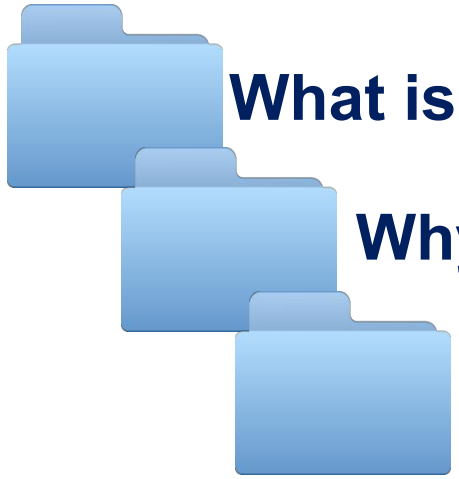
Decomposition

Lesson Objectives

At the end of this lesson, you should be able to:

- Describe the concept of decomposition
- Explain the importance of decomposition
- Decompose complex problems

Topic Outline



What is Decomposition?

Why is Decomposition Important?

How does Decomposition Help to Solve Problem?

What is Decomposition?

Natural way to solve problems

Decomposition is the process of breaking down a complex problem into smaller manageable parts (subproblems).

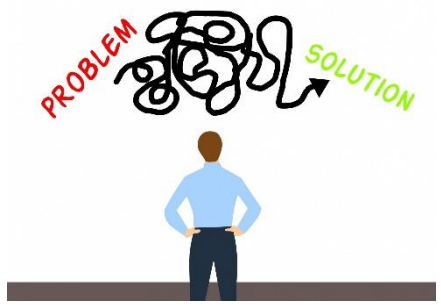
- Each subproblem can then be examined or solved individually, as they are simpler to work with.

is also known as **Divide and Conquer**

Click icon to play

Why is Decomposition Important?

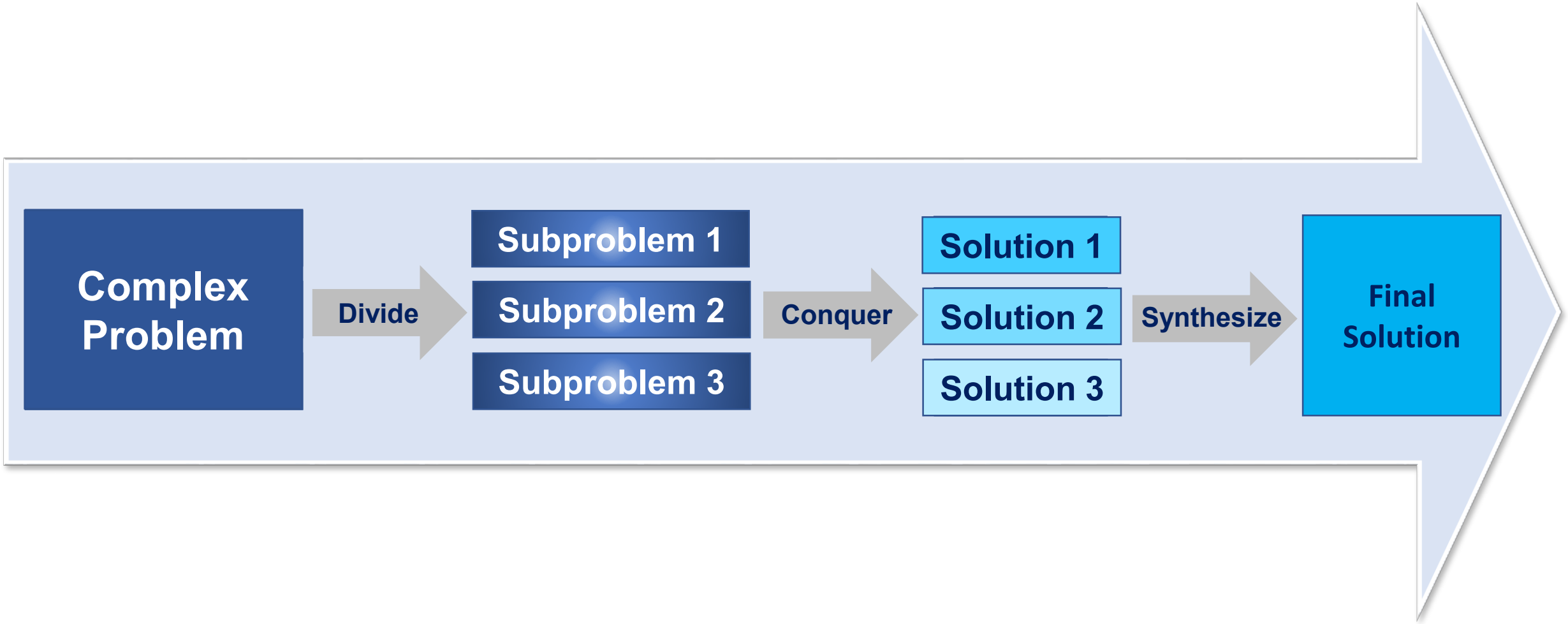
- **Solve complex problems**
  - If a complex problem is not decomposed, it is much harder to solve at once. Subproblems are usually easy to tackle.



- **Enable collaboration and teamwork**
  - Each subproblem can be solved by different parties.

- **Analysis**
  - Decomposition forces you to analyze your problem from different aspects

How does Decomposition Help to Solve Problems?



Example

How do you calculate the result of 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10?

Complex Problem

1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = ?

Divide the problem into five subproblems

- 1 + 10
- 2 + 9
- 3 + 8
- 4 + 7
- 5 + 6

Conquer each subproblem

Solution to each subproblem: 11

Synthesize the final solution

11 * 5 = 55

Practice

How do you calculate the result of 1 + 2 + 3 +…+ 98 + 99 + 100?

Answer **Result: 101 * 50 = 5050**

Summary

In this lesson, we have learned:

- What decomposition is
- The importance of decomposition
- How to apply decomposition

References for Images

No.	Slide No.	Image	Reference
1	5		Problem Puzzle [Online Image]. Retrieved June 27, 2018 from https://www.maxpixel.net/Concept-Problem-Puzzle-3d-Render-Solve-Jigsaw-2636254.
2	6		Problem Solved [Online Image]. Retrieved June 27, 2018 from https://www.publicdomainpictures.net/en/view-image.php?image=238047&picture=problem-solved.
3	6		Teamwork [Online Image]. Retrieved June 27, 2018 from https://pixabay.com/en/teamwork-together-objectives-create-3276694/.
4	6		Bright Contemplation [Online Image]. Retrieved June 27, 2018 from https://pixabay.com/en/bright-contemplation-idea-1296538/.

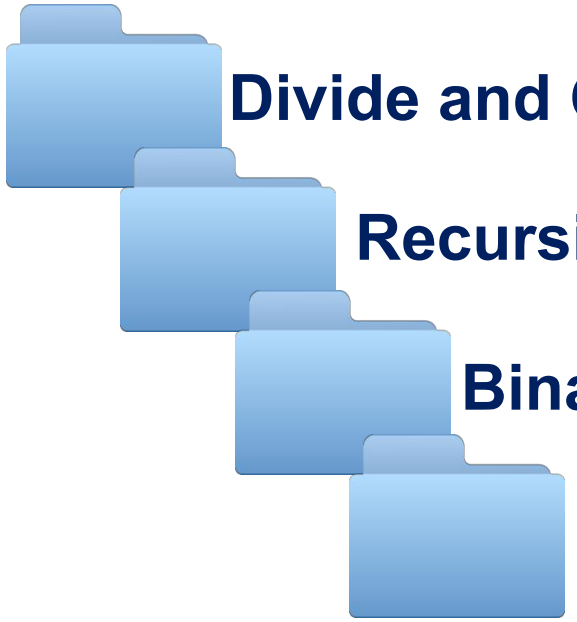
Decomposition in Python

Lesson Objectives

At the end of this lesson, you should be able to:

- Describe Divide-and-Conquer and Recursion as a decomposition process
- Apply the method of Divide-and-Conquer and Recursion in Python coding

Topic Outline



Divide and Conquer

Recursion

Binary Tree

Complete Binary Tree (CBT) Representation and Operations

Decomposition

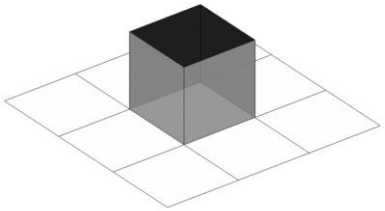
Divide-and-Conquer

- Decompose a problem into several sub-problems
- Solve each sub-problem
- Compose the solution to sub-problems

Recursion naturally supports divide-and-conquer.

Recursive Function

- A function that invokes itself
- Very useful and important in computer science



Example:

Factorial of n

$$n! = \begin{cases} 1, & n = 0 \\ n \times (n-1)!, & n > 0 \end{cases}$$

```
def f(n):
    if n == 0:
        return 1
    else:
        return n * f(n - 1)
```

Recursion (Cont'd)

Recursive Function: General Form

```
def recursiveFunc(param1, param2, …):
      if exp:       # base case (conquer)
                    …
            return value

      else:         # recursive step (divide)
            recursiveFunc(subproblem1)
            recursiveFunc(subproblem2)
                    …
            return value
```

How to Write a Recursive Function

Determine the interface (signature) of the function

- How many **parameters**? What are they?
- What is the **return object**?
- What is the **functionality** of the function?

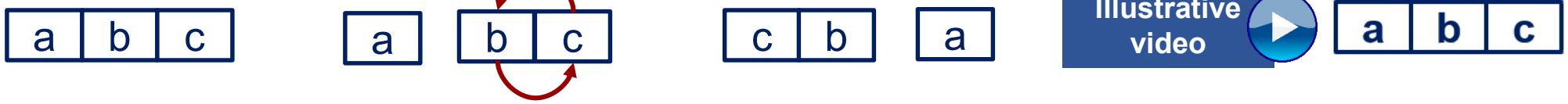
Assume you had finished the implementation of the function

Develop the function body

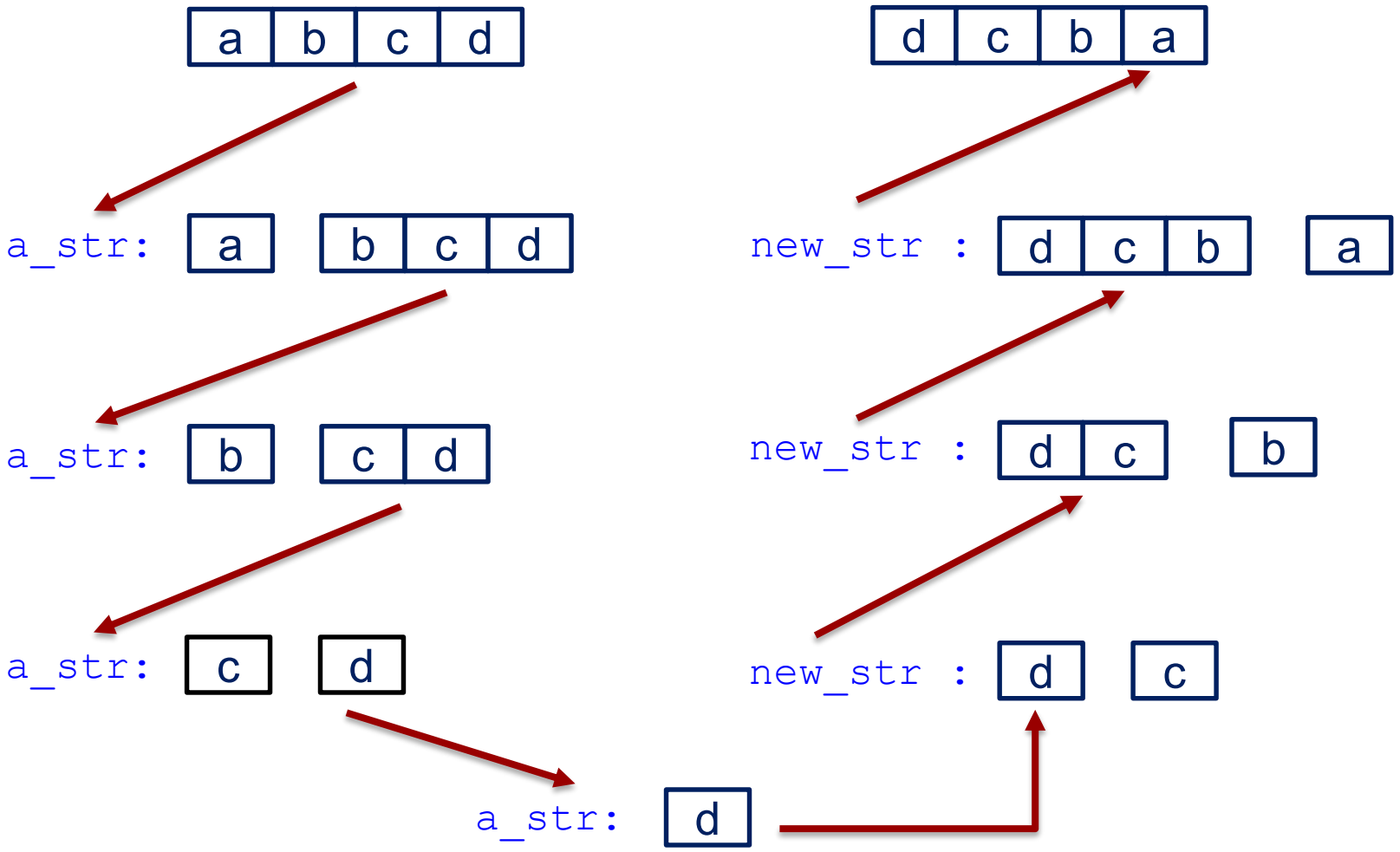
- **Base Case** (**Conquer**):
  Solve the primitive case, and then return the result
- **Recursive Step** (**Divide**)
  - Decompose the problem into subproblems (with the same structure)
  - Call the function to solve each subproblem
  - Compose the final result from subproblems, and then return it

Example: Reversing a String

```
def reverser(a_str):
    if len(a_str) == 1:     # base case
        return a_str

    else:                   # recursive step
        new_str = reverser(a_str[1:])+ a_str[0]
        return new_str
```



Example: Reversing a String (Cont'd)



Illustrative video

Performance of Recursion

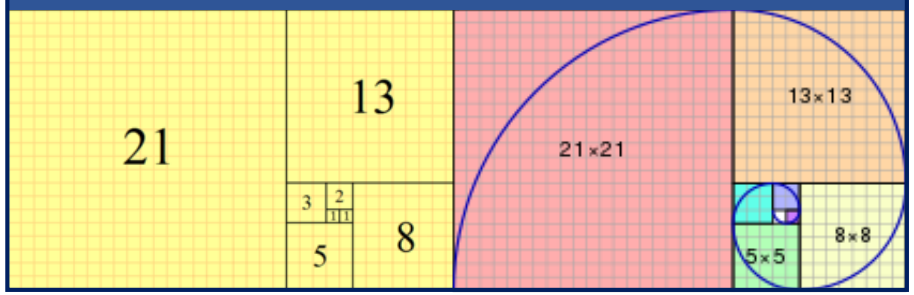
Recursive function may be inefficient!

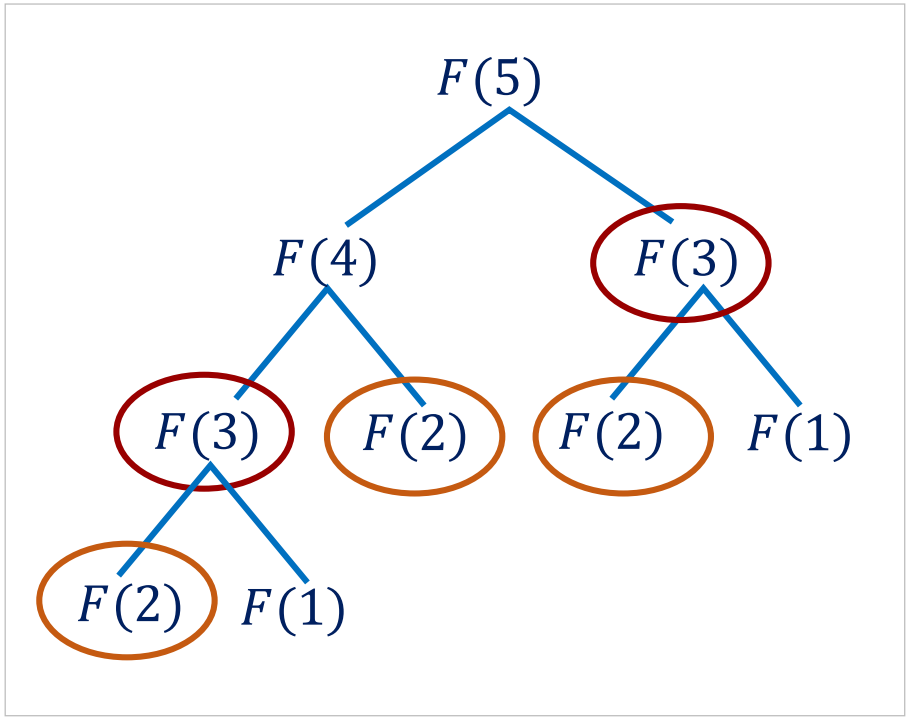
- Redundant computation!

Fibonacci Number:

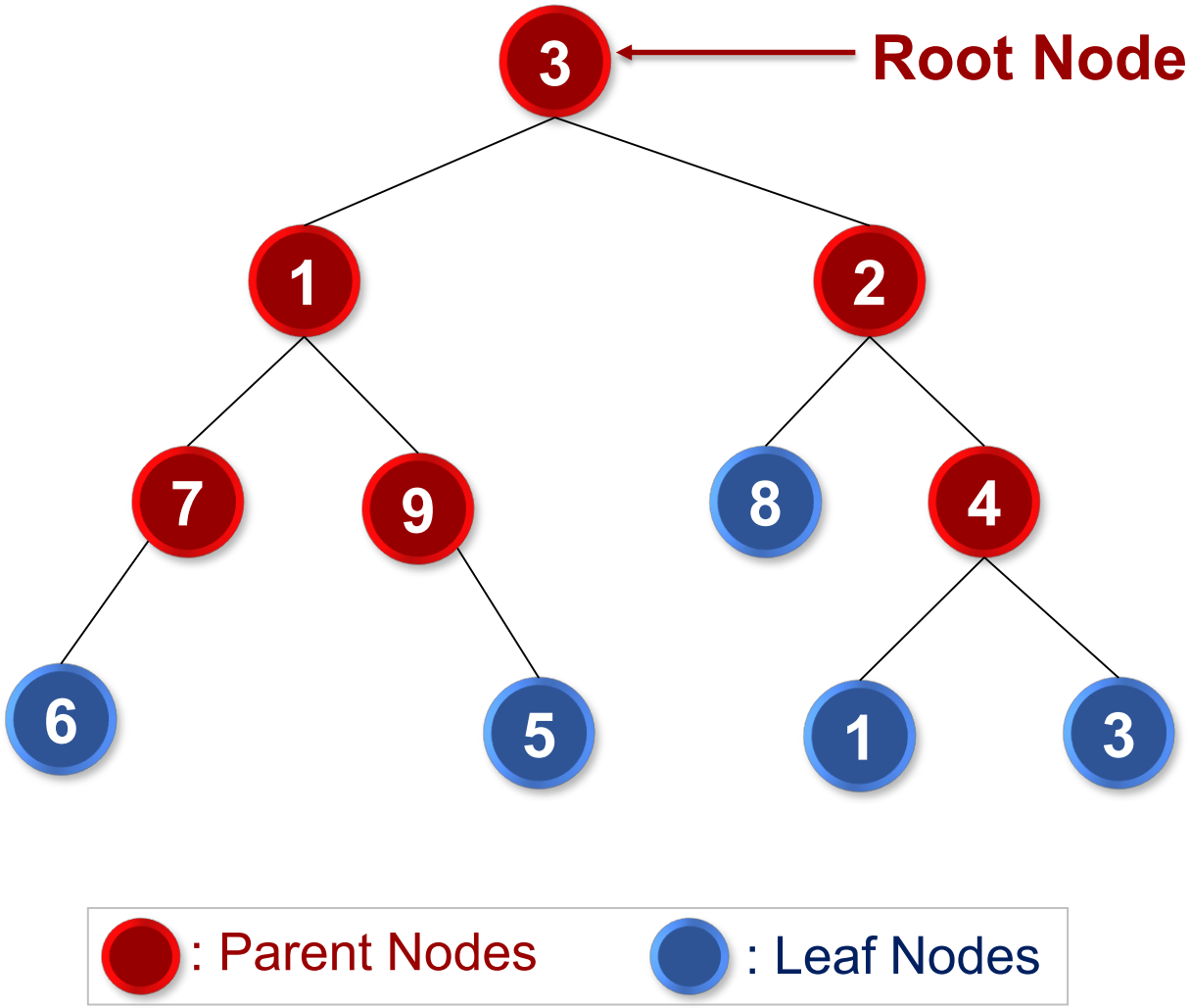
$F(1) = 1$ and $F(2) = 1$

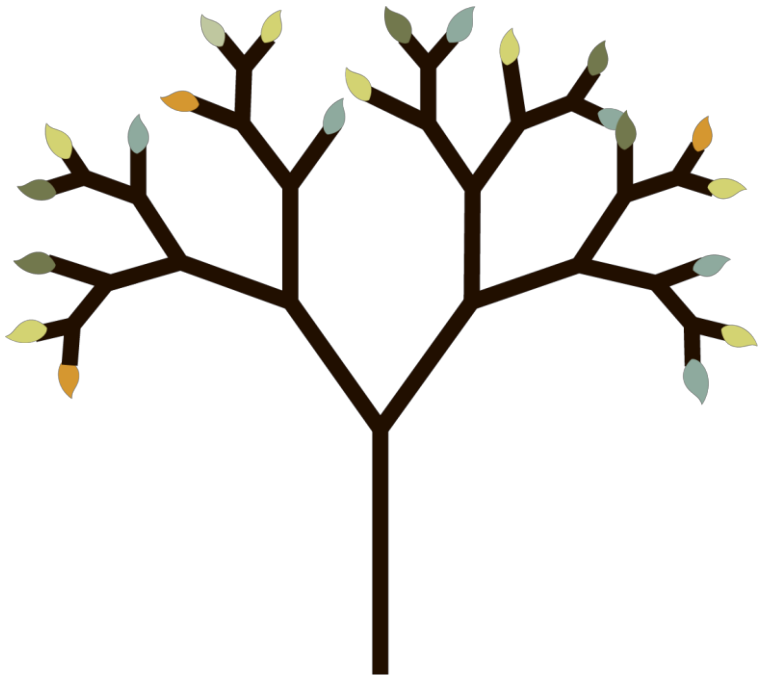
$F(n) = F(n-1) + F(n-2)\ , n \geq 2$





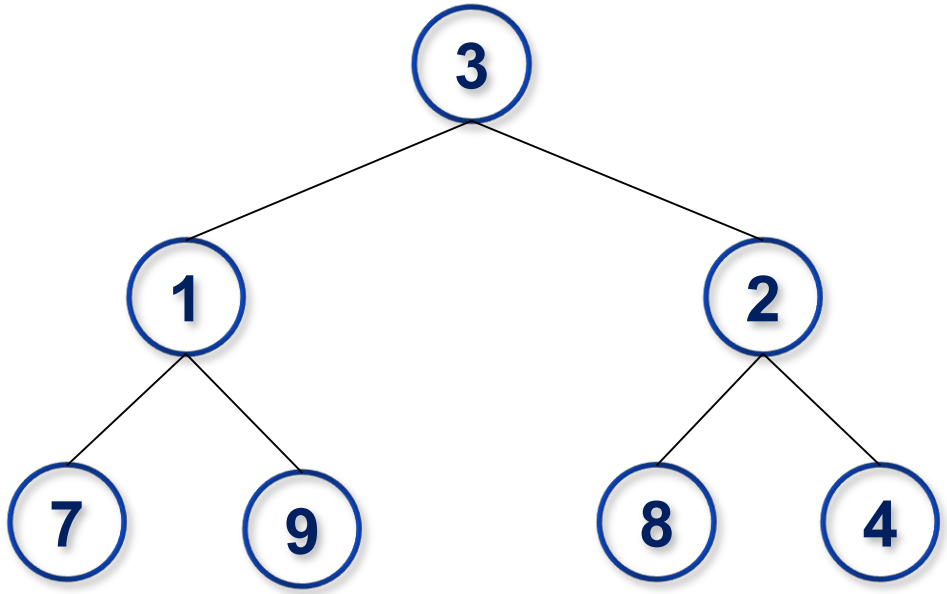
Binary Tree





Complete Binary Tree (CBT)

Every parent node in a ***complete binary tree*** (**CBT**) has exactly **two** child nodes.



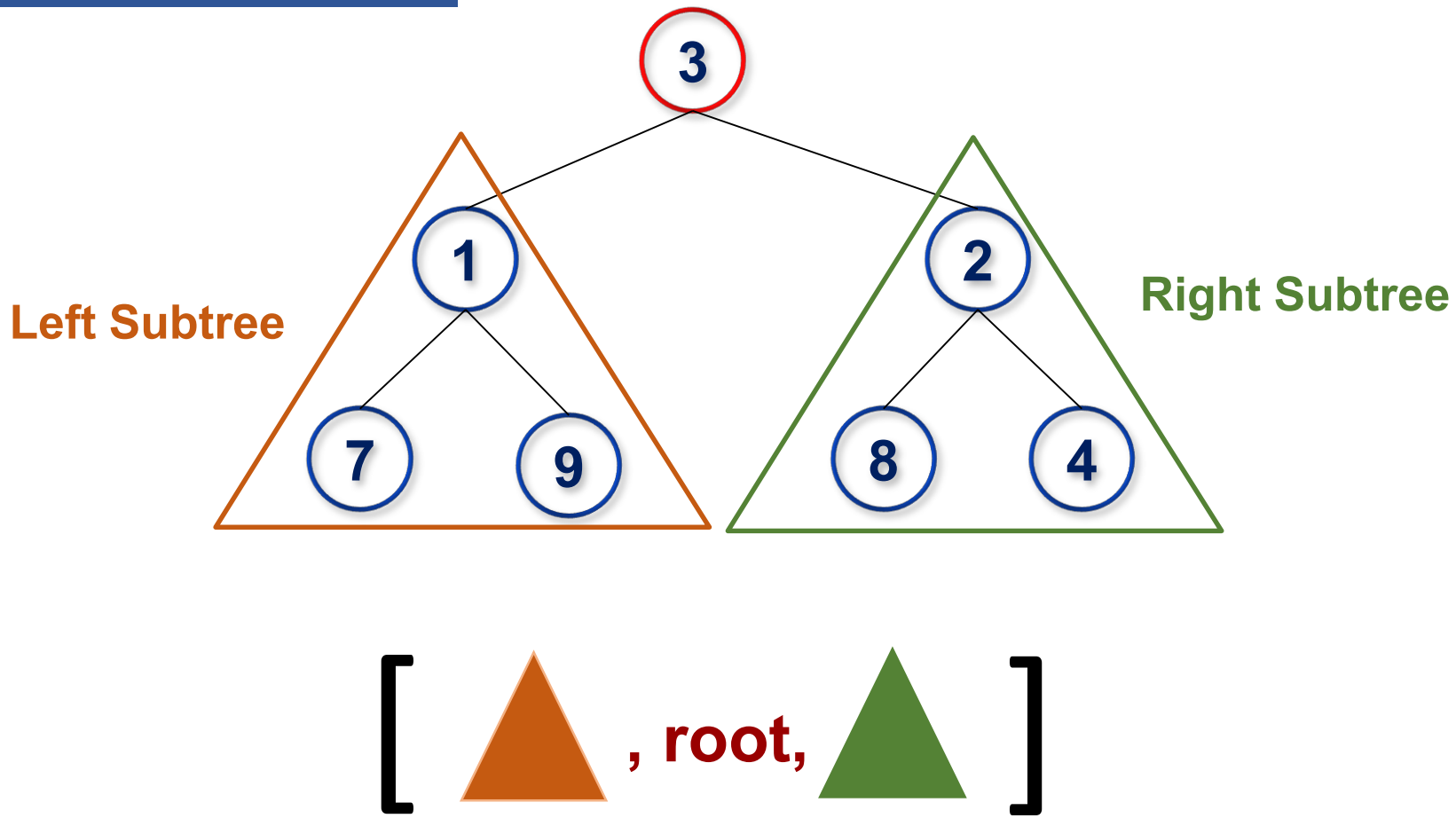
CBT is the focus in this session…

CBT Representation

- How do we represent a CBT?
- What data structures do we have now?
  - List
  - Tuple
  - Dictionary
- Which one is better?

CBT Representation (Cont'd)

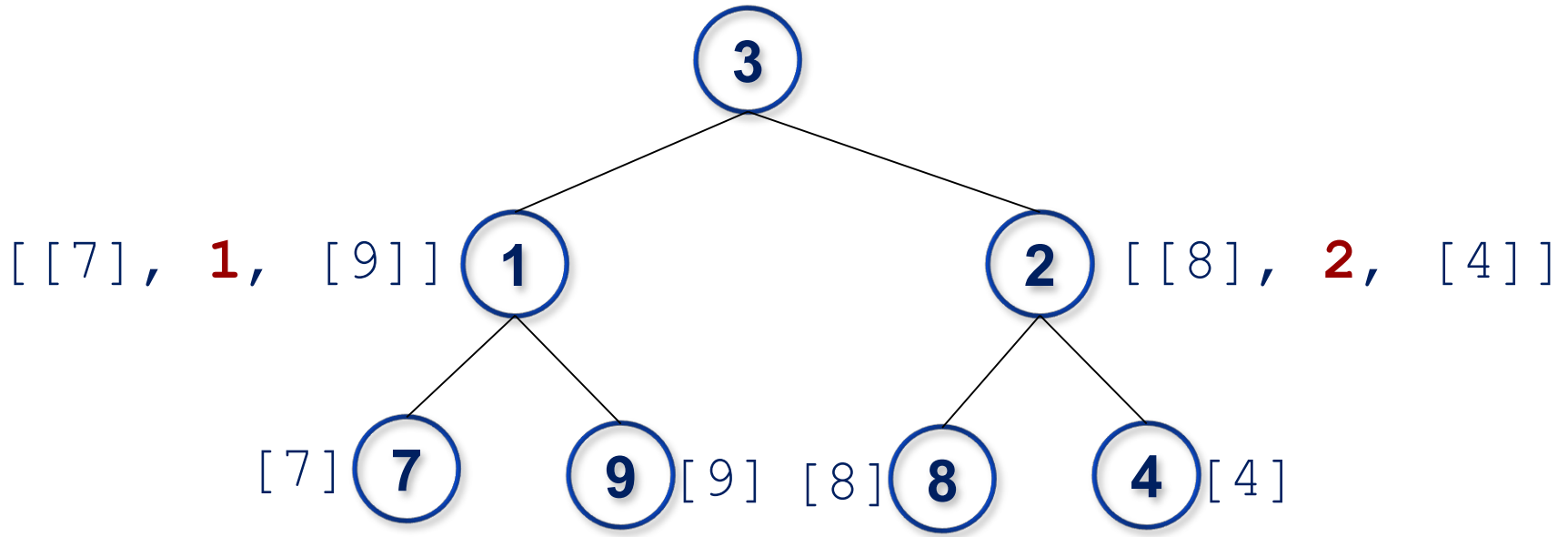
Using list maybe a good idea



CBT Representation (Cont'd)

Using list maybe a good idea

```
[[[7], 1, [9]], 3, [[8], 2, [4]]]
```



Creating CBT from the List: Example

What does the following CBT look like?

[[[1],6,[3]],2,[[5],8,[7]]],7,[[[2],3,[4]],9,[[6],5,[8]]]]

Left subtree of 7

Right subtree of 7

Left subtree of 2

Right subtree of 2

Left subtree of 9

Right subtree of 9

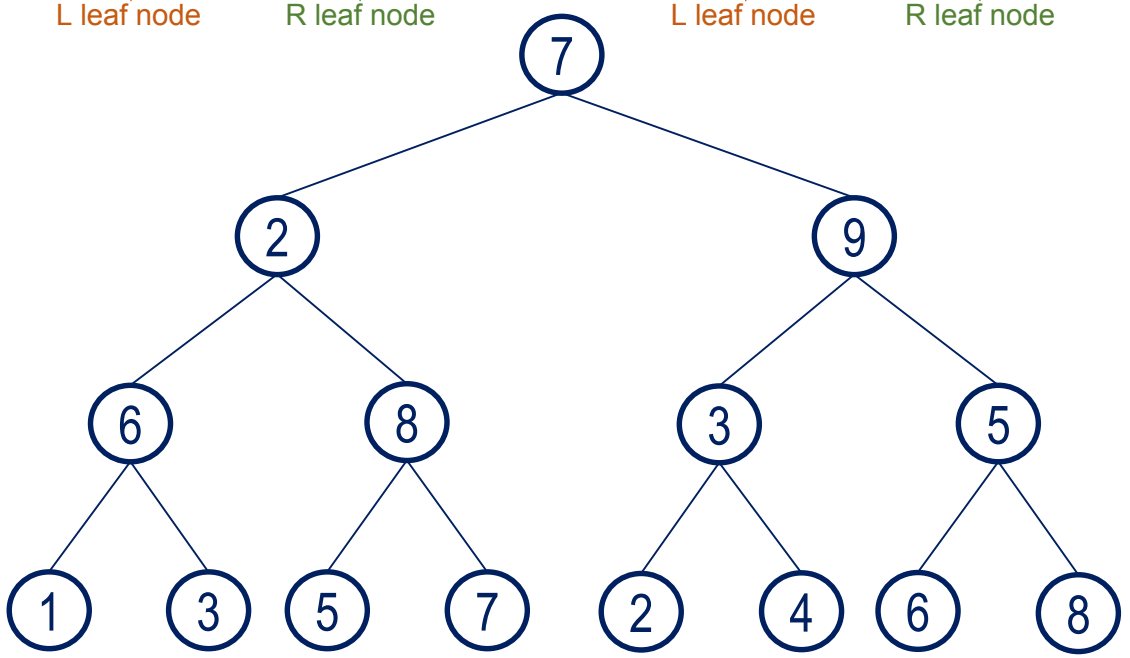
L leaf node R leaf node L leaf node R leaf node L leaf node R leaf node L leaf node R leaf node

Root node: 7

Parent nodes: Red nos.

Left leaf nodes: Orange nos.

Right leaf nodes: Green nos.



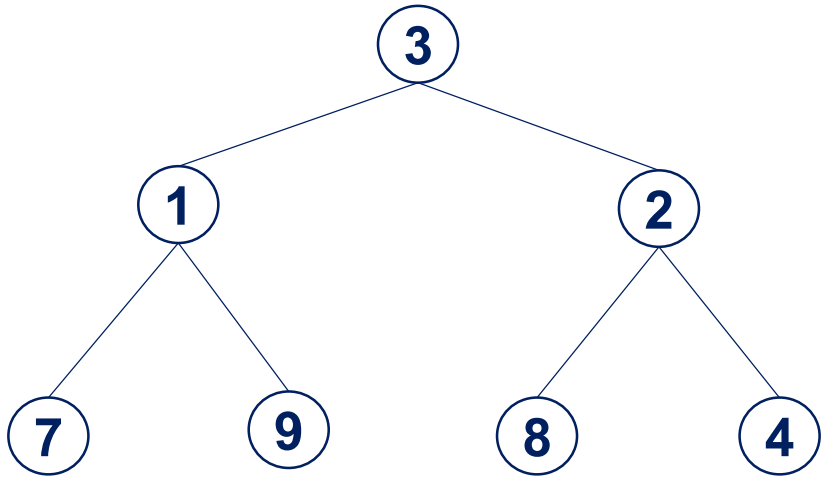
Operations in CBT

`numOfNodes(t)`	returns the total number of nodes in a CBT t
`sumNodes(t)`	returns the summation of all nodes in a CBT t
`maxNode(t)`	returns the maximum value of nodes in a CBT t
`minNode(t)`	returns the minimum value of nodes in a CBT t
`mirror(t)`	returns the mirrored CBT of a CBT t

Operations in CBT (Cont'd)

numOfNodes(t)

```
tree = [[[7], 1, [9]], 3, [[8], 2, [4]]]

print("# of Nodes: ", end='')
print( numOfNodes(tree) )
```



# of Nodes: 7

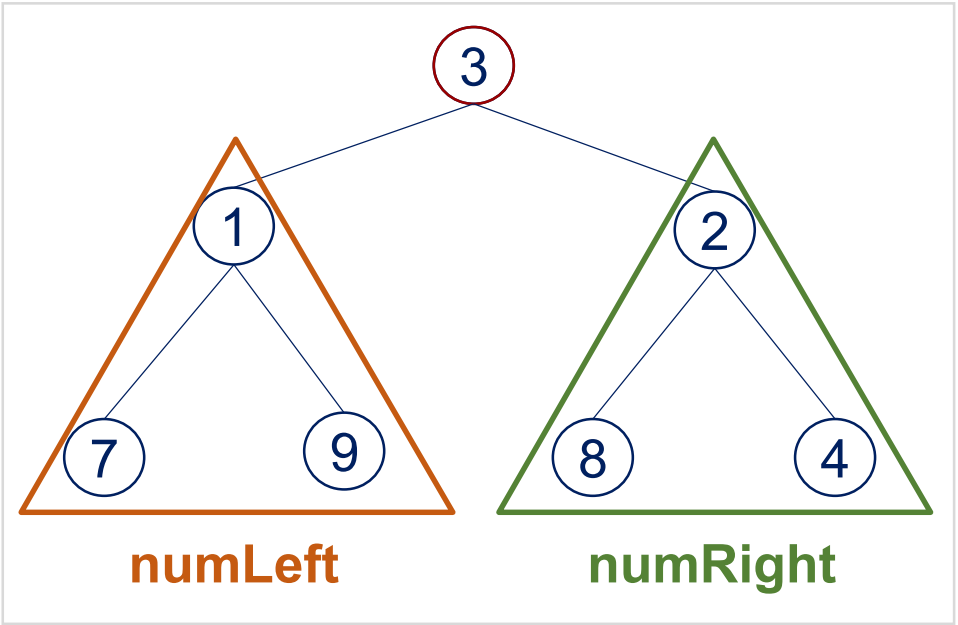
Operations in CBT (Cont'd)

`numOfNodes(t)`

Decompose the problem

- The **root node**
- The **left subtree**
- The **right subtree**

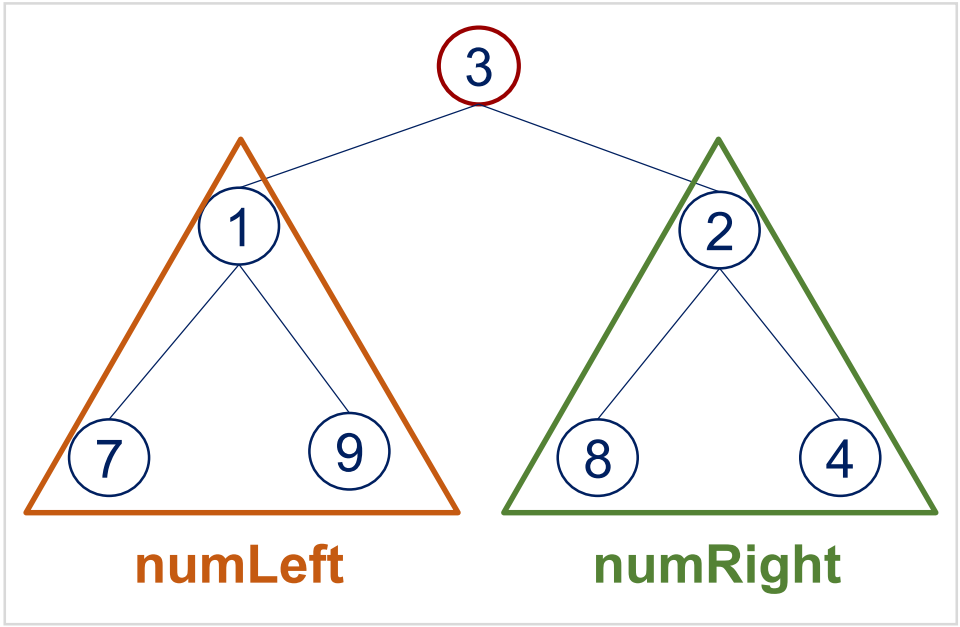
Result = **numLeft** + **1** + **numRight**



Operations in CBT (Cont'd)

numOfNodes(t)

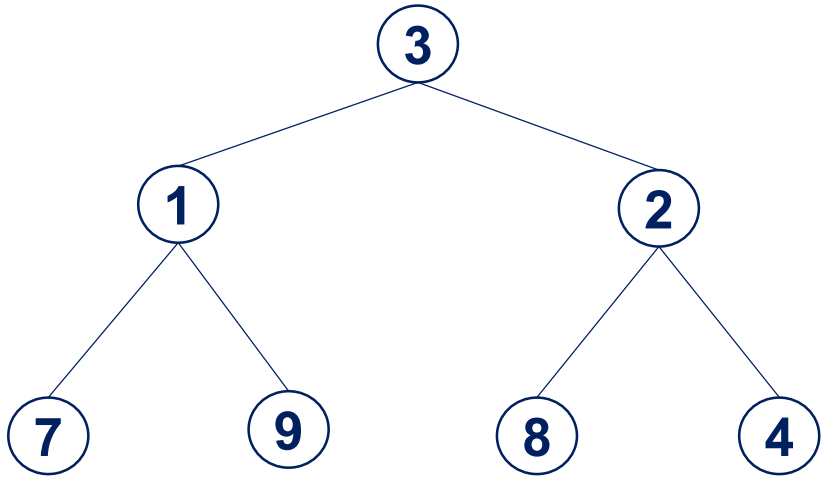
```
def numOfNodes(t):
    if len(t) == 1:
        return 1;

    else:
        numLeft = numOfNodes(t[0])

        numRight = numOfNodes(t[2])

        return ( numLeft + numRight + 1 )
```



Operations in CBT (Cont'd)

sumNodes(t)

```
tree = [[[7], 1, [9]], 3, [[8], 2, [4]]]

print("sum of Nodes: ", end='')
print( sumNodes(tree) )
```



sum of Nodes: 34

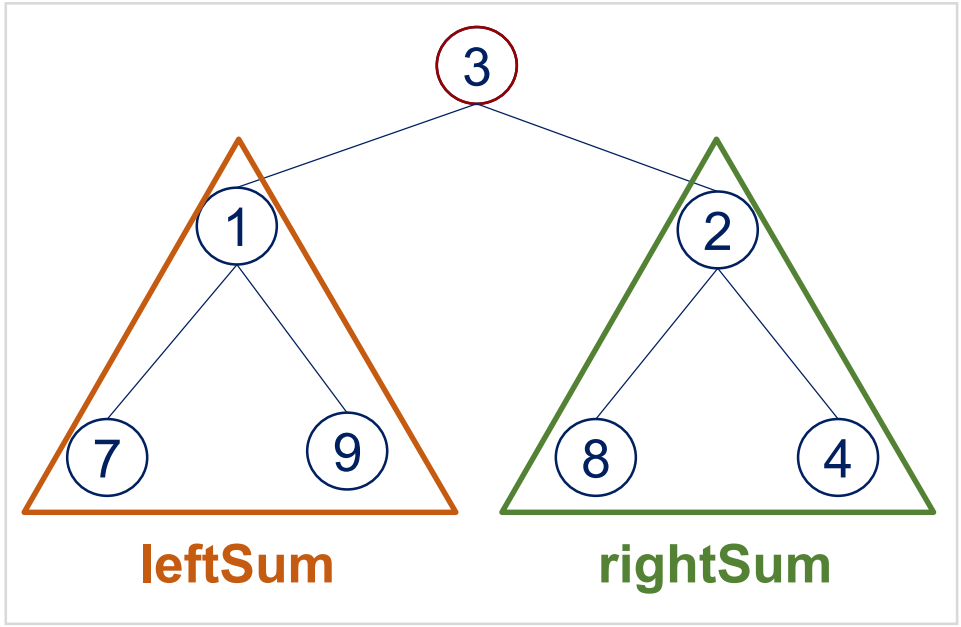
Operations in CBT (Cont'd)

`sumNodes(t)`

Decompose the problem

- The **root node**
- The **left subtree**
- The **right subtree**

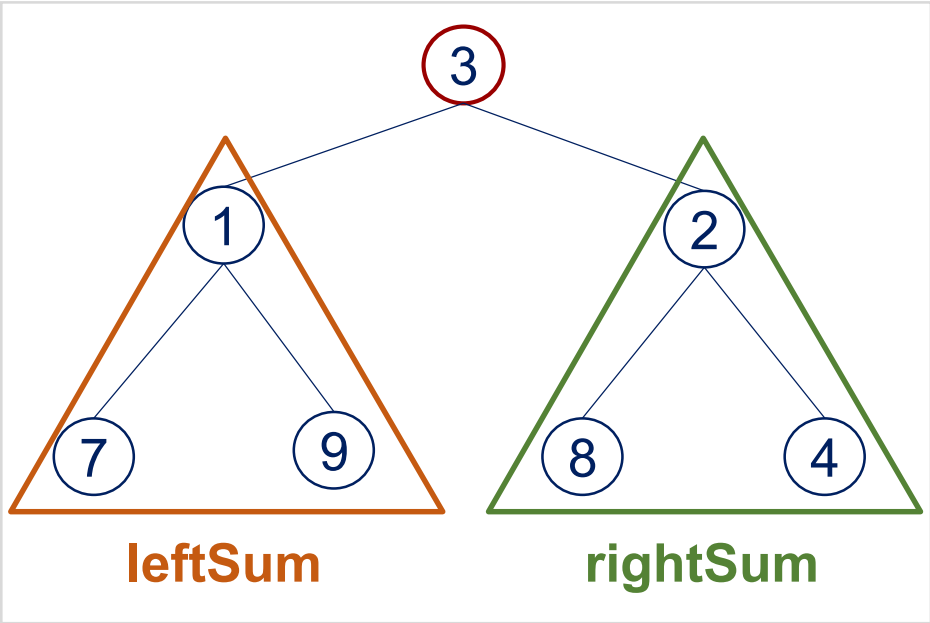
Result = leftSum + 3 + rightSum



Operations in CBT (Cont'd)

sumNodes(t)

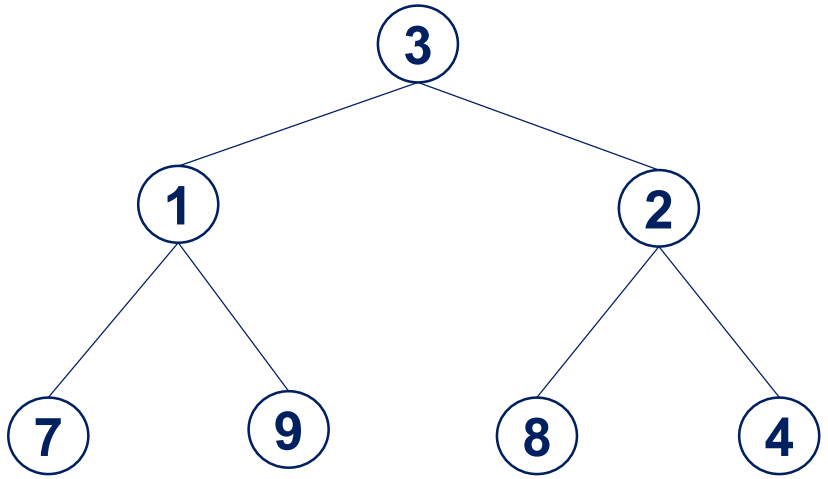
```
def sumNodes(t):
    if len(t) == 1:
        return t[0];

    else:
        leftSum = sumNodes(t[0])

        rightSum = sumNodes(t[2])

        return ( t[1] + leftSum + rightSum)
```



Operations in CBT (Cont'd)

maxNode(t)

```
tree = [[[7], 1, [9]], 3, [[8], 2, [4]]]

print("max of Nodes: ", end='')
print( maxNodes(tree) )
```



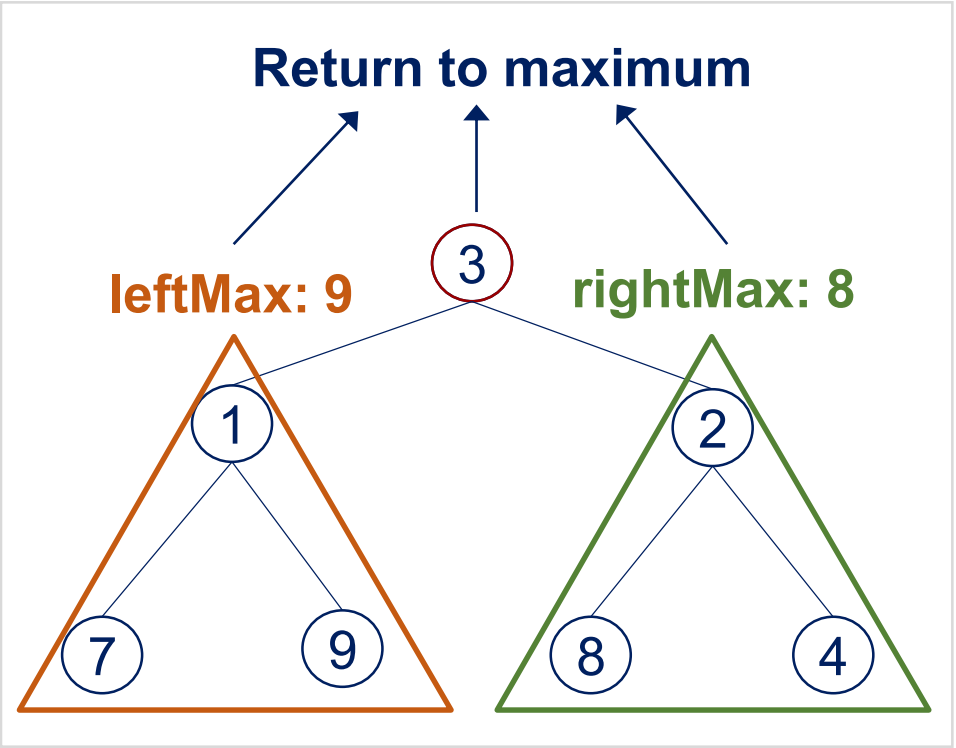
max of Nodes: 9

Operations in CBT (Cont'd)

`maxNode(t)`

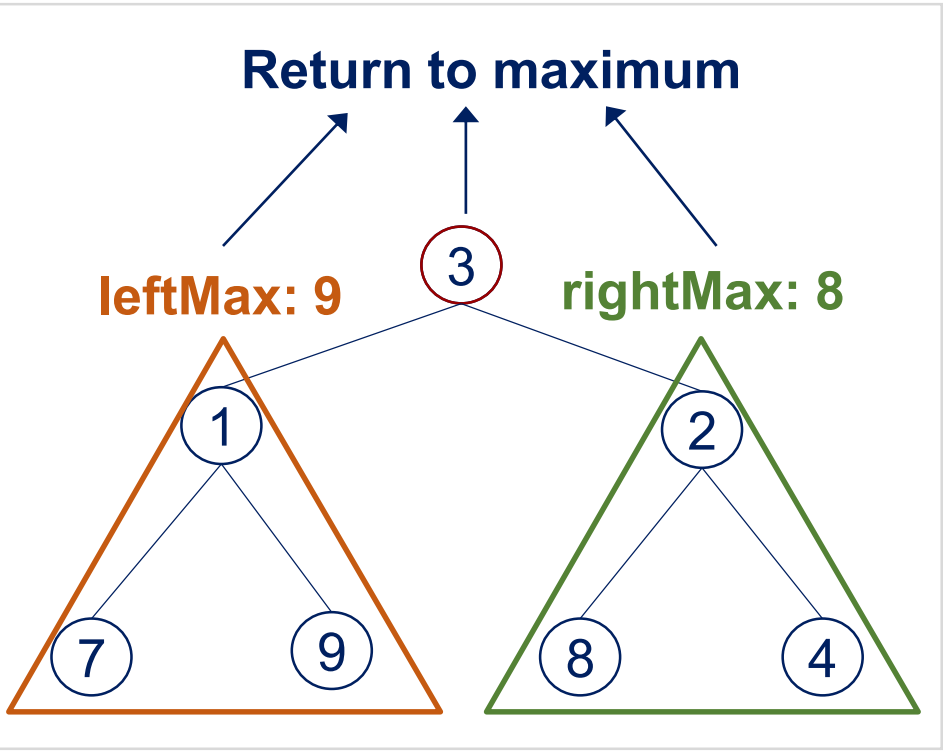
Decompose the problem

- The **root node**
- The **left subtree**
- The **right subtree**



Operations in CBT (Cont'd)

maxNode(t)

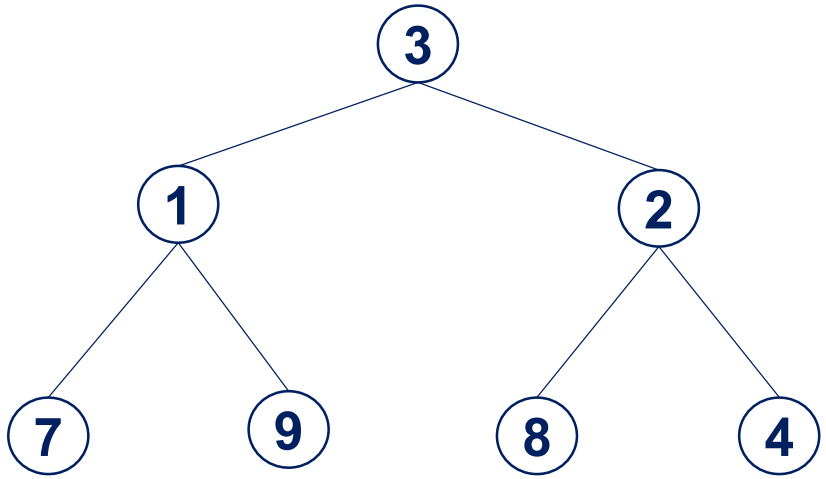



```
def maxNode(t):
    if len(t) == 1:
        return t[0]
    else:
        leftMax = maxNode(t[0])
        rightMax = maxNode(t[2])

        maxValue = t[1]
        if leftMax > maxValue:
            maxValue = leftMax

        if rightMax > maxValue:
            maxValue = rightMax

        return maxValue
```

Operations in CBT (Cont'd)

minNode(t)

```
tree = [[[7], 1, [9]], 3, [[8], 2, [4]]]

print("min of Nodes: ", end='')
print( minNodes(tree) )
```



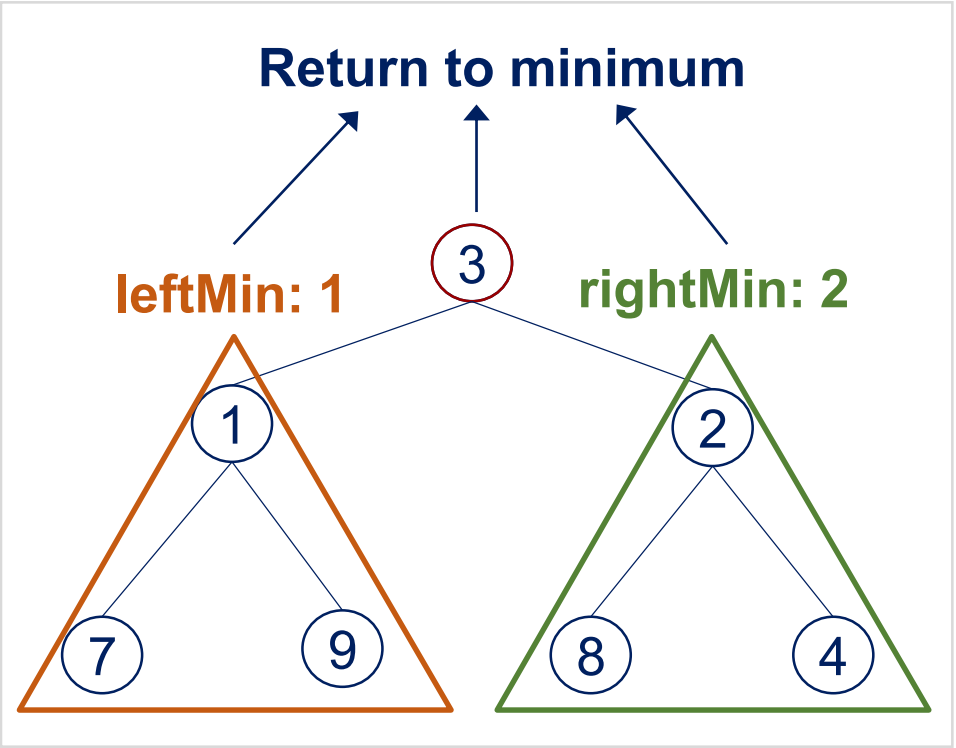
min of Nodes: 1

Operations in CBT (Cont'd)

`minNode(t)`

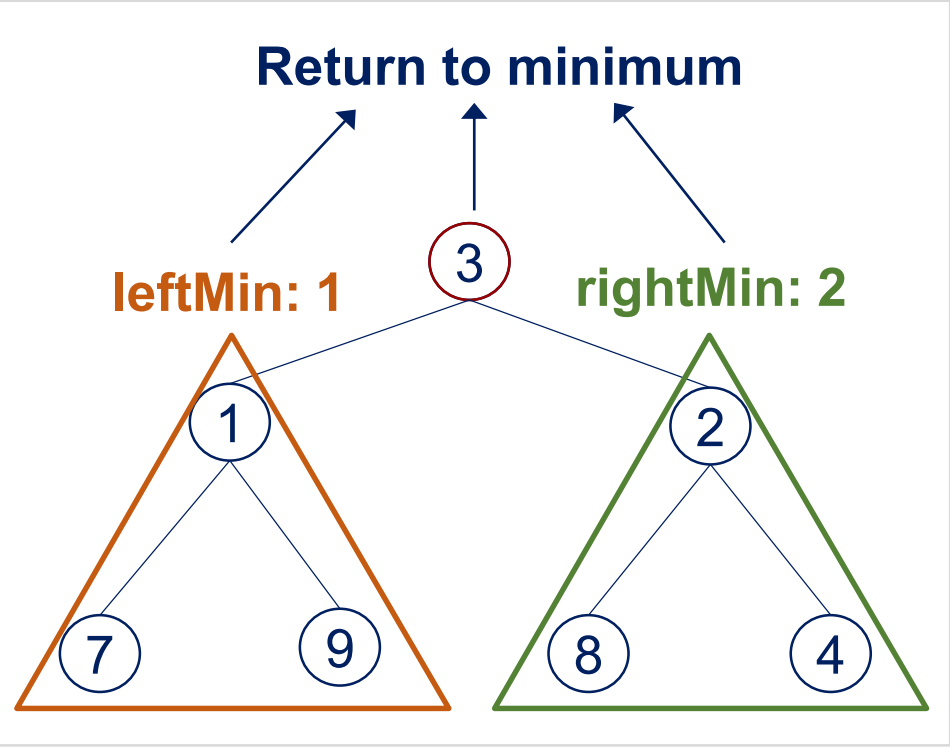
Decompose the problem

- The **root node**
- The **left subtree**
- The **right subtree**



Operations in CBT (Cont'd)

minNode(t)

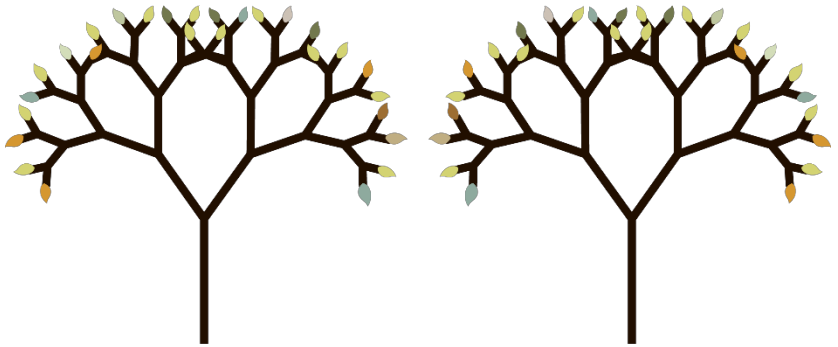


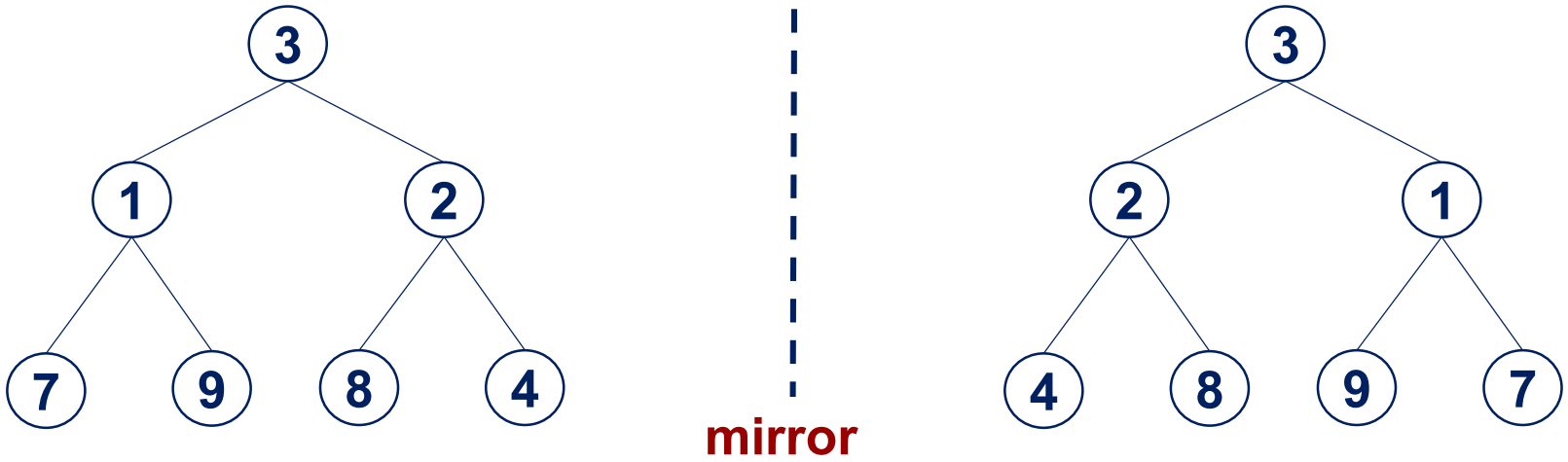

```
def maxNode(t):
    if len(t) == 1:
        return t[0]
    else:
        minValue = t[1]
        leftMin = minNode(t[0])
        rightMin = minNode(t[2])

        if leftMin < minValue:
            minValue = leftMin

        if rightMin < minValue:
            minValue = rightMin

        return minValue
```

Operations in CBT (Cont'd)

mirror(t)

```
tree = [[[7], 1, [9]], 3, [[8], 2, [4]]]

mirrortree = mirror(tree)
```

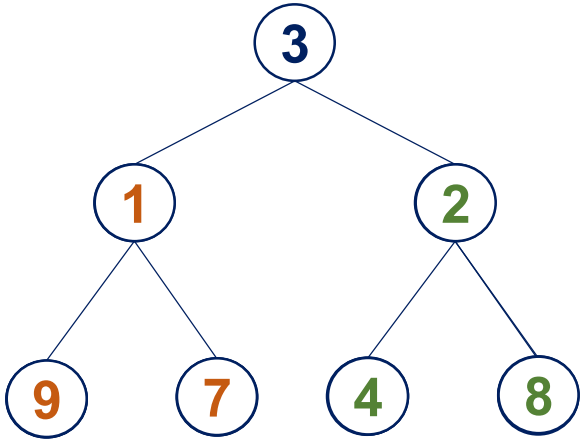


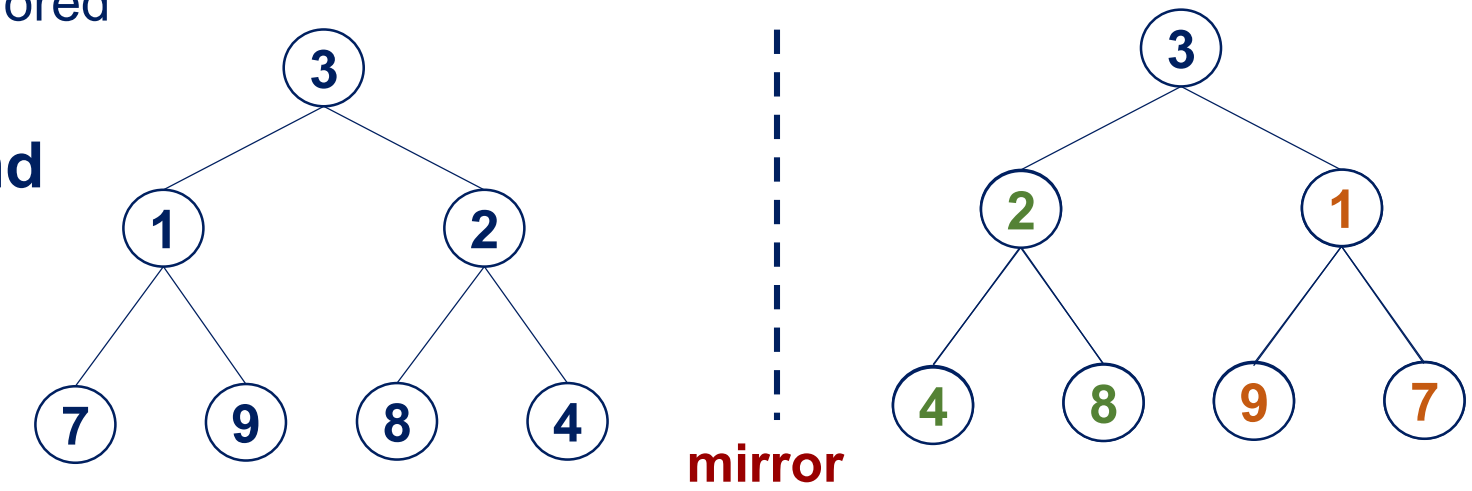


Operations in CBT (Cont'd)

`mirror(t)`

- **Decompose the problem:**
  - The **left subtree**
    - Make the left subtree mirrored
  - The **right subtree**
    - Make the right subtree mirrored
- **Switch the mirrored left and right subtree**

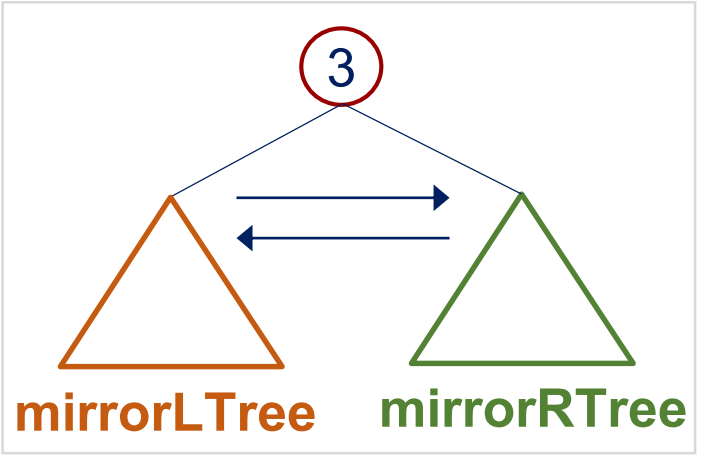




Operations in CBT (Cont'd)

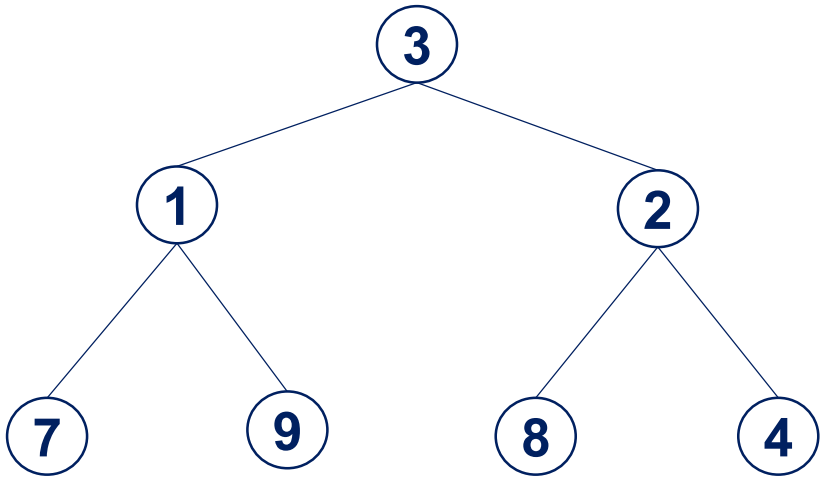
mirror(t)

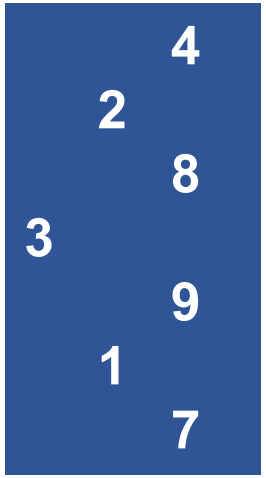
```
def mirror(t):
    if len(t) == 1:
        return t

    else:
        parent = t[1]
        mirrorLTree = mirror(t[0])
        mirrorRTree = mirror(t[2])

        return [ mirrorRTree, parent, mirrorLTree ]
```



Print Out a CBT

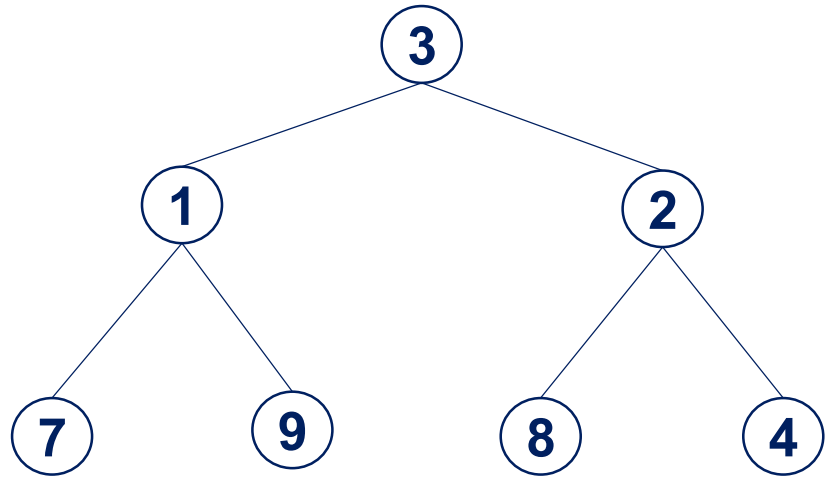
```
tree = [[[7], 1, [9]], 3, [[8], 2, [4]]]

printTree(tree, 0)
```



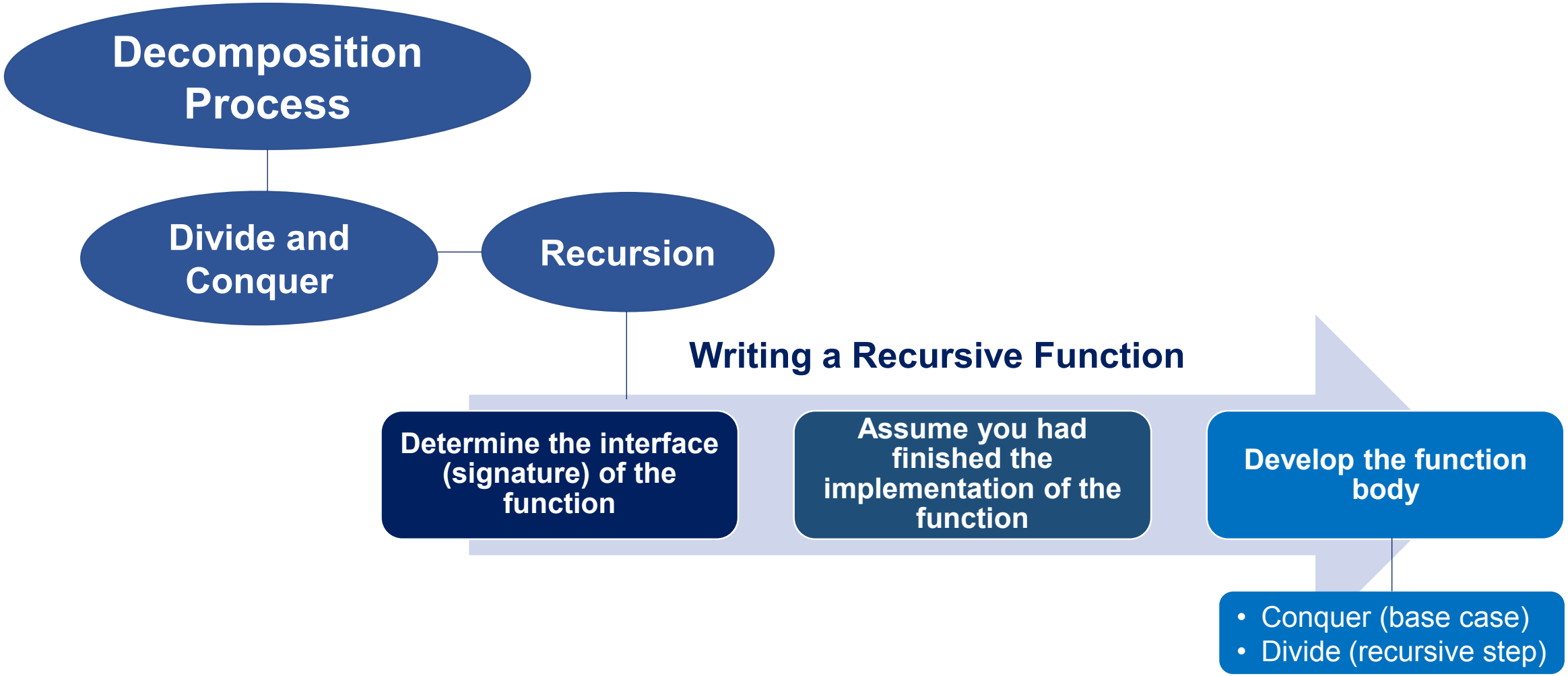


Print Out a CBT

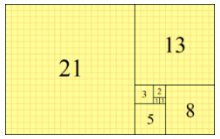
```
def printTree(t, level):
    if len(t) == 1:
        print("  " * level, end="")
        print(t[0])

    else:
        printTree(t[2], level + 1)

        print("  " * level, end="")
        print(t[1])

        printTree(t[0], level + 1)
```



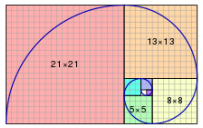
Summary


Decomposition Process
Divide and Conquer
Recursion
Writing a Recursive Function
Determine the interface (signature) of the function
Assume you had finished the implementation of the function
Develop the function body
• Conquer (base case)
• Divide (recursive step)

References for Image

No.	Slide No.	Image	Reference
1	5		Online Image. Retrieved April 24, 2018 from https://www.flickr.com/photos/epublicist/8718123610.
2	6		By Guillaume Jacquenot - Own work, CC BY-SA 3.0,retrieved April 24, 2018 from https://commons.wikimedia.org/w/index.php?curid=11678451.
3	7, 9, 21, 24, 27, 30, 33, 35		Python Logo [Online Image]. Retrieved April 24, 2018 from https://pixabay.com/en/language-logo-python-2024210/.
4	9, 10		Play Button [Online Image]. Retrieved April 24, 2018 from https://pixabay.com/en/play-button-round-blue-glossy-151523/.
5	11		By 克勞棣 - Own work, CC BY-SA 4.0,retrieved April 24, 2018 from https://commons.wikimedia.org/w/index.php?curid=38708516.

References for Images

No.	Slide No.	Image	Reference
6	11		By Jahobr - Own work, CC0,retrieved April 24, 2018 from https://commons.wikimedia.org/w/index.php?curid=58460223,
7	14		Question problem [Online Image]. Retrieved April 24, 2018 from https://pixabay.com/en/question-problem-think-thinking-622164/,