NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I (CA1)

MH1812 – Discrete Mathematics

February 2020 TIME ALLOWED: 50 minutes

Name:

Matric. no.: Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1. **(30 marks)**

(a) Which integer $a \in \{0, 1, \ldots, 14\}$ is congruent to $2020+1010+550+225$ modulo 15? (10 marks)

(b) Write down each integer $a \in \{0, 1, 2\}$ for which there exists an integer n such that $a \equiv n^2 + n - 1 \pmod 3$. (10 marks)

(c) Let $S = \{\text{integers congruent to 1 modulo 5}\}$ and Δ be multiplication. Is S closed under Δ? Justify your answer. (10 marks)

a) $2020 + 1010 + 550 + 225 = 10 \bmod 15$

$a = 10$ //

b) $a \equiv n(n+1) - 1 \bmod 3$

for mod3, 3 possible n

n

$0,\ 0^2 + 0 - 1 \equiv 2 \bmod 3$

$1,\ 1^2 + 1 - 1 \equiv 1 \bmod 3$

$2,\ 2^2 + 2 - 1 \equiv 2 \bmod 3$

$\therefore a = \{1, 2\}$ //

c) $S = 5(n) + 1, \quad n \in \mathbb{Z} \quad m, 6 \mathbb{Z}$

$(5(n)+1)(5(m)+1) = 25(n)(m) + 5n + 5m + 1$

$= 5(5nm + n + m) + 1$

$\therefore$ is closed

Blank page for writing

QUESTION 2. **(40 marks)**

(a) Prove or disprove the following logical equivalences:

(i) (10 marks)

$$p \wedge (T \rightarrow p) \equiv p$$

(ii) (10 marks)

$$(p \wedge q \wedge r) \rightarrow (p \vee s) \equiv (p \rightarrow s) \vee (q \rightarrow s) \vee (r \rightarrow s)$$

(b) Decide whether or not the following argument is valid (20 marks):

$$\begin{array}{l} \neg q \vee p; \\ \neg q \rightarrow F; \\ p \rightarrow (\neg r \rightarrow s); \\ q \rightarrow \neg r \\ \therefore s \end{array}$$

Briefly justify your answers.

a) i) IF $P = F$, $T \rightarrow P = F$, $P \wedge F = F$

$P = T$, $T \rightarrow P = T$, $P \wedge T = T$

$\therefore$ valid

ii) LHS is T when P, q, r is T, P v S is T

S is F

RHS if P, q, r, is T & S is F,

RHS is F.

$\therefore$ disproven

Blank page for writing

(b) Decide whether or not the following argument is valid (20 marks):

$$\begin{array}{l} \neg q \vee p; \\ \neg q \rightarrow F; \\ p \rightarrow (\neg r \rightarrow s); \\ q \rightarrow \neg r \\ \therefore s \end{array}$$

$\neg q$ is F due to (2)

q is T ✓

$\neg r$ is T, (4)

p is T, (1) ✓

$(\neg r \rightarrow s)$ is T, (3) ✓

$\therefore$ s is T // ✓

QUESTION 3. **(30 marks)**

(a) Let X and Y be domains, and let $P(x)$ and $Q(y)$ be predicates. Which of the following statements is the *negation* of the statement

$\forall x \in X,\ \exists y \in Y,\ P(x) \vee \neg Q(y)$? (10 marks)

(i) $\forall y \in Y,\ \exists x \in X,\ \neg P(x) \wedge Q(y)$;

(ii) $\exists x \in X,\ \forall y \in Y,\ P(x) \to \neg Q(y)$;

(iii) $\exists y \in Y,\ \forall x \in X,\ \neg P(x) \to \neg Q(y)$;

(iv) $\exists x \in X,\ \forall y \in Y,\ \neg P(x) \wedge Q(y)$;

(v) none of the above.

(b) Consider the domains $A = \{3, 4\}$ and $B = \{0, 3, 6\}$ and the predicate $P(x, y) =$ "$x^2 - y \geqslant 9$".

Determine the truth value of the following statements:

(i) $\forall x \in A,\ \exists y \in B,\ P(x, y)$; (10 marks)

(ii) $\exists x \in A,\ \forall y \in B,\ P(x, y)$. (10 marks)

Briefly justify your answers.

a) $\neg(P(x) \vee \neg Q(y)) \equiv \neg P(x) \wedge Q(y)$

b) $x = 3,\ y = 0 \quad 3^2 - 0 \geq 9$

$x = 4,\ y = 0 \quad 4^2 - 0 \geq 9$

i is T

$x = 3,\ y = 0 \quad 3^2 - 0 \geq 9$

$y = 3 \quad 3^2 - 3 < 9$

$y = 6 \quad 3^2 - 6 < 9$

ii is F

For $x = 4$, the predicate is T for all B.

Blank page for writing

Blank page for writing