Lab 0

Due February 22, 2021 before class

Cassandra, Shashank, Sherrie, and Manu were starting to get bored because they didn't have enough Advanced Algorithms work, so they decided they needed to find themselves a hobby. They found a local badminton league and each joined separate teams. Everyone on the team with the most wins at the end of the season will win a Dunkin Donuts gift card. Shashank has stated that once his team can no longer win the most games, he is going to quit. Therefore he wants to keep track of when his team has been mathematically eliminated from the competition.

Part 1 - Setting up the problem

The current standings are as follows:

				Against			
Team	**Wins**	**Losses**	**Left**	**Sherrie**	**Shashank**	**Manu**	**Cassandra**
Sherrie	83	71	8	-	1	6	1
Shashank	80	79	3	1	-	0	2
Manu	78	78	6	6	0	-	0
Cassandra	77	82	3	1	2	0	-

Problem 1:

Question 1: Which teams have been eliminated from getting the Dunkin Donuts prize? Which teams have not been eliminated? Why or why not?

Solution: Cassandra's team is definitely eliminated from getting the Dunkin Donuts prize. This is because she has only three games left, and even if she wins all her games, the most wins she could get is a total of eighty wins. Since Sherrie's current wins is above eighty, Cassandra has no possible way of getting the most number of wins. Therefore, Cassandra is eliminated. Shashank is going to be eliminated since even if he wins against both Sherrie and Cassandra, he can only tie with Sherrie (if Sherrie loses against Manu and Cassandra) which means that Shashank would not get the most points.

Manu and Sherrie are still in the running since they have enough games left where Manu can either tie with Sherrie or achieve the most number of wins. Manu only has six games left against Sherrie. If he wins all of his six games, he will get the most number of wins (or tie with Sherrie if she wins against Shashank or Cassandra). If she wins against both Shashank and Cassandra, Manu cannot win. Sherrie is not eliminated since she currently has the most wins and has the most games left.

Problem 2:

Sherrie decides that she's going to be smarter than the rest of the Advanced Algorithms team and create an easy way to tell who's been eliminated. Due to bad record-keeping, Sherrie has access to none of the scores. However, she does have a 5 minute window to look at the standings to quickly determine what teams are eliminated. She decides she is going to set this up as a network flow problem.

The games won will be represented by w_{name} and the games remaining will be represented by r_{name}. For instance, Sherrie has won $w_{Sherrie}$ games and has $r_{Sherrie}$ games remaining. The teaching team trusts you can figure out the variable representation for the other players.

She sets it up as the following network flow:

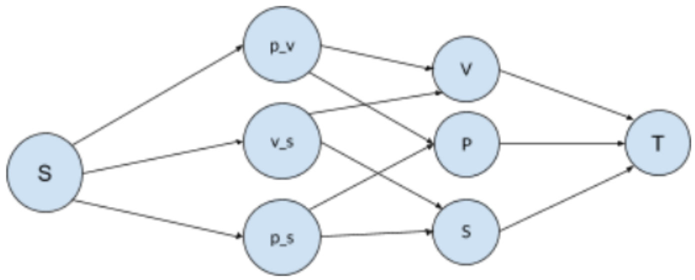

Figure 1: Sherrie's network flow

Figure 1 denotes the network flow diagram that Sherrie constructed to figure out if she was eliminated (note: the flow diagram is specific to her), without any of the capacities. The first node is a source node. The second series of nodes represent the matches between each of the other teams (i.e. Cassandra vs Shashank, Cassandra vs Manu). The third series of nodes represent the other teams (i.e. Cassandra, Shashank). The final node is a sink node.

Construct a procedure for determining if teams are eliminated. Note that to do this, you will have to determine what the capacities of the graph depicted in Figure 1 are. For this question, you must:

Question 1 and 2: Draw out the graph with the capacities represented in variable form (explain what the variables represent. Identify what the values of the variables would be for this specific problem

Solution:

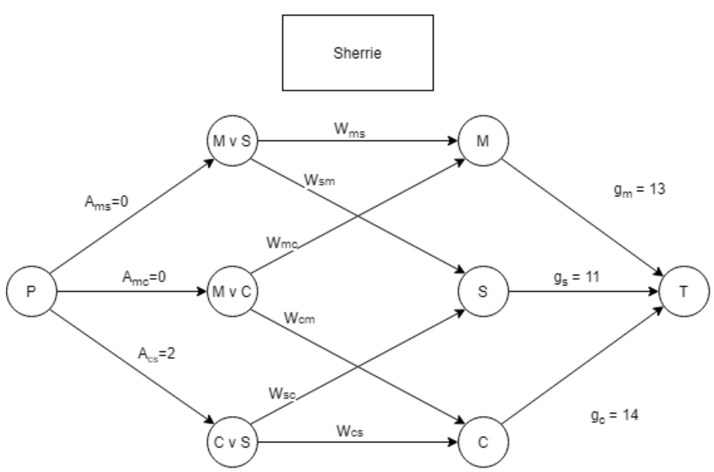

Figure 2: Sherrie's network flow

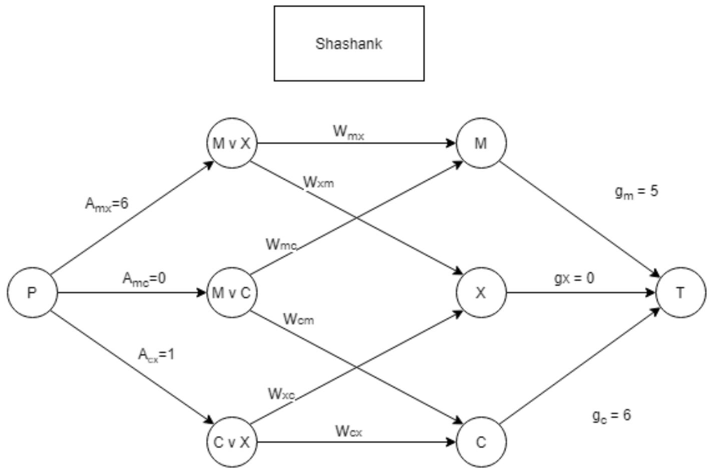

Figure 3: Shashank's network flow

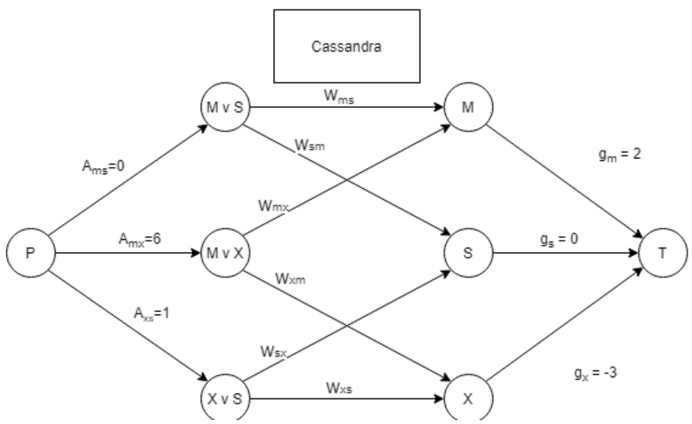

Figure 4: Cassandra's network flow

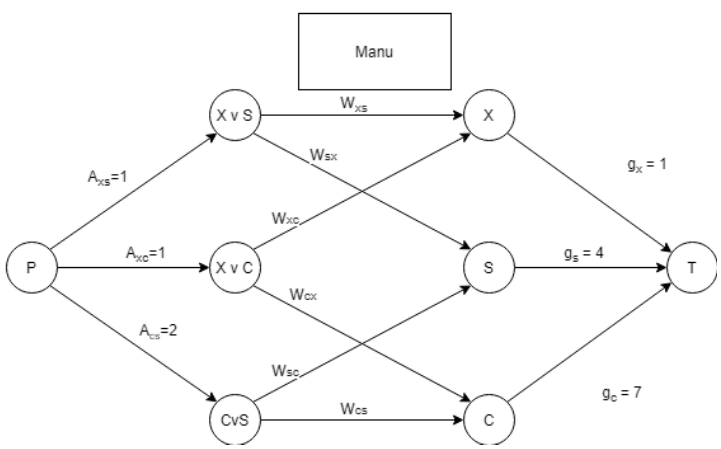

Figure 5: Manu's network flow

Figure 2 shows Sherrie's network flow. The source, P, is the total number of games left that does not involve

Sherrie. M v S is the number of games left between Manu and Shashank. Thus, the edge between P and M v S is the number of games left between Manu and Shashank, and the capacities are denoted by A_{ms}. Similarly, M v C represents the games left between Manu and Cassandra and C v S represents the number of games left between Cassandra and Shashank. Likewise A_{mc} and A_{cs} are the respective capacities for edges between P and the two aforementioned nodes. The nodes M, S, and C represents the wins that each person could possibly get based on the number of games they have left. The edges between the games left and the games won, denoted by $w_{winnerloser}$ are not limited by any constraints, so their capacities are infinite. Finally, we have the sink T. The capacity between the nodes M, S, C and T is the total wins from Sherrie plus the games remaining minus the total games each respective other participants have won. The variable that denotes this value is g_{name}.The figure also shows the values of the variables. Sherrie's network flow shows whether or not the games she is not a part of will affect whether she is eliminated or not. Note that S is Shashank, M is Manu, C is Cassandra, and X is Sherrie.

Figure 3 shows Shashank's network flow, figure 4 shows Cassandra's network flow, and figure 5 shows Manu's network flow. They are similar to Sherrie's network flow in that they all show if the person of focus will be eliminated or not. The names of the specific variables and their values are different from Sherrie's network flow (which are reflected in their respective figures), but the general network is the same.

Question 3 and 4: Write out and explain the strategy for solving the problem.

Solution: To solve this problem, we can look at each person's network flow. If there are no residual edges leaving the source (meaning flow is at maximum capacity on all edges leaving the source), then they are not eliminated since the games the other players played without the person of interest are not enough for them to beat the player of interest. This works because if there are no residual edges leaving the source, it means that the number of potential games a player can win from games not involving the person of interest is less than the number of games they need to win to beat the person of interest. Thus, by taking into account the different scenarios in which winning points can be distributed to each player from games that don't involve the person of interest, one can evaluate whether or not games outside the control of the person of interest can outrule the chance the person of interest has a chance of winning or not.

Problem 3:

Question 1: For the network flow diagram you finished above, convert it into a linear program (using variables, not the values). If you aren't sure how to do this, check out this link.

Solution:

Sherrie's Linear Program:

To see if Sherrie is eliminated, we have to maximize:

$$A_{ms} + A_{mc} + A_{cs}$$

which is subject to the flow capacity constraints:

$$A_{ms} \leq 0$$

$$A_{mc} \leq 0$$

$$A_{cs} \leq 2$$

$$g_m \leq 13$$

$$g_s \leq 11$$

$$g_c \leq 14$$

and is also subject to the flow conservation constraints:

Node M v S:

$$A_{ms} = W_{ms} + W_{sm}$$

Node M v C:

$$A_{mc} = W_{mc} + W_{cm}$$

Node S v C:

$$A_{cs} = W_{sc} + W_{cs}$$

Node M:

$$W_{mc} + W_{ms} = g_m$$

Node S:

$$W_{sm} + W_{sc} = g_s$$

Node C:

$$W_{cm} + W_{cs} = g_c$$

Shashank's Linear Program:

To see if Shashank is eliminated, we have to maximize

$$A_{mx} + A_{mc} + A_{cx}$$

which is subject to the flow capacity constraints:

$$A_{mx} \leq 6$$
$$A_{mc} \leq 0$$
$$A_{cx} \leq 1$$
$$g_m \leq 5$$
$$g_x \leq 0$$
$$g_c \leq 6$$

and is also subject to the flow conservation constraints:

Node M v X:

$$A_{mx} = W_{mx} + W_{xm}$$

Node M v C:

$$A_{mc} = W_{mc} + W_{cm}$$

Node C v X:

$$A_{cx} = W_{cx} + W_{xc}$$

Node M:

$$W_{mx} + W_{mc} = g_m$$

Node X:

$$W_{xm} + W_{xc} = g_x$$

Node C:

$$W_{cm} + W_{cx} = g_c$$

Cassandra's Linear Program:

To see if Cassandra is eliminated, we have to maximize

$$A_{ms} + A_{mx} + A_{xs}$$

which is subject to the flow capacity constraints:

$$A_{ms} \leq 0$$

$$A_{mx} \leq 6$$

$$A_{xs} \leq 1$$

$$g_m \leq 2$$

$$g_s \leq 0$$

$$g_x \leq 3$$

and is also subject to the flow conservation constraints:

Node M v S:

$$A_{ms} = W_{ms} + W_{sm}$$

Node M v X:

$$A_{mx} = W_{mx} + W_{xm}$$

Node X v S:

$$A_{xs} = W_{xs} + W_{sx}$$

Node M:

$$W_{mx} + W_{ms} = g_m$$

Node S:

$$W_{sm} + W_{sx} = g_s$$

Node X:

$$W_{xm} + W_{xs} = g_x$$

Manu's Linear Program:
To see if Manu is eliminated, we have to maximize:

$$A_{xs} + A_{xc} + A_{cs}$$

which is subject to the flow capacity constraints:

$$A_{xs} \leq 1$$

$$A_{xc} \leq 1$$
$$A_{cs} \leq 2$$
$$g_x \leq 1$$
$$g_s \leq 4$$
$$g_c \leq 7$$

and is also subject to the flow conservation constraints:

Node X v S:
$$A_{xs} = W_{xs} + W_{sx}$$
Node X v C:
$$A_{xc} = W_{xc} + W_{cx}$$
Node C v S:
$$A_{cs} = W_{sc} + W_{cs}$$
Node X:
$$W_{xc} + W_{xs} = g_x$$
Node S:
$$W_{sx} + W_{sc} = g_s$$
Node C:
$$W_{cx} + W_{cs} = g_c$$

Question 2: Provide an explanation of why this formulation makes sense, given the original context.

Solution: This formulation makes sense because if we maximize the worst case where all the focused person's opponents are able to get their max number of points in the games without that focused person, then the person of focus can see whether or not the games within their control will allow them to still be in the running or not.

The flow capacity constraints show the number of wins an opponent could get (without the focused person) as well as the likelihood of the opponents being too far away in terms of score which would lead to the person of focus being eliminated. This make sense because opponents cannot win more points than the number of games they have left against other opponents. And, the opponents with zero or negative capacity means that it is impossible for the person of focus to win against that opponent. Since $g_{opponent}$ is equal to the number of wins and games left for the person of focus minus the possible games won for the opponent, this means that a zero or negative number would mean that the number of possible games won for the opponent meets or exceeds the amount of wins the person of focus can achieve.

The flow conservation constraints show that the number of possible wins for each person's match-up has to equal the number of games left for those match-ups. It also shows that the total number of wins for each respective opponent (excluding the games left against the person of focus) represents the difference in wins between the opponent and the person of focus.

Part 2 - Implementation

Question 1: Implement the network flows portion of of the algorithm.

Code can be found here

Question 2: Feel free to add your own test cases if you'd like to robustly test your code. If you think the test cases are sufficient, please explain.

Solution: We think the test cases are sufficient because they cover a wide range of situations, where teams are eliminated and where they are not. It also covers cases where there are negative edges or edges with capacity is zero in the network flow. Thus, it is rather robust and is acceptable for the purposes of this lab, though of course there are always other test cases that can be run.