① Consider matrix $A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 2 & 17 \\ 0 & 2 & 4 & 23 \end{pmatrix}$. For every statement write whether it is true or false

1) Columns are linear independant

2) Rows are linear dependant

3) Homogeneous system with matrix A has only trivial solution

4) System $Ax=b$ has a solution for every $b \in \mathbb{R}$

5) Row echelon form of A has a pivot in every row.

6) Row echelon form of A has a pivot in every column.

② Given the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$. Which of the following matrix multiplication terms correctly represent the LU factorisation for the matrix A above

A) $\begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

E) None of the above.

③ Coefficient matrices of systems given in LU factorisation form. For each matrix determine whether it is true or false that system $Ax=0$ has non-trivial solution.

B) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 7 & 1 & 1 \\ 0 & 5 & 6 & 1 \\ 0 & 0 & 12 & 1 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 0 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}$

① Consider matrix $A = \begin{pmatrix} 1 & 2 & 1 & 3 \\ 0 & 1 & 2 & 17 \\ 0 & 2 & 4 & 23 \end{pmatrix}$. For every statement write whether it is true or false.

1) Columns are linear independant **F**

2) Rows are linear dependant **F**

3) Homogeneous system with matrix A has only trivial solution **F**

4) System $Ax=b$ has a solution for every $b \in \mathbb{R}^3$ **T**

5) Row echelon form of A has a pivot in every row. **T**

6) Row echelon form of A has a pivot in every column. **F**

$$\begin{bmatrix} 1 & 2 & 1 & 3 & 0 \\ 0 & 1 & 2 & 17 & 0 \\ 0 & 2 & 4 & 23 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 1 & 2 & 4 & 0 \\ 3 & 17 & 23 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ independent

$x_1 - 3x_3 = 0$

$x_2 + 2x_3 = 0$

$x_4 = 0$

let $x_3 = 1$

$x_1 = 3$

$x_2 = -2$

$3V_1 - 2V_2 + 1V_3 = 0$

$\therefore$ Dependent

② Given the matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$. Which of the following matrix multiplication terms correctly represent the LU factorisation for the matrix A above

~~A)~~ $\begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & -2 \end{bmatrix}$

~~D)~~ $\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

③ Coefficient matrices of systems given in LU factorisation form. For each matrix determine whether it is true or false that system $Ax=0$ has non-trivial solution.

B) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 7 & 1 & 1 \\ 0 & 5 & 6 & 1 \\ 0 & 0 & 12 & 1 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 0 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ 0 & 0 \end{pmatrix}$

B $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\left[\begin{array}{cccc|c} 1 & 7 & 1 & 1 & 0 \\ 0 & 5 & 6 & 1 & 0 \\ 0 & 0 & 12 & 1 & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 13/60 & 0 \\ 0 & 1 & 0 & 1/10 & 0 \\ 0 & 0 & 1 & 1/12 & 0 \end{array}\right]$

$X_1 = -13/60\, t$

$X_2 = -1/10\, t$

$X_3 = -1/12\, t$

$\therefore$ non trival

c) $\begin{bmatrix} 1 & 0 & 0 \\ 7 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$\begin{bmatrix} 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$

$x_1 + x_2 = 0$

non trival solution

d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 7 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right] = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$\left[\begin{array}{ccc|c} 2 & 3 & 5 & 0 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right] = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$

$\therefore$ Only trivcl

E) $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 5 & 1 & 0 \end{array}\right] = \begin{matrix} 0 \\ 0 \end{matrix}$

$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 3 & 0 \end{array}\right] = \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$

$\therefore$ only trival

D) $\begin{pmatrix} 1 & 0 & 0 \\ 7 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 8 \end{pmatrix}$ E) $\begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

4. Which of the transformations A, B, C, D in the sequence below were used correctly (i.e., preserved the determinant) while calculating matrix determinant:

$$\begin{vmatrix} 2 & 4 & 3 \\ 2 & 5 & 4 \\ 0 & 1 & 1/2 \end{vmatrix} \underset{A}{\xrightarrow{R_3 \leftarrow 2R_3}} \begin{vmatrix} 2 & 4 & 3 \\ 2 & 5 & 4 \\ 0 & 2 & 1 \end{vmatrix} \underset{B}{\xrightarrow{R_2 \leftarrow R_2 - R_1}} \begin{vmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow{R_2 \text{ swap } R}$$

$$\underset{C}{\xrightarrow{R_2 \text{ swap } R_3}} \begin{vmatrix} 2 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} \underset{D}{\xrightarrow{col_2 \leftarrow col_2 - col_3}} \begin{vmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2 \cdot 1 \cdot 1 = 2$$

5. Determine value(s) of h such that the following matrix is the augmented matrix of a consistent linear system: $A = \left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right]$

B) $h \neq 2$ C) $h \neq 3$ D) $h = 2$

6. Find the matrix product AB if it is defined

$A = \begin{bmatrix} -1 & 3 \\ 5 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 2 \end{bmatrix}$

C) $\begin{bmatrix} 0 & -6 \\ 15 & 5 \\ -18 & 12 \end{bmatrix}$ D) $\begin{bmatrix} 3 & -7 & 1 \\ 6 & -28 & 37 \end{bmatrix}$ E) AB is undefined

F) $\begin{bmatrix} 3 & 6 & -7 \\ -28 & 1 & 37 \end{bmatrix}$

④ Which of the transformations A, B, C, D in the sequence below were used correctly (i.e., preserved the determinant) while calculating matrix determinant:

$$\begin{vmatrix} 2 & 4 & 3 \\ 2 & 5 & 4 \\ 0 & 1 & 1/2 \end{vmatrix} \xrightarrow[A]{R_3 \leftarrow 2R_3} \begin{vmatrix} 2 & 4 & 3 \\ 2 & 5 & 4 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow[B]{R_2 \leftarrow R_2 - R_1} \begin{vmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} \xrightarrow{R_2 \text{ swap } R}$$

$$\xrightarrow[C]{R_2 \text{ swap } R_3} \begin{vmatrix} 2 & 4 & 3 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{vmatrix} \xrightarrow[D]{col_2 \leftarrow col_2 - col_3} \begin{vmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 2 \cdot 1 \cdot 1 = 2$$

1) multiplying changes determinant
2) correct
3) change determint (−ve)
4) correct

⑤ Determine value(s) of h such that the following matrix is the augmented matrix of a consistent linear system: $A = \left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right]$

B) $h \neq 2$ C) $h \neq 3$ D) $h = 2$

⑥ Find the matrix product AB if it is defined

$A = \begin{bmatrix} -1 & 3 \\ 5 & 6 \end{bmatrix}$ · $B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 2 \end{bmatrix}$

~~C)~~ $\begin{bmatrix} 0 & -6 \\ 15 & 5 \\ -18 & 12 \end{bmatrix}$ D) $\begin{bmatrix} 3 & -7 & 1 \\ 6 & -28 & 37 \end{bmatrix}$ ~~E)~~ AB is undefined

F) $\begin{bmatrix} 3 & 6 & -7 \\ -28 & 1 & 37 \end{bmatrix}$

2×2 2×3

⑦ Which matrix will be matrix of linear trasformation L if $L(e_1) = 3e_1 + e_3$, $L(e_2) = e_1 + 2e_2$, $L(e_3) = -e_1 - e_2 + e_3$, where e_1, e_2 and e_3 - unit vector

A) $\begin{pmatrix} 1 & 1 & -1 \\ 3 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 3 & 1 & -1 \\ 0 & 2 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ C) $\begin{pmatrix} 7 & 10 & 1 \\ 8 & 11 & 2 \\ 1 & 12 & 3 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 \\ 5 & 8 \end{pmatrix}$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \to \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \to \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \to \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

⑧ Find matrix of linear transformation $L(v) = \begin{pmatrix} 3 \\ -9 \\ 9 \end{pmatrix} \times v$ ($\times$ is vector product).

A) $\begin{pmatrix} 0 & -9 & -9 \\ 9 & 0 & -3 \\ 9 & 3 & 0 \end{pmatrix}$ B) $\begin{pmatrix} 0 & 3 & 0 \\ 9 & 0 & 9 \\ 9 & 3 & 9 \end{pmatrix}$ C) $\begin{pmatrix} 0 & 9 & 3 \\ 9 & 3 & 9 \end{pmatrix}$

D) $\begin{pmatrix} 3 & 9 \\ -9 & 3 \end{pmatrix}$

$$\begin{bmatrix} 3 \\ -9 \\ 9 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ -9 \\ 9 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -9 \\ 0 \\ 3 \end{bmatrix}$$

$$\begin{pmatrix} 3 \\ -9 \\ 9 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -9 \\ -3 \\ 0 \end{pmatrix}$$

⑨ Given matrix $A = \frac{1}{26}\begin{pmatrix} 24 & -6 & 8 \\ -6 & 8 & 24 \\ 8 & 24 & -6 \end{pmatrix}$.

Which linear transformation has this matrix?

A) $L(e_1) = e_1 + e_2 + e_3$, $L(e_2) = e_3 + 24e_2$, $L(e_3) = -6e_1 + 8e_2$, where e_1, e_2, e_3 - unit vectors and $L(ae_1 + be_2 + ce_3) = aL(e_1) + bL(e_2) + cL(e_3)$.

B) $L(v) = \begin{pmatrix} -6 \\ 8 \\ 24 \end{pmatrix} \times v$, where $\times$ is vector product

C) $L(v) = v - 2\frac{(a, v)}{(a, a)}a$, where $a = \begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix}$ and $(\ ,\ )$ is scalar product

D) none of above

let $v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$= \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - 2\left[\frac{\begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}{\begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix}}\right] \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 2\left[\frac{-3}{1 + 9 + 16}\right] \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \frac{6}{26}\begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix} = \frac{1}{26}\begin{bmatrix} 0 \\ 26 \\ 0 \end{bmatrix} + \frac{1}{26}\begin{bmatrix} -6 \\ -18 \\ 24 \end{bmatrix}$$

(7) Which matrix will be matrix of linear transformation L if $L(e_1) = 3e_1 + e_3$, $L(e_2) = e_1 + 2e_2$, $L(e_3) = -e_1 - e_2 + e_3$, where e_1, e_2 and e_3 - unit vector

A) $\begin{pmatrix} 1 & 1 & -1 \\ 3 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 3 & 1 & -1 \\ 0 & 2 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ C) $\begin{pmatrix} 7 & 10 & 1 \\ 8 & 11 & 2 \\ 1 & 12 & 3 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 2 \\ 5 & 8 \end{pmatrix}$

(8) Find matrix of linear transformation $L(v) = \begin{pmatrix} 3 \\ -9 \\ 9 \end{pmatrix} \times v$ ($\times$ is vector product).

A) $\begin{pmatrix} 0 & -9 & -9 \\ 9 & 0 & -3 \\ 9 & 3 & 0 \end{pmatrix}$ B) $\begin{pmatrix} 0 & 3 & 0 \\ 9 & 0 & 9 \\ 9 & 3 & 9 \end{pmatrix}$ C) $\begin{pmatrix} 0 & 9 & 3 \\ 9 & 3 & 9 \end{pmatrix}$

D) $\begin{pmatrix} 3 & 9 \\ -9 & 3 \end{pmatrix}$

(9) Given matrix $A = \frac{1}{26} \begin{pmatrix} 24 & -6 & 8 \\ -6 & 8 & 24 \\ 8 & 24 & -6 \end{pmatrix}$.

Which linear transformation has this matrix?

A) $L(e_1) = e_1 + e_2 + e_3$, $L(e_2) = e_3 + 24e_2$, $L(e_3) = -6e_1 + 8e_2$, where e_1, e_2, e_3 - unit vectors and $L(ae_1 + be_2 + ce_3) = aL(e_1) + bL(e_2) + cL(e_3)$.

B) $L(v) = \begin{pmatrix} -6 \\ 8 \\ 24 \end{pmatrix} \times v$, where $\times$ is vector product

C) $L(v) = v - 2\frac{(a;v)}{(a,a)} a$, where $a = \begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix}$ and $(,)$ is scalar product

D) none of above

Aswers!

1

1	2	3	4	5	6
F	F	F	T	T	F

2 C)

3

B	C	D	E
T	T	F	F

4

A	B	C	D
F	T	F	T

5 B)

6 D)

7 B)

8 A)

9 C)