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IŞIK ÜNİVERSİTESİ



AO bench design

June 27, 2018

AO bench design Zemax model and optimization post-PDR design

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Issue	Date	Sect./Page	Reason/Remarks
1.0	Oct. 10, 2017	All	New
2.0	May 18, 2018	All	Review of all the design after decisions and Paolo Spano advices
3.0	June 08, 2018	All	Review of all the design without the FCL



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Acronyms

aO	active Optics	NGS	Natural Guide Star
AO	Adaptive Optics	OTF	Optical Transfer Function
CCD	Charge Coupled Device	P2V	Peak-to-Valley
CMOS	Complementary Metal Oxide Semiconductor	PSF	Point Spread Function
DAG	Dogu Anadolu Gözlemevi (East Anatolian Observatory)	PSF-R	PSF Reconstruction
DM	Deformable Mirror	PWFS	Pyramid WFS
FCL	Field Correction Lens	RMS	Root Mean Square
FoV	Field-of-View	SH-WFS	Shack-Hartmann WFS
FWHM	Full Width at Half Maximum	SNR	Signal-to-Noise Ratio
NCPA	Non Common Path Aberrations	TT	Tip-Tilt
		WFS	Wavefront Sensor
		WFE	Wavefront Error



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1 Scope of this document

This document presents the DAG adaptive optics (AO) optical train design and optimization.

2 Introduction

There are several mandatory points we have to handle :

- the exit pupil has to be imaged on the DM (Deformable Mirror);
- the DM diameter is fixed to the DM-468 clear aperture from ALPAO; that means Ø33 mm;
- the exit pupil of the telescope has to be imaged on the TT (Tip-Tilt) mirror for the P-WFS modulation;
- the beam has to converge on the pyramid apex;
- the angle of the beam that arrives onto the pyramid apex is calculated in order to get a diffraction limited PSF size of 2 times the pyramid roof;
- the exit pupil has to be imaged onto the Nuvu EMCCD detector;
- the beam footprint diameter onto the detector has to be defined accordingly to the oversampling criterion (explained section ??).

We use off axis parabolas (OAPs) for all our design to reduce the aberrations (compared to lenses). We have added one more constraint on the design which is to have parallel beam arriving or leaving each off axis parabola in order to limit the spherical aberrations.

The fixed parameters are resumed here :

- exit pupil diameter : $\varnothing_{\text{ExtP}} = 727.4046 \text{ mm}$;
- distance from the exit pupil to the focal plane : $\overline{\text{ExP-FP}} = 10338.74 \text{ mm}$;
- DM diameter : $\varnothing_{\text{DM}} = 33.0 \text{ mm}$;
- pixel size of the Nuvuu EMCCD AO : $\text{PxSize} = 24 \mu\text{m}$



In order to design the AO bench we started with the telescope model. Indeed, we would try to compensate for the field of curvature introduced by the mirrors of the telescope with the off-axis parabolas (OAP) of the AO. The Zemax model of the telescope shows a curvature radius of about 1255 mm.

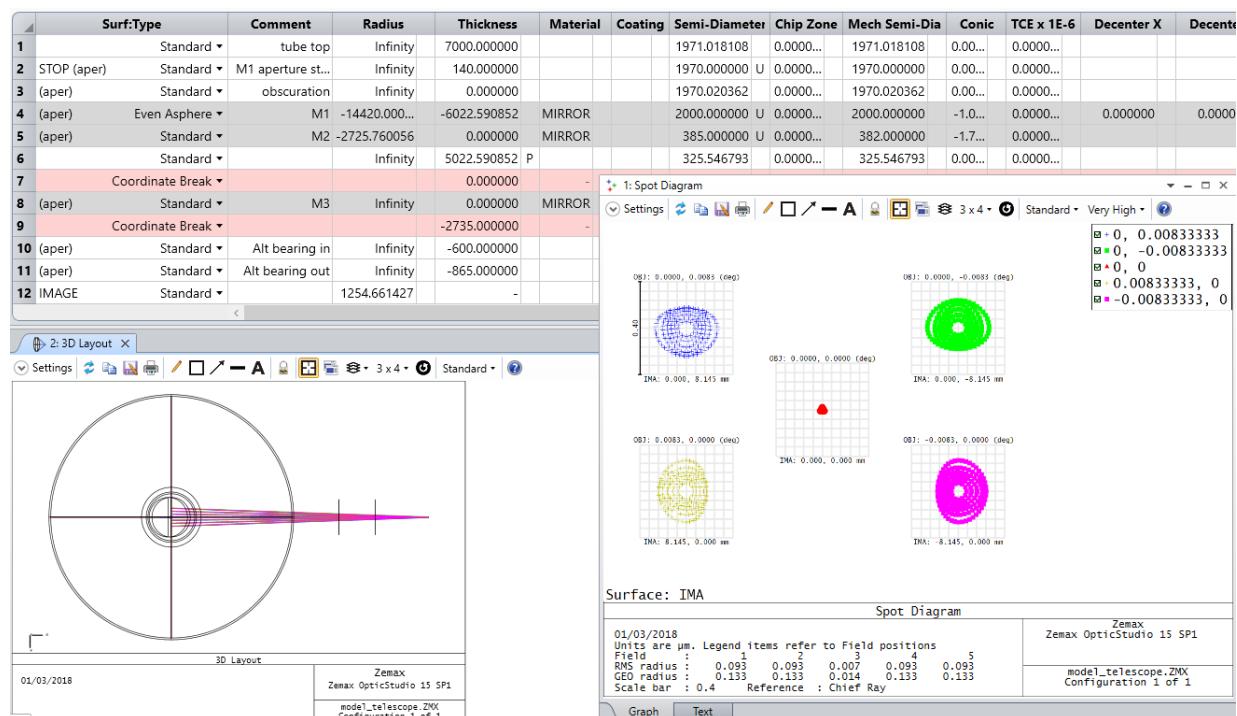


Figure 1: Zemax model of the telescope

Concerning off-axis aberrations for a future GLAO mode, they are totally dominated by the atmospheric aberrations through this telescope (30 nm for 2' FoV), so we do not need a field correction lens. In a first approach, we thought we should have one nearby the focal plane but this is not the case anymore.



3 Design development and optimization

A preliminary design have been set for the PDR and the main points are summarized at each optimization step of this report.

3.1 Imaging the exit pupil of the telescope onto the DM

The first section consists on an imaging system of the telescope exit pupil onto the DM. The beam has to be collimated and the footprint has to take the entire clear aperture of the DM (for the DM-468 from ALPAO $\phi_{DM} = 33$ mm).

The beam on the DM has to be reflected with a certain angle otherwise the reflected beam comes back on itself. The maximum acceptable angle can be calculated considering the position error of the beam per actuator on the DM. We can align the beam with a registration error due to this elliptical beam footprint of $1/10^{\text{th}} \Lambda$ (Λ the actuator pitch = 1.5 mm for the ALPAO DM-468). In order for the projected beam diameter to be no more than 10% smaller than the DM diameter on both side, the tilt of the DM must be no more than α (figure 2) :

$$\begin{aligned}\alpha &= \arccos\left(1 - \frac{1}{5} \frac{\Lambda}{\phi_{DM}}\right) \\ \alpha &= \arccos\left(1 - \frac{1}{5} \frac{1.5}{33}\right) \\ \alpha &= 7.73^\circ \\ 2\alpha &= 15.46^\circ\end{aligned}\quad (1)$$

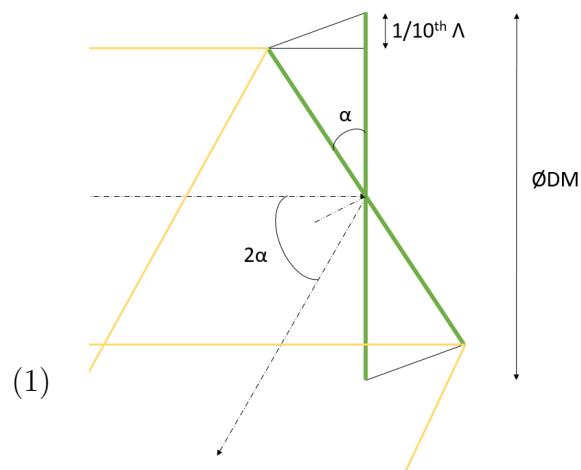


Figure 2: Sketch of the input and output beam depending on the tilt angle of the DM



The input beam diameter is then :

$$\varnothing_{\text{DM-beam}} = \varnothing_{\text{DM}} \cos \alpha = 33 \times \cos (7.73) = 32.7 \text{ mm} \quad (2)$$

In order to have a collimated beam on the DM with a diameter of $\varnothing_{\text{DM-beam}}$, the focal length of the OAP0 is calculated by the following sequence of equations. To visualize the parameters needed and the context we can look at figure 3. The OAP is tilted with an angle θ and its corresponding reference coordinates is $(0, x', y')$. The ray ΔCR corresponds to the chief ray, $\Delta\alpha u$ is the "upper" marginal ray and $\Delta\alpha b$ is the "bottom" marginal ray. The origin is placed at the focal plane of the telescope.

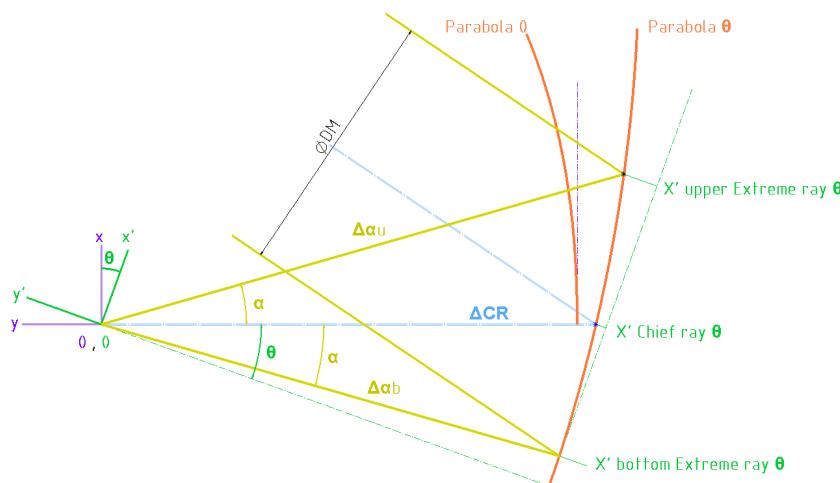


Figure 3: Ray tracing of a tilted parabola

In the reference coordinates $(0, x', y')$, we have (with PFL the Parental Focal Length of the OAP):

- equation of the parabola θ :

$$y' = \frac{x'^2}{4 \text{PFL}} - \text{PFL} \quad (3)$$

- equation of $\Delta\alpha u$:

$$y' = -\frac{1}{\tan(\theta + \alpha)} x' \quad (4)$$

- equation of $\Delta\alpha b$:

$$y' = -\frac{1}{\tan(\theta - \alpha)} x' \quad (5)$$



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Determination of $x'_{\text{Ex}\theta u}$ (intersection between parabola θ and $\Delta\alpha u$) :

(3) = (4)

$$x'_{\text{Ex}\theta u} = 2 \text{PFL} \left(-\frac{1}{\tan(\theta + \alpha)} + \sqrt{\frac{1}{\tan^2(\theta + \alpha)} + 1} \right) \quad (6)$$

Determination of $x'_{\text{Ex}\theta b}$ (intersection between parabola θ and $\Delta\alpha b$) :

(3) = (5)

$$x'_{\text{Ex}\theta b} = 2 \text{PFL} \left(-\frac{1}{\tan(\theta - \alpha)} + \sqrt{\frac{1}{\tan^2(\theta - \alpha)} + 1} \right) \quad (7)$$

In order to have the OAP0 output beam diameter equal to the DM clear aperture we have :

$$x'_{\text{Ex}\theta u} - x'_{\text{Ex}\theta b} = \varnothing_{\text{DM-beam}} \quad (8)$$

The parental focal length (see appendix A) of OAP0 is :

$$\text{PFL} = \frac{1}{2} \frac{\varnothing_{\text{DM-beam}}}{-\frac{1}{\tan(\theta+\alpha)} + \sqrt{\frac{1}{\tan^2(\theta+\alpha)} + 1} + \frac{1}{\tan(\theta-\alpha)} - \sqrt{\frac{1}{\tan^2(\theta-\alpha)} + 1}} \quad (9)$$

Using the equation (20) described in appendix A we can transform the parental focal length into the effective focal length.

For the OAP0 the angle α_{OAP0} is

$$\alpha_{\text{OAP0}} = \arctan \left(\frac{\varnothing_{\text{ExP}/2}}{\overline{\text{ExP-FF}}} \right) \quad (10)$$

The numerical application of (10) with $\varnothing_{\text{ExP}} = 727.4046$ mm and $\overline{\text{ExP-FF}} = 10338.74$ mm gives $\alpha_{\text{OAP0}} = 2.015^\circ$, the aperture demi angle of the beam arriving on the first OAP.

After the discussion we had with Paolo Spano, we took his advice into account and choose the parabola tilt angle according to the following precept : $\theta = 35^\circ$ is a critical choice and $\theta > 35^\circ$ is a nightmare (manufacturing and alignment). We should take an tilt angle smaller than 30° to be realistic and the smaller the best.



We set the parabola tilt angle to $\theta = 10^\circ$ in order to keep this angle very small. This way we can see that the DM is quite close to the telescope focal plane (about 2 cm). We choose to increase this distance with a view to update the entire design to a GLAO one (and a larger FoV). We finally set the tilt angle of the OAP0 to $\theta = 15^\circ$ to keep it as small as possible to limit aberrations.

Using these inputs the focal length of the OAP0 is (appendix C): $PFL_{OAP0} = 456.99$ mm. This value is too specific so we round it to¹ :

$$PFL_{OAP0} = 455 \text{ mm}$$

The image of the exit pupil given by OAP0 gives the position of the DM. Its position p_i relative to the OAP0 vertex is given by (we use Gauss law) :

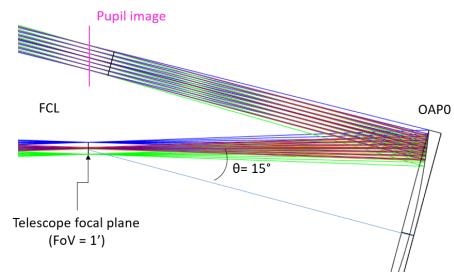
$$p_i = \frac{p_o PFL_{OAP0}}{p_o + PFL_{OAP0}} \quad (11)$$

$$p_i = \frac{-(10338.74 + 455)455}{-10338.74} \quad (12)$$

$$p_i = 477 \text{ mm} \quad (13)$$

The Zemax "pick up pupil position" macro is used to get the perfect DM position wrt the OAP0. The DM is then at $p_i = 491.50$ mm. Even if we understand that we cannot align the DM reflecting surface with this accuracy, we keep it in the Zemax model and we will work on that during the tolerancing analysis².

The image of the pupil through a tilted OAP is also tilted (see [1]). This creates pupil aberrations. Here we want to place the DM at this pupil position. When we work with OAPs the image stop after an OAP is not perpendicular to the optical



¹In Zemax, we enter the PFL_{OAP0} and not the effective focal length in the coordinate break surface because using it, the translation is done before the tilt angle. This is why if we enter the translation in a surface before the coordinate break we cannot use the same length (We would then write the EFL_{OAP0})

²We always keep all the digits during an AO design in order to see what are the best results we can obtain with an ideal system.



axis.

The OAP introduce a tilt angle of the image plane. If we tilt the DM in the opposite position of this OAP-introduced tilt angle we add pupil aberrations whereas if we tilt it the same direction we can compensate for it.

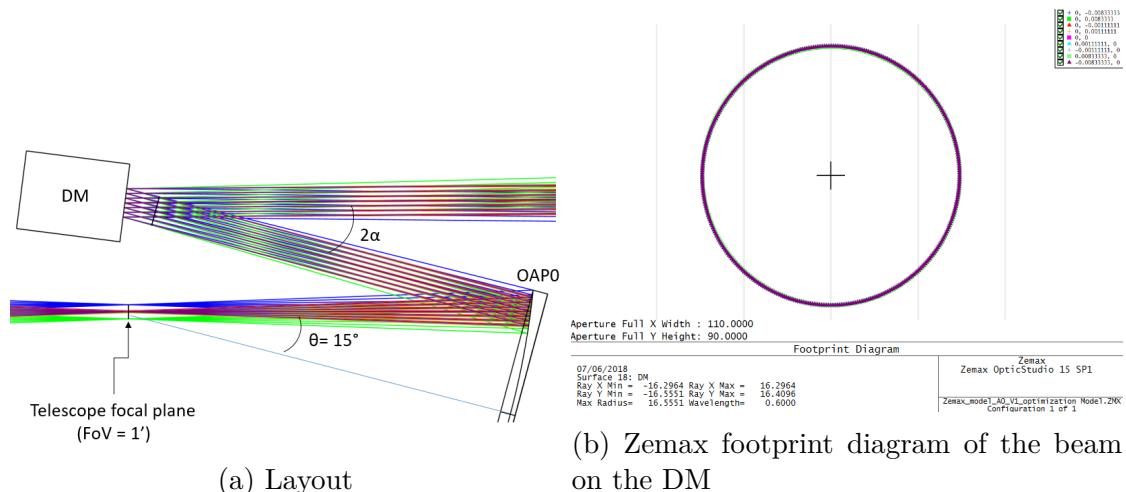


Figure 5: Zemax model of the 1rst part of the AO design

The beam footprint on the DM shows that all the field fits inside the clear aperture. The maximum diameter is equal to 32.9656 mm (< 33 mm **To Paolo : is this okay ? because we are close to the limit... should I take a focal length of 450 mm which gives us a footprint diameter on the DM of 32.603 mm ? maybe it is okay for modelling an ideal case but I should review this value for our real case during the tolerancing analysis..**). The angle of the DM is set to -7.73° (equation (1)), and not 7.73° as explained above (pupil aberration compensation).

3.2 Imaging the pupil on the TT modulation mirror and focus on the pyramid apex

3.2.1 Create an image of the pupil

We need to image the pupil on the TT modulation mirror. Moreover, we know that to compensate for the aberration introduced by OAP0 we can use an OAP1 with the same



focal length. The F/D ratio would stay the same as the telescope one which is favourable for the science path.

Figure 6 shows that to model this combination we use the "chief ray" solve on Zemax (Lens data) to keep the coordinates following the beam path (using the on-axis ray). OAP1 is placed at the exact inverse position of OAP0 in order to compensate for its aberrations so the intermediate focal plane is near the telescope focal plane.

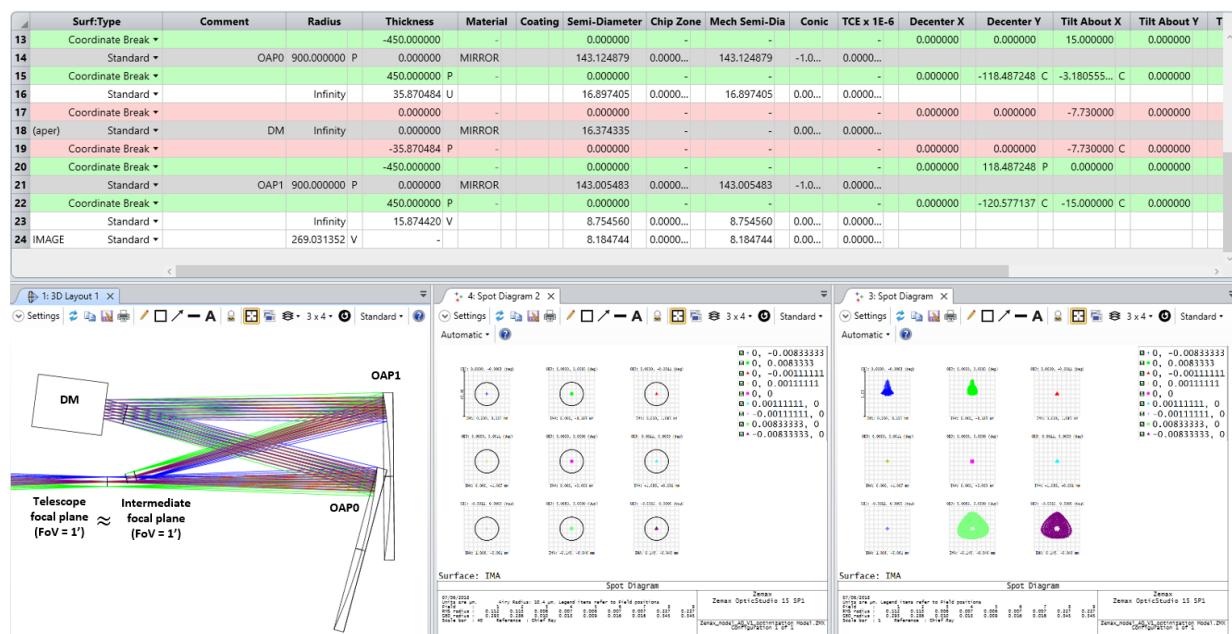


Figure 6: Zemax model from the telescope beam to the intermediate focal plane

We use an optimization (smallest spot radius) to go to the best focal plane (surface #23 thickness is set as variable) and in order to consider the field curvature we set the radius of the image plane variable. The optimization using a merit function (Type : P2V, Criteria : Spot Radius, Reference : Chief Ray) is done on the image surface to get the smallest spot radius of the central beam (FoV = 0'). We can see that the spot diameter varies between 0.345 μm and 0.009 μm (the center of the FoV) but when we look the spots with the Airy disk we can say that the beam is well focused.

However, the radius of the image curvature is smaller than the telescope output field of curvature so we did not compensate for that, instead we increase it (we will see how we



could work on that or not Paolo : how could I compensate for it ?).

3.2.2 Bend the beam

In order to avoid to send back the beam on the focal plane and because we would like to place the dichroic nearby the intermediate focal plane, we fold the beam after the DM with a flat mirror.

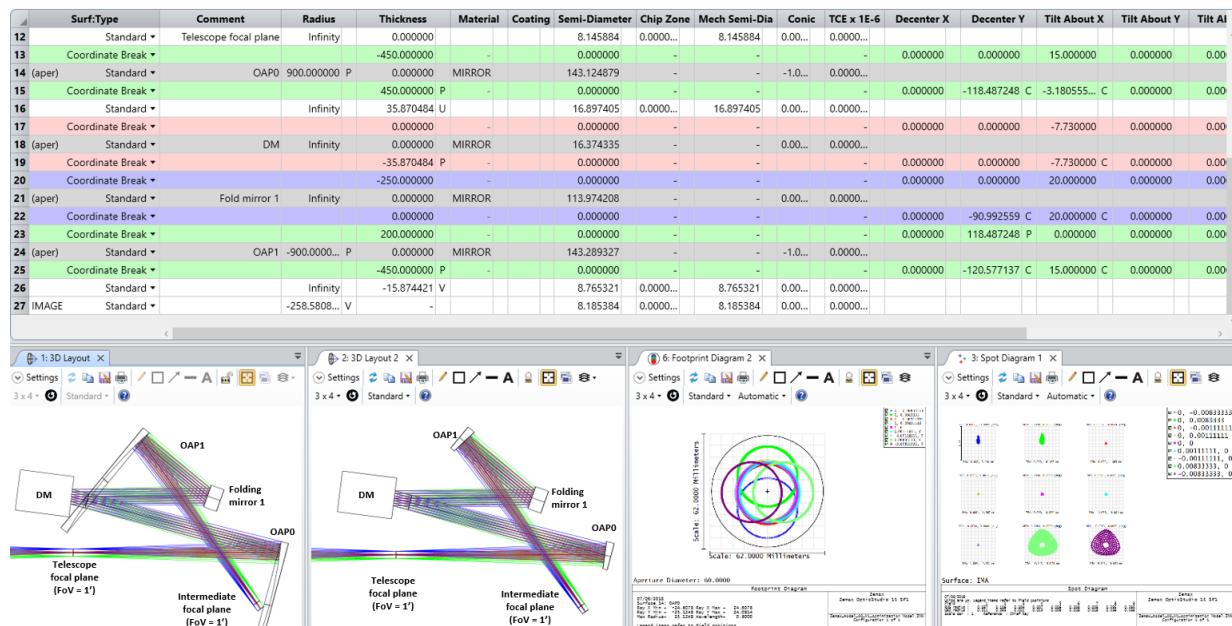


Figure 7: Zemax model from the telescope beam to the intermediate focal plane with a fold mirror after the DM

Figure 7 shows the new design with a folding mirror tilted at 20°. For more clarity we apply the real aperture size of the OAPs on layout 2. We can see on the footprint diagram that we can use two OAPs of 30 mm of diameter. Actually, manufacturers can guaranty the surface quality on 90% of the diameter so the useful optical surface diameter is 27 mm (according to the discussion with Mirsad). The maximum diameter of the beam footprint (for a FoV = 1') is about 50.25 mm. However, we should be careful and verify during the tolerance analysis that when we move the OAP in the alignment precision range we can do, the entire beam stays reflected on the OAP (no vignetting). Concerning the folding



mirror, we can use a diameter of 2 inches (footprint diameter = 45.72 mm) it is positioned at 286 mm from the DM (same as above, we have to check during the tolerancing that we do not have vignetting when moving around this position).

After redoing the optimization of the intermediate focal plane position and the image radius of curvature we found spot diameters vary from $0.009 \mu\text{m}$ (on-axis) to $0.436 \mu\text{m}$ ($0.5'$) across the FoV (see figure 8).

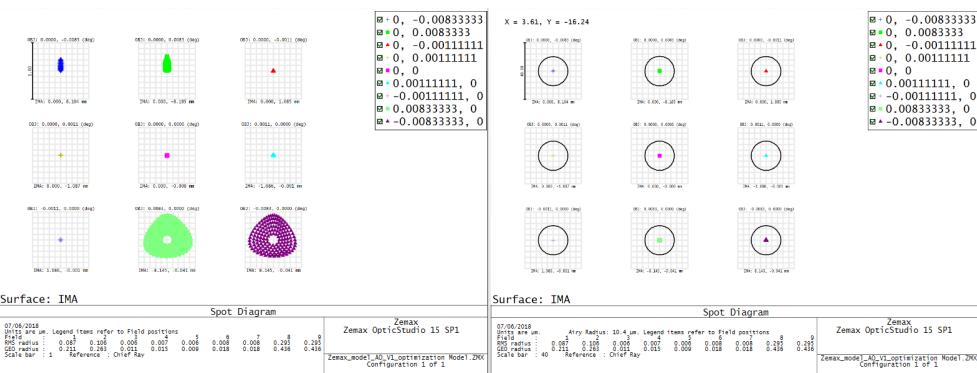


Figure 8: Spot diagram at the intermediate focal plane for the AO FoV and the WFS path FoV

3.2.3 Sending the light to science instruments

We want to introduce a dichroic membrane nearby the intermediate focal plane to reduce its diameter. Then a folding mirror is added to send the light to the instruments. This allows avoiding a large incidence angle on the dichroic which would generate a large reflection loss due to polarisation. The dichroic tilt angle is set to 20° and in order to send the beam collinear to the output of the telescope, the folding mirror is put at 7° of tilt.

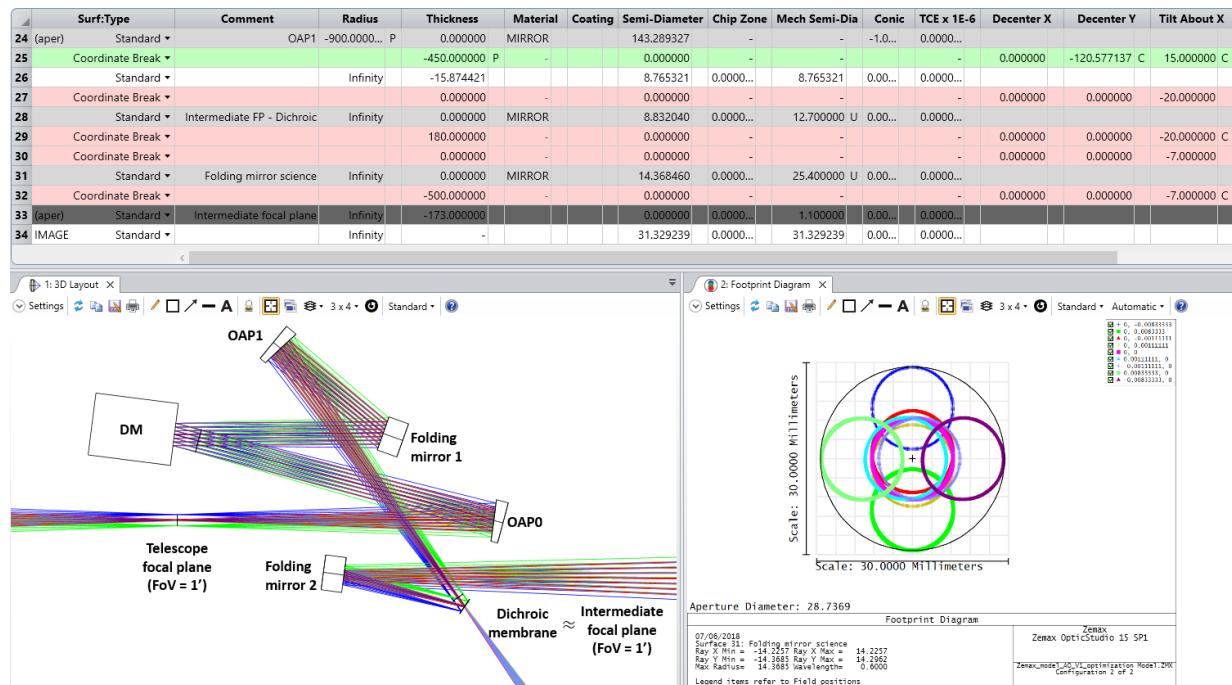


Figure 9: Zemax model from the telescope focal plane to the intermediate science light

The dichroic membrane send the infrared light to the science and let the rest going through the WFS sensor path. The cutting line should be around 900 nm (according to the Nuvu camera QE). The FoV at the WFS path is about 8" which corresponds (for Zemax) to a demi-FoV of $8/(2 \times 3600) = 0.001111^\circ$.

We could also replace the folding mirror with an OAP in order to make the beam collimated again to avoid having very large diameter beam arriving at the instrument entrance. **analyse this possibility !!!!!!!!!!!!!!!**

3.2.4 Imaging the pupil on the TT modulation mirror and focus on the pyramid apex

Now, as the OAP0 and OAP1 are the same, this optical configuration creates an image of the pupil of the same diameter at the same distance from the FP as the exit pupil.

It is so not possible to use it to place the TT modulation mirror which has to be 10 mm in diameter and at a reasonable distance from FP. **FOR these reasons.....**



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we take a TT modulation mirror of $\phi 0.5$ inch mounted on the fast tip-tilt platform S-331 from PI.

We know that we need to arrive on the pyramid roof (P-WFS) with :

$$\begin{aligned} \text{PSF FWHM} &> 2\text{pyramid roof} \\ \lambda \frac{f}{D} &> 2\text{pyramid roof} \\ \frac{f}{D} &> 2 \frac{\text{pyramid roof}}{\lambda} \end{aligned} \quad (14)$$

Taking a pyramid roof of about $20 \mu\text{m}$ according to Jean-Pierre Veran (private communication) at $\lambda = 0.6 \mu\text{m}$ we have : $F\# = 66.7$

We round this value to $F\# = 60$ (we need to investigate the minimum pyramid roof size that can be manufactured).

We want then a power that takes the beam with a $F\#$ of 14 and brings it to 60. The TT modulation mirror is placed in this converging beam at the pupil position. We choose to take an imaging doublet lens because the wavelength range in the WFS path is 500 nm to 900 nm and achromatism can be easily compensated.

At first approximation, we can do a geometrical dimensioning. We use a drawing software and apply the constraints to calculate which focal length and distance from the intermediate focal plane we need. In order to build this ray tracing properly we calculate the following parameters :

$$\begin{aligned} F\#\text{input} &= \frac{\text{ExP-FP}}{\phi_{\text{ExtP}}} \\ F\#\text{output} &= \frac{f_{\text{lens}}}{\phi_{\text{beam on the lens}}} \\ \gamma_{\text{input}} &= \frac{1}{2F\#\text{input}} \\ \gamma_{\text{output}} &= \frac{1}{2F\#\text{output}} \end{aligned}$$

The image of the pupil is nearby the foci (at 0.06 mm) and its radius has to be 5 mm. Taking the angles and the beam diameter on the TT modulation mirror we find the focal



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length of the lens and the distance from the intermediate focal plane described figure 10.

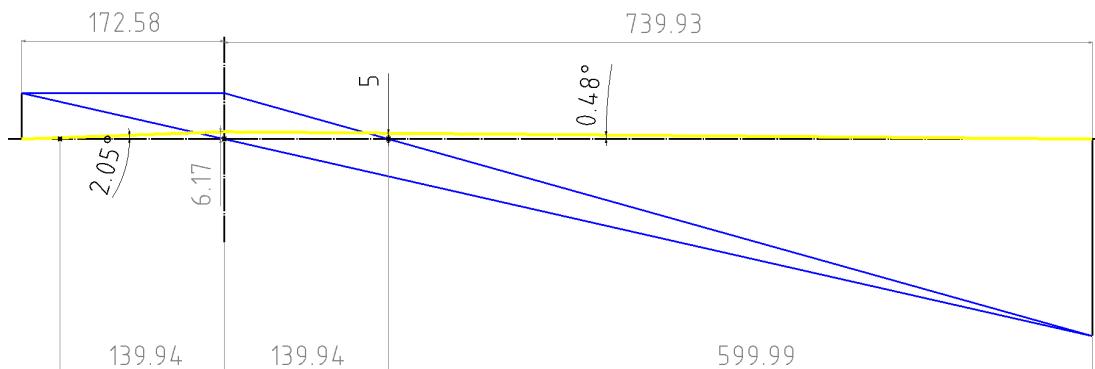


Figure 10: Ray tracing using SolidWorks to approximate the converging lens before the TT modulation mirror

The focal length is rounded to $f = 140$ mm. We introduce these results in Zemax using a paraxial lens (figure 11). We use the optimization tool to find the best focal plane (distance and radius of curvature at the image plane). We can see that the beam diameter on the TT modulation mirror surface is about 5 mm as expected. Taking a distance from the intermediate focal plane to the lens of 173 mm and the beam footprint diameter on the paraxial lens (on-axis case) we have a $F\# = \frac{136.58+597.36}{6.0897*2} = 60.2$ which is larger than the $F\#\text{fixed}$ before at 60.



Figure 11: Zemax model from the telescope beam to the TT modulation mirror using a paraxial lens

In order to correct for the atmospheric chromatic dispersion we need to introduce an ADC (Atmospheric Dispersion Compensator, see 3.6) in our system. This one has to be as close as possible to a pupil plane so to avoid to reimagine the pupil and add optical surfaces we choose to insert it around the TT modulation mirror. For now, we have placed the ADC in the design but it is only made of air. It is just here to have an idea of its size and the room we need for it around the TT modulation mirror (see figure 12).

We want to tilt the TT modulation mirror to send the beam in a convenient direction. Moreover we want to add the ADC around it so we can tilt the TT modulation mirror with an angle of about 20°. The diameter of the beam footprint on the TT modulation mirror (tilted to 20°) measured on the Zemax model is 10.63 mm (figure 12) at maximum (which is smaller than 0.5 inch).

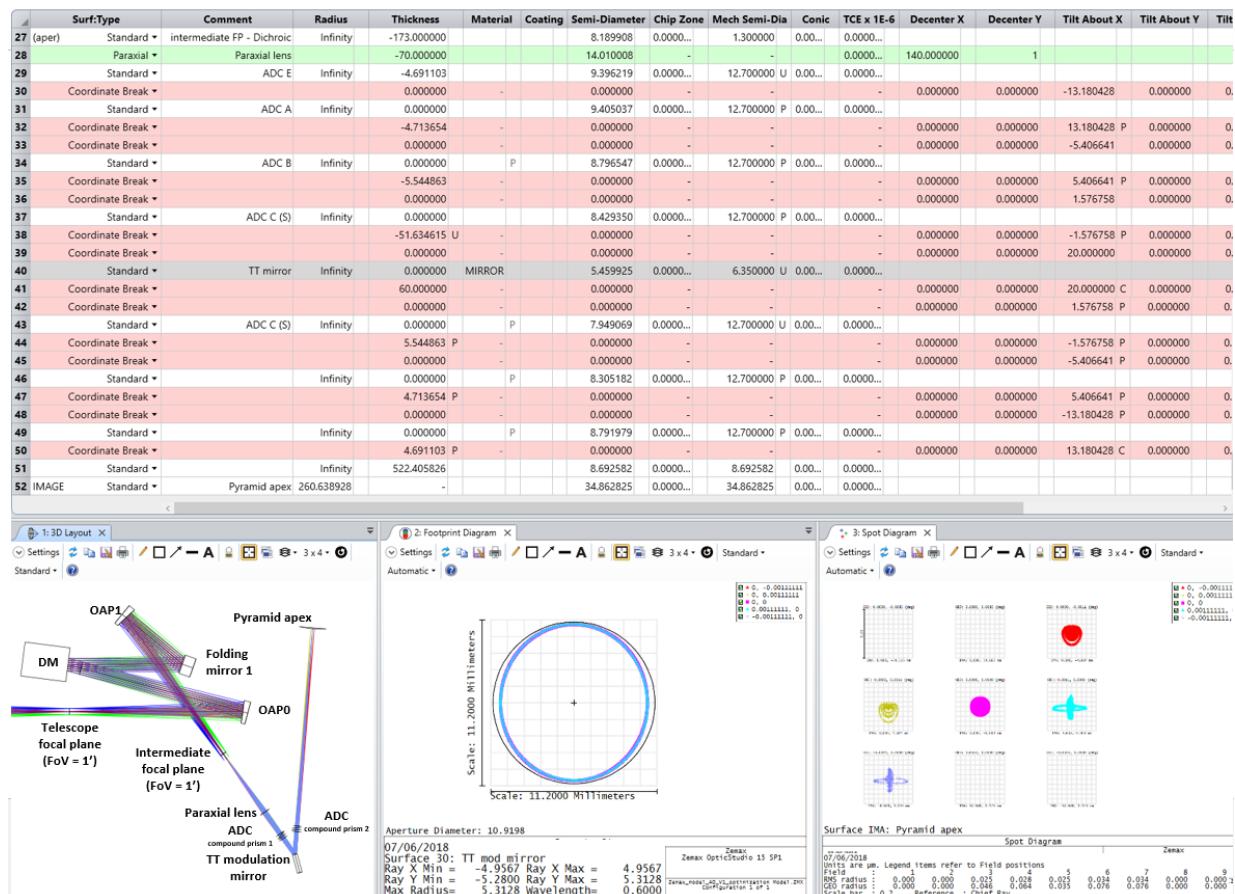


Figure 12: Zemax model from the telescope beam to the apex of the pyramid WFS.

Actually, we could play with the angle of the TT modulation mirror to place the ADC closer to the pupil plane. This would be investigate according to the ADC design.

3.3 Imaging the pupil on the CCD detector

The detector is the Nuvu EMCCD with 128^2 pixels and a pixel size of $24 \mu\text{m}$. We have to sample the beam with at least 1 pixel per actuator. In order to relax the alignment specifications we can oversample the beam. According to Jean-Pierre Veran advice an oversampling of 1.5 should be enough. The ALPAO DM-468 has 22 actuators pitchs across the clear aperture diameter (so 23 actuators). Then we can calculate the beam diameter



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on the detector :

$$\varnothing_{CCD} = \#_{\text{actuator across } \varnothing} \times \text{PxSize} \times \text{Oversampling factor} \quad (15)$$

$$\varnothing_{CCD} = 22 \times 24 \mu\text{m} \times 1.5 \quad (16)$$

$$\varnothing_{CCD} = 0.792 \text{ mm} \quad (17)$$

The pupil has to be imaged on the camera through a relay lens. The pyramid does not act for the ray tracing and so the optical design, it separates the beam in four images only. We can look for the on-axis rays to dimension the relay lens and its position.

In order to play with the variables and have a direct result of what could be done or not, we used a sketch on the drawing software SolidWorks (figure 13). Then, when we find something roughly fine we introduce the values in Zemax and optimize the distances in order to find the proper pupil plane and diameter.



Figure 13: SolidWorks ray tracing from the TT modulation mirror to the CCD.

We choose to fix the lens focal length in order to have a feasible lens. Playing with this distance we found that a lens of $f = 60 \text{ mm}$ would be our best choice. We have changed the distance between the pyramid apex and the lens from 181 mm (according to figure 13) to 213.615 mm to adjust for the pupil diameter. The distance from the lens to the CCD detector is fixed by the pupil position solver in Zemax. We can see figure 14 that at the pupil plane, the pupil image has a diameter of $2 * 0.396 = 0.792 \text{ mm}$ as expected.



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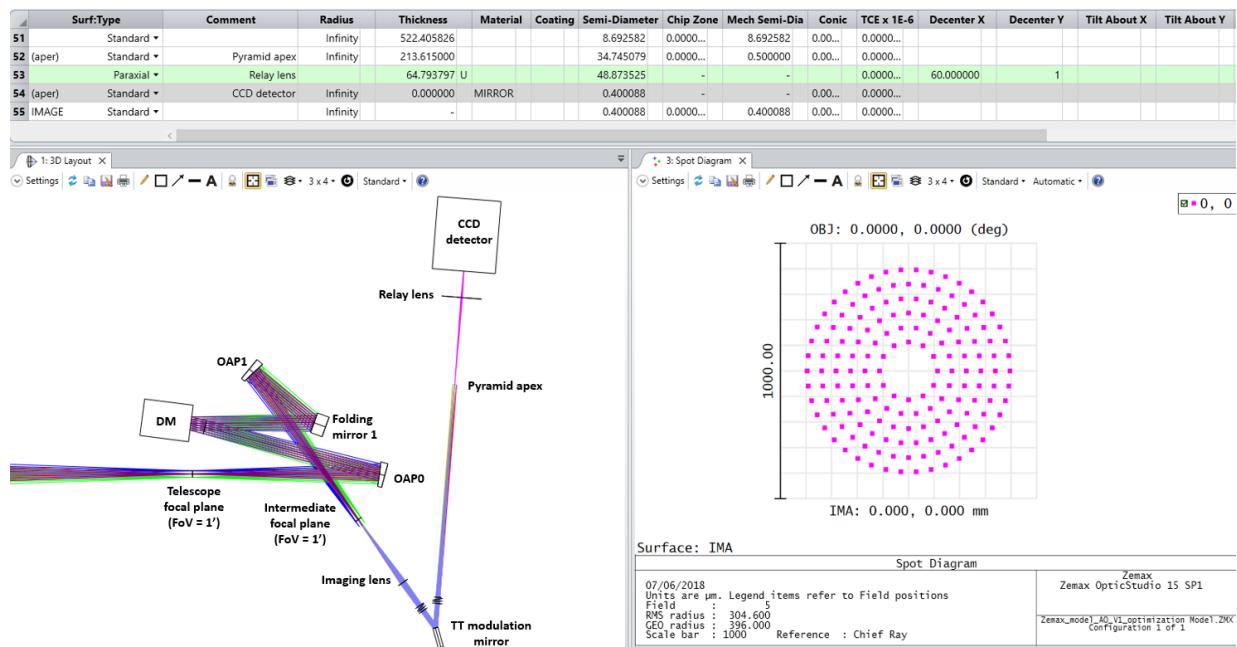


Figure 14: Zemax model of the bench until the CCD camera

The natural guide star selection in the FoV is made using a XYZ stage where the WFS path is mounted. The TT modulation mirror is on a pupil plane so it can modulate the beam around the apex of the pyramid irrespectively of the star position. However, the pyramid has to be placed at the star focus in the FoV. The XY stage allows the NGS selection while the Z axis allows realigning the pupil when needed.

3.4 The entire design

Here is the entire design of the AO system.



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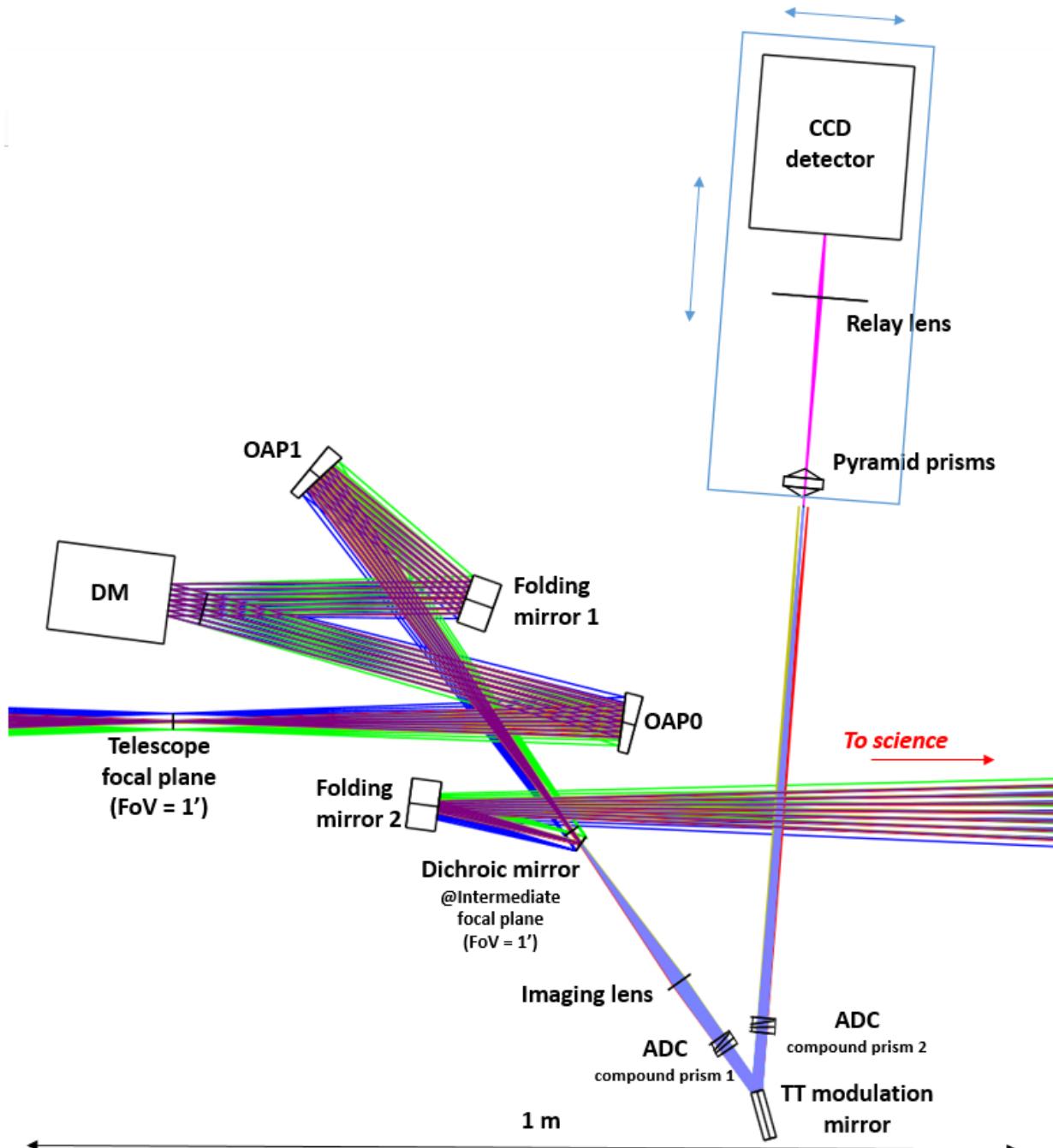


Figure 15: Zemax model of the entire optical train



3.5 Aberrations

The AO optical model on Zemax shows that if the alignment and optical surfaces are ideals we should have a total amount of aberration less than 1 nm (largely dominated by spherical aberrations). However, we know that it is not going to be the case and some tolerance analyses are on process to give a realistic approximation of the system aberrations.

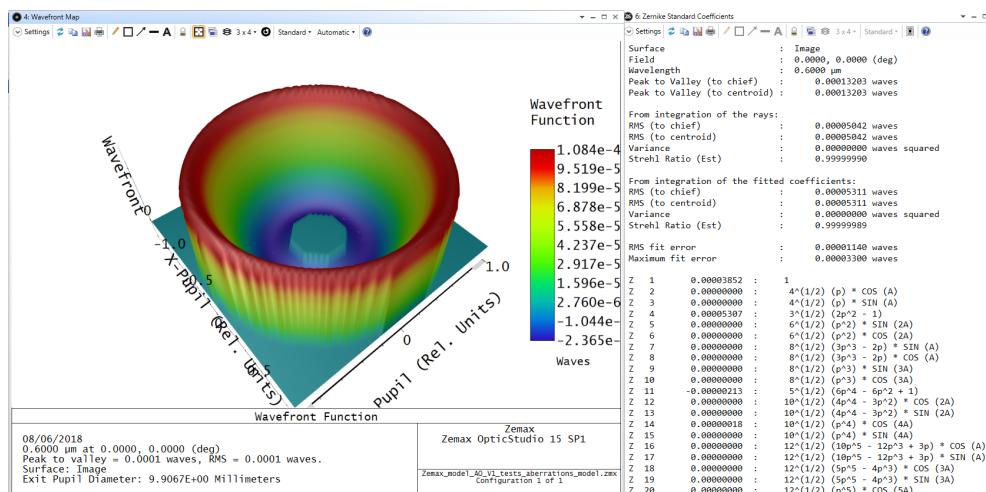


Figure 16: Wavefront and Zernike coefficients at the pyramid apex focal plane.

3.6 Atmopsheric Dispersion Compensator (ADC)

We are designing an ADC to compensate the atmospheric dispersion (see appendixB). The geometric parameters and the glass are not set yet this is why we do not have implemented it in the Zemax model for now.

3.7 Further development and optimization

The current design will be completed with the optimized ADC, real lens models, the pyramid double prisms and an optional neutral density filter placed nearby the dichroic membrane to protect the camera for the bright stars.

This design could be slightly modified in order to update it to a GLAO (Ground Layer Adaptive Optics) system. We could add a beam splitter before the PWFS shoe box and duplicate this PWFS.



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A OAPs focal lengths relations

This section describe the relation between the parental and the effective focal length for an off-axis parabola. The sketch figure 17 describes the situation with :

- EFL the effective focal length
- PFL the parental focal length
- (x_s, y_s) the coordinate of the ray intersection with the parabola
- the parabola of equation

$$y = \frac{x^2}{4PFL} - PFL \quad (18)$$

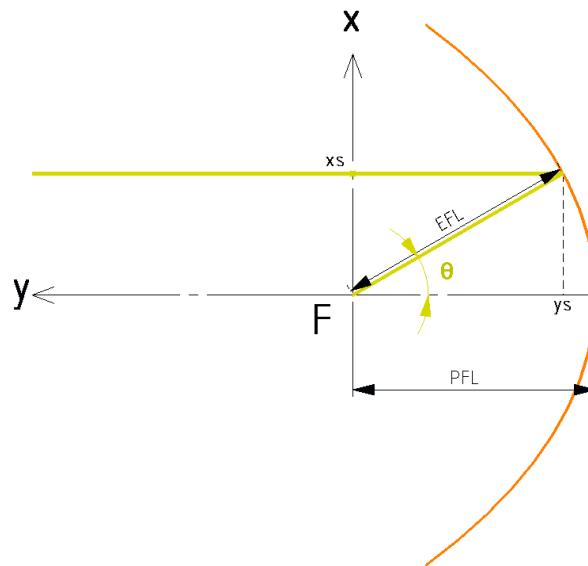


Figure 17: Sketch of a ray reflected on an OAP

The relation between EFL and PFL can be determined with the following set of equations.

$$\begin{aligned} x_s &= EFL \sin \theta \\ y_s &= EFL \cos \theta \end{aligned} \quad (19)$$



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Using (18) and (19) we have :

$$\begin{aligned} 4PFL(y_S + PFL) &= x_S^2 \\ 4PFL(EFL \cos \theta + PFL) &= (EFL \sin \theta)^2 \\ 4PFL^2 - 4PFL EFL \cos \theta - EFL^2 \sin^2 \theta &= 0 \end{aligned}$$

$$\begin{aligned} PFL_{1,2} &= \frac{4EFL \cos \theta \pm \sqrt{(-4EFL \cos \theta)^2 + 16EFL^2 \sin^2 \theta}}{2 * 4} \\ &= \frac{1}{2} \left[EFL \cos \theta \pm \sqrt{EFL^2 \cos^2 \theta + EFL^2 \sin^2 \theta} \right] \end{aligned}$$

The final equation is :

$$2PFL = EFL(1 + \cos \theta) \quad (20)$$



B ADC design

B.1 Scope

The previous report explains the different possibilities to configure an ADC. Here we are presenting the design we are going to implement for the case of the DAG telescope AO. This ADC will be for visible wavelength. The range is limited by the camera Nuvu spectral range and the star spectrum studied. We are taking only the Nuvu camera bandwidth into account for a first iteration (500 nm to 900 nm).

B.2 Amici principle

As said in the previous report, the Amici prisms are commonly used in the ADC systems. The ADCs are composed by a 2-doublet design which mixes two Amici prisms. This kind of prism is an alliance of two pieces of glass which have different dispersion (different refractive indexes). The materials must have the same refractive number for a mean wavelength so that at this frequency the incident and emergent rays are parallel (zero-deviation at λ_{av}). The two prisms can be rotated around the optical axis in order to change the dispersion and compensate it for each zenith angle. When the angle between them is 180° the dispersion is reduced at its minimum.

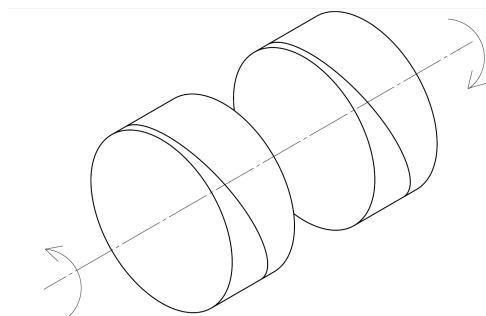


Figure 18: 2-doublet Amici prisms design

Another design can be a three-glass Amici prisms, which is called triplet-design. It consists of the insertion of a anomalous dispersion glass between the two firstly introduced surfaces.



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According to Kopon thesis [2], the doublet corrects only the first order of chromatism while the triplet acts on both primary and secondary chromaticism aberration.

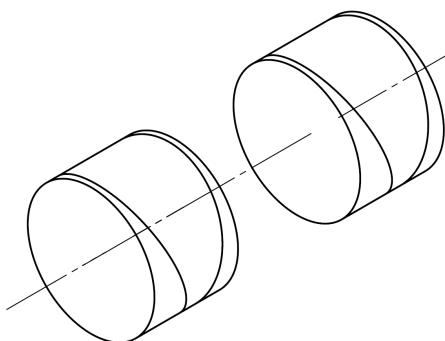


Figure 19: 2-triplet Amici prisms design [2]

The triplet-design seems to be appropriate for our case because it gives the best performance. However, its cost (time to design it + manufacture) is much higher than the gain we can have adding one wedge in our Amici prism. This is why we are going to design a doublet prisms ADC.

The system is made only by plane surfaces so that a plane front which go through the prisms should leave them as a plane front. In a collimated beam with a very narrow field of view the prisms combined thickness can be larger than the beam diameter [3].

We can also use vertex with a certain radius in a converging beam. This way we can change the F# leaving the ADC.

The doublet glass has to be designed in order to have a thermal expansion rate as close as possible between each glass to not break with the temperature changes. The internal reflection is also a parameter that we have to take into account because we want the ADC to transmit as much as possible light.

In order to design our ADC, we need to collect some information about its position in the AO bench, the dispersion of the atmosphere for our parameters, the field of view (FoV) and wavelength bandwidth it has to work in.



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B.3 Model of the atmosphere chromatic dispersion

The atmosphere chromatic dispersion can be computed looking at the refraction index for each interesting wavelength. The following model has been implemented in order to decide which dispersion the ADC has to compensate for.

B.3.1 Atmosphere refraction index

First, we need to model to atsmophere dispersion (refractive index) depending on the wavelength. The simulation of the atmosphere refractive index is based on Ciddor's approach [4] which is a compilation of all previous equations for the visible and near infrared. Ciddor develops a process that we can easily follow in order to model correctly the atmosphere refractive index represented by :

$$n_{final} = (\rho_a / \rho_{axs}) (n_{axs} - 1) + (\rho_\omega / \rho_{\omega s}) (n_{\omega s} - 1) \quad (21)$$

The procedure to arrive to equation (37) is described by the following sections.

Point 1: svp, f, x_w

Saturated water vapour pressure in the humid air svp

$$svp = \exp \left(AT^2 + BT + C + \frac{D}{T} \right) \text{ [Pa]} \quad (22)$$

With the coefficients : $A = 1.2378847 \times 10^{-5}$ [K⁻²], $B = -1.9121316 \times 10^{-2}$ [K⁻¹], $C = 33.93711047$ [-], $D = -6.4341645 \times 10^3$ [K])

Enhancement factor f of water vapour in the air

The molar mass of dry air containing x_{CO_2} ppm of CO₂ :

$$M_a = (28.9635 + 12.011 \times 10^{-6}(x_{CO_2} - 400)) \times 10^{-3} \text{ [kg/mol]} \quad (23)$$

Here we take $x_{CO_2} = 400$ [ppm].

The total pressure P :

$$P = P_0 \left(1 - \frac{\Delta T H}{T} \right)^{\frac{g M_a}{R \Delta T}} \text{ [Pa]} \quad (24)$$

With $\Delta T = 0.0065$ [K/100 m] the vertical gradient of temperature 0.65 K for 100 m in the 28



troposphere [5], H [m] the altitude of the site (3170 m for Karakaya), $g = 9.80665$ [m/s²] the earth-surface gravitational acceleration and $R = 8.314510$ [J/mol/K] the universal gas constant. The enhancement factor :

$$f = \alpha + \beta P + \gamma T^2 \quad (25)$$

With $\alpha = 1.00062$ [-], $\beta = 3.14 \times 10^{-8}$ [Pa⁻¹], $\gamma = 5.6 \times 10^{-7}$ [°C⁻²]

Molar fraction of water vapor in moist air x_w

$$x_w = \text{RH} f \frac{\text{SVP}}{P} \quad (26)$$

With RH the relative humidity (RH = 0.8 for DAG).

Point 2: n_{axs}

Constants involved in the standard phase and group refractivities of dry air $k_0 = 238.0185$, $k_1 = 5792105$, $k_2 = 57.362$, $k_3 = 167917$ [μm⁻²].

The refractive index of standard air at T=20°C, 101325 Pa, 0 % humidity, 450 ppm of CO₂ :

$$10^8 (n_{as} - 1) = k_1 / (k_0 - \sigma^2) + k_3 / (k_2 - \sigma^2) \quad (27)$$

With $\sigma = 1/\lambda$ [μm⁻¹]. For us it is not useful to have the following equation because we are in a standard case so $n_{axs} = n_{as}$, but I let it in order to remember the sequence of equations :

$$(n_{axs} - 1) = (n_{as} - 1) \left[1 + 0.534 \times 10^{-6} (x_c - 450) \right] \quad (28)$$

With $x_c = 450$ [-ppm] number of ppm of CO₂ (standard).

Point 3: M_a see equation (23).

Point 4: Z_a

The compressibility of dry air :

$$Z_{as} = 1 - \frac{P_{as}}{T_{as}} \left[a_0 + a_1 T_{as} + a_2 T_{as}^2 + (b_0 + b_1 T_{as}) * x_{was} + (c_0 + c_1 T_{as}) * x_{was}^2 \right] + \frac{P_{as}^2}{T_{as}^2} \left(d + e * x_{was}^2 \right) \quad (29)$$



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With $x_{was} = 0$ the molar fraction of water vapor in moist air (here we are in dry air so 0), P_{as} [Pa] the pressure, T_{as} [K] temperature at standard conditions and the following coefficients the compressibility factor Z :

$$a_0 = 1.58123 \times 10^{-6} [\text{K}^* \text{Pa}^{-1}], b_0 = 5.707 \times 10^{-6} [\text{K}^* \text{Pa}^{-1}], c_1 = -2.376 \times 10^{-6} [\text{Pa}^{-1}], \\ a_1 = -2.9331 \times 10^{-8} [\text{Pa}^{-1}], b_1 = -2.051 \times 10^{-8} [\text{Pa}^{-1}], d = 1.83 \times 10^{-11} [\text{K}^2 \text{Pa}^{-2}], \\ a_2 = 1.1043 \times 10^{-10} [(\text{KPa})^{-1}] c_0 = 1.9898 \times 10^{-4} [\text{K}^* \text{Pa}^{-1}], e = -0.765 \times 10^{-8} [\text{K}^2 \text{Pa}^{-2}]$$

Point 5: Z_w

The compressibility of pure water vapour :

$$Z_{ws} = 1 - \frac{P_{ws}}{T_{ws}} \left[a_0 + a_1 T_{ws} + a_2 T_{ws}^2 + (b_0 + b_1 T_{ws}) * x_{w_{ws}} + (c_0 + c_1 T_{ws}) * x_{w_{ws}}^2 \right] + \frac{P_{ws}^2}{T_{ws}^2} \left(d + e * x_{w_{ws}}^2 \right) \quad (30)$$

With $x_{w_{ws}} = 1$ the molar fraction of water vapor in moist air (here we are in moist air so 1), $P_{ws} = 1333$ [Pa] the pressure, $T_{ws} = 293.15$ [K] the temperature.

Point 6: ρ_{axs}, ρ_{ws}

The density of standard dry air [kg/m^3] :

$$\rho_{axs} = \frac{P_{as} M_a}{Z_{as} R T_{as}} \left(1 - x_{was} \left(1 - \frac{M_w}{M_a} \right) \right) \quad (31)$$

The density of standard moist air [kg/m^3] :

$$\rho_{ws} = \frac{P_{ws} M_a}{Z_{ws} R T_{ws}} \left(1 - x_{w_{ws}} \left(1 - \frac{M_w}{M_a} \right) \right) \quad (32)$$

With $M_w = 18.01528 \times 10^{-3}$ [kg/mol].

Point 7: Z

The compressibility of moist air under experimental conditions :

$$Z = 1 - \frac{P_w}{T_w} \left[a_0 + a_1 T_w + a_2 T_w^2 + (b_0 + b_1 T_w) * x_{w_w} + (c_0 + c_1 T_w) * x_{w_w}^2 \right] + \frac{P_w^2}{T_w^2} \left(d + e * x_{w_w}^2 \right) \quad (33)$$

Point 8: ρ_a



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The density of the dry component of the moist air [kg/m³] :

$$\rho_a = P Ma \frac{(1 - x_a)}{Z R T_a} \quad (34)$$

Point 9: ρ_w

The density of the dry component of the moist air [kg/m³] :

$$\rho_a = P Ma \frac{(1 - x_a)}{Z R T_a} \quad (35)$$

Point 10: n_{final}

Constants involved in the standard phase and group refractivities of water vapour :

$\omega_0 = 295.235 \text{ } [\mu\text{m}^{-2}]$, $\omega_1 = 2.6422 \text{ } [\mu\text{m}^{-2}]$, $\omega_2 = -0.032380 \text{ } [\mu\text{m}^{-4}]$, $\omega_3 = 0.004028 \text{ } [\mu\text{m}^{-6}]$.

The refractive index of water vapour at 20°C, 1335Pa :

$$10^8 (n_{ws} - 1) = 1.022 \times (\omega_0 + \omega_1 \sigma^2 + \omega_2 \sigma^4 + \omega_3 \sigma^6) \quad (36)$$

The refractive index of moist air (atmosphere) :

$$n_{final} = (\rho_a / \rho_{axs}) (n_{axs} - 1) + (\rho_w / \rho_{ws}) (n_{ws} - 1) \quad (37)$$

B.3.2 Atmosphere refraction equation

The refraction of the atmosphere is computed with the equation [6]:

$$R(\lambda, z) = \kappa (n(\lambda) - 1) (1 - \beta) \tan(z) - \kappa (n(\lambda) - 1) \left(\beta - \frac{(n(\lambda) - 1)}{2} \right) \tan^3(z) \text{ [rad]} \quad (38)$$

with

- $\beta = 0.001254 \left(\frac{T(K)}{273.15} \right)$ the effective height of the observatory above the surface of the earth [6]
- $\kappa = 1$ for a spherical Earth surface [6] or instrumental correction no more useful [7]

This equation (38) is valid only for zenith angle $< 75^\circ$. We can even neglect the second term for zenith angles $< 65^\circ$ [8]. The limit of zenith angle of our computation is set to



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70° so the entire equation is set.

The model applied for the wavelength range [500, 900] nm shows :

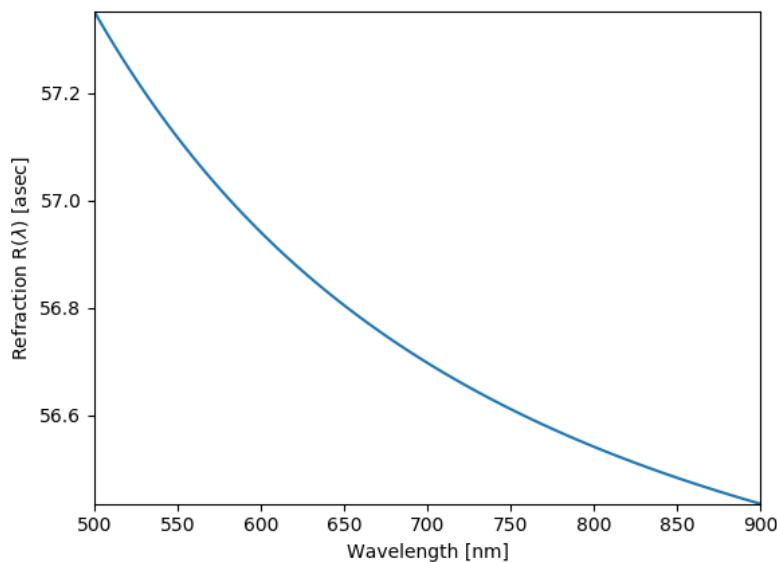


Figure 20: Angular dispersion of the atmosphere (appendix E)

B.3.3 Wavelength range

For our case, the wavelength range is set by the Nuvu camera [9] bandwidth limits $\lambda_{min} = 500$ [nm] and $\lambda_{max} = 900$ [nm] and the maximum quantum efficiency that fixes the central wavelength to $\lambda_{cen} = 600$ [nm]. We use these 3 reference wavelengths to compute the ADC efficiency.

B.3.4 Glasses for the ADC

The choice of glass depends on the range of zenith angles, the wavelength interval and the cost [10]. Most of the glass couple in the Amici conception are flint/crown pairs.

The glass choice is made in a data base built from Schott and Ohara catalogues. The data are taken from [11] where we can download glasses properties from many different suppliers. The data are sorted to extract Sellmeier coefficients [12] in order to compute



the refractive index of each glass depending on the wavelength with the equation (39).

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \frac{B_3\lambda^2}{\lambda^2 - C_3} \quad (39)$$

with B_i and C_i Sellmeier coefficients for λ in μm [12].

B.3.5 Beam propagation through the ADC

In order to compute de dispersion through the ADC a geometrical and refraction set of equations have been implemented. The sequence of these equations is listed bellow. I will follow the light on the figure 21 to describe each step.

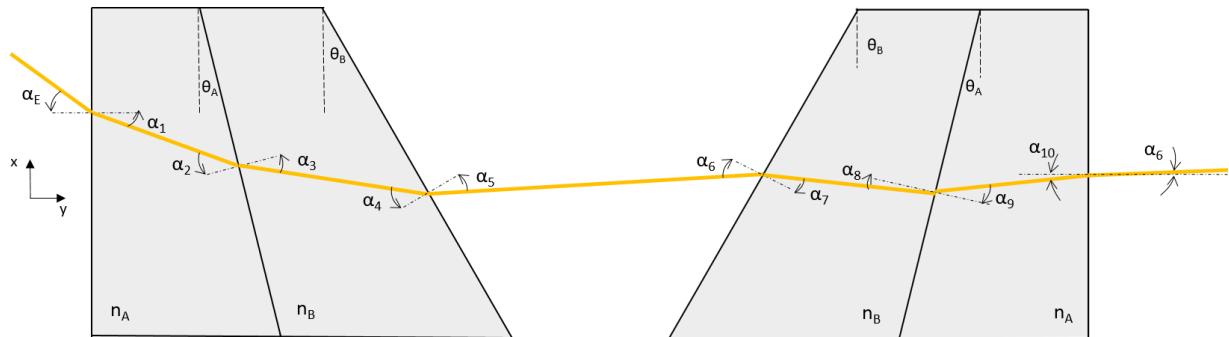


Figure 21: Sketch of the refraction through the ADC

The refractive index of the I -th glass is $n_I(\lambda_i)$. To simplify the notation, we just write n_I . We apply the sign convention described in the course material [13].

The function `exitAngle.py` D computes the exit angle depending on the glass refractive index, the geometrical parameters of the prisms and the entrance angle.

B.3.6 The metric definition

Now we have computed the dispersion of the atmosphere and the ADC. We want the smallest total dispersion after the ADC so we investigate all configurations in order to minimize it. The metric we use to calculate the efficiency of our prism combination (appendix F) is



defined by the name BDCQ (Beam Dispersion Correction Quality) :

$$BDCQ^2 = (\alpha_{\text{Exit}}(\lambda_{\min}) - \alpha_{\text{Exit}}(\lambda_{av}))^2 + (\alpha_{\text{Exit}}(\lambda_{\max}) - \alpha_{\text{Exit}}(\lambda_{av}))^2 + (\alpha_{\text{Exit}}(\lambda_{av}) - \alpha_{\text{Entry}}(\lambda_{av}))^2 \quad (40)$$

B.3.7 Internal reflection

In parallel of the dispersion computation, we calculate the total internal reflection of the ADC. If the refractive index of two joint glasses are too different then we will loose a lot of intensity reflected on the interface. The external faces of the ADC will be coated. The determination of the total internal reflectivity is developed below.

The parameters are :

- $\alpha_Z = [\alpha_E; \alpha_1; \alpha_A; \alpha_B; \alpha_S]$ [rad] array of the rays angle wrt the optical axis
- $\alpha_I = [\alpha_1; \alpha_2; \alpha_3; \alpha_4; \alpha_5; \alpha_6; \alpha_7]$ [rad] array of the rays angles wrt the normal the vertex (refraction angle)
- $n = [n_0; n_A; n_B; n_C; n_0]$ medium refractive index
- $d = [z_A, z_B - z_A, z_C - z_B]$ [mm] distances on the optical axis between each medium change on the ray path

At the 1rst interface AB, we have [14]:

$$r[1] = \frac{n[3] \cos \alpha_I[2] - n[2] \cos \alpha_I[3]}{n[3] \cos \alpha_I[2] + n[2] \cos \alpha_I[3]} \quad (41)$$

Taking the recursive initialization term $U(1) = r(1)$ we have for $p = 2$:

$$r[p] = \frac{n[p+1] \cos(\alpha_I[p]) - n[p] \cos(\alpha_I[p+1])}{n[p+1] \cos(\alpha_I[p]) + n[p] \cos(\alpha_I[p+1])} \quad (42)$$

$$U[p] = \frac{U[p-1] + r[p] \exp\left(\frac{2\pi}{\lambda} \cos(\alpha_Z[p]) d[p-1]\right)}{1 + U[p-1] r[p] \exp\left(\frac{2\pi}{\lambda} \cos(\alpha_Z[p]) d[p-1]\right)} \quad (43)$$

Using equation (43) we can calculate the reflection coefficient for $(p+1)$ number of glasses. In our case we have only 3 glasses so the total reflection (without taking external faces into account) is $U[2]$.



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B.4 ADC design optimization

The efficiency of a combination (glass, angle) is computed by the script *glassChoice.py*. The function *fmin_l_bfgs_b* from the optimize package in Python is used to find minimum of this constrained nonlinear multivariable function. The parameters are the geometrical coefficients for one set of glass. When the optimum is found for a combination of glass, the efficiency and internal reflection are given as outputs.

The global matrix is made from two for-loops that look for all glasses from the catalogues Schott and Ohara. The wavelength for which the refraction is calculated are taken from the Nuvu camera quantum efficiency curve : $\lambda_{\text{min}} = 500$ nm, $\lambda_{\text{average}} = 600$ nm and $\lambda_{\text{max}} = 900$ nm. The zenith angle is set to 70°.



C DimensionnementOAPs.py

```
1 # -*- coding: utf-8 -*-
"""
3 Created on Fri May 18 10:27:43 2018
This code compute the focal length of the OAPs and the image position.
5 @author: audrey.bouxin
"""

7 import numpy as np
import matplotlib.pyplot as plt
9
" Parameters"
11 DEG2RAD = np.pi/180;
theta_OAP0 = 15*DEG2RAD; #[rad] pour l'OAP0
13 ExP2FP = 10338.74; #[mm]
dExP = 727.4046; #[mm]
15 DDM = 32.7; #[mm]

17 # for OAP0
alpha = np.arctan(dExP*0.5/ExP2FP); #[rad]
19 PFL_OAP0 =(DDM)/2/(-1/np.tan(theta_OAP0+alpha)+(1/(np.tan(theta_OAP0+alpha)))
**2+1)**0.5+1/np.tan(theta_OAP0-alpha)-(1/(np.tan(theta_OAP0-alpha))
**2+1)**0.5); #[mm] pour l'OAP0
EFL_OAP0 = 2*PFL_OAP0/(1+np.cos(theta_OAP0));
21 print('PFL_OAP0 (estim) : ', PFL_OAP0)

23
#PFL_OAP0 = 445.0;
25 print('PFL_OAP0 : ', PFL_OAP0)
EFL_OAP0 = 2*PFL_OAP0/(1+np.cos(theta_OAP0))
27 print('EFL_OAP0 : ', EFL_OAP0)
po = -(PFL_OAP0+ExP2FP);
29 pi = po*PFL_OAP0/(po+PFL_OAP0);
print('pi (DM) : ', pi)
31

33 #Aberrations : Wabefront MAP
import zernike as zer
```



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```
35 aa = zer.calc_zern_j(4,200,0.1,1)  
37 print(aa)
```

../DimensionnementOAPs.py

D modelisationADCfunctions.py

E test_Refraction_atmosphere.py

F glassChoice.py