KTOM Febuari 2019 Nomor 9
Diberikan bilangan real positif a dan b yang memenuhi

$$a^2 = 6 + b^2 \quad \text{dan} \quad \frac{12b}{6 - b^2} = \sqrt{2019}$$

Tentukan nilai dari

$$\frac{\sqrt{6}a^2}{12 - a^2}$$

Solusi
Perhatikan bahwa $12 - a^2 = 6 - b^2 = \frac{12b}{\sqrt{2019}}$. Perhatikan juga bahwa

$$\frac{6}{b} - b = \frac{6 - b^2}{b} = \frac{12}{\sqrt{2019}}$$

$$\left(\frac{6}{b}\right)^2 - 12 + b^2 = \frac{12^2}{2019}$$

$$\left(\frac{6}{b}\right)^2 + 12 + b^2 = \frac{12^2}{2019} + 24$$

$$\implies \frac{6}{b} + b = \sqrt{\frac{12^2}{2019} + 24}$$

Maka,

$$\frac{\sqrt{6}a^2}{12 - a^2} = \frac{\sqrt{6}(6 + b^2)}{\frac{12b}{\sqrt{2019}}} = \left(\frac{\sqrt{6 \cdot 2019}}{12}\right)\left(\frac{6}{b} + b\right)$$

$$= \sqrt{\left(\frac{6 \cdot 2019}{12^2}\right)\left(\frac{12^2}{2019} + 24\right)} = \sqrt{6 + 2019} = \sqrt{2025} = 45$$