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Tutorial group: SC2

Matriculation number: U 2 1 2 3 3 2 1 H

18 September 2021 **MH1812 Test 1** 60 minutes

QUESTION 1. (30 marks)

Use mathematical induction to show that

$$5+4+3+\cdots+(6-n)=\frac{1}{2}n(11-n).$$

whenever n is a positive integer.

P(n) is true if n = 1

$(6-1) = \frac{1}{2}(11-1)$

$5 = 5$

P(n) is true if n = k

$(6-k) = \frac{1}{2}(k)(11-k)$ No....

P(n) is true if n = k+1

$(6-k) + (6-(k+1)) = \frac{1}{2}(k+1)(11-(k+1))$ No.....

$\frac{1}{2}(k)(11-k) + (6-k-1) = \frac{1}{2}(k+1)(11-k-1)$

~~10~~ 15

RHS

$= \frac{1}{2}(9k-k^2+10)$

$= \frac{9k}{2} - \frac{k^2}{2} + 5$

LHS

$\frac{1}{2}(11k-k^2)+(5-k)$

$= \frac{11k}{2} - \frac{k^2}{2} + 5 - k$

$= \frac{9k}{2} - \frac{k^2}{2} + 5$

$\therefore$ LHS = RHS

$\therefore$ Using mathematical Induction, we can show that $5+4+3+\ldots+(6-n) = \frac{1}{2}n(11-n)$ when n is a positive integer.

For graders only	Question	1	2(a)	2(b)	3(a)	3(b)	Bonus	Total
	Marks	~~10~~ 15	9	3	12	10		49

QUESTION 2. (30 marks)

(a) (10 points) Find $(18^{12}\cdot 20^{21})$ mod 60.

$(18^{12}\cdot 20^{21}) \bmod 60 \equiv [(18^{12} \bmod 60)\cdot(20^{21} \bmod 60)] \bmod 60$

$\equiv [(18^3 \bmod 60 \times 18^3 \bmod 60 \times 18^3 \bmod 60 \times 18^3 \bmod 60) \bmod 60 \times (20^5 \bmod 60 \times 20^5 \bmod 60 \times 20^5 \bmod 60 \times 20^5 \bmod 60 \times 20 \bmod 60) \bmod 60$

$\equiv [(12\times12\times12\times12) \bmod 60 \times (20\times20\times20\times20 \bmod 60 \times 20) \bmod 60] \bmod 60$

$\equiv (36\times 20) \bmod 60$

$(18^{12}\cdot 20^{21}) \bmod 60 = 12$?

9

(b) Let $\mathbb{R}$ denote the set of reals. For $x,y\in\mathbb{R}$, let $P(x,y)$ denote the predicate "$x\cdot(y^2-\frac{1}{1812})<0$". What are the truth values of these statements?

(i) (10 points) $\forall x\in\mathbb{R},\exists y\in\mathbb{R},P(x,y)$.

(ii) (10 points) $\exists x\in\mathbb{R},\forall y\in\mathbb{R},P(x,y)$.

(i) For all of x that is a real number there exists a y.

$x\cdot(y^2-\frac{1}{1812})<0$

$xy^2 - \frac{x}{1812} < 0$

$\frac{x}{1812} > \frac{1}{xy^2}$

$\frac{x^2}{1812} > \frac{1}{y^2}$

For any value of x, there exists a y

0

$\therefore \forall x \in \mathbb{R}, \exists y \in \mathbb{R}, P(x,y)$ is true

(ii) There exists a x that is real for all values of y.

$x > \frac{1}{y^2-\frac{1}{1812}}$

$\frac{x^2}{1812^2} + \frac{1}{1812} > y^2$

Sub in any value of y, x will continuously change hence

$\therefore \exists x \in \mathbb{R}, \forall y \in \mathbb{R}, P(x,y)$ is false

3

Use mathematical induction to show that

$$5 + 4 + 3 + \cdots + (6 - n) = \frac{1}{2}n(11 - n).$$ $\frac{1}{2}(n)(11-n) + 5 - n$

whenever n is a positive integer.

let $P(n) = \frac{1}{2}(n)(11-n)$

Base step: $P(1)$: $5 = \frac{1}{2}(1)(11-1)$

$5 = 5$ //

inductive step : let $P(n)$ be true

finding $P(n+1)$ $= \frac{1}{2}(n+1)(11-n-1)$

$= \frac{1}{2}\left[(n+1)(n-10)(-1)\right]$

$= \frac{1}{2}\left[-n^2 + 10n - n + 10\right]$

$= \frac{1}{2}\left[-n^2 + 9n + 10\right]$

$= \frac{1}{2}\left[-n^2 + 11n + 10 - 2n\right]$

$= \frac{1}{2}\left[n(11-n)\right] + 5 - n$ //

(a) (10 points) Find $(18^{12} \cdot 20^{21}) \bmod 60$.

$(18^{12} \cdot 20^{21}) \bmod 60 = (18^{12} \cdot 20^9 \cdot 20^9 \cdot 20^3) \bmod 60$

$= (18^{12} \cdot 20^3) \bmod 60$

$= (18^9 \cdot 18^3 \cdot 20^3) \bmod 60$

$= (18^3 \cdot 20^3) \bmod 60$

$= (360^3) \bmod 60$

$= 0^3 \bmod 60$

$= 0$

(b) Let $\mathbb{R}$ denote the set of reals. For $x, y \in \mathbb{R}$, let $P(x, y)$ denote the predicate "$x \cdot (y^2 - \frac{1}{1812}) < 0$". What are the truth values of these statements?

(i) (10 points) $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, P(x, y)$.

(ii) (10 points) $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, P(x, y)$. that

i) False, as $P(0, y)$ is F for all y

iii) False, as $P(x, \sqrt{\frac{1}{1812}})$ is false for all x

QUESTION 3. **(40 marks)**

(a) (20 points) Determine whether $p \to (q \to (p \wedge q))$ is a tautology.

$$p \to ((q \to (p \wedge q)))$$

$$\equiv \neg p \vee ((q \to (p \wedge q)))$$ conversion

$$\equiv \neg p \vee ((\neg q \vee (p \wedge q)))$$ conversion

$$\equiv (\neg p \vee \neg q) \vee (p \wedge q)$$ Associative $\equiv \neg(p \wedge q) \vee (p \wedge q)$

$$\equiv (F) \vee (T)$$ Tautology $\equiv T$

$$\equiv \text{True}$$

$\therefore$ $p \to (q \to (p \wedge q))$ is a tautology

(b) (20 points) Determine whether the following argument is valid[1].

$$(q \wedge r) \to \neg p;$$
$$q \vee \neg r;$$
$$\neg r \to q;$$
$$\therefore \neg p.$$

$\neg(\neg r) \vee q$ $\quad$ $\neg(q \wedge r) \vee \neg p$

p	q	r	$\neg r$	premise $(q \vee \neg r)$	$(q \wedge r)$	premise $\neg r \to q$	premise $(q \wedge r) \to \neg p$	$\neg p$
F	F	F	T	T	F	F	T	T
F	F	T	F	F	F	T	T	T
F	T	F	T	T	F	T	T	T
F	T	T	F	T	T	T	T	T
T	F	F	T	T	F	F	T	F
T	F	T	F	F	F	T	T	F
T	T	F	T	T	F	T	T	F
T	T	T	F	T	T	T	F	F

Truth table looks ok.

10

$\therefore \neg p$ is valid when all the premises in the critical row are true, $\neg p$ is also true.

[1]The inference rules you may use are: modus ponens, modus tollens, conjunctive simplification, conjunctive addition, disjunctive addition, disjunctive syllogism, hypothetical syllogism, disjunction elimination and rule of contradiction.

BONUS QUESTION. **(10 marks)**

[Points will be given to *fully* correct solutions only. The total mark of this test is capped at 100 marks.]

Prove that $(5^{69} - 1)/4$ is odd and composite. (Recall that a positive integer which can be factored into smaller positive integers, neither of which is one, is a composite.)

(a) (20 points) Determine whether $p \to (q \to (p \wedge q))$ is a tautology.

$$p \to (q \to (p \wedge q))$$

$$\equiv \neg p \vee (q \to (p \wedge q))$$

(b) (20 points) Determine whether the following argument is valid[1].

$$(q \wedge r) \to \neg p;$$
$$q \vee \neg r;$$
$$\neg r \to q;$$
$$\therefore \neg p.$$

(a) (20 points) Determine whether $p \to (q \to (p \wedge q))$ is a tautology.

$$p \Rightarrow (q \Rightarrow (p \wedge q)) = p \Rightarrow (\neg q \vee (p \wedge q))$$

$$= \neg p \vee (\neg q \vee (p \wedge q))$$

$$= \neg p \vee ((p \vee \neg q) \wedge (\neg q \vee q))$$

$$= \neg p \vee ((p \vee \neg q) \wedge T)$$

$$= \neg p \vee (p \wedge T) \vee (\neg q \wedge T)$$

$$= \neg p \vee p \vee \neg q$$

$$= T$$

(b) (20 points) Determine whether the following argument is valid[1].

$$(q \wedge r) \to \neg p;$$
$$q \vee \neg r;$$
$$\neg r \to q;$$
$$\therefore \neg p.$$

$\neg r \Rightarrow q$

$r \vee q$

$\neg r \vee q$

q is T

$\neg (q \wedge r) \vee \neg p$

$(q \vee r) \vee \neg p$

$\therefore$ counter arg $\neg p$ is F, q is T, r is F