MH1812 Discrete Mathematics: Quiz (CA) 2

Name: Tutorial Group:

NTU Email:

There are 3 (THREE) questions, please try all of them, and justify all your answers! Best of luck!

Question 1 (35 points)

a) Let A, B, C be sets. Prove or disprove the following set equality (20 points):

$$A \times (B - C) = (A \times B) - (A \times C).$$

b) Prove the following set equality (15 points):

$$\{12a + 25b, a, b \in \mathbb{Z}\} = \mathbb{Z}.$$

Question 2 (40 points)

a) Prove by mathematical induction that

$$1^2 + 2^2 + \ldots + n^2 = \frac{1}{6}n(n+1)(2n+1).$$

b) How many subsets of $\{1, \ldots, n\}$ are there with an even number of elements? Justify your answer.

Question 3 (25 points)

Solve the following linear recurrence relation:

$$b_n = 4b_{n-1} - b_{n-2},\ b_0 = 2,\ b_1 = 4.$$