NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I (CA1)

MH1812 – Discrete Mathematics

February 2017 TIME ALLOWED: 40 minutes

Name:

Matric. no.: Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1. **(40 marks)**

(a) Which integer $a \in \{0, 1, 2\}$ is congruent to $2017 + 2020 + 2023$ modulo 3? (10 marks)

(b) Write down each integer $a \in \{0, 1, 2, 3\}$ for which there exists an integer n such that $a \equiv n^2 \pmod 4$? (10 marks)

(c) Decide whether the set S is closed under the operation Δ when

- $S = \{\text{even integers}\}$ and Δ is subtraction. (10 marks)
- $S = \{\text{irrational numbers}\}$ and Δ is multiplication. (10 marks)

Briefly justify your answers.

a) $0 \bmod 3$, 0

b) $a \bmod 4$.

a	a^2
0	0
1	1
2	0
3	1

c) $\sqrt{10} \times \sqrt{10} = 10$

QUESTION 2. **(40 marks)**

(a) Prove or disprove the following statement (20 marks):

$$(p \wedge \neg q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow q.$$

(b) Decide whether or not the following argument is valid (20 marks):

$$p \rightarrow q;$$
$$\neg p \rightarrow r;$$
$$\neg(r \wedge q);$$
$$\therefore \neg p$$

Briefly justify your answer.

a) $\neg(p \wedge \neg r) \vee q \equiv \neg p \vee r \vee q$ (T)

$\neg(p \wedge \neg q) \vee r \equiv \neg p \vee q \vee r$

b) $\neg(r \wedge q)$

$\neg r \vee \neg q$

$\neg r$ is T or/and $\neg q$ is T

r is F q is F

$\neg p$ is F p is F

$\neg p$ is T

Counter: $\neg p$ is F when q is T, r is F

QUESTION 3. **(20 marks)**

Consider the domains $X = \mathbb{Z} = \{\text{integers}\}$ and $Y = \{0, 1, 2\}$, and the predicate $P(x, y) =$ "3 divides $x - y$". Determine the truth values of the following statements:

(a) $\forall x \in X,\ \exists y \in Y,\ P(x, y)$; (10 marks)

(b) $\neg\,(\forall y \in Y,\ \exists x \in X,\ \neg P(x, y))$. (10 marks)

Briefly justify your answers.

a) True, as for $P(x, (x+3))$ is T

b) $\neg\,(\forall y \in Y,\ \exists x \in X,\ \neg P(x,y)) = \exists y \in Y,\ \forall x \in X,\ P(x,y)$

False as for any y, $P(y,y)$ is F