① Which set is a basis for $\mathbb{R}^4$?

A) $\begin{pmatrix}1\\2\\3\\4\end{pmatrix}, \begin{pmatrix}1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\\1\end{pmatrix}$ B) $\begin{pmatrix}1\\2\\0\\0\end{pmatrix}, \begin{pmatrix}2\\0\\1\\0\end{pmatrix}, \begin{pmatrix}4\\3\\2\\1\end{pmatrix}, \begin{pmatrix}1\\2\\10\\7\end{pmatrix}, \begin{pmatrix}8\\9\\10\\11\end{pmatrix}$

C) $\begin{pmatrix}1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\\0\end{pmatrix}$ D) $\begin{pmatrix}4\\0\\0\\0\end{pmatrix}, \begin{pmatrix}2\\1\\0\\0\end{pmatrix}, \begin{pmatrix}3\\1\\1\\0\end{pmatrix}, \begin{pmatrix}7\\5\\2\\1\end{pmatrix}$

Which set is a basis for space of matrices 2×2?

② A) $\begin{pmatrix}2&3\\1&0\end{pmatrix}, \begin{pmatrix}2&0\\0&1\end{pmatrix}, \begin{pmatrix}2&0\\0&0\end{pmatrix}$ B) $\begin{pmatrix}2&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&1\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}$

C) $\begin{pmatrix}2&0\\2&0\end{pmatrix}, \begin{pmatrix}1&0\\1&0\end{pmatrix}, \begin{pmatrix}0&3\\0&3\end{pmatrix}, \begin{pmatrix}0&1\\0&1\end{pmatrix}$ D) none of above

③ A) $1, 1+t, t^2$ is a basis for $\mathbb{P}_2[t]$

B) $1+t, t^2, t^3$ is a basis for $\mathbb{P}_3[t]$

C) $t+t^2, t^2, t^3$ is a basis for space of polynomials of degree 3 with root 0.

D) $\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}$ is a basis for space of all matrices 2×2 with $Tr=0$.

④ Which pair is Null and Column Spaces for matrix

$A=\begin{pmatrix}1&0&2&0\\0&-1&1&0\\0&0&3&6\end{pmatrix}$.

B) $N(A)=\left\langle\begin{pmatrix}1\\0\\1\end{pmatrix}\right\rangle$, $C(A)=\mathbb{R}^3$

C) $N(A)=\left\langle\begin{pmatrix}4\\2\\-2\\2\end{pmatrix}, \begin{pmatrix}8\\4\\-4\\4\end{pmatrix}\right\rangle$, $C(A)=\mathbb{R}^3$

D) $N(A)=\left\langle\begin{pmatrix}4\\2\\-2\\2\end{pmatrix}\right\rangle$; $C(A)=\mathbb{R}^3$

① Which set is a basis for R^4?

A) $\begin{pmatrix}1\\2\\3\\4\end{pmatrix}, \begin{pmatrix}1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\\1\end{pmatrix}$

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C) $\begin{pmatrix}1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\\0\end{pmatrix}$

D) $\begin{pmatrix}4\\0\\0\\0\end{pmatrix}, \begin{pmatrix}2\\1\\0\\0\end{pmatrix}, \begin{pmatrix}3\\1\\1\\0\end{pmatrix}, \begin{pmatrix}7\\5\\2\\1\end{pmatrix}$

Which set is a basis for space of matrices 2×2?

② A) $\begin{pmatrix}2&3\\1&0\end{pmatrix}, \begin{pmatrix}2&0\\0&1\end{pmatrix}, \begin{pmatrix}2&0\\0&0\end{pmatrix}$

B) $\begin{pmatrix}2&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&1\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}$

C) $\begin{pmatrix}2&0\\2&0\end{pmatrix}, \begin{pmatrix}1&0\\1&0\end{pmatrix}, \begin{pmatrix}0&3\\0&3\end{pmatrix}, \begin{pmatrix}0&1\\0&1\end{pmatrix}$

D) none of above

③ A) $1, 1+t, t^2$ is a basis for $\mathbb{P}_2[t]$ **T**

B) $1+t, t^2, t^3$ is a basis for $\mathbb{P}_3[t]$ **F, cant be zero**

C) $t+t^2, t^2, t^3$ is a basis for space of polynomials of degree 3 with root 0. **F, $\underline{\underline{c}} + x t^1$**

D) $\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}$ is a basis for space of all matrices 2×2 with $tr = 0$. $\rightarrow \begin{bmatrix}a&b\\c&-a\end{bmatrix}$ $a-a=0$ ∴ only 3 element.

∴ Valid basis

④ Which pair is Null and Column Spaces for matrix $A = \begin{pmatrix}1&0&2&0\\0&-1&1&0\\0&0&3&6\end{pmatrix}$.

~~B)~~ $N(A) = \left\langle \begin{pmatrix}1\\0\\1\end{pmatrix} \right\rangle$, $C(A) = \mathbb{R}^3$

~~C)~~ $N(A) = \left\langle \begin{pmatrix}4\\2\\-2\\2\end{pmatrix}, \begin{pmatrix}8\\4\\-4\\4\end{pmatrix} \right\rangle$, $C(A) = \mathbb{R}^3$

D) $N(A) = \left\langle \begin{pmatrix}4\\2\\-2\\2\end{pmatrix} \right\rangle$; $C(A) = \mathbb{R}^3$

① Which set is a basis for R^4?

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C) $\begin{pmatrix}1\\1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\2\\0\end{pmatrix}$ D) $\begin{pmatrix}4\\0\\0\\0\end{pmatrix}, \begin{pmatrix}2\\1\\0\\0\end{pmatrix}, \begin{pmatrix}3\\1\\1\\0\end{pmatrix}, \begin{pmatrix}7\\5\\2\\1\end{pmatrix}$

$A \to 3$ elements $\Rightarrow R^3$ ✗

$B \to 5$ elements $\Rightarrow R^5$ ✗

$C \to$ Row of zero $\Rightarrow$ linear dependent $\Rightarrow$ not basis ✗

basis are linear independent and spans whole space

$\therefore$ D

Which set is a basis for space of matrices 2×2?

② A) $\begin{pmatrix}2&3\\1&0\end{pmatrix}, \begin{pmatrix}2&0\\0&1\end{pmatrix}, \begin{pmatrix}2&0\\0&0\end{pmatrix}$ B) $\begin{pmatrix}2&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&1\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}$

C) $\begin{pmatrix}2&0\\2&0\end{pmatrix}, \begin{pmatrix}1&0\\1&0\end{pmatrix}, \begin{pmatrix}0&3\\0&3\end{pmatrix}, \begin{pmatrix}0&1\\0&1\end{pmatrix}$ D) none of above

2×2 matrices $\Rightarrow R^4$ $\begin{pmatrix}a&b\\c&d\end{pmatrix}$

$A \to$ Only 3 elements $\Rightarrow R^3$ ✗

$B \to$ linear independent did not explain why

$C \to \begin{pmatrix}2&0\\2&0\end{pmatrix}, \begin{pmatrix}1&0\\1&0\end{pmatrix}, \begin{pmatrix}0&3\\0&3\end{pmatrix}, \begin{pmatrix}0&1\\0&1\end{pmatrix}$ (×2, ×3)

$\Rightarrow$ linear dependent ?? ✗

$\therefore \boxed{B}$

③ A) $1, 1+t, t^2$ is a basis for $P_2[t]$

B) $1+t, t^2, t^3$ is a basis for $P_3[t]$

C) $t+t^2, t^2, t^3$ is a basis for space of polynomials of degree 3 with root 0.

D) $\begin{pmatrix}1&0\\0&-1\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}$ is a basis for space of all matrices 2×2 with $Tr=0$.

A) $P_2[t] = c + at + bt^2$

$= c + a(1+t) + bt^2$

$= c - a + at + bt^2$

$\therefore$ A is true

B) $P_3[t] = c + at + bt^2 + dt^3$

No c $\therefore$ B is false

C) C is also basis ??

D) $T_r = 0 \Rightarrow \begin{pmatrix}a&b\\c&-a\end{pmatrix} \Rightarrow R^3$??

$\therefore$ D is true

④ Which pair is Null and Column Spaces for matrix

$A = \begin{pmatrix}1&0&2&0\\0&-1&1&0\\0&0&3&6\end{pmatrix}$.

B) $N(A) = \langle \begin{pmatrix}1\\0\\1\end{pmatrix} \rangle$, $C(A) = \mathbb{R}^3$

C) $N(A) = \langle \begin{pmatrix}4\\2\\-2\\2\end{pmatrix}, \begin{pmatrix}8\\4\\-4\\4\end{pmatrix} \rangle$, $C(A) = \mathbb{R}^3$

D) $N(A) = \langle \begin{pmatrix}4\\2\\-2\\2\end{pmatrix} \rangle$, $C(A) = \mathbb{R}^3$

②

B) $N(A) = R^3 \neq R^4 \therefore$ wrong

$C(A) = R^3 \Rightarrow 3$ rows

$N(A) = R^4 \Rightarrow 4$ elements 4 columns