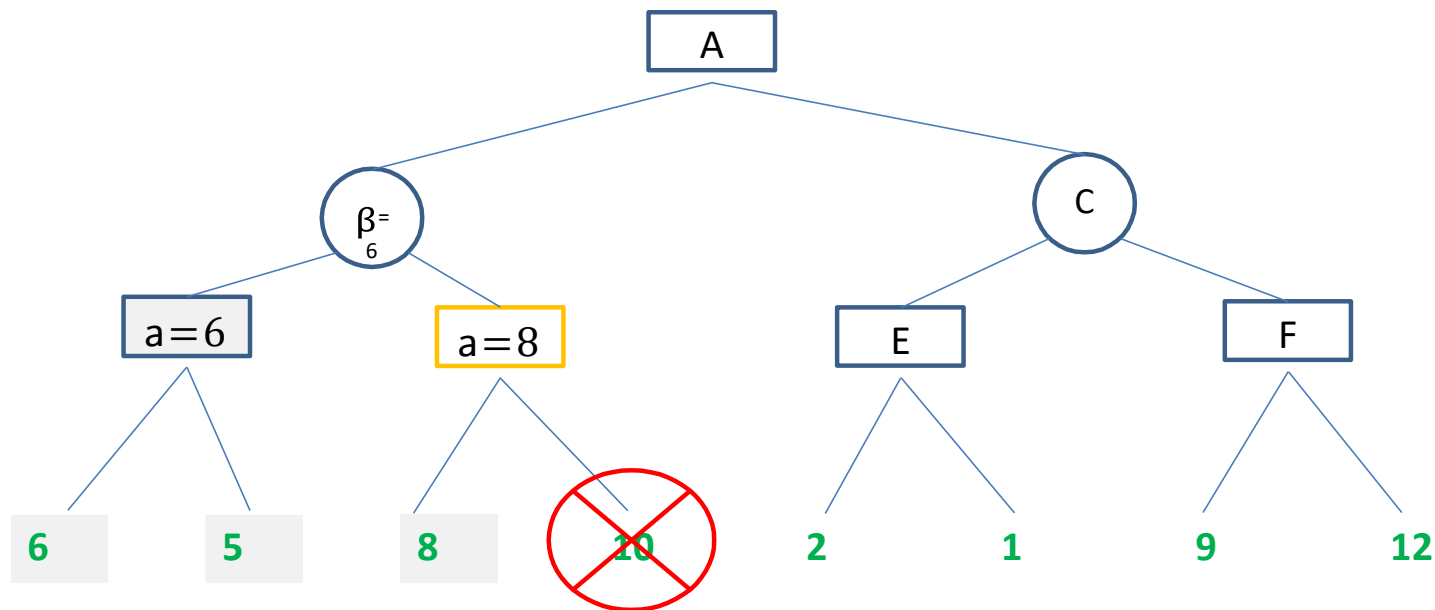




Alpha Beta Pruning



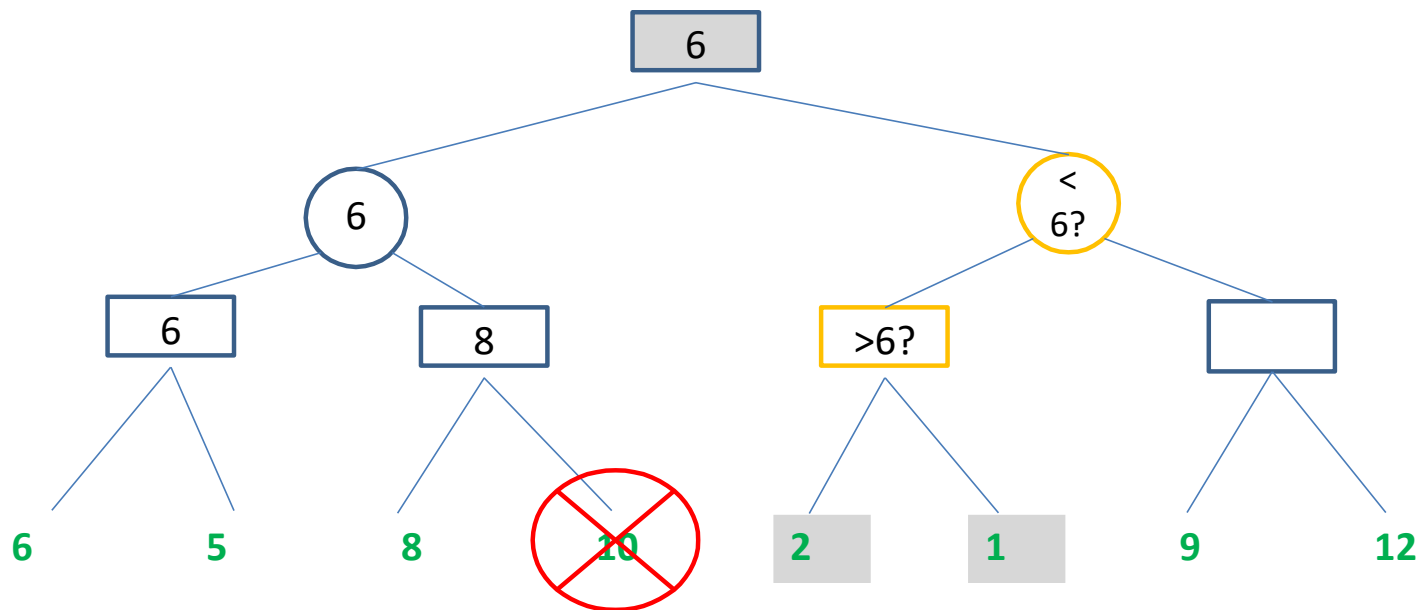
Idea –Pruning



Alpha Beta Pruning



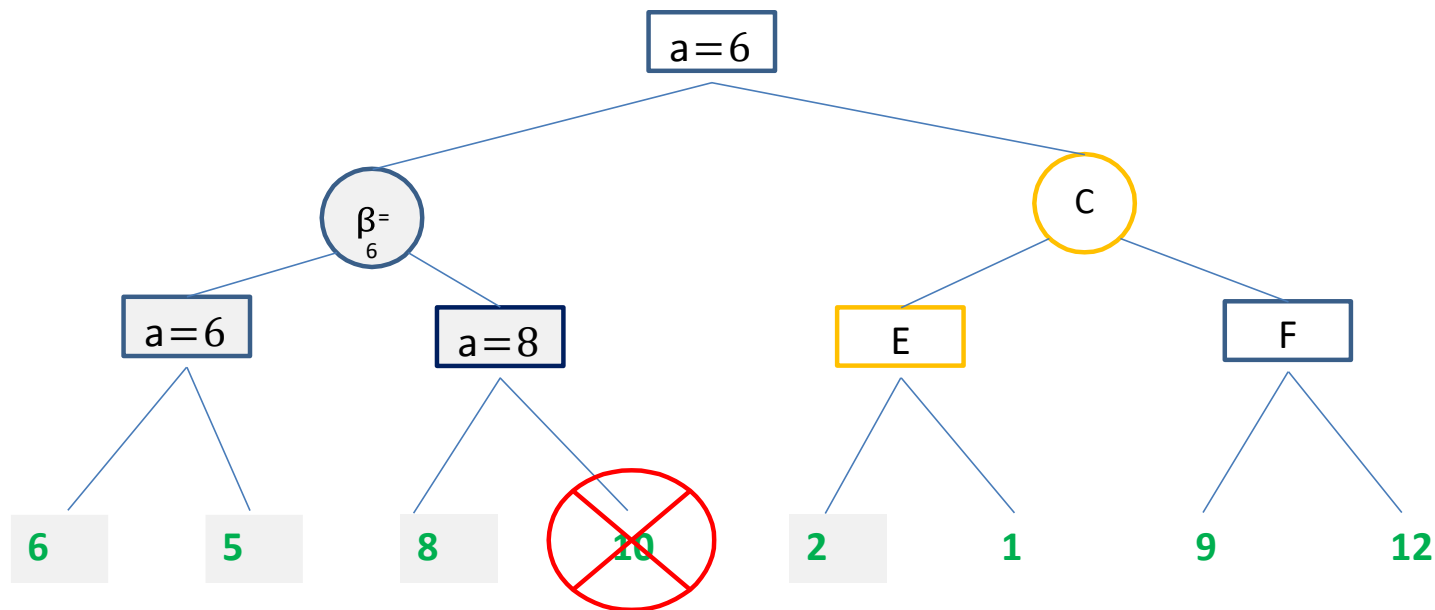
Idea –Pruning



Alpha Beta Pruning



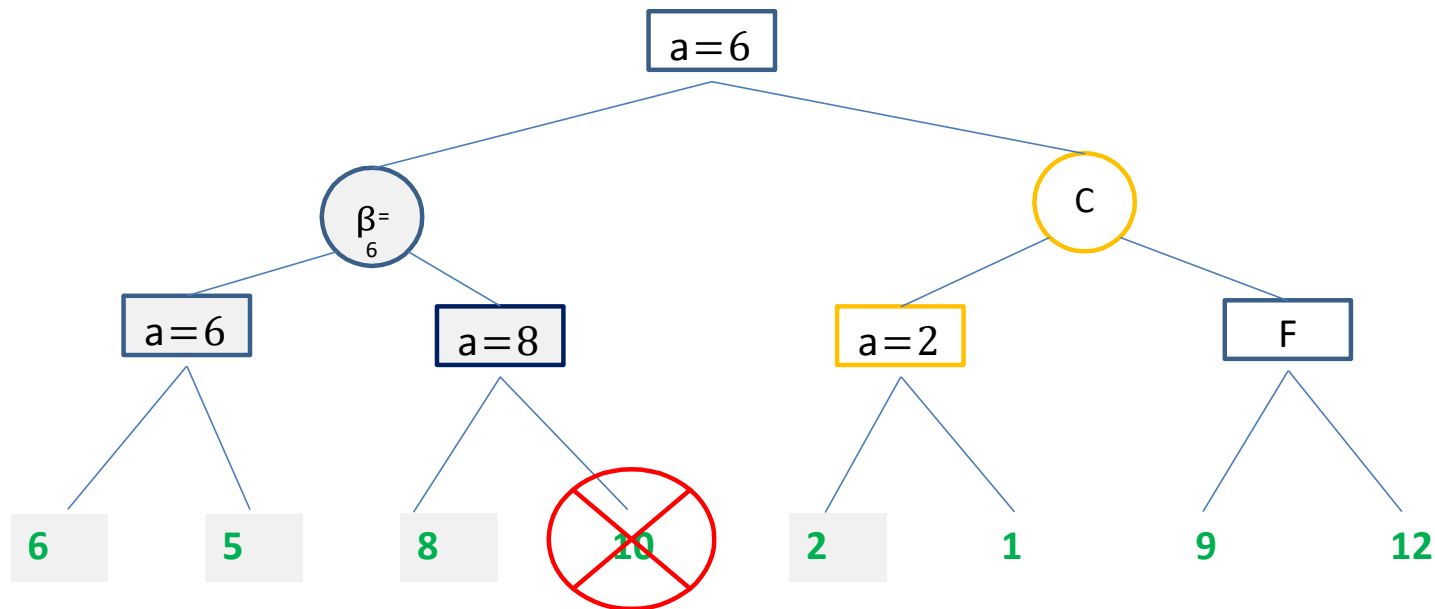
Idea –Pruning



Alpha Beta Pruning



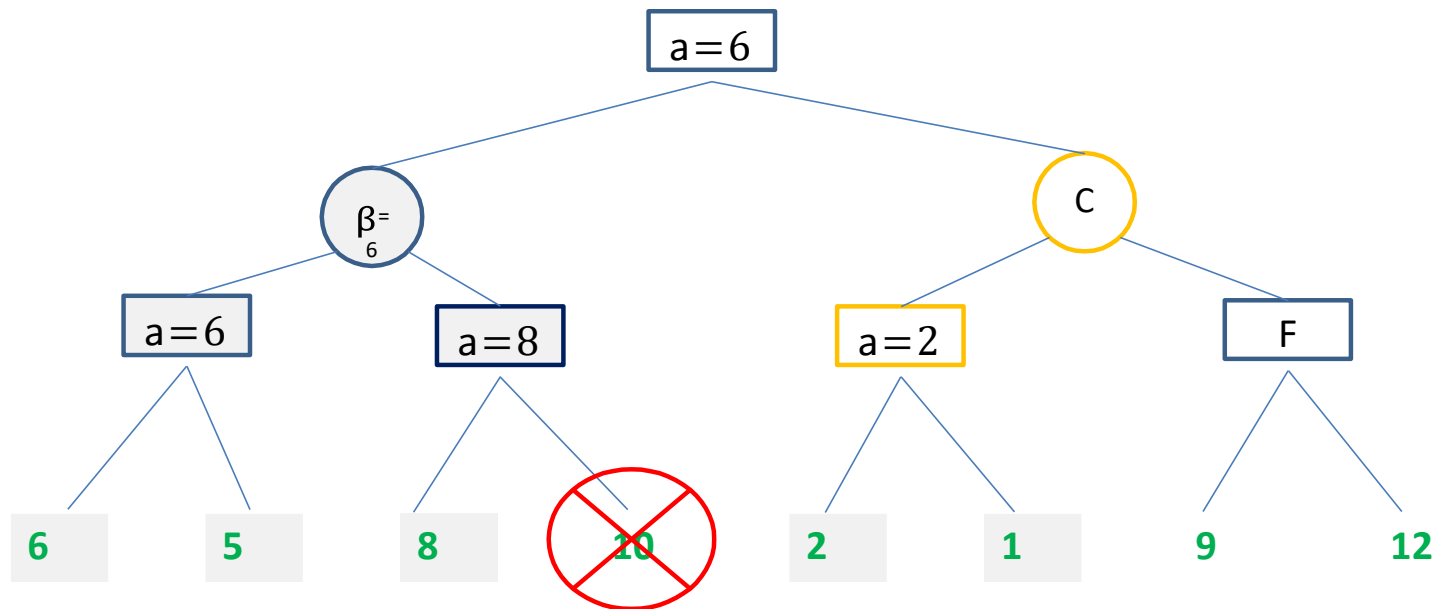
Idea –Pruning



Alpha Beta Pruning



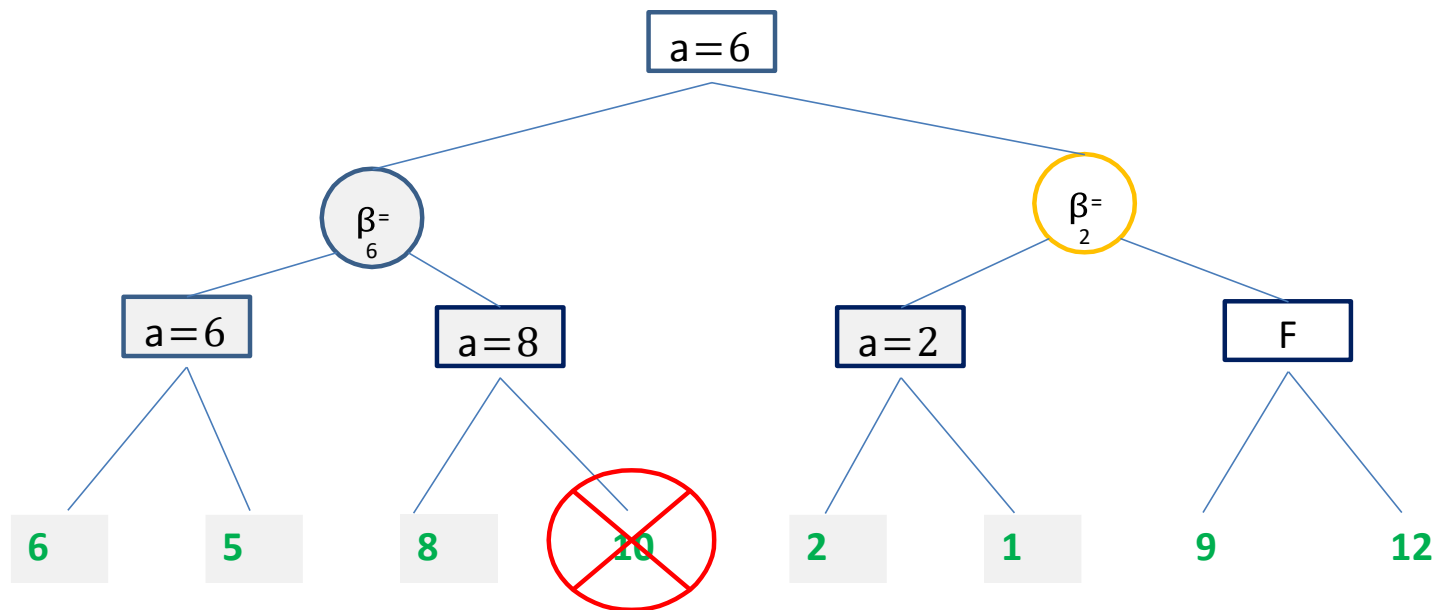
Idea –Pruning



Alpha Beta Pruning



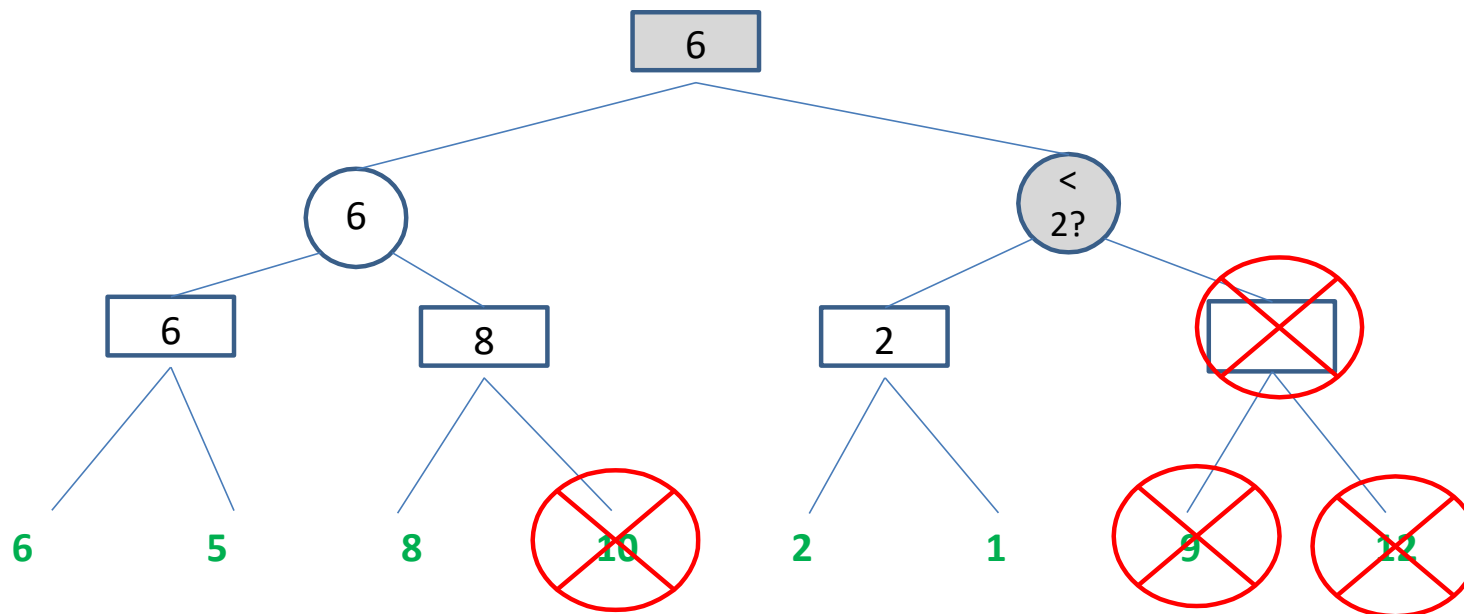
Idea –Pruning



Alpha Beta Pruning



Idea – Alpha Pruning



Alpha – Lower bound of Maximizer's value. Perceived value that Maximizer hopes to achieve against a competitive Minimizer

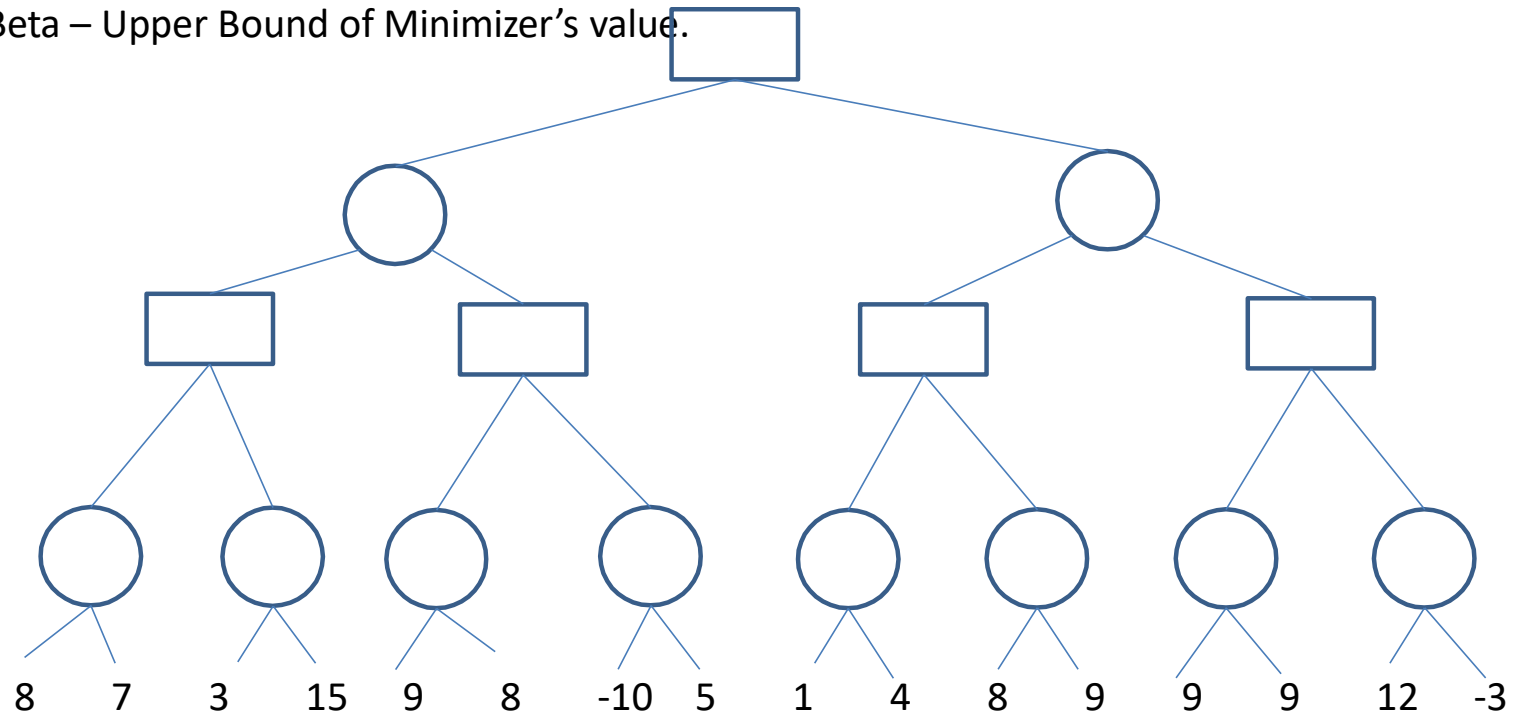
Alpha – beta Pruning – Example -4



Do for practice.

Alpha – Lower bound of Maximizer's value. Perceived value that Maximizer hopes to get with a competitive Minimizer

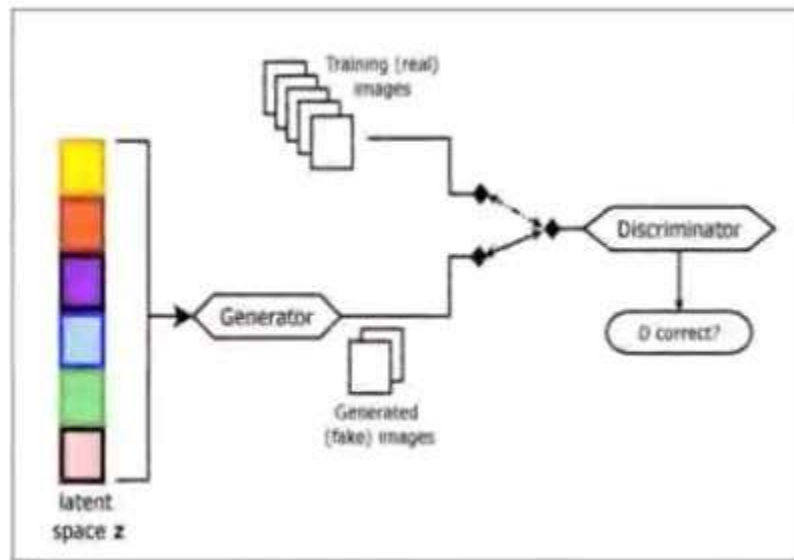
Beta – Upper Bound of Minimizer's value.





Game Playing (Interesting Case Studies)

Games in Image Processing



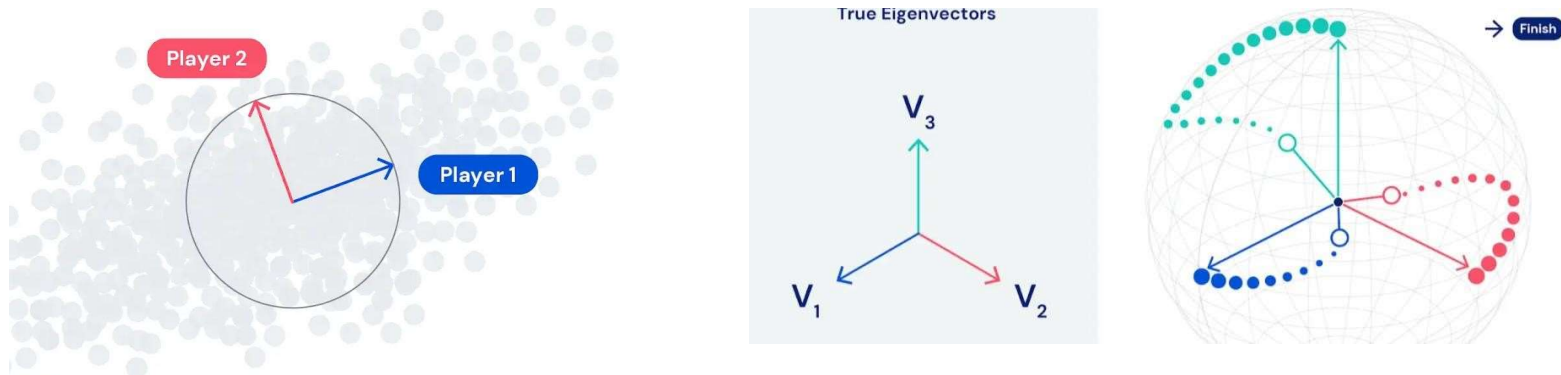
Source Credit:

[2019 - Analyzing and Improving the Image Quality of StyleGAN](#)

[Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, Timo Aila](#)

<https://thispersondoesnotexist.com/>

Games in Feature Engineering



Source Credit:

<https://deepmind.com/blog/article/EigenGame>

2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel



Games in Feature Engineering

$$\text{utility}(v_i | v_{j < i}) = \boxed{\text{Var}(v_i)} - \sum_{j < i} \boxed{\text{Align}(v_i, v_j)}$$

Source Credit:

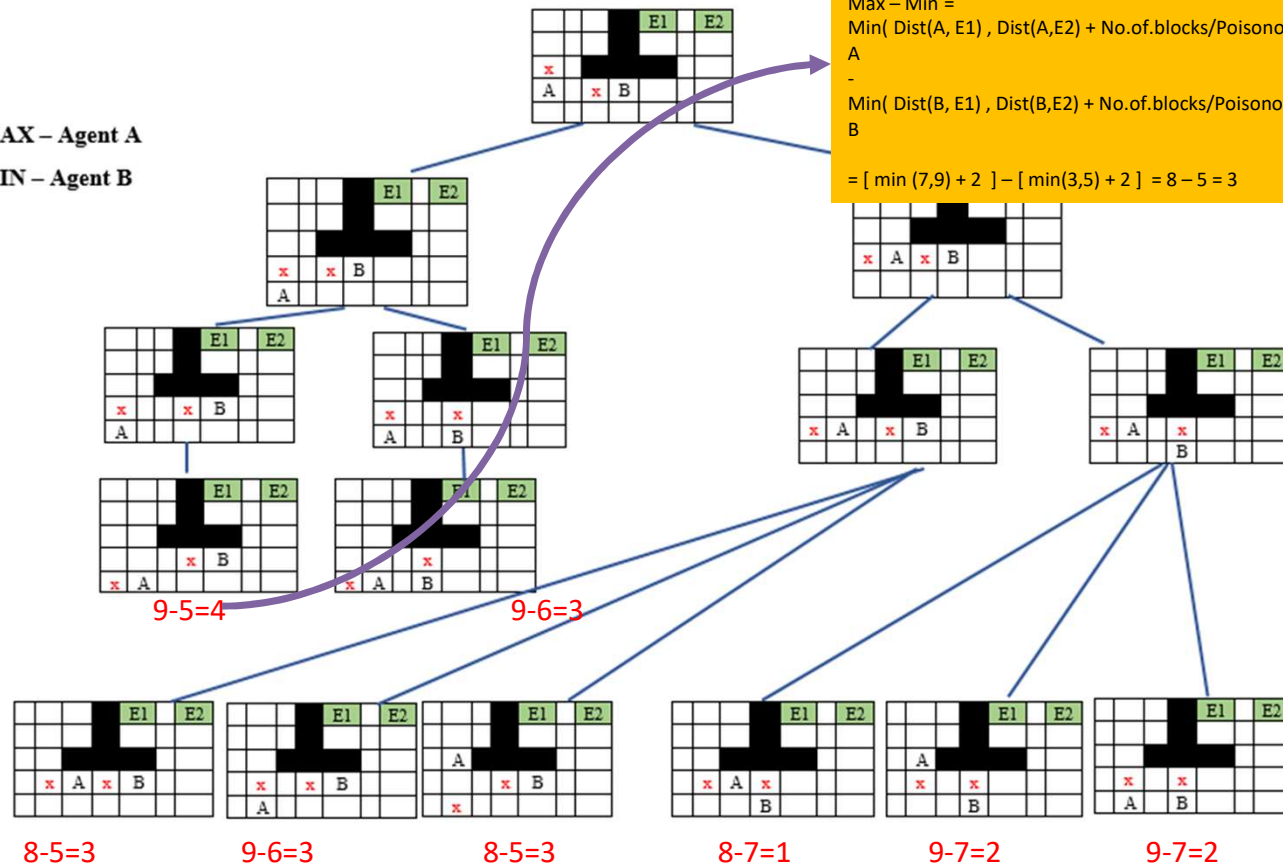
<https://deepmind.com/blog/article/EigenGame>

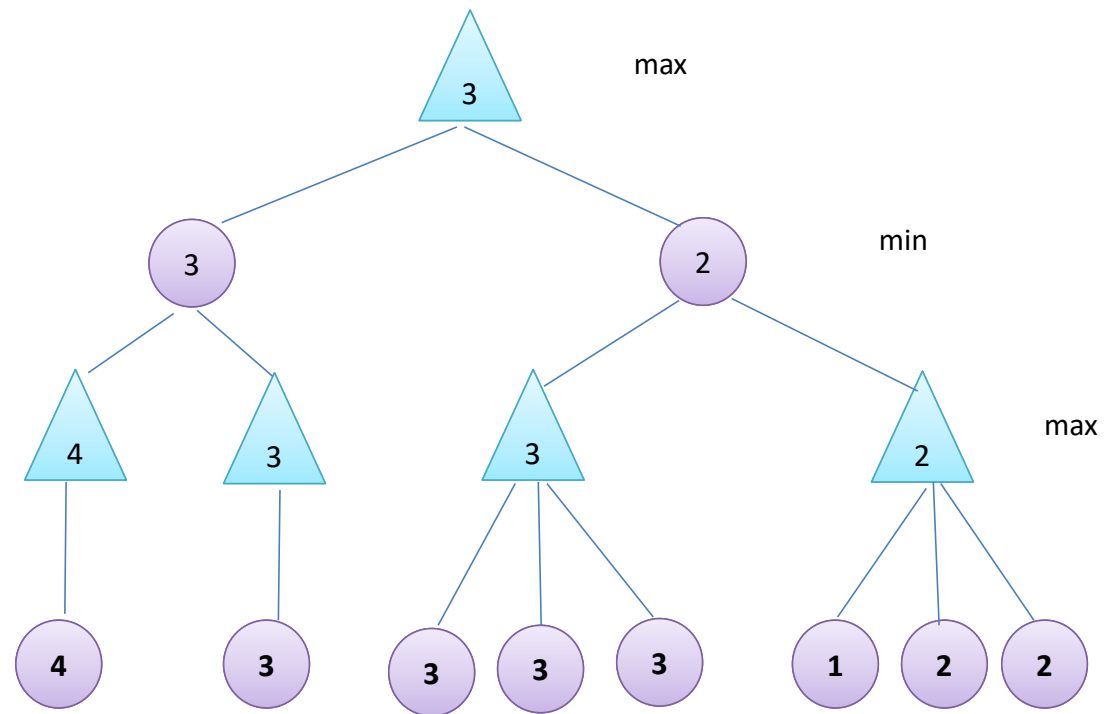
2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel

					E1		E2
x							
A		x	B				

Penalty = Number of unsafe cells (blockage+ poisonous squares) with 4 degree of freedom(Up, Down, Right, Left) in the immediate neighborhood of the Player "Z's" position.

MAX – Agent A
MIN – Agent B







Required Reading: AIMA - Chapter # 5.1, 5.2, 5.3

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials