# Introduction to Data Science and Artificial Intelligence

## Solving Problems by Search: Uninformed Search and Informed Search

Assoc Prof Bo AN

*Research area*: artificial intelligence, computational game theory, optimization
www.ntu.edu.sg/home/boan
*Email*: boan@ntu.edu.sg
*Office*: N4-02b-55

## Lesson Outline

- Uninformed search strategies
- Informed search strategies
    - Greedy search
    - A * search

# Review: Well-Defined Formulation

| | |
|---|---|
| Definition of a problem | The information used by an agent to decide what to do |
| Specification | • Initial state<br>• Action set, i.e. available actions (successor functions)<br>• State space, i.e. states reachable from the initial state<br>  • Solution path: sequence of actions from one state to another<br>• Goal test predicate<br>  • Single state, enumerated list of states, abstract properties<br>• Cost function<br>  • Path cost *g(n),* sum of all (action) step costs along the path |
| Solution | A path (a sequence of operators leading) from the Initial-State to a state that satisfies the Goal-Test |

# Search Algorithms

- Exploration of state space by generating successors of already-explored states
  - Frontier: candidate nodes for expansion
  - Explored set

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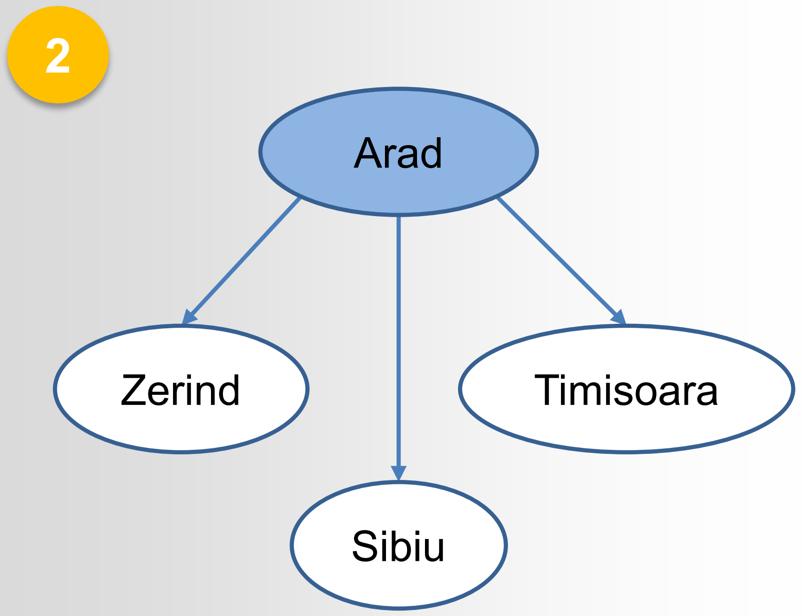

# Search Algorithms

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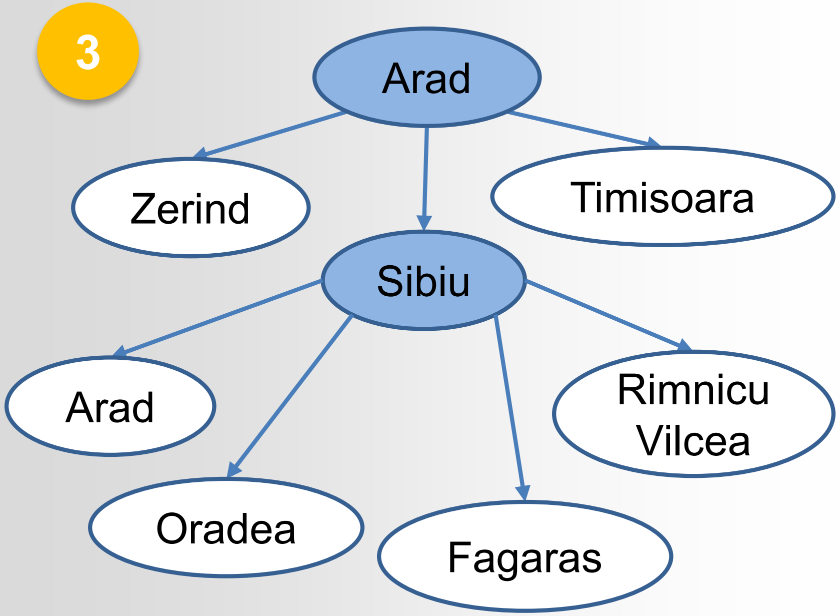

# Search Algorithms

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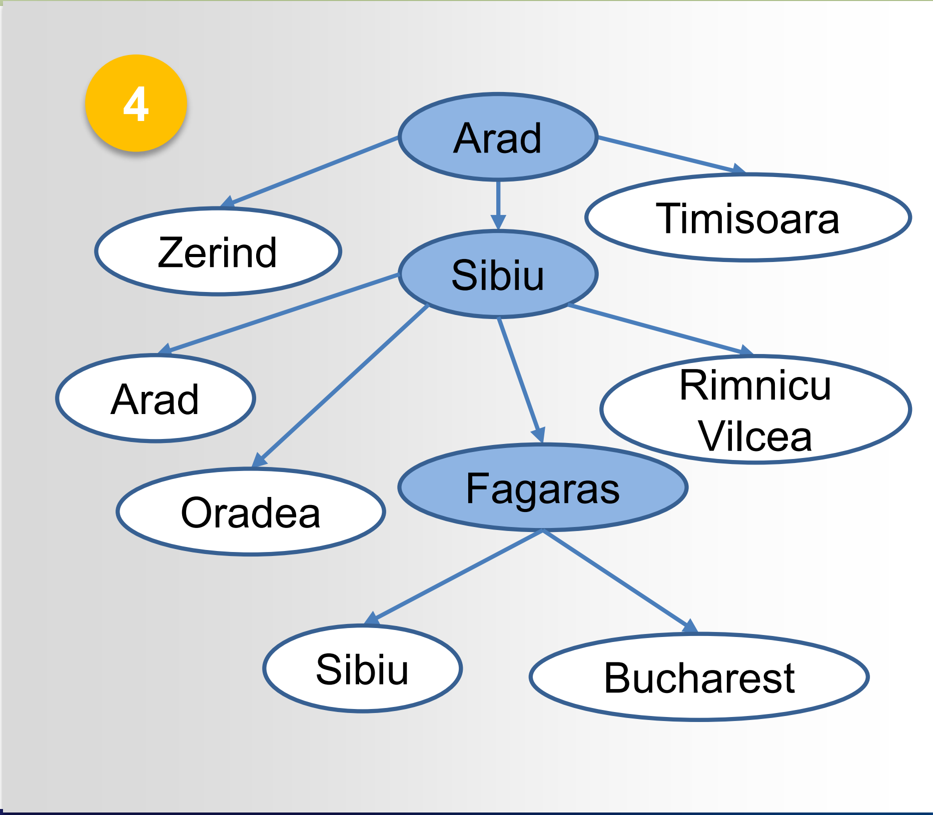


## Search Strategies

- A strategy is defined by picking the order of node expansion.
- Strategies are evaluated along the following dimensions:

| | |
|---|---|
| Completeness | Does it always find a solution if one exists? |
| Time Complexity | How long does it take to find a solution: the number of nodes generated |
| Space Complexity | Maximum number of nodes in memory |
| Optimality | Does it always find the best (least-cost) solution? |

## Search Strategies

- Branching factor
  - Maximum number of successors of any node
  - Or average branching factor

# Uninformed vs Informed

## Uninformed search strategies

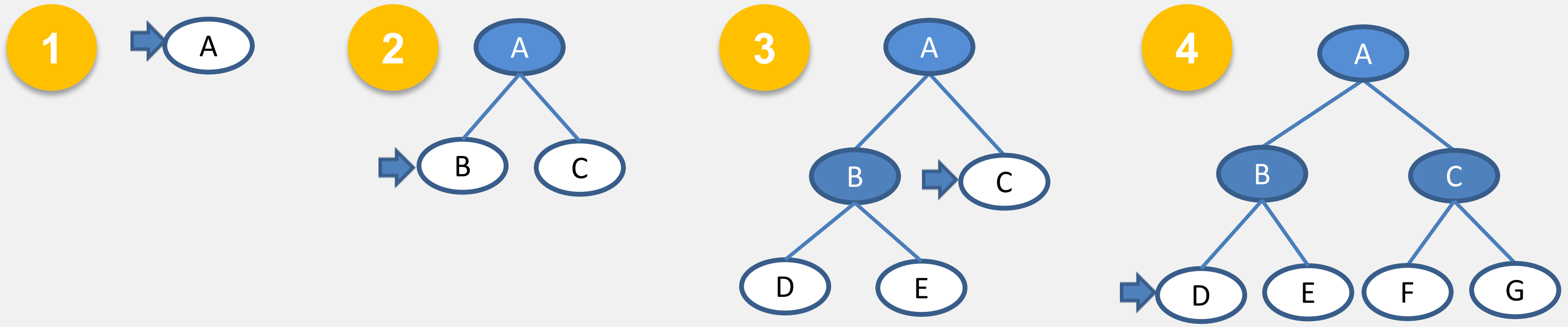
- Use only the information available in the problem definition
  1. Breadth-first search
  2. Uniform-cost search
  3. Depth-first search
  4. Depth-limited search
  5. Iterative deepening search

## Informed search strategies

- Use problem-specific knowledge to guide the search
- Usually more efficient

# Breadth-First Search

Expand shallowest unexpanded node which can be implemented by a First-In-First-Out (FIFO) queue



Denote

- b: maximum branching factor of the search tree
- d: depth of the least-cost solution
- Complete: Yes
- Optimal: Yes when all step costs equally

## Complexity of BFS

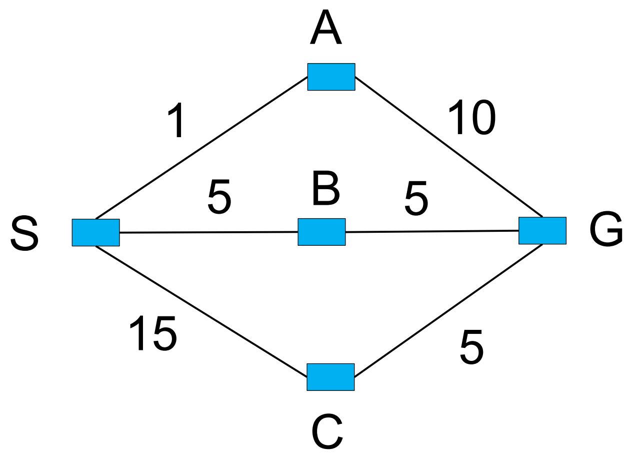
- Hypothetical state-space, where every node can be expanded into *b* new nodes, solution of path-length *d*
- Time: $1 + b + b^2$+$b^3$+ ... + $b^d$ = $O(b^d)$
- Space: (keeps every node in memory) $O(b^d)$are equal

| Depth | Nodes | Time | | Memory | |
|---:|---:|---:|---:|---:|---:|
| 0 | 1 | 1 | millisecond | 100 | bytes |
| 2 | 111 | 0.1 | seconds | 11 | kilobytes |
| 4 | 11111 | 11 | seconds | 1 | kilobytes |
| 6 | $10^6$ | 18 | minutes | 111 | megabyte |
| 8 | $10^8$ | 31 | hours | 11 | gigabytes |
| 10 | $10^{10}$ | 128 | days | 1 | terabyte |
| 12 | $10^{12}$ | 35 | years | 111 | terabytes |
| 14 | $10^{14}$ | 3500 | years | 11111 | terabytes |

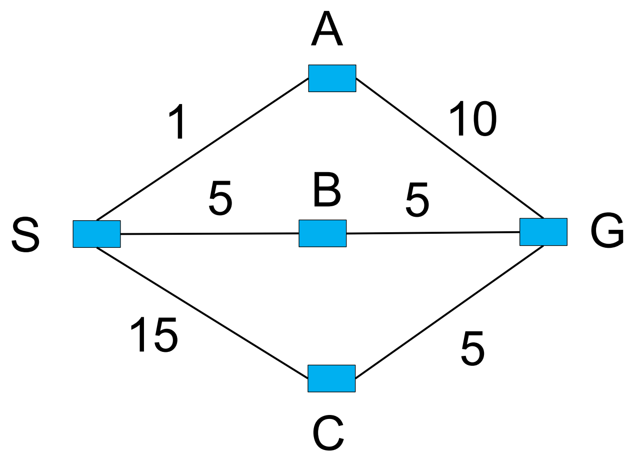
# Uniform-Cost Search

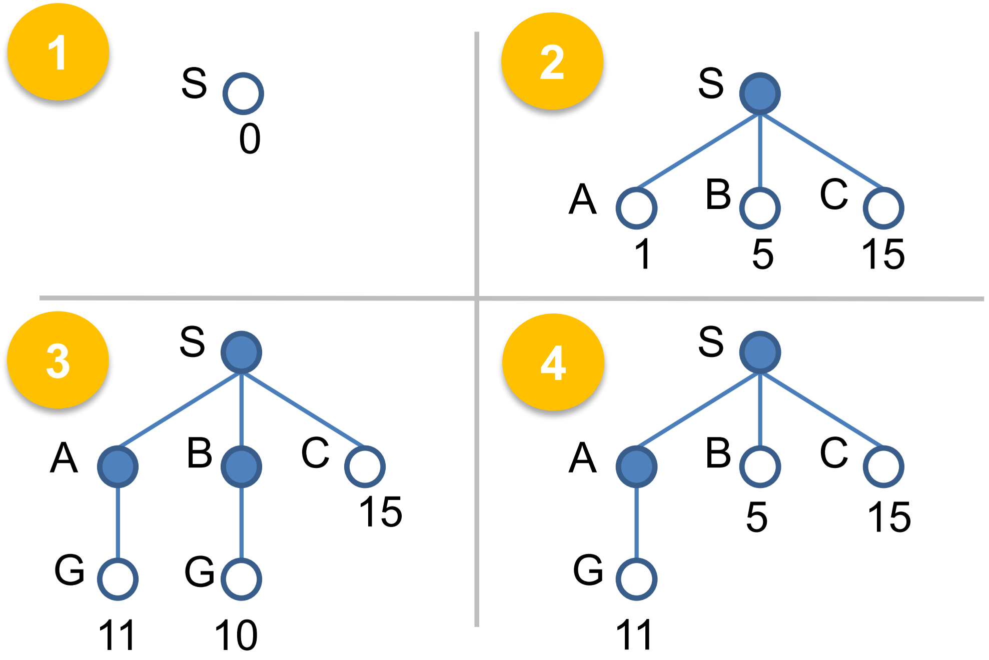
To consider edge costs, expand unexpanded node with the least path cost *g*

- Modification of breath-first search
- Instead of First-In-First-Out (FIFO) queue, using a priority queue with path cost *g(n)* to order the elements
- BFS = UCS with *g(n)* = Depth(n)



# Uniform-Cost Search





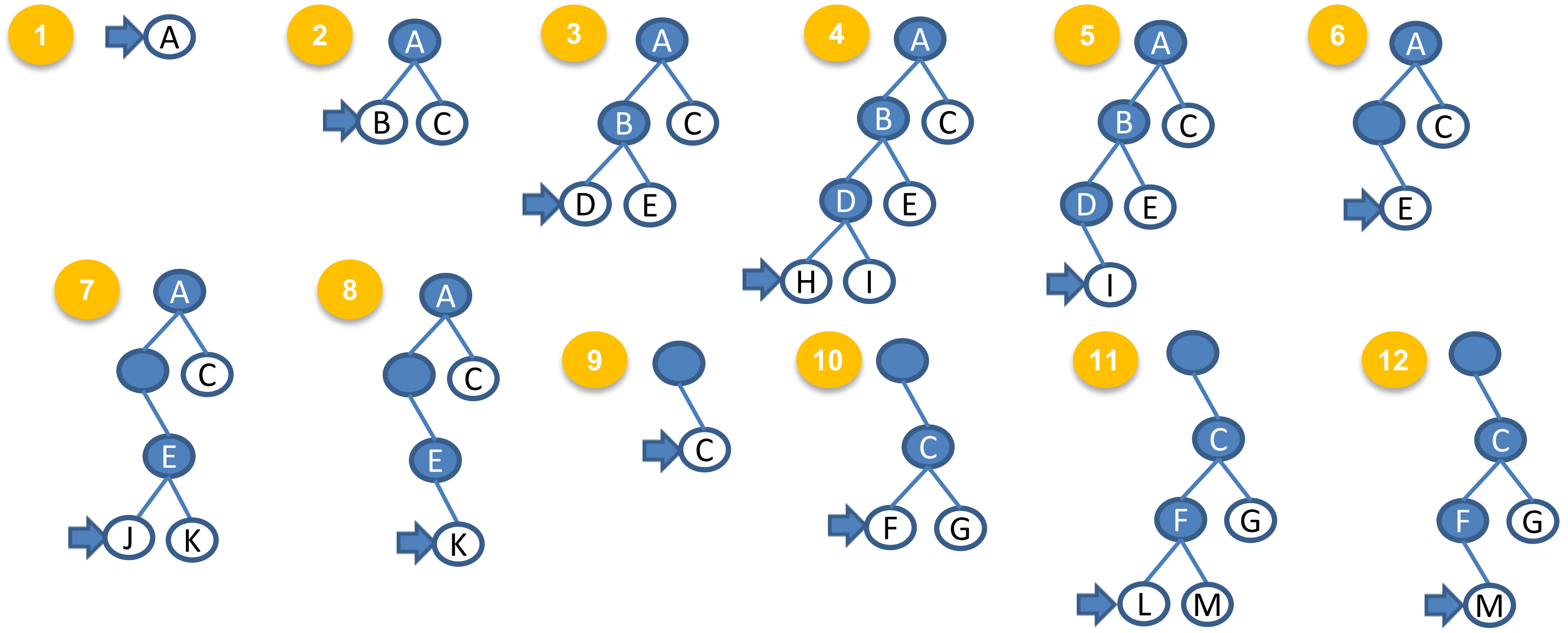
Here we do not expand notes that have been expanded.

## Uniform-Cost Search

| | |
|---|---|
| Complete | Yes |
| Time | # of nodes with path cost g <= cost of optimal solution (eqv. # of nodes pop out from the priority queue) |
| Space | # of nodes with path cost g <= cost of optimal solution |
| Optimal | Yes |

# Depth-First Search

Expand deepest unexpanded node which can be implemented by a Last-In-First-Out (LIFO) stack, Backtrack only when no more expansion



# Depth-First Search

Denote

- m: maximum depth of the state space

| | |
|---|---|
| Complete | • infinite-depth spaces: No<br>• finite-depth spaces with loops: No<br>  • with repeated-state checking: Yes<br>• finite-depth spaces without loops: Yes |
| Time | $O(b^m)$<br>If solutions are dense, may be much faster than breadth-first |
| Space | $O(bm)$ |
| Optimal | No |

# Depth-Limited Search

To avoid infinite searching, Depth-first search with a cutoff on the max depth $l$ of a path

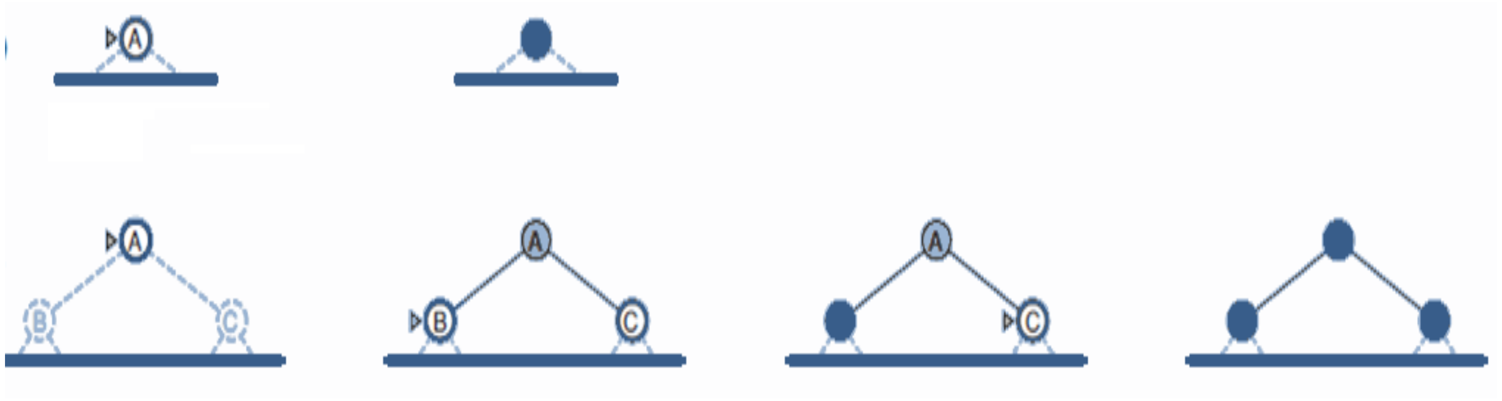
| | |
|---|---|
| Complete | Yes, if $l \geq d$ |
| Time | $O(b^{l})$ |
| Space | $O(bl)$ |
| Optimal | No |

# Iterative Deepening Search

Iteratively estimate the max depth *l* of DLS one-by-one

Limit = 0

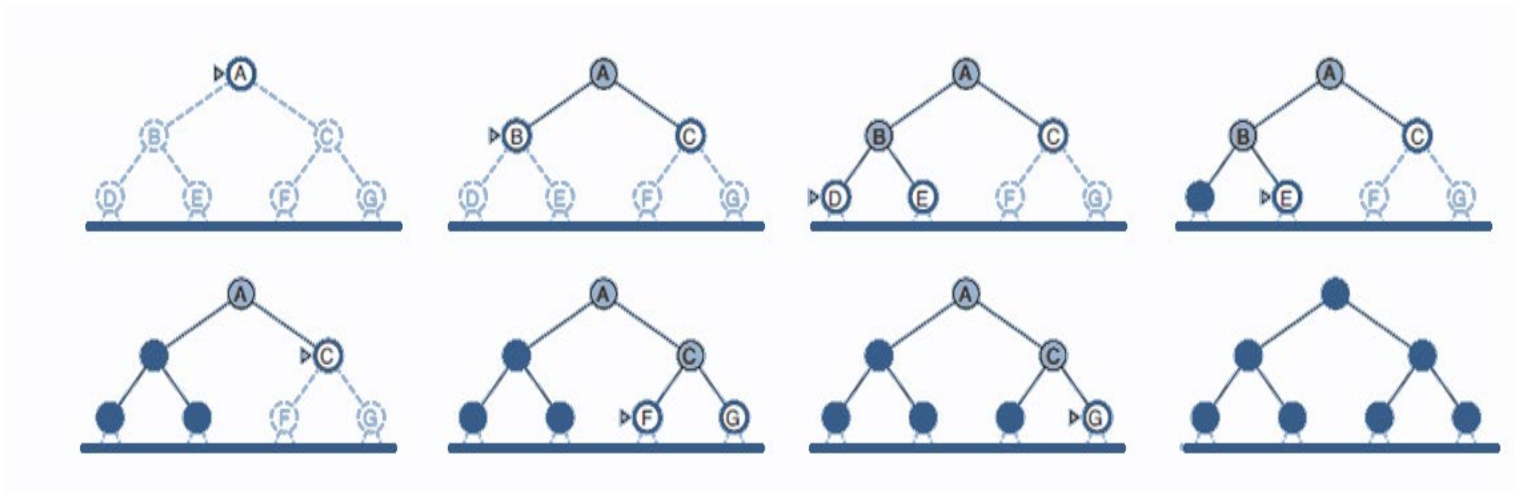
Limit = 1



# Iterative Deepening Search

Iteratively estimate the max depth *l* of DLS one-by-one

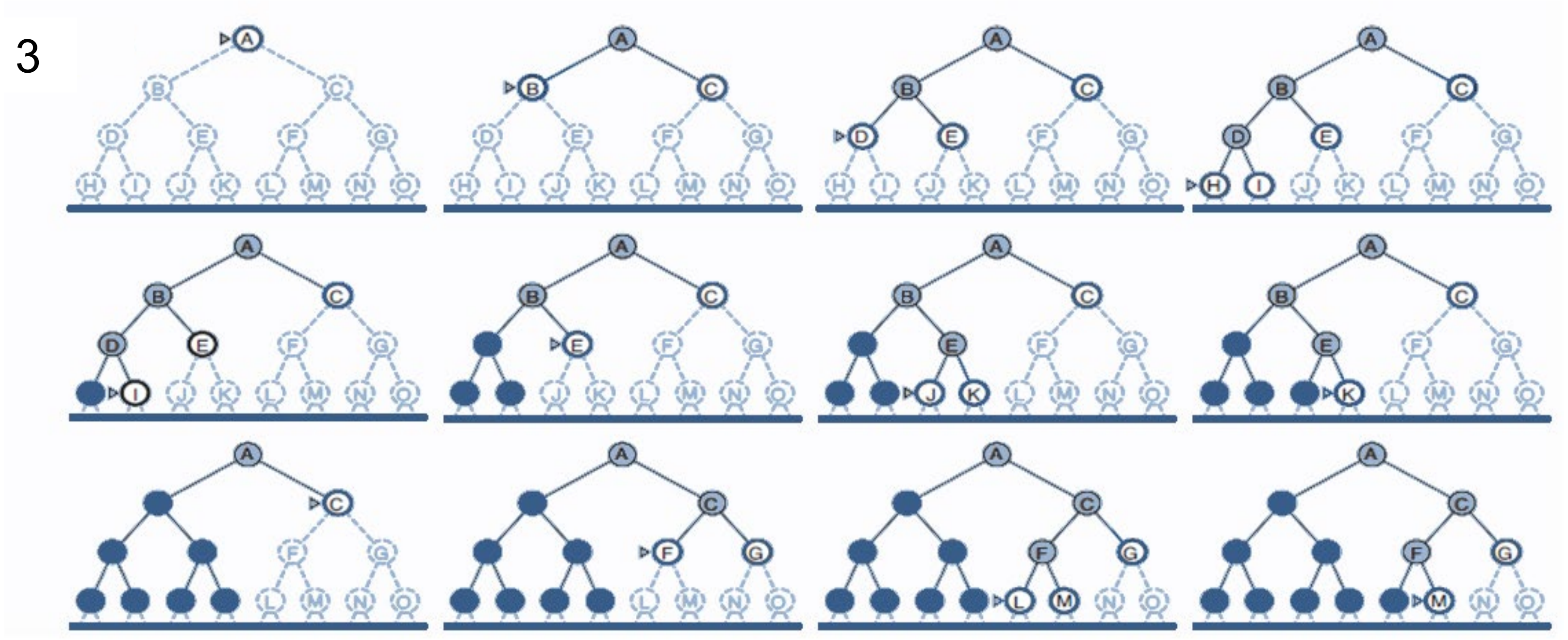
Limit = 2



## Iterative Deepening Search

Iteratively estimate the max depth *l* of DLS one-by-one

Limit = 3



# Iterative Deepening Search...

```
Function ITERATIVE-DEEPENING-SEARCH(problem) returns a solution sequence
   inputs: problem, a problem

   for depth   0 to ∞ do
    if DEPTH-LIMITED-SEARCH(problem, depth) succeeds then return its result
   end
   return failure
```

| | |
|---|---|
| Complete | Yes |
| Time | $O(b^d)$ |
| Space | $O(bd)$ |
| Optimal | Yes |

## Summary (we make assumptions for optimality)

| Criterion | Breadth-first | Uniform-Cost | Depth-First | Depth-Limited | Iterative Deepening | Bidirectional (if applicable) |
|---|---|---|---|---|---|---|
| Time | $b^d$ | $b^d$ | $b^m$ | $b^l$ | $b^d$ | $b^{d/2}$ |
| Space | $b^d$ | $b^d$ | $bm$ | $bl$ | $bd$ | $b^{d/2}$ |
| Optimal | Yes | Yes | No | No | Yes | Yes |
| Complete | Yes | Yes | No | Yes, if $l \geq d$ | Yes | Yes |

## Question to think:

- If a search strategy is optimal, is it also complete?

# General Search

Uninformed search strategies

- Systematic generation of new states (→Goal Test)

- Inefficient (exponential space and time complexity)

Informed search strategies

- Use problem-specific knowledge
  - To decide the order of node expansion

- Best First Search: expand the most desirable unexpanded node
  - Use an evaluation function to estimate the "desirability" of each node

# Evaluation function

- Path-cost function $g(n)$
  - Cost from initial state to current state (search-node) $n$
  - No information on the cost toward the goal
- Need to estimate cost to the closest goal
- "Heuristic" function $h(n)$
  - Estimated cost of the cheapest path from $n$ to a goal state $h(n)$
    - Exact cost cannot be determined
  - depends only on the state at that node
  - $h(n)$ is not larger than the real cost (admissible)

## Greedy Search

Expands the node that appears to be closest to goal

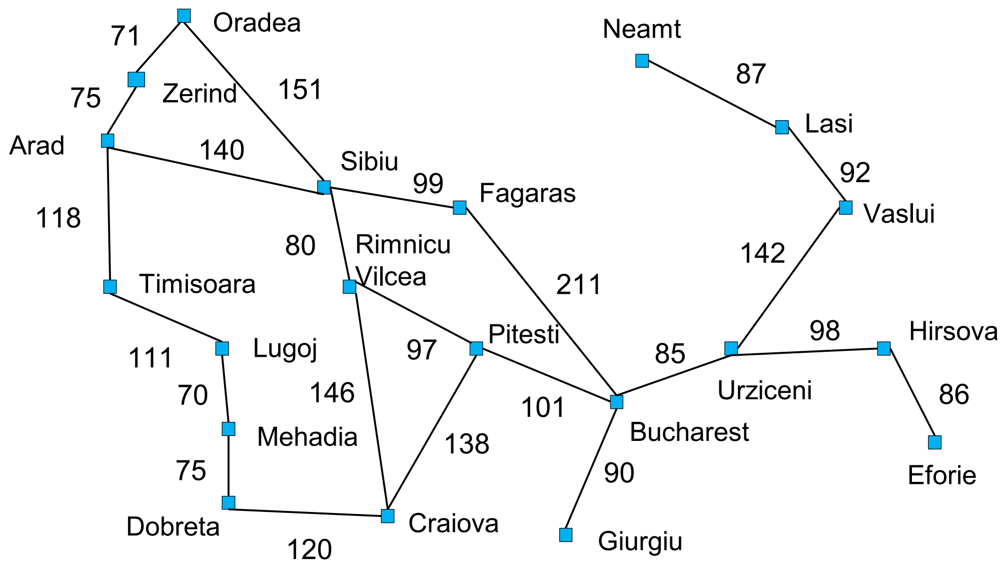
- Evaluation function $h(n)$:estimate of cost from $n$ to $goal$
- Function Greedy-Search(problem) returns solution
  - Return Best-First-Search(problem, $h$)   // $h(goal) = 0$

**Question:** How to estimation the cost from $n$ to goal?

**Answer:** Recall that we want to use problem-specific knowledge

# Example: Route-finding from Arad to Bucharest

$h(n)$ = straight-line distance from $n$ to Bucharest



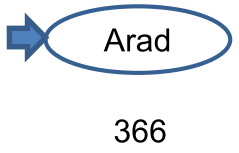
- Useful but potentially fallible (heuristic)
- Heuristic functions are problem-specific

Straight-line distance to Bucharest

| | |
|---|---|
| **Arad** | 366 |
| **Bucharest** | 0 |
| **Craiova** | 160 |
| **Dobreta** | 242 |
| **Efoire** | 161 |
| **Fagaras** | 176 |
| **Giurgiu** | 77 |
| **Hirsova** | 151 |
| **Lasi** | 226 |
| **Lugoj** | 244 |
| **Mehadia** | 241 |
| **Neamt** | 234 |
| **Oradea** | 380 |
| **Pitesti** | 98 |
| **Rimnicu Vilcea** | 193 |
| **Sibiu** | 253 |
| **Timisoara** | 329 |
| **Urziceni** | 80 |
| **Vaslui** | 199 |
| **Zerind** | 374 |

## Example

a) The initial state



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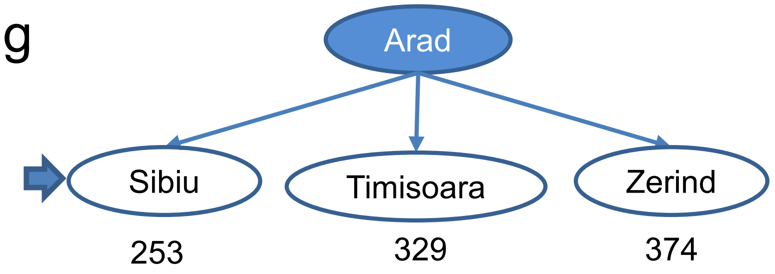
## Example

b) After expanding Arad



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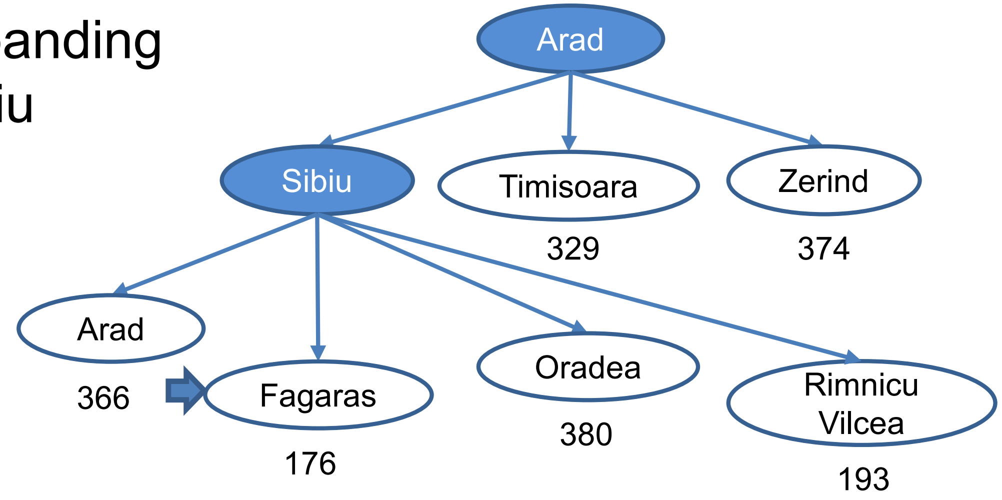
## Example

c) After expanding Sibiu



Straight-line distance to Bucharest

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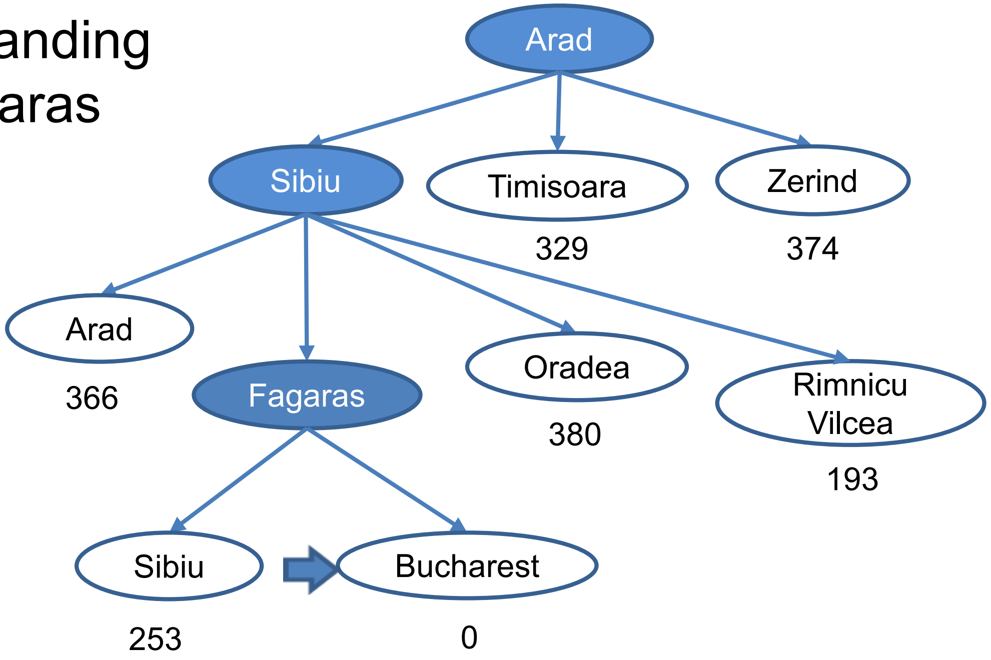
## Example

d) After expanding Fagaras



Straight-line distance to Bucharest

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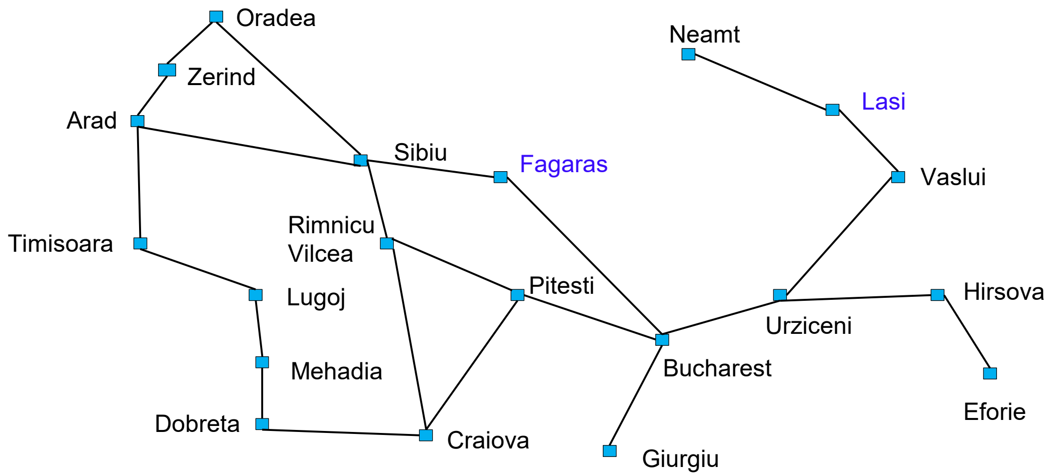
## Complete?

**Question:** Is this approach complete?

**Example: Find a path from Lasi to Fagaras**



**Answer:** No

## Greedy Search...

- m: maximum depth of the search space

| | |
|---|---|
| Complete | No |
| Time | $O(b^m)$ |
| Space | $O(b^m)$(keeps all nodes in memory) |
| Optimal | No |

## Question to think:

- Is it possible to combine functions g(n) and h(n) in one search strategy?

# A * Search

- Uniform-cost search
  - *g(n):* cost to reach n (Past Experience)
  - optimal and complete, but can be very inefficient
- Greedy search
  - *h(n):* cost from n to goal (Future Prediction)
  - neither optimal nor complete, but cuts search space considerably

# A * Search

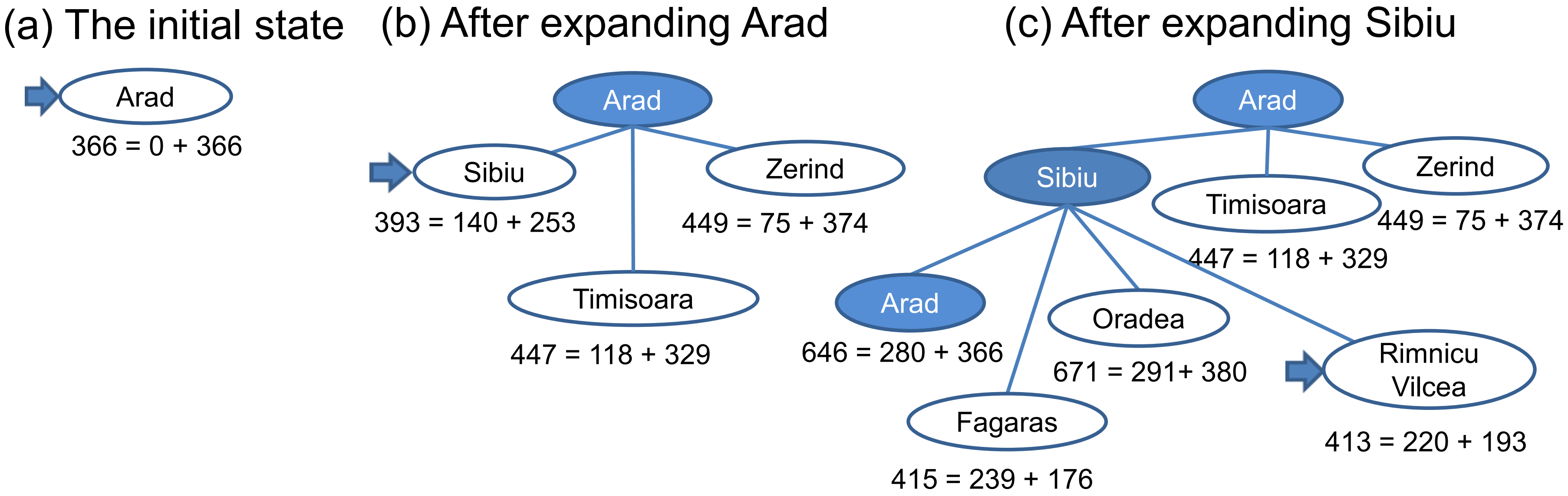
Idea: Combine Greedy search with Uniform-Cost search

Evaluation function: $f(n) = g(n) + h(n)$

- *f (n):* estimated total cost of path through *n* to goal (Whole Life)
- If g = 0 → greedy search;     If h = 0 → uniform-cost search
- Function A* Search(problem) returns solution
    - Return Best-First-Search(problem, $g + h$)

# Example: Route-finding from Arad to Bucharest

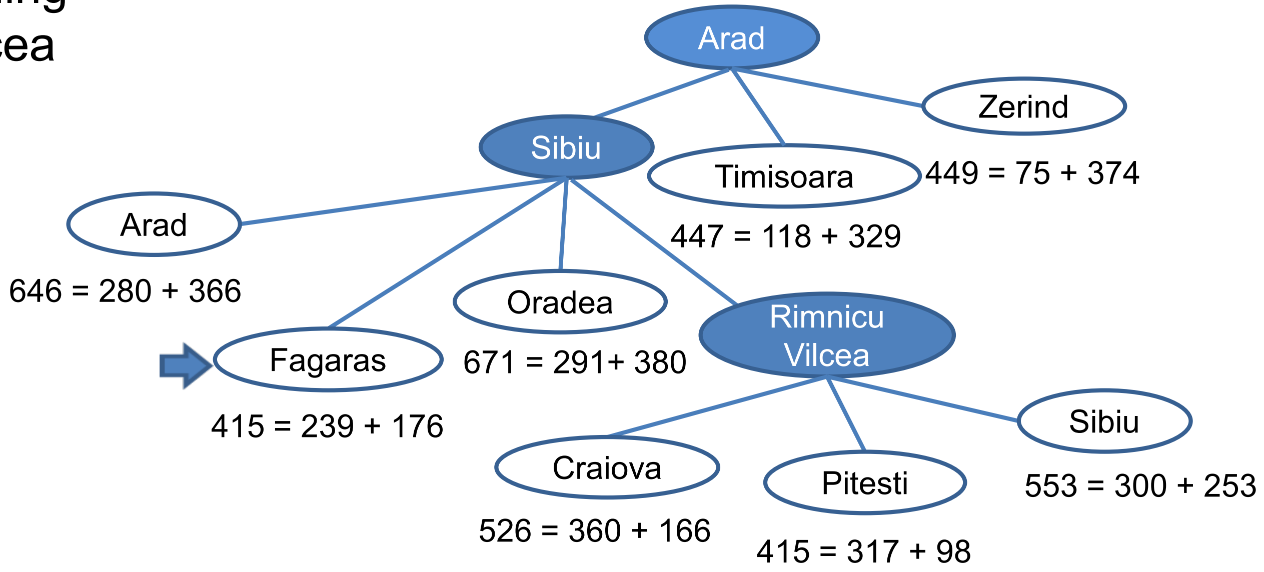
Best-first-search with evaluation function $g + h$



# Example: Route-finding from Arad to Bucharest

Best-first-search with evaluation function $g + h$

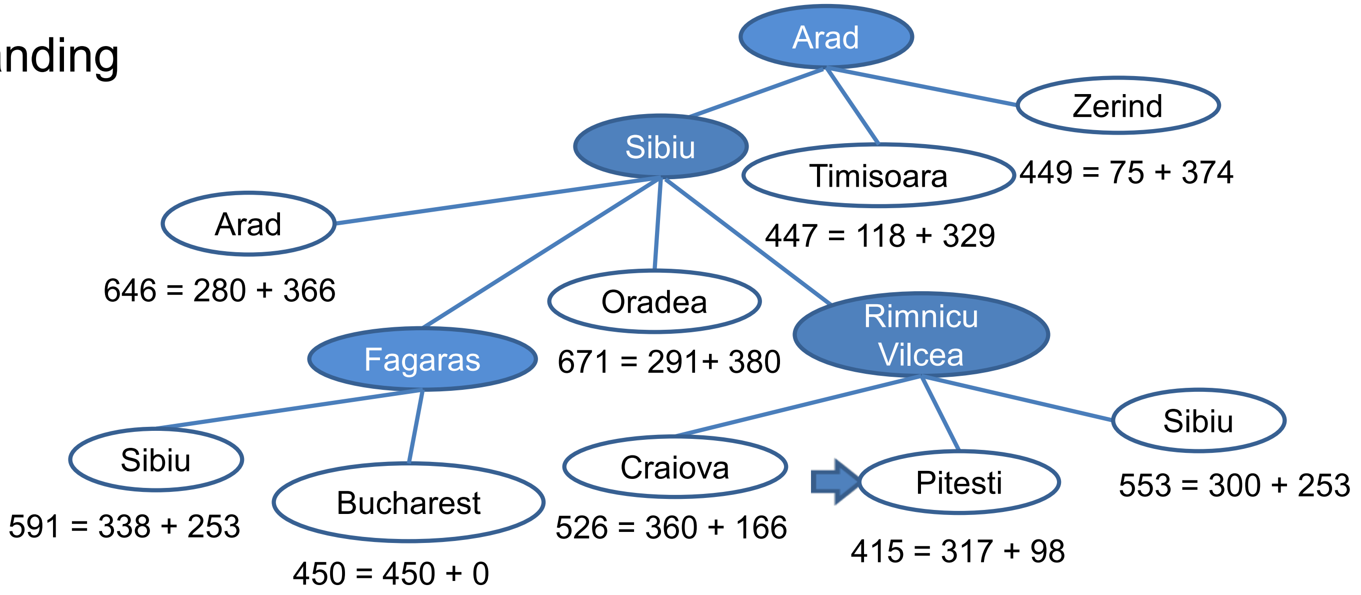
(d) After expanding
Rimnicu Vilcea



# Example: Route-finding from Arad to Bucharest

Best-first-search with evaluation function $g + h$

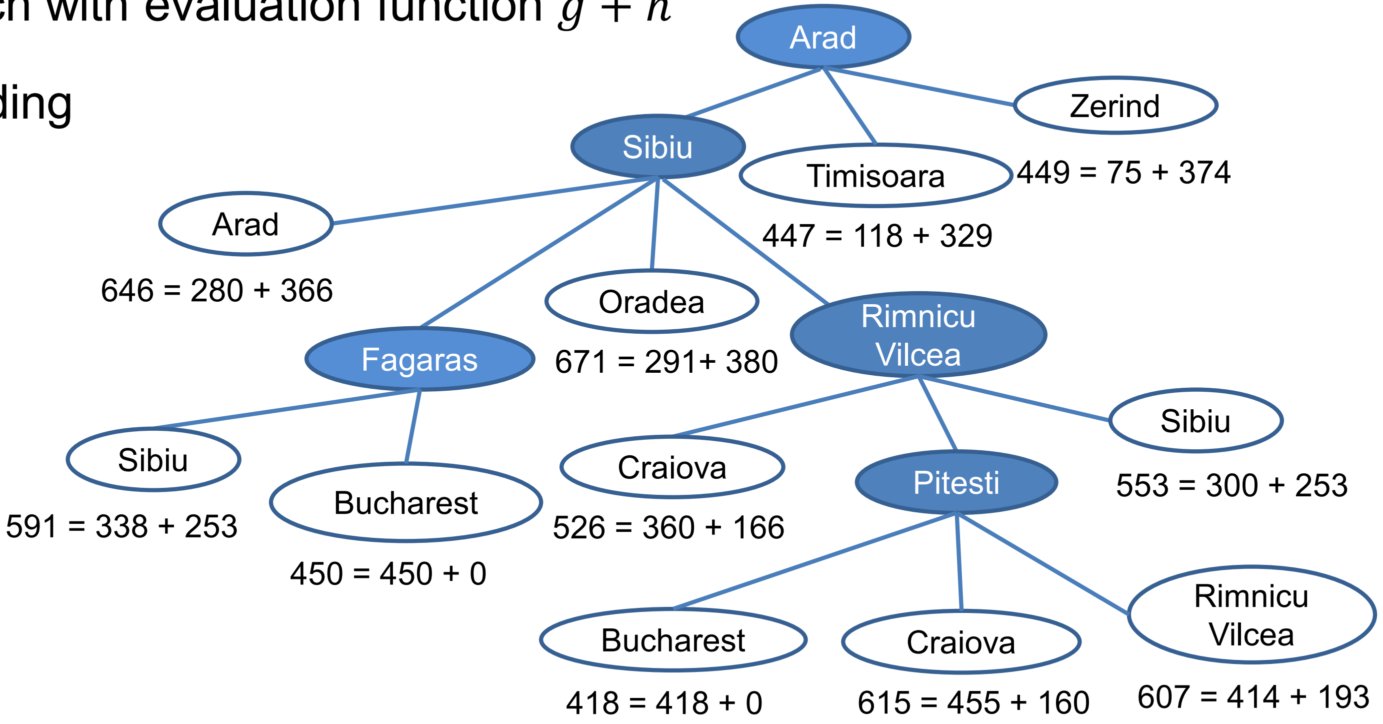
(e) After expanding Fagaras



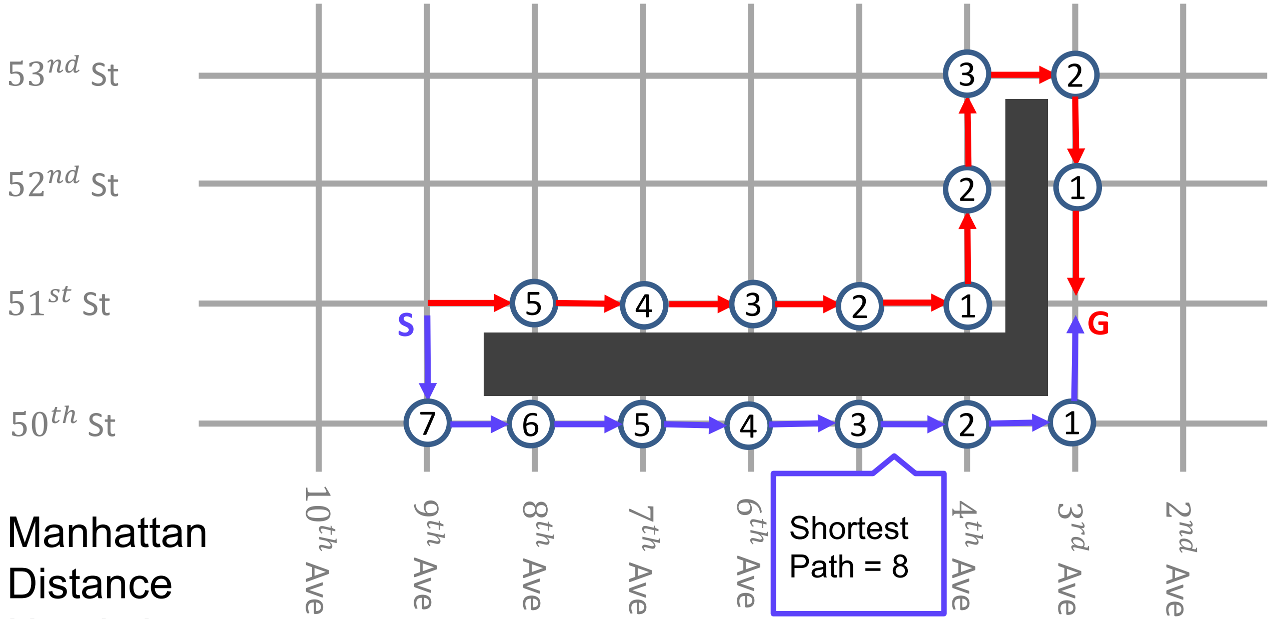
## Example: Route-finding from Arad to Bucharest

Best-first-search with evaluation function $g + h$

(f) After expanding Pitesti

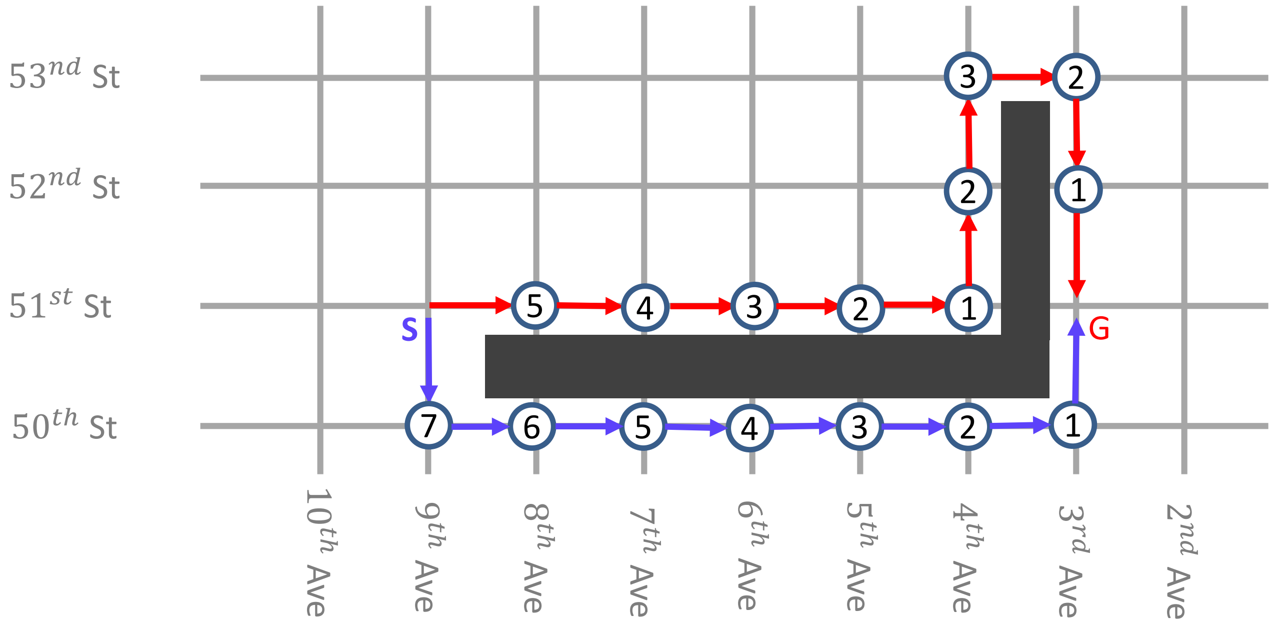


# Example: Route-finding in Manhattan

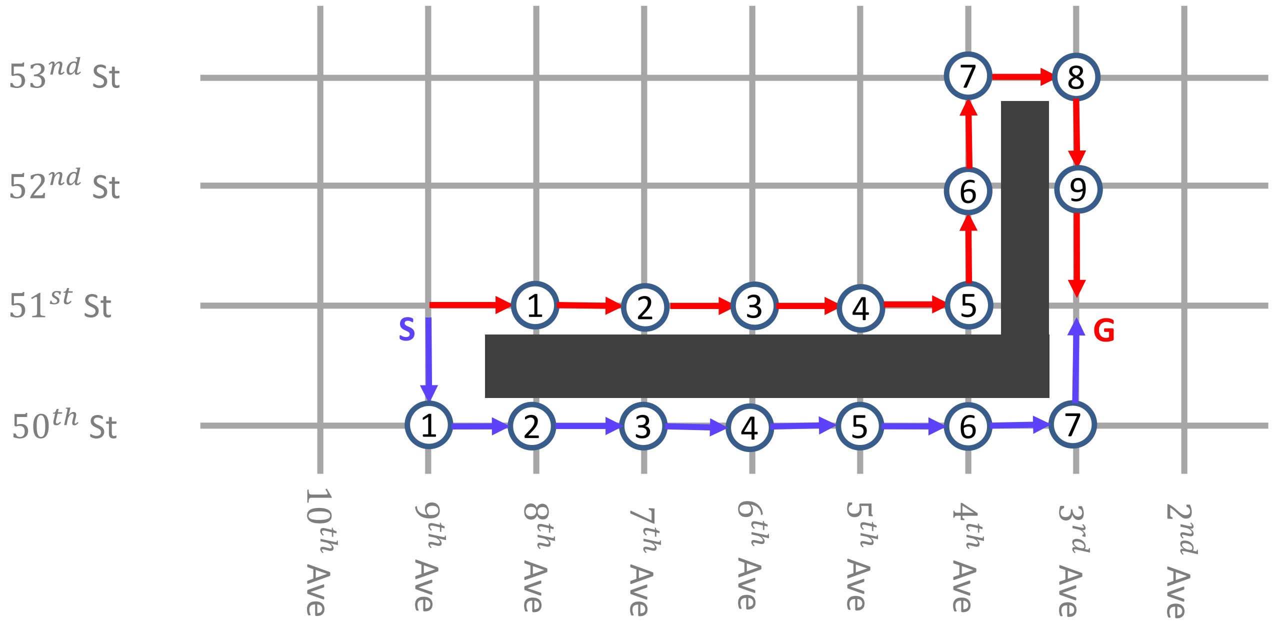


Manhattan Distance Heuristic

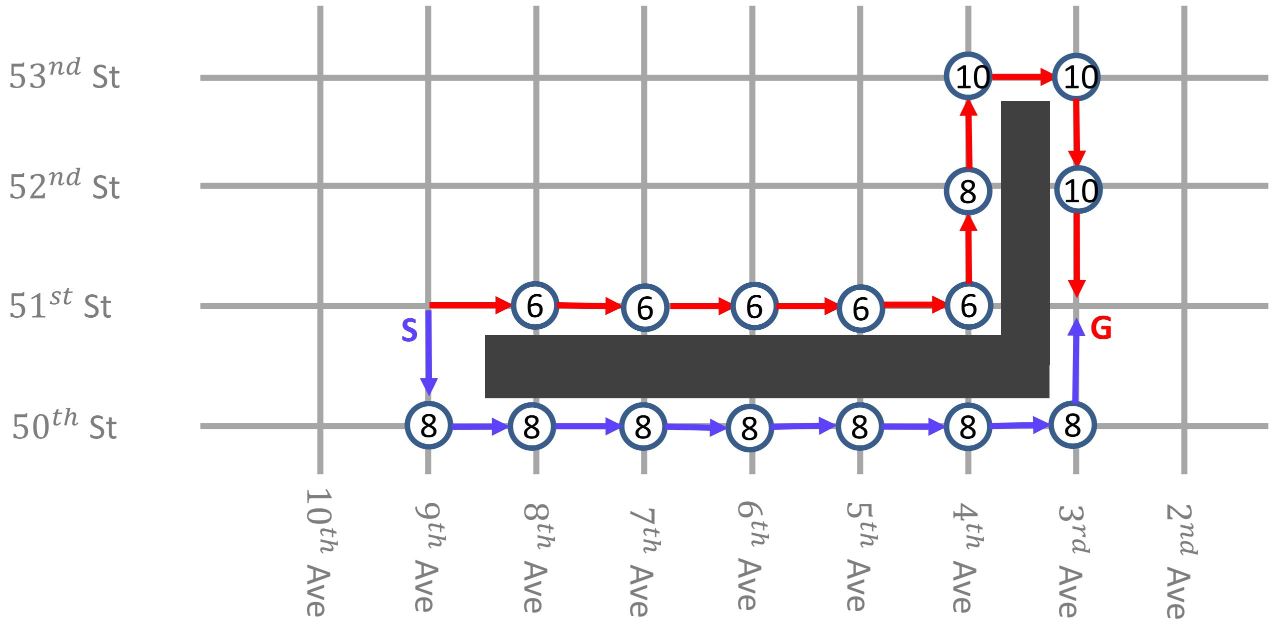
## Example: Route-finding in Manhattan (Greedy)



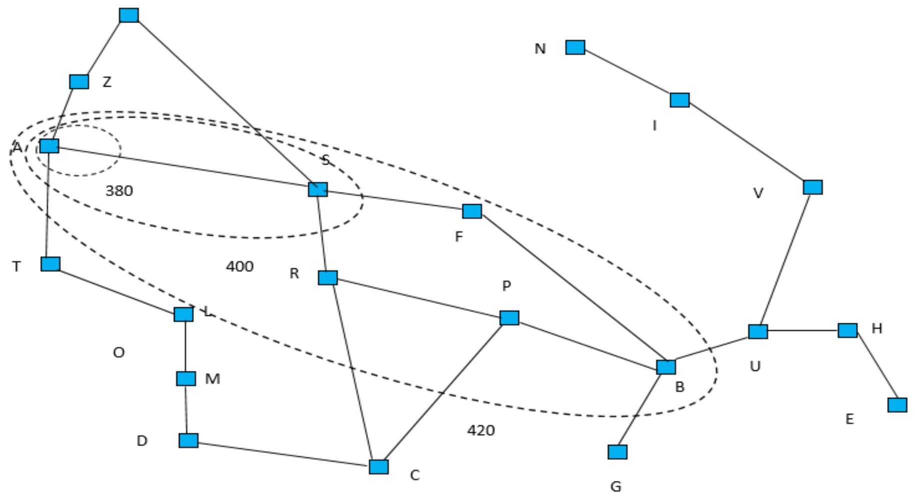
# Example: Route-finding in Manhattan (UCS)



# Example: Route-finding in Manhattan (A*)



# Complexity of A*



| | |
|---|---|
| Time | Exponential in length of solution |
| Space | (all generated nodes are kept in memory) Exponential in length of solution |

With a good heuristic, significant savings are still possible compared to uninformed search methods