Artificial & Computational Intelligence

DSECSZG557

M4 : Knowledge Representation using Logics

Dr. Sudheer Reddy

BITS Pilani

Pilani Campus

Course Plan

M1 Introduction to AI

M2 Problem Solving Agent using Search

M3 Game Playing

M4 Knowledge Representation using Logics

M5 Probabilistic Representation and Reasoning

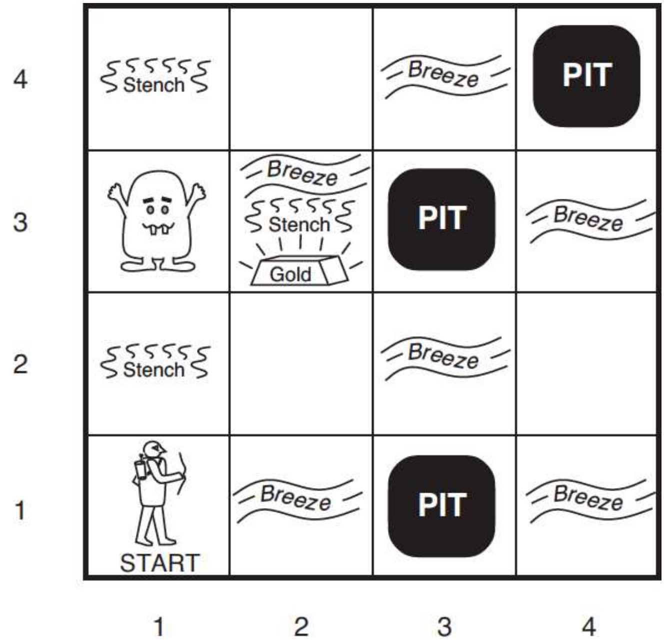
M6 Reasoning over time

M7 Ethics in AI

PL-Resolution

Horn Clause

1. Definite Clause : A horn clause with exactly one positive literal
2. Fact : Definite clause with no negative literal / assertion
3. Rule
4. Inference by Chaining



PL-Resolution : CNF conversion

Wumpus world Book example

$R_1 : \neg P_{1,1}$

$\mathbf{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$

$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

Conjunctive Normal Form :

(A V ~B) ^ (A V B V ~C) ^ ~A

Unit Resolution : ~A

Query : Is 'C' true?

Propositional Logic

Proof by Contradiction

Inference = FALSE

↓

Assumption is False

↓

Given Query is TRUE

Inference = TRUE

↓

Assumption is TRUE

↓

Given Query is FALSE

PL-Resolution : CNF conversion

Wumpus world Book example

$R_1 : \neg P_{1,1}$

$\mathbf{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$

$\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}$

$R_4 : \neg B_{1,1}$

$R_5 : B_{2,1}$

Query: $\neg P_{1,2}$

Conjunctive Normal Form :

(A V ~B) ^ (A V B V ~C) ^ ~A

Unit Resolution : ~A

Query : Is 'C' true?

PL-Resolution

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of $\wedge$
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of $\vee$
$\neg(\neg \alpha) \equiv \alpha$ double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg \beta \Rightarrow \neg \alpha)$ contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg \alpha \vee \beta)$ implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$ De Morgan
$\neg(\alpha \vee \beta) \equiv (\neg \alpha \wedge \neg \beta)$ De Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ distributivity of $\vee$ over $\wedge$

$R_1 : \neg P_{1,1}$
$\cancel{R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})}$
$\cancel{\mathbf{R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})}}$
$R_4 : \neg B_{1,1}$
$R_5 : B_{2,1}$
Query: $\neg P_{1,2}$

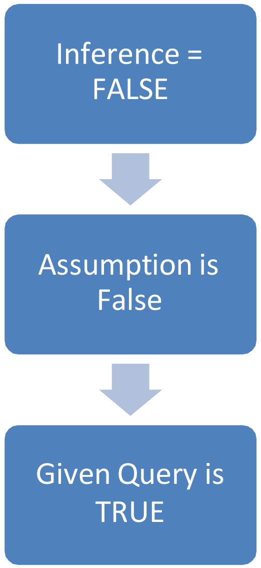
$R_6 : \neg B_{1,1} \vee P_{1,2} \vee P_{2,1}$
$R_7 : \neg P_{1,2} \vee B_{1,1}$
$R_8 : \neg P_{2,1} \vee B_{1,1}$
$R_9 : \neg B_{2,1} \vee P_{1,1} \vee P_{2,2} \vee P_{3,1}$
$R_{10} : \neg P_{1,1} \vee B_{2,1}$
$R_{11} : \neg P_{2,2} \vee B_{2,1}$
$R_{12} : \neg P_{3,1} \vee B_{2,1}$

Eliminate		$R_2 : B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$	$R_3 : B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
←→	Biconditional Elimination	$(B_{1,1} \Rightarrow (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \Rightarrow B_{1,1})$	$(B_{2,1} \Rightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})) \wedge ((P_{1,1} \vee P_{2,2} \vee P_{3,1}) \Rightarrow B_{2,1})$
→	Implication Elimination	$\neg B_{1,1} \vee (P_{1,2} \vee P_{2,1})$ $\neg(\mathbf{P_{1,2}} \vee \mathbf{P_{2,1}}) \vee \mathbf{B_{1,1}}$	$\neg B_{2,1} \vee (P_{1,1} \vee P_{2,2} \vee P_{3,1})$ $\neg(\mathbf{P_{1,1}} \vee \mathbf{P_{2,2}} \vee \mathbf{P_{3,1}}) \vee \mathbf{B_{2,1}}$
Clause level ¬	De Morgan	$(\neg \mathbf{P_{1,2}} \wedge \neg \mathbf{P_{2,1}}) \vee \mathbf{B_{1,1}}$	$(\neg \mathbf{P_{1,1}} \wedge \neg \mathbf{P_{2,2}} \wedge \neg \mathbf{P_{3,1}}) \vee \mathbf{B_{2,1}}$
CNF Form	Distributivity of ∨ over ∧	$(\neg \mathbf{P_{1,2}} \vee \mathbf{B_{1,1}}) \wedge (\neg \mathbf{P_{2,1}} \vee \mathbf{B_{1,1}})$	$(\neg \mathbf{P_{1,1}} \vee \mathbf{B_{2,1}}) \wedge (\neg \mathbf{P_{2,2}} \vee \mathbf{B_{2,1}}) \wedge (\neg \mathbf{P_{3,1}} \vee \mathbf{B_{2,1}})$

PL-Resolution

Unit Resolution: Query: $\neg P_{1,2}$

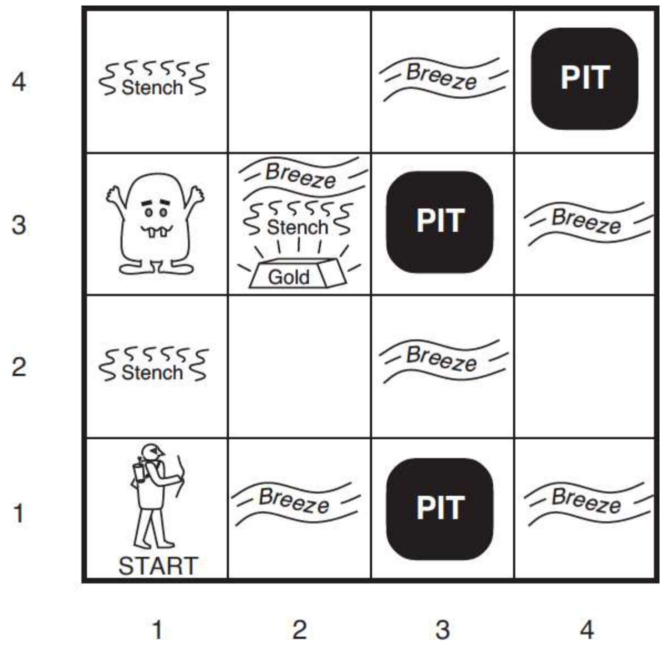
To find: Is there a pit in location (1,2) using the CNF obtained in previous slide



$\neg B_{1,1}$ $\quad$ $\neg P_{1,2} \vee B_{1,1}$

$P_{1,2}$ $\quad$ $\neg P_{1,2}$

{ }



DPLL Algorithm

In logic and computer science, the Davis–Putnam–Logemann–Loveland (**DPLL**) **algorithm** is a complete, backtracking-based search **algorithm** for deciding the satisfiability of propositional logic formulae in conjunctive normal form

Improvements:

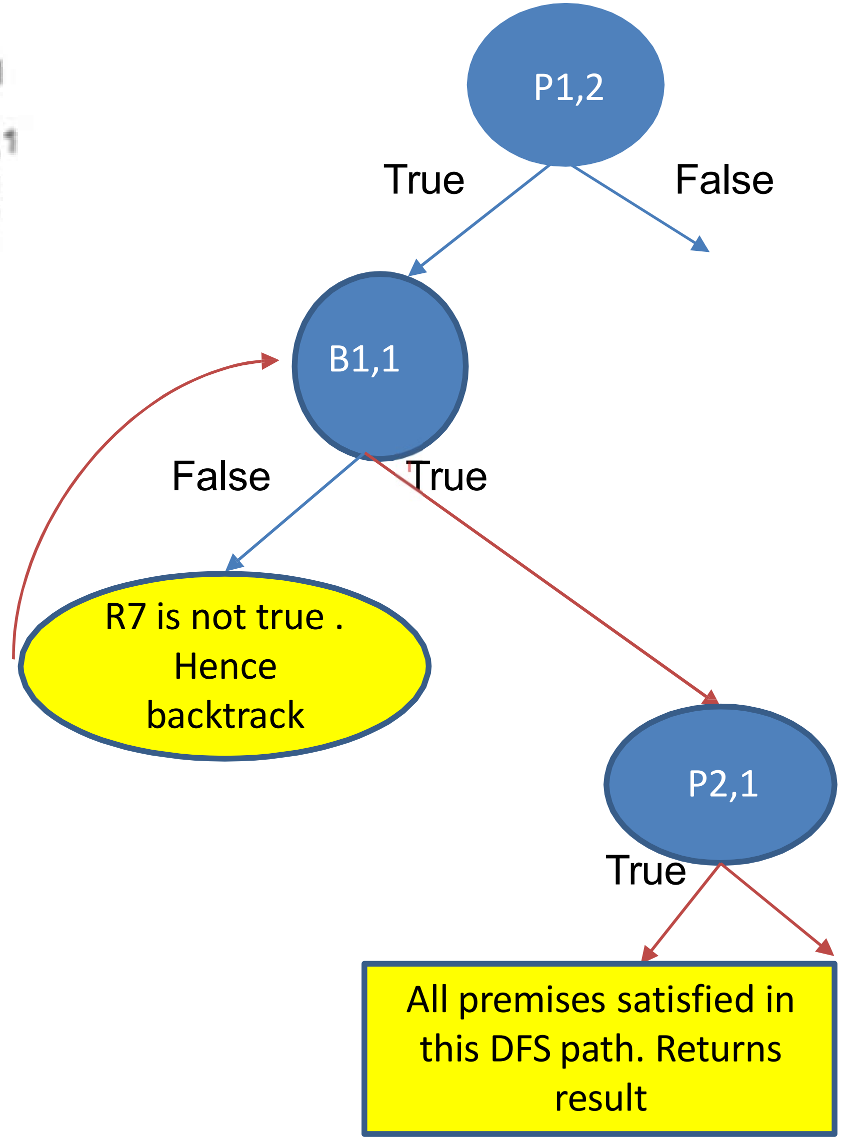
1. Early Termination
2. Pure Symbolic Heuristic
3. Unit Clause Heuristic

DPLL Algorithm

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$\{P_{1,2}, B_{1,1}, P_{2,1}\}$



$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

$R_7 : \neg P_{1,2} \vee B_{1,1}$

$R_8 : \neg P_{2,1} \vee B_{1,1}$

–

Towards Predicate Logic

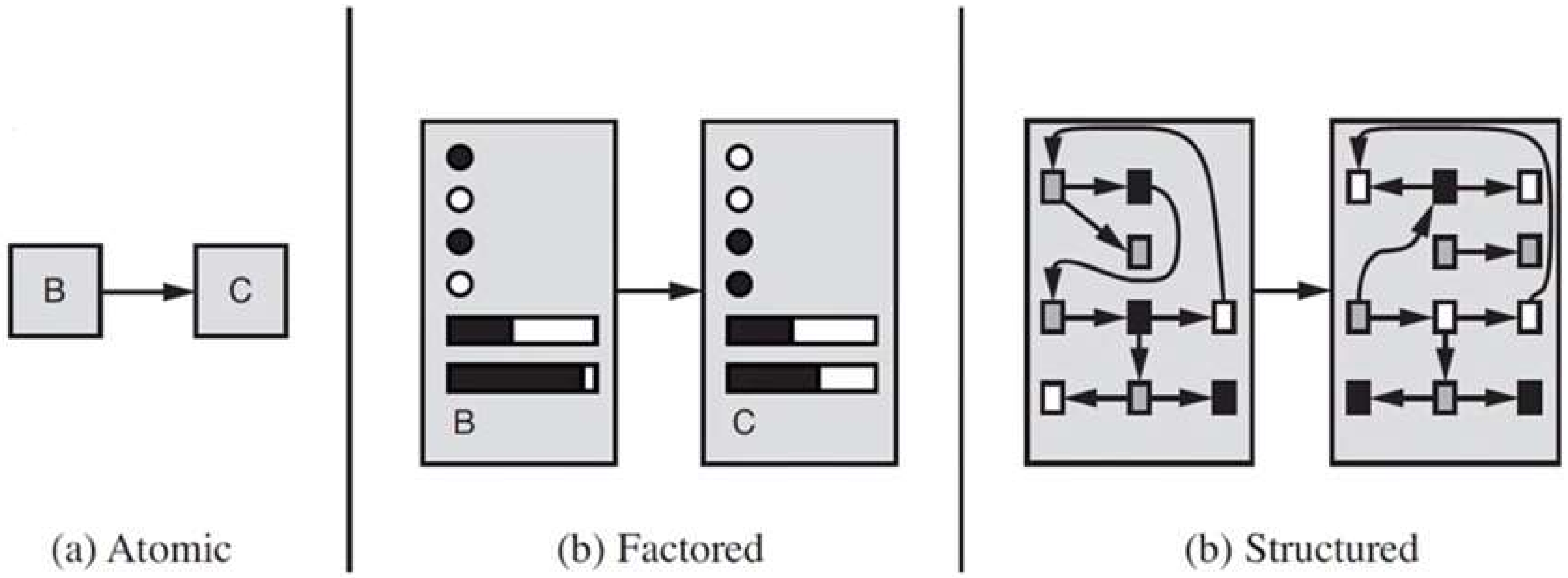
All courses are offered and interesting

All offered courses are interesting

Some of the courses are offered and interesting [Atleast one of the offered courses is interesting]

Some of the offered courses are interesting

Towards Predicate Logic



Predicate Logic

Squares neighboring the wumpus are smelly

Objects: squares, wumpus

Unary Relation (properties of an object): smelly N-ary

Relation (between objects): neighboring

Function: Neighbouring of Wumpus are Smelly

Primary difference between propositional and first-order logic lies in "ontological commitment" – the assumption about the nature of reality.

Predicate Logic – Sample Modelling

1. “Squares neighboring the wumpus are smelly”

$\forall x$,y *Neighbour*(x,y) ∧ *Wumpus*(y)⟹Smelly(x)

Order of quantifiers is important

Predicate Logic – Sample Modelling

2. “Everybody loves somebody”

$\forall x\ \exists y\ Loves(x, y)$

3. “There is someone who is loved by everyone”

$\exists y\ \forall x\ Loves(x, y)$

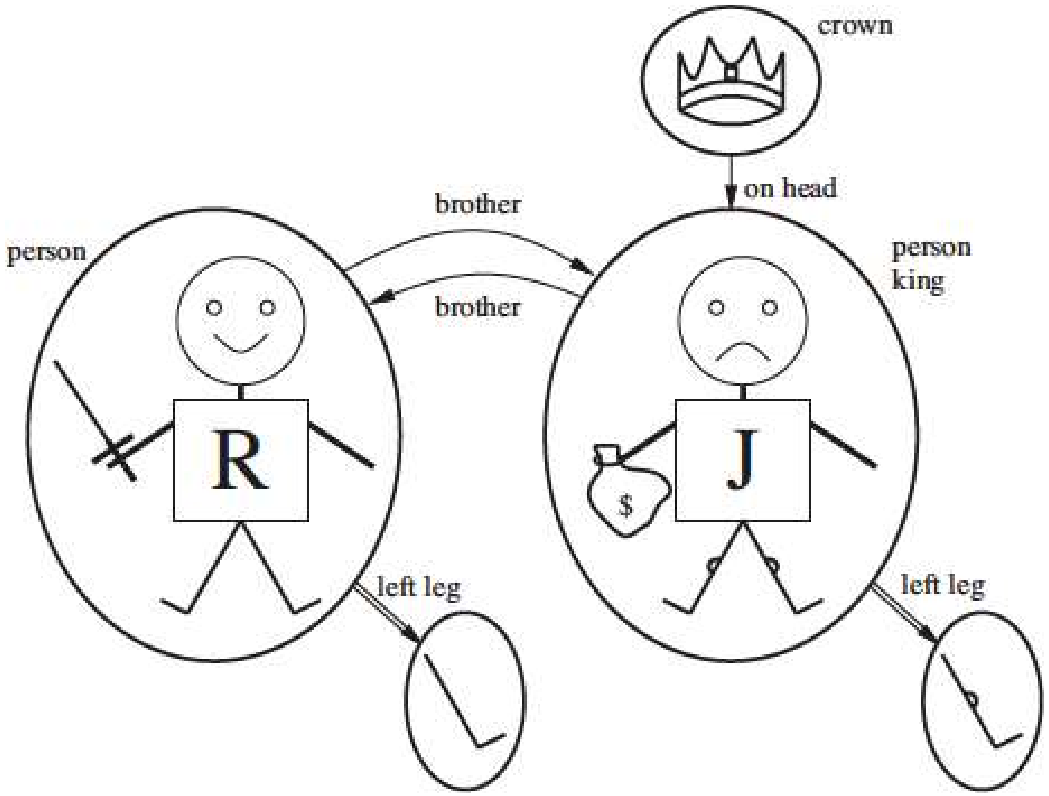
Order of quantifiers is important

Predicate Logic – Sample Modelling

Brother(Richard, John) $\wedge$ Brother(John, Richard)

King(Richard) $\vee$ King(John)

$\neg$King(Richard) $\Rightarrow$ King(John)

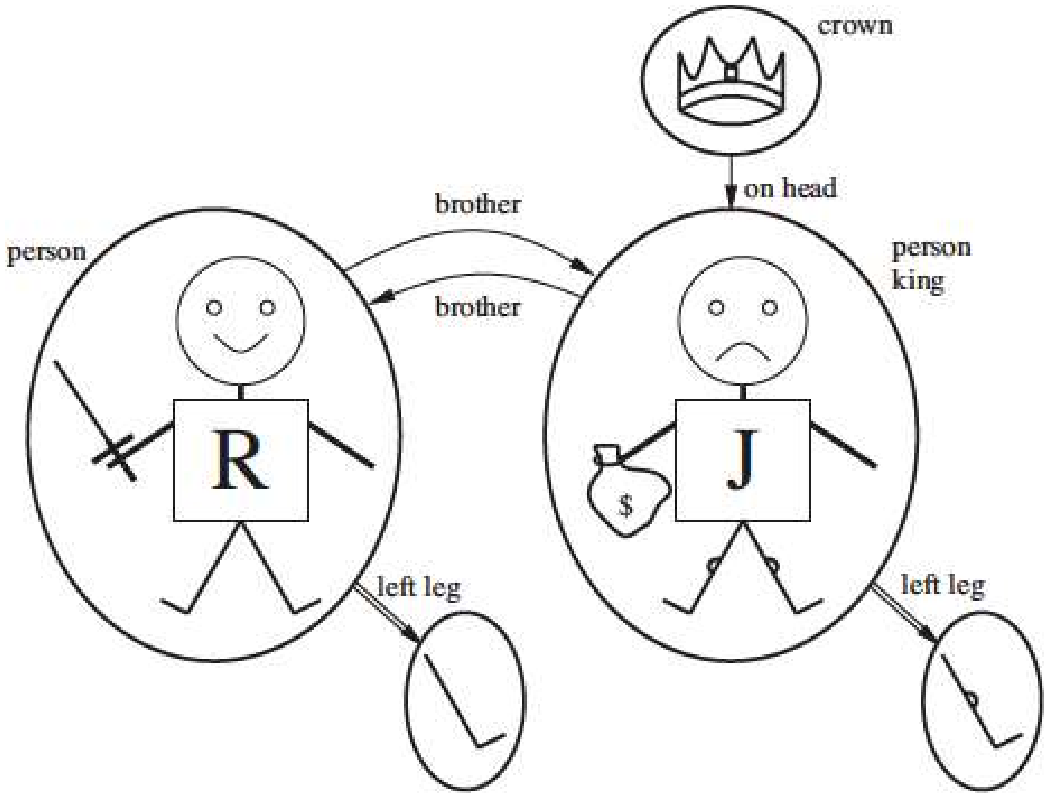


Unification & Lifting

Brother(Richard, John) ∧ Brother(John, Richard)

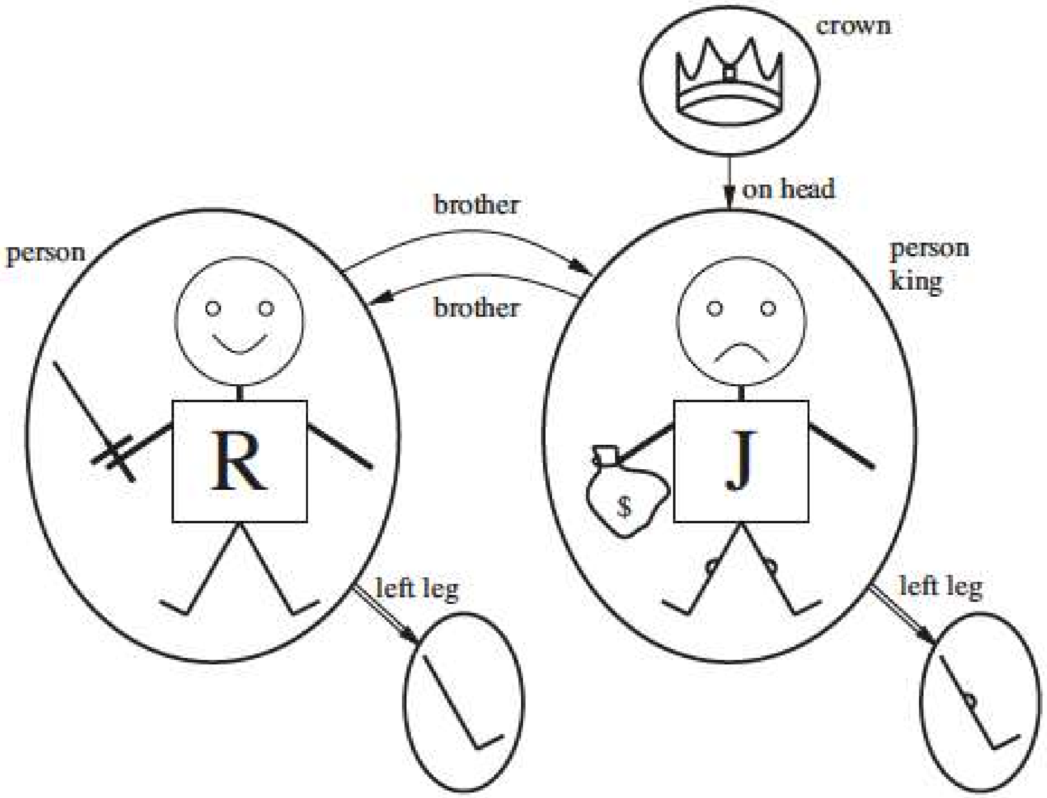
King(Richard) ∨ King(John)

¬King(Richard) ⟹ King(John)



Predicate Logic – Sample Modelling Quantifiers

Brother(Richard, John) ∧ Brother(John, Richard)
King(Richard) ∨ King(John)
¬King(Richard) ⟹ King(John)



”All Kings are persons”

$\forall x\ King(x) \Longrightarrow Person(x)$

“King John has a crown on his head”

$\exists x\ Crown(x)\ \wedge OnHead(x, John)$

Ground Term: A term with no variables. E.g., King(Richard)

Predicate Logic – Inference

1. Substitute for Quantifiers
2. Convert into Propositional Logic
3. Apply inference tech

$\forall x\ King(x) \Longrightarrow Person(x)$

Richard the Lionheart is a king ⟹ Richard the Lionheart is a person

King John is a king ⟹ King John is a person

$\exists x\ Crown(x)\ \ \mathrm{A} OnHead(x, John)$

Crown(C_1) A OnHead(C_1, John) ||C1 is imputed assumed fact

Forward Chaining

- Consider the following problem:

The law says it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

- We will prove that West is a criminal

Forward Chaining

- “All of its missiles were sold to it by Colonel West”

$$Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$

- Missiles are weapons

$$Missile(x) \Rightarrow Weapon(x)$$

- Hostile means enemy

$$Enemy(x, America) \Rightarrow Hostile(x)$$

- ”West, who is American”

$$American(West)$$

- “The country Nono, an enemy of America”

$$Enemy(Nono, America)$$

Forward Chaining

- First, we will represent the facts in First Order Definite Clauses

" … it is a crime for an American to sell weapons to hostile nations"

$$American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

"Nono … has some missiles"

$$\exists x \; Owns(Nono, x) \wedge Missile(x)$$

is transformed into two definite clauses by Existential Instantiation

$$Owns(Nono, M_1)$$
$$Missile(M_1)$$

Forward Chaining

- Consider the following problem:

The law says it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all of its missiles were sold to it by Colonel West, who is American.

- We will prove that West is a criminal

Algorithm:

1. Start from the facts
2. Trigger all rules whose premises are satisfied
3. Add the conclusion to known facts
4. Repeat the steps till no new facts are added or the query is answered

Forward Chaining

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)

Forward Chaining

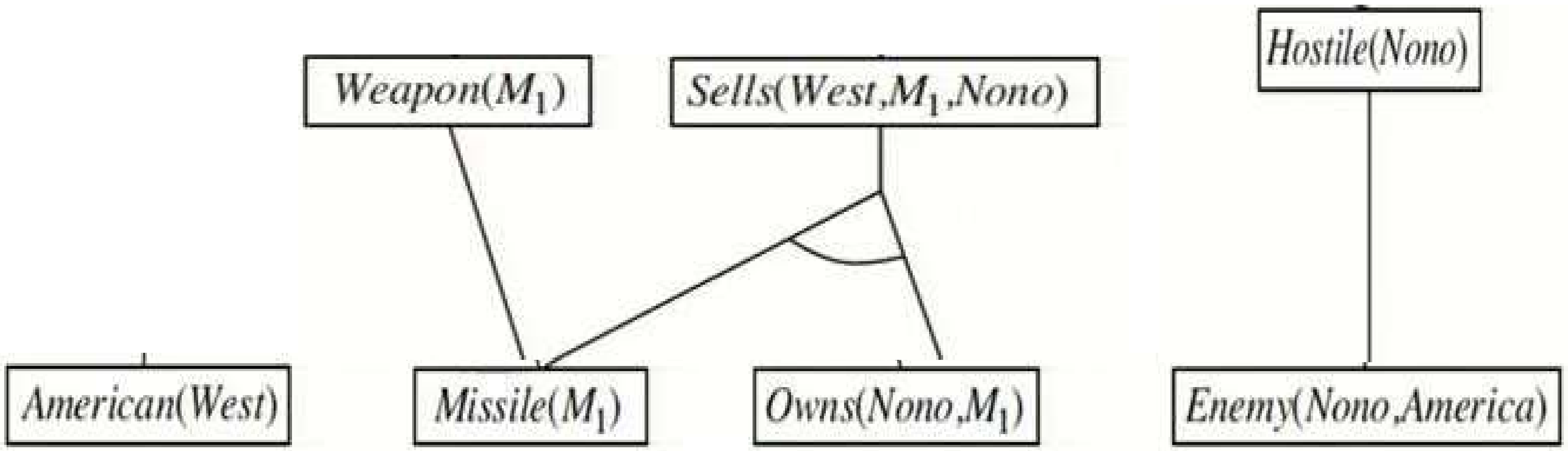
1. $American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)

Owns(Nono, M1)

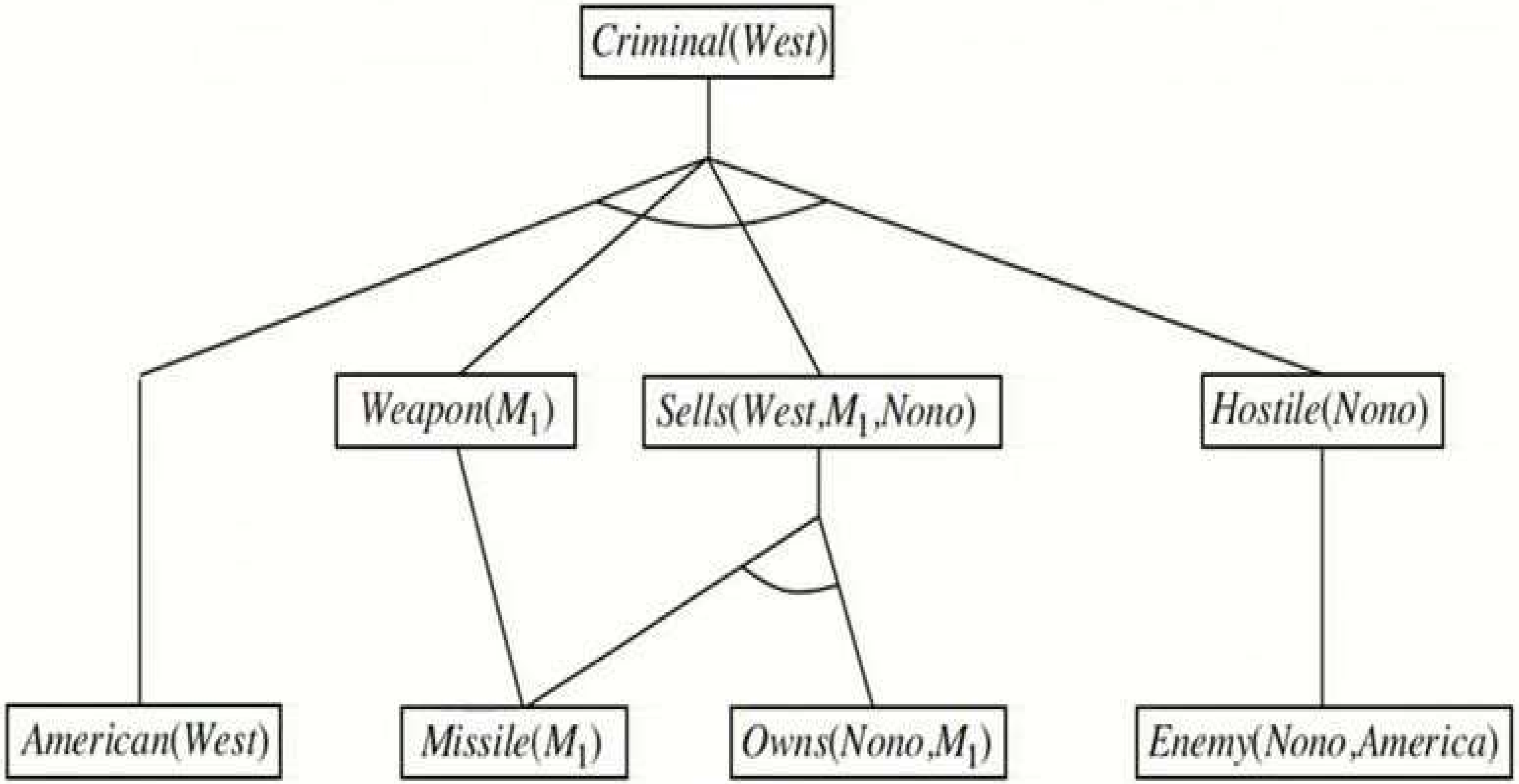
American (West)

Enemy (Nono, America)



Forward Chaining

1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$



Backward Chaining

Algorithm:

1. Form Definite Clause
2. Start from the Goals
3. Search through rules to find the fact that support the proof
4. If it stops in the fact which is to be proved → Empty Set- LHS

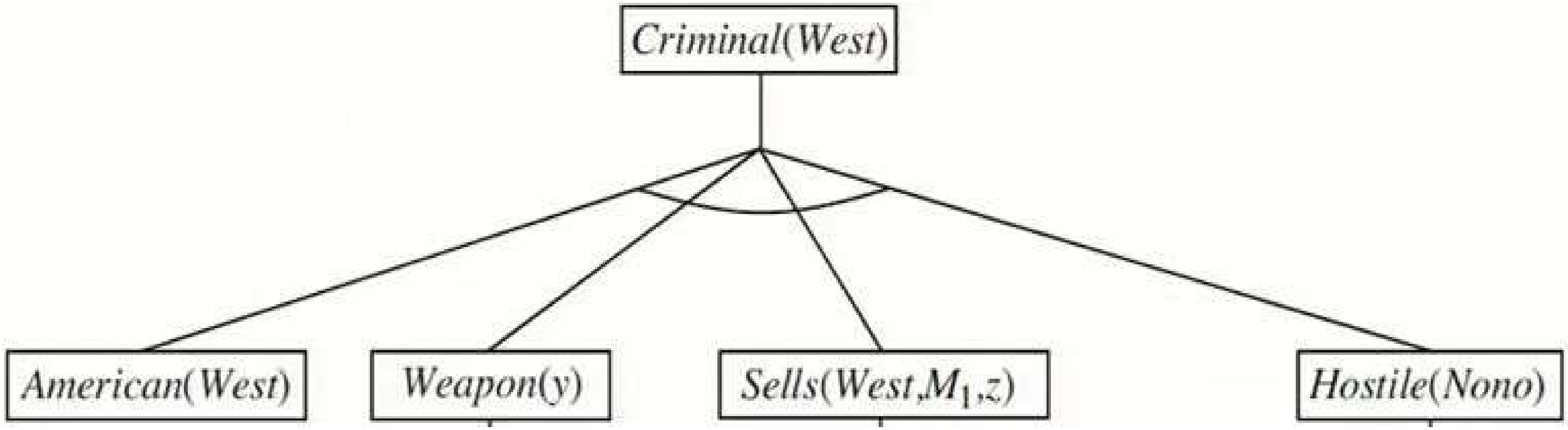
Divide & Conquer Strategy

Substitution by Unification

Backward Chaining

1. $American(x) \wedge Weapon(y) \wedge Sells(x,y,z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono,x) \Rightarrow Sells(West,x,Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

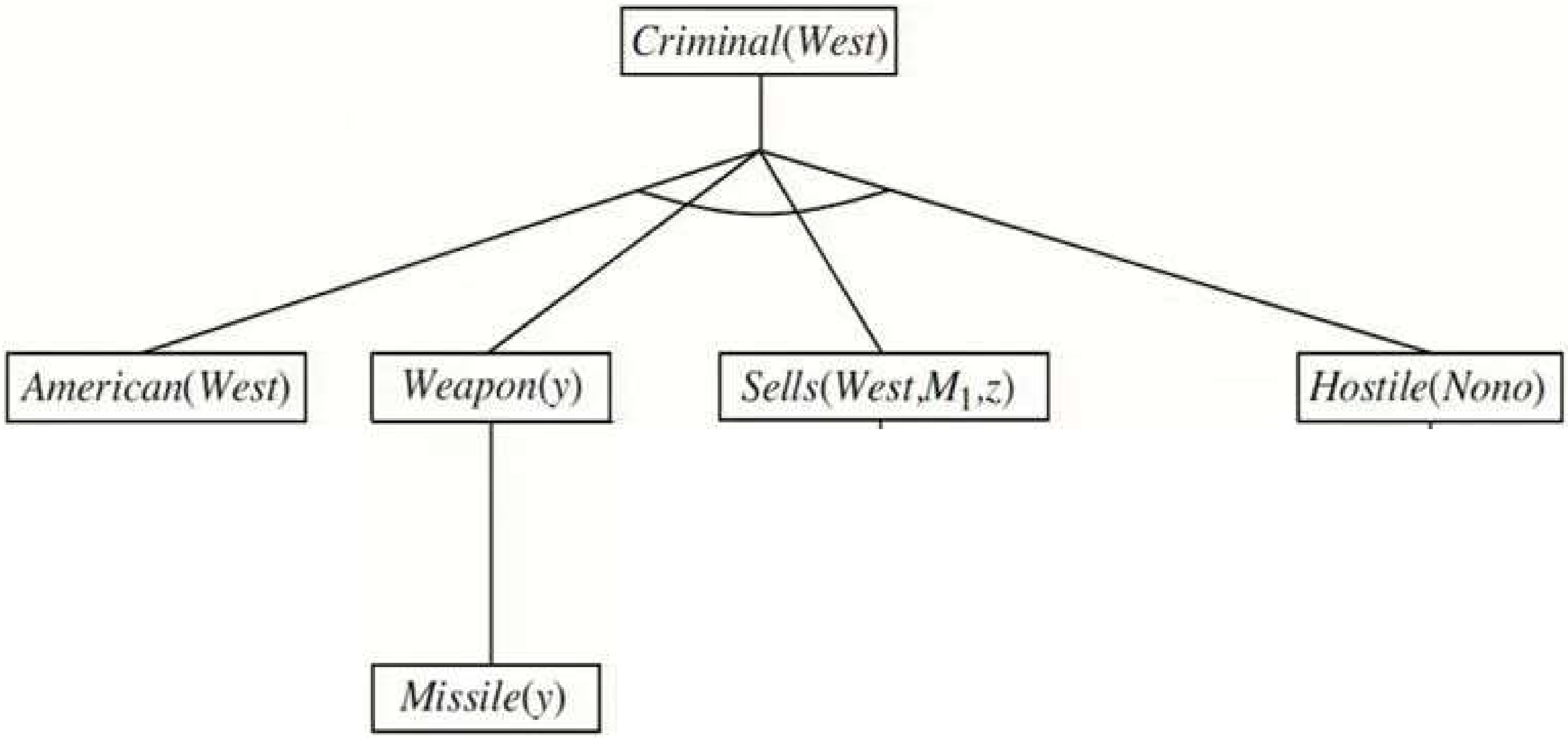
Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)



Backward Chaining

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)

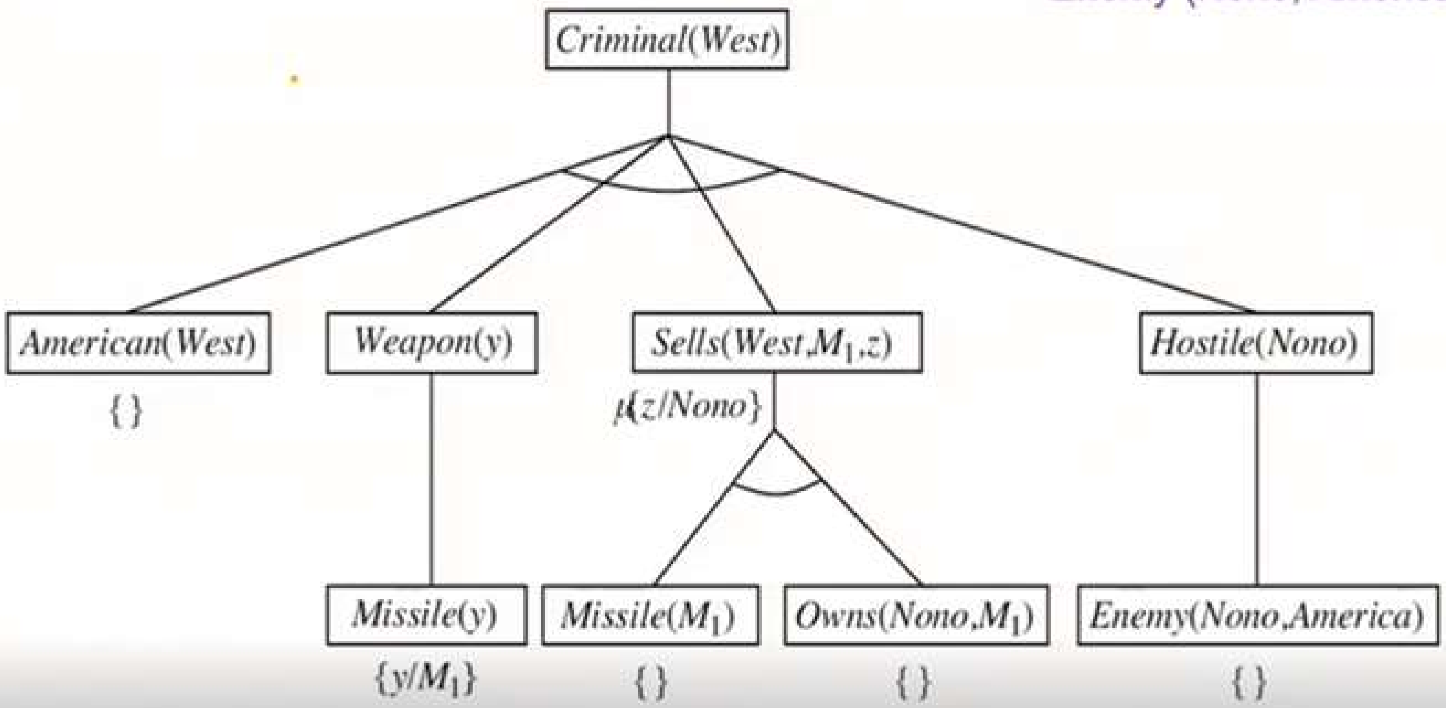
1. $American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \wedge Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$



Backward Chaining

1. $American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$
2. $Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$
3. $Missile(x) \Rightarrow Weapon(x)$
4. $Enemy(x, America) \Rightarrow Hostile(x)$

Missile(M1)
Owns(Nono, M1)
American (West)
Enemy (Nono, America)



Required Reading: AIMA - Chapter # 7, # 8, # 9

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials