SC1007
Data Structures and Algorithms

Week 10: Backtracking Algorithm

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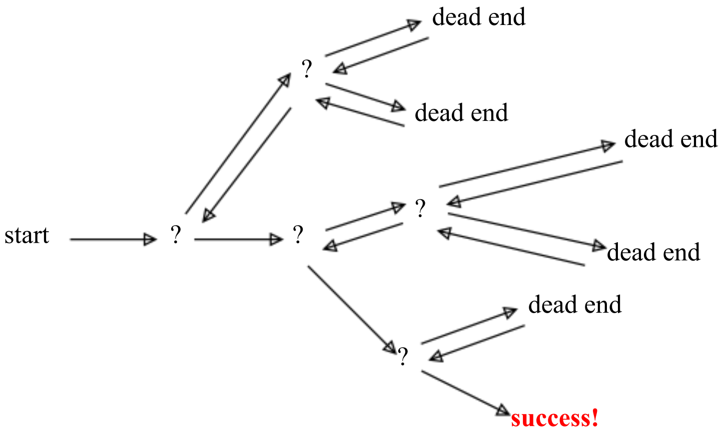
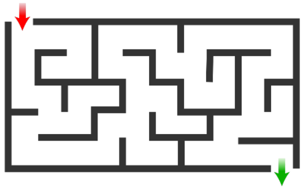
Backtracking

- Suppose you have to make a series of *decisions,* among various *choices,* where:
  - You don’t have enough information to know what to choose
  - Each decision leads to a new set of choices
  - Some sequence of choices (possibly more than one) may be a solution to your problem
- Backtracking is a methodical way of trying out various sequences of decisions, until you find one that “works”

Backtracking (animation)

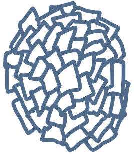
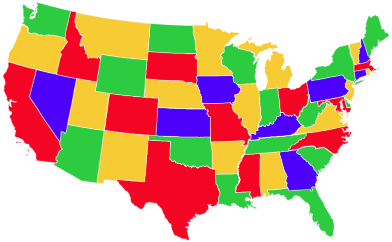


Solving a maze



- Given a maze, find a path from start to finish
- At each intersection, you have to decide:
  - Go straight
  - Go left
  - Go right
- You don’t have enough information to choose correctly
  - Each choice leads to another set of choices
  - One or more sequences of choices may (or may not) lead to a solution
- Many types of maze problem can be solved with backtracking

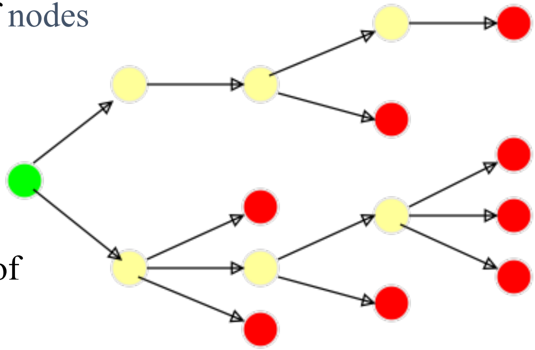
Coloring a map

- You wish to color a map with not more than n colors
- Adjacent areas must be in different colors
- You don’t have enough information to choose colors
- Each choice leads to another set of choices
- One or more sequences of choices may (or may not) lead to a solution
- Many coloring problems can be solved with backtracking

Terminology

A tree is composed of nodes



There are three kinds of nodes:

- The (one) root node
- Internal nodes
- Leaf nodes

Backtracking can be thought of as searching a tree for a particular "goal" leaf node

The backtracking algorithm

- Backtracking is really quite simple--we “explore” each node, as follows:
- To “explore” node N:

```
If N is a goal node, return “success”
Else if N is a leaf node, return “failure”
For each child C of N:
    Explore C
    If C was successful, return “success”
Return “failure”
```

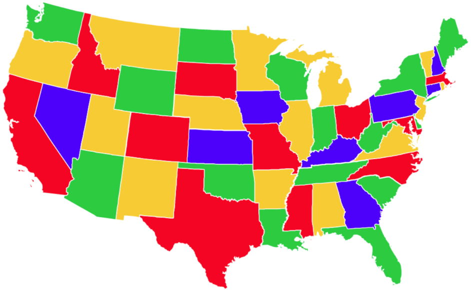
Backtracking Algorithm

- How to backtrack?
  - Recursive function

```
Backtracking(N)
    If N is a goal node, return "success"
    Else if N is a leaf node, return "failure"
    For each child C of N,
        If Backtracking(C) == "success"
            Return "success"
    Return "failure"
```

Coloring problem

- Input format:
  - 2D adjacency matrix representation of the graph [V][V]
  - Number of colors m
- Output format:
  - array color[V] that should have numbers from 1 to m



Coloring problem: Backtracking

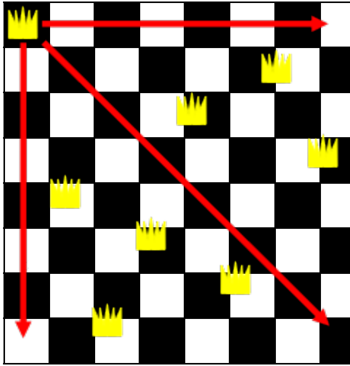
- Create a recursive function that takes current index
- If the current index is equal to the number of vertices
  - Print the color configuration in output array.
- Assign each color to a vertex (1 to m).
- For every assigned color, check if the configuration is safe, recursively call the function with next index and number of vertices
  - If any recursive function returns true break the loop and return true.
- If no recursive function returns true then return false.

```
bool graphColoringUtil(
    bool graph[V][V], int m,
    int color[], int v)
{
    /* base case: */
    if (v == V)
        return true;

    /* Consider this vertex v and
    try different colors */
    for (int c = 1; c <= m; c++) {
        /* Check if color c to v is fine*/
        if (isSafe(
                v, graph, color, c)) {
            color[v] = c;

            /* recur to assign colors to
            rest of the vertices */
            if (
                graphColoringUtil(
                    graph, m, color, v + 1)
                == true)
                return true;

            /* If c is not successful -> remove it */
            color[v] = 0;
        }
    }

    /* If no color can be assigned \*/
    return false;
}
```

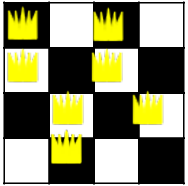
```
bool isSafe(
    int v, bool graph[V][V],
    int color[], int c)
{
    for (int i = 0; i < V; i++)
        if (
            graph[v][i] && c == color[i])
            return false;
    return true;
}
```

The Eight Queens Problem

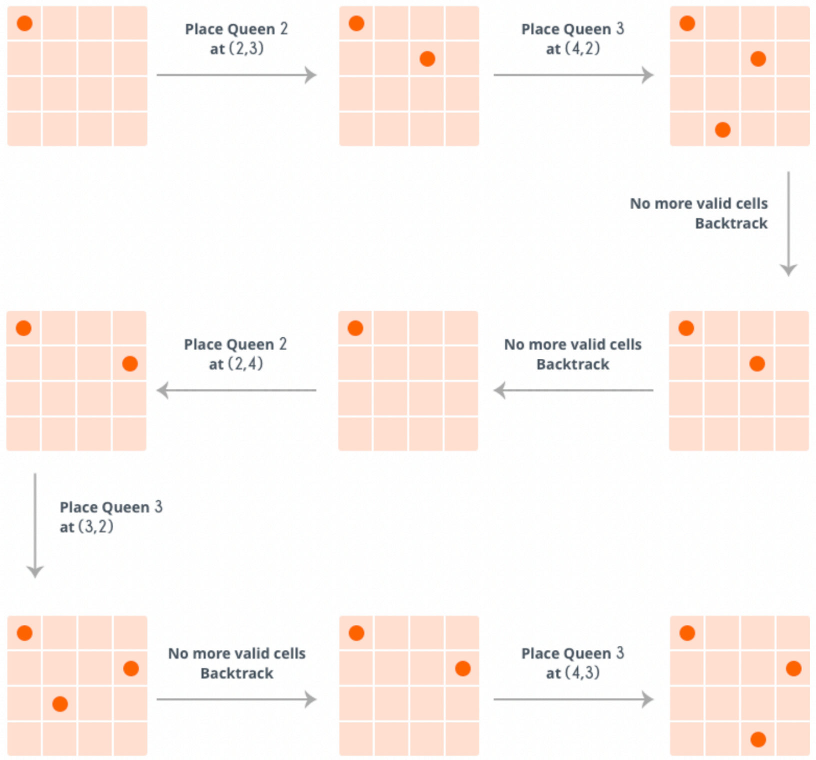
- A chessboard has 8 rows
- A queen can move within its diagonal
- Place 8 queens on the bo
  - No queen can attack any



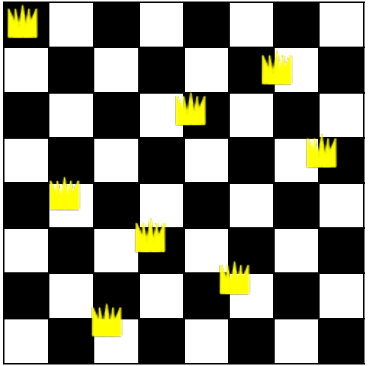
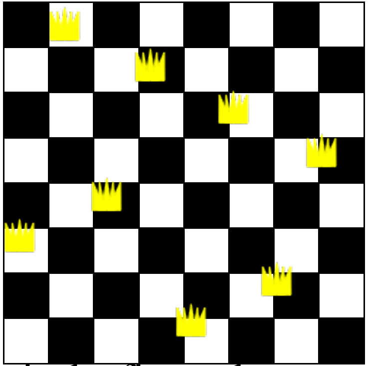
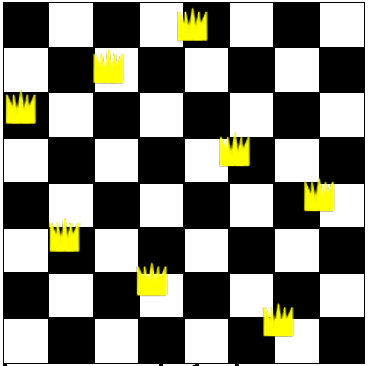
Backtracking Algorithm



-
  1. Starts by placing a queen on the tc
  2. Places a queen on the second colu
     cannot be hit by the queen on the
  3. Places a queen on the third colum
     either of the first two queens and


Place Queen 2
at (2,3)
Place Queen 3
at (4,2)
No more valid cells
Backtrack
Place Queen 2
at (2,4)
No more valid cells
Backtrack
Place Queen 3
at (3,2)
No more valid cells
Backtrack
Place Queen 3
at (4,3)

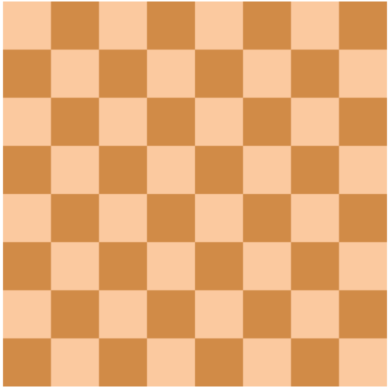
Backtracking Algorithm

- If the current column is the first column and its queen is being moved off the board then all possible configurations have been examined, all solutions have been found, and the algorithm terminates.
- This puzzle has **92** solutions.

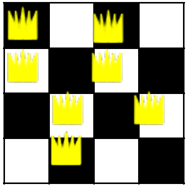
N-Queens Problem

n	Possible Solutions
4	2
5	10
6	4
7	40
8	92
10	724
12	14,200
15	2,279,184
20	39,029,188,884



The Eight Queens Problem’s Algorithm

```
function NQUEENS(Board[N][N], Column)
    if Column >= N then return true                              ▷ Solution is found
    else
        for i ← 1, N do
            if Board[i][Column] is safe to place then
                Place a queen in the square
                if NQueens(Board[N][N], Column + 1) then return true   ▷ Solution is found
                end if
                Delete the queen
            end if
        end for
    end if
return false                                                     ▷ no solution is found
end function
```




Q&A