



Artificial & Computational Intelligence

DSECSZG557

M5 : Probabilistic Representation and Reasoning

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BITS Pilani

Pilani Campus

Course Plan



- M1 Introduction to AI
- M2 Problem Solving Agent using Search
- M3 Game Playing
- M4 Knowledge Representation using Logics
- M5 Probabilistic Representation and Reasoning
- M6 Reasoning over time
- M7 Ethics in AI



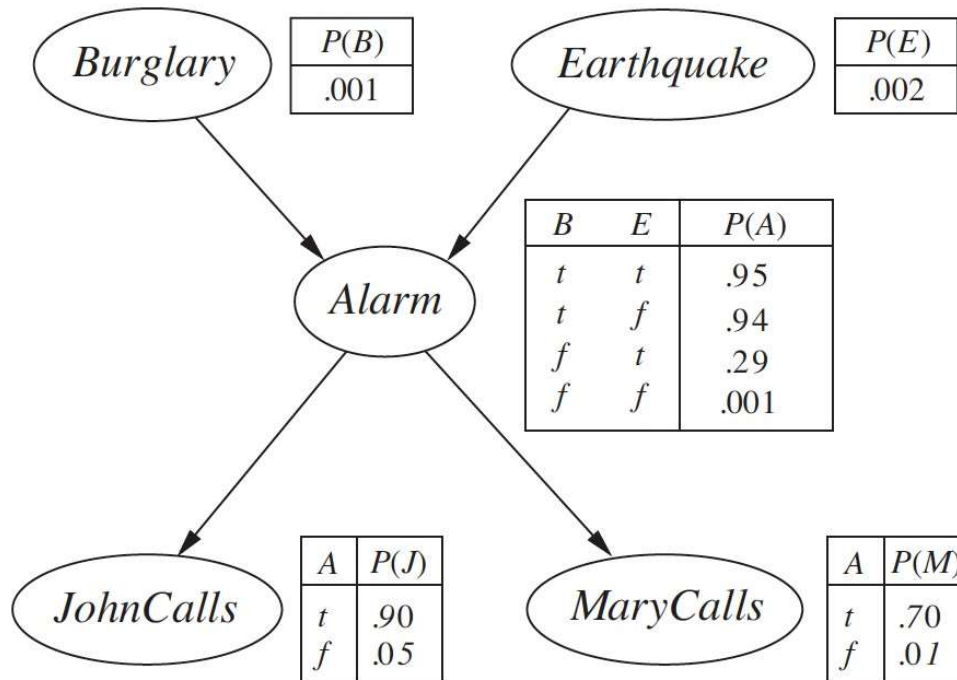
Inferences in Bayesian Nets

Enumeration

Examples



What is the probability that Burglary happened given John & Mary called the police



$$P(B | J, M) = \frac{P(B, J, M)}{P(J, M)}$$

$$P(B | J, M) = \frac{\sum_{A, E} P(J, M, A, B, E)}{\sum_{A, B, E} P(J, M, A, B, E)}$$

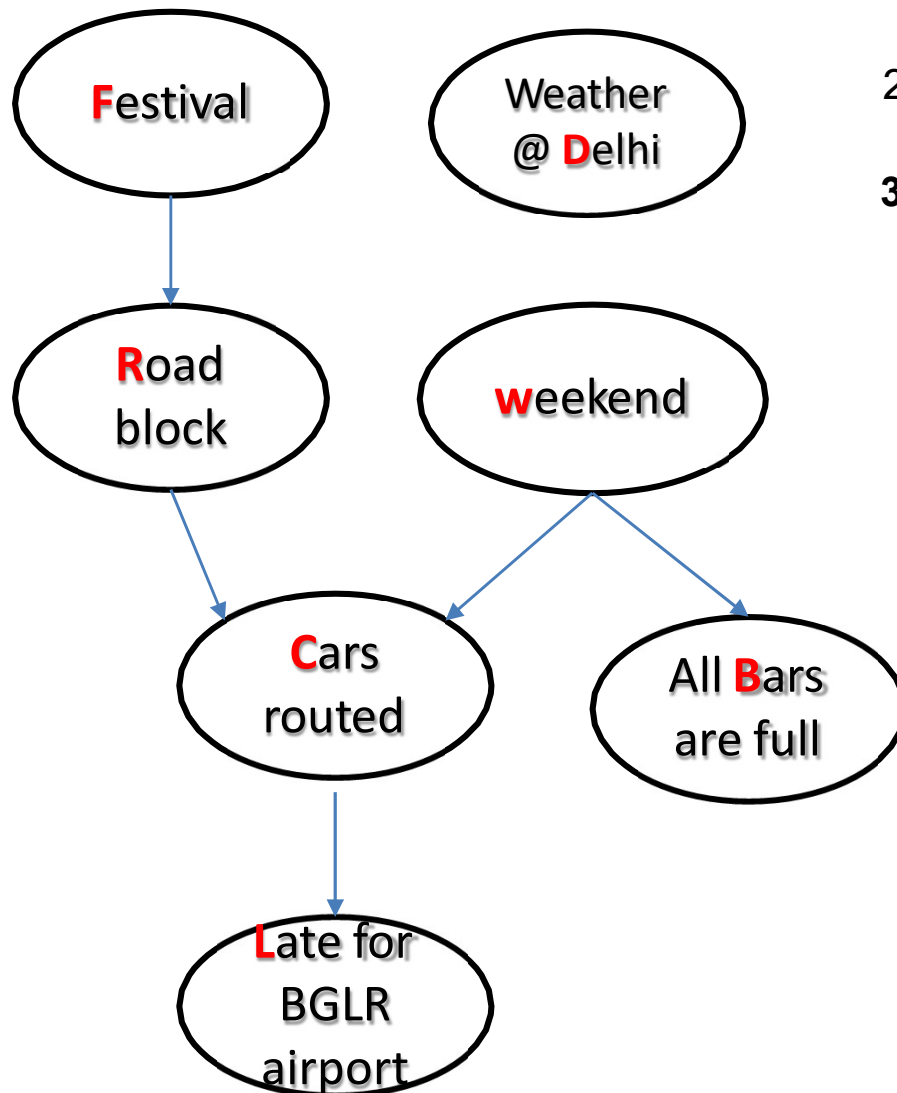


Inferences in Bayesian Nets

Variable Elimination

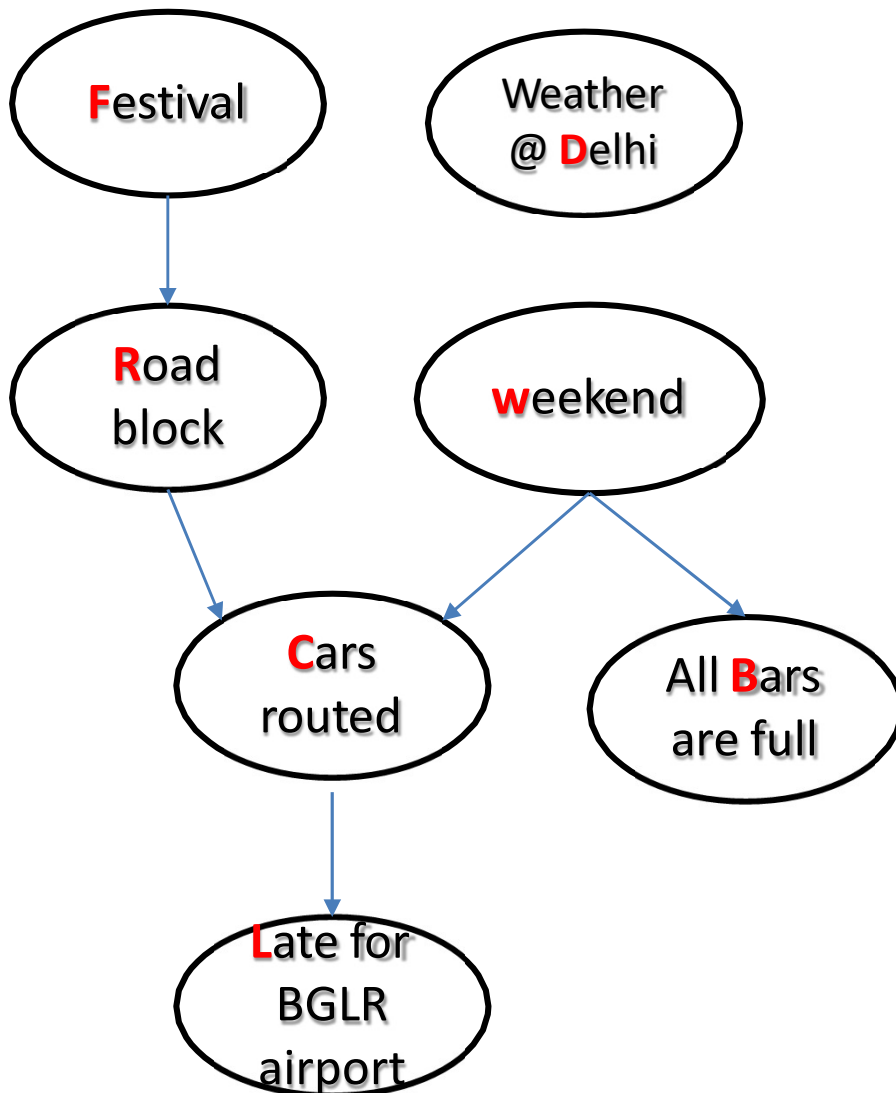
Reduce Guaranteed Independent nodes

D-Connectedness Vs D-Separation



1. Each variable is conditionally independent of its non-descendants, given its parents
2. Eliminate the hidden variables that is neither a query nor an evidence
3. **Two variables are d-separated if they are conditionally independent given evidences**

Try it & Test

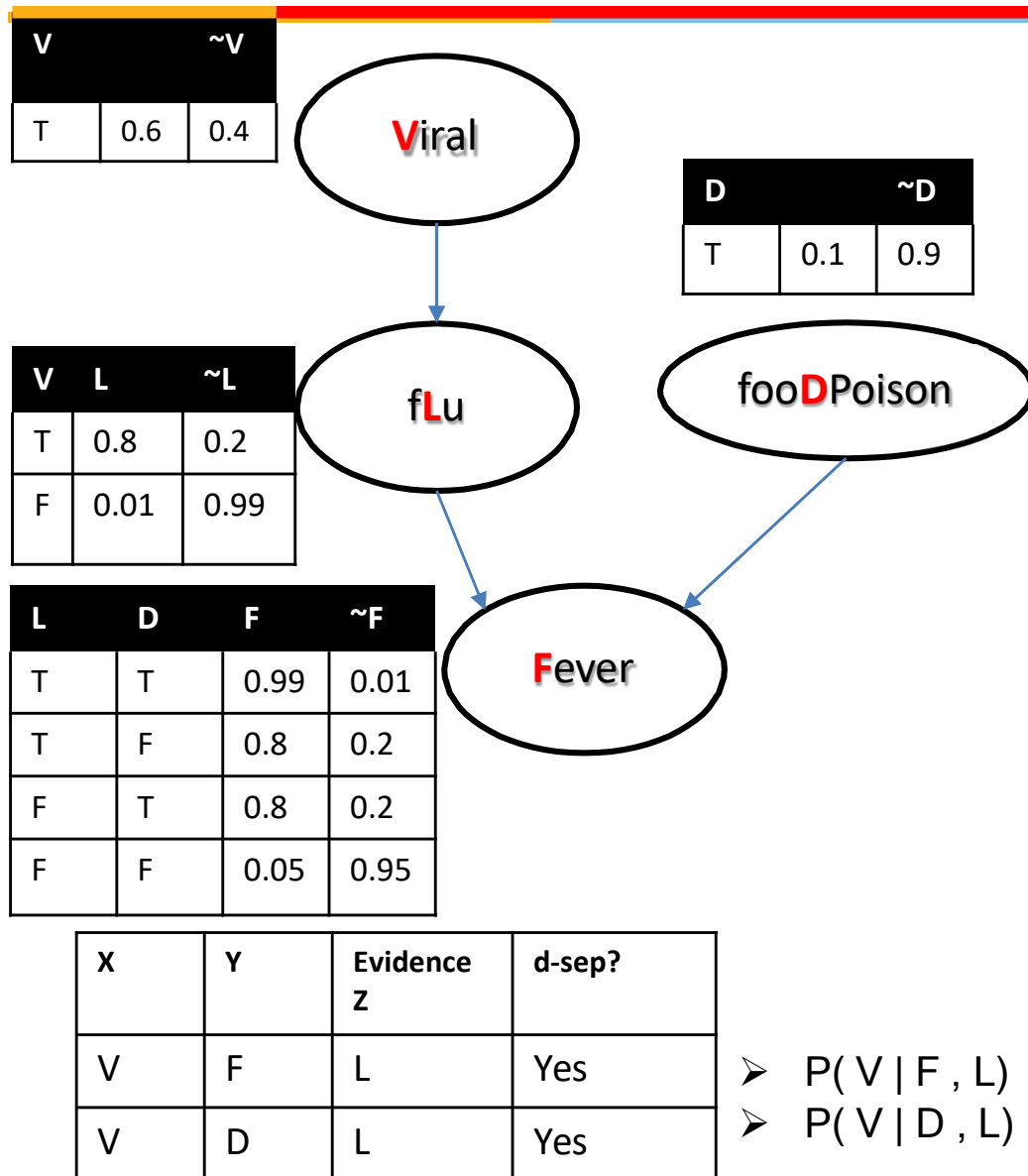


| X | Y | Evidence Z | d-sep? |
|----------|----------|---------------|------------|
| F | W | C | No |
| L | W | R | No |
| R | L | C | Yes |
| B | R | C | No |

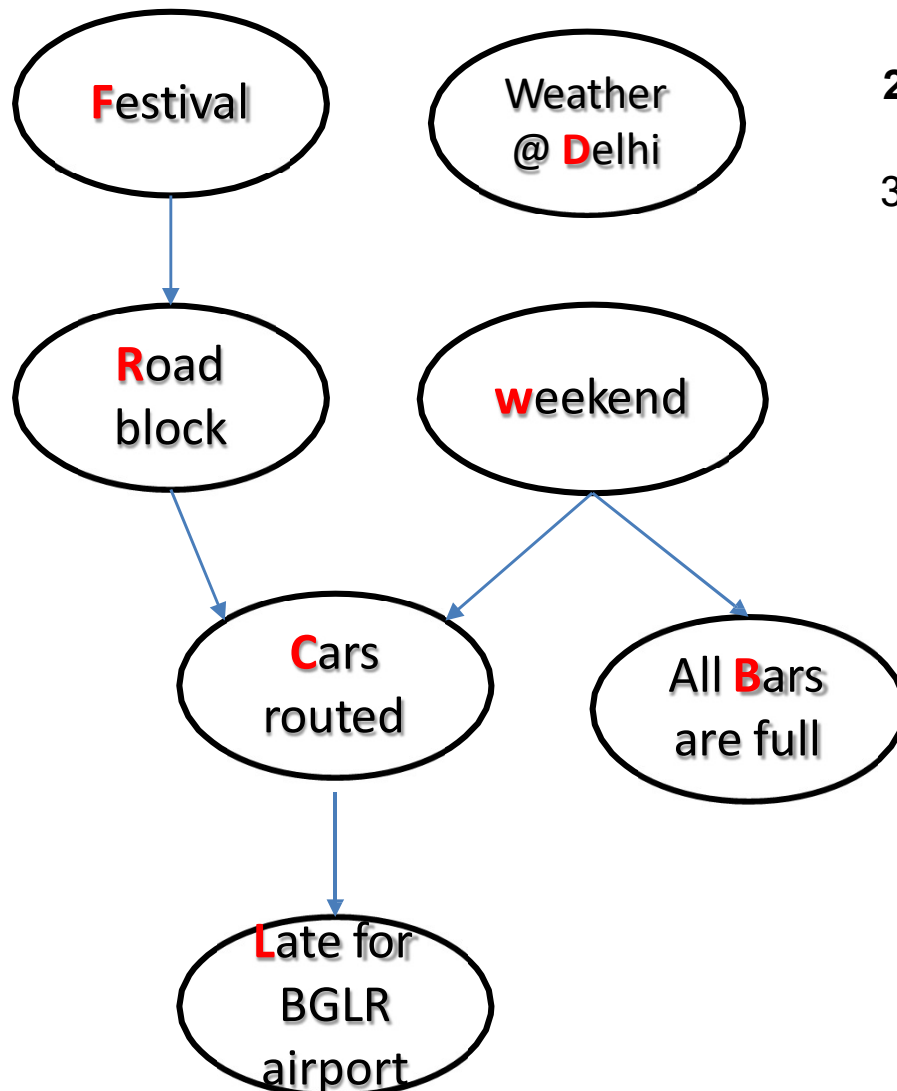
➤ $P(R | L, C) = P(R | L)$

R & L are d-separated ie., conditionally independent given C

D-Separation in Inference



Variable Elimination



1. Each variable is conditionally independent of its non-descendants, given its parents
2. **Eliminate the hidden variables that is neither a query nor evidence**
3. Two variables are d-separated if they are conditionally independent given evidences

$$\begin{aligned}
 \text{➤ } P(B) &= \sum_{L, B, W, R, F} P(L, C, B, W, R, F) \\
 &= \sum_L \sum_B P(L|C) \cdot P(B|W) \cdot \sum_W P(C|W, R) \cdot \sum_R P(R|F) \cdot \sum_F P(F) \\
 &= \mathbf{P(B|W)}
 \end{aligned}$$

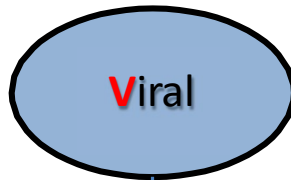
All other variables are hidden w.r.t to B as (L, C, R, F) are neither evidence nor query nor $(L, C, R, F) \in \text{Ancestors}(W, B)$

This is variable elimination example targeting irrelevant nodes

Inference

Variable Elimination: V

| V | ~V |
|---|-----|
| T | 0.6 |
| F | 0.4 |

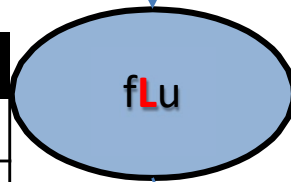


| D | ~D |
|---|-----|
| T | 0.1 |
| F | 0.9 |

| V | ~V |
|---|-----|
| T | 0.6 |
| F | 0.4 |

| V | L | ~L |
|---|------|------|
| T | 0.8 | 0.2 |
| F | 0.01 | 0.99 |

| V | L | ~L |
|---|------|------|
| T | 0.8 | 0.2 |
| F | 0.01 | 0.99 |



| V | L | |
|---|---|-------|
| T | T | 0.48 |
| T | F | 0.12 |
| F | T | 0.004 |
| F | F | 0.396 |

| L | D | F | ~F |
|---|---|------|------|
| T | T | 0.99 | 0.01 |
| T | F | 0.8 | 0.2 |
| F | T | 0.8 | 0.2 |
| F | F | 0.05 | 0.95 |

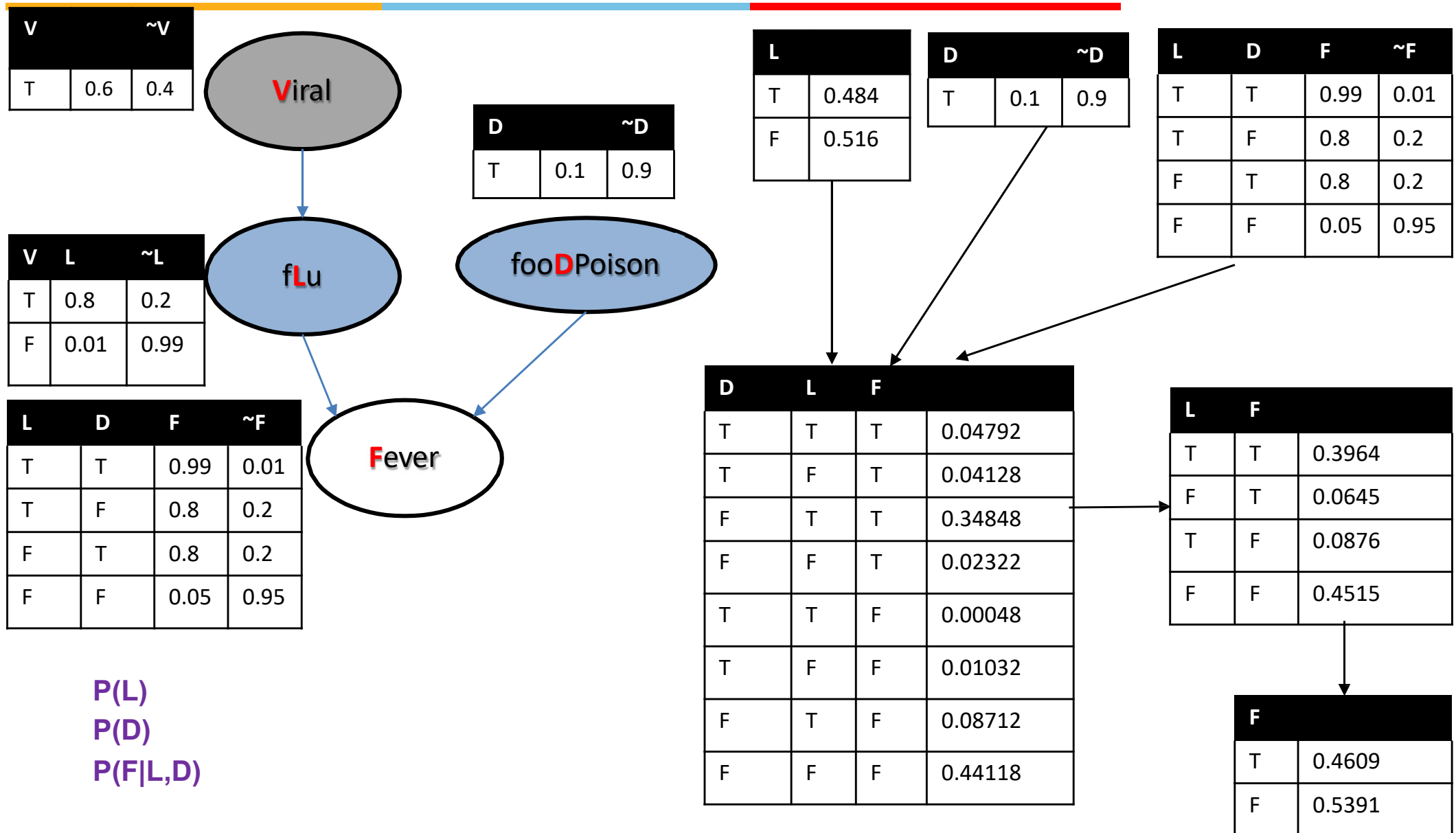


| L | |
|---|-------|
| T | 0.484 |
| F | 0.516 |

$P(V)$
 $P(L|V)$
 $P(D)$
 $P(F|L,D)$

Inference

Variable Elimination: L,D





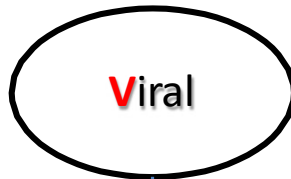
Approximate Inferences in Bayesian Nets

Introduction

Prior Sampling

Sample Generation by Randomization

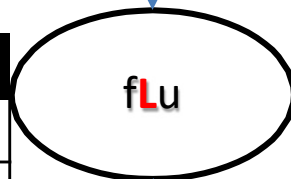
| V | ~V |
|---|-----|
| T | 0.6 |
| | 0.4 |



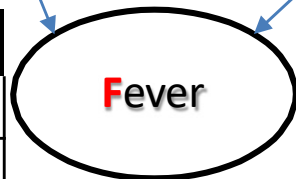
| D | ~D |
|---|-----|
| T | 0.1 |
| | 0.9 |



| V | L | ~L |
|---|------|------|
| T | 0.8 | 0.2 |
| F | 0.01 | 0.99 |



| L | D | F | ~F |
|---|---|------|------|
| T | T | 0.99 | 0.01 |
| T | F | 0.8 | 0.2 |
| F | T | 0.8 | 0.2 |
| F | F | 0.05 | 0.95 |



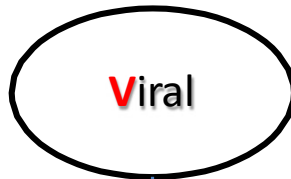
| V | L | D | F |
|---|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

0.3, 0.2, 0.6, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.9, 0.55.....

Prior Sampling

Sample Generation by Randomization

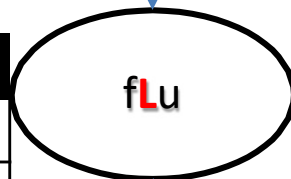
| V | ~V |
|---|-----|
| T | 0.6 |
| F | 0.4 |



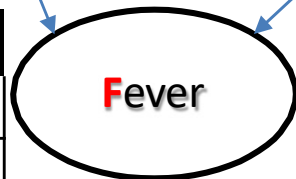
| D | ~D |
|---|-----|
| T | 0.1 |
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| V | L | ~L |
|---|------|------|
| T | 0.8 | 0.2 |
| F | 0.01 | 0.99 |



| L | D | F | ~F |
|---|---|------|------|
| T | T | 0.99 | 0.01 |
| T | F | 0.8 | 0.2 |
| F | T | 0.8 | 0.2 |
| F | F | 0.05 | 0.95 |

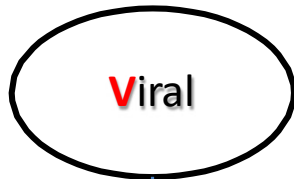


| V | L | D | F |
|------|---|---|---|
| T | T | F | T |
| F | F | F | F |
| T | F | F | T |
| F | T | F | T |
| .. | | | |
| | | | |

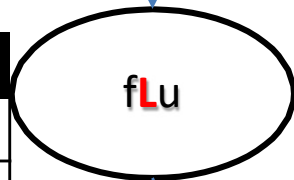
0.3, 0.2, 0.6, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.9, 0.55.....

Prior Sampling

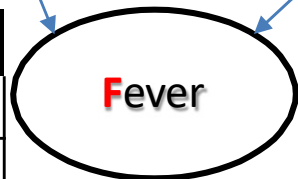
| V | ~V |
|---|-----|
| T | 0.6 |
| F | 0.4 |



| V | L | ~L |
|---|------|------|
| T | 0.8 | 0.2 |
| F | 0.01 | 0.99 |



| L | D | F | ~F |
|---|---|------|------|
| T | T | 0.99 | 0.01 |
| T | F | 0.8 | 0.2 |
| F | T | 0.8 | 0.2 |
| F | F | 0.05 | 0.95 |



| D | ~D |
|---|-----|
| T | 0.1 |
| F | 0.9 |



$$\begin{aligned}
 P(L) &= 3/8 \\
 P(FL) &= 3/8 \\
 P(L|F) &= 3/5 \\
 P(\sim V|F) &= 2/5 \\
 P(L|V\sim F) &= 0 \\
 P(F|D) &= \text{?????}
 \end{aligned}$$

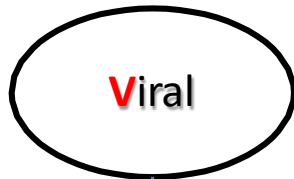
Inference

| V | L | D | F |
|---|---|---|---|
| T | T | F | T |
| F | F | F | F |
| T | F | F | T |
| F | T | F | T |
| T | T | F | T |
| T | F | F | F |
| F | F | F | T |
| T | F | F | F |

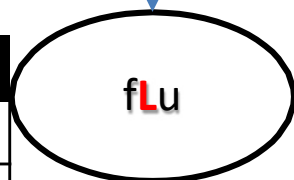
Rejection Sampling

Sample Generation by Randomization

| V | ~V |
|---|-----|
| T | 0.6 |
| | 0.4 |



| V | L | ~L |
|---|------|------|
| T | 0.8 | 0.2 |
| F | 0.01 | 0.99 |



| D | ~D |
|---|-----|
| T | 0.1 |
| | 0.9 |



| L | D | F | ~F |
|---|---|------|------|
| T | T | 0.99 | 0.01 |
| T | F | 0.8 | 0.2 |
| F | T | 0.8 | 0.2 |
| F | F | 0.05 | 0.95 |



| V | L | D | F |
|------|---|---|---|
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

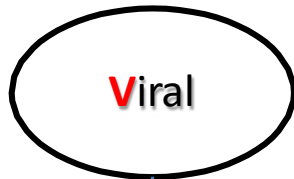
$$\begin{aligned}
 P(L) &= 3/8 \\
 P(FL) &= 3/8 \\
 P(L|F) &= 3/5 \\
 P(\sim V|F) &= 2/5 \\
 P(L|V\sim F) &= 0 \\
 \mathbf{P(F|D)} &= \text{?????}
 \end{aligned}$$

0.3, 0.2, 0.6, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.9, 0.555, 0.38.....

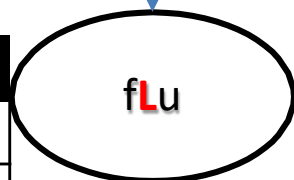
Rejection Sampling

Sample Generation by Randomization

| V | ~V |
|---|-----|
| T | 0.6 |
| | 0.4 |



| V | L | ~L |
|---|------|------|
| T | 0.8 | 0.2 |
| F | 0.01 | 0.99 |



| D | ~D |
|---|-----|
| T | 0.1 |
| | 0.9 |



| L | D | F | ~F |
|---|---|------|------|
| T | T | 0.99 | 0.01 |
| T | F | 0.8 | 0.2 |
| F | T | 0.8 | 0.2 |
| F | F | 0.05 | 0.95 |

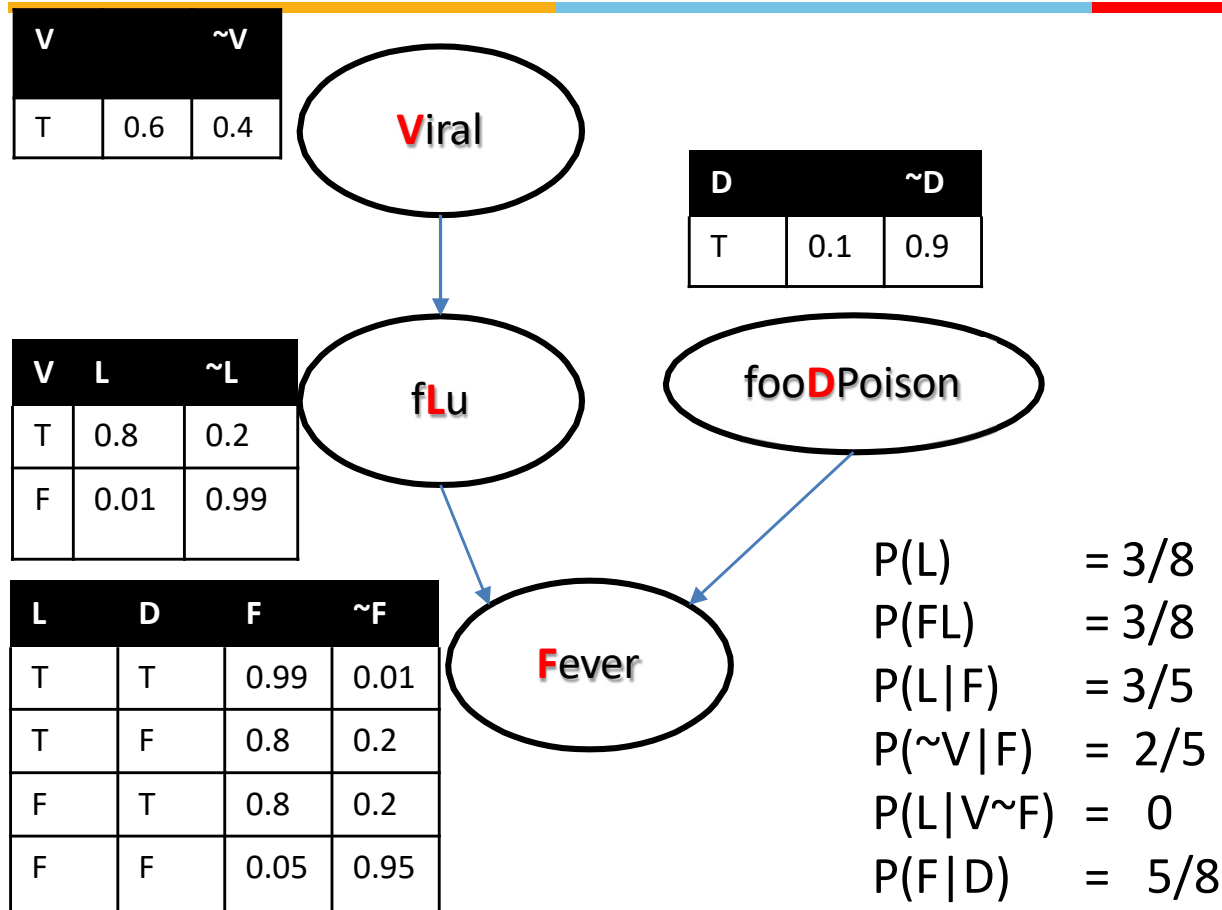


| V | L | D | F |
|------|---|---|---|
| T | T | F | |
| T | T | F | |
| T | T | F | |
| F | F | T | F |
| T | F | T | T |
| | | | |

$$\begin{aligned}
 P(L) &= 3/8 \\
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 P(L|V\sim F) &= 0 \\
 \mathbf{P(F|D)} &= \text{?????}
 \end{aligned}$$

0.3, 0.2, 0.6, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.9, 0.555, 0.38.....

Rejection Sampling

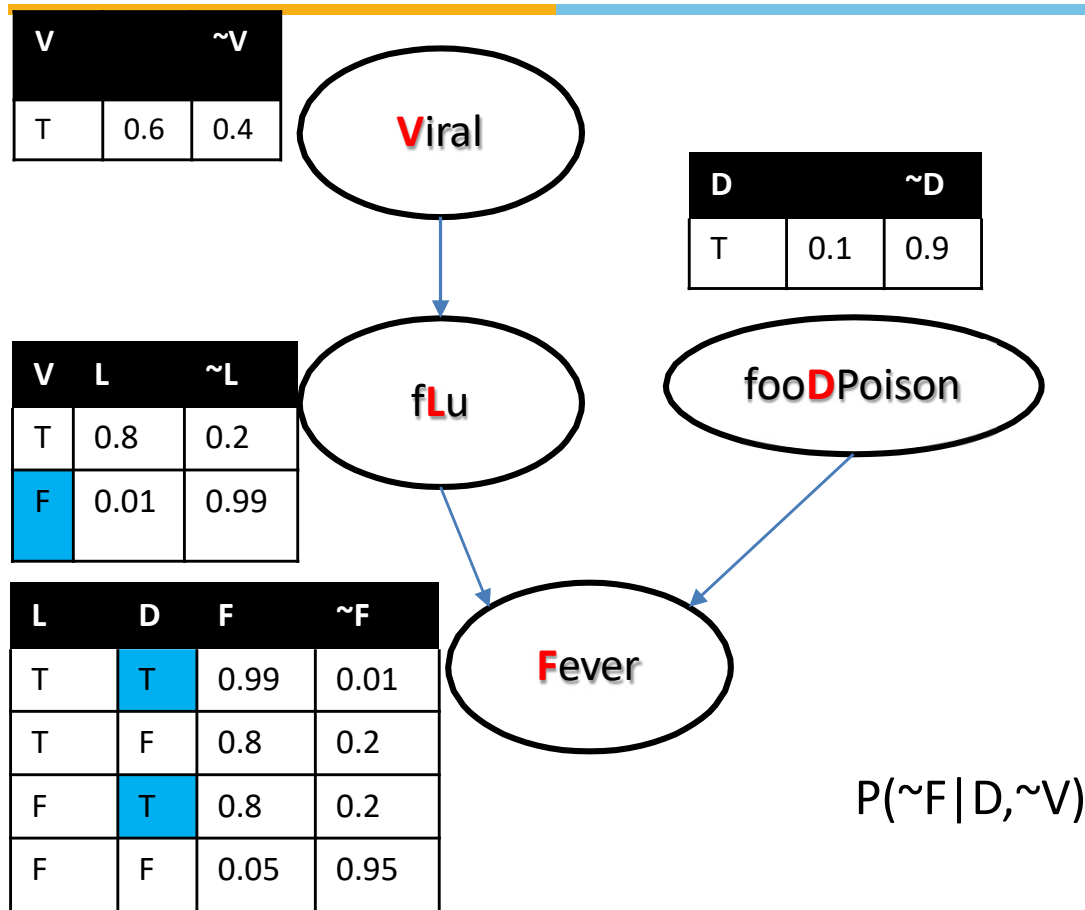


Inference

| V | L | D | F |
|---|---|---|---|
| T | T | T | T |
| F | F | T | F |
| T | F | T | T |
| F | T | T | T |
| T | T | T | T |
| T | F | T | F |
| F | F | T | T |
| T | F | T | F |

Likelihood Weighing

Sample Generation by Randomization

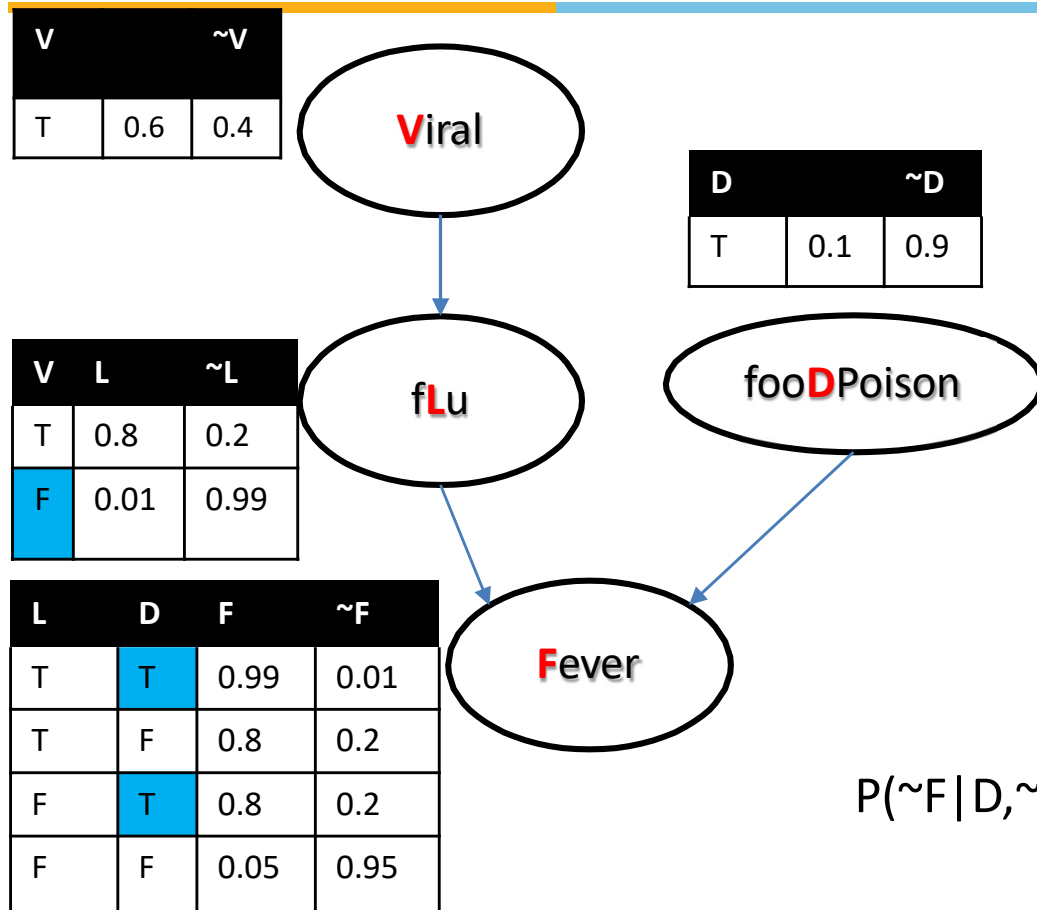


| V | L | D | F | wgt |
|---|---|---|---|-----|
| F | | T | | |
| F | | T | | |
| F | | T | | |
| F | | T | | |
| F | | T | | |
| F | | T | | |
| F | | T | | |

0.3, 0.2, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.99, ,.....

Likelihood Weighing

Sample Generation by Randomization



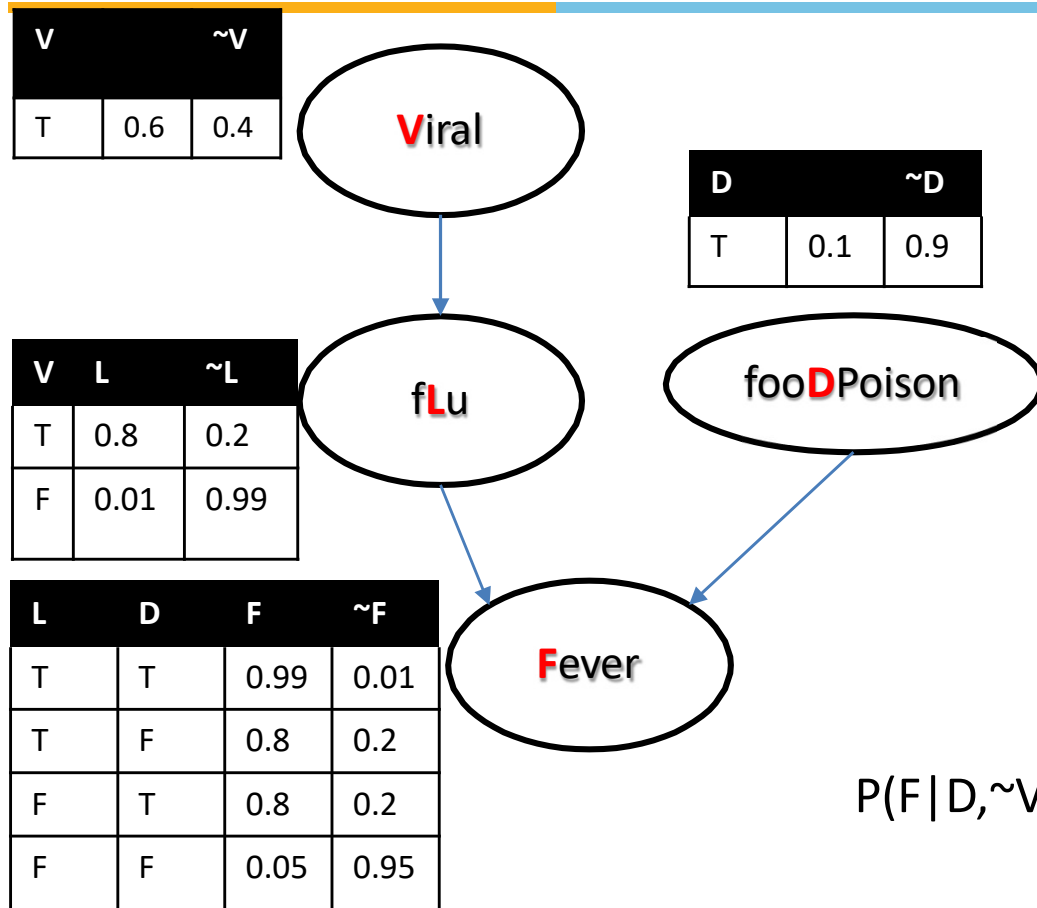
| V | L | D | F | wgt |
|---|---|---|---|-----------------------|
| F | F | T | T | $0.4 * 1 * 0.1 * 1 =$ |
| F | F | T | T | |
| F | F | T | T | |
| F | F | T | T | |
| F | F | T | T | |
| F | T | T | T | |
| F | T | T | F | |

$$P(\sim F | D, \sim V) = 0.04 / 7 * 0.04$$

0.3, 0.2, 0.58, 0.73, 0.87, 0.15, 0.6, 0.57, 0.85, 0.12, 0.004, 0.93, 0.0002, 0.99, ,



Likelihood Weighing



Inference

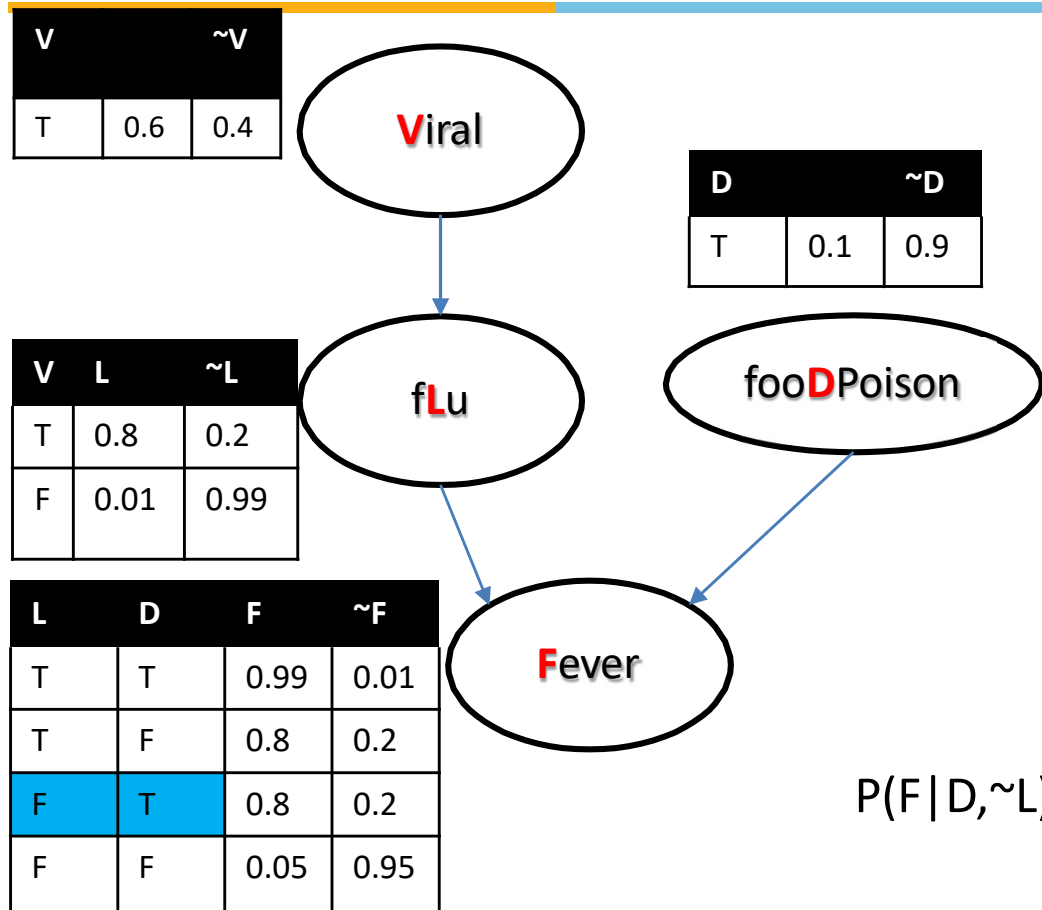
| V | L | D | F | wgt |
|---|---|---|---|-----------------------|
| F | F | T | F | $0.4 * 1 * 0.1 * 1 =$ |
| F | T | T | T | $0.4 * 1 * 0.1 * 1 =$ |
| F | F | T | T | $0.4 * 1 * 0.1 * 1 =$ |
| F | F | T | F | $0.4 * 1 * 0.1 * 1 =$ |

$$P(F|D, \sim V)$$

$$= 0.04 + 0.04 / 4 * 0.04$$



Likelihood Weighing

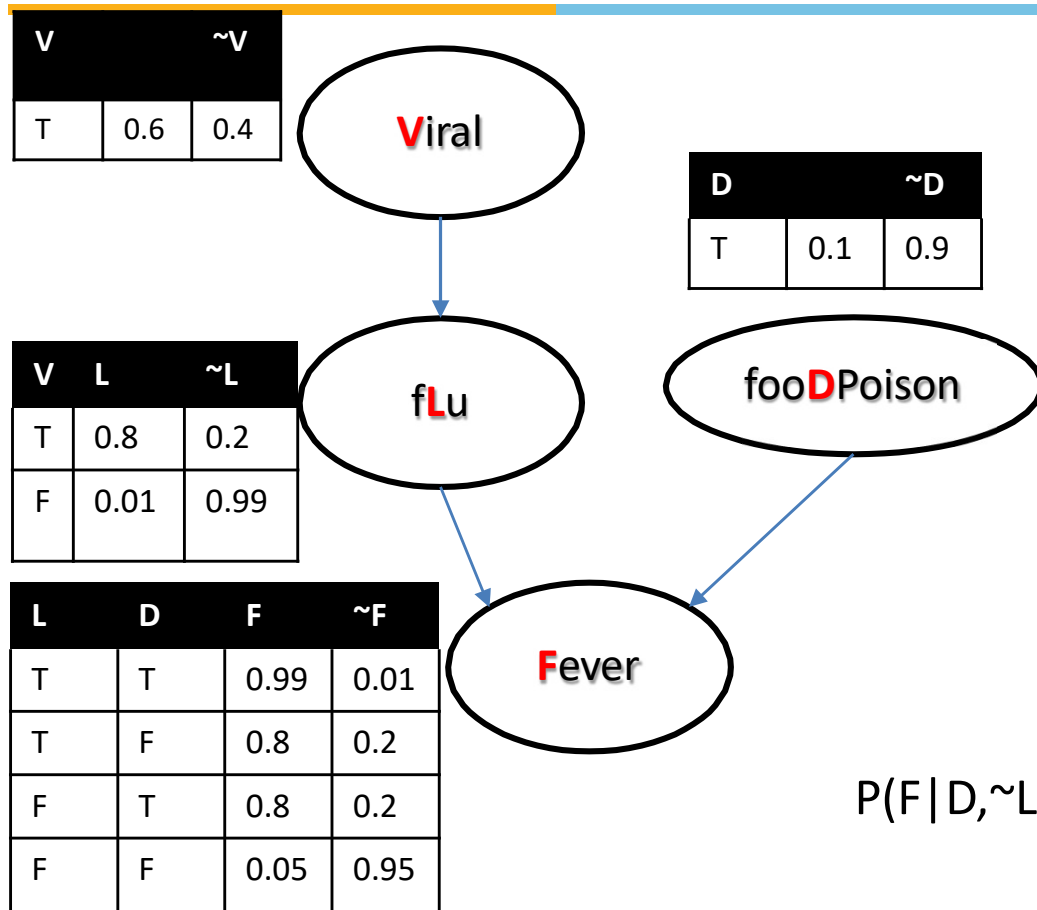


Inference

| V | L | D | F | wgt |
|---|---|---|---|-----|
| F | F | T | F | |
| F | F | T | T | |
| F | F | T | T | |
| T | F | T | F | |



Likelihood Weighing



Inference

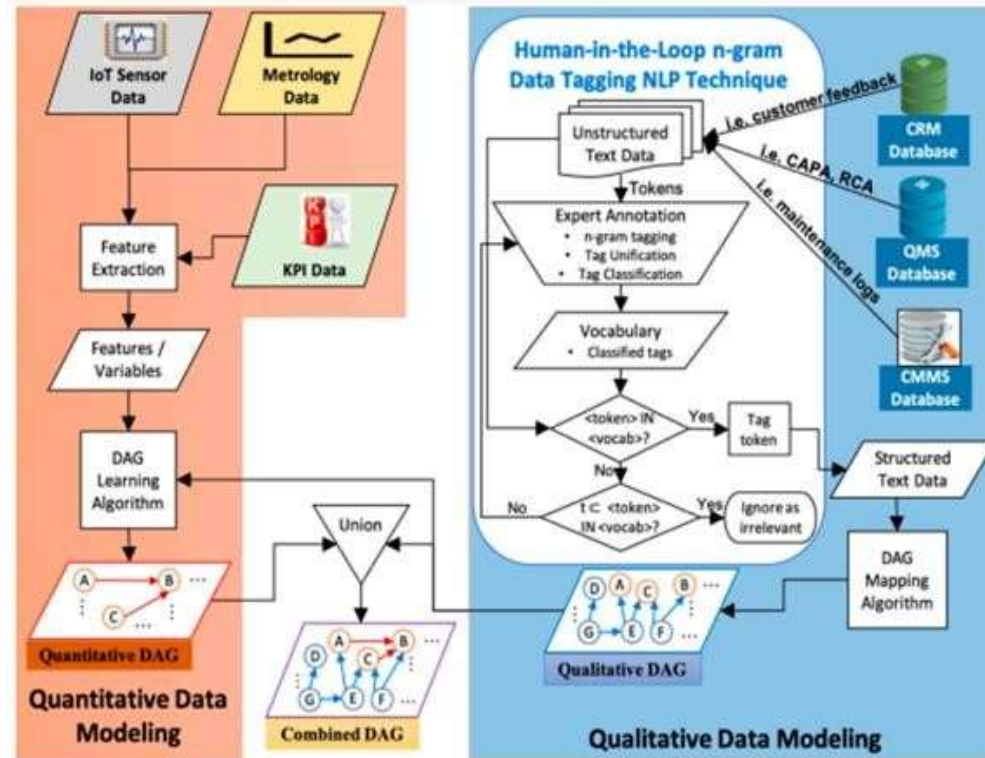
| V | L | D | F | wgt |
|---|---|---|---|-----------------|
| F | F | T | F | 1*0.99* 0.1 *1= |
| F | F | T | T | 1*0.99* 0.1 *1= |
| F | F | T | T | 1*0.99* 0.1 *1= |
| T | F | T | F | 1*0.2* 0.1 *1= |

$P(F|D, \sim L)$

$$= 0.099+0.099 \quad / \quad (3*0.099 + 0.02)$$

Bayesian Network

Fault Diagnostic System

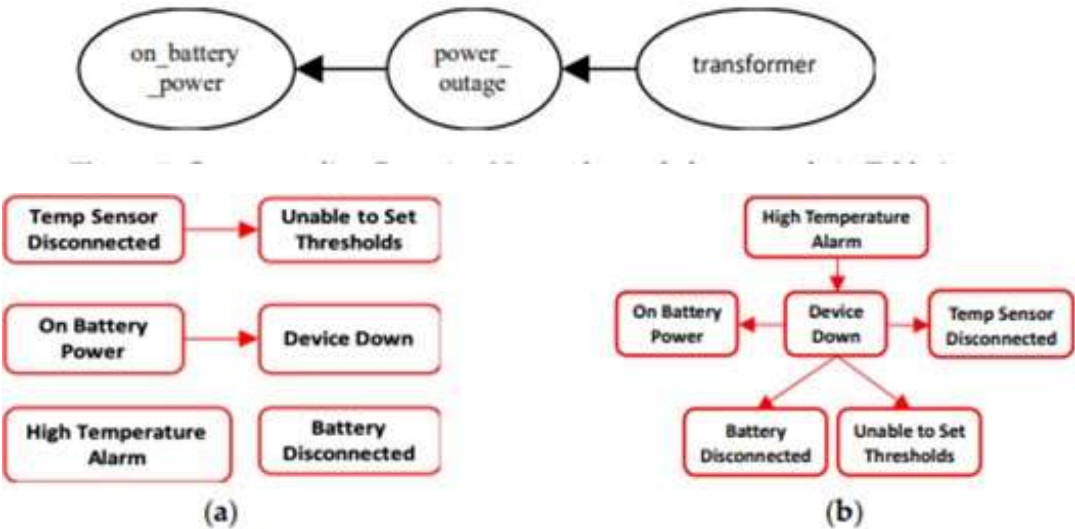


Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)

Bayesian Network

Fault Diagnostic System

| | | | | | | |
|-----------------|-------------------|-------------|--------------------------------------|--------------|-------------------|----------------|
| Raw Data | Short Description | | Resolution Notes | | | |
| | On battery power | | Power outage due to transformer fire | | | |
| Classified Tags | Symptom | | Cause(s) | | Link | |
| | on_battery_power | | power_outage, transformer_fire | | due_to | |
| BN Mapping | Child Variable | Child State | Parent Variable | Parent State | Ancestor Variable | Ancestor State |
| | on_battery_power | yes | power_outage | yes | transformer | Fire |



Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)

Bayesian Network



Fault Diagnostic System

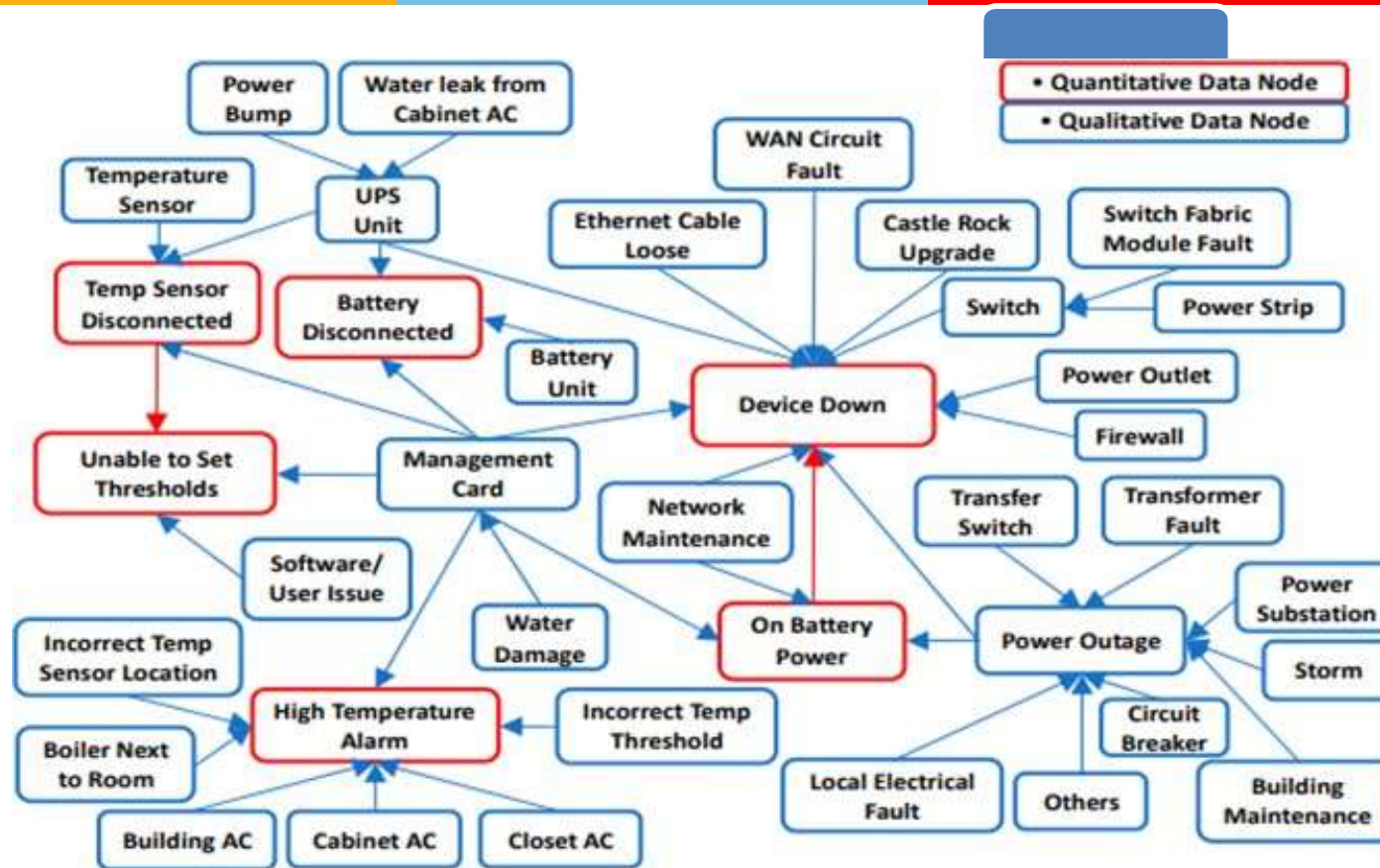


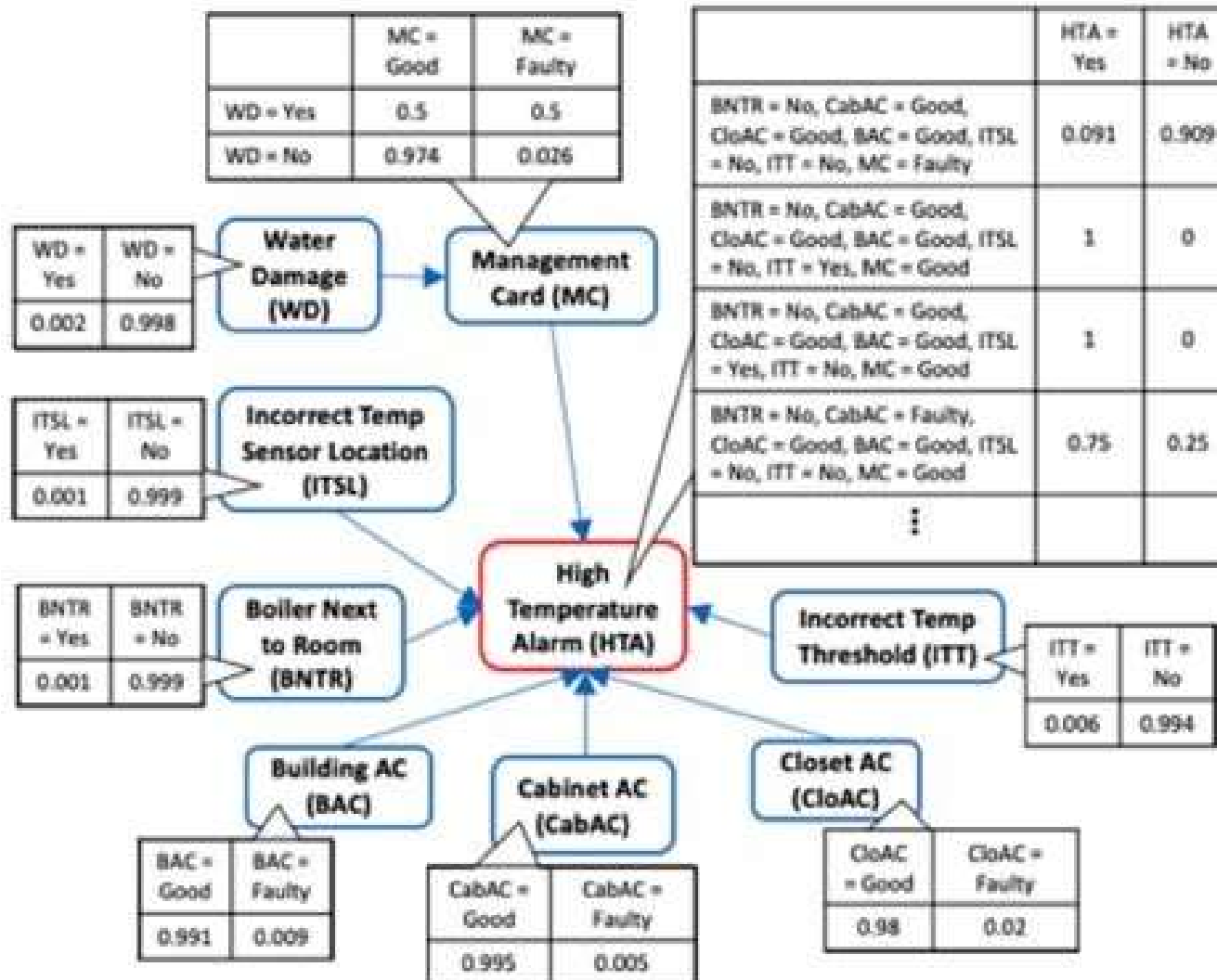
Figure 8. Fused Bayesian Network structure for top six occurring UPS messages.

Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)

Bayesian Network



Fault Diagnostic System



Source Credit : [Sensors 2021 : Fusion-Learning of Bayesian Network Models for Fault Diagnostics](#)



Required Reading: AIMA - Chapter # 14

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials