Artificial & Computational Intelligence

AIML CLZG557

M4 : Knowledge Representation using Logics

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Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

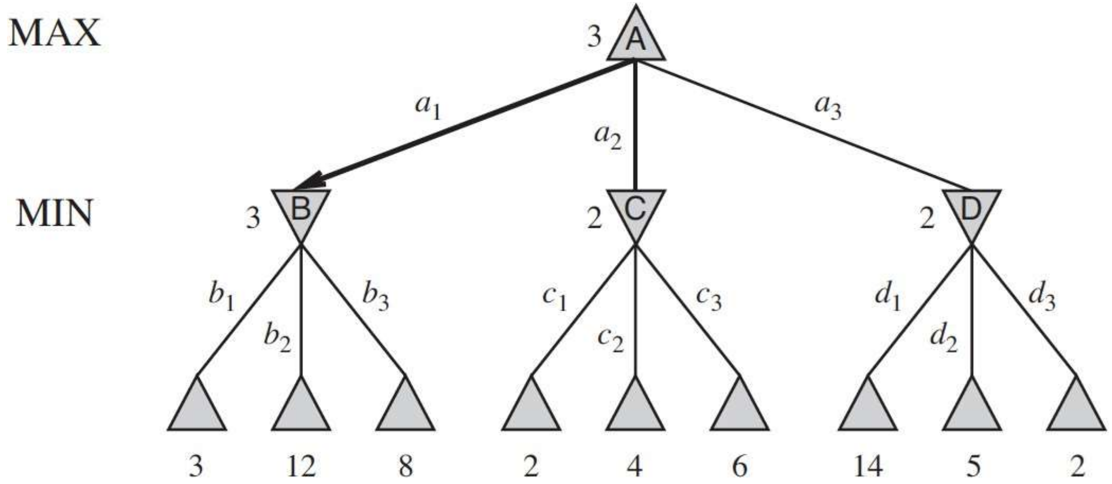
M6 Reasoning over time

M7 Ethics in AI

Gaming
(Imperfect Decisions)

Computational Efficiency

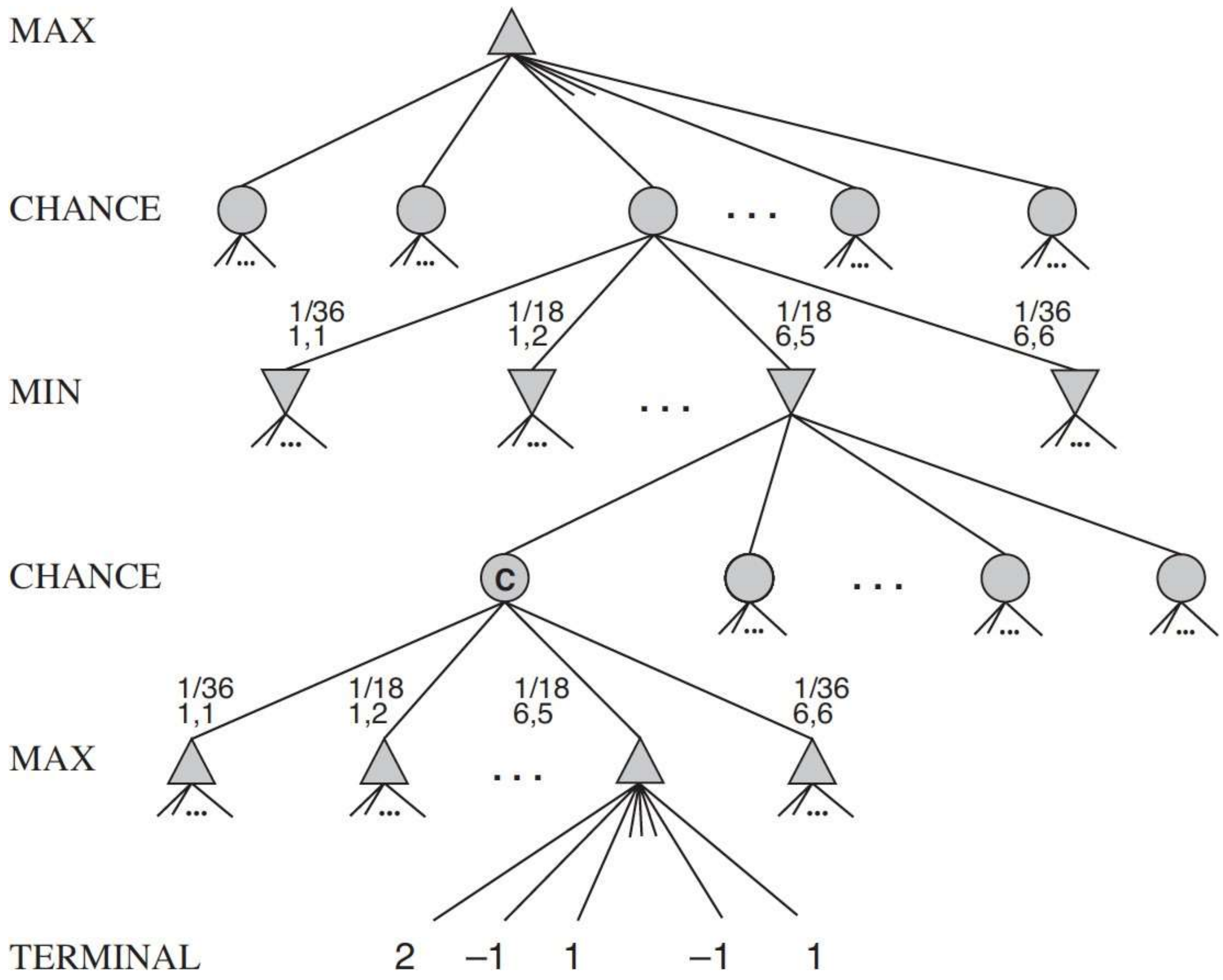
How games can be designed to handle imperfect decisions in real-time?



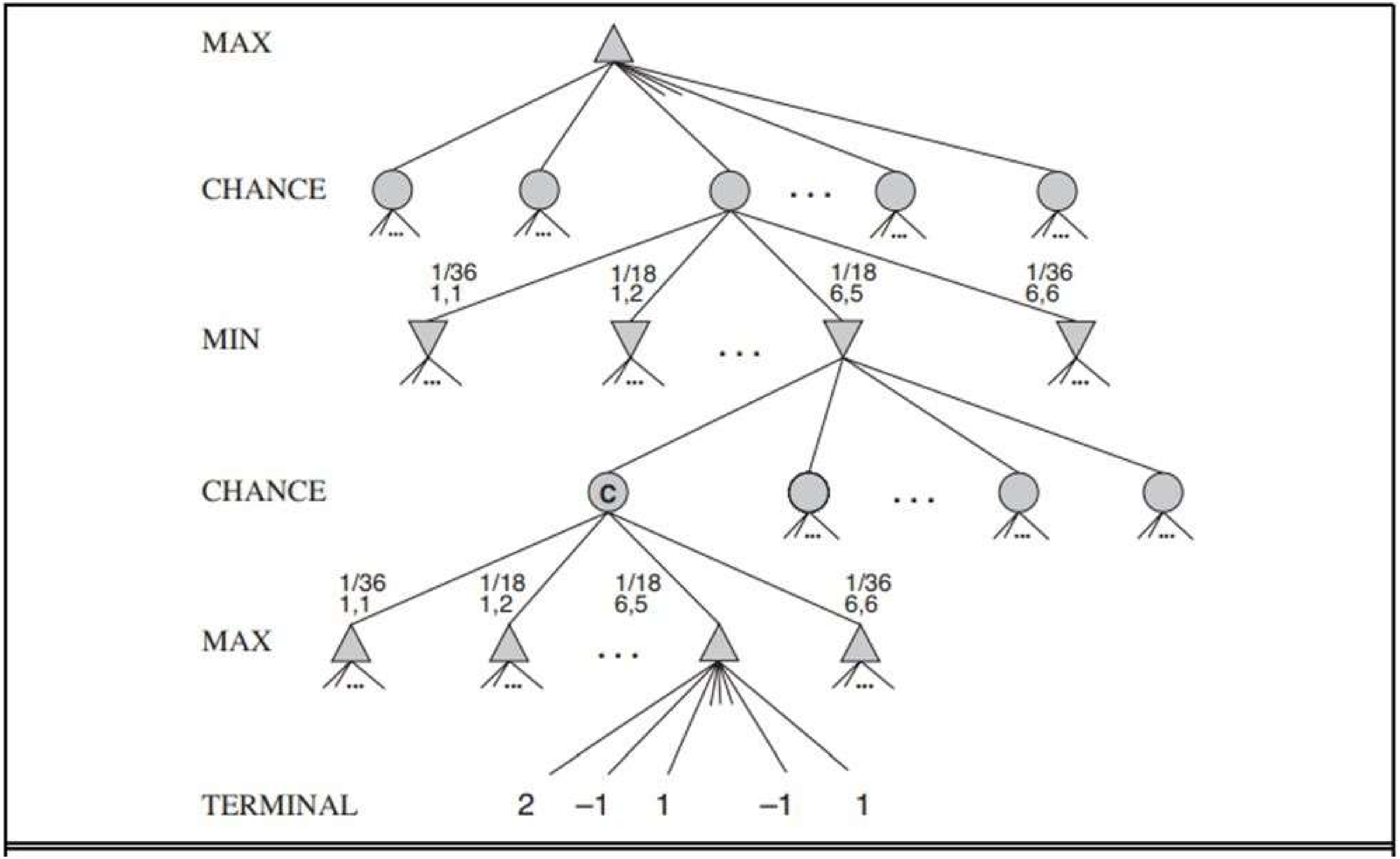
Computational Efficiency

Idea : Chance Node:

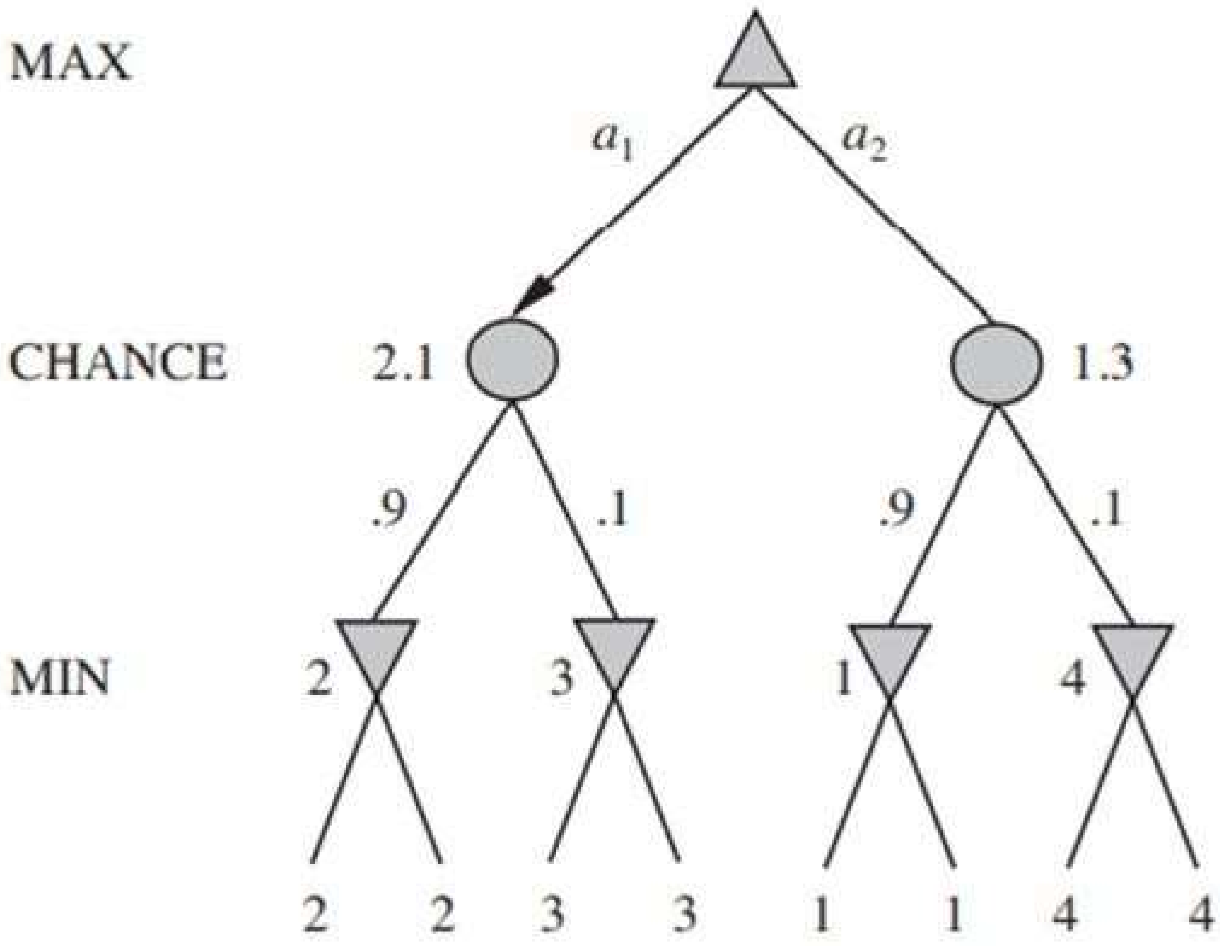
Holds the expected values that are computed as a sum of all outcomes weighted by their probability (of dice roll)

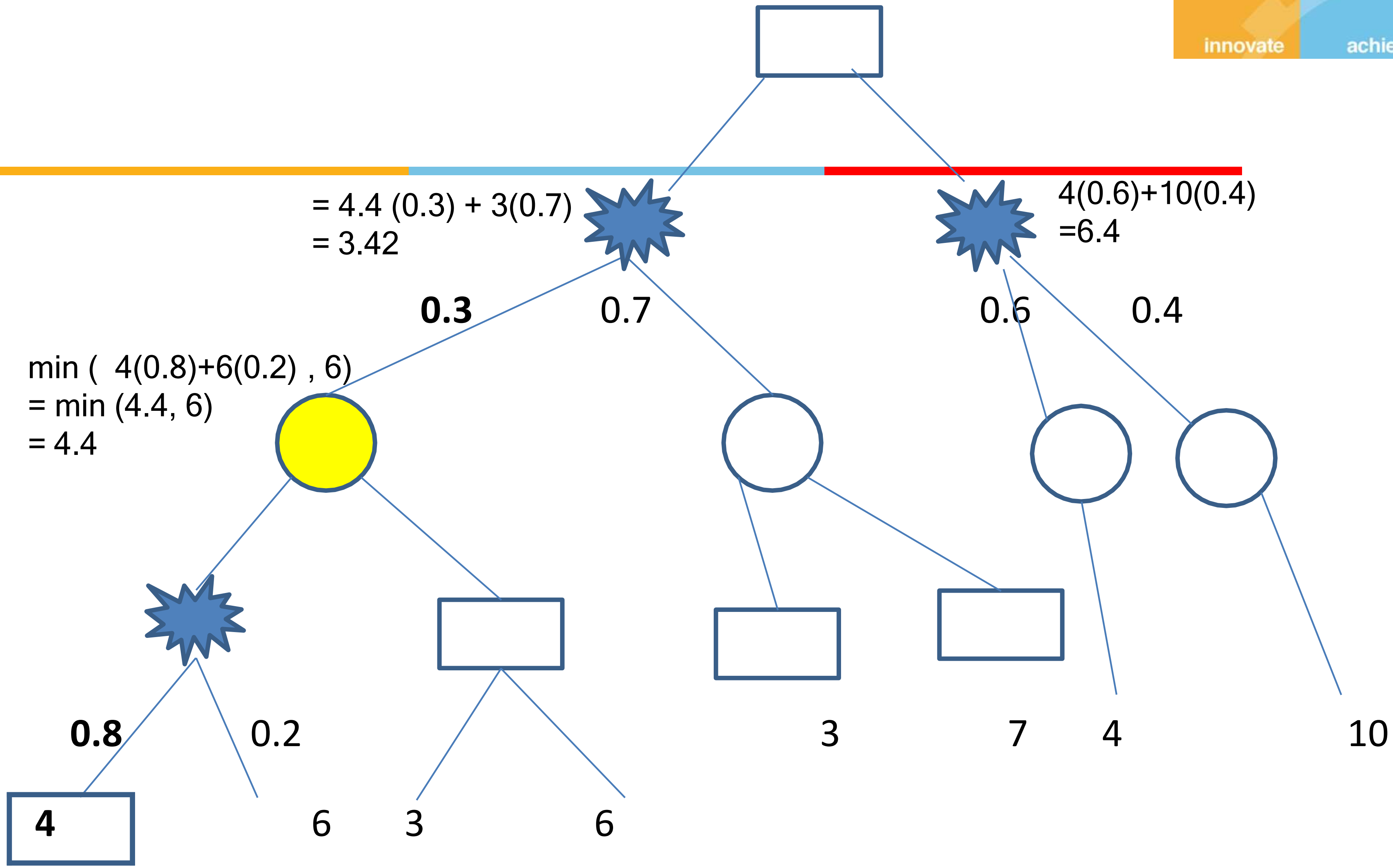


Expecti Mini Max Algorithm



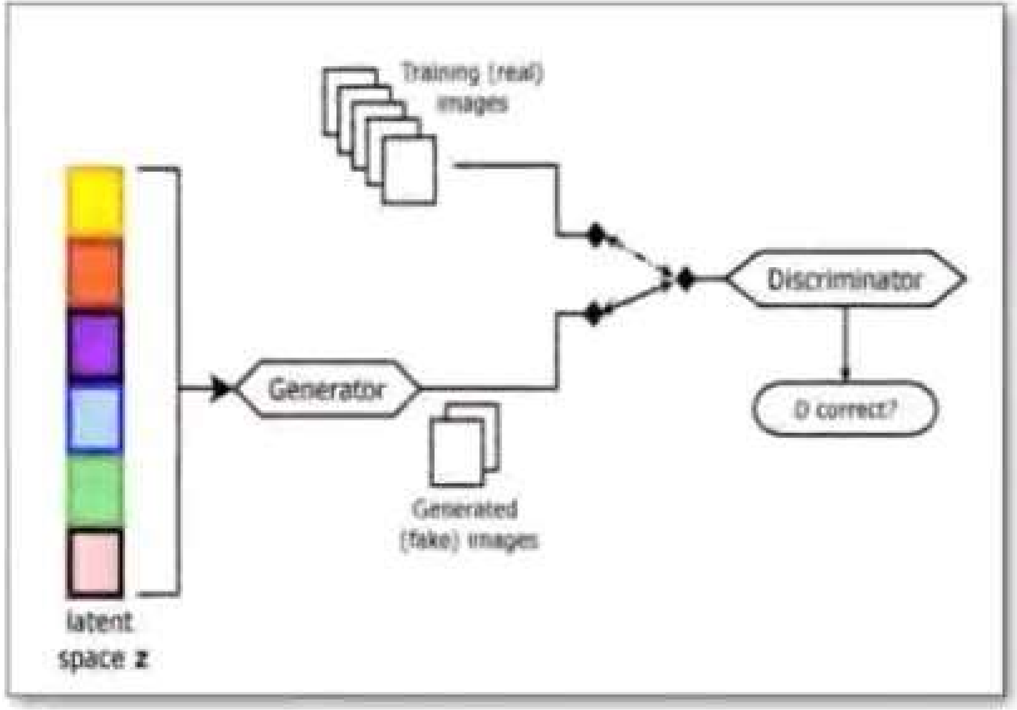
Expecti Mini Max Algorithm




= 4.4 (0.3) + 3(0.7)
= 3.42
4(0.6)+10(0.4)
=6.4
0.3
0.7
0.6
0.4
min (4(0.8)+6(0.2) , 6)
= min (4.4, 6)
= 4.4
0.8
0.2
3
7
4
10
4
6
3
6

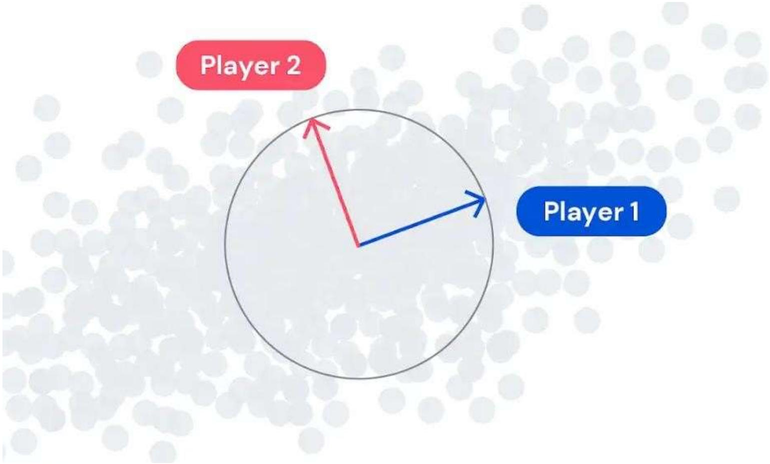
Game Playing
(Interesting Case Studies)

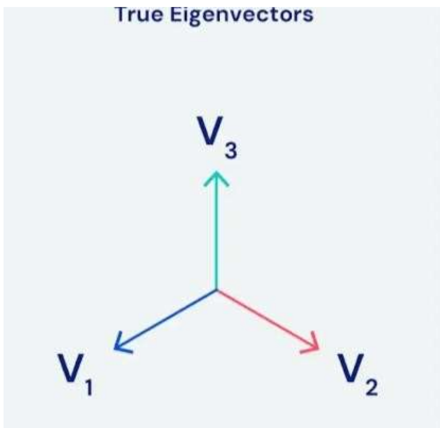
Games in Image Processing

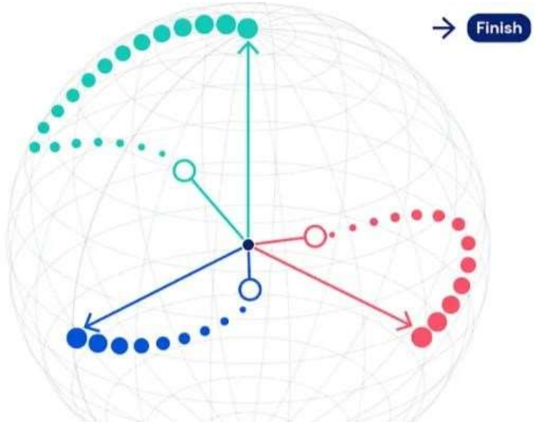


Source Credit:
2019 - Analyzing and Improving the Image Quality of StyleGAN
Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, Timo Aila
https://thispersondoesnotexist.com/

Games in Feature Engineering







Source Credit:
https://deepmind.com/blog/article/EigenGame
2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel

Games in Feature Engineering

$$\mathtt{Utility}(v_i \mid v_{j<i}) = \boxed{\mathtt{Var}(v_i)} - \sum_{j<i} \boxed{\mathtt{Align}}(v_i, v_j)$$

Source Credit:
https://deepmind.com/blog/article/EigenGame
2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel

Knowledge Representation Using Logics

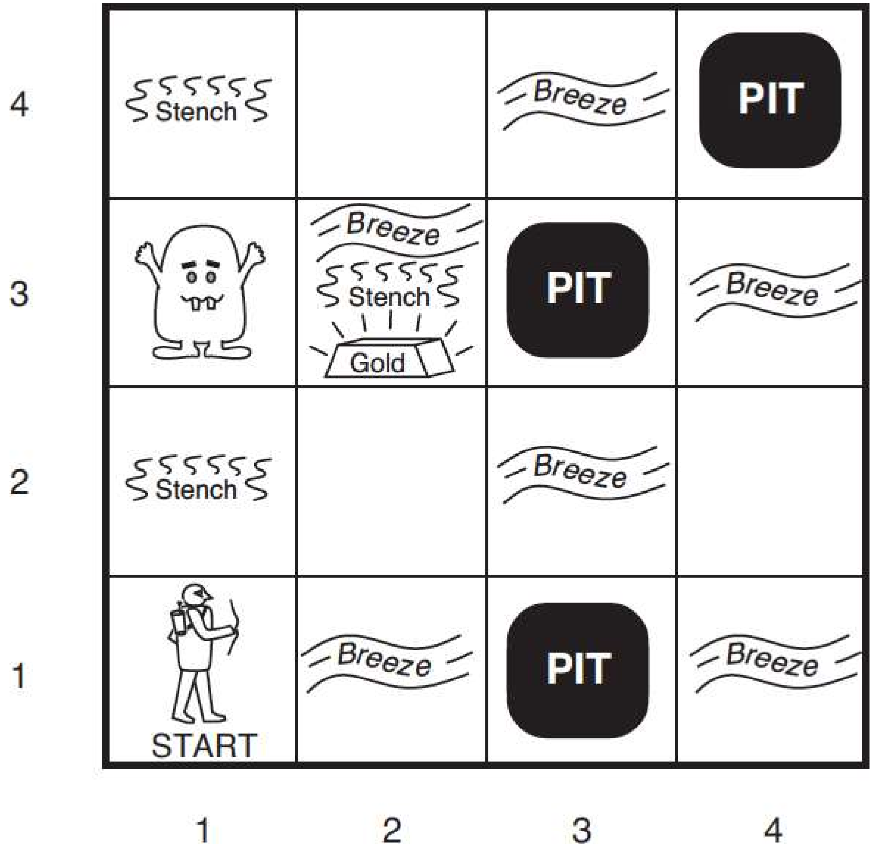
Learning Objective

At the end of this class , students should be able to:

1. Represent a given knowledge base into logic formulation
2. Infer facts from KB using Resolution
3. Infer facts from KB using Forward Chaining
4. Infer facts from KB using Backward Chaining

Knowledge based Agent : Model & Represent

Concepts, logic Representation of a sample agent



Wumpus World Problem:

PEAS:

Performance Measure:

+1000 for climbing out with gold,

-1000 for falling into a pit or being eaten by Wumpus,

-1 for each action taken and

-10 for using an arrow

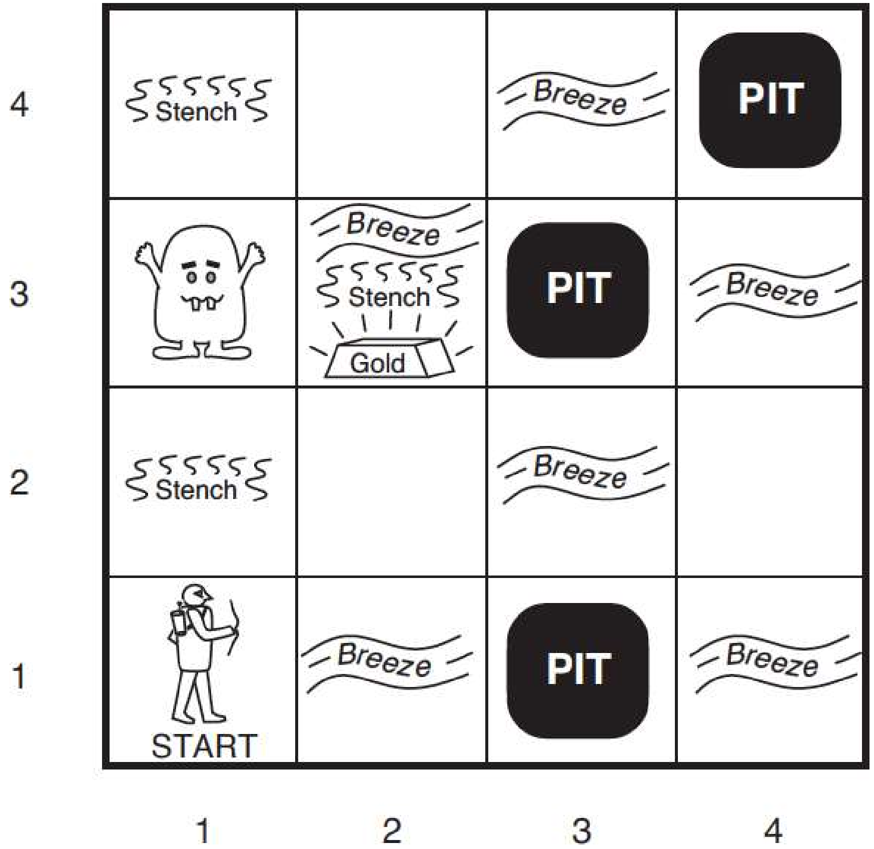
Environment: 4x4 grid of rooms. Always starts at [1, 1] facing right.

The location of Wumpus and Gold are random.

Agent dies if entered a pit or live Wumpus.

Knowledge based Agent : Model & Represent

Concepts, logic Representation of a sample agent



Wumpus World Problem:

PEAS:

Actuators –

Forward,

TurnLeft by 90,

TurnRight by 90,

Grab – pick gold if present,

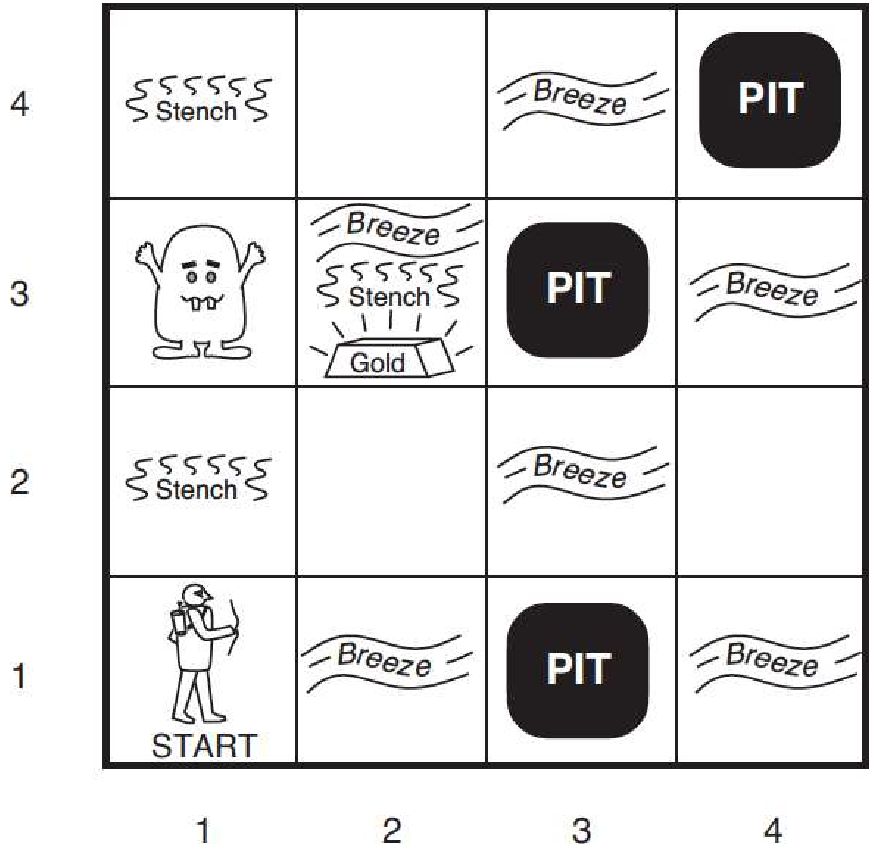
Shoot – fire an arrow, it either hits a wall or kills wumpus. Agent has only one arrow.

Climb – Used to climb out of cave, only from [1, 1]

Knowledge based Agent : Model & Represent

innovate achieve lead

Concepts, logic Representation of a sample agent



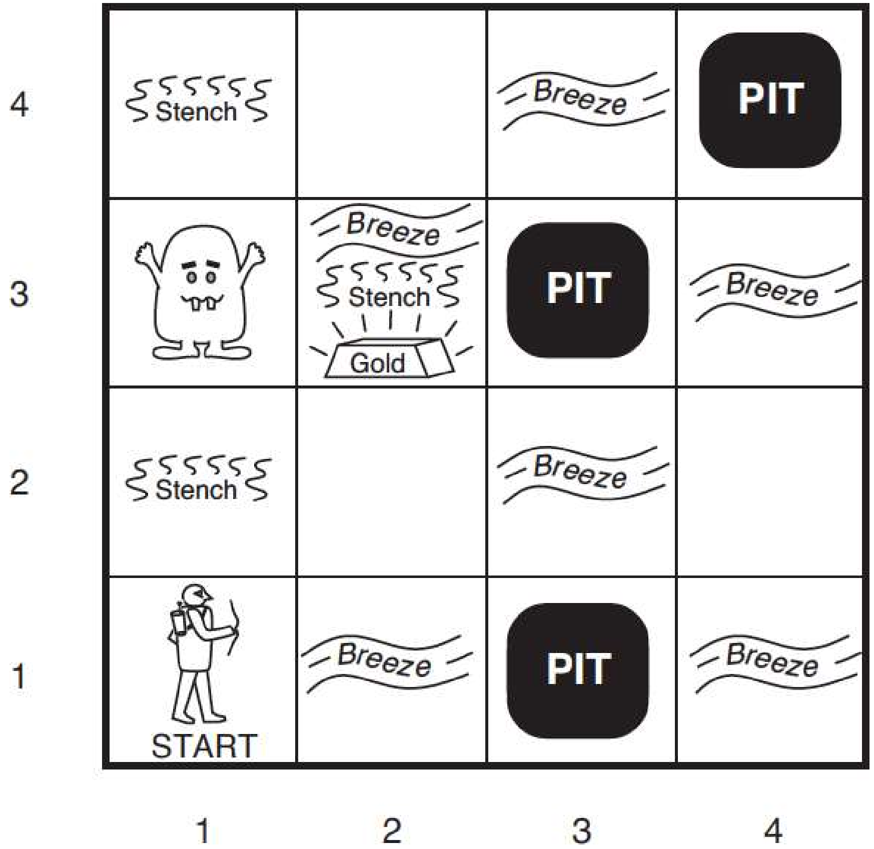
Why do we need Factored representation

- To reason about steps
- To learn new knowledge about the environment
- To adapt to changes to the existing knowledge
- Accept new tasks in the form of explicit goals
- To overcome partial observability of environment

Knowledge based Agent : Model & Represent

innovate achieve lead

Concepts, logic Representation of a sample agent



Wumpus World Problem:

PEAS:

Sensors. The agent has five sensors

- **S**tench: In all adjacent (but not diagonal) squares of Wumpus
- **B**reeze: In all adjacent (but not diagonal) squares of a pit
- **G**litter: In the square where gold is
- **B**ump: If agent walks into a wall
- **S**cream: When Wumpus is killed, it can be perceived everywhere

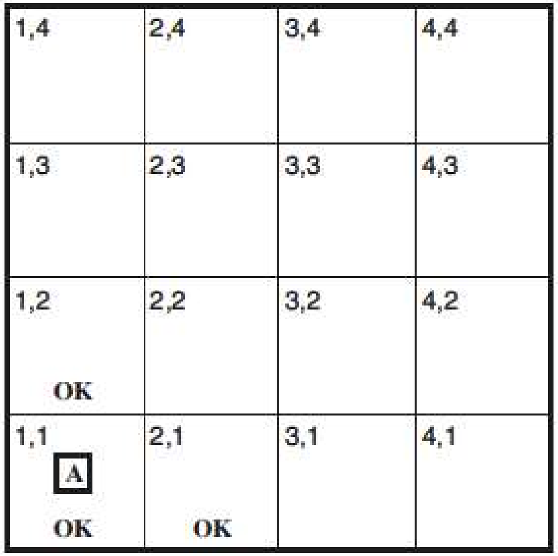
Percept Format:

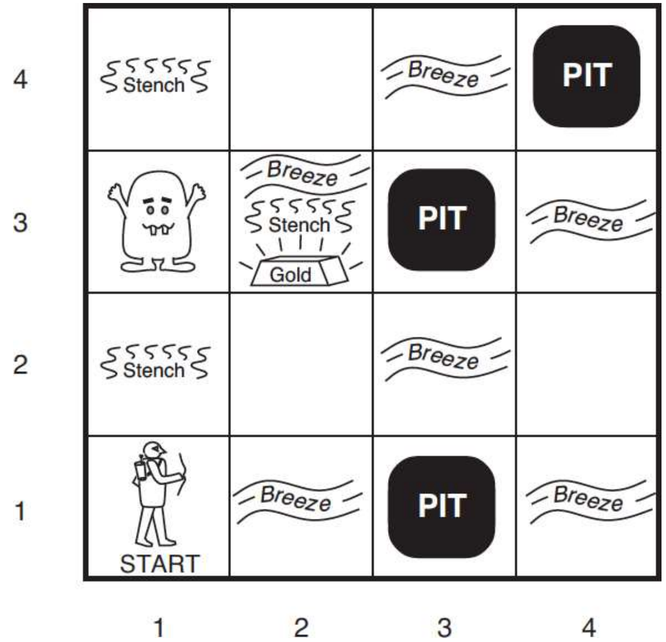
[Stench?, Breeze?, Glitter?, Bump?, Scream?]

E.g., [Stench, Breeze, None, None, None]

Percept 1: [None, None, None, None, None]

Action: Forward





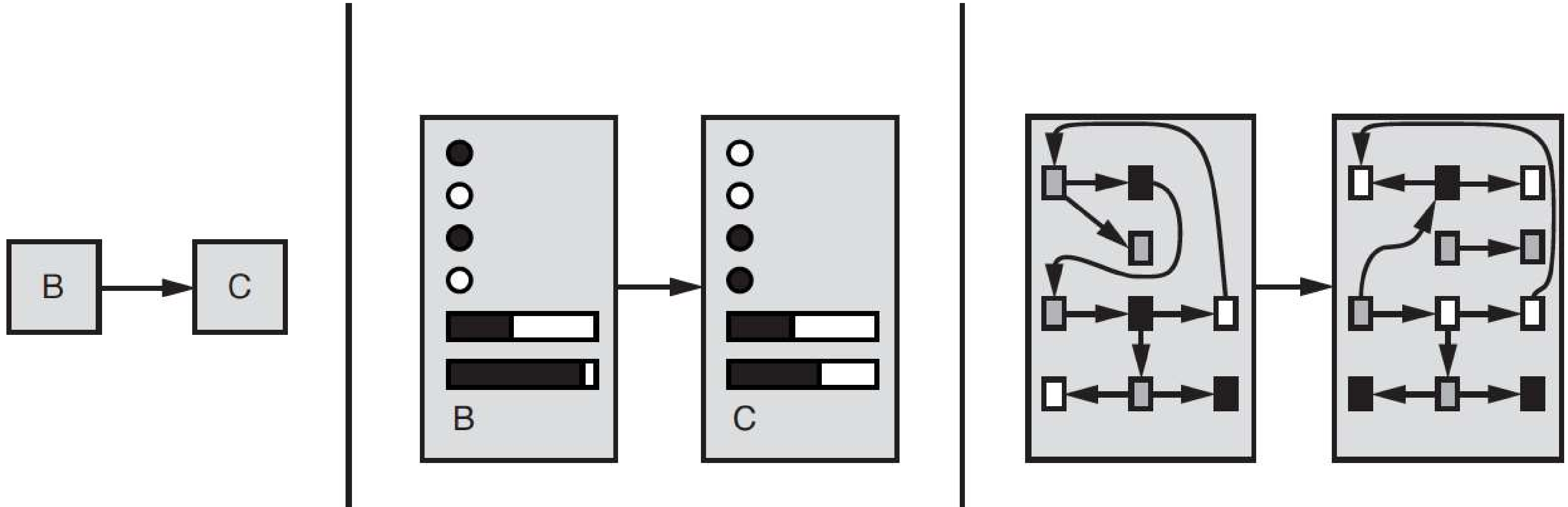
Percept Format:
[Stench?, Breeze?, Glitter?, Bump?, Scream?]

Representation

Agents based on Propositional logic, TT-Entail for inference from truth table

Syntax
Semantics
Model

→ Logic → Propositional Logic / Predicate Logic



(a) Atomic — (b) Factored — (b) Structured

Search Strategies — Propositional Logic — First Order Logic

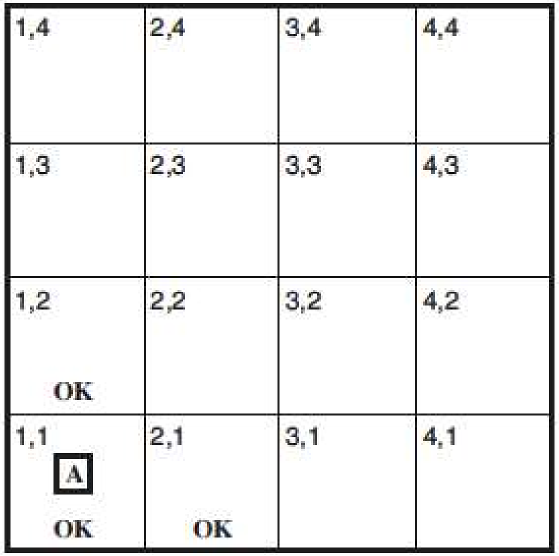
Propositional Logic

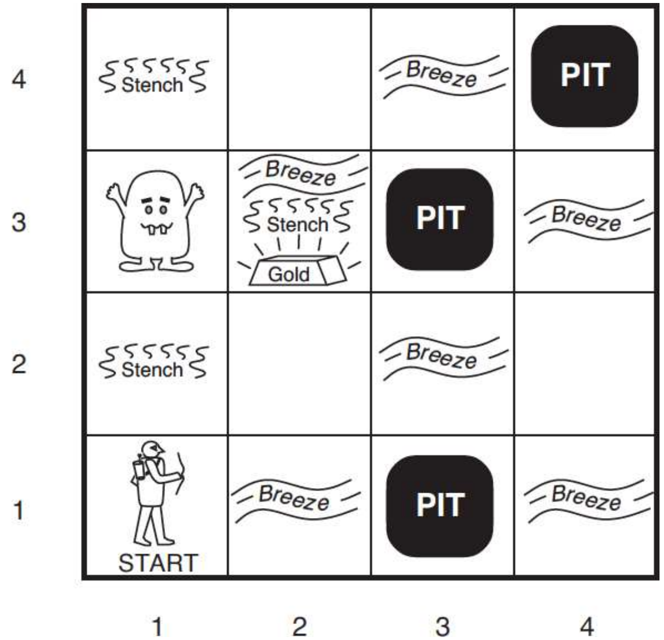
Agentsbasedon Propositionallogic,TT-Entailforinferencefromtruthtable

A simple representation language for building knowledge-based agents

Proposition Symbol - A symbol that stands for a proposition.
E.g., W1,3 - "Wumpus in [1,3]" is a proposition and $W_{1,3}$ is the symbol
Proposition can be true or false

Atomic : $W_{1,3}$
Conjuncts : $W_{1,3} \wedge P_{3,1}$
Disjuncts : $W_{1,3} \vee P_{3,1}$
Implications :
$(W_{1,3} \wedge P_{3,1}) \Rightarrow \neg W_{2,2}$
Biconditional : $W_{1,3} \Leftrightarrow \neg W_{2,2}$





Propositional Logic

Tie break in search:

$\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$

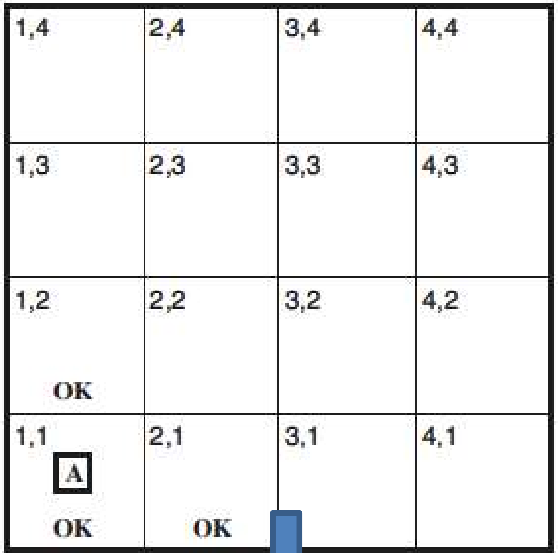
$(\neg A) \wedge B$ has precedence over $\neg (A \wedge B)$

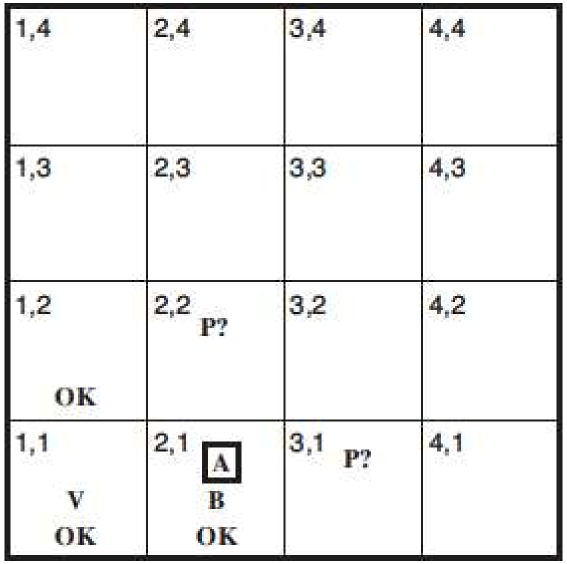
P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

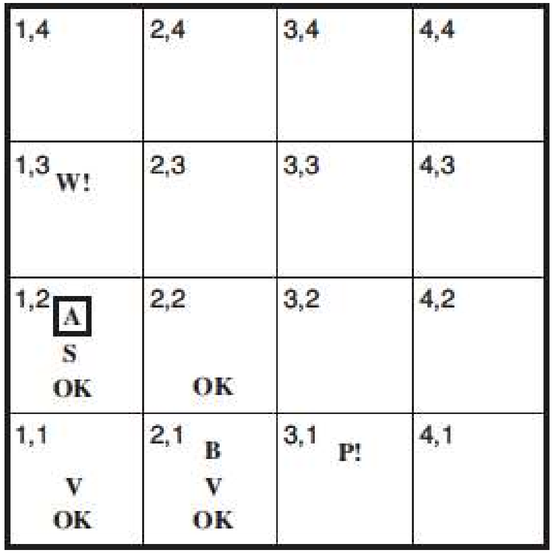
Percept 3: [Stench, None, None, None, None]

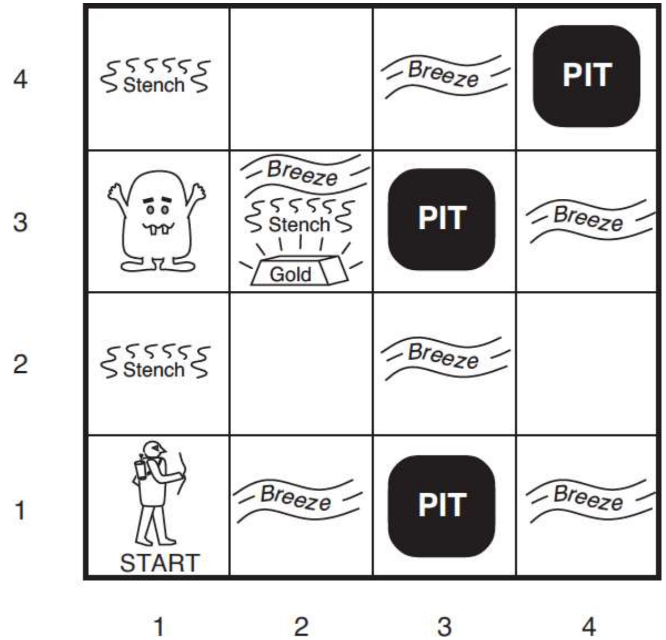
Action: Move to [2, 2]

Remembers (2,2) as possible PIT and no Stench.









Representation by Propositional Logic

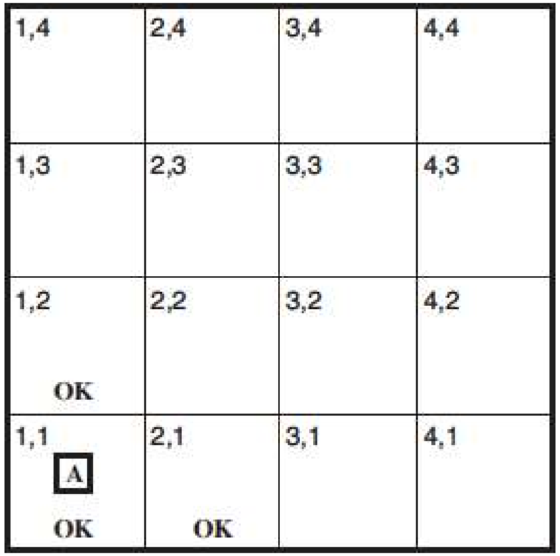
For each [x, y] location

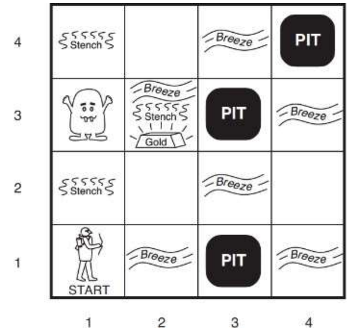
$P_{x,y}$ is true if there is a pit in [x, y]

$W_{x,y}$ is true if there is a wumpus in [x, y]

$B_{x,y}$ is true if agent perceives a breeze in [x, y]

$S_{x,y}$ is true if agent perceives a stench in [x, y]





------------------------ R is the sentence in KB

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query : $\neg P_{1,2}$ entailed by our KB?

TT – Entails Inference – Example

$\neg P_{1,2}$ entailed by our KB?

Way – 1 :

1. Get sufficient information $B_{1,1}$, $B_{2,1}$, $P_{1,1}$, $P_{1,2}$, $P_{2,1}$, $P_{2,2}$, $P_{3,1}$
2. Enumerate all models with combination of truth values to propositional symbols
3. In all the models, find those models where KB is true, i.e., every sentence R_1, R_2, R_3, R_4, R_5 are true
4. In those models where KB is true, find if query sentence $\neg P_{1,2}$ is true
5. If query sentence $\neg P_{1,2}$ is true in all models where KB is true, then it entails, otherwise it won't

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	true
false	true	false	false	false	true	false	true	true	true	true	true	true
false	true	false	false	false	true	true	true	true	true	true	true	true
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

TT – Entails Inference – Example

Agents based on Propositional logic, TT-Entail for inference from truth table

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	*false*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*false*	*false*
false	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*false*	*true*	*false*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	*true*	*false*	*false*	*false*	*false*	*false*	*true*	*true*	*false*	*true*	*true*	*false*
false	*true*	*false*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*false*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*false*	*true*	*true*	*true*	*true*	*true*	*true*	*true*	*true*
false	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*false*	*false*	*true*	*true*	*false*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	*true*	*true*	*true*	*true*	*true*	*true*	*false*	*true*	*true*	*false*	*true*	*false*

Inference : Properties

1. Entailment : $\alpha \models \beta$
2. Equivalence : $\alpha \equiv \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$
3. Validity
4. Satisfiability

Inference : Example – Theorem Proving (Self Study)

Propositional theorem proving - Proof by resolution

Logical Equivalence rules can be used as inference rules

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Inference : Example – Theorem Proving

1. **Modes Ponens**
2. AND Elimination

α : I walk in rain without the umbrella

β : I get wet

α → β　　　　**α**

　　　β

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) && \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) && \text{associativity of } \lor \\
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\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) && \text{De Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Inference : Example – Theorem Proving

1. Modes Ponens
2. **AND Elimination**

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
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((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
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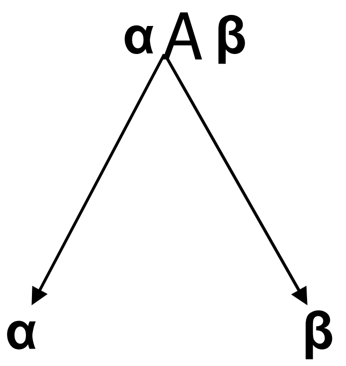
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

α : I walk in rain without the umbrella

β : I get wet



Inference : Example – Theorem Proving

$R_1 : \neg P_{1,1}$

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \lor P_{2,2} \lor P_{3,1})$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
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(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Query: $\neg P_{1,2}$. Can we prove if this sentence be entailed from KB using inference rules?_______________

$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \lor P_{2,1})$

$R_6 : (B_{1,1} \Rightarrow (P_{1,2} \lor P_{2,1})) \land ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$ Biconditional Elimination

$R_7 : ((P_{1,2} \lor P_{2,1}) \Rightarrow B_{1,1})$ And Elimination

$R_8 : (\neg B_{1,1} \Rightarrow \neg (P_{1,2} \lor P_{2,1}))$ Contraposition

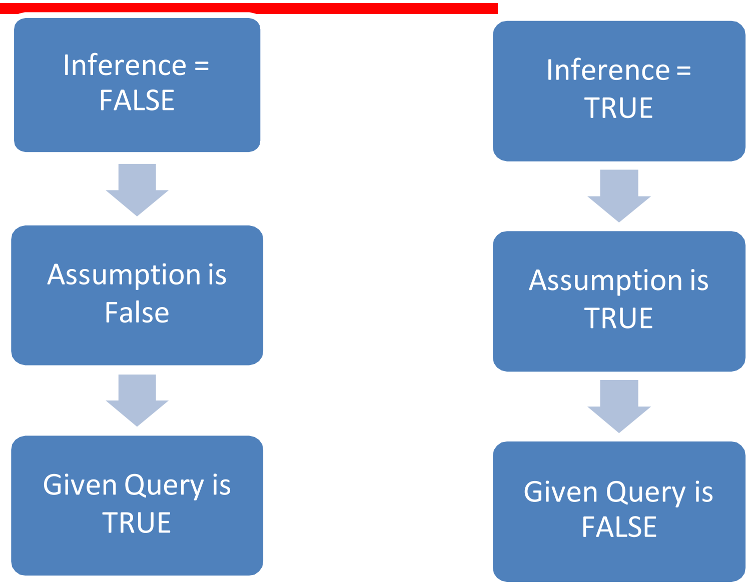
$R_9 : \neg (P_{1,2} \lor P_{2,1})$ Modus Ponens

$R_{10} : \neg P_{1,2} \land \neg P_{2,1}$ Demorgans

R11: $\neg P_{1,2}$ And Elimination

Propositional Logic

Proof by Contradiction



Required Reading: AIMA - Chapter # 5.4, # 7

Note : Some of the slides are adopted from AIMA TB materials

Thank You for all your Attention