Artificial & Computational Intelligence

AIML CLZG557

M6: Reasoning over time

Dr. Sudheer Reddy

BITS Pilani
Pilani Campus

Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time

M7 Ethics in AI

Module 6:
Reasoning over time

Reasoning Over Time

A. Time and Uncertainty

B. Inference in temporal models

C. Overview of HMM

D. Learning HMM Parameters using EM Algorithm

E. Applications of HMM

Learning Objective

- Understand the relationship between Time & Uncertainty
- Recognize the transition model of Markov Model
- Relate to the application of the Hidden Markov Model

Sequential Decision Problems & Markov Decision Process

Markov Decision Process

Sequential Problem | Partial Observability | Belief System

Modelling sequences of random events and transitions between states over time is known as Markov chain

Agents in partially observable environment should keep a track of current state to the extent allowed by sensors

E.g., Robot moving in a new maze

Agent maintains a **belief state** representing the current possible world states

Transition Model / Probability Matrix :
Using belief state and transition model,
the agent can how the world might evolve in
next time step. To capture the degree of belief
we will use Probability Theory. We model the change in world
using a variable for each aspect of state and
at each point in time.
Current state depends only finite number of previous states.

$$\begin{array}{cc}
\begin{array}{cc} \mathbf{C} & \mathbf{M} \end{array} & \\
\left|\begin{array}{cc} 0.40 & 0.20 \\ 0.60 & 0.80 \end{array}\right| & \begin{array}{c} \mathbf{C} \\ \mathbf{M} \end{array}
\end{array}$$

Markov Decision Process

Time - Uncertainty | States - Observations

Static World: Each random variable would have a single fixed value

E.g., Diagnosing a broken car

Dynamic World: The state information keeps changing with time

E.g., treating a diabetic patient, tracking the location of robot, tracking economic activity of a nation

Time slices: World is observed in time slices. Each slice has a set of random variables, some observable and some not.

Assumption: We will assume same subset of random variables are observable in each time slice

E_t - set of observable random variables at time t

X_t - set of unobserved random variables at time t

$$
\begin{array}{cc}
\begin{array}{cc} C & M \end{array} & \\
\left|\begin{array}{cc} 0.40 & 0.20 \\ 0.60 & 0.80 \end{array}\right| & \begin{array}{c} C \\ M \end{array}
\end{array}
$$

Markov Process

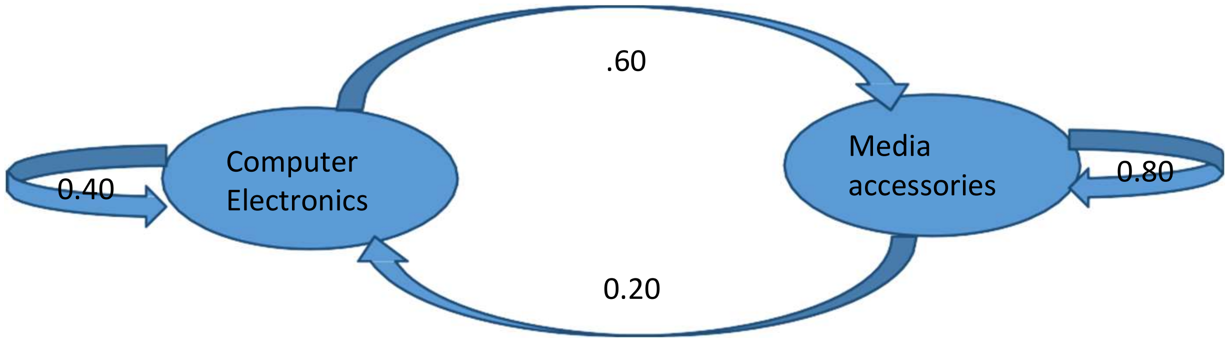
States | Observations | Assumptions

Modelling sequences of random events and transitions between states over time is known as Markov chain

Transition Model / Probability Matrix :
Current state depends only finite number of previous states.

$$
\begin{array}{cc}
\begin{array}{cc} C & M \end{array} & \\
\left|\begin{array}{cc} 0.40 & 0.20 \\ 0.60 & 0.80 \end{array}\right| & \begin{array}{c} C \\ M \end{array}
\end{array}
$$

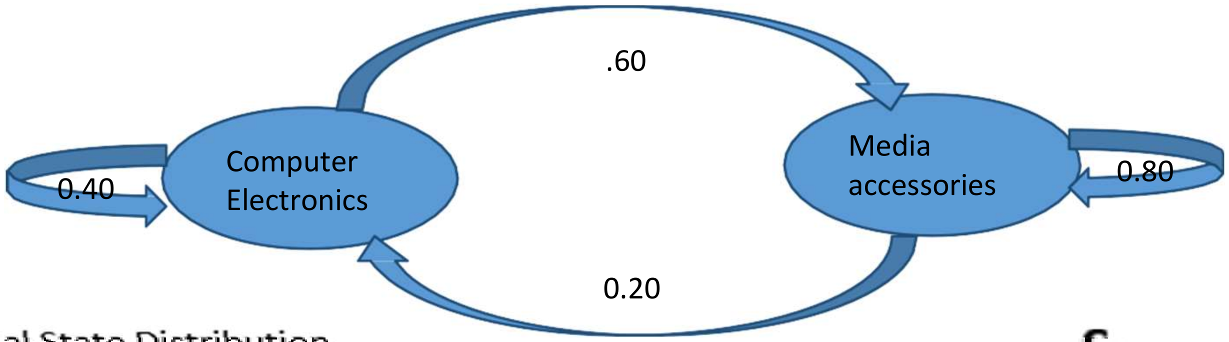
Markov Model- Example 1



Transition Model

$$
\begin{array}{cc}
\begin{array}{cc} C & M \end{array} & \\
\left|\begin{array}{cc} 0.40 & 0.20 \\ 0.60 & 0.80 \end{array}\right| & \begin{array}{c} C \\ M \end{array}
\end{array}
$$

Markov Model



Current State: Initial State Distribution

$$\begin{array}{c|c|c} & 1 & C \\ & 0 & M \end{array}$$

$$\begin{array}{cc} C & M \\ \end{array}$$

$$\begin{array}{c|cc|c} & C & M & \\ \hline & 0.40 & 0.20 & C \\ & 0.60 & 0.80 & M \end{array}$$

Next State : Likely to buy Media accessories on next visit

$$\begin{array}{c|c|c} & 0.40 & C \\ & 0.60 & M \end{array}$$

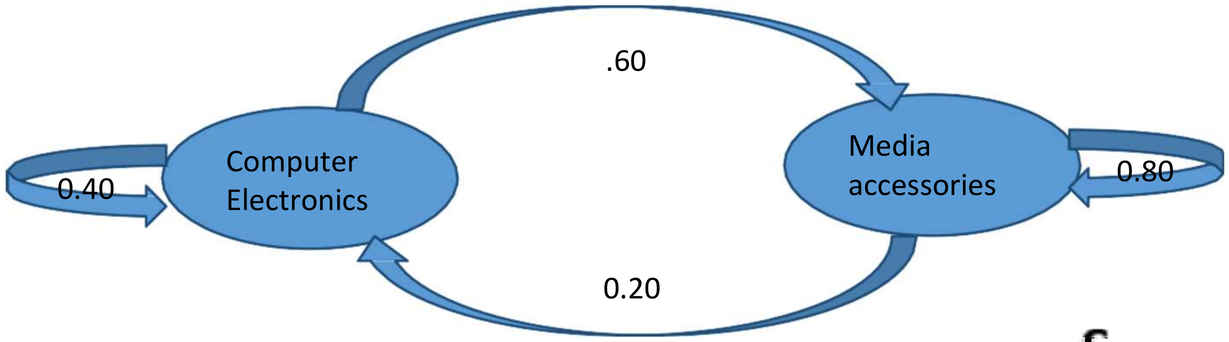
Next State : Likely to buy Media accessories on next visit

$$\begin{array}{c|c|c} & 0.28 & C \\ & 0.72 & M \end{array}$$

Inference in temporal Models

Markov Model

Inference Type 1



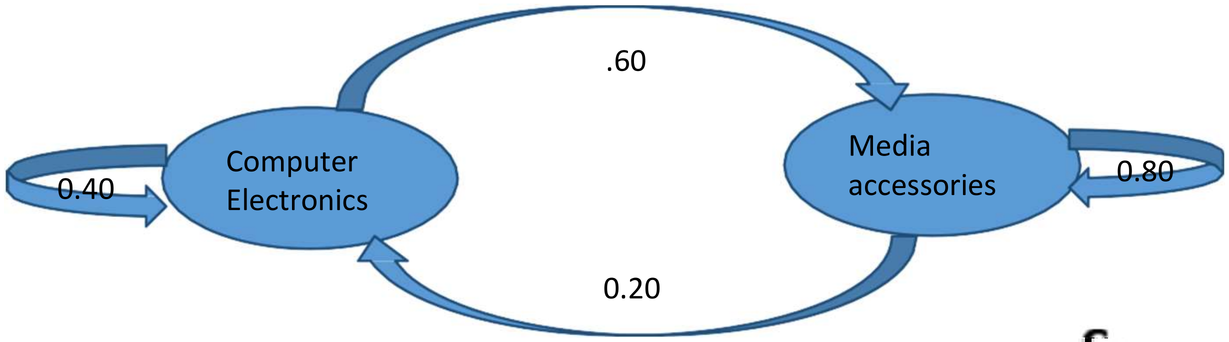
$$\begin{array}{ccc} C & M & \\ \left|\begin{matrix} 0.40 & 0.20 \\ 0.60 & 0.80 \end{matrix}\right| & & \begin{matrix} C \\ M \end{matrix} \end{array}$$

What is the probability that the purchasing behaviour of the customer is in below sequential order only?Initial Probability Matrix is P(C) = 1 , P(M) = 0
(Computer, Media, Media, Computer)

<u>Apply Bayes chain rule:</u>
P(Computer, Media, Media, Computer) = P(C) * P(M|C) * P(M|M) * P(C|M) = 0.096

Markov Model

Inference Type 2



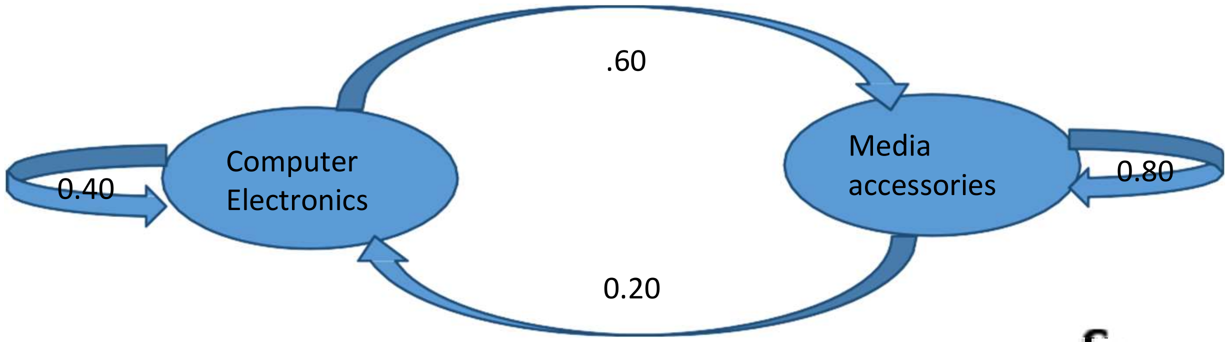
$$
\begin{array}{cc|c}
C & M & \\
\hline
0.40 & 0.20 & C \\
0.60 & 0.80 & M
\end{array}
$$

What is the probability that the customer who purchased Media accessories will keep coming back to purchase media accessories in the next 2 consecutive visits only?

<u>Derive Initial prob values & Apply Bayes chain rule on the pattern exhibited:</u>
Initial Probability Matrix is P(M) = 1 , P(C) = 0

Markov Model

Inference Type 2



$$\begin{array}{cc}
\begin{array}{cc} C & M \end{array} & \\
\left|\begin{array}{cc} 0.40 & 0.20 \\ 0.60 & 0.80 \end{array}\right| & \begin{array}{c} C \\ M \end{array}
\end{array}$$

What is the probability that the customer who purchased Media accessories will keep coming back to purchase media accessories in the next 2 consecutive visits only?

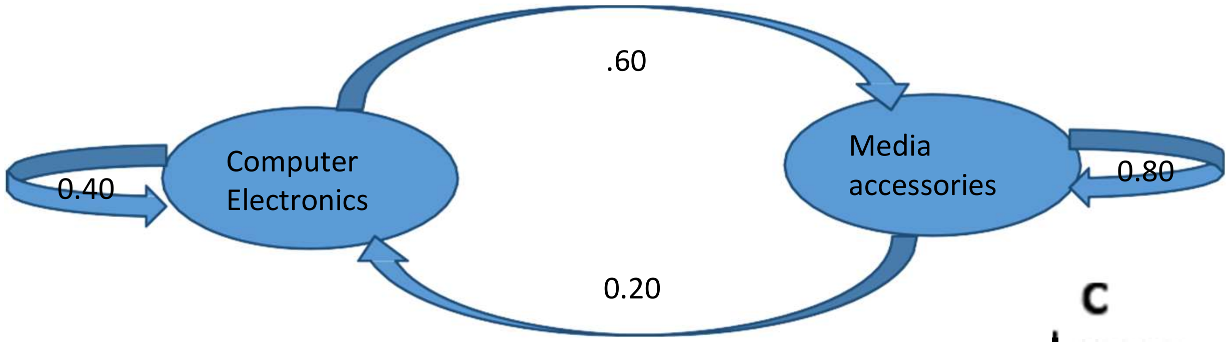
<u>Derive Initial prob values & Apply Bayes chain rule on the pattern exhibited:</u>

Initial Probability Matrix is P(M) = 1 , P(C) = 0

P(Media, Media, Media, Computer) = P(M) * P(M|M) * P(M|M) * P(C|M) = 0.128

Markov Model

Inference Type 3



$$\begin{array}{cc|c} C & M & \\ \left|\begin{matrix} 0.40 \\ 0.60 \end{matrix}\right. & \left.\begin{matrix} 0.20 \\ 0.80 \end{matrix}\right| & \begin{matrix} C \\ M \end{matrix} \end{array}$$

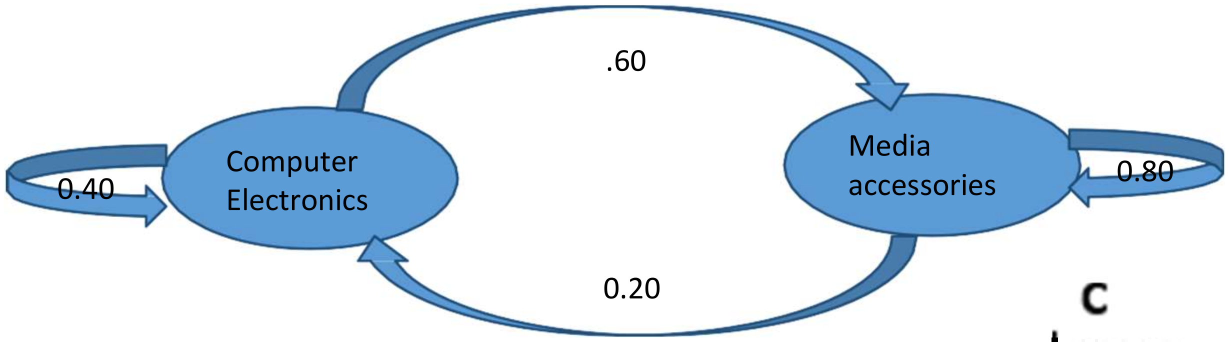
Given the evidence that the customer walked into the store and bought a computer electronics, find the expected purchase pattern in the next 3 visits

<u>Derive Initial prob values & Apply Bayes chain rule and reverse predict the combination on the most likely pattern:</u>

Initial Probability Matrix is P(C) = 1 , P(M) = 0

Markov Model

Inference Type 3



$$\begin{array}{ccc} C & M & \\ \left|\begin{matrix} 0.40 & 0.20 \\ 0.60 & 0.80 \end{matrix}\right| & & \begin{matrix} C \\ M \end{matrix} \end{array}$$

Given the evidence that the customer walked into the store and bought a computer electronics, find the expected purchase pattern in the next 3 visits

<u>Derive Initial prob values & Apply Bayes chain rule and reverse predict the combination on the most likely pattern:</u>

Initial Probability Matrix is P(C) = 1 , P(M) = 0

P(Computer , X, Y, Z) = P(Computer) * P(X|Computer) * P(Y|X) * P(Z|X) = 1*0.6*0.8*0.8 → Produces max values

Ans : Pattern = (Computer, Media, Media, Media)

Hidden Markov Process

States | Observations | Assumptions

Modelling sequences of random events and transitions between states over time is known as Morkov chain

Hidden Markov Process models events as the state sequences that are not directly observable but only be approximated from the sequence of observations produced by the system

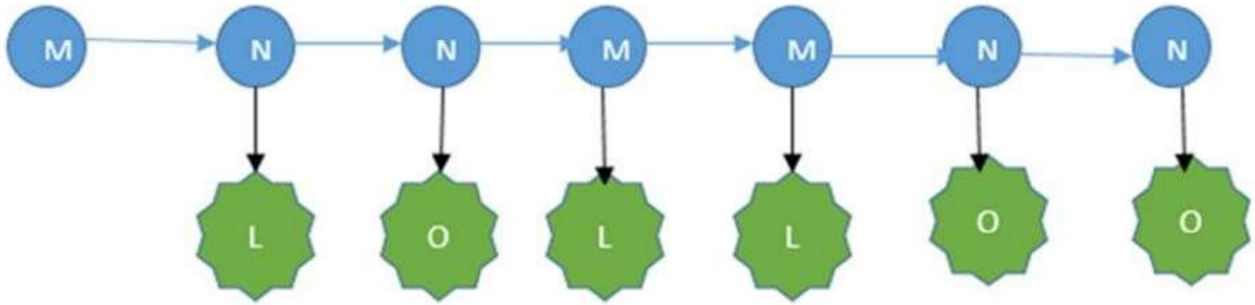
Transition Model / Probability Matrix :
Current state depends only finite number of previous states. :

Evidence / Sensor Model/ Emission Probability Matrix :
Current Evidence or Observation depends Current State of the world. Given the Current State Knowledge of the world, observation doesn't depend on history:

Hidden Markov Model

States | Observations | Assumptions

Time Slice **(t)**	0	1	2	3	4	5	6	$P(O_t \mid O_{t-1})$
Observed Evidence **(O_t/ E_t)**	-	Late	OnTime	Late	Late	Ontime	Ontime	……
Unobserved State **(U_t /X_t/ Q_t)**	Meeting	No Meeting	No Meeting	Meeting	Meeting	No Meeting	No Meeting	……



Transition Model / Probability Matrix

$P(U_{t-1}$ = **No Meeting)**	$P(U_{t-1}$ = **Meeting)**	←Previous
0.5	0.67	$P(U_t$ = **No Meeting)**
0.5	0.33	$P(U_t$ = **Meeting)**

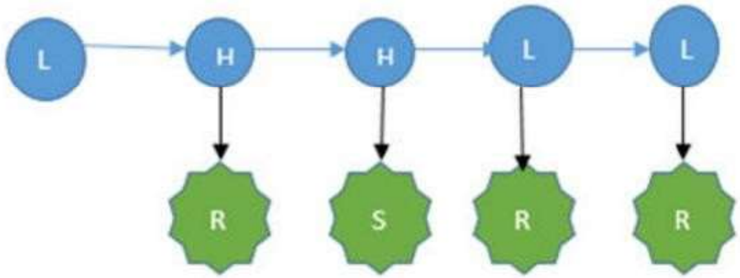
Evidence / Sensor Model/ Emission Probability Matrix

$P(U_t$ = **No Meeting)**	$P(U_t$ = **Meeting)**	←Unobserved Evidence v
0.9	0.3	$P(O_t$ = **OnTime)**
0.1	0.7	$P(O_t$ = **Late)**

Hidden Morkov Model

States | Observations | Assumptions

Time Slice (t)	0	1	2	3	4	....	$P(O_t \mid O_{t-1}, O_{t-2})$
Observed Evidence (O_t)	-	Rainy	Sunny	Rainy	Rainy		
Unobserved State(U_t)	Low Pressure	High Pressure	High Pressure	Low Pressure	Low Pressure		



Transition Model / Probability Matrix

$P(U_{t-2} = LP, U_{t-1} = HP)$	$P(U_{t-2} = HP, U_{t-1} = HP)$	$P(U_{t-2} = HP, U_{t-1} = LP)$	$P(U_{t-2} = LP, U_{t-1} = LP)$	←Previous
0.2	0.40	0.85	0.5	$P(U_t = LP)$
0.8	0.60	0.15	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Hidden Markov Model

Filtering

$P(L_3 \mid R\text{-}S\text{-}R\text{-}R)$

$P(X_t \mid E_{1 \ldots t})$

Prediction

$P(L_3 \mid R\text{-}S)$

$P(X_{t+k} \mid E_{1 \ldots t})$

Smoothing

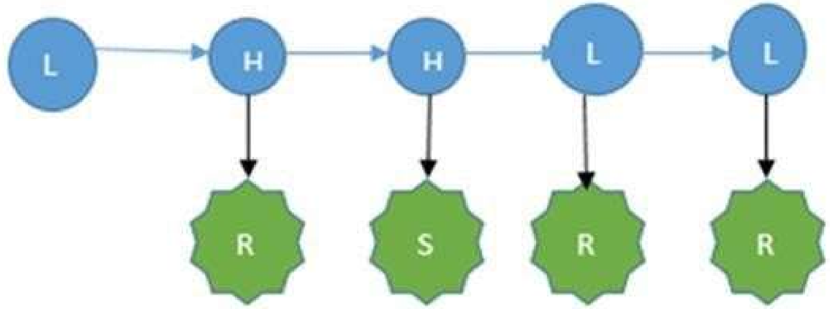
$P(H_2 \mid R\text{-}S\text{-}R\text{-}R)$ $P(X_{k,\ o>k>t} \mid E_{1 \ldots t})$

Most Likely Explanation

$P(H\text{-}H\text{-}L\text{-}L \mid R\text{-}S\text{-}R\text{-}R)$

$\text{argmax}\ X_{1 \ldots t} : P(X_{1 \ldots t} \mid E_{1 \ldots t})$

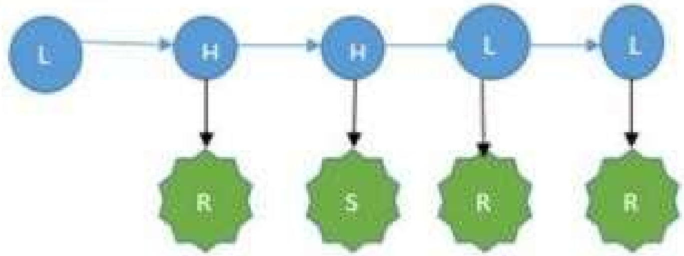
In your Text book another example for these inferences is explained "Task of predicting the weather condition by a security personnel sitting in an underground secret installation by observing the state of an employee who either umbrella or don't" Kindly check it and work it out as additional practice



Hidden Markov Model

Sequence Evaluation : Likely hood Computation : Forward Algorithm

Find the probability of occurrence of
this weather sequence observation: **S-S-R**



Intuition: $P(E_{1\ldots t}) = \sum_{i=1}^{N} P(E_{1\ldots t} \mid X_{1\ldots t}) * P(X_{1\ldots t}) =$

$= \sum_{i=1}^{N} \prod_{j=1}^{t} P(Ej \mid Xj) * P(Xj \mid Xj_{-1})$

Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

P(SSR)

$$= \sum_{X} P(SSR, X) = \sum_{X} P(SSR, X_1X_2X_3)$$

$$= \sum_{X} P(R, X_3, S, X_2, S, X_1) = \sum_{X} P(R \mid X_3) * P(X_3 \mid X_2) * P(S \mid X_2) * P(X_2 \mid X_1) * P(S \mid X_1) * P(X_1 \mid X_0)$$

$$= \sum_{X} P(R \mid X_3) * P(S \mid X_2) * P(S \mid X_1) * P(X_3 \mid X_2) * P(X_2 \mid X_1) * P(X_1 \mid X_0)$$

$$= \sum_{X} \prod_{j=1}^{t} P(Ej \mid X_j) * P(Xj \mid X_{j-1})$$

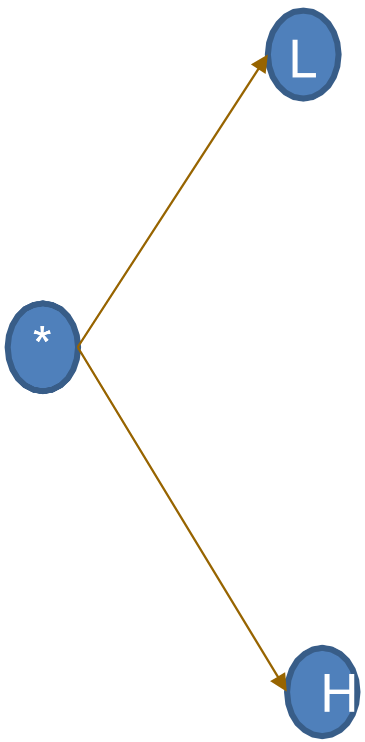
Hidden Markov Model

Forward Propagation Algorithm

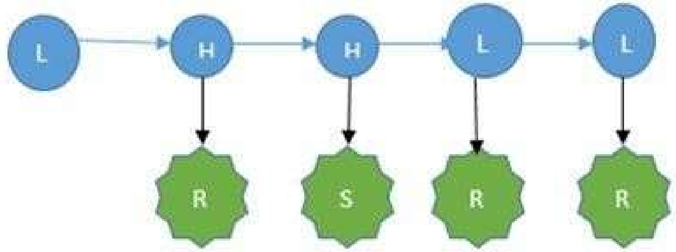
Find the probability of occurrence of this whether sequence observation: **S-S-R**

<u>Initialization Phase:</u>

P(L)*P(S|L) = 0.5*0.2 = 0.1



P(H)*P(S|H) = 0.5*0.6 = 0.3



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

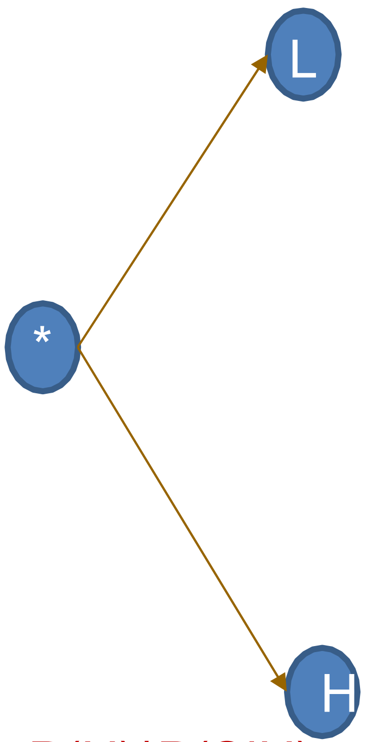
Hidden Markov Model

Forward Propagation Algorithm

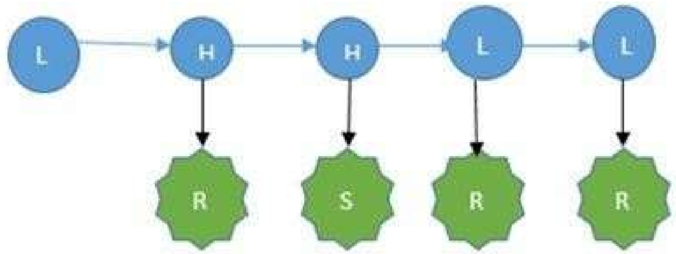
Find the probability of occurrence of this whether sequence observation: **S-S-R**

Initialization Phase:

P(L)*P(S|L) = 0.5*0.2 = 0.1 -> 0.25



P(H)*P(S|H) = 0.5*0.6 = 0.3 -> 0.75



Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

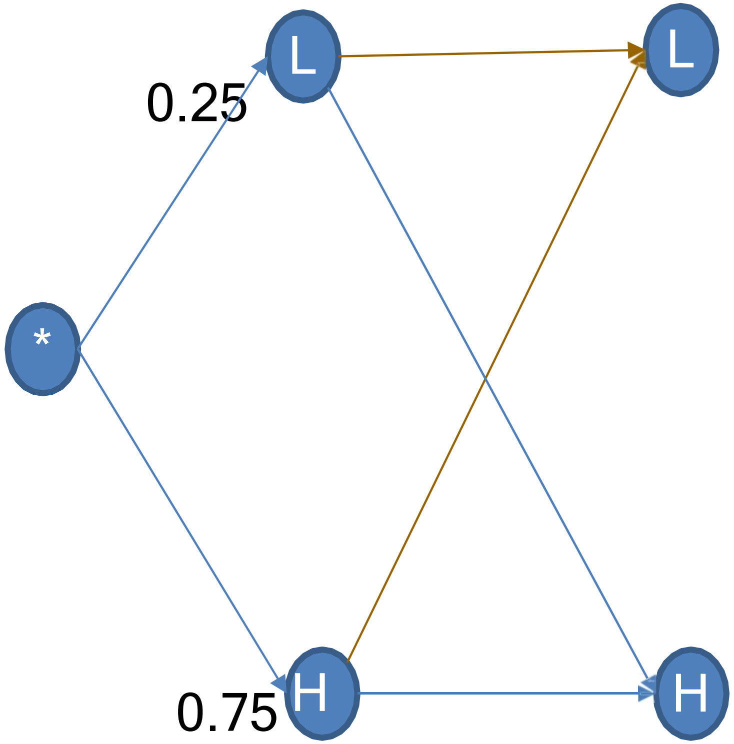
$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

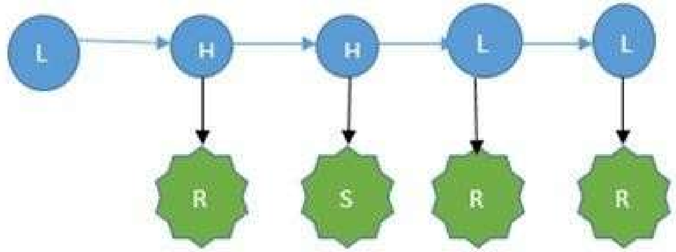
Hidden Markov Model

Forward Propagation Algorithm : S-S-R

P(L)*P(L|L)*P(S|L) = 0.25*0.5*0.2 = **0.025**

P(H)*P(L|H)*P(S|L) = 0.75*0.2*0.2 = **0.03**





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = \text{Rainy})$
0.2	0.6	$P(E_t = \text{Sunny})$

P(L)*P(H|L)*P(S|H) = 0.25*0.5*0.6 = **0.075**

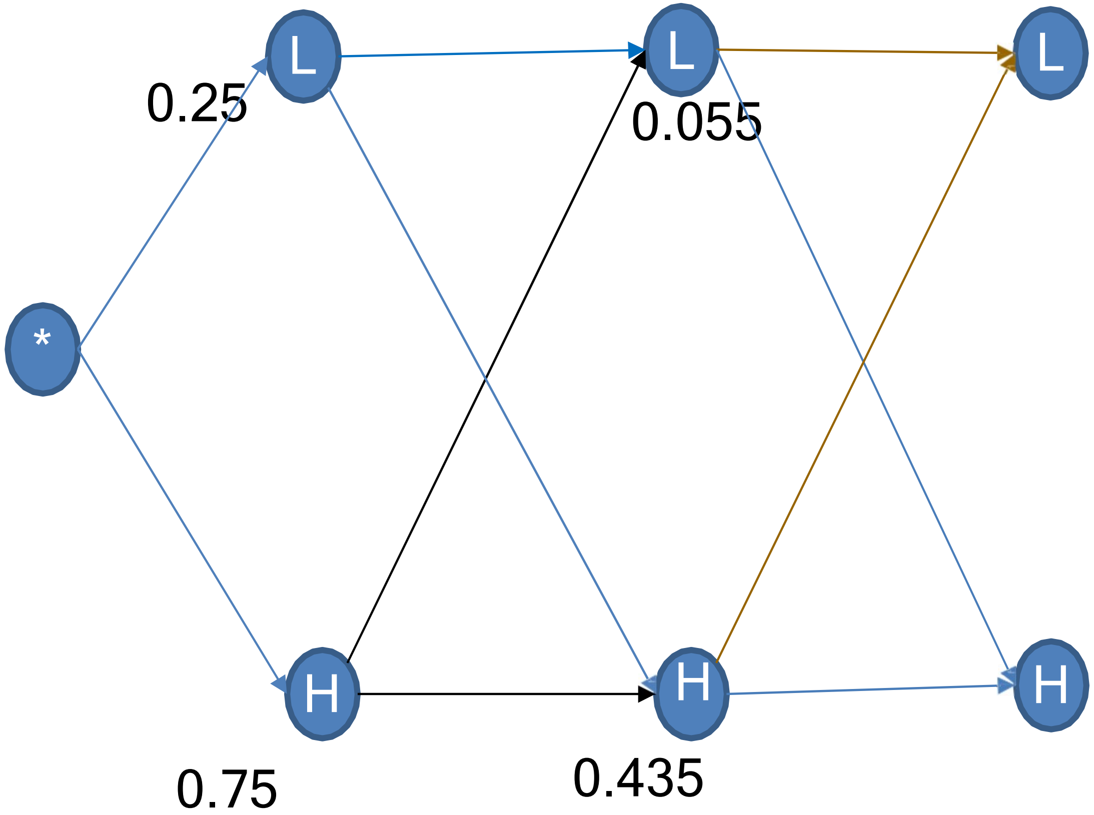
P(H)*P(H|H)*P(S|H) = 0.75*0.8*0.6 = **0.36**

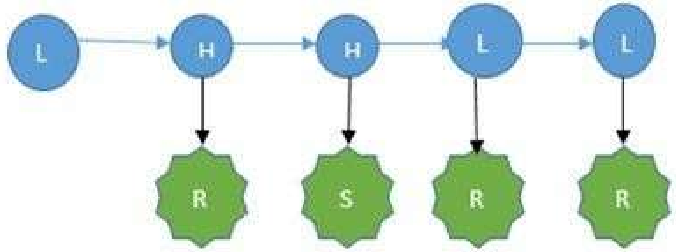
Hidden Markov Model

Forward Propagation Algorithm : S-S-R

P(L)*P(L|L)*P(R|L) = 0.055*0.5*0.8 = **0.022**

P(H)*P(L|H)*P(R|L) = 0.435*0.2*0.8 = **0.0696**





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = \text{Rainy})$
0.2	0.6	$P(E_t = \text{Sunny})$

P(L)*P(H|L)*P(R|H) = 0.055*0.5*0.4 = **0.011**

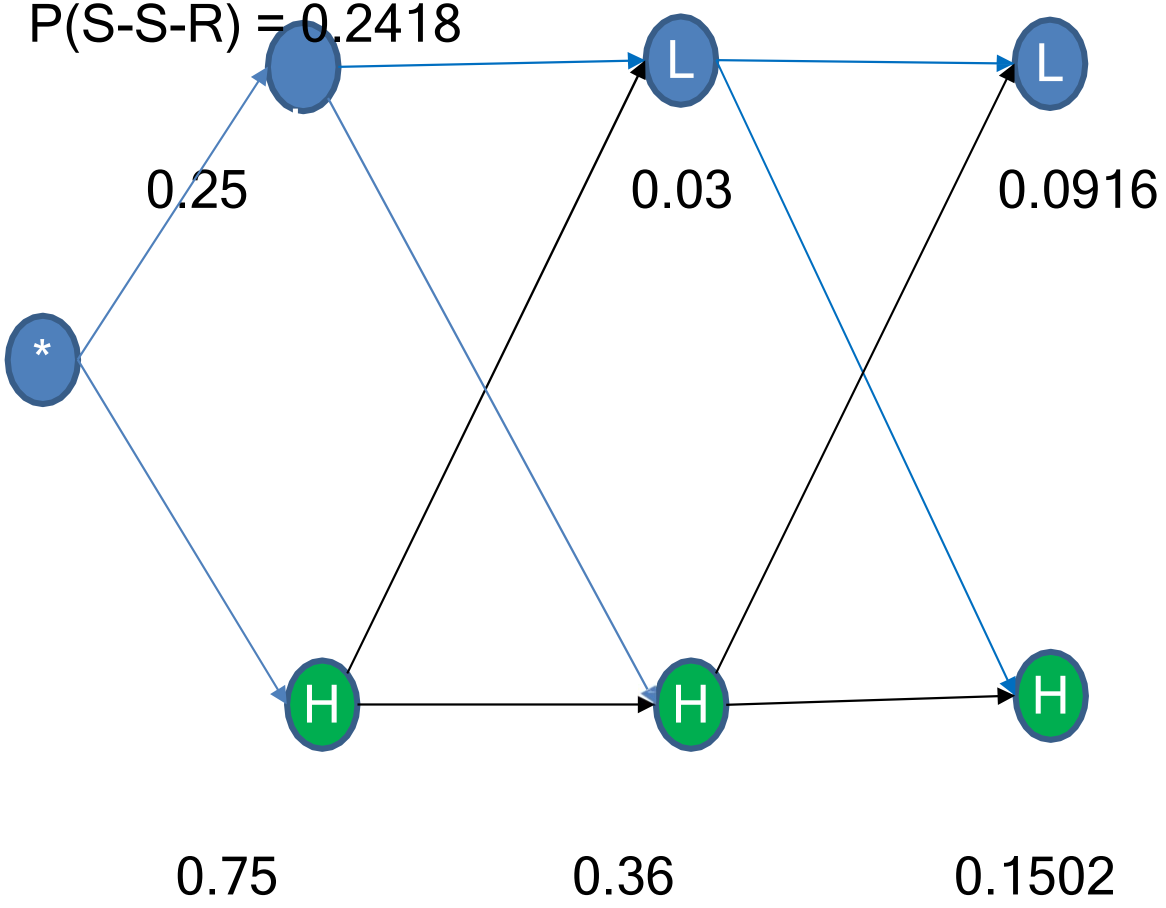
P(H)*P(H|H)*P(R|H) = 0.435*0.8*0.4 = **0.1392**

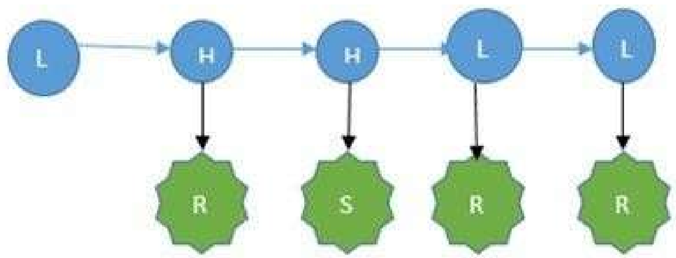
Hidden Markov Model

Forward Propagation Algorithm : S- S-R

Termination Phase:

P(S-S-R) = 0.2418





Transition Model / Probability Matrix

$P(U_{t-1} = HP)$	$P(U_{t-1} = LP)$	←Previous
0.2	0.5	$P(U_t = LP)$
0.8	0.5	$P(U_t = HP)$

Evidence / Sensor Model/ Emission Probability Matrix

$P(X_t = LP)$	$P(X_t = HP)$	←Unobserved Evidence v
0.8	0.4	$P(E_t = Rainy)$
0.2	0.6	$P(E_t = Sunny)$

Required Reading: AIMA - Chapter #15.1, #15.2, #15.3

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials