



# Artificial & Computational Intelligence

**AIML CZG557**

**M2 : Problem Solving Agent using Search**

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**BITS Pilani**

Pilani Campus

# Course Plan



M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

M6 Reasoning over time

M7 Ethics in AI

# Learning Objective



At the end of this class , students Should be able to:

1. Design problem solving agents
2. Create search tree for given problem
3. Apply uninformed search algorithms to the given problem
4. Compare performance of given algorithms in terms of completeness, optimality, time and space complexity
5. Differentiate for which scenario appropriate uninformed search technique is suitable and justify



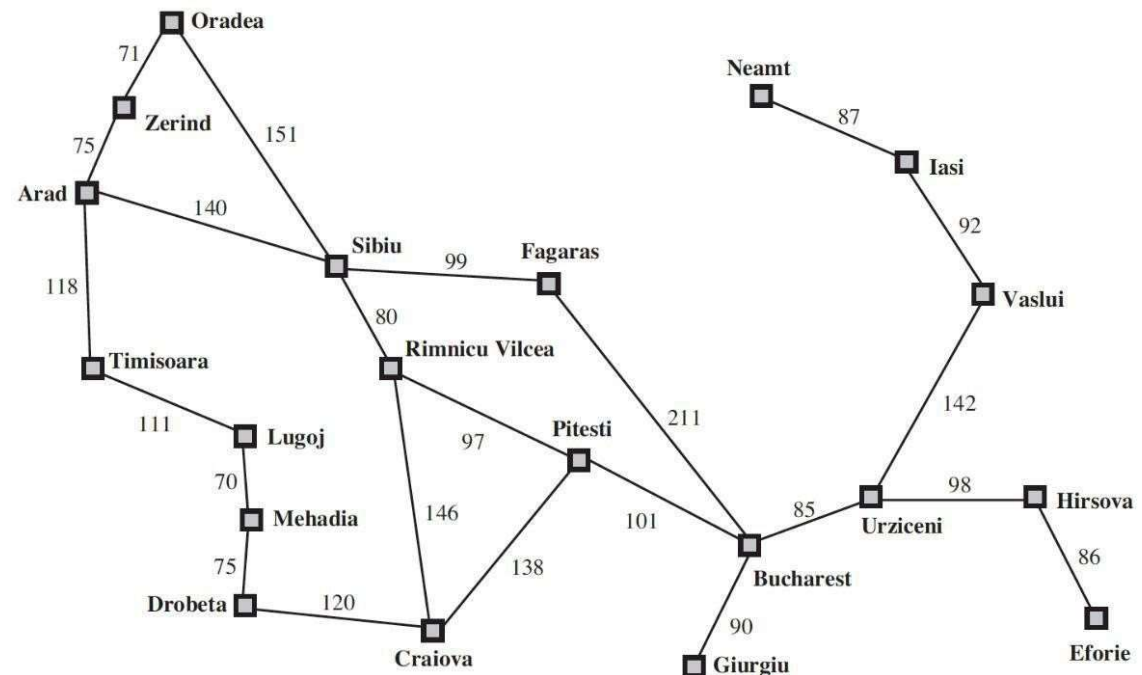
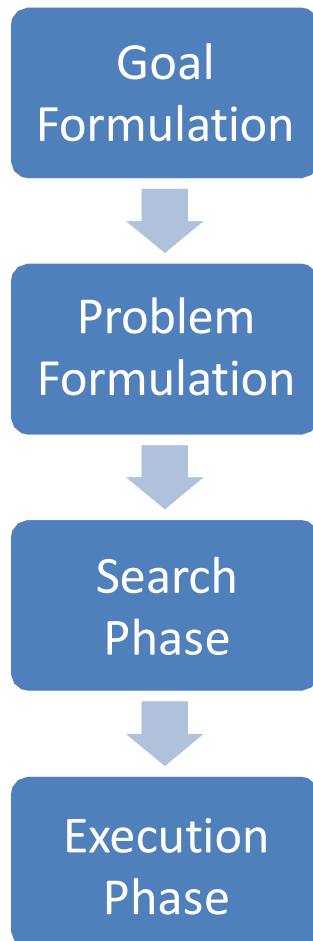
# Problem Formulation

# Problem Solving Agents

Goal based decision making agents finds sequence of actions that leads to the desirable state.

## Phases of Solution Search by PSA

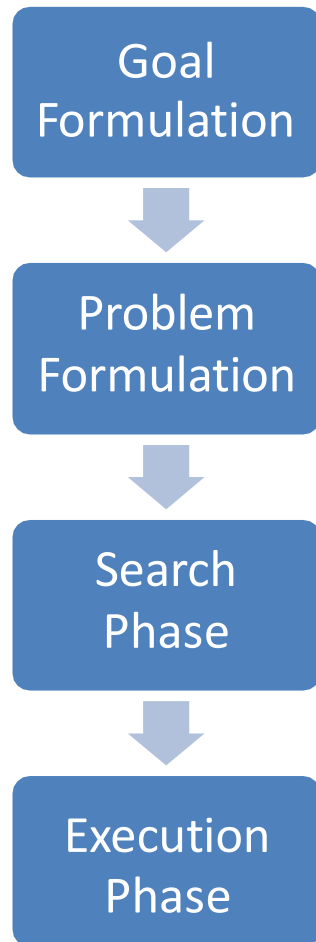
Optimizes the Objective (Local | Global) Limits the Actions



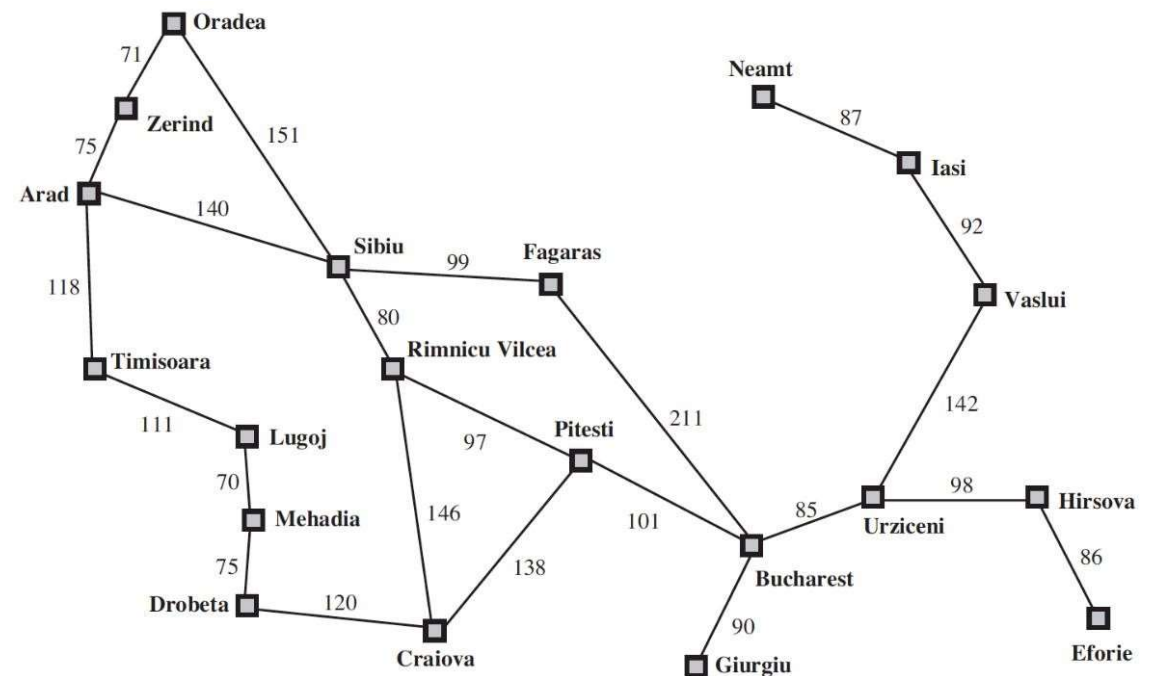
# Problem Solving Agents



## Phases of Solution Search by PSA

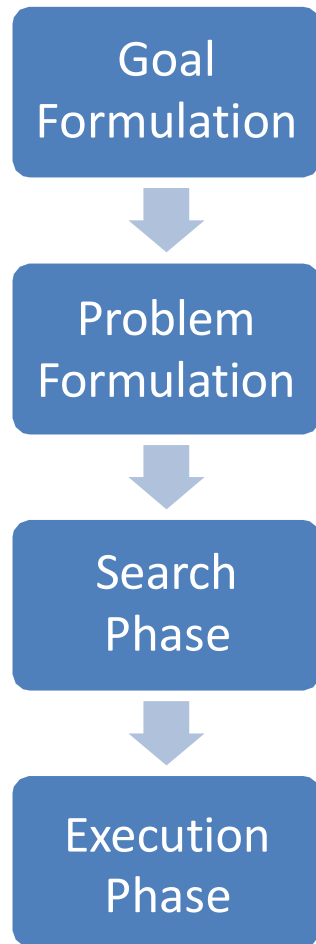


State Space Creations [in the path of Goal]  
Lists the Actions

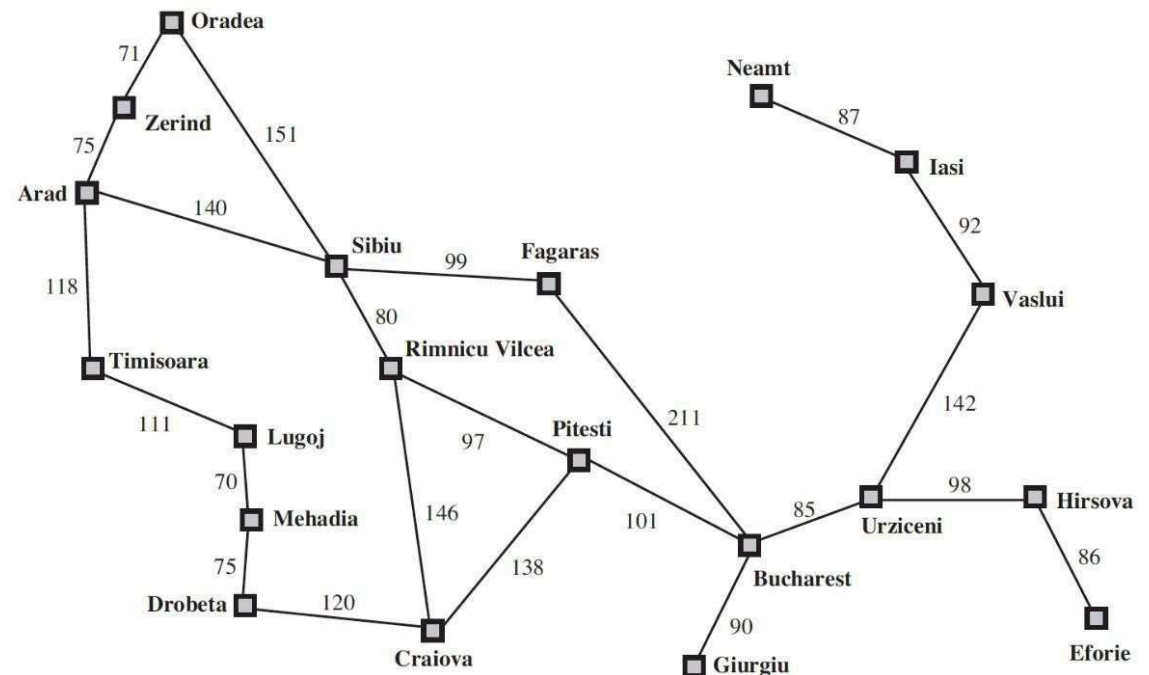


# Problem Solving Agents

## Phases of Solution Search by PSA

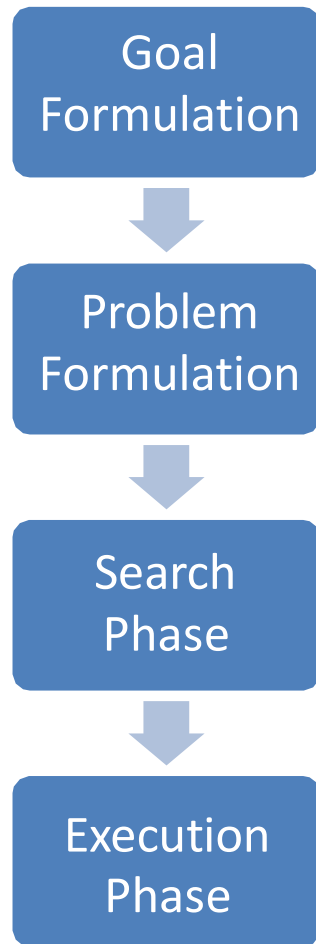


**Assumptions – Environment :**  
 Static  
 Observable Discrete  
 Deterministic

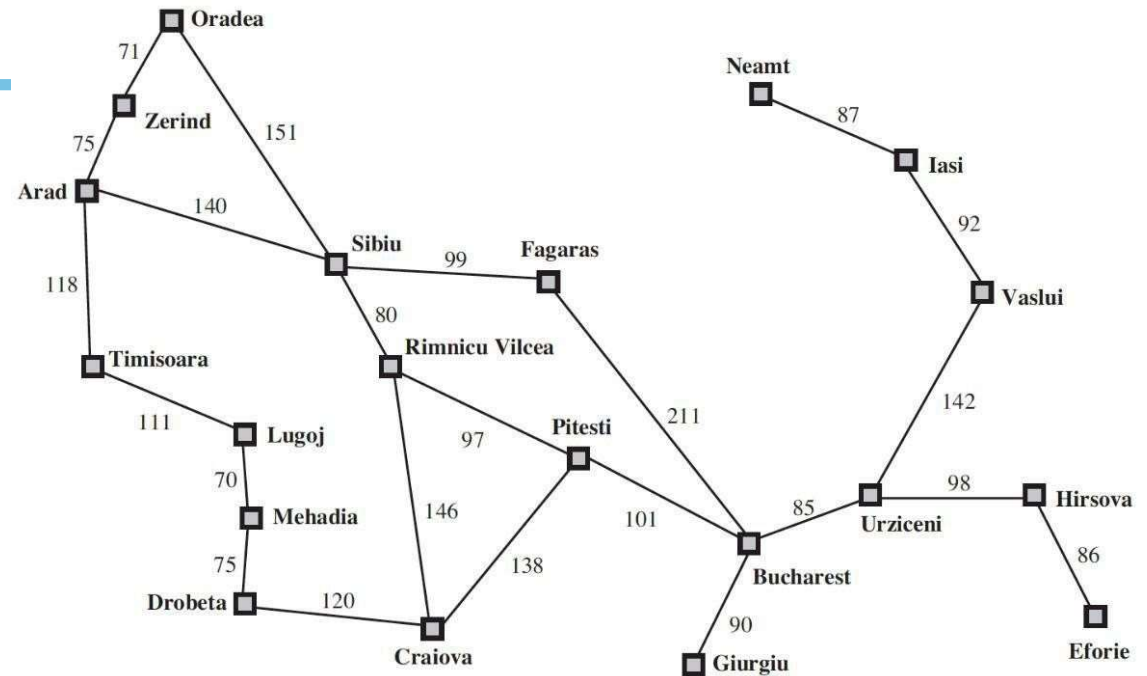


# Problem Solving Agents

## Phases of Solution Search



Examine all sequence  
Choose best |  
Optimal



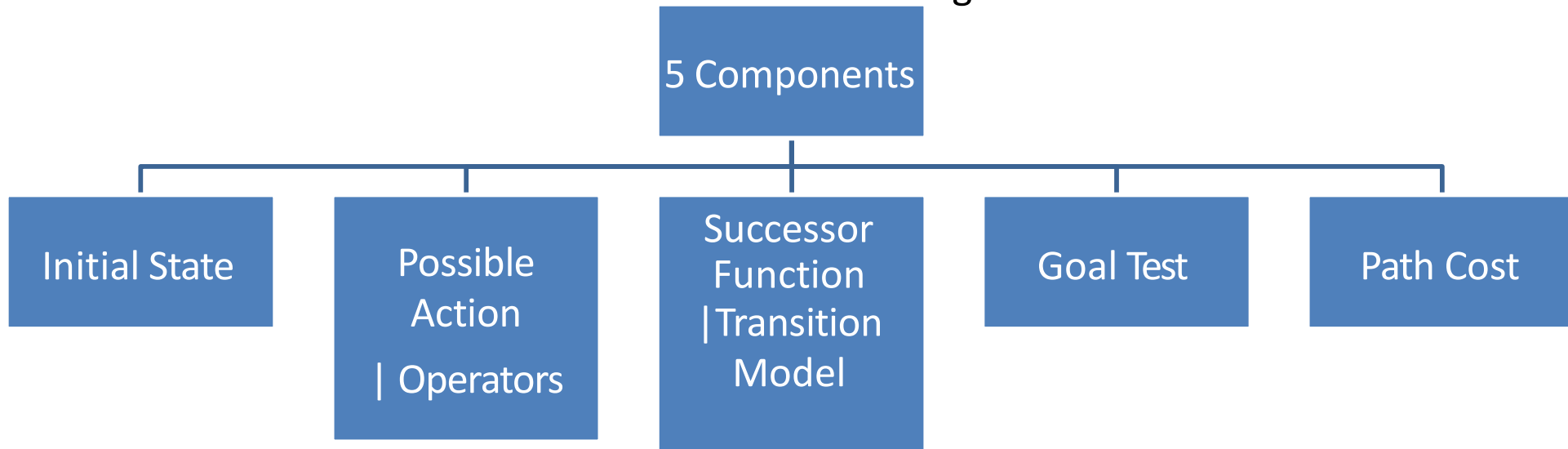




# Problem Solving Agents – Problem Formulation

## Abstraction Representation

Decide what actions under states to take to achieve a goal

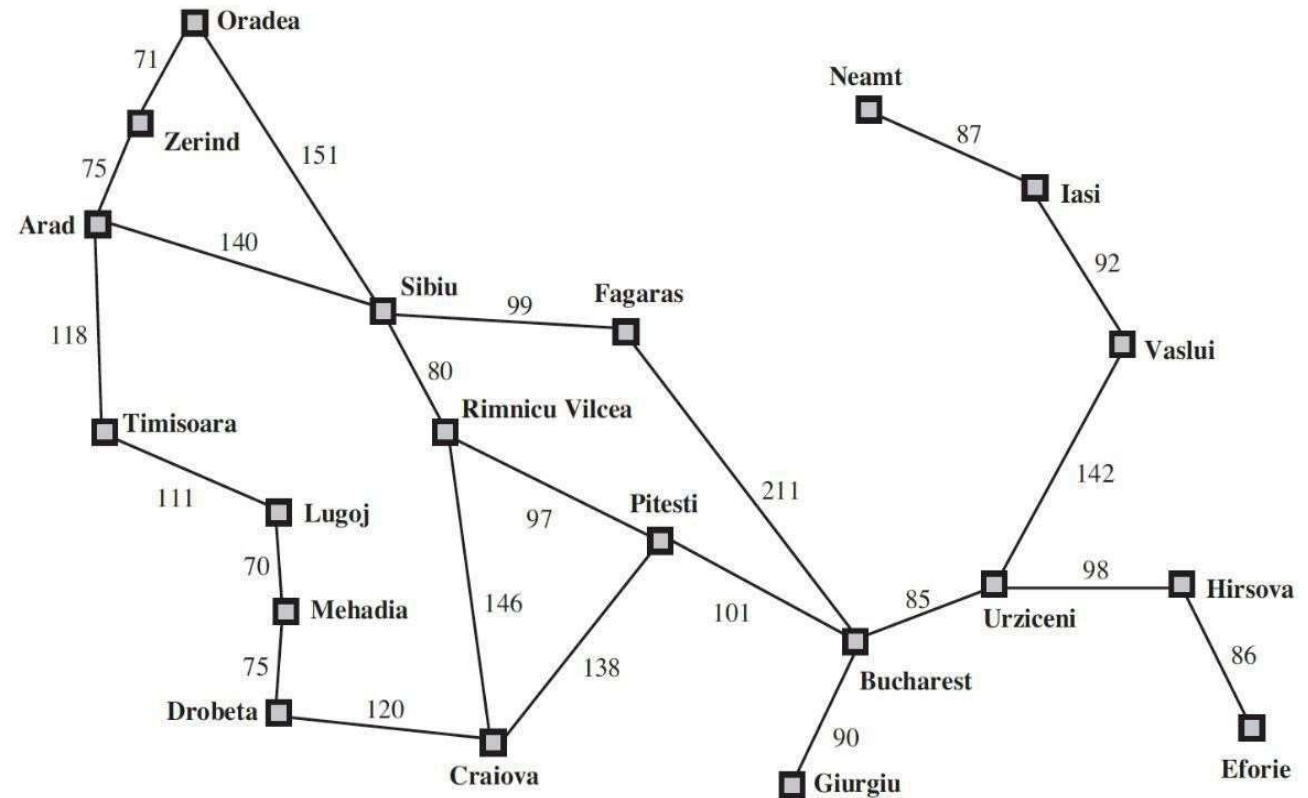


A function that assigns a numeric cost to each path. A path is a series of actions. Each action is given a cost depending on the problem.

**Solution = Path Cost Function + Optimal Solution**

# Problem Solving Agents – Problem Formulation:

## Book Example



**Initial State** – E.g.,  $In(Arad)$

**Possible Actions** –  $ACTIONS(s) \sqsubseteq \{Go(Sibiu), Go(Timisoara), Go(Zerind)\}$

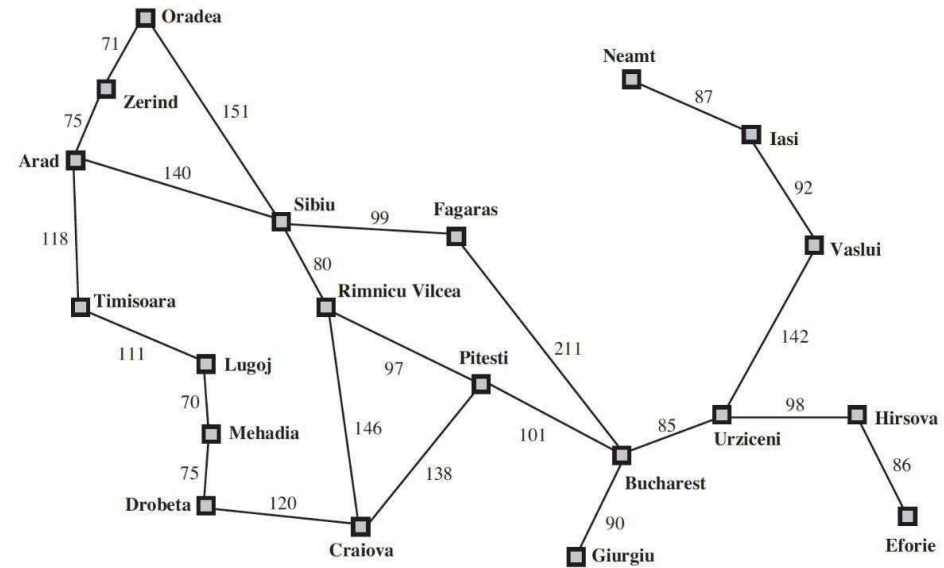
**Transition Model** –  $RESULT(In(Arad), Go(Sibiu)) = In(Sibiu)$

**Goal Test** –  $IsGoal(In(Bucharest)) = Yes$

**Path Cost** –  $cost(In(Arad), go(Sibiu)) = 140 \text{ kms}$

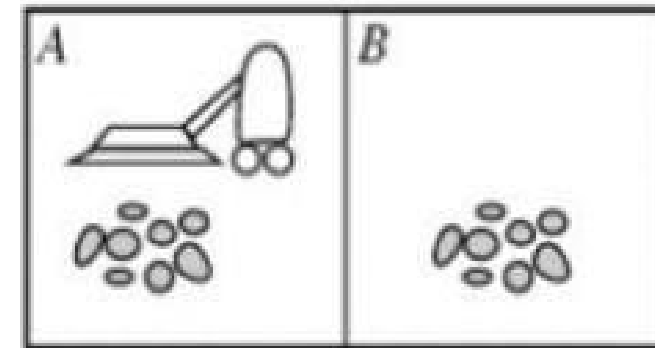
# Example Problem Formulation

	Travelling Problem
Initial State	Based on the problem
Possible Actions	Take a flight   Train   Shop
Transition Model/ Successor Function	$[A, \text{Go}(A \rightarrow S)] = [S]$
Goal Test	Is current = B (destination)
Path Cost	Cost + Time + Quality



## Example Problem Formulation

	Vacuum World
Initial State	Any
Possible Actions	[Move Left, Move Right, Suck, NoOps]
Transition Model/ Successor Function	[A, ML] = [B, Dirty] [A, ML] = [B, Clean]
Goal Test	Is all room clean? [A, Clean] [B, Clean]
Path Cost	No of steps in path



# Example Problem Formulation



	N-Queen
Initial State	Empty   Partial   Full
Possible Actions	
Transition Model/ Successor Function	
Goal Test	
Path Cost	

	0	1	2	3
0			♔	
1	♔			
2				♔
3		♔		

board[r][c]

# Path finding Robot

## Successor Function Design

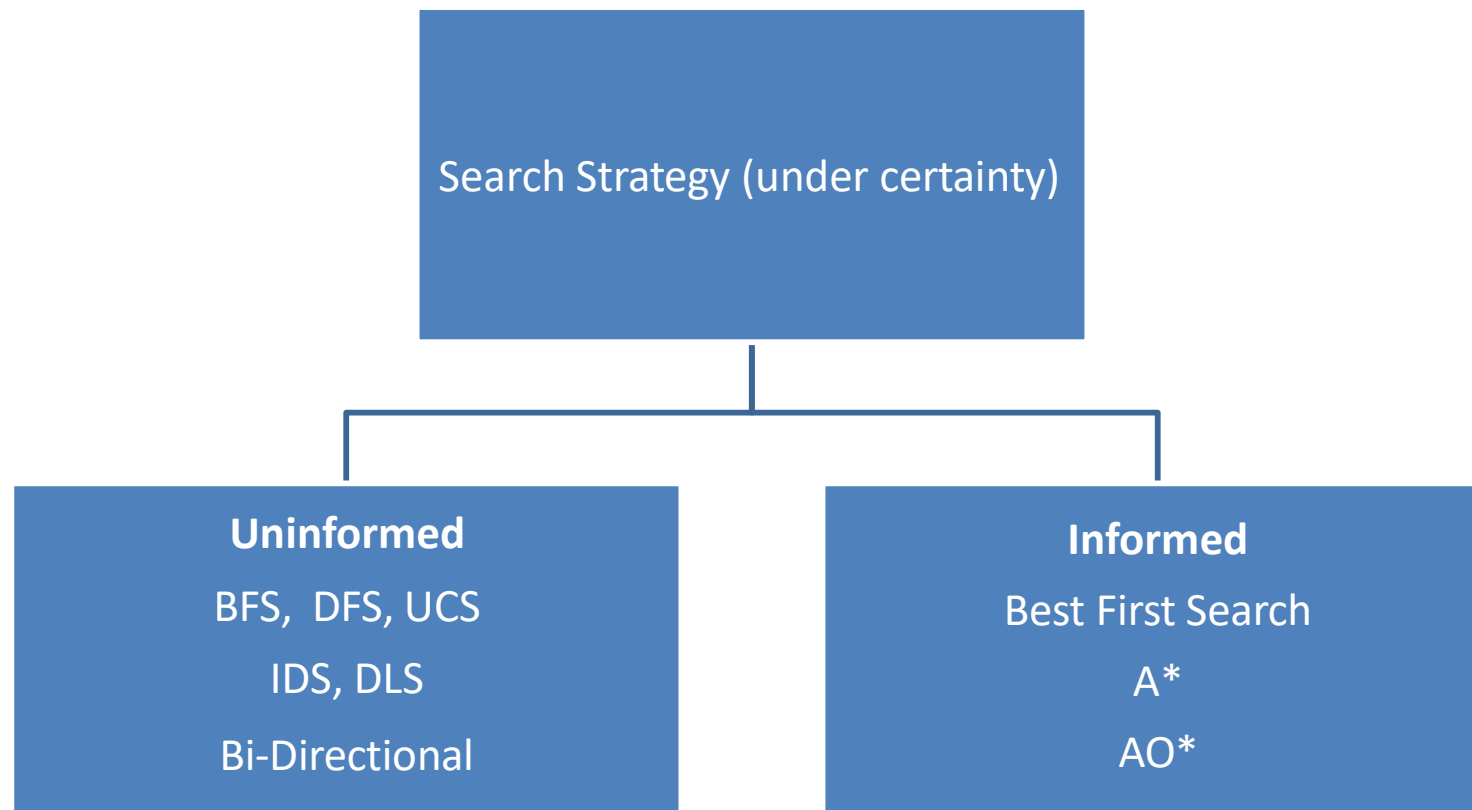
1	2	3	4	5	6	0
	8		10	11	12	1
13	14		16	17	18	2
19	20		22	23	24	3
25	26	27			30	4
	32	33		35	36	5
37	38	39	40	41	42	6
0	1	2	3	4	5	

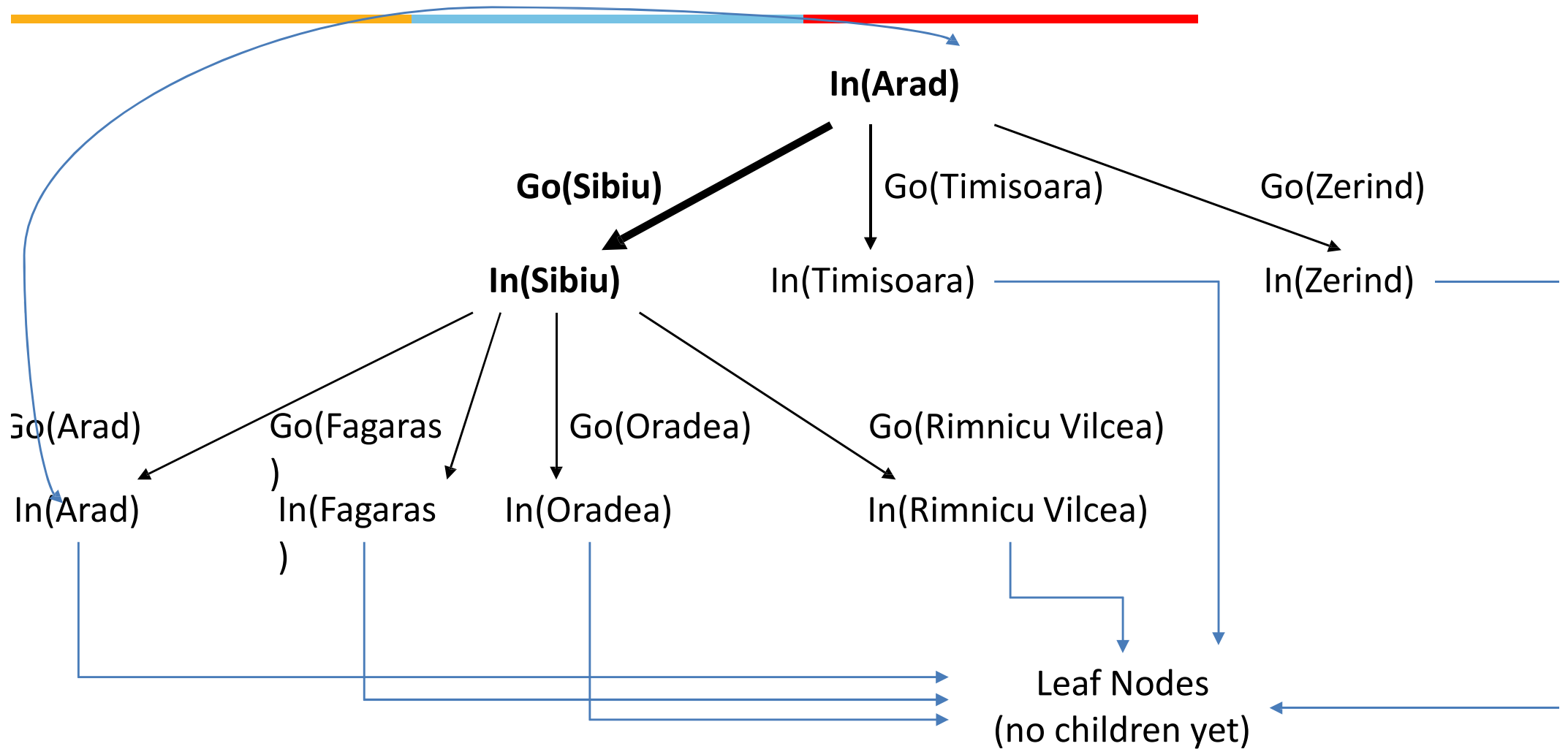
N-W-E-S

# Searching for Solutions



Choosing the current state, testing possible successor function, expanding current state to generate new state is called Traversal. Choice of which state to expand – Search Strategy





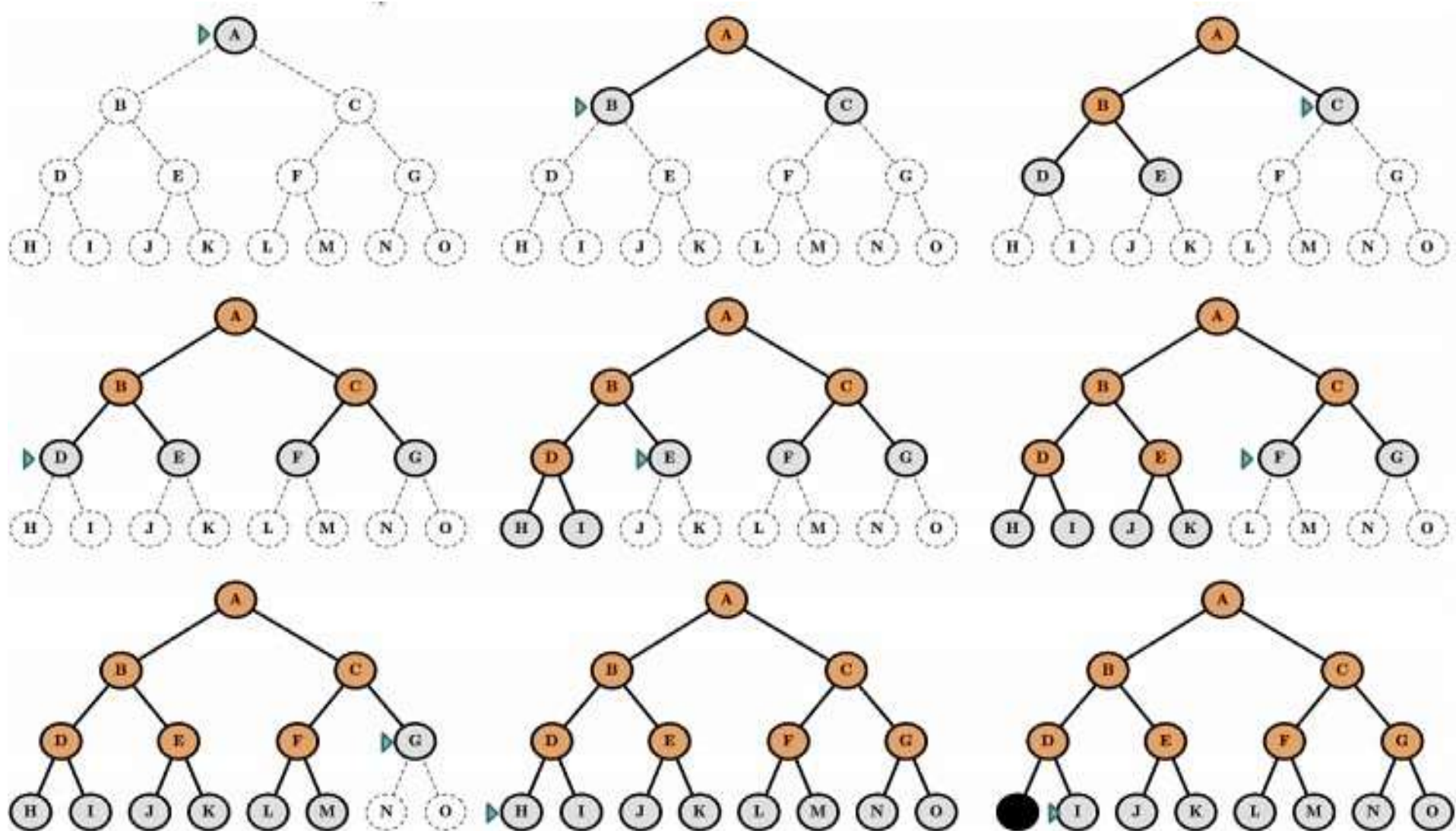




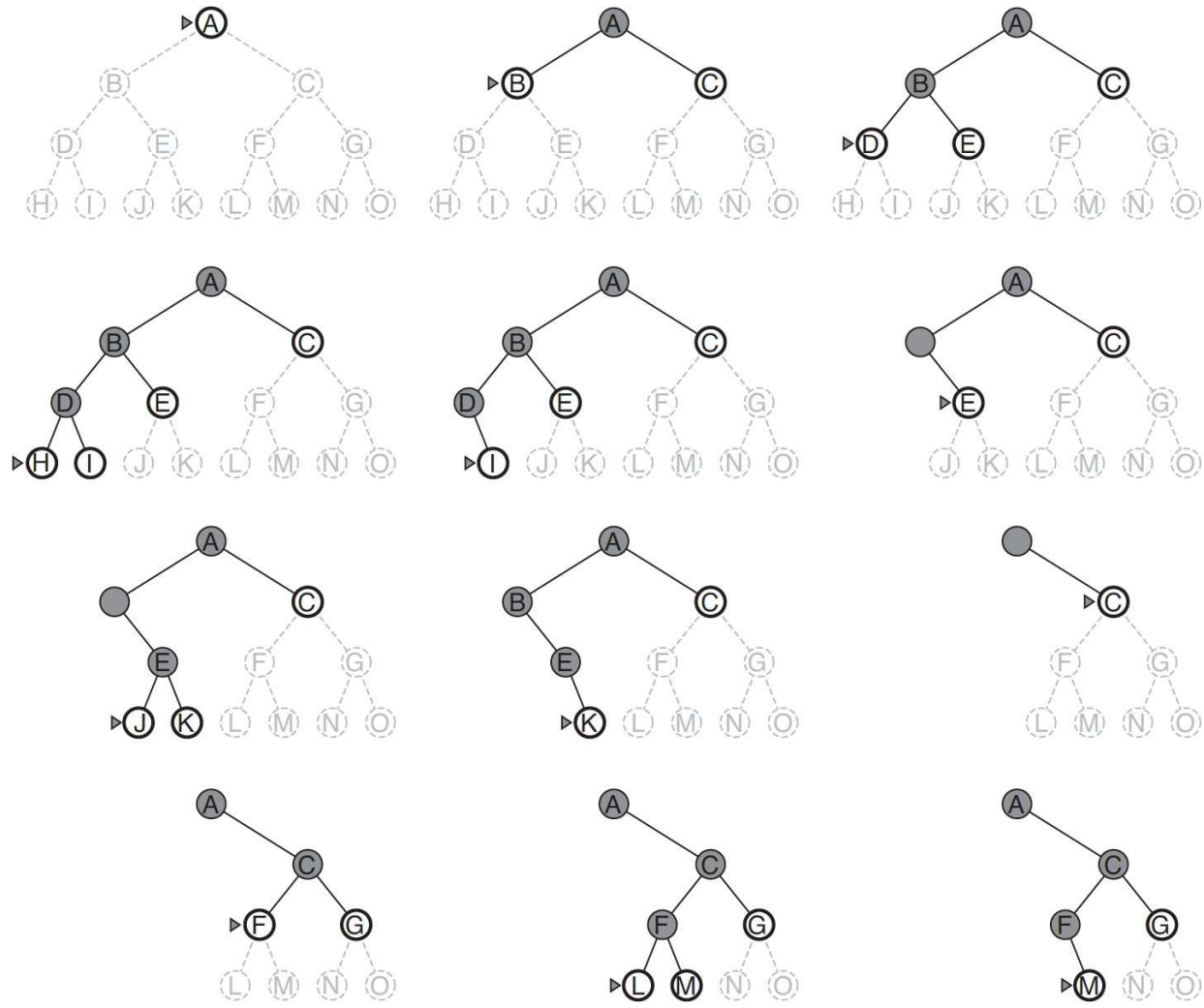
# Uninformed Search Overview



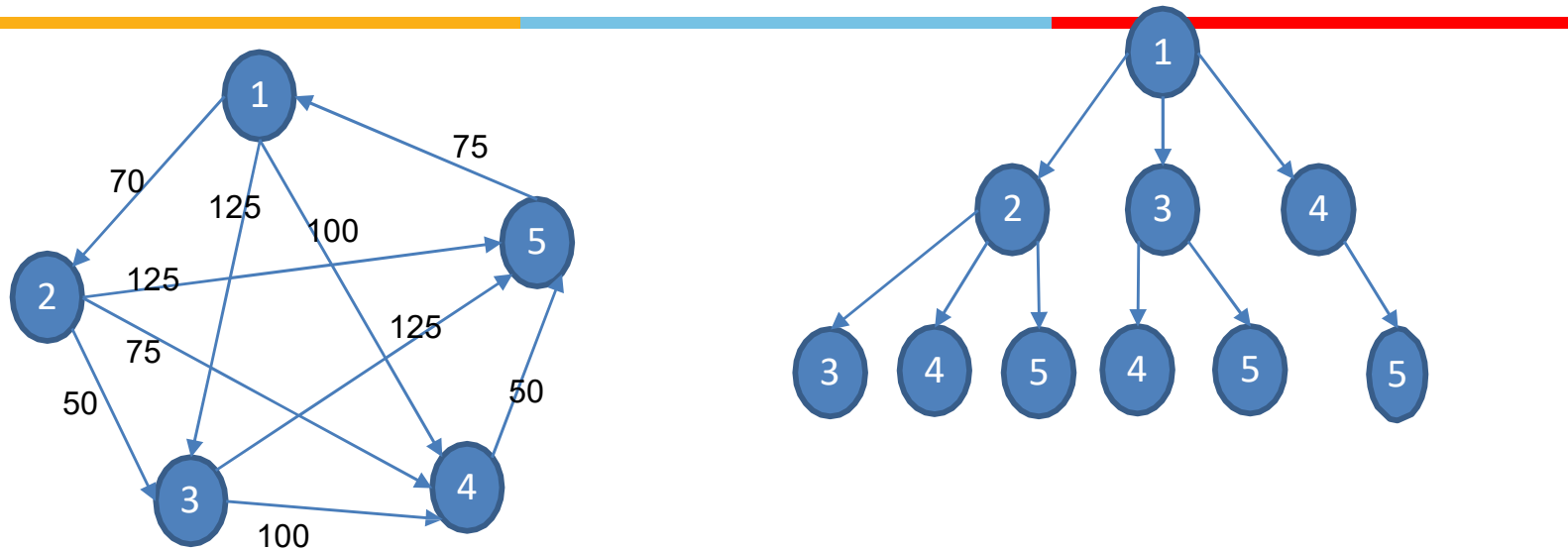
# Breadth First Search (BFS)



# Depth First Search (DFS)

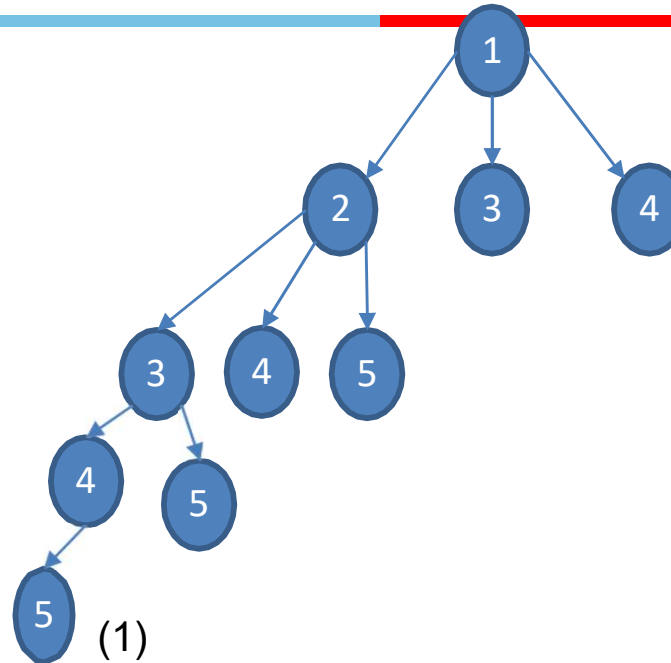
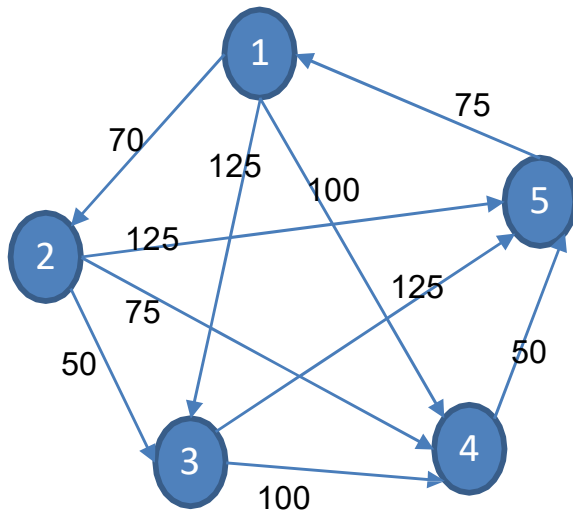


# Search Tree – Sample Generation



**Each NODE in in the search tree denotes an entire PATH through the state space graph.**

# Search Algorithm – Uninformed Example - 1



(1)  
 (1 2) (1 3) (1 4)  
 (1 2 3) (1 2 4) (1 2 5) (1 3) (1 4)  
 (1 2 3 4) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)  
**(1 2 3 4 5)** (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)

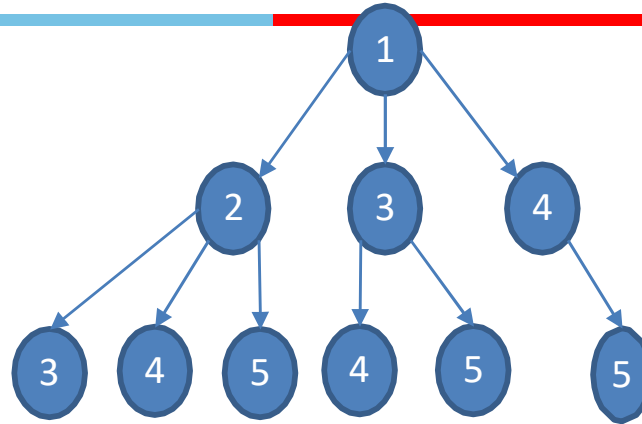
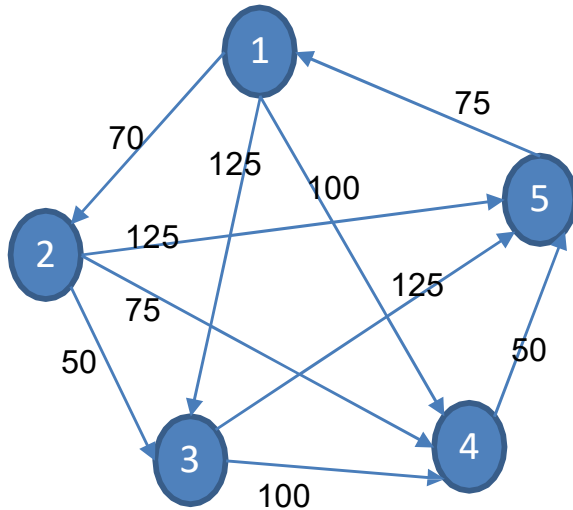
$$C(1-2-3-4-5) = 70 + 50 + 100 + 50 = 270$$

Expanded : 4

Generated : 10

Max Queue Length : 6

## Search Algorithm – Uninformed Example - 2



(1)  
(1 2) (1 3) (1 4)  
TEST FAILED

(1 3) (1 4) (1 2 3) (1 2 4) (1 2 5)  
(1 2 3) (1 2 4) (1 2 5) (1 3 4) (1 3 5) (1 4 5)  
TEST PASSED

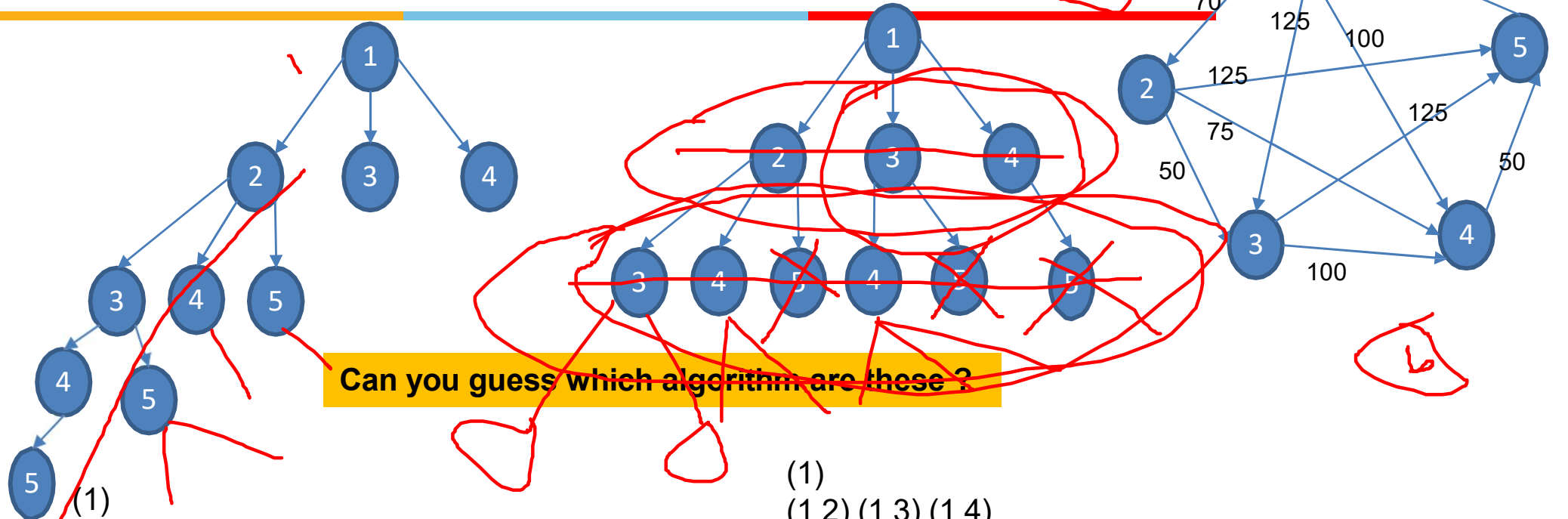
$C(1-2-5) = 70 + 125 = 195$

Expanded : 4

Generated : 10

Max Queue Length : 6

# Search Algorithm – Uninformed Example - 2



Can you guess which algorithm are these ?

(1)  
 (1 2) (1 3) (1 4)  
 (1 2 3) (1 2 4) (1 2 5) (1 3) (1 4)  
 (1 2 3 4) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)  
 (1 2 3 4 5) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)

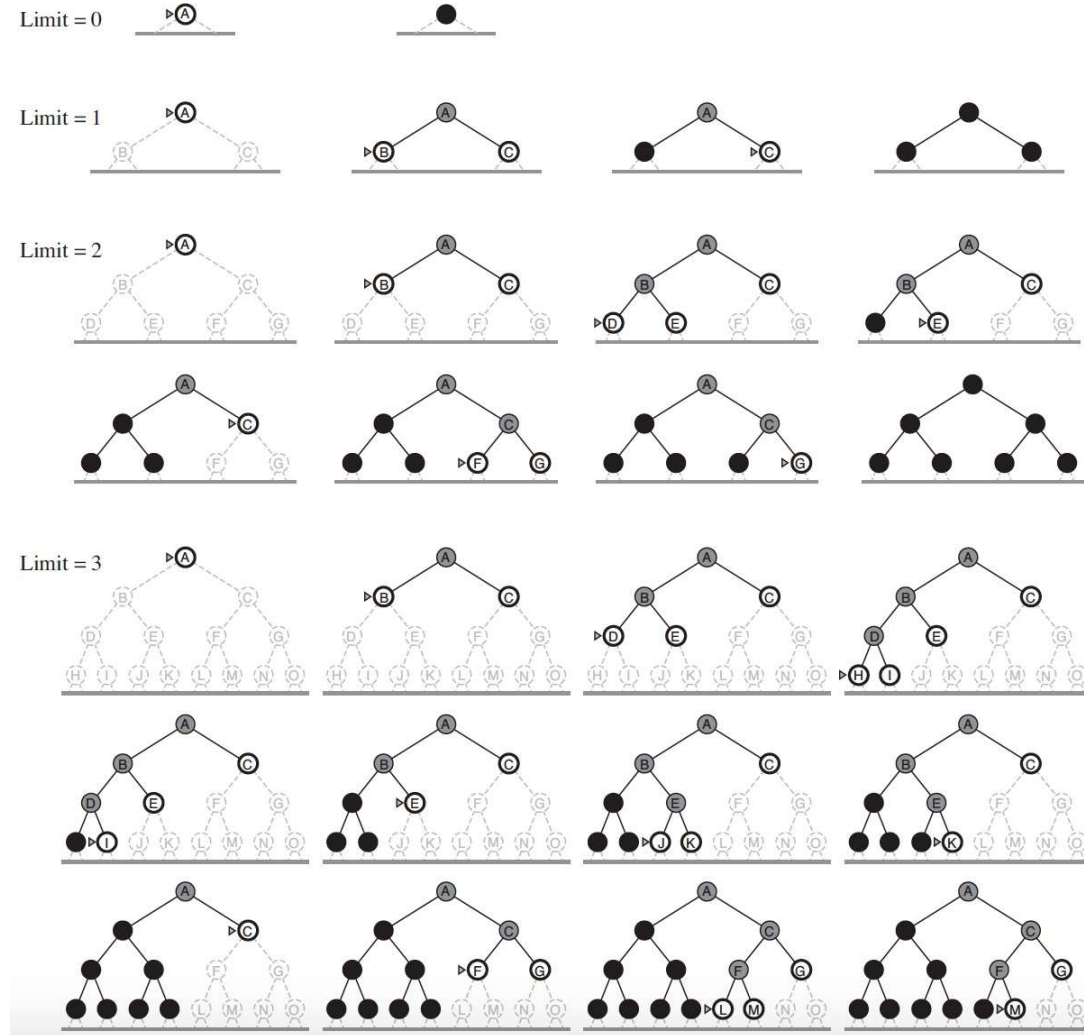
$C(1-2-3-4-5) = 70 + 50 + 100 + 50 = 270$   
 Expanded : 4  
 Generated : 10  
 Max Queue Length : 6

(1)  
 (1 2) (1 3) (1 4)  
 TEST FAILED

(1 3) (1 4) (1 2 3) (1 2 4) (1 2 5)  
 (1 2 3) (1 2 4) (1 2 5) (1 3 4) (1 3 5) (1 4 5)  
 TEST PASSED

$C(1-2-5) = 70 + 125 = 195$   
 Expanded : 4  
 Generated : 10  
 Max Queue Length : 6

# Iterative Deepening Depth First Search (IDS)



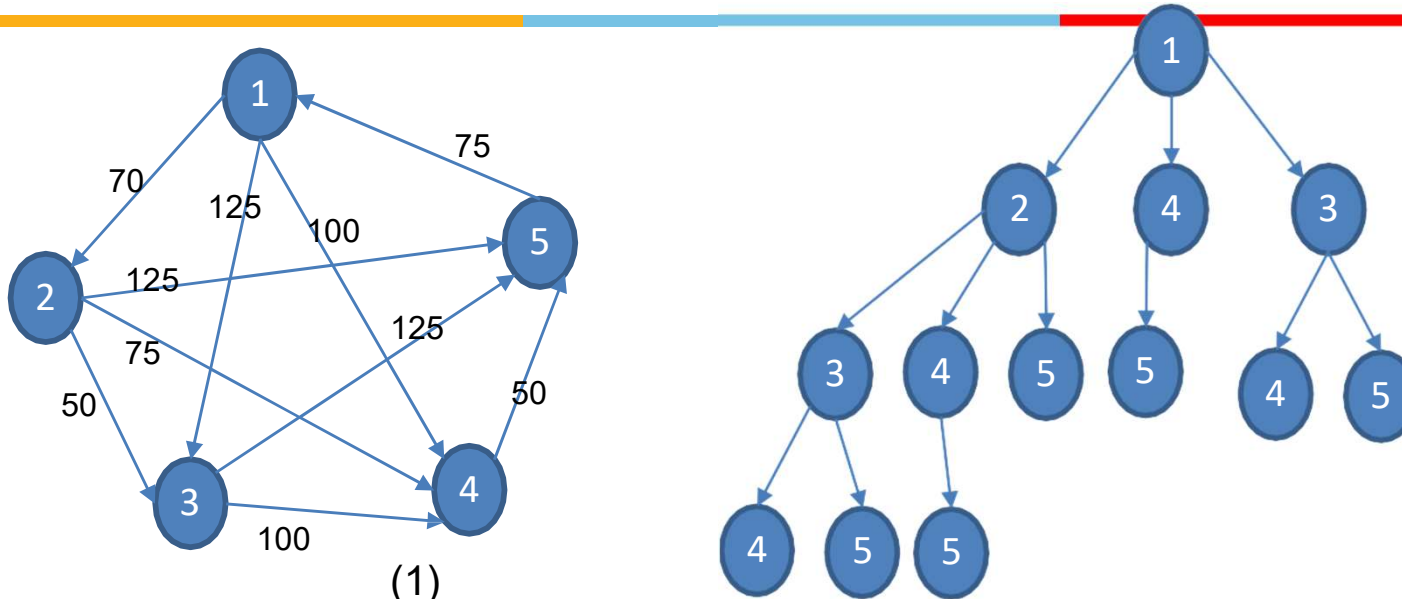


# Algorithm Tracing



Students must follow this in the exams for all the search algorithms in addition to the search tree constructions. The ordering of the Open Lists must be in consistent with the algorithm with a note on the justification of the order expected!

Iter	Open List / Frontiers / Fringes	Closed List	Goal Test
1.	(1)		Fail on (1)
2.	(1 3), (1 4), (1 2)	(1)	Fail on (1 3)



(1)

**(1 2 : 70)** (1 4 : 100) (1 3 : 125)

TEST-F

**(1 4 : 100)** (1 2 3 : 120) (1 3 : 125) (1 2 4 : 145) **(1 2 5 : 195)**

TEST-F

**(1 2 3 : 120)** (1 3 : 125) (1 2 4 : 145) **(1 4 5 : 150)** **(1 2 5 : 195)**

TEST-F

**(1 3 : 125)** (1 2 4 : 145) **(1 4 5 : 150)** (1 2 3 4 : 170) (1 2 5 : 195) (1 2 3 5 : 245)

TEST-F

**(1 2 4 : 145)** **(1 4 5 : 150)** (1 2 3 4 : 170) **(1 2 5 : 195)** (1 3 4 : 225) (1 2 3 5 : 245) **(1 3 5 : 250)**

TEST-F

**(1 4 5 : 150)** (1 2 3 4 : 170) (1 2 4 5 : 195) **(1 2 5 : 195)** (1 3 4 : 225) (1 2 3 5 : 245) (1 3 5 : 250)

**TEST - P**

# Uniform Cost Search



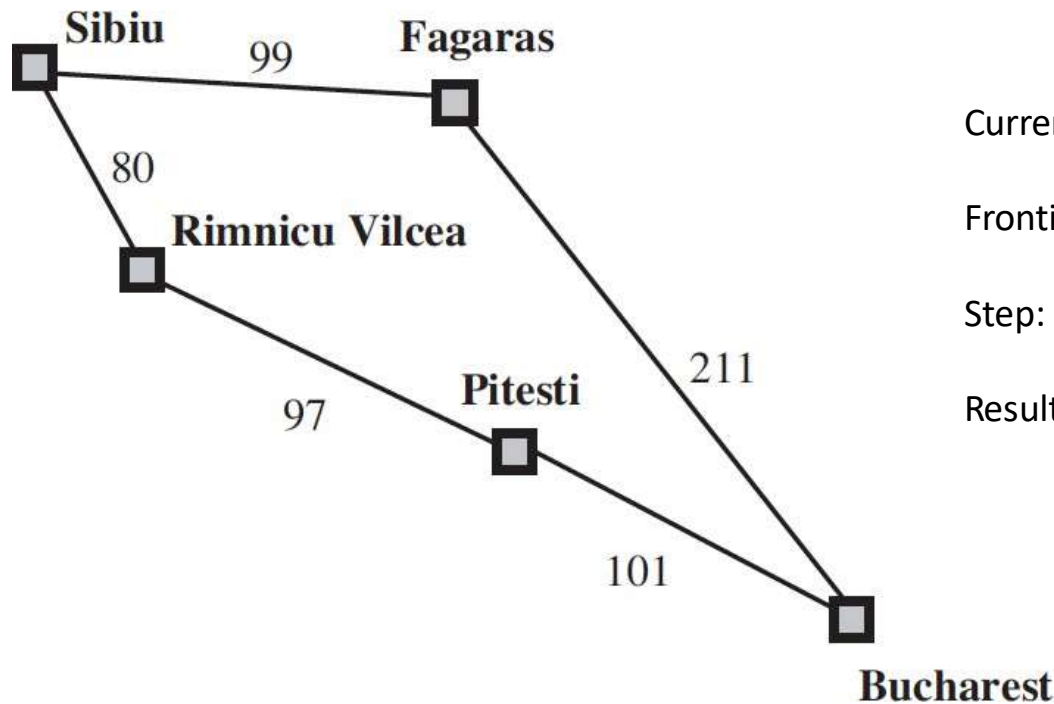
Instead of expanding the shallowest node, Uniform-Cost search expands the node  $n$  with the lowest path cost  $g(n)$

Sorting the Frontier as a priority queue ordered by  $g(n)$

Goal test is applied during expansion

- The goal node if generated may not be on the optimal path
- Find a better path to a node on the Frontier

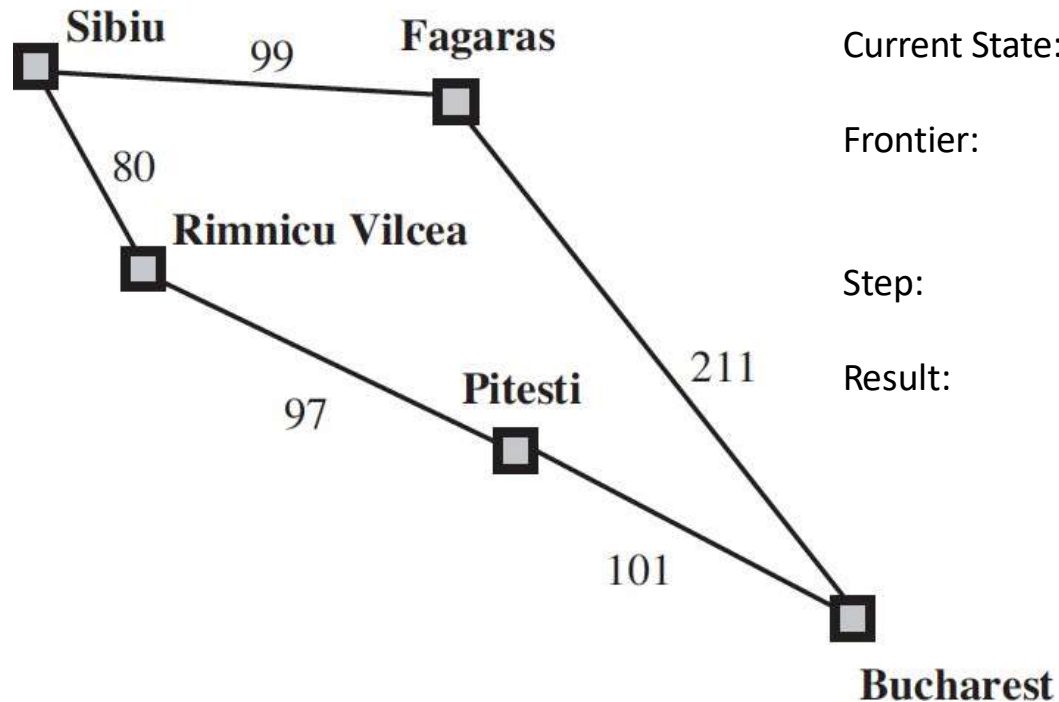
# Uniform Cost Search



Current State:	Sibiu
Frontier:	[]
Step:	Expand Sibiu
Result:	Generates ("Rimnicu Vilcea" 80) ("Fagaras", 99) Add to Frontier

Initial State:	Sibiu
Goal State:	Bucharest

# Uniform Cost Search



Current State:

Sibiu

Frontier:

[("Rimnicu Vilcea" 80)  
("Fagaras", 99)]

Step:

Expand "Rimnicu Vilcea" (least cost)

Result:

Generates ("Pitesti", 177)  
Add to Frontier

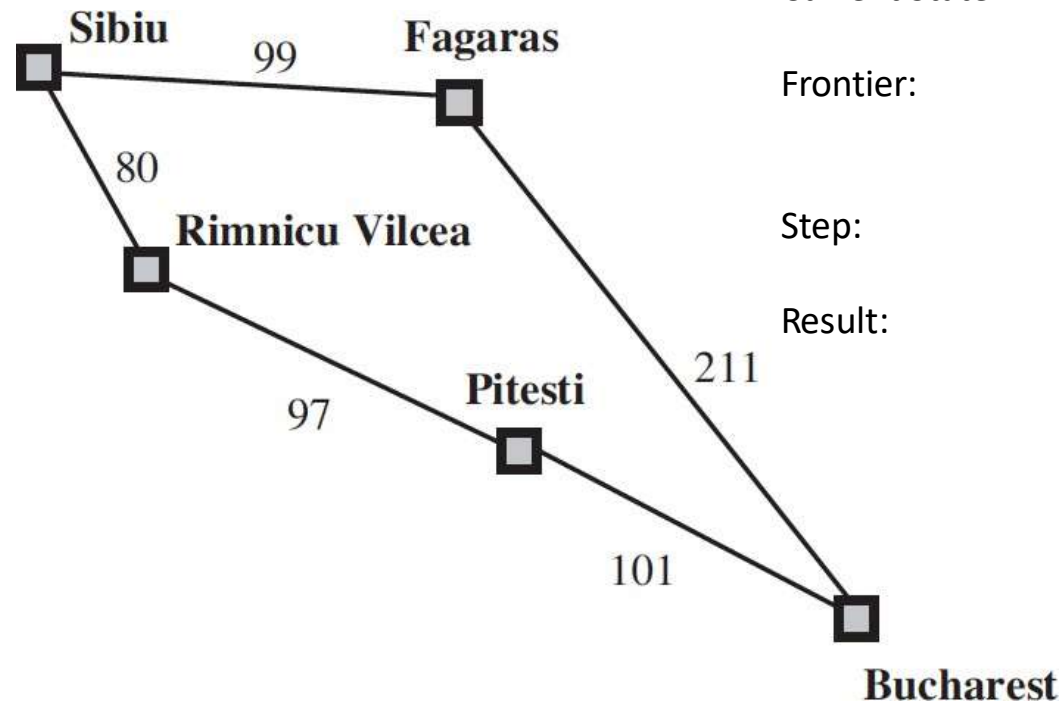
Initial State:

Sibiu

Goal State:

Bucharest

# Uniform Cost Search



Current State:

Rimnicu Vilcea (not a Goal state)

Frontier:

[ ("Fagaras", 99)  
("Pitesti", 177)]

Step:

Expand "Fagaras" (least cost)

Result:

Generates ("Bucharest", 310)  
Add to Frontier  
(It's a Goal State but we won't  
test during generation)

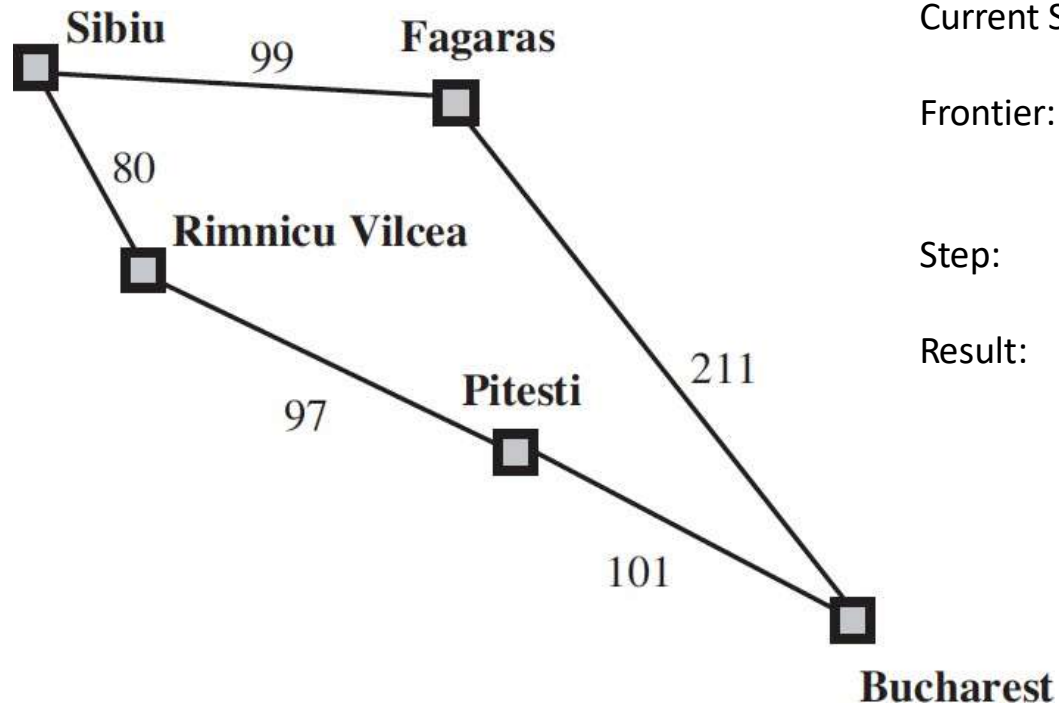
Initial State:

Sibiu

Goal State:

Bucharest

# Uniform Cost Search



Current State:

Fagaras (not a goal state)

Frontier:

[ ("Pitesti", 177)  
("Bucharest", 310)]

Step:

Expand "Pitesti" (least cost)

Result:

Generates ("Bucharest", 278)  
Replace in Frontier  
(It's a Goal State but we won't  
test during generation)

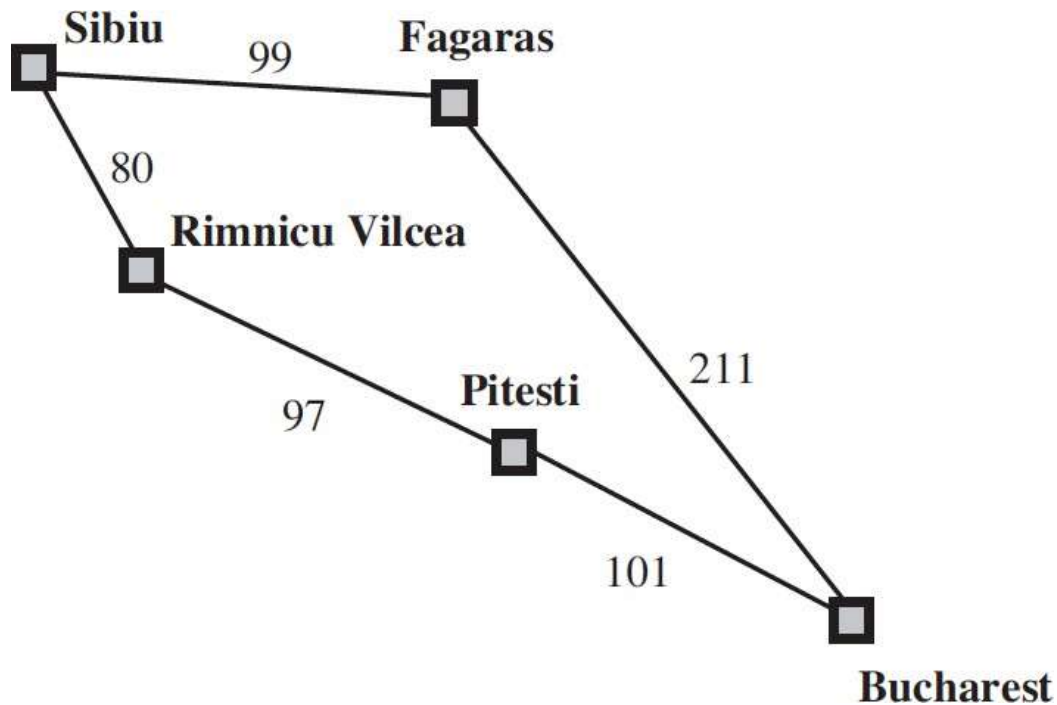
Initial State:

Sibiu

Goal State:

Bucharest

# Uniform Cost Search



Current State:	Pitesti (not a goal state)
Frontier:	[ ("Bucharest", 278)]
Step:	Expand "Bucharest"
Result:	No further generation as Goal Test satisfied Return Solution

Initial State:	Sibiu
Goal State:	Bucharest



## Sample Evaluation of the Algorithm

**Complete** – If the shallowest goal node is at a depth  $d$ , BFS will eventually find it by generating all shallower nodes

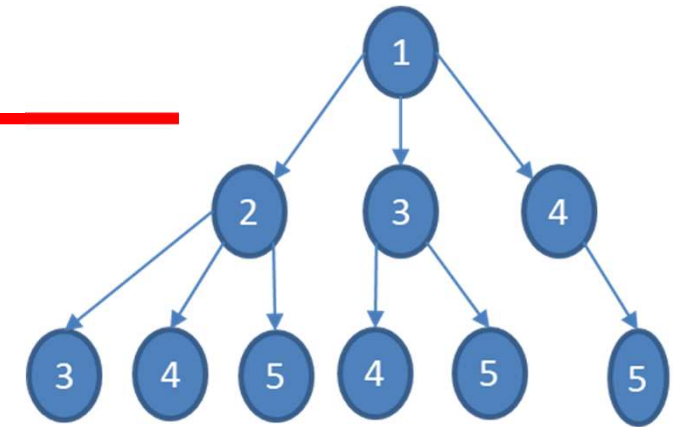
**Optimal** – Not necessarily. Optimal if path cost is non-decreasing function of depth of node.  
E.g., all actions have same cost

**Time Complexity** –  $G(b^d)$   $b$  - branching factor,  $d$  – depth

- Nodes expanded at depth 1 =  $b$
- Nodes expanded at depth 2 =  $b^2$
- Nodes expanded at depth  $d$  =  $b^d$
- Goal test is applied during generation, time complexity would be  $G(b^{d+1})$

**Space Complexity** –  $G(b^d)$

- $G(b^{d-1})$  in explored set
- $G(b^d)$  in frontier set



## Uniform Cost Search – Evaluation

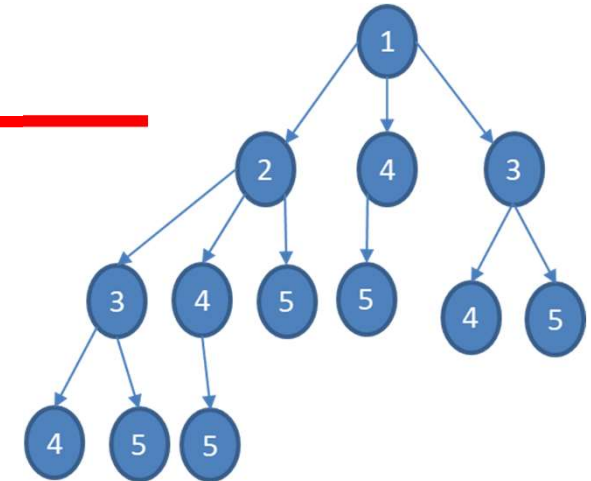
**Completeness** – It is complete if the cost of every step  $>$  small +ve constant  $\epsilon$

- It will stuck in infinite loop if there is a path with infinite sequence of zero cost actions

**Optimal** – It is Optimal. Whenever it selects a node, it is an optimal path to that node.

**Time and Space complexity** – Uniform cost search is guided by path costs not depth or branching factor.

- If  $C^*$  is the cost of optimal solution and  $\epsilon$  is the min. action cost
- Worst case complexity =  $G(b^{1+\frac{C^*}{\epsilon}})$ ,
- When all action costs are equal  $\rightarrow G(b^{d+1})$ , the BFS would perform better
  - As Goal test is applied during expansion, Uniform Cost search would do extra work

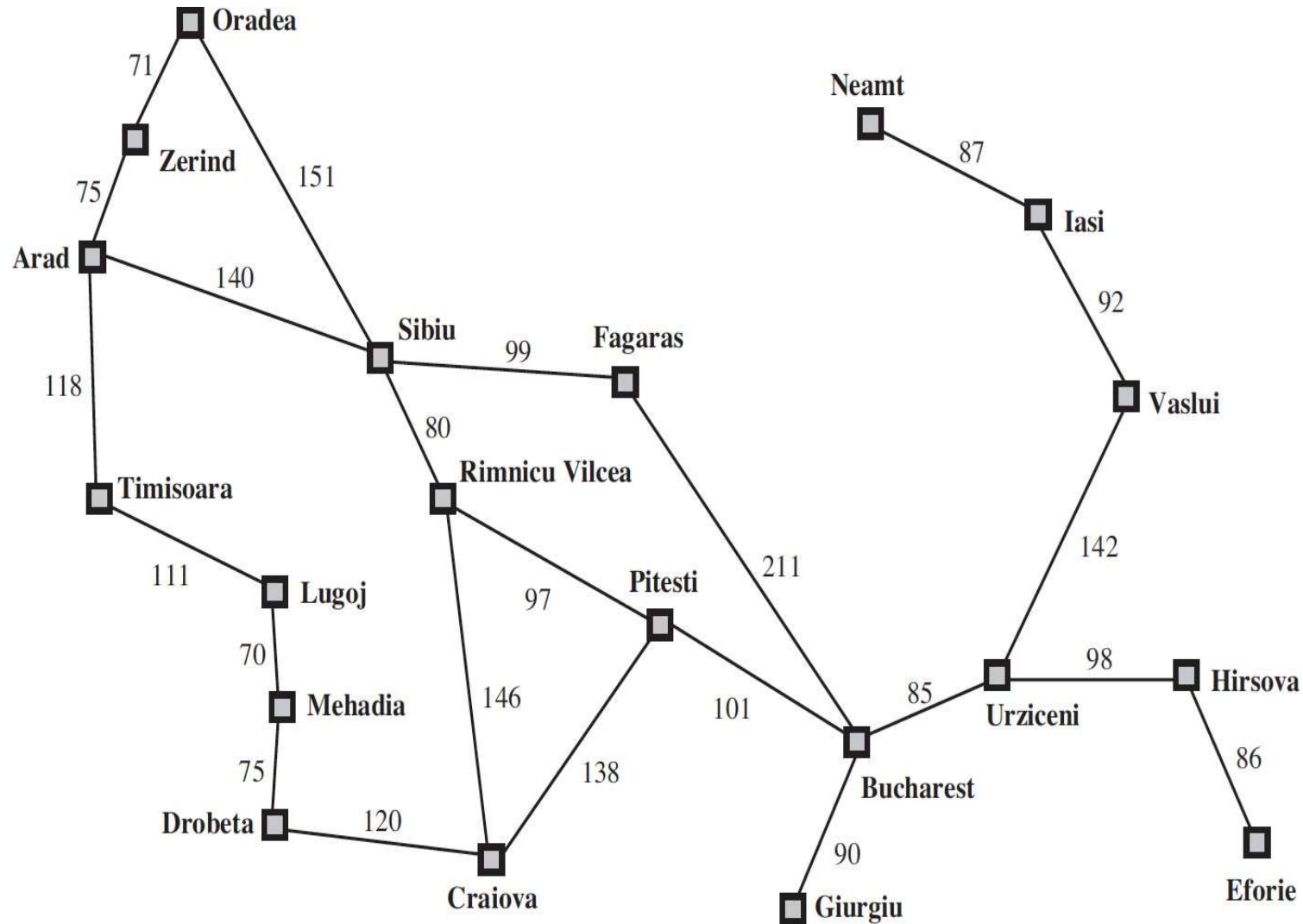




# Terminologies – Learnt Today

- Nodes
- States
- Frontier | Fringes
- Search Strategy : LIFO | FIFO | Priority Queue
- Performance Metrics
  - Completeness
  - Optimality
  - Time Complexity
  - Space Complexity
- Algorithm Terminology
  - d Depth of a node
  - b Branching factor
  - n – nodes
  - l – level of a node
  - m – maximum
  - C\* - Optimal Cost
  - E – least Cost
  - N –total node generated

# Tree Search Vs Graph Search



## Coding Aspects

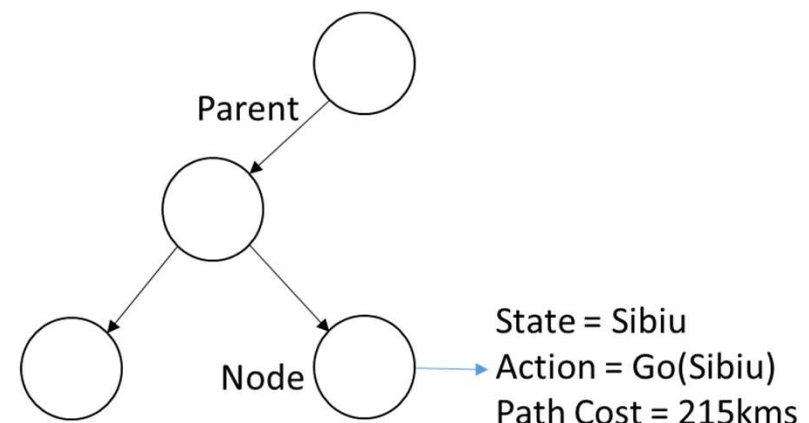
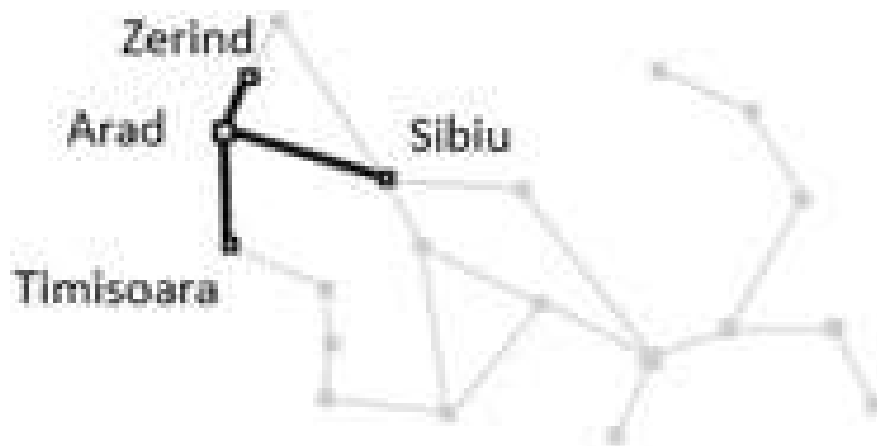
For each node  $n$  of the tree,

**n.STATE** : the state in the state space to which node corresponds

**n.PARENT** : the node in the search tree that generated this node

**n.ACTION** : the action that was applied to parent to generate the node

**n.PATH-COST** : the cost, denoted by  $g(n)$ , of the path from initial state to node



# Algorithm Tracing



Students must follow this in the exams for all the search algorithms in addition to the search tree constructions. The ordering of the Open Lists must be in consistent with the algorithm with a note on the justification of the order expected!

Iter	Open List / Frontiers / Fringes	Goal Test
1.	(1)	Fail on (1)
2.	(1 3), (1 4), (1 2)	Fail on (1 3)



# Tree Search Algorithms

```
function Tree-Search (problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problems
  loop do
    if there are no candidate for expansion
      then return failure
    choose: leaf node for expansion according to strategy
    if the node contains a goal state
      then return the corresponding solution
    else
      Expand the node
      Add the resulting nodes to the search tree
  end
```



# Tree Search Vs Graph Search Algorithms

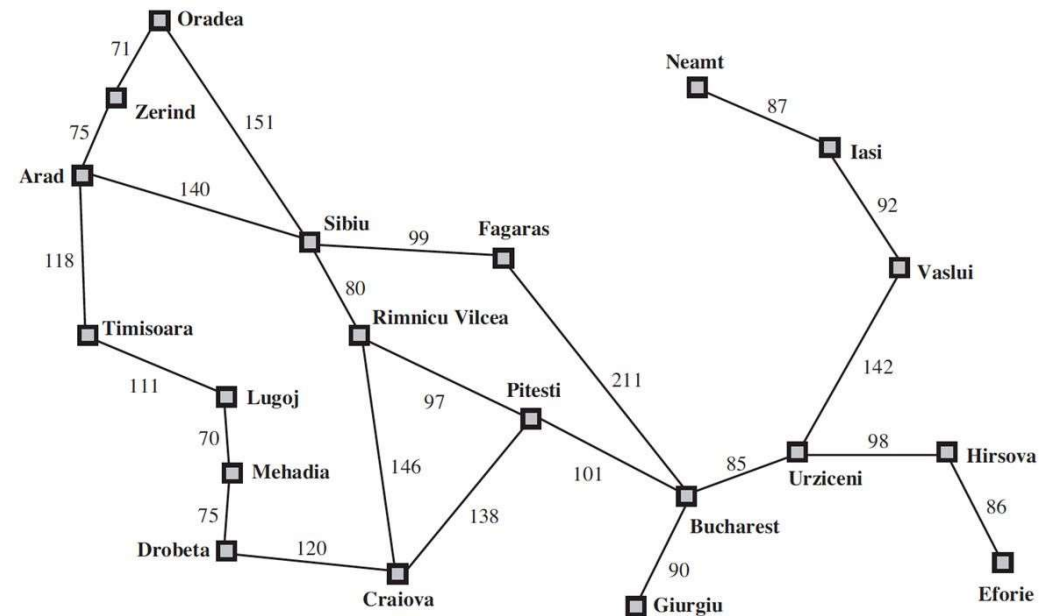


## Coding Aspects

**Need:**

**Redundant Path Problem** : More than one way to reach a state from another.

Infinite Loop Path Problem



Start : Arad

Goal : Craiova

# Tree Search Vs Graph Search Algorithms

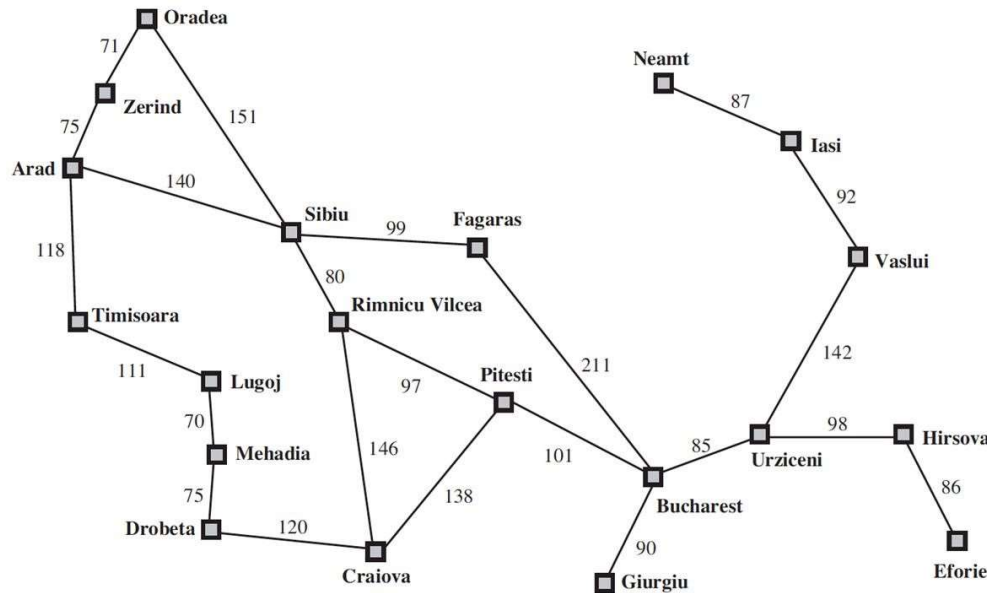


## Coding Aspects

### Need:

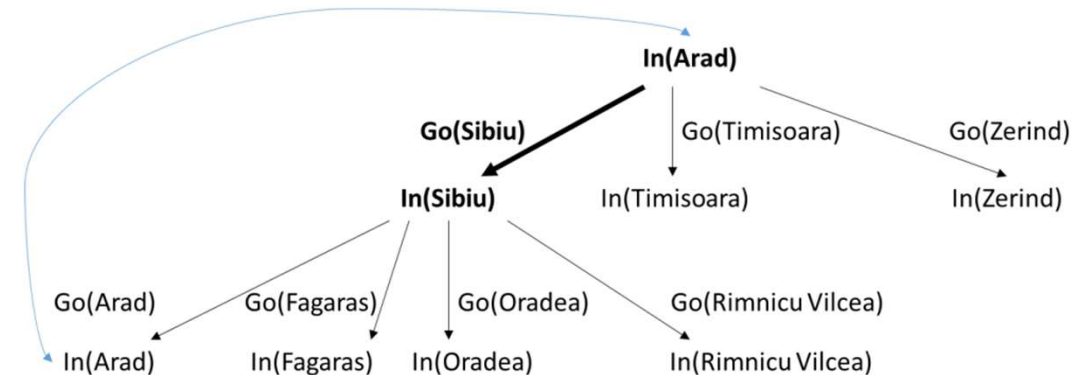
Redundant Path Problem

**Infinite Loop Path Problem:** Repeated State generated by looped path existence.



Start : Arad

Goal : Craiova



# Algorithm Tracing



Students must follow this in the exams for all the search algorithms in addition to the search tree constructions. The ordering of the Open Lists must be in consistent with the algorithm with a note on the justification of the order expected!

Iter	Open List / Frontiers / Fringes	Closed List	Goal Test
1.	(1)		Fail on (1)
2.	(1 3), (1 4), (1 2)	(1)	Fail on (1 3)

## Coding Aspects

For each node  $n$  of the tree,

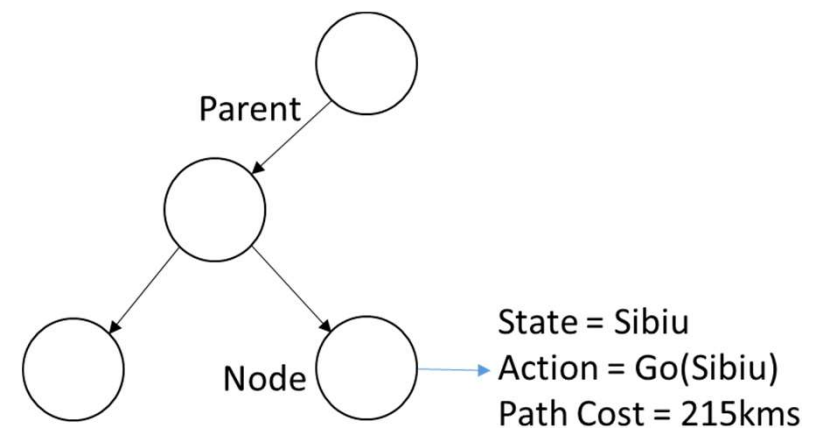
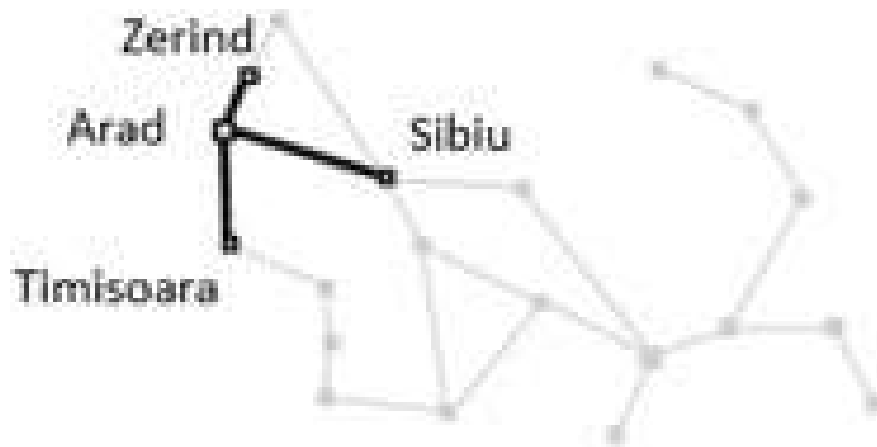
**n.STATE** : the state in the state space to which node corresponds

**n.PARENT** : the node in the search tree that generated this node

**n.ACTION** : the action that was applied to parent to generate the node

**n.PATH-COST** : the cost, denoted by  $g(n)$ , of the path from initial state to node

**n.VISITED** : the boolean indicating if the node is already visited and tested (**or**)  
a global SET of visited nodes



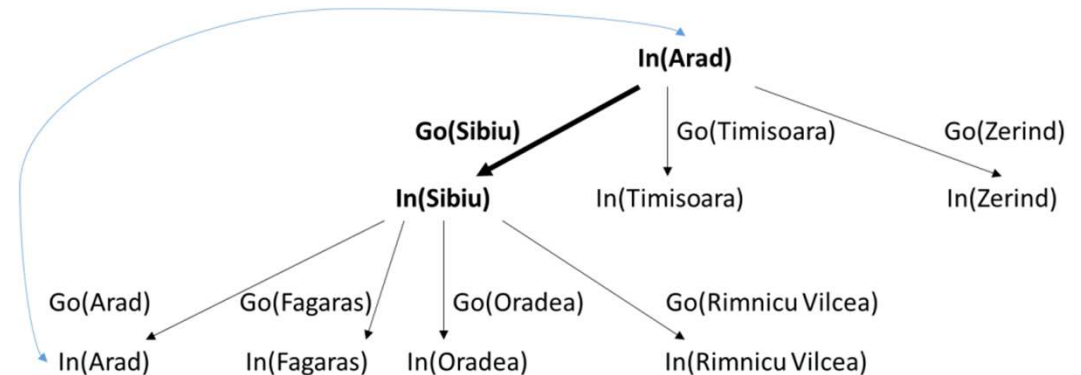
# Tree Search Vs Graph Search Algorithms



## Coding Aspects

### Graph-Search Algorithm

Augments the Tree-Search algorithm to solve redundancy by keeping track of states that are already visited called as **Explored Set**. **Only one copy of each state is maintained/stored.**





# Graph Search Algorithms

```
function Graph-Search (problem, fringe) returns a solution, or failure
    initialize the search space using the initial state of problems memory to store
    the visited fringe
    closed ? an empty set
    fringe ? Insert(Make-Node(Initial-State[problem]), fringe)
    loop do
        if fringe is empty
            then return failure
        node? Remove-Front(fringe)
        if the node contains a goal state
            then return the corresponding solution
        else
            if the node is not in closed ie., not visited yet
                Add the node to the closed set
                Expand all the fringe of the node
                Add all expanded sorted successors into the fringe
    end
```



# Informed Search

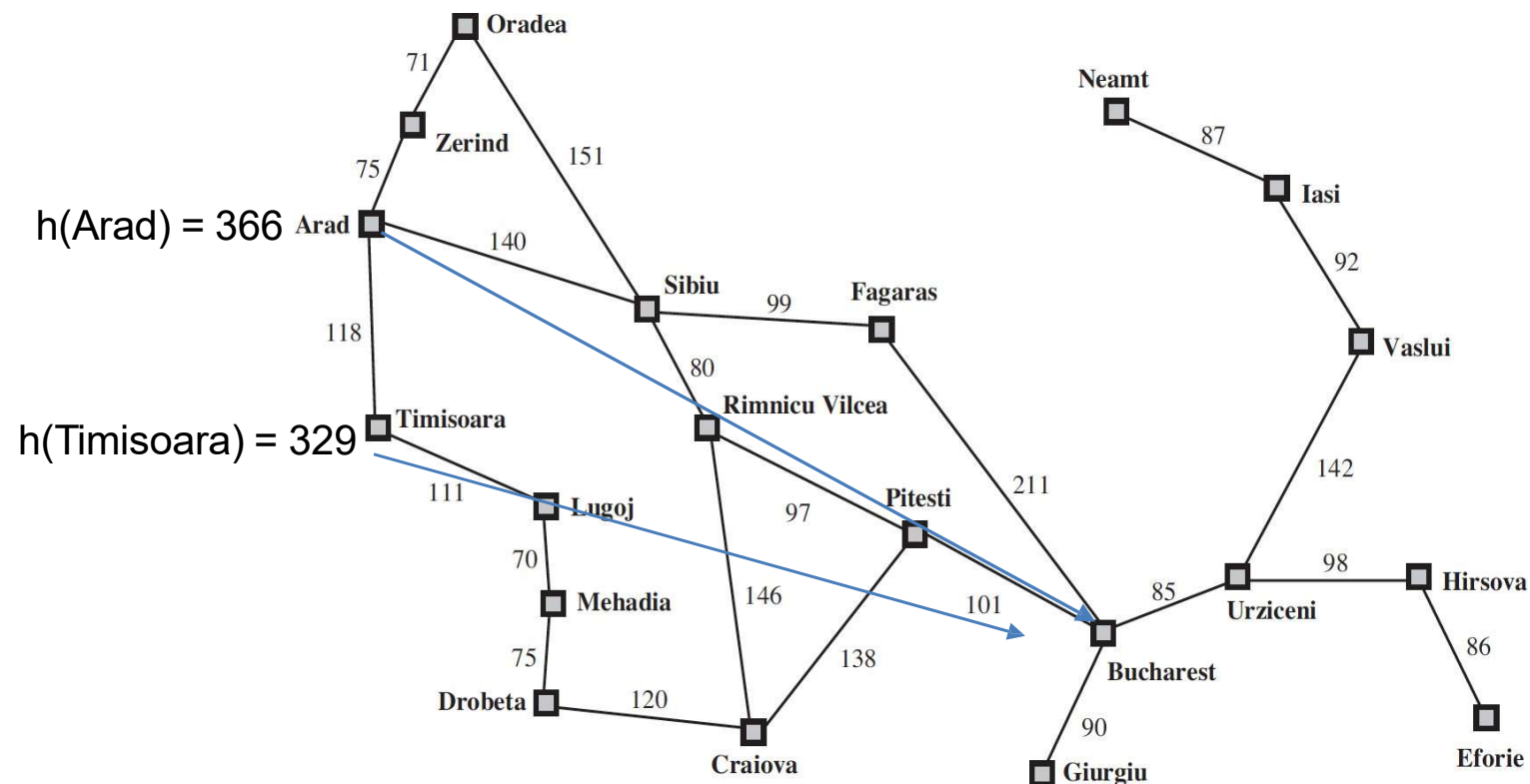
Greedy Best First

A\*

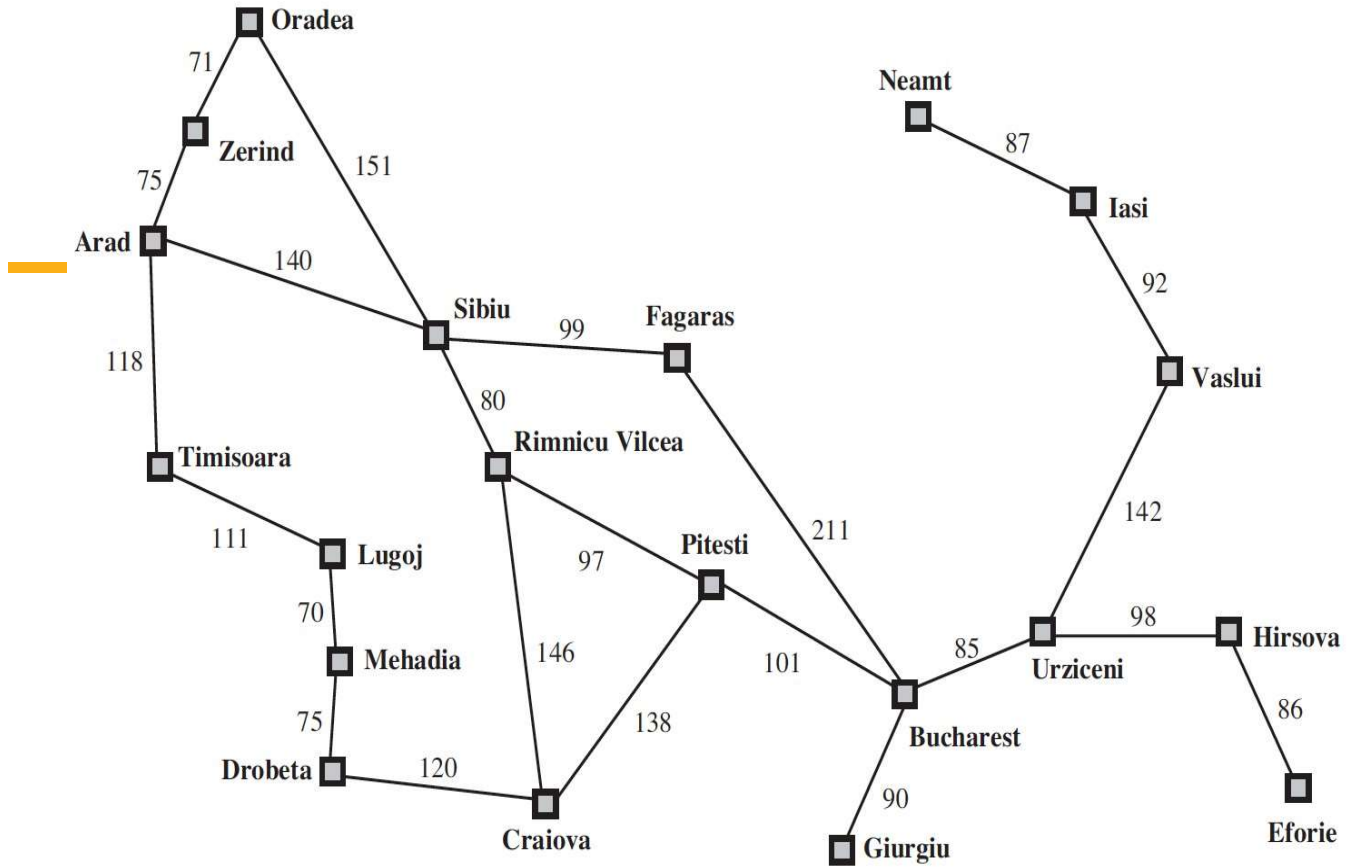
# Informed /Heuristic Search



Strategies that know if one non-goal state is more promising than another non-goal state







Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374



# Greedy Best First Search

Expands the node that is closest to the goal

Thus,  $f(n) = h(n)$

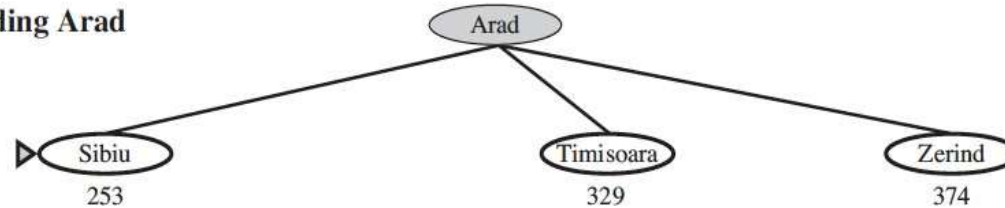
<b>Arad</b>	366	<b>Mehadia</b>	241
<b>Bucharest</b>	0	<b>Neamt</b>	234
<b>Craiova</b>	160	<b>Oradea</b>	380
<b>Drobeta</b>	242	<b>Pitesti</b>	100
<b>Eforie</b>	161	<b>Rimnicu Vilcea</b>	193
<b>Fagaras</b>	176	<b>Sibiu</b>	253
<b>Giurgiu</b>	77	<b>Timisoara</b>	329
<b>Hirsova</b>	151	<b>Urziceni</b>	80
<b>Iasi</b>	226	<b>Vaslui</b>	199
<b>Lugoj</b>	244	<b>Zerind</b>	374



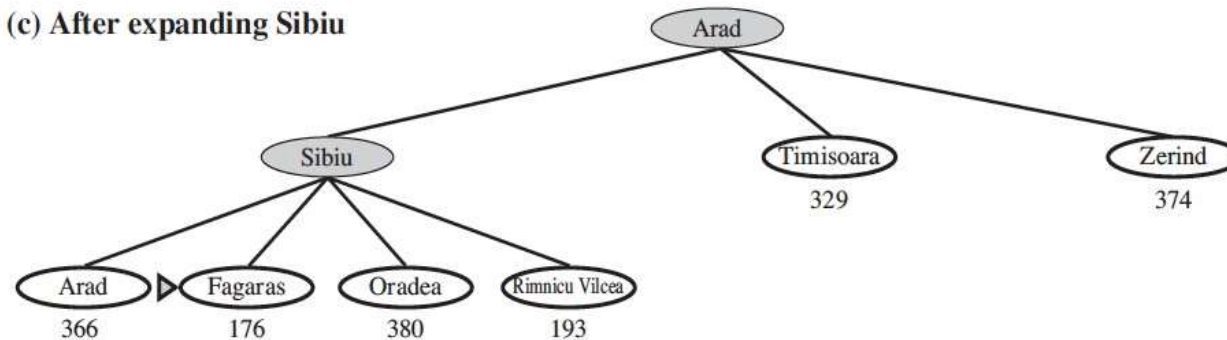
(a) The initial state



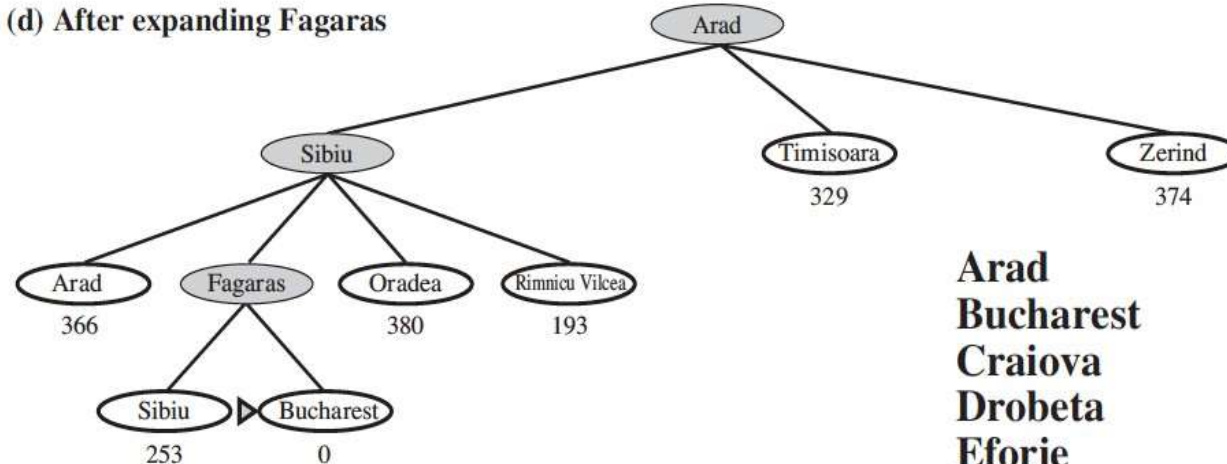
(b) After expanding Arad



(c) After expanding Sibiu



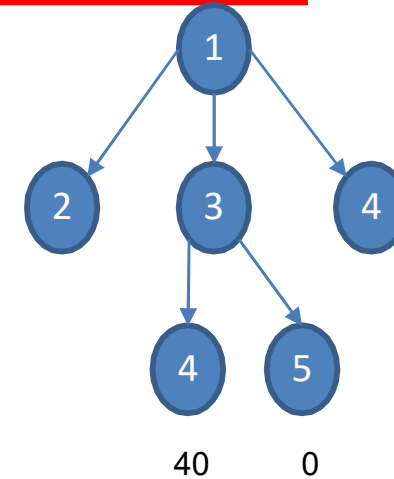
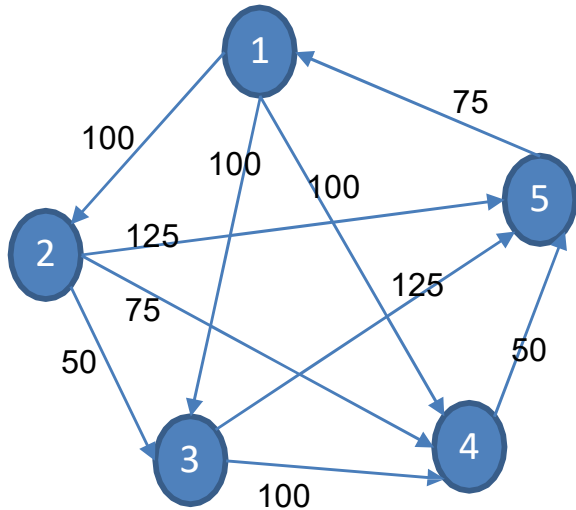
(d) After expanding Fagaras



Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244

Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

# Greedy Best First Search

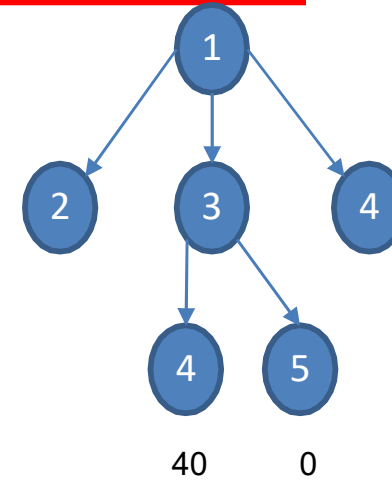
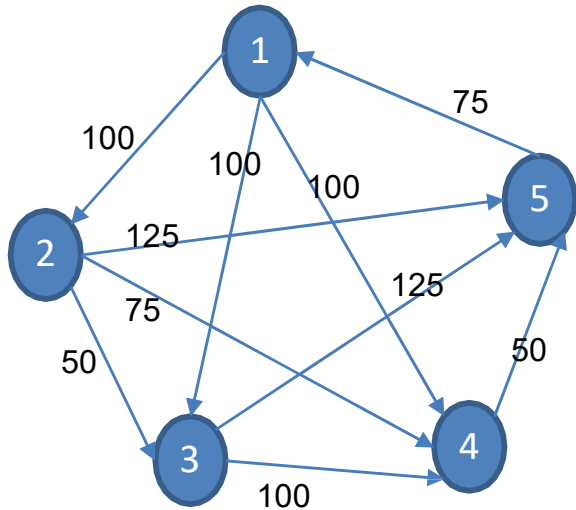


n	h(n)
1	60
2	120
3	30
4	40
5	0

(1)  
 (1 3) (1 4) (1 2)  
 (1 3 5) (1 3 4)

$C(1-3-5) = 100 + 125 = 225$   
 Expanded : 2  
 Generated : 6  
 Max Queue Length : 3

# Greedy Best First Search



n	h(n)
1	60
2	120
3	30
4	40
5	0

(1)  
 (1 3) (1 4) (1 2)  
 (1 3 5) (1 3 4)

$C(1-3-5) = 100 + 125 = 225$   
 Expanded : 2  
 Generated : 6  
 Max Queue Length : 3

# A\* Search



Expands the node which lies in the closest path (estimated cheapest path) to the goal

Evaluation function  $f(n) = g(n) + h(n)$

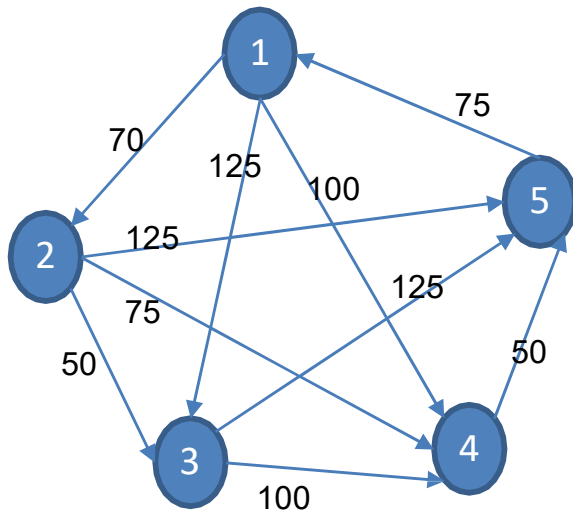
$g(n)$  – the cost to reach the node

$h(n)$  – the expected cost to go from node to goal

$f(n)$  – estimated cost of cheapest path through node  $n$

<b>Arad</b>	366	<b>Mehadia</b>	241
<b>Bucharest</b>	0	<b>Neamt</b>	234
<b>Craiova</b>	160	<b>Oradea</b>	380
<b>Drobeta</b>	242	<b>Pitesti</b>	100
<b>Eforie</b>	161	<b>Rimnicu Vilcea</b>	193
<b>Fagaras</b>	176	<b>Sibiu</b>	253
<b>Giurgiu</b>	77	<b>Timisoara</b>	329
<b>Hirsova</b>	151	<b>Urziceni</b>	80
<b>Iasi</b>	226	<b>Vaslui</b>	199
<b>Lugoj</b>	244	<b>Zerind</b>	374

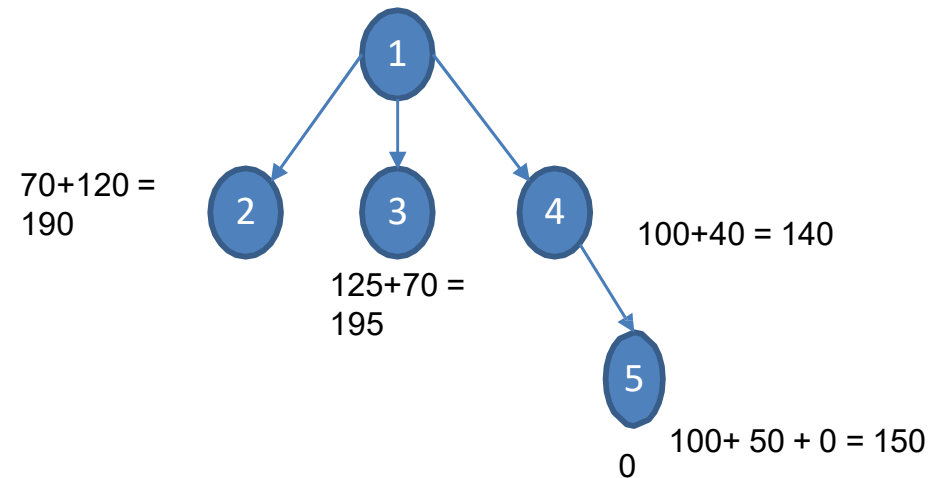
# A\* Search



n	h(n)
1	60
2	120
3	70
4	40
5	0

(1)  
 (1 4) (1 2) (1 3)  
 (1 4 5) (1 2) (1 3)

C(1-4-5) = 100 + 150 =150  
 Expanded : 2  
 Generated : 5  
 Max Queue Length : 3





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**Required Reading:** AIMA - Chapter #3: 3.1, 3.2, 3.3, 3.4, 3.5

Next Class Plan :  
Informed Search : GFBS & A\*  
Heuristic Design

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials