



F E Y Z İ Y E M E K T E P L E R İ V A K F I
IŞIK ÜNİVERSİTESİ



AO bench design

May 18, 2018

AO bench design Zemax model and optimization post-PDR design

	Name	Date	Signature
Prepared	A. Bouxin	May 18, 2018	
Approved	L. Jolissaint	May 18, 2018	
Released	Name	October 10, 2017
Released	Name	May 18, 2018	<div style="border: 2px solid red; padding: 5px; text-align: center;">SIGNATURE if you want to import a scanned one</div>
	Name	May 18, 2018



F E Y Z İ Y E M E K T E P L E R İ V A K F I
IŞIK ÜNİVERSİTESİ



AO bench design

May 18, 2018

Issue	Date	Section/Page affected	Reason/Remarks
1.0	October 10, 2017	All	New
2.0	May 18, 2018	All	Review of all the design after decisions and Paolo S.



AO bench design

May 18, 2018

Acronyms

aO	active Optics	NGS	Natural Guide Star
AO	Adaptive Optics	OTF	Optical Transfer Function
CCD	Charge Coupled Device	P2V	Peak-to-Valley
CMOS	Complementary Metal Oxide Semiconductor	PSF	Point Spread Function
DAG	Dogu Anadolu Gözlemevi (East Anatolian Observatory)	PSF-R	PSF Reconstruction
DM	Deformable Mirror	PWFS	Pyramid WFS
FoV	Field-of-View	RMS	Root Mean Square
FWHM	Full Width at Half Maximum	SH-WFS	Shack-Hartmann WFS
NCPA	Non Common Path Aberrations	SNR	Signal-to-Noise Ratio
		TT	Tip-Tilt
		WFS	Wavefront Sensor
		WFE	Wavefront Error



AO bench design

May 18, 2018

1 Scope of this document

This document presents the DAG adaptive optics (AO) optical train design and optimization.

2 Introduction

There are several mandatory points we have to handle :

- the exit pupil has to be imaged on the DM;
- the DM diameter is fixed to the DM-468 from ALPAO; that means $\phi 33$ mm;
- the exits pupil has to be imaged on the TT mirror for the P-WFS modulation;
- the beam has to converge on the pyramid apex;
- the angle of the beam that arrives onto the pyramid apex is calculated in order to get a diffraction limited PSF size of 2 times the pyramid roof;
- the exit pupil has to be imaged onto the EMCCD detector
- the beam footprint diameter onto the detector has to be defined accordingly to the oversampling criterion (explained section 3.5).

We use off axis parabolas (OAPs) for all our design to reduce the aberrations (compared to lenses). We have added one more constraint on the design which is to have parallel beam arriving or leaving each off axis parabola in order to limit the spherical aberrations.

The fixed parameters are resumed here :

- exit pupil diameter : $\phi_{ExtP} = 727.4046$ mm;
- distance exit pupil to focal plane : $\overline{ExP-FP} = 10338.74$ mm;
- DM diameter : $\phi_{DM} = 33.0$ mm;
- pixel size of the Nuvuu EMCCD AO : PxSize = $24 \mu\text{m}$



AO bench design

May 18, 2018

In order to design the AO bench we started with the telescope model. Indeed, we would try to compensate for the field of curvature introduced by the mirrors of the telescope with the off-axis parabolas (OAP) of the AO. The Zemax model of the telescope shows a curvature radius of about 1255 mm.

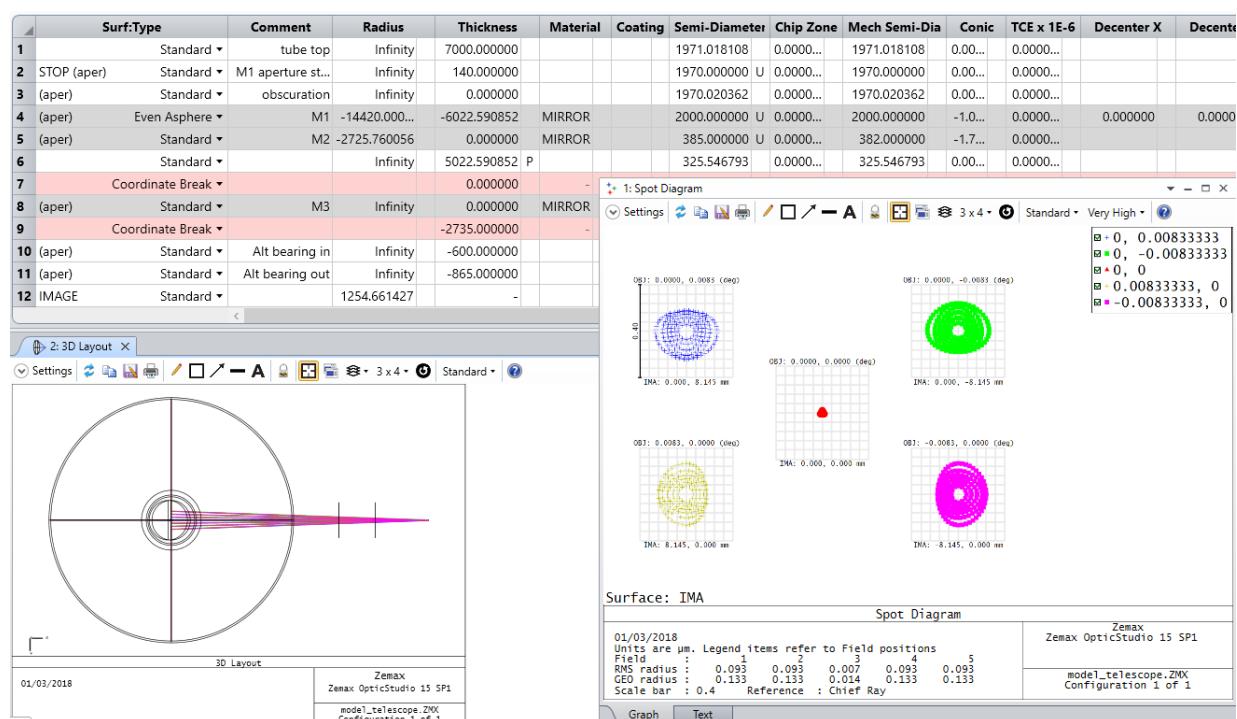


Figure 1: Zemax model of the telescope



3 Design development and optimization

A preliminary design have been set for the PDR and the main points are summarized at each optimization step of this report.

3.1 Imaging the exit pupil of the telescope onto the DM

The first branch consists on an imaging system of the telescope pupil onto the DM. The beam has to be collimated and the footprint has to take the entire clear aperture of the DM (for the DM-468 from ALPAO $\phi_{DM} = 33$ mm).

The beam on the DM has to be reflected with a certain angle otherwise the reflected beam comes back on itself. The maximum acceptable angle can be calculated considering the position error of the beam per actuator on the DM. We can align the beam with an error of $1/10^{\text{th}} \Lambda$ (Λ the actuator pitch = 1.5 mm for the ALPAO DM-468). In order for the projected beam diameter to be no more than 10% smaller than the DM diameter on both side, the tilt of the DM must be no more than α (figure 2) :

$$\alpha = \arccos \left(1 - \frac{1}{5} \frac{\Lambda}{\phi_{DM}} \right) \quad (1)$$

$$\alpha = \arccos \left(1 - \frac{1}{5} \frac{1.5}{33} \right) \quad (2)$$

$$\alpha = 7.73^\circ \quad (3)$$

$$2\alpha = 15.46^\circ \quad (4)$$

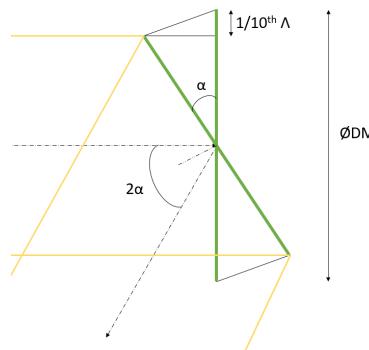


Figure 2: Sketch of the input and output beam depending on the tilt angle of the DM



AO bench design

May 18, 2018

The input beam diameter is then :

$$\varnothing_{\text{DM-beam}} = \varnothing_{\text{DM}} \cos \alpha = 33 \times \cos (7.73) = 32.7 \text{ mm} \quad (5)$$

In order to have a collimated beam on the DM with a diameter of $\varnothing_{\text{DM-beam}}$, the focal length of the OAP0 is calculated by the following sequence of equations. To visualize the parameters needed and the context we can look at figure 3. The with an angle θ with the corresponding reference coordinates $(0, x', y')$. The ray ΔCR corresponds to the chief ray, $\Delta\alpha u$ is the "upper" marginal ray and $\Delta\alpha b$ is the "bottom" marginal ray. The origin is placed at the focal plane of the telescope.

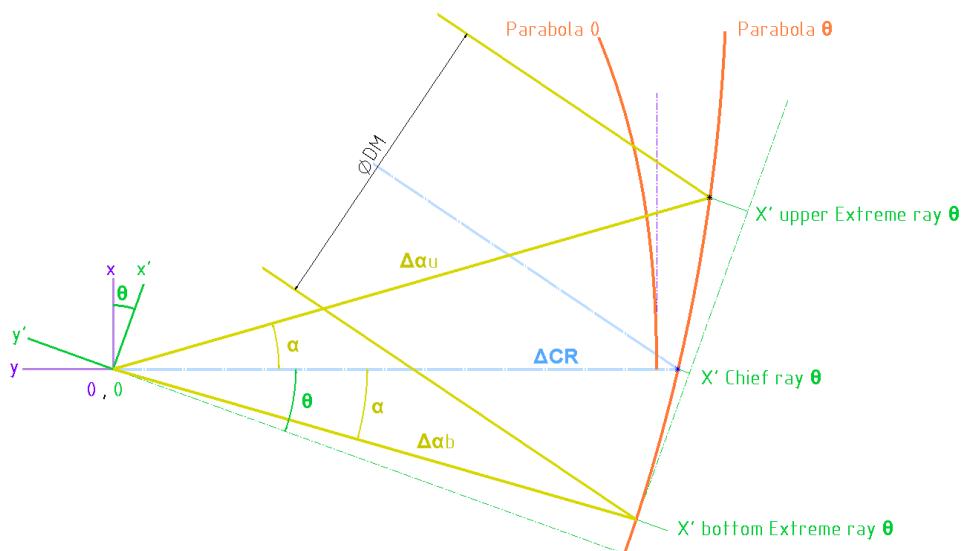


Figure 3: Ray tracing of a tilted parabola

In the reference coordinates $(0, x', y')$, we have (with PFL the Parental Focal Length of the OAP):

- equation of the parabola θ :

$$y' = \frac{x'^2}{4 \text{PFL}} - \text{PFL} \quad (6)$$

- equation of $\Delta\alpha u$:

$$y' = -\frac{1}{\tan(\theta + \alpha)} x' \quad (7)$$



AO bench design

May 18, 2018

- equation of $\Delta\alpha b$:

$$y' = -\frac{1}{\tan(\theta - \alpha)}x' \quad (8)$$

Determination of $x'_{\text{Ex}\theta u}$ (intersection between parabola θ and $\Delta\alpha u$) :

$$(6) = (7)$$

$$x'_{\text{Ex}\theta u} = 2 \text{PFL} \left(-\frac{1}{\tan(\theta + \alpha)} + \sqrt{\frac{1}{\tan^2(\theta + \alpha)} + 1} \right) \quad (9)$$

Determination of $x'_{\text{Ex}\theta b}$ (intersection between parabola θ and $\Delta\alpha b$) :

$$(6) = (8)$$

$$x'_{\text{Ex}\theta b} = 2 \text{PFL} \left(-\frac{1}{\tan(\theta - \alpha)} + \sqrt{\frac{1}{\tan^2(\theta - \alpha)} + 1} \right) \quad (10)$$

In order to have the OAP0 output beam diameter equal to the DM clear aperture we have :

$$x'_{\text{Ex}\theta u} - x'_{\text{Ex}\theta b} = \varnothing_{\text{DM-beam}} \quad (11)$$

The parental focal length (see appendix A) of OAP0 is :

$$\text{PFL} = \frac{1}{2} \frac{\varnothing_{\text{DM-beam}}}{-\frac{1}{\tan(\theta + \alpha)} + \sqrt{\frac{1}{\tan^2(\theta + \alpha)} + 1} + \frac{1}{\tan(\theta - \alpha)} - \sqrt{\frac{1}{\tan^2(\theta - \alpha)} + 1}} \quad (12)$$

Using the equation (30) described in appendix A we can transform the parental focal length into the effective focal length.

For the OAP0 the angle α_{OAP0} is

$$\alpha_{\text{OAP0}} = \arctan \left(\frac{\varnothing_{\text{ExP}/2}}{\overline{\text{ExP-FP}}} \right) \quad (13)$$

The numerical application of (13) with $\varnothing_{\text{ExP}} = 727.4046$ mm and $\overline{\text{ExP-FP}} = 10338.74$ mm gives $\alpha_{\text{OAP0}} = 2.015^\circ$, the aperture demi angle of the beam arriving on the first OAP.

After the discussion we had with Paolo Spano, we took his advice into account and choose the parabola tilt angle according to the following precept : $\theta = 35^\circ$ is a critical choice and $\theta > 35^\circ$ is a nightmare (manufacturing and alignment). We should take an tilt angle



AO bench design

May 18, 2018

smaller than 30° to be realistic and the smaller the best. However, we are limited by the space around the focal plane because of the field correction lens (represented by the 2 surfaces around the FP on figure ?????????????????????).

We can set the parabola tilt angle to $\theta = 20^\circ$ in order to keep this angle as small as possible to limit aberrations.

Using these inputs the focal length of the OAP0 is :

$$PFL_{OAP0} = 443.13 \text{ mm}$$

This value is too specific so we round it to 460 mm. The image of the exit pupil given by OAP0 gives the position of the DM. Its position p_i relative to the OAP0 vertex is given by (we use Gauss law) :

$$p_i = \frac{p_o PFL_{OAP0}}{p_o + PFL_{OAP0}} \quad (14)$$

$$p_i = \frac{-(10338.74 + 464.77)464.77}{-10338.74} \quad (15)$$

$$p_i = 480.47 \text{ mm} \quad (16)$$

The Zemax "pick up pupil position" macro is used to get the perfect DM position wrt the OAP0. The DM is then at $p_i = 487.823$ mm. Even if we understand that we cannot align the DM reflecting surface with this accuracy, we keep it in the Zemax model and we will work on that during the tolerancing analysis.



AO bench design

May 18, 2018

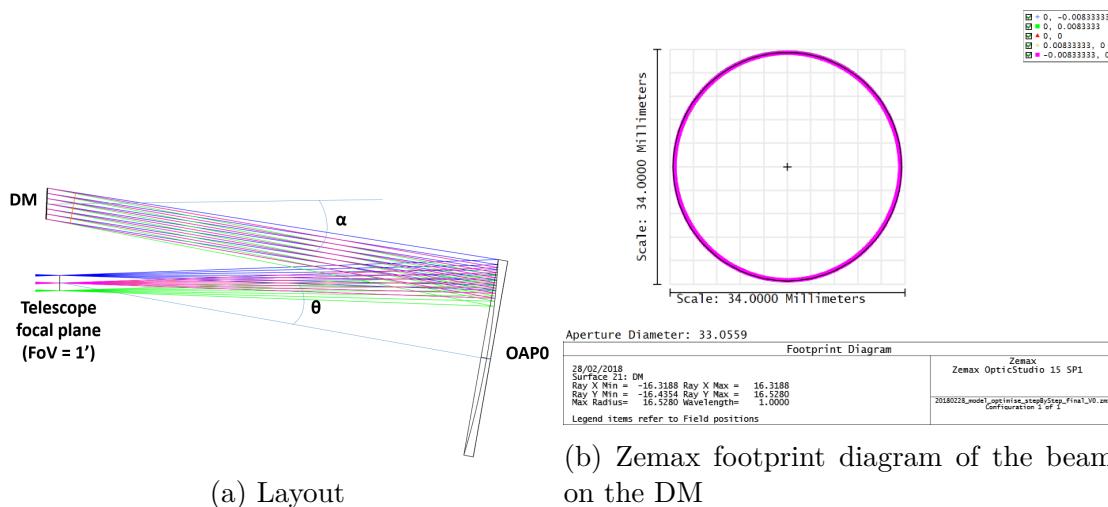


Figure 4: Zemax model of the 1rst part of the AO design

The beam footprint on the DM shows that all the field fits inside the clear aperture. The maximum diameter is equal to 32.963 mm (< 33 mm). The angle of the DM is set to -7.73°(and not 7.73°) because the footprint of the beam is larger when the DM tilt angle does not follow the stop angle (see [1]). Indeed when we work with OAPs the image stop after an OAP is not perpendicular to the optical axis. The OAP introduce a tilt angle of the image plane. If we tilt the DM in the opposite position of this OAP-introduced tilt angle we add aberrations whereas if we tilt it the same direction we can compensate for it.

3.2 Option to let the beam going through the bench

When the AO is off we want the beam to go through the bench without taking the AO path. In order to do that we place a folding mirror nearby the focal plane (to size it as small as possible) to pick up the beam and send it to the AO. When the AO if off, the pick up mirror is put down and the beam goes straight. The angle of the pickup mirror is determined to make its size as small possible and to allow that all optics we have to implement after can be mounted on mechanical structure properly.

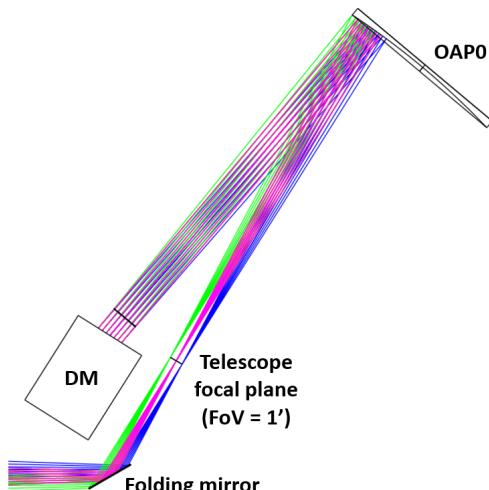


Figure 5: Zemax model from the telescope beam to the DM using a folding mirror

We could also add this fold mirror when the AO is off to bend the non corrected beam. The percentage of time the AO is off is less than when it is on. The light would reflect on the two mirror surfaces fewer times when it is off. This option will be studied for the next model of the AO (it is not implemented here).

3.3 Imaging the pupil on the TT modulation mirror

3.3.1 First approach

We need to image the pupil on the TT modulation mirror. Moreover, we know that to compensate for the introduced aberration from OAP0 we can use an OAP1 with the same focal length. The F/D ratio would stay the same as the telescope one which is favourable for the science path.

Figure 6 shows that to model this combination we use the "chief ray" solve on Zemax (Lens data) to keep the coordinates following the beam path. OAP1 is placed at the exact inverse position of OAP0 in order to compensate for its aberrations so the intermediate focal plane is near the telescope focal plane.



AO bench design

May 18, 2018



Figure 6: Zemax model from the telescope beam to the intermediate focal plane

We use an optimization (smallest spot radius) to go to the best focal plane (surface # 26 thickness is set as variable) and in order to consider the field curvature we apply a image radius of variable curvature. The optimization using a merit function is done on the image surface to get the smallest spot radius of the central beam (FoV = 0'). We can see that the spot diameter varies between 0.262 μm and 0.009 μm (the center of the FoV) but when we look the spots with the Airy disk we can say that the beam well focused.

However, the radius of the image curvature is smaller than the telescope output field of curvature so we did not compensate for that, instead we increase it (we will see how we could work on that or not).

Now, to find the tip-tilt modulation mirror diameter we need to go a step further. We know that we need to arrive on the pyramid roof (P-WFS) with a corresponding :

$$F\# = 2 \frac{\text{pyramid roof}}{\lambda} \quad (17)$$

Taking a pyramid roof of about 20 μm according to Jean-Pierre Veran (private communica-



AO bench design

May 18, 2018

cation) at $\lambda = 0.6 \mu\text{m}$ we have :

$$F\# = 66.7 \quad (18)$$

We round this value to $F\# = 60$ (we need to investigate the pyramid roof size that can be manufactured). We know we do not want a long focal distance for the converging optical system because it would take a lot of space and we want to reduce the number of optical surface as much as possible. Arbitrarily, we add the constraint that $f_{\text{OAP}3} < 800 \text{ mm}$. Then we have :

$$\begin{aligned} \varnothing_{\text{TT mod mirror}} &= \frac{f_{\text{OAP}3}}{F\#} \text{ mm} \\ \varnothing_{\text{TT mod mirror}} &\leq \frac{800}{60} \text{ mm} \\ \varnothing_{\text{TT mod mirror}} &\leq 13 \text{ mm} \end{aligned} \quad (19)$$

We know also that the magnification of OAP1 and OAP2 is given by :

$$G_t = \frac{f_{\text{OAP}2}}{f_{\text{OAP}1}} = \frac{\varnothing_{\text{TT mod mirror}}}{\varnothing_{\text{DM-beam}}} \quad (20)$$

Now, if we look at the current Zemax model (figure 6), we can see that if the focal length of OAP2 is smaller than 175 mm, we could not let the beam go through the bench (removing the 1rst folding mirror) because it would touch the OAP2 mount. This constraint and equation (20) bring to :

$$\begin{aligned} \frac{\varnothing_{\text{TT mod mirror}}}{\varnothing_{\text{DM-beam}}} f_{\text{OAP}1} &\geq 175 \text{ mm} \\ \varnothing_{\text{TT mod mirror}} &\geq 175 \frac{\varnothing_{\text{DM-beam}}}{f_{\text{OAP}1}} \\ \varnothing_{\text{TT mod mirror}} &\geq 12.5 \text{ mm} \end{aligned} \quad (21)$$

Looking at conditions (19) and (21) we can chose a standard tip tilt modulation mirror diameter of 0.5" (12.7 mm).

We choose $\varnothing_{\text{TT mod mirror}} = 12.7 \text{ mm}$ so $f_{\text{OAP}2} = 178.6 \text{ mm}$. We round this value to 180 mm. The OAP2 tip angle is set arbitrarily to 10° to avoid aberrations as much as possible.



AO bench design

May 18, 2018

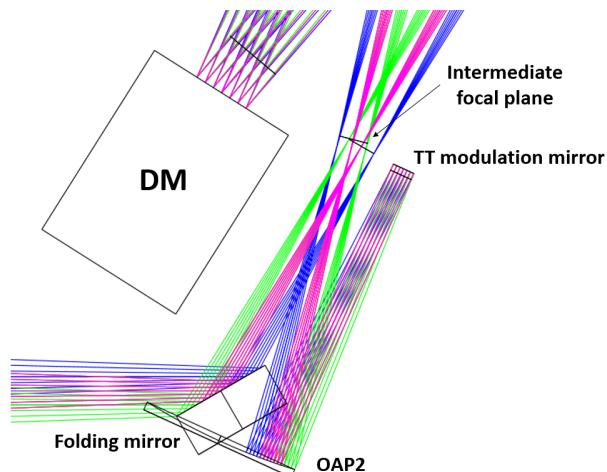


Figure 7: Zemax model from the telescope beam to the tip-tilt modulation mirror, where we see the space problem

Looking at figure 7 we can see that some room for the first folding mirror mount we can not add the OAP2 at this position because the beam would be cut off. Moreover, the TT modulation mirror would be close to the intermediate focal plane and there would be no position where to place a dichroic filter to send the light to the science.

3.3.2 Second approach

We can bend the path before arriving on the DM in order to save space for the optical elements (figure 8). This way we can avoid the troubles we have with the first approach.



AO bench design

May 18, 2018

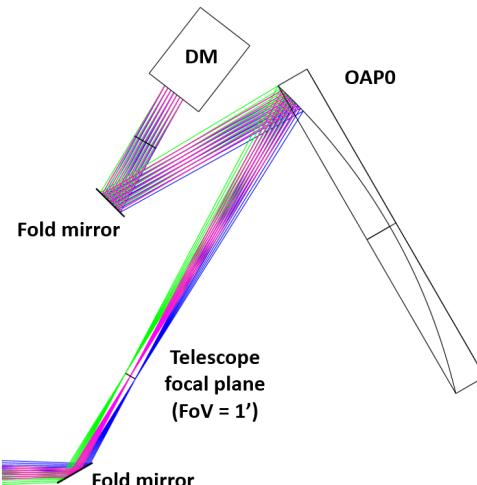


Figure 8: Zemax model from the telescope beam to the DM using a folding mirror to bend the path before the DM

The tilt angle of the DM becomes negative in the system coordinates to keep it as close as possible to the OAP0 stop plane. The new beam diameter on the DM is equal to 32.98 mm at its maximum.

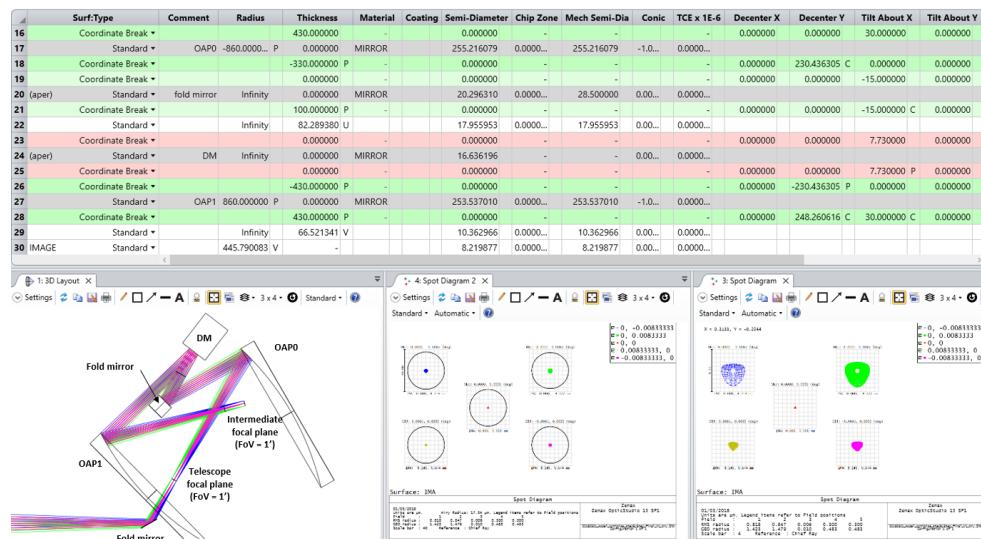


Figure 9: Zemax model from the telescope beam to the intermediate focal plane

The OAPs angle has been set to 30° because if we take the fold mirror volume into account



AO bench design

May 18, 2018

we need some room for its mount (the mirror mount volume is taken from [2] to have a first estimate). After redoing the same optimization as before, the intermediate focal plane aberrations are larger than with the first version. However the radius of the field curvature is larger than the first one (but still smaller than the telescope field curvature).

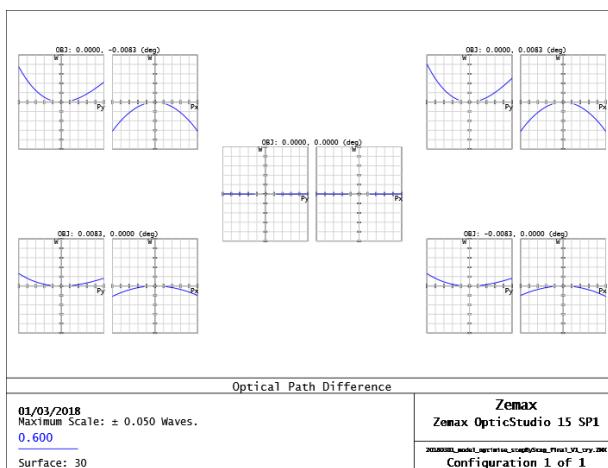


Figure 10: OPD Fan at the intermediate focal plane

To design the rest of the optic we can keep the constraint given by equation (19) repeated here because the F# does not change :

$$\varnothing_{TT \text{ mod mirror}} \leq 13 \text{ mm}$$

However, the constraint that limited the focal length of OAP2 (equation (21)) can be released because the beam does not go near the fold mirror.

This constraint lifter, with a TT modulation mirror diameter of 0.5" (set arbitrarily) we can determine the focal length of OAP2 of 175 mm. The TT modulation beam footprint maximum diameter is 12.64 mm for a TT angle of 10°. Figure 11 shows the Zemax model and beam footprint on the TT modulation mirror.

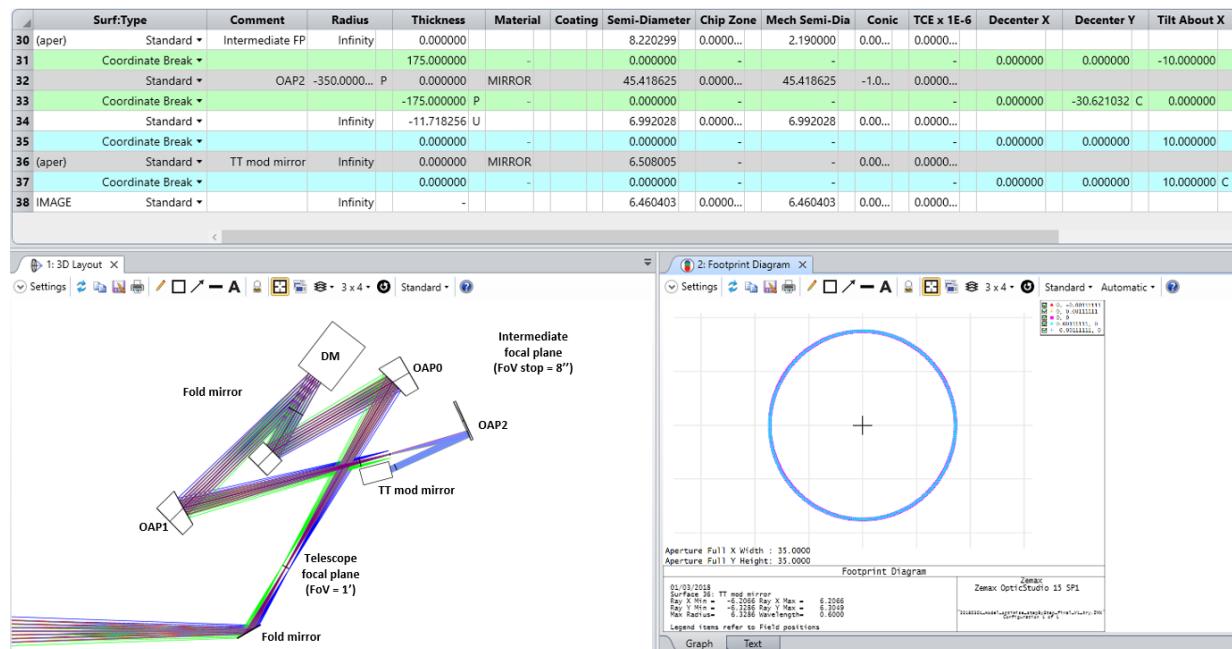


Figure 11: Zemax model from the telescope beam to the tip-tilt modulation mirror

We can see that there is not so much space for the tip-tilt modulation mirror (the dimensions are taken from [3] just to have an idea). We know that we will implement a dichroic mirror nearby the intermediate focal plane to send visible light to the WFS path and the rest to the science. The idea is then to bend the WFS path and leave the science light going straight (fig 12).

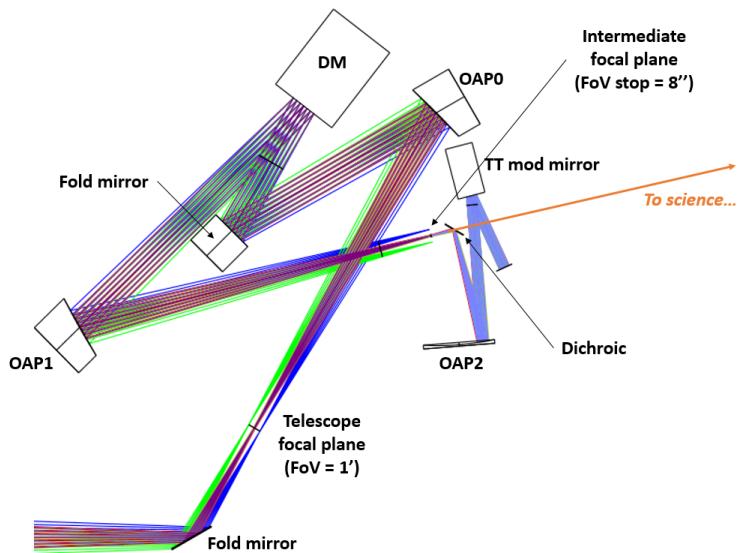


Figure 12: Zemax model from the telescope beam to the tip-tilt modulation mirror with the dichroic

3.4 Focus of the beam on the pyramid apex

Now we can calculate the OAP3 properties. Taking the TT modulation diameter of 12.7 mm and the F/D ratio equal (at least) to 60 (equation (18)) we find that : $f_{OAP3} \geq 762$ mm. We chose $f_{OAP3} = 800$ mm arbitrarily. In order to keep the image of the exit pupil of the telescope (so the image of the TT modulation mirror) real the TT modulation mirror has to be at least at f_{OAP3} from OAP3. To shorten as much as possible the distances we chose to place the TT modulation mirror at a distance $\geq f_{OAP3}$ from OAP3, its image is then at its focal plane. The OAP3 is decentered with a pick up of OAP2 output beam decenter to reduce aberrations.

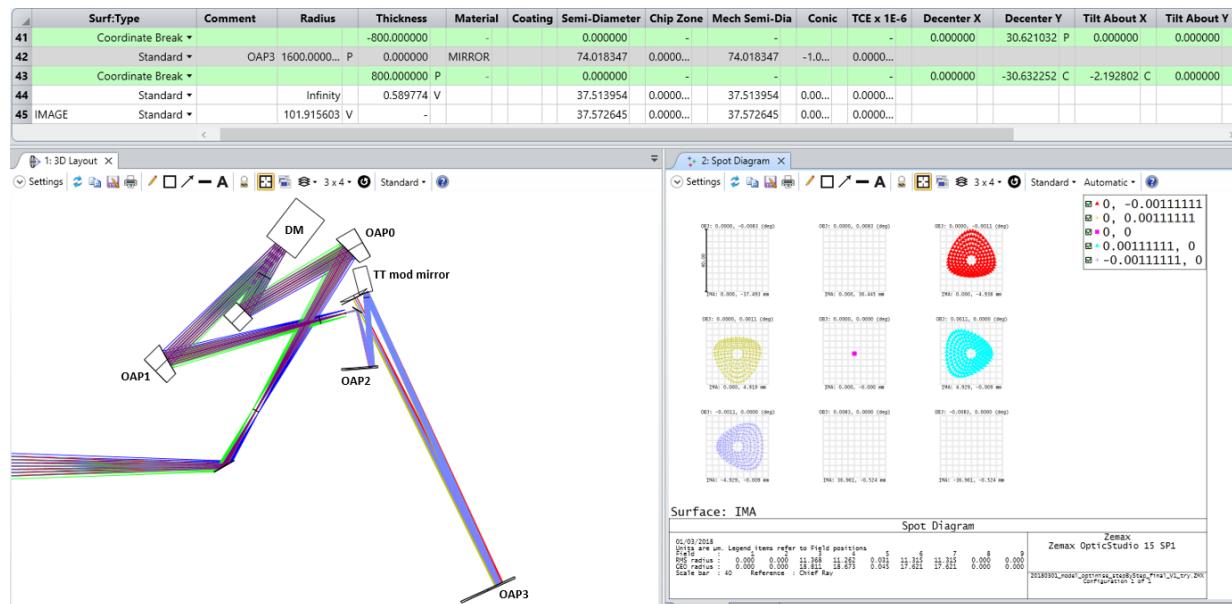


Figure 13: Zemax model from the telescope beam to focal plane of OAP3

We can see in figure 13 that the focal plane of the OAP3 is on the TT mirror, and the space is large between the TT modulation mirror and the OAP3. Moreover, the angle between the input and output beam on the OAP3 is about 2° which is too small to insert a fold mirror. We change the OAP3 decentring value to the number that correspond approximately to 10° of angular separation between in/out beams and we add a fold mirror to save space.

AO bench design

May 18, 2018

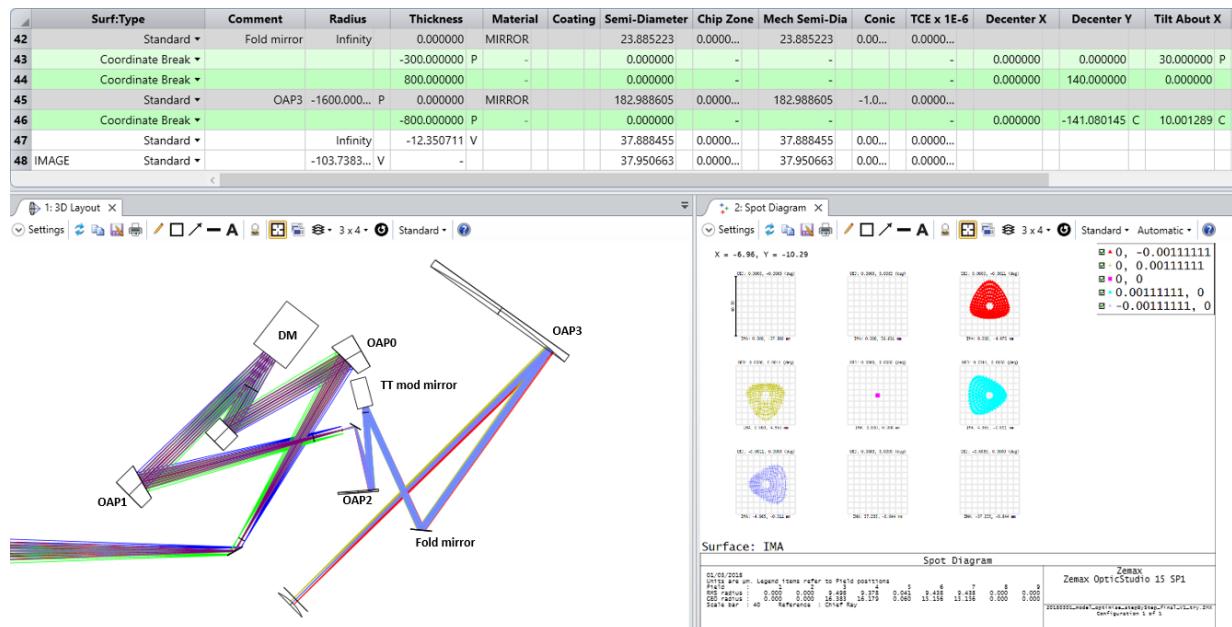


Figure 14: Zemax model from the telescope beam to focal plane of OAP3 with a fold mirror and decentring optimization

The pyramid apex will be at the focal plane of OAP3. The final OPD fan is given figure 15.

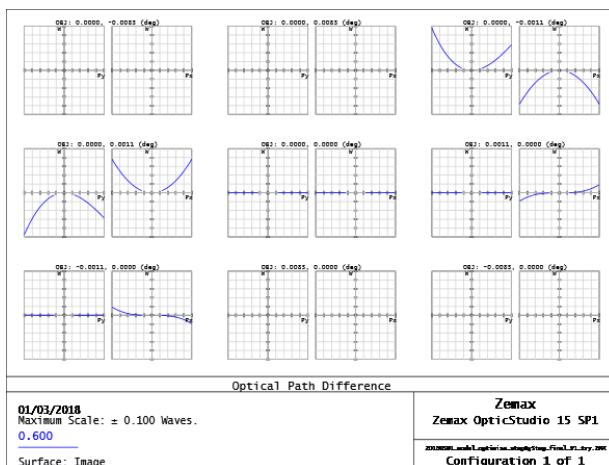


Figure 15: OPD fan at the focal plane of OAP3 (pyramid apex)



3.5 Image the pupil on the CCD detector

The detector is the Nuvuu EMCCD with 128 pixels and a pixel size of $24 \mu\text{m}$. We have to sample the beam with at least 1 pixel per actuator. In order to relax the alignment problems we can oversample the beam. According to Jean-Pierre Veran advice an oversampling of 1.5 should be enough. The ALAPO DM-468 has 22 actuators across the clear aperture diameter (so 23 pitches). Then we can calculate the beam diameter on the detector :

$$\varnothing_{\text{CCD}} = \#_{\text{actuator across } \varnothing} \times \text{PxSize} \times \text{Oversampling factor} \quad (22)$$

$$\varnothing_{\text{CCD}} = 22 \times 24 \mu\text{m} \times 1.5 \quad (23)$$

$$\varnothing_{\text{CCD}} = 0.792 \text{ mm} \quad (24)$$

The pupil has to be imaged on the camera. The TT mirror (pupil plane) image through the OAP3 is at the infinity. So when we image it through the relay lens the pupil plane arrives at the focal plane of the relay lens. Moreover, in order to get the beam size on the camera, the focal length of the relay lens (f_{LR} , see appendix B) has to be :

$$f_{\text{LR}} = \text{F\#} \times \varnothing_{\text{CCD}} \quad (25)$$

$$f_{\text{LR}} = 60 \times 0.792 \quad (26)$$

$$f_{\text{LR}} = 47.52 \text{ mm} \quad (27)$$

The remaining optics have been implemented in the Zemax model just to have an estimation of the space no optimization has been done (we can already see that the diameter is not precisely of \varnothing_{CCD}).

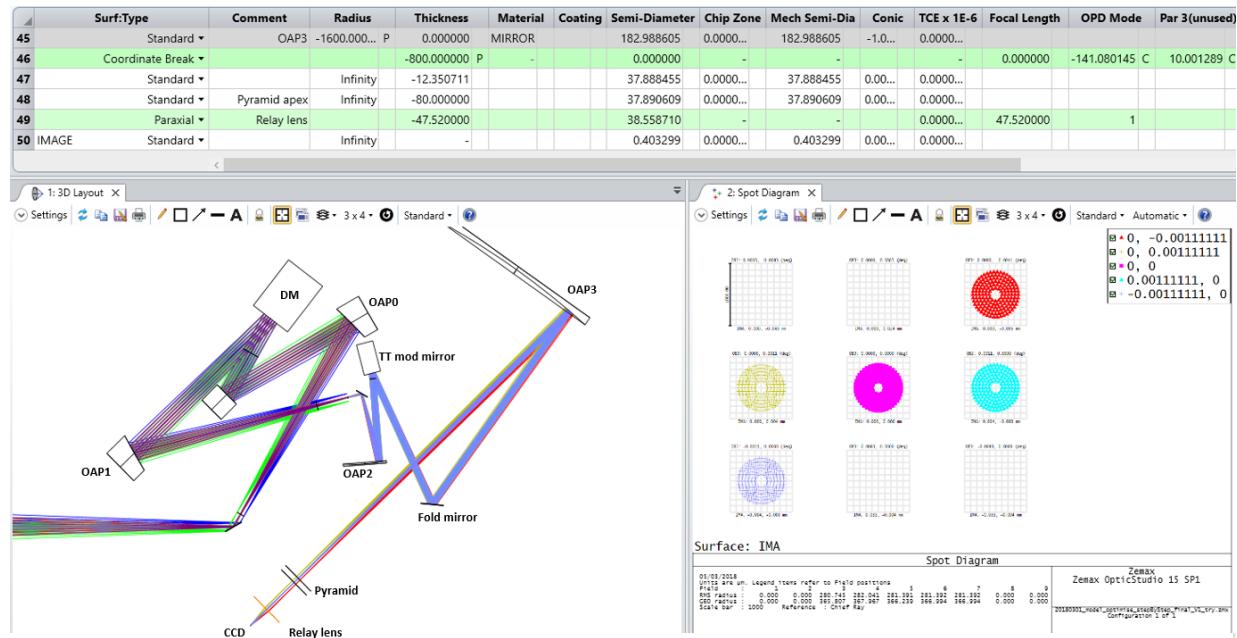


Figure 16: Zemax model of the bench until the CCD camera

3.6 Atmopsheric Dispersion Compensator (ADC)

We are designing an ADC to compensate the atmospheric dispersion (see appendixC). The geometric parameters and the glass are not set yet this is why we do not have implemented it in the Zemax model for now. However, we can already say that the ADC would probably be inserted between OAP2 and OAP3 because the beam is collimated.



AO bench design

May 18, 2018

References

- [1] R. M. Malone, D. Dolan, G. Stevens, and W. Turley, “Design of a thermal imaging diagnostic using 90 degree off-axis parabolic mirrors,” *Proceedings of SPIE*, 2006.
- [2] Thorlabs, “Precision mount 2 inch ks2,” 2018.
- [3] P. Instrument, “S335 fast tiptilt platform,” 2018.
- [4] D. Kopon, L. M. Close, and V. Gasho, “An advanced atmospheric dispersion corrector: the magellan visible ao camera,” tech. rep., University of Arizona, 2012.
- [5] C. Wynne, “A new form of atmospheric dispersion corrector,” *Monthly Notices of the Royal Astronomical Society*, no. 262, pp. 741–748, 1992.
- [6] P. Ciddor, “Refractive index of air: new equations for the visible and near infrared,” *Applied Optics*, vol. 35, no. 9, 1996.
- [7] W. . V. de la pression atmosphérique avec l'altitude.
- [8] Davis, “Equation for the determination of the density of moist air (1981/91),” tech. rep., Metrologia, 1992.
- [9]
- [10] R. C. Stone, “An accurate method for computing atmospheric refraction,” *Astronomical Society of the Pacific*, 1996.
- [11] R. C. Stone, “A new method for computing differential color refraction,” *The Astronomical Society of the Pacific*, 2002.
- [12] S. Tendulkar, *Beyond the Blur: Construction and Characterization of the First Autonomous AO System and an AO Survey of Magnetar Proper Motion*. PhD thesis, California Institute of Technology (Pasadena), 2014.
- [13] S. W. C. Wynne, “Atmospheric dispersion correctors at the cassegrain focus,” *Monthly Notices of the Royal Astronomical Society*, no. 220, pp. 657–670, 1986.
- [14] refractiveindex.info.



AO bench design

May 18, 2018

- [15] S. AG, “Refractive index and dispersion -technical information,” tech. rep., Schott AG, 2018.
- [16] N. cameras, “Hnu128ao,” 2018.
- [17] L. Jolissaint, *Introduction a l'optique pour les ingenieurs.* HEIG VD, 2017.



A OAPs focal lengths relations

This section describe the relation between the parental and the effective focal length for an off-axis parabola. The sketch figure 17 describes the situation with :

- EFL the effective focal length
- PFL the parental focal length
- (x_s, y_s) the coordinate of the ray intersection with the parabola
- the parabola of equation

$$y = \frac{x^2}{4PFL} - PFL \quad (28)$$

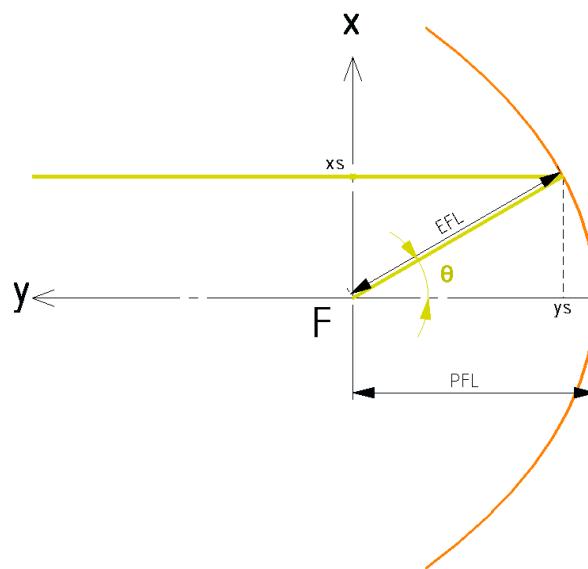


Figure 17: Sketch of a ray reflected on an OAP

The relation between EFL and PFL can be determined with the following set of equations.

$$\begin{aligned} x_s &= EFL \sin \theta \\ y_s &= EFL \cos \theta \end{aligned} \quad (29)$$



AO bench design

May 18, 2018

Using (28) and (29) we have :

$$\begin{aligned} 4PFL(y_S + PFL) &= x_S^2 \\ 4PFL(EFL \cos \theta + PFL) &= (EFL \sin \theta)^2 \\ 4PFL^2 - 4PFL EFL \cos \theta - EFL^2 \sin^2 \theta &= 0 \end{aligned}$$

$$\begin{aligned} PFL_{1,2} &= \frac{4EFL \cos \theta \pm \sqrt{(-4EFL \cos \theta)^2 + 16EFL^2 \sin^2 \theta}}{2 * 4} \\ &= \frac{1}{2} \left[EFL \cos \theta \pm \sqrt{EFL^2 \cos^2 \theta + EFL^2 \sin^2 \theta} \right] \end{aligned}$$

The final equation is :

$$2PFL = EFL(1 + \cos \theta) \quad (30)$$



AO bench design

May 18, 2018

B Focal length of the relay lens between the pyramid and the CCD

Gauss :

$$\begin{aligned}\frac{1}{p_i} - \frac{1}{p_o} &= \frac{1}{f_{LR}} \\ 1 - \frac{f_{LR}}{p_i} &= -\frac{f_{LR}}{p_o}\end{aligned}\quad (31)$$

Thales :

$$\begin{aligned}\frac{\varnothing_{CCD}}{\varnothing_L} &= \frac{|p_i| - |f_{LR}|}{|p_i|} \\ \frac{\varnothing_{CCD}}{\varnothing_L} &= 1 - \frac{f_{LR}}{p_i}\end{aligned}\quad (32)$$

Mixing (31) and (32) :

$$\frac{\varnothing_{CCD}}{\varnothing_L} = \frac{f_{LR}}{p_o} \quad (33)$$

In another hand :

$$F\# = \frac{f_{OAP3}}{\varnothing_{TT}} = \frac{p_o}{\varnothing_L} \quad (34)$$

Mixing (33) and (34) :

$$f_{LR} = F\# \varnothing_{CCD} \quad (35)$$



C ADC design

C.1 Scope

The previous report explains the different possibilities to configure an ADC. Here we are presenting the design we are going to implement for the case of the DAG telescope AO. This ADC will be for visible wavelength. The range is limited by the camera Nuvu spectral range and the star spectrum studied. We are taking only the Nuvu camera bandwidth into account for a first iteration.

C.2 Amici principle

As said in the previous report, the Amici prisms are commonly used in the ADC systems. The ADCs are composed by a 2-doublet design which mixes two Amici prisms. This kind of prism is an alliance of two pieces of glass which have different dispersion (different refractive indexes). The materials must have the same refractive number for a mean wavelength so that at this frequency the incident and emergent rays are parallel (zero-deviation at λ_{mean}). The two prisms can be rotated around the optical axis in order to change the dispersion and compensate it for each wavelength. Shorter or longer wavelength than the middle one are deflected in opposite directions. When the angle between them is 180° the dispersion is reduced at its minimum.

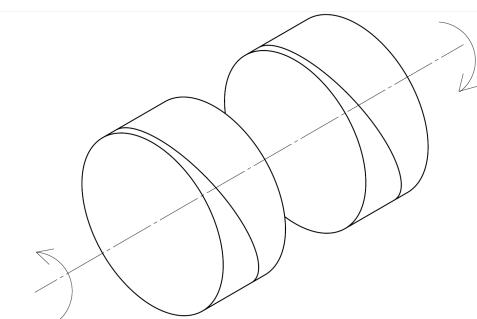


Figure 18: 2-doublet Amici prisms design

Another design can be a three-glass Amici prisms, which is called triplet-design. It consists of the insertion of a anomalous dispersion glass between the two firstly introduced surfaces.



AO bench design

May 18, 2018

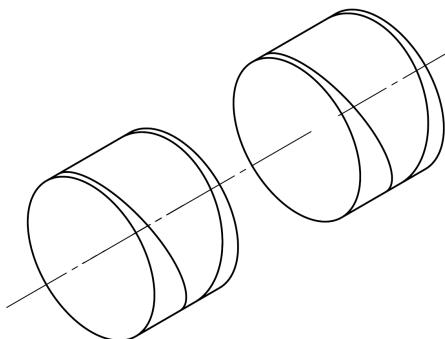


Figure 19: 2-triplet Amici prisms design [4]

The triplet-design seems to be appropriate for our case because it gives the best performance. According to Kopon thesis [4], the doublet corrects only the first order of chromatism while the triplet acts on both primary and secondary chromaticism aberration.

The system is made only by plane surfaces so that a plane front which go through the prisms should leave them as a plane front. In a collimated beam with a very narrow field of view the prisms combined thickness can be larger than the beam diameter [5].

We can also use vertex with a certain radius in a converging beam. This way we can change the F# leaving the ADC.

The doublet glass has to be designed in order to have a thermal expansion rate as close as possible between each glass to not break with the temperature changes. The internal reflection is also a parameter that we have to take into account because we want the ADC to transmit as much as possible light.

ADC design In order to design our ADC, we need to collect some information about its position in the AO bench, the dispersion of the atmosphere for our parameters, the field of view (FoV) and wavelength bandwidth it has to work in.

C.3 Model of the atmosphere refraction

C.3.1 Atmosphere refraction index

First, we need to model the atmosphere dispersion (refractive index) depending on the wavelength. The simulation of the atmosphere refractive index is based on Ciddor's ap-



proach [6] which is a compilation of all previous equations for the visible and near infrared. The following set of equations are used to model the atmosphere refraction :

$$10^8 (n_{as} - 1) = k_1 / (k_0 - \sigma^2) + k_3 / (k_2 - \sigma^2) \quad (36)$$

$$(n_{asx} - 1) = (n_{as} - 1) [1 + 0.534 \times 10^{-6} (x_c - 450)] \quad (37)$$

$$10^8 (n_{ws} - 1) = 1.022 \times (\omega_0 + \omega_1 \sigma^2 + \omega_2 \sigma^4 + \omega_3 \sigma^6) \quad (38)$$

$$n_{final} = (\rho_a / \rho_{axs}) (n_{axs} - 1) + (\rho_\omega / \rho_{ws}) (n_{ws} - 1) \quad (39)$$

where the parameters are defined as follow :

- the wave number
 $\sigma = 2\pi/\lambda [\mu\text{m}^{-1}]$
- Constants involved in the standard phase and group refractivities of dry air [6]
 $k_0 = 238.0182$, $k_1 = 5792105$, $k_2 = 57.362$, $k_3 = 167917 [\mu\text{m}^{-2}]$
- n_{as} the refractive index of standard air at $T=20^\circ\text{C}$, 101325 Pa, 0 % humidity, 450 ppm of CO_2
- for now, I took the concentration of CO_2 in the air of $x_c = 450$ ppm (standard) so equation (37) becomes $n_{asx} = n_{as}$ but we can change that easily changing the x_c parameter in the code
- n_{asx} the refractive index of x_c ppm of CO_2 at $T=20^\circ\text{C}$, 101325 Pa, 0 % humidity
- Constants involved in the standard phase and group refractivities of water vapor [6]
 $\omega_0 = 295.235 [\mu\text{m}^{-2}]$, $\omega_1 = 2.6422 [\mu\text{m}^{-2}]$, $\omega_2 = -0.032380 [\mu\text{m}^{-4}]$, $\omega_3 = 0.004028 [\mu\text{m}^{-6}]$
- n_{ws} the refractive index of water vapour at $T=20^\circ\text{C}$, 1333 Pa
- $\rho_a [\text{kg}/\text{m}^3]$ the humid air density calculated equation (40)
- $\rho_{axs} [\text{kg}/\text{m}^3]$ the density of dry air at standard conditions calculated equation (41)
- $\rho_\omega [\text{kg}/\text{m}^3]$ the density of water vapour calculated equation (42)
- $\rho_{ws} [\text{kg}/\text{m}^3]$ the density of water vapour at standard conditions calculated equation (43)



AO bench design

May 18, 2018

Calcul of the humid air density

$$\begin{aligned}\rho_a &= \frac{P M_a}{Z RT} \left[1 - x_V \left(1 - \frac{M_V}{M_a} \right) \right] \\ P &= P_0 \left(1 - \frac{\Delta T H}{T} \right) \\ M_a &= (28.9635 + 12.011(xCO2 - 0.0004)) \times 10^{-3} \\ Z &= 1 - \frac{P}{T} \left[a_0 + a_1 t + a_2 T^2 + (b_0 + b_1 t) * x_v + (c_0 + c_1 t) * x_v^2 \right] + \frac{P^2}{T^2} \left(d + e * x_v^2 \right)\end{aligned}\quad (40)$$

with :

- $P_0 = 1.01325 \times 10^5$ [Pa] the normal pressure at altitude 0 m;
- $\Delta T = 0.0065$ [K] the vertical gradient of temperature (0.65K for 100 m) [7]
- $T_0 = -10$ [°C] mean temperature from DAG-AWOS1
- $H = 3170$ [m] Karakaya altitude
- M_a [kg/mol] the density of dry air with $xCO2 = 0.0004$ [8]
- $M_V = 18.01528 \times 10^{-3}$ [kg/mol] the mole mass of water
- $R = 8.314510$ [J/mol/K] the molar gas constant
- Z the compressibility with t the temperature in [°C] and the following constants and parameters [8] :

- $a_0 = 1.58123 \times 10^{-6}$ [K*Pa $^{-1}$], $a_1 = -2.9331 \times 10^{-8}$ [Pa $^{-1}$], $a_2 = 1.1043 \times 10^{-10}$ [(K*Pa) $^{-1}$], $b_0 = 5.707 \times 10^{-6}$ [K*Pa $^{-1}$], $b_1 = -2.051 \times 10^{-8}$ [Pa $^{-1}$], $c_0 = 1.9898 \times 10^{-4}$ [K*Pa $^{-1}$] $c_1 = -2.376 \times 10^{-6}$ [Pa $^{-1}$], $d = 1.83 \times 10^{-11}$ [K 2 Pa $^{-2}$], $e = -0.765 \times 10^{-8}$ [K 2 Pa $^{-2}$]
- $x_v = \text{RH} f \frac{P_{sv}}{P}$ mole fraction of water vapour, RH the relative humidity (taken here as 0.8)
- $f = \alpha + \beta P + \gamma T^2$ increasing factor ($\alpha = 1.00062$ [-], $\beta = 3.14 \times 10^{-8}$ [Pa], $\gamma = 5.6 \times 10^{-7}$ [K $^{-2}$])
- $P_{sv} = \exp \left(AT^2 + BT + C + \frac{D}{T} \right)$ the saturation vapour pressure of moist air ($A = 1.2378847 \times 10^{-5}$ [K $^{-2}$], $B = -1.9121316 \times 10^{-2}$ [K $^{-1}$], $C = 33.93711047$ [-], $D = -6.4341645 \times 10^3$ [K])



AO bench design

May 18, 2018

Calcul of density of dry air at standard conditions

$$\rho_{axs} = \frac{P_0}{R_{gas}T_0} \quad (41)$$

with $R_{gas} = 287.05 \text{ [J/kg/K]}$ [9].

Calcul of the water vapour density

$$\rho_w = \frac{P_w M_V}{R T} \quad (42)$$

With :

- $M_V = 18.01528 \times 10^{-3} \text{ [kg/mol]}$ the mole mass of water
- $P_w \text{ [mb]}$ partial pressure of water vapour ($\text{mmHg2Pa} = 133.322365$ and P_{sat} [mmHg] valid between -50°C and 200°C)

$$P_{sat} = \exp \left(46.784 - \frac{6435}{T} - 3.868 \log(T) \right)$$
$$P_w = P_{sat} * RH * \text{mmHg2Pa}$$

Calcul of the water vapour density at standard conditions

$$\rho_{ws} = \frac{P_{w0} M_V}{R T_0} \quad (43)$$

With :

- $M_V = 18.01528 \times 10^{-3} \text{ [kg/mol]}$ the mole mass of water
- $P_{w0} \text{ [mb]}$ partial pressure of water vapour ($\text{mmHg2Pa} = 133.322365$ and P_{sat0} [mmHg] valid between -50°C and 200°C)

$$P_{sat0} = \exp(46.784 - 6435/(T0) - 3.868 \log(T0))$$

$$P_{w0} = P_{sat0} * RH * \text{mmHg2Pa}$$



C.3.2 Atmosphere refraction equation

The refraction of the atmosphere is computed with the equation [10]:

$$R(\lambda, z) = \kappa (n(\lambda) - 1) (1 - \beta) \tan(z) - \kappa (n(\lambda) - 1) \left(\beta - \frac{(n(\lambda) - 1)}{2} \right) \tan^3(z) \quad (44)$$

with

- $\beta = 0.001254 \left(\frac{T(K)}{273.15} \right)$ the effective height of the observatory above the surface of the earth [10]
- $\kappa = 1$ for a spherical Earth surface [10] or instrumental correction no more useful [11]

This equation (44) is valid only for zenith angle $\geq 75^\circ$. We can even neglect the second term for zenith angles $\geq 65^\circ$ [12]. The limit of zenith angle of our computation is set to 70° so the entire equation is set.

C.3.3 Glasses for the ADC

The choice of glass depends on the range of zenith angles, the wavelength interval, the maximum size of the blanks and the cost [13]). Most of the glass couple in the Amici conception are flint/crown pairs plus an anomalous dispersion glass inserted for the triplet design.

The glass choice is made in a data base built from Schott and Ohara catalogues. The data are taken from [14] where we can download glasses properties from many different suppliers. The data are sorted to extract Sellmeier coefficients [15] in order to compute the refractive index of each glass depending on the wavelength with the equation (45).

$$n^2(\lambda) = 1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3} \quad (45)$$

with B_i and C_i Sellmeier coefficients for λ in μm [15].

The final glass data base with the refractive index reported is generated for three wavelengths : the smallest, the highest and the mean wavelength. These are for our case the Nuvu camera [16] bandwidth limits $\lambda_{min} = 0.3 \text{ } \mu\text{m}$ and $\lambda_{max} = 1.0 \text{ } \mu\text{m}$ and the maximum quantum efficiency corresponding wavelength $\lambda_{mean} = 0.6 \text{ } \mu\text{m}$.



AO bench design

May 18, 2018

C.3.4 Beam propagation through the ADC

In order to compute de dispersion through the ADC a geometrical and refraction set of equations have been implemented. The sequence of these equations is listed bellow. I will follow the light on the figure 20 to describe each step.

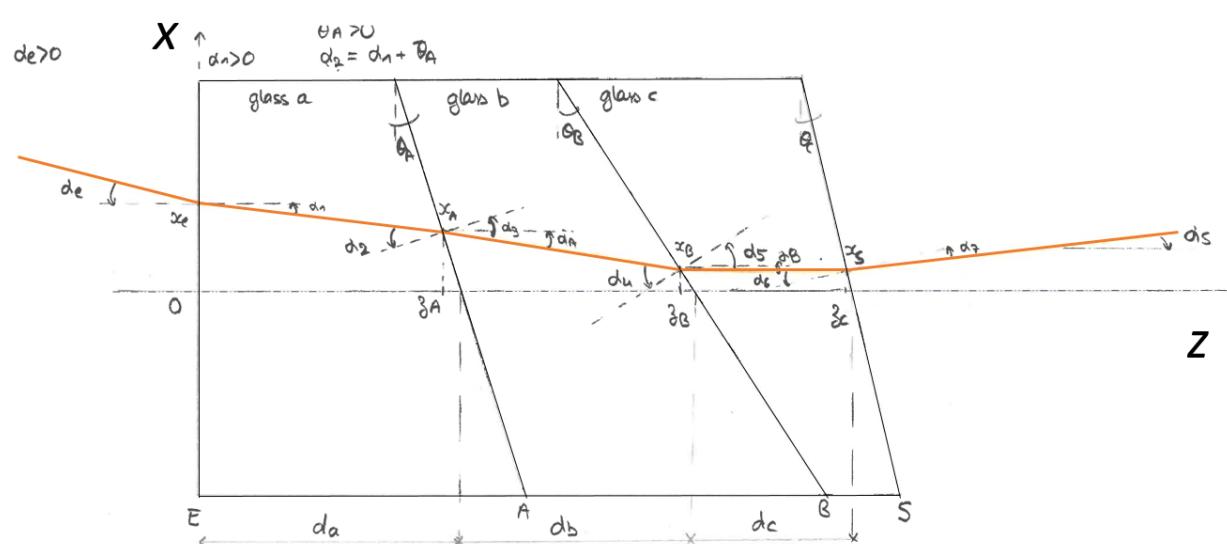


Figure 20: Sketch of the refraction through the ADC

The refractive index of the I-th glass is $n_I(\lambda_i)$. To simplify the notation, we just write n_I . We apply the sign convention described in the course material [17].

At the entrance we have the vector

$$\begin{bmatrix} x_E \\ z_E = 0 \\ n_0 \sin \alpha_E \end{bmatrix}$$

At the entrance 0E, we have a refraction

$$\begin{bmatrix} x_E \\ z_E = 0 \\ n_A \sin \alpha_1 \end{bmatrix}$$

which gives :

$$\alpha_1 = \arcsin \left(\frac{n_0}{n_A} \sin \alpha_E \right)$$



AO bench design

May 18, 2018

At the first interface AB, we arrive with the coordinates

$$\begin{bmatrix} x_A \\ z_A \\ n_A \sin(\alpha_2) \end{bmatrix}$$

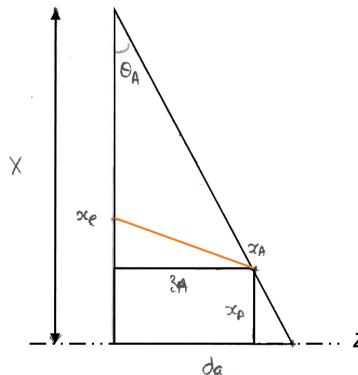


Figure 21: Decomposition of the parameters at an interface

We can determine this vector using the following equations from figure 21 :

$$X = \frac{d_A}{\tan \theta_A} ; \frac{X-x_A}{X} = \frac{z_A}{d_A} ; z_A = \frac{x_E - x_A}{\tan \alpha_1}$$

Then we find :

$$\begin{cases} x_A &= \frac{x_E - d_A \tan \alpha_1}{1 - \tan \alpha_1 \tan \theta_A} \\ z_A &= \frac{x_E - x_A}{\tan \alpha_1} \\ \alpha_2 &= \alpha_1 + \theta_A \end{cases}$$

At the interface AB, we have a refraction

$$\begin{bmatrix} x_A \\ z_A \\ n_B \sin \alpha_3 \end{bmatrix}$$

which gives :

$$\alpha_3 = \arcsin \left(\frac{n_A}{n_B} \sin \alpha_2 \right)$$



AO bench design

May 18, 2018

At the interface BC, we arrive with the coordinates

$$\begin{bmatrix} x_B \\ z_B \\ n_B \sin(\alpha_4) \end{bmatrix}$$

We can determine this vector using the following equations from figure 21 :

$$X = \frac{x_A}{\tan \alpha_A} ; \frac{x_B}{X - Q - d_B + m} = \tan \alpha_A ; m = d_A + d_B - z_B ; m = x_B \tan \theta_B ; Q = d_A - z_A ; \alpha_A = \alpha_3 + \theta_A$$

Then we find :

$$\begin{cases} x_B = \frac{x_A - \tan \alpha_A (d_A + d_B - z_A)}{1 - \tan \alpha_A \tan \theta_B} \\ z_B = d_A + d_B - x_B \tan \theta_B \\ \alpha_4 = \alpha_A + \theta_B \end{cases}$$

At the interface BC, we have a refraction

$$\begin{bmatrix} x_B \\ z_B \\ n_C \sin \alpha_5 \end{bmatrix}$$

which gives :

$$\alpha_5 = \arcsin \left(\frac{n_B}{n_C} \sin \alpha_4 \right)$$

At the interface C0, we arrive with the coordinates

$$\begin{bmatrix} x_C \\ z_C \\ n_C \sin(\alpha_6) \end{bmatrix}$$

We can determine this vector using the following equations from figure 21 :

$$X = \frac{x_B}{\tan \alpha_B} ; \frac{x_C}{X - Q - d_C + m} = \tan \alpha_B ; m = d_A + d_B - z_B ; m = x_C \tan \theta_C ; Q = d_A + d_B - z_B ; \alpha_B = \alpha_5 + \theta_B$$

Then we find :

$$\begin{cases} x_C = \frac{x_B - \tan \alpha_B (d_A + d_B + d_C - z_B)}{1 - \tan \alpha_B \tan \theta_C} \\ z_C = d_A + d_B + d_C - x_C \tan \theta_C \\ \alpha_6 = \alpha_B + \theta_C \end{cases}$$



At the interface C0, we have a refraction

$$\begin{bmatrix} x_C \\ z_C \\ n_0 \sin \alpha_7 \end{bmatrix}$$

which gives :

$$\alpha_7 = \arcsin \left(\frac{n_C}{n_0} \sin \alpha_6 \right)$$

The output angle with respect to the optical axis $\alpha_S = \alpha_7 - \theta_C$ This α_S is the dispersion angle of the prism : $R_{prism} = \alpha_S$.

C.3.5 The metric definition

Now we have computed the dispersion of the atmosphere and the ADC. We want the smallest total dispersion so we investigate all ADC configurations in order to minimize it. The metric we use comes from [12] to calculate the efficiency of our prism combination :

$$\text{Eff (prism parameters)} = \sum_{\lambda_i} (R_{prism}(\lambda_i; \text{prism parameters}) - R_{atm}(\lambda_i))^2 \quad (46)$$

C.3.6 Internal reflection

In parallel of the dispersion computation, we calculate the total internal reflection of the ADC. If the refractive index of two joint glasses are too different then we will loose a lot of intensity reflected on the interface. The external faces of the ADC will be coated. The determination of the total internal reflectivity is developed below.

The parameters are :

- $\alpha_Z = [\alpha_E; \alpha_1; \alpha_A; \alpha_B; \alpha_S]$ [rad] array of the rays angle wrt the optical axis
- $\alpha_I = [\alpha_1; \alpha_2; \alpha_3; \alpha_4; \alpha_5; \alpha_6; \alpha_7]$ [rad] array of the rays angles wrt the normal the vertex (refraction angle)
- $n = [n_0; n_A; n_B; n_C; n_0]$ medium refractive index
- $d = [z_A, z_B - z_A, z_C - z_B]$ [mm] distances on the optical axis between each medium change on the ray path



At the 1rst interface AB, we have :

$$r[1] = \frac{n[3] \cos \alpha_I[2] - n[2] \cos \alpha_I[3]}{n[3] \cos \alpha_I[2] + n[2] \cos \alpha_I[3]} \quad (47)$$

Taking the recursive initialization term $U(1) = r(1)$ we have for $p = 2$:

$$r[p] = \frac{n[p+1] \cos(\alpha_I[p]) - n[p] \cos(\alpha_I[p+1])}{n[p+1] \cos(\alpha_I[p]) + n[p] \cos(\alpha_I[p+1])} \quad (48)$$

$$U[p] = \frac{U[p-1] + r[p] \exp\left(\frac{2\pi}{\lambda} \cos(\alpha_Z[p]) d[p-1]\right)}{1 + U[p-1] r[p] \exp\left(\frac{2\pi}{\lambda} \cos(\alpha_Z[p]) d[p-1]\right)} \quad (49)$$

Using equation (49) we can calculate the reflection coefficient for $(p+1)$ number of glasses. In our case we have only 3 glasses so the total reflection (without taking external faces into account) is $U[2]$.

C.4 ADC design optimization

The efficiency of a combination (glass, angle, thickness) is computed by the function *Refraction_calculs_geometriques_20180302.m*. The *fmincon* Matlab algorithm is used to find minimum of this constrained nonlinear multivariable function. The parameters are the geometrical coefficients for one set of glass. When the optimum is found for a combination of glass, the efficiency and internal reflection are given as outputs.

The global matrix is made from two for-loops that look for all glasses from the catalogues Schott and Ohara. We decide to have the same glass type for the external wedges in order to simplify the anti-reflection coating process. This would give us the possibility to coat the prism at once and so reduce the cost. The refractive index of the middle wedge glass is chosen larger than the middle one to save effort and cost on the coating. In the optimization process on Matlab, when this refractive index (for λ_{mean} is larger than external one the loop stops. The outputs are set to 20'000 (arbitrarily large to oust them)).

The wavelength for which the refraction is calculated are taken from the Nuvu camera quantum efficiency curve : $\lambda_{\text{min}} = 300$ nm, $\lambda_{\text{mean}} = 600$ nm and $\lambda_{\text{max}} = 1000$ nm. The zenith angle is set to 70°.