

Find volume of tetrahedron with vertices

a) $A_1(2, 3, 1), A_2(4, 1, -2), A_3(6, 3, 7), A_4(7, 5, 3)$

b) $A_1(-3, 4, -7), A_2(1, 5, 4), A_3(-5, -2, 0), A_4(2, 5, 4)$

Find volume of parallelepiped with one vertex at the origin and adjacent vertices

c) $\begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$

d) $\begin{pmatrix} -2 \\ 4 \\ 6 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \\ -8 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -5 \\ 4 \end{pmatrix}$

How volumes in a), b), c) and d) will change after transformation with matrix:

e) $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

f) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 0 & 0 \\ 3 & 1 & 0 \end{pmatrix}$

Answers:

a) $\frac{140}{6}$ b) $\frac{151}{6}$ c) 33 d) 74

a)+e) $\frac{140}{6}$ b)+e) $\frac{151}{6}$ c)+e) 33 d)+e) 74

a)+f) $\frac{280}{6}$ b)+f) $\frac{302}{6}$ c)+f) 66 d)+f) 148

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$$A = \begin{bmatrix} 2 & 4 & 6 & 7 \\ 3 & 1 & 3 & 5 \\ 1 & -2 & 7 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 5 \\ -2 & 0 & 2 \\ -3 & 6 & 2 \end{bmatrix} \xrightarrow{\substack{L_{12} \\ R_2 = R_2 + (R_1)(1)}} \begin{bmatrix} 2 & 4 & 5 \\ 0 & 4 & 7 \\ 0 & 12 & 5 \end{bmatrix}$$

tag one point to zero

$\frac{92}{6}$? what

$$\xrightarrow{\substack{L_{13} \\ R_3 = R_3 + R_1(\frac{3}{2}) \\ (-3)}} \begin{bmatrix} 2 & 4 & 5 \\ 0 & 4 & 7 \\ 0 & 0 & -11\frac{1}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} -5 & 1 & -3 & 2 \\ -2 & 5 & 4 & 5 \\ 0 & 4 & -7 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 7 \\ 7 & 6 & 7 \\ 4 & -7 & 4 \end{bmatrix} \quad \text{Det} = -73 \quad \dots \quad \frac{73}{6}$$

Find volume of parallelepiped with one vertex at the origin and adjacent vertices

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c) $\det = |c| = 33$

d) $\det = |d| = 74$

How volumes in a), b), c) and d) will change after transformation with matrix:

e) $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \rightarrow 1$ no transformation

f) $\begin{pmatrix} 1 & 0 & 2 \\ 1 & 0 & 0 \\ 3 & 1 & 0 \end{pmatrix} \rightarrow 2$ multiplied by 2