Mathematical Foundations for Machine Learning

Key Points

Lecture 1

1. Understand the key properties of addition and multiplication and note that in general $AB \neq BA$.
2. Determine rank of a matrix using elementary row operations and converting to REF / RREF.
3. Use rank to deduce the consistency of linear systems and derive general solution(s) of linear systems.
4. Properties of transpose and inverse operations.
5. Gauss elimination method of solving a system of linear equations of the form $A_{n\times n}x = b$.

Lecture 2

1. Define groups and fields.
2. Understand the concept of a vector space over a field F.
3. Determine whether a given subset of a vector space is a subspace or not.
4. Linear span, linear independence and dependence of vectors.
5. Basis and dimension of a vector space.

Lecture 3

1. Define norms of vectors and matrices.
2. Define innerproduct and differentiate from dot product.
3. Understand innerproduct space as a vector space with an innerproduct defined.
4. Derive positive definiteness of A from the positive definiteness of Ω.
5. Angles and orthogonality of vectors and the Cauchy-Schwarz inequality.
6. Gram Schmidt orthogonalization process.
7. LU decomposition through elementary matrices.

Lecture 4

1. Determinants and its properties.
2. Determine eigenvalues and eigenvectors of a square matrix.
3. Diagonalize a matrix using linearly independent eigenvectors (eigenvalue decomposition (EVD)).
4. Use the fact that a symmetric matrix has an orthogonal (and hence) an orthonormal set of eigenvectors.
5. Observe that eigenvectors corresponding to distinct eigenvalues are linearly independent.
6. Understand the nature of eigenvalues for Hermitian, Skew-Hermitian and Unitary matrices.
7. Apply the properties of eigenvalues and eigenvectors in solving problems.

Lecture 5

1. Derive SVD of any given matrix and understand it geometrically.
2. Note the key differences between EVD and SVD.
3. Write a matrix A of rank r and a sum of r rank 1 matrices.
4. Understand the spectral norm of a matrix.