NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER I EXAMINATION 2017–2018

MH1810 – Mathematics 1

NOVEMBER 2017 TIME ALLOWED: 2 HOURS

Matriculation Number:

Seat Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **SEVEN (7)** questions and comprises **NINETEEN (19)** pages, including an Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This **IS NOT** an **OPEN BOOK** exam. However, a list of formulae is provided in the attachments.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners only

Questions	Marks
1 (10)	
2 (10)	
3 (10)	
4 (15)	

Questions	Marks
5 (15)	
6 (15)	
7 (25)	

Total (100)	

QUESTION 1. **(10 Marks)**

(a) In Figure 1, AB is the diameter of a circle and C is a point on the arc joining A and B. Let $\mathbf{u} = \overrightarrow{OC}$ and $\mathbf{v} = \overrightarrow{OB}$, where O is the center of the circle.

(i) Draw, on the diagram, the vectors $\mathbf{v} - \mathbf{u}$ and $-\mathbf{u} - \mathbf{v}$, with C as the initial point (starting point).

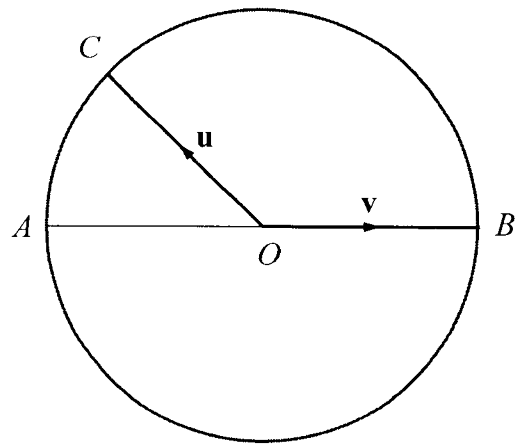

Figure 1

(ii) Use dot product to show that $\overrightarrow{CA}$ is perpendicular to $\overrightarrow{CB}$.

Question 1 continues on Page 3.

(b) Find the area of triangle whose vertices are $A\,(1,0,0)$, $B\,(0,1,0)$, and $C\,(0,0,1)$. **Hence**, deduce the distance from A to the line BC.

QUESTION 2. **(10 Marks)**

(a) Let $z = a + ai$, where a is a *negative* real number.

(i) Find the modulus and principal argument of z.

(ii) Show that z^{16} is a real number. What is that number?

Question 2 continues on Page 5.

(b) Let $z = -1 - i$. Plot the points z, z^3, z^5, z^7 on the Argand diagram below (Figure 2).

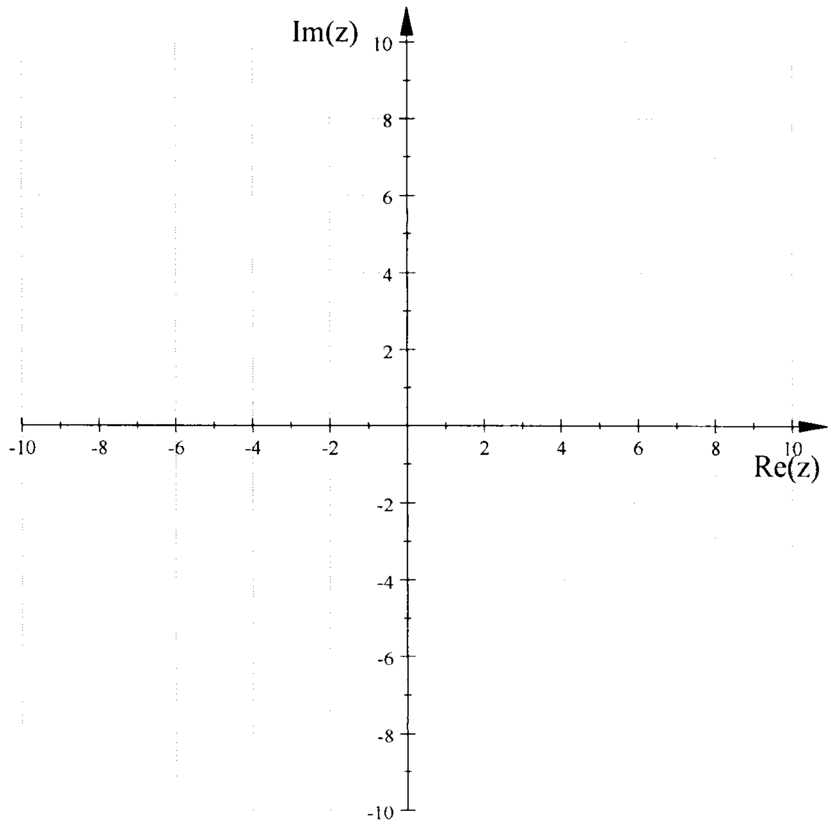

Figure 2

QUESTION 3. **(10 Marks)**

Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x & 0 & 1 & x \\ 1 & 0 & x & 0 \\ x & 0 & 0 & 1 \end{bmatrix}, \text{ where } x \text{ is a real number.}$$

(a) Show that determinant of A is $x^3 - x^2 + 1$.

Question 3 continues on Page 7.

(b) Show that A is singular for some value of $x \in (-1, 0)$. State the theorem used.

QUESTION 4. **(15 Marks)**

(a) Figure 3 shows the graph of a differentiable function $f(x)$. Given that $(1, -2)$ and $(-1, 2)$ are the local minimum and maximum of the graph, and the graph has a minimum gradient of -3 at $x = 0$, **sketch the graph of $f'(x)$ on the same diagram** (Figure 3).

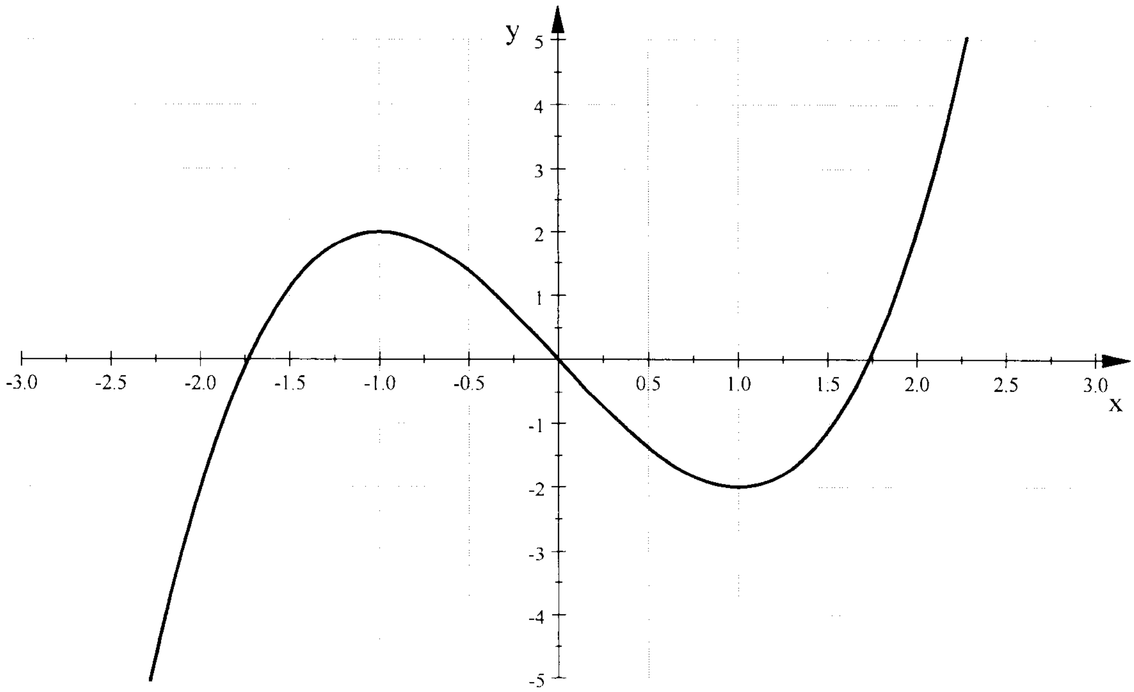

Figure 3

Question 4 continues on Page 9.

(b) Figure 4 is the graph of a piecewise linear function $f(x)$ (i.e., the graph of $y = f(x)$ is made up of straight lines).

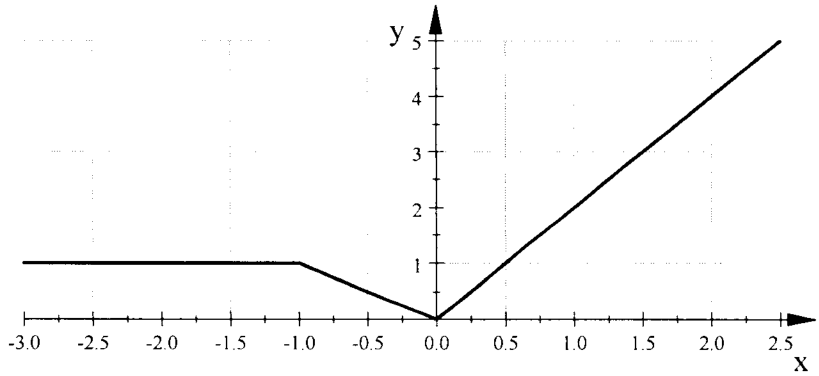

Figure 4

(i) State the definition of the derivative $f'(c)$ of $f(x)$ at $x = c$. Use **the definition of derivative** to show that $f'(0)$ does not exist.

(ii) Evaluate $\int_{-2}^{2} f(x)\, dx$.

QUESTION 5 **(15 Marks)**

(a)(i) State, without proof, the Mean Value Theorem.

(ii) If $f'(x) > 0$ for all $x \in \mathbb{R}$, **use the Mean Value Theorem** to show that f is an increasing function.

Question 5 continues on Page 11.

(b) Compute the following limits.

(i) $\displaystyle\lim_{x\to-\infty} \frac{x}{\sqrt{x^2+1}}$

(ii) $\displaystyle\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{x}{x-1}\right)$

QUESTION 6. **(15 Marks)**

(a) A street light is mounted at the top of a 6-metre-tall pole. A man 2 m tall is walking away from the pole at a speed of 2 m/s along a straight path. How fast is the tip of his shadow moving when he is 10 m away from the pole?

Question 6 continues on Page 13.

(b) Find the global maximum and minimum of the function $f(x) = \ln(x^2+1) + x$ on the interval $[-2, 2]$.

QUESTION 7. **(25 Marks)**

(a) Evaluate the following integrals.

(i) $\displaystyle\int \frac{x+2}{x^2+5x-6}\,dx,$

(ii) $\displaystyle\int x\tan^{-1} x dx.$

Question 7 continues on Page 15.

(b) Find the values of p for which the improper integral $\int_e^{\infty} \frac{1}{x(\ln x)^p}dx$ converges and evaluate the integrals for those values of p.

Question 7 continues on Page 16.

(c) Use Simpson's Rule and Trapezoidal Rule with $n = 10$ to approximate $\int_0^1 \frac{1}{x+1} dx$.

Show that S_{10} is a better approximation to the actual value of $\int_0^1 \frac{1}{x+1} dx$ than T_{10}.

END OF PAPER

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\left[y_0 + 2\,(y_1 + y_2 + \cdots + y_{n-1}) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \quad \approx \quad S_n = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right],$$

where n is even.

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \text{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\text{csch}^2 x$$

$$\frac{d}{dx}(\text{sech } x) = -\text{sech} x \tanh x \qquad \frac{d}{dx}(\text{csch } x) = -\text{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin x \, dx = -\cos x + C \qquad \int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C \qquad \int \csc^2 x \, dx = -\cot x + C$$

$$\int \tan x \sec x \, dx = \sec x + C \qquad \int \cot x \csc x \, dx = -\csc x + C$$

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \cot x \, dx = \ln|\sin x| + C$$

$$\int e^x \, dx = e^x + C \qquad \int a^x \, dx = \frac{a^x}{\ln a} + C,\ a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C,\ |x| < |a| \qquad \int \frac{1}{x^2+a^2} \, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}} \, dx = \sinh^{-1} x + C \qquad \int \frac{1}{\sqrt{x^2+a^2}} \, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$