Question No: 01

Let $(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \ldots (\boldsymbol{x}_N, y_N)$ be N points on which we find a SVM classifier using the dual SVM formulation which is given below:

$$\text{maximize} \sum_{i=1}^{i=N} \alpha_i - \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} \alpha_i \alpha_j y_i y_j \boldsymbol{x}_i . \boldsymbol{x}_j \text{ subject to}$$

$$\sum_{i=1}^{N} \alpha_i y_i = 0$$

$$\alpha_i \geq 0 \forall i$$

Let O_A be the value of the objective function at the optimal solution returned by the dual SVM formulation for this problem. Now we add a new point $(\boldsymbol{x}_{N+1}, y_{N+1})$ and find a SVM classifier by solving the dual formulation again. Let O_B be the value of the objective function at the optimal solution for this problem. Considering the following three relationships (a) $O_A > O_B$, (b) $O_A = O_B$, (c) $O_A < O_B$ determine which of these relationships are possible and give a mathematical argument for

$$\text{maximize} \sum_{i=1}^{i=N} \alpha_i - \sum_{i=1}^{i=N} \sum_{j=1}^{j=N} \alpha_i \alpha_j y_i y_j \boldsymbol{x}_i . \boldsymbol{x}_j \text{ subject to}$$

$$\sum_{i=1}^{N} \alpha_i y_i = 0$$

$$\alpha_i \geq 0 \forall i$$

Let O_A be the value of the objective function at the optimal solution returned by the dual SVM formulation for this problem. Now we add a new point $(\boldsymbol{x}_{N+1}, y_{N+1})$ and find a SVM classifier by solving the dual formulation again. Let O_B be the value of the objective function at the optimal solution for this problem. Considering the following three relationships (a) $O_A > O_B$, (b) $O_A = O_B$, (c) $O_A < O_B$ determine which of these relationships are possible and give a mathematical argument for your answer in each case.

This is a subjective question, hence you have to write your answer in the Text-Field given below.

Solve the System of equations by Gaussian elimination method

$2x_1 + 5x_2 + 2x_3 - 3x_4 = 3$

$3x_1 + 6x_2 + 5x_3 + 2x_4 = 2$

$4x_1 + 5x_2 + 14x_3 + 14x_4 = 11$

$5x_1 + 10x_2 + 8x_3 + 4x_4 = 4$

Question No: 04

This is a subjective question, hence you have to write your answer in the Text-Field given below.

A linear Algebra student arrived at an $n \times n$ real matrix given below

$$A = \begin{pmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} \end{pmatrix}$$

Help the student to find singular value decomposition of full rank matrix A if the columns of A are orthogonal.

Question No: 05

This is a subjective question, hence you have to write your answer in the Text-Field given below.

Let $\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots \boldsymbol{x}_N$ be N points on which we perform Principle Components Analysis leading to the discovery of the principle component directions $\boldsymbol{b}_1, \boldsymbol{b}_2 \ldots \boldsymbol{b}_D$. The given data is now transformed to the points $\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots \boldsymbol{y}_N$ where $\boldsymbol{y}_i = \boldsymbol{Q}\boldsymbol{x}_i + \boldsymbol{\mu}$ where $\boldsymbol{Q}^T\boldsymbol{Q} = \boldsymbol{I}$ and $\boldsymbol{\mu}$ is a constant vector. Determine the principle components for the transformed set of points in terms of the old principle components. How much variance is accounted for by the first principal component for the transformed set of points in terms of the variance accounted for by the first principle component for the original set of points? Justify your answer with mathematical arguments.

Question No: 06

This is a subjective question, hence you have to write your answer in the Text-Field given below.

A data analyst modeled the objective function $f : \mathbb{R}^n \to \mathbb{R}$ as product of squares of n feature x_i s and $n \geq 2$. He has to maximize the objective function such that sum of squares of n features is less than or equal to c^2 where $c \in \mathbb{R}$. Write the mathematical formulation of the problem and solve it. Using the above result prove the inequality

$$(a_1 a_2 \cdots a_n)^{1/n} \leq \frac{a_1 + \cdots + a_n}{n} \text{ for any } a_i > 0, i = 1, \cdots, n$$

Question No: 07

This is a subjective question, hence you have to write your answer in the Text-Field given below.

Consider two sets named $\mathcal{H}_1$ and $\mathcal{H}_2$. It is known that these two sets are convex sets.

(a) Prove or disprove that $\mathcal{H}_1 \cap \mathcal{H}_2$ is a convex set. Here $\cap$ represents the set intersection operation.

(b) Prove or disprove that $\mathcal{H}_1 \cup \mathcal{H}_2$ is a convex set. Here $\cup$ represents the set union operation.

Question No: 08

This is a subjective question, hence you have to write your answer in the Text-Field given below.

Consider three linearly independent vectors in $\mathbb{R}^n$ named a_1, a_2 and a_3. Now construct three vectors $b_1 = a_2 - a_3$, $b_2 = a_1 - a_3$ and $b_3 = a_1 - a_2$. Now consider the set $Q = \{b_1, b_2, b_3\}$. Prove or disprove that the set Q is linearly independent.

[5 Marks]

This is a subjective question, hence you have to write your answer in the Text-Field given below.

You are given the quadratic polynomial $f(x, y, z) = 2x^2 - 2xy - 4xz + y^2 + 2yz + 3z^2 - 2x + 2z$:

(a) Write $f(x, y, z)$ in the form $f(x, y, z) = x^T Ax - b^T x$ where $x = (x, y, z)$, A is a real symmetric matrix, and b is constant vector.

(b) Find the point (x, y, z) where f(x, y, z) is at an extremum.

(c) Is this point a minimum, maximum, or a saddle point of some kind?

[5 Marks]