

Digital
Logic

Analog vs Digital

- Changes in a continuous manner

↳ Day to night

↳ Temperature increase

Sampling & quantisation

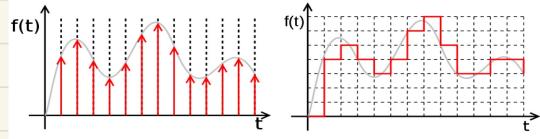
- Δ of quantity are discrete

↳ 12:34 pm \rightarrow 12:35 pm

$f(t)$ is an analog signal continuously varying with time (gray curve)

Sampling $f(t)$ at periodic intervals will generate the discrete time signal (red arrows)

Quantisation of the discrete time signal will produce the digital signal (red lines)



Logic Gates

Boolean Constants

↳ 2 values

TRUE / FALSE

Logic HI / logic LO

1 / 0

Waveform diagrams



Boolean Laws

$$a + ab = a(1+b)$$

$$= a$$

$$a + a'b = (a+ab) + a'b$$

$$= a + b(a+a')$$

$$= a + b$$

$$\begin{aligned} ab + a'c + bc &= ab + a'c + bc(a+a') \\ &= a(b+b)c + a'(c+bc) \\ &= ab + a'c \end{aligned}$$

$$X = a + b$$

$$X' = (a+b)'$$

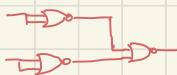
$$= a' \cdot b'$$

$$Y = a \cdot b$$

$$Y' = (a \cdot b)'$$

$$= a' + b'$$

NOR



$$(a' + b')' = a \cdot b$$

NA And

0	0	1
0	1	1
1	0	0
1	1	0

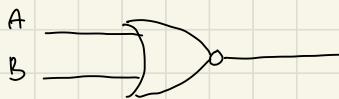
$$X = (A \cdot B)'$$



NOR

0	0	1
0	1	0
1	0	0
1	1	0

$$X = (A + B)'$$



Not Gate

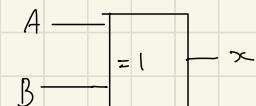
$$\begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \quad X = A'$$



XOR Exclusive-OR Gate

$$X = AB' + A'B$$

$$X = A \oplus B$$



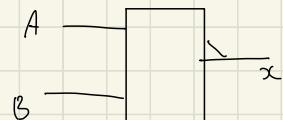
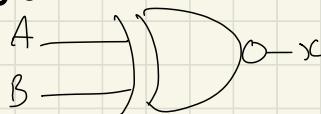
0	0	0
0	1	1
1	0	0
1	1	0

When $A \neq B, X=1$
 $A = B, X=0$

XNOR Exclusive-NOR Gate

$$X = AB + A'B'$$

$$X = (A \oplus B)'$$

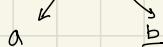


0	0	1
0	1	0
1	0	0
1	1	1

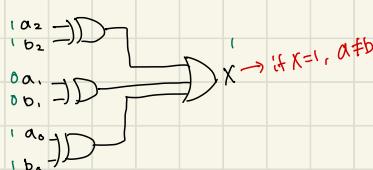
When $A = B, X=1$
 $A \neq B, X=0$

XOR Application

Compare two circuits : Bitwise comparator



Output 1 when
 $a \neq b$
// let $a : a_2, a_1, a_0$
let $b : b_2, b_1, b_0$
Comparing multi-bit input

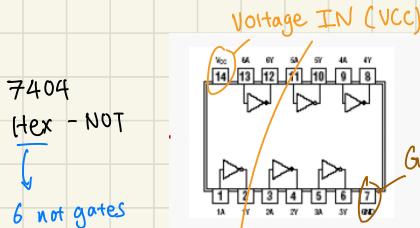
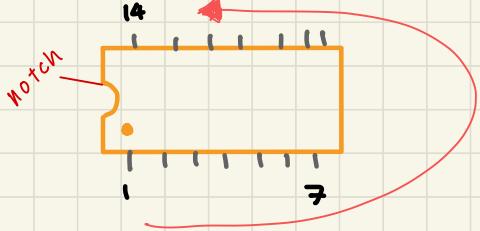


$$A \oplus B \oplus C = \text{Only } 1$$

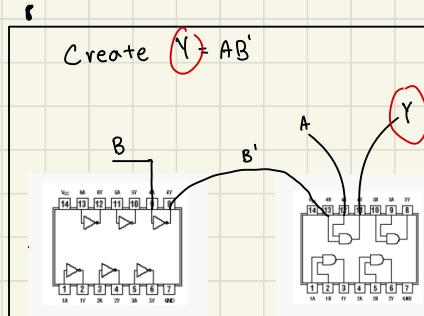
when odd number
of ones!

Physical Logic Devices

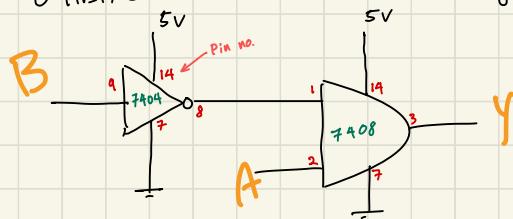
integrated circuits



7408
QUAD - AND
4 and gates



① first, Create circuit connection diag.



Parity Bit

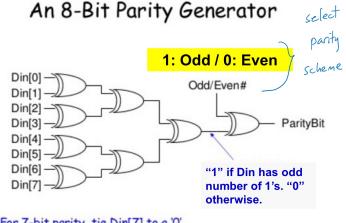
Even parity: Add a bit that makes it Even

Odd parity: Add a bit that makes it Odd

Even: 011011

Odd:
111011
now become
odd num. of ones

An 8-Bit Parity Generator



If odd number of

For 7-bit parity, tie Din[7] to a '0'

Digital Arithmetic

* Begin with LSB

$$10111_2 + 1010_2 = \frac{?}{2}$$

$$\begin{array}{r} 10111_2 \\ + 1010_2 \\ \hline 100001_2 = 33_{10} \end{array}$$

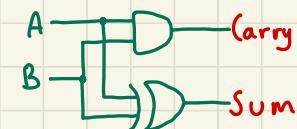
Digital circuits can also do this

> HALF adder (HA)

- logic circuit that adds two bits

Carry : AB
Sum : $A \oplus B$

		Inputs	Outputs		
		A	B	Carry	Sum
0	0			0	0
0	1			0	1
1	0			0	1
1	1			1	0



> Full Adder (FA)

Sum : $A \oplus B \oplus C$

Carry out : $A \cdot B + B \cdot C + A \cdot C$

as long as two inputs are 1,
carry out = 1

Inputs			Outputs	
A	B	Cin	Cout	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

Carry input

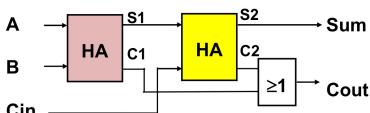
$$1+1+1=3$$

$$11_2=3$$

Carry Out : $A \cdot B + B \cdot C + A \cdot C$

$$\begin{aligned}
 &= A \cdot B + (A'B + AB)C + (B'A + BA)C \\
 &= A \cdot B + CAB + CBA + CA'B + CB'A \\
 &= A \cdot B + CAB + C(A'B + AB') \\
 &= A \cdot B + C(A \oplus B)
 \end{aligned}$$

Full adder circuit implemented using
2 half-adders and an OR gate

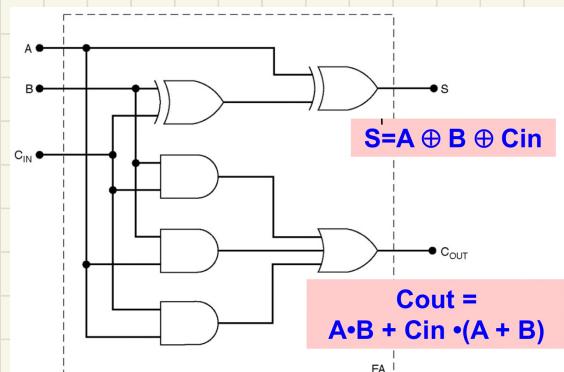


$S1 = A \oplus B$

$C1 = A \cdot B$

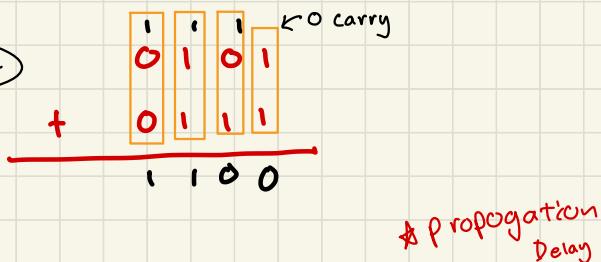
$S2 = (A \oplus B) \oplus Cin$

$C2 = Cin \cdot (A \oplus B)$

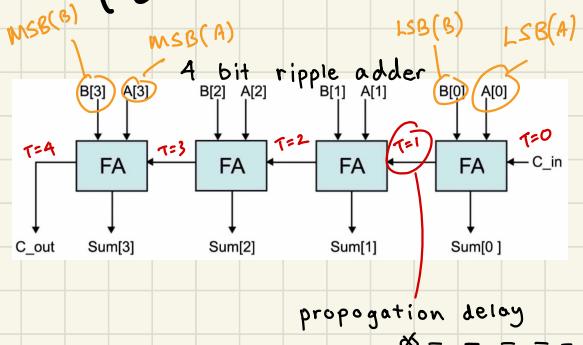


Binary Addition of: $0101_2 + 0111$

Each FA has 3 values, the carry and the 2 bits



Parallel Adder



Ripple adder, where full adders are cascaded...

Signed numbers

Sign Magnitude System

Sign bit = MSB : 0 for +ve numbers
MSB: 1 for -ve numbers

Range For N bits
 $-(2^{N-1})$ to $(2^{N-1} - 1)$

14: 0 1110
-14: 1 1110

2's complement system

Sign bit = MSB: 0 for zero/+ve
MSB: 1 for -ve

Range For N bits
 $-(2^{N-1})$ to $(2^{N-1} - 1)$

14: 0 1110

-14: 1 0010

- ① Invert every bit
... perform 1's complement

$$1110 \rightarrow 0111$$

- ② Add (arithmetic) 1 to it

$$\begin{array}{r}
 0111 \\
 + \quad 1 \\
 \hline
 0010
 \end{array}$$

1110 is the 2's complement of 0010

- ① From LHS,
copy 0s until you hit
the first 1 bit
- ② Invert the remaining bits

Shortcut method for 2's complement
This comes from $x 1176$

$$\begin{array}{r}
 010010011000 \\
 \downarrow \downarrow \downarrow \downarrow \\
 101101101000 \\
 \hline
 -1176
 \end{array}$$

Two's Complement

Decimal	Binary magnitude	2's comp	Decimal	2's comp	Decimal Values Special Bit Patterns
-8	1000	1000	$-(2^{N-1})$	1000...00	$-(2^{N-1})$ 100...000, N-1 zeros
-7	0111	1001	$-(2^{N-1}) + 1$	1000...01	-1 All Ones
-6	0110	1010	$-(2^{N-1}) + 2$	1000...10	0 All zeros
-5	0101	1011	.	.	$2^{N-1} - 1$ With N-1 ones
-4	0100	1100	.	.	
-3	0011	1101	.	.	
-2	0010	1110	.	.	
-1	0001	1111	-1	1111...11	
0	0000	0000	0	0000...00	
1	0001	0001	1	0000...01	
2	0010	0010	2	0000...10	
3	0011	0011	.	.	
4	0100	0100	.	.	
5	0101	0101	.	.	
6	0110	0110	$(2^{N-1}) - 2$	0111...10	
7	0111	0111	$(2^{N-1}) - 1$	0111...11	

Converting 2's Comp to Binary

+ve, Perform normal conversion $01001 = 9$

-ve, Perform 2's Comp to convert to +ve $10011 = 01101$
 Perform normal conversion $= -13$

Sign extension

When there are more bits necessary to represent binary,

> MSB will be filled with sign bits.

$$101 = 1101 = 11101 \quad 011 = 0011 = 00011$$

Also, 2's Compliment will Δ -ve/+ve w/o Δ magnitude

> Except for most -ve value (10000)

> -8 in 4-bit system } Because 8, 16 & 128
 -16 " 5-bit " } don't exist
 -128 " 8-bit .. }

Why use 2's?

- Same circuit for addition & subtraction

Universality!
Less space!
Cheaper!

$$A - B = A + (-B)$$

↓
2's complement

ADDITION IN 2's

+10	0	1	0	1	0	(augend)
+3	0	0	0	1	1	(addend)
+13	0	1	1	0	1	(sum)

Sign bit must be aligned

+10	0	1	0	1	0	
-3	1	1	1	0	1	
+7	1	0	0	1	1	

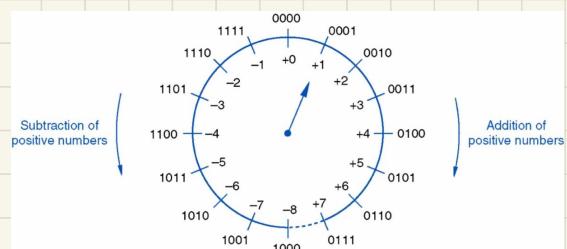
Ignore carry out sign bit

11101 Sign extension

Only look at relevant bits

+10	0	1	0	1	0
-3	1	1	1	0	1
+7	1	0	0	1	1

Ignore carry out sign bit



Arithmetic Overflow

- When operation between two numbers produces result

that exceeds current bit. 4-bit: (-8 to 7) cannot do $-7 - 5 = \underbrace{-12}_{5\text{-bit}}$

$$\begin{array}{r}
 101010 \\
 + 01000 \\
 \hline
 10010
 \end{array}$$

Case 1  $x - y$ Always OK

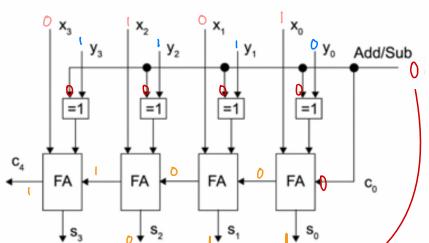
Case 2  $-x - y$ Overflow!
Sign bit +ve

Case 3  $x + y$ Overflow!
Sign bit -ve

Using full adders

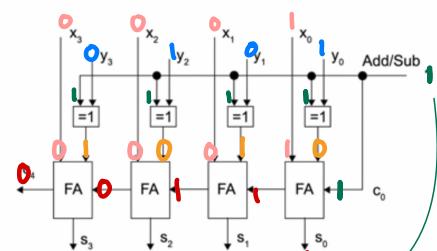
Addition

$$x_3 x_2 x_1 x_0 + y_3 y_2 y_1 y_0$$



$$\begin{array}{r} & & & 0 \\ & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 & 1 \end{array}$$

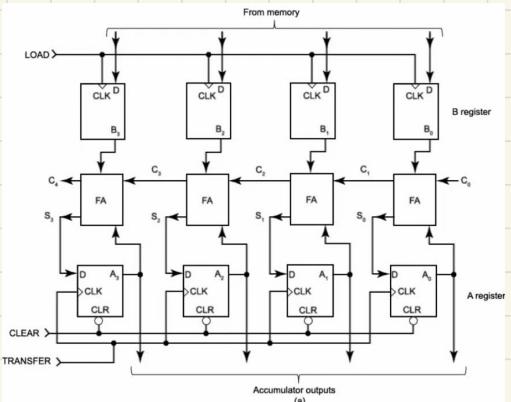
Subtraction



$$\begin{array}{r} & & & 0 \\ & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ \hline 1 & 1 & 0 & 0 \\ \hline 0 & 1 & 1 & 1 \end{array} \Rightarrow 0100 = -4$$

Register

- Memory storage to store bits
- Timing signals control circuit sequence

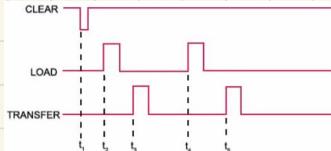


t1: CLEAR* clears the contents of A register to 0's

t2: PGT of first LOAD pulse transfers operand X from memory into B register

t3: PGT of first TRANSFER pulse transfers FA output (=X) into A register
t4: PGT of second LOAD pulse transfers operand Y from memory into B register

t5: PGT of second TRANSFER pulse transfers FA output (=X+Y) into A register



Overflow if

$$B[3] = A[3] \text{ but } \neq S[3]$$

OR

$$C_{\text{out}} \neq C_{\text{in}}$$

Binary Multiplication

$$\begin{array}{r}
 10100 \\
 \times 1011 \\
 \hline
 10100 \\
 + 10100 \quad \text{add individually} \\
 + 00000 \quad \text{copy 0, all zeros.} \\
 + 10100 \quad \text{X X X} \\
 \hline
 10100
 \end{array}$$

* Multiplying two numbers of m-bits will not exceed 2^m -bits of memory.

-Ve Binary

Option One: Convert Signed to Unsigned, then multiply

Option Two: If multiplier is -ve

i) Treat value as sum of 2 parts

$$\begin{array}{r}
 -3_{10}: 1101 \\
 \text{-ve part} \qquad \text{+ 5} \\
 \hline
 -8, 1000 \qquad +5, 0101
 \end{array}$$

$$\begin{array}{r}
 1011 \\
 \times 1(101) \\
 \hline
 "11111011 \\
 0000000x \\
 111011xx \\
 00101xx \\
 \hline
 00000111
 \end{array}$$

shifted multiplicands with sign extension
a change in sign through 2's compliment of 1011

BCD addition

adding bcd, the result may be more than 9

> a correction needs to be made by adding 6 (0110)

Two Steps

The correction involves two steps:

- a carry of decimal value 1 is brought forward and added to the next higher digit
- the decimal value 6 is added to the sum to obtain the correct BCD digit

$$24_{10} + 47_{10} = 71_{10}$$

$$\begin{array}{r}
 0010 \quad 0100 \\
 2 \quad 4 \\
 + 0100 \quad -0111 \\
 \hline
 0110' \quad 1'011 \quad \text{Exceeds 9,} \\
 + 0110 \quad \text{Thus add 6 to it,} \\
 \hline
 0111 \quad 0001 \quad \text{Skipping the illegal part}
 \end{array}$$

How many FA needed?

xxxx xxxx xxxx ← Base FA

(x)(x) (x)(x) (x)(x) ← Correction FA

These can be half adders

These FA's are not used due to +0110

Combinational logic circuit

- No memory, it is a "reactive" circuit
- Unlike sequential circuits that have memory
- USE TRUTH TABLE, then can implement

Self study
Review

Boolean Expression

- ALSO KNOWN AS
 - ↳ Boolean Equation
 - ↳ logic function

Canonical Form

① Sum of minterm expression

② Product of maxterm expression

Minterm: All possible combinations formed by AND

maxterm: All possible combinations formed by OR

Using AND, what result gives $m_0 = 1$

Convert number to
dec = M2

inputs			minterms		maxterms	
X	Y	Z	$X' \cdot Y' \cdot Z'$	m_0	$X + Y + Z$	M_0
0	0	0	$X' \cdot Y' \cdot Z'$	m_0	$X + Y + Z$	M_0
0	0	1	$X' \cdot Y' \cdot Z$	m_1	$X + Y + Z'$	M_1
0	1	0	$X' \cdot Y \cdot Z'$	m_2	$X + Y' + Z$	M_2
0	1	1	$X' \cdot Y \cdot Z$	m_3	$X + Y' + Z'$	M_3
1	0	0	$X \cdot Y' \cdot Z$	m_4	$X' + Y + Z$	M_4
1	0	1	$X \cdot Y' \cdot Z$	m_5	$X' + Y + Z'$	M_5
1	1	0	$X \cdot Y \cdot Z'$	m_6	$X' + Y' + Z$	M_6
1	1	1	$X \cdot Y \cdot Z$	m_7	$X' + Y' + Z'$	M_7

Minterm: Achieve 1.

any 0s will prime

Maxterm: Achieve 0

any 1s will prime

Using OR, what will
make $M_2 = 0$

For N-inputs, there will be 2^N minterms

- e.g. 4 inputs: a, b, c, d
 - 13 in decimal = 1101 in binary
 - Then minterm $m_{13} = a \cdot b \cdot c \cdot d'$
 - maxterm $M_{13} = a' + b' + c + d'$
 - 2 in decimal = 0010 in binary
 - Then minterm $m_2 = a \cdot b' \cdot c \cdot d'$
 - maxterm $M_2 = a + b + c' + d$

inputs			minterms		maxterms		F
X	Y	Z	$X' \cdot Y' \cdot Z'$	m_0	$X + Y + Z$	M_0	0
0	0	0	$X' \cdot Y' \cdot Z'$	m_0	$X + Y + Z$	M_0	0
0	0	1	$X' \cdot Y' \cdot Z$	m_1	$X + Y + Z'$	M_1	1
0	1	0	$X' \cdot Y \cdot Z'$	m_2	$X + Y' + Z$	M_2	1
0	1	1	$X' \cdot Y \cdot Z$	m_3	$X + Y' + Z'$	M_3	0
1	0	0	$X \cdot Y' \cdot Z'$	m_4	$X' + Y + Z$	M_4	1
1	0	1	$X \cdot Y' \cdot Z$	m_5	$X' + Y + Z'$	M_5	0
1	1	0	$X \cdot Y \cdot Z'$	m_6	$X' + Y' + Z$	M_6	0
1	1	1	$X \cdot Y \cdot Z$	m_7	$X' + Y' + Z'$	M_7	1

How to compute?

① Find relevant $m_n \& M_n$

② For minterms, $\&$ Look for output = 1

$$m_1 + m_2 + m_4 + m_7, \sum_{xyz} (1, 2, 4, 7)$$

\hookrightarrow Odd term, active High

③ For maxterms, $\&$ Look for output = 0

$$(M_0) (M_3) (M_5) (M_6), \prod_{xyz} (0, 3, 5, 6)$$

\hookrightarrow even term, active Low

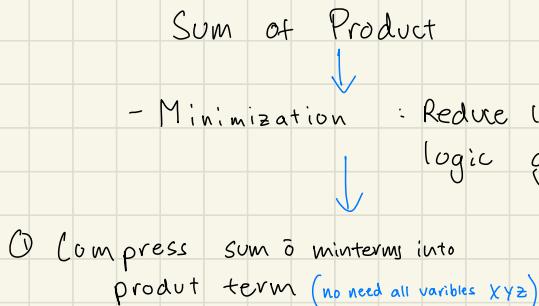
$$F = X'Y'Z + X'YZ' + XY'Z' + XYZ \quad \text{SOM}$$

$$= \Sigma_{XYZ} (1, 2, 4, 7)$$

$$F = (X+Y+Z) (X+Y'+Z') (X'+Y+Z') (X'+Y'+Z) \quad \text{POM}$$

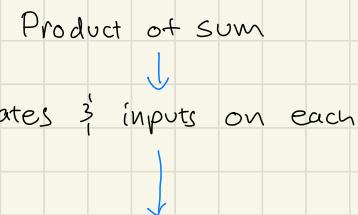
$$= \prod_{XYZ} (0, 3, 5, 6)$$

Standard Form of Boolean Exp



example:
This is a sum-of-minterms expression:
 $f(x, y, z) = xyz' + xyz + x'y'z + xy'z$

Simplifying, we get
 $f(x, y, z) = xy(z' + z) + (x' + x)y'z$
 $= xy + y'z$



example:
This is a product-of-maxterms expression:
 $f(x, y, z) = (x+y'+z')(x+y'+z)(x+y'+z)(x'+y+z)$

Simplifying, we get
 $f(x, y, z) = (x + y')(x' + z)$

These are **neither SOP nor POS expressions:**

$$f = (xy)'z + xz'$$
 not a product term

$$f = xy(x' + z)'$$
 not a sum term

$$f = (xy + z)(x' + y)$$
 not a sum term

WAYS TO SIMPLIFY

- Algebraic method
- Karnaugh map (K-map)
- Quine-McCluskey method (Q-M method or tabulation method)
- Heuristic methods, e.g. Espresso-II

Karnaugh Map

I) Based on $AB + AB' = A(B+B') = A$

$$\begin{aligned} (A+B)(A+B') &= AA + AB + AB' + BB' \\ &= A + A + 0 \\ &= A \end{aligned}$$

- K-map squares are labelled such that adjacent squares differ only in one variable.
- SOP expression for output X can be obtained by ORing together those squares that contain a 1.
- Can also obtain POS expression by ANDing together those squares that contain a 0.
- Note the correspondence with SOM (think 1) and POM (think 0).

A	B	C	D	X
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

		$X = A'B'C'D + A'BC'D + ABC'D + ABCD$				
		X	00	01	11	10
		C, D				
	A, B					
		00	0	1	0	0
		01	0	1	0	0
		11	0	1	1	0
		10	0	0	0	0

look! Only one var change:

$C, D = 0, 0$
 $C, D = 0, 1$ Change
 $C, D = 1, 1$ Done at a Time
 $C, D = 1, 0$

loop neighbouring Ones

X	00	01	11	10
00	0	1	0	1
01	0	1	0	0
11	0	1	1	0
10	1	0	0	1

A, B

Loop 4 neighbouring Ones

X	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	1

A, B

Loop 8 neighbouring Ones

X	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

A, B

- Only can loop 2^n neighbours together.
- No diagonal looping
- Loops can be more than once, "double dip"
- Similarly, looping zeros to obtain POS is possible, giving a sum term

Don't Care Conditions

Example: Design a logic circuit whose input is a BCD digit, and whose output goes HIGH if the input is smaller than 6₁₀

A	B	C	D	Z
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X
1	1	1	1	X

$Z \leq 6$

Treat X as one \neq zero:
it doesn't matter!

CD				
Z	00	01	11	10
00	1	1	1	1
01	1	1	0	0
11	X	X	X	X
10	0	0	X	X

AB

$$Z = A'B' + A'C'$$

CD				
Z	00	01	11	10
00	1	1	1	1
01	1	1	0	0
11	X	X	X	X
10	0	0	X	X

AB

$$Z = B'C + A'C'$$

CD				
Z	00	01	11	10
00	1	1	1	1
01	1	1	0	0
11	X	X	X	X
10	0	0	X	X

AB

$$Z = A'(B'+C')$$

Enable / Disable Circuits

Enable: Output can Δ based on Input

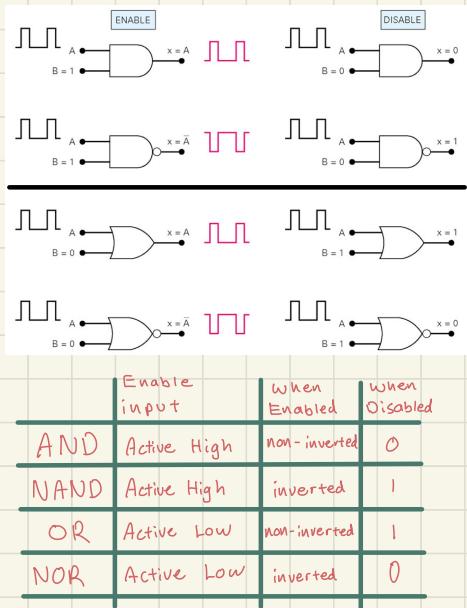
Disable / Inhibit: Output cannot Δ , fixed @ 0 or 1

Basically, it's like a "child lock"

Inputs		Output	Effect of output
Child Safe	Unlock	Open	
0	X	0	Door closed
1	0	0	Door closed
1	1	1	Door opened

X: "don't care", i.e. can be 0 or 1

- Circuit is enabled when ChildSafe=1; the output Open changes with input Unlock.
- Circuit is disabled when ChildSafe=0; Open is stuck at 0.



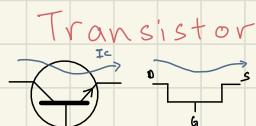
Digital Circuits

- Transistors, resistors, diodes on silicone chip.

Diodes



- One direction
- Includes LED



- Can be used as switch

TTL \rightarrow old, but

- 1) Simple fabrication
- 2) Low power dissipation
- 3) Transistor only
- 4) packing density
- 5) Greater fan out

TTL family

- $V_{CC} @ 5\text{V}$

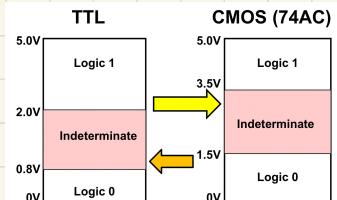
- Unconnected (floating) will act as logic 1, 1.4 to 1.8 volt

CMOS family

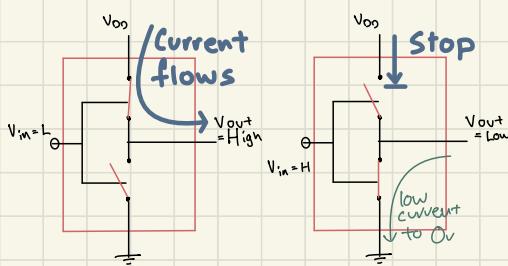
- $V_{DD} @ 3\text{V to } 18\text{V}$ (use 5V?)

- Floating pins are BAD
 - ↳ Overheating. All must connect to ground/V_{DD}

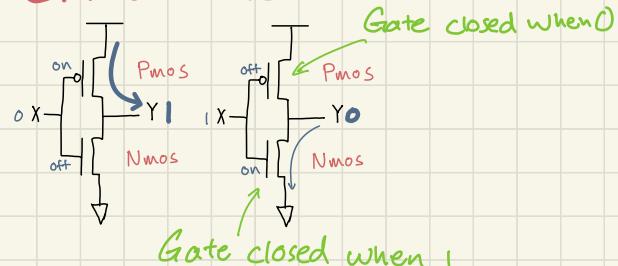
\rightarrow Some CMOS in compatible
operating in indeterminate



TTL Inverter



CMOS Inverter



Active high / Active low

	Active Low	Active High
READY-	READY+	
ERROR.L	ERROR.H	
ADDR15(L)	ADDR15(H)	
RESET*	RESET	RESET
ENABLE-	ENABLE	
-GO	GO	
/RECEIVE	RECEIVE	
TRANSMIT_L	TRANSMIT	

for this mod,
reset* = low
reset = high

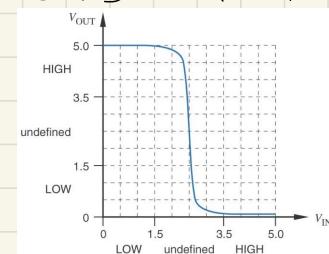
	Active High	Active Low
asserted	1	0
negated	0	1

Signal name	Effect/result when signal=0	Effect/result when signal=1	Active high/low
Subtract	Add Y to X	Subtract Y from X	Active high
Add*	Add Y to X	Subtract Y from X	Active low
Unmute	Mute a speaker	Unmute a speaker	Active high
Unmute*	Unmute a speaker	Mute a speaker	Active low
Read	Write data	Read data	Active high
Write*	Write data	Read data	Active low

Active high or active low depends on the signal name and effect/result

Transfer Characteristics

CMOS IN/OUT



* CURRENT is similar

V_{OH} min out volt for High e.g. 3.5v and above

V_{IH} min input volt for High e.g. 2.0v

V_{OL} max output volt for Low

V_{IL} max input volt for Low

Basically: $V_{OH} > V_{IH}$ Output > input for High

$V_{OL} < V_{IL}$ Output < input for Low

Noise Margin

$V_{OH} \rightarrow$ Noise ↑ voltage → V_{IH}
 $\hookrightarrow V_{IL}$ (noise dips volt)

$$\text{High (Noise margin)} = V_{OH}(\text{min}) - V_{IH}(\text{min}) \\ = 3.5 - 2.0 \\ = 1.5 \text{ volt margin}$$

Logic Level	Driving		Receiving			
	STTL	5-V TTL	5-V CMOS	3.3-V LVTTL	2.5-V CMOS	1.8-V CMOS
0 (L)	Yes	No	Yes*	Yes*	Yes*	Yes*
1 (H)	Yes	Yes	Yes*	Yes*	Yes*	Yes*
5-V STTL	Yes	Yes	No	Yes	Yes	Yes
3.3-V LVTTL	Yes	No	Yes	Yes	Yes	Yes
2.5-V CMOS	Yes	No	Yes	Yes	Yes	Yes
1.8-V CMOS	No	No	No	No	No	Yes

* Requires V_{IH} Tolerance

5 V	V_{CC}	5 V	V_{CC}	Is V_{OH} higher than V_{OH} ? Is V_{OL} less than V_{OL} ?
4.44	V_{OH}	3.5	V_{IH}	D \rightarrow R
2.7	V_{OH}	2.5	V_{I}	
2.0	V_{IH}	2.0	V_{IL}	
1.5	V_I	1.5	V_{OL}	
0.8	V_{IL}	0.8	V_{OL}	
0.5	V_{OL}	0.5	V_{OL}	
0	GND	0	GND	

5-V LSTTL	V_{CC}	3.3-V LVTTL	V_{CC}	2.5-V CMOS	V_{CC}
2.7	V_{OH}	2.4	V_{OH}	2.5	V_{CC}
2.0	V_{IH}	2.0	V_{IL}	2.3	V_{OH}
1.5	V_I	1.5	V_{OL}	1.7	V_{OI}
0.8	V_{IL}	0.8	V_{OL}	0.9	V_{IL}
0.5	V_{OL}	0.4	V_{OL}	0.7	V_{OL}
0	GND	0	GND	0	GND

* for overall, will be smallest of the two

Fan Out (current related)

: How much gates can you connect to one output:
Current is "spread" amongst ICs

↑ connections will ↑ noise & ↓ switching speed

$$\text{DC fan out} = \frac{I_{OH}}{I_{IH}}$$

Always, $I_{out} \geq I_{in}$

Dynamic Switching

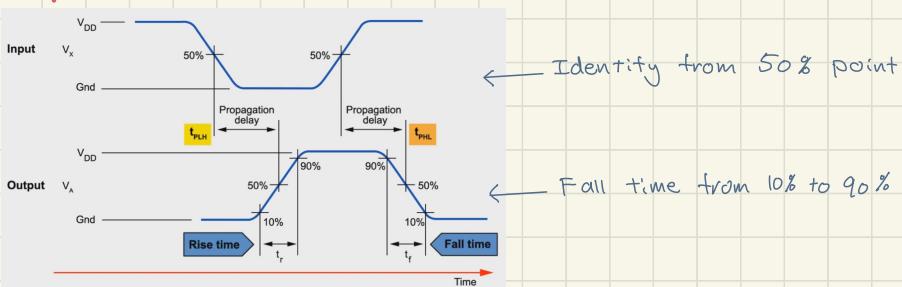
P, Power Dissipation: When IC switches between $1\frac{1}{2} \Omega$, power consumed
When not switching, Low dissipation

$$P = CV^2 f$$

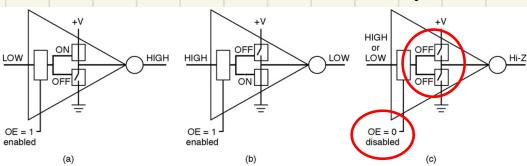
C = Constant f = switching frequency
V = Power supply voltage

propagation delay: time to transition between input & output (nanoseconds)

$$t_{PHL} = \text{High to Low time} \quad t_{PLH} = \text{Low to High time}$$



Tri state Output



3 logic states

① logic 0

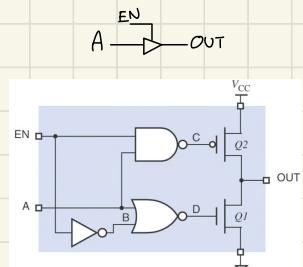
② logic 1

③ High impedance (Hi-Z)
→ Like an "open" circuit

EN	A	B	C	D	Q1	Q2	OUT
L	L	H	H	L	off	off	Hi-Z
L	H	H	H	L	off	off	Hi-Z
H	L	H	H	H	on	off	Hi-Z
H	H	L	L	L	off	on	H

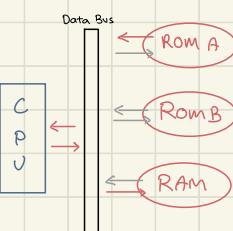
Enable High

CMOS Tristate Buffer

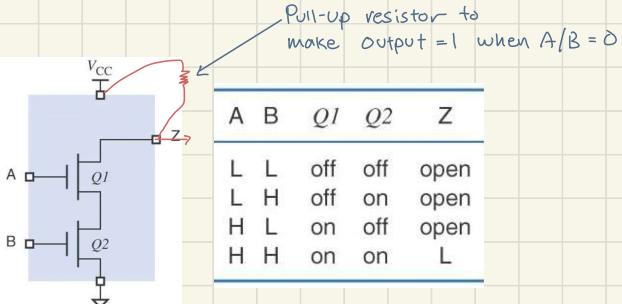
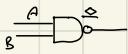


Advantage

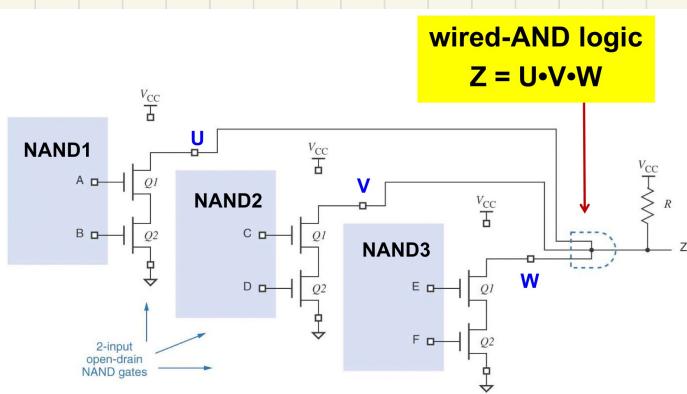
- Can connect output together
- ↳ Only one output is enabled (max)
- ↳ Bus Contention, damage to devices
(Used to connect memory modules, share bus)



Open Collector / Drain output



When A & B are high, $Z = 0$



Constructing "AND" gates using only wires?



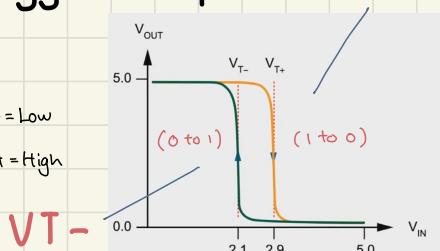
- 1) If any of the drains are open, provides flow through ground.
- 2) When all are 'closed', current has to flow through Σ

Schmitt-Trigger Inputs $VT+$

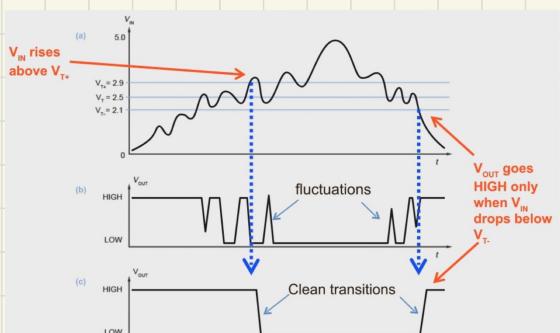
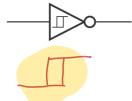
Input volt rise above $VT+$, $V_{out} = \text{Low}$

Input volt drops below $VT-$, $V_{out} = \text{High}$

* Hysteresis



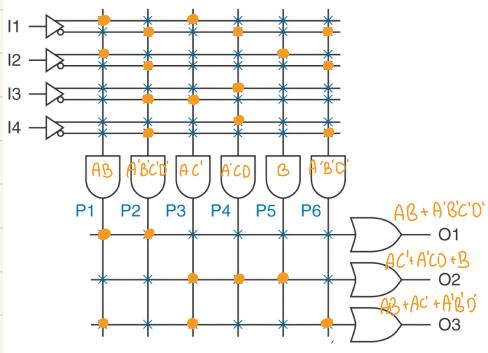
(b)



Usefull when V_{in} rises slowly, or fluctuating due to noise. Clean transition, Stable

* Useful: when output connected to state sensitive component that "activates" stuff

Combinational PLA



PLD = programmable logic devices

PLA = Programmable logic array

PAL = Programmable array logic

CPLD = Complex PLG

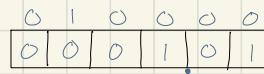
FPGA = Field Programmable Gate Array

Fixed-Point Numbers

→ Small numbers / integers
loose decimal places!

n number of bits, m number of bits

→ Predetermine number of bits for front & back



The system stores the decimal

Floating-Point Numbers

$$S \times R^E$$

Where

- S is the **significand**, usually a signed fractional fixed-point number
- R is the **radix** (e.g. 2 for binary, 8 for octal, 10 for decimal, etc.)
- E is the **exponent**, usually a signed fixed-point integer

Fixed Point

0 1 0 1 . 1 1 1 1

Example: the IEEE Standard 754 on floating-point arithmetic

Single precision (32 bits):



Double precision (64 bits):



- * is the sign bit of significand
- radix is 2
- significand is in sign-magnitude notation
- exponent is in 2's complement notation

Floating Point

1.0111 $\times 2^{(2)}$

- 1) First one is removed, as don't need to store bcz all numbers will start with 1
- 2) Take Radix, 0010
- 3) Take first 4 bits, 0111
- 4) Number = 0010 0111

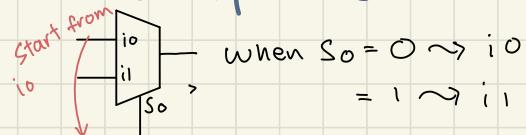
Problems of floating Point

- i) It is an "estimate" or range

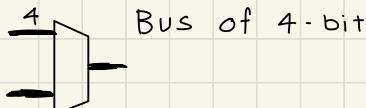
Combinational Circuits

- The input determine the output

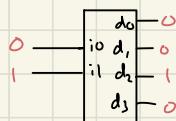
Multiplexer



when line is thick



Decoder



Sequential Circuits

Verilog (Hardware Design Language)

We declare a module in Verilog using the **module** keyword and a list of ports:

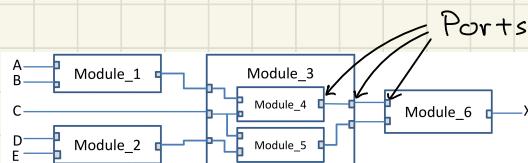
```
module somename (port1, port2, port3);
```

- The above declaration describes a module with three **ports**, each a single wire

We can indicate the direction of the ports using the keywords **input** and **output**:

```
module somename (input port1, port2,  
                  output port3);
```

- The above describes a module with two inputs and one output



Something like a function



```
module some_name (  
    input a, b, c,  
    output X);
```

// Describe your circuit here

```
endmodule
```

VL Structural Design

- This allows us to **structurally** describe any circuit we might otherwise use a circuit diagram for
- The principles learnt in the first half of the course allow us to describe any circuit in terms of these gates

x	y	c _i	c _o
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0
1	0	0	1
0	1	1	0
1	0	1	0
1	1	1	1

$S = X \oplus Y \oplus C_i$
 $C_o = X \cdot Y + X \cdot C_i + Y \cdot C_i$

Minimize using Boolean Algebra or K-Maps

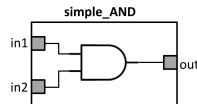
```
module full_adder (
    input X, Y, Ci,
    output S, Co);
    // Describe your structural
    // design for full adder here
endmodule
```

Truth table for Full Adder

For example: Case sensitive
end with ;

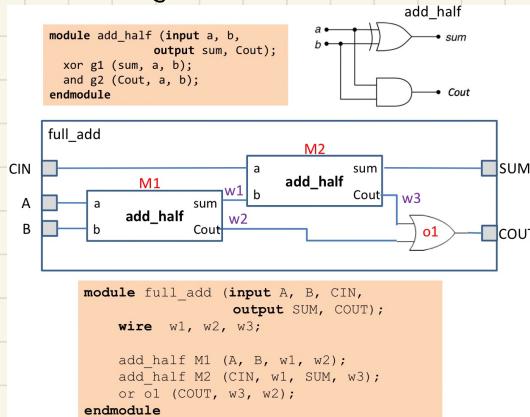
A simple module that contains just a two-input **and** gate would be written like this:

```
module simple_AND (
    input in1, in2,
    output out);
    and (out, in1, in2);
endmodule
```



Module instantiation

- Something like functions



```
module full_add (input A, B, CIN,
                  output SUM, COUT);
    wire w1, w2, w3;

    add_half M1 (A, B, w1, w2);
    add_half M2 (CIN, w1, SUM, w3);
    or o1 (COUT, w3, w2);
endmodule
```

Using Single Assign

$$X = (a \cdot b) + ((\neg b) \cdot (\neg d + e))'$$

$$\text{assign } x = (a \cdot b) \mid \sim(\neg b \cdot c) \cdot \sim(\neg d \cdot e);$$

Steps

A module definition - done

Some gates

Wires to connect those gates

Primitives Available

and(y, a, b)

or(z, a, b, c, d)

Wires (one bit)
wire int-signal

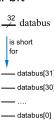
Multibit Wire

Verilog has a special construct for handling multi-bit signals (wires). Formed by specifying a range:

wire [31:0] databus;

Also for specifying multi-bit module ports

```
module add16 (input [15:0] a, b,
              output [15:0]sum,
              output cout);
    // add module internals here
endmodule
```

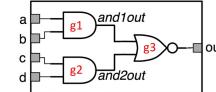


Using it together:

Let us consider an **and-or-inverter**

Boolean equation: $\text{out} = ((a \cdot b) + (\neg c \cdot \neg d))'$

- Note you can also use a unique **identifier** for each gate (this can help in testing):



```
module andorinv (input a, b, c, d,
                  output out);
    wire and1out, and2out;
    and g1 (and1out, a, b);
    and g2 (and2out, c, d);
    or g3 (out, and1out, and2out);
endmodule
```

Ordered instantiation

```
module add_half (input a, b,
                  output sum, cout);
    xor g1 (sum, a, b);
    and g2 (cout, a, b);
endmodule
```

add_half M1 (.a(A), .b(B), .sum(w1), .cout(w2));

add_half M2 (.a(CIN), .b(w1), .sum(SUM), .cout(w3));

Named instantiation (Better)

```
module add_half (input a, b,
                  output sum, cout);
    xor g1 (sum, a, b);
    and g2 (cout, a, b);
endmodule
```

add_half M1 (.a(A), .b(B), .sum(w1), .cout(w2));

add_half M2 (.a(CIN), .b(w1), .sum(SUM), .cout(w3));

Verilog Assignments

- We can use a range of operators:

- &&** : and (bitwise is: **&**)
- ||** : or (bitwise is: **|**)
- !** : not (bitwise is: **~**)
- ^** : xor (bitwise is: **^**)

Operator	Name
~	Bitwise NOT
&	Bitwise AND
 	Bitwise OR
^	Bitwise XOR
~&	Bitwise NAND
~ 	Bitwise NOR

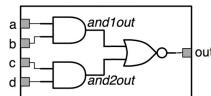
- The and-or-inverter example previously shown:

- The **assign statement** allows the use of more complex operators and operands – more on this later

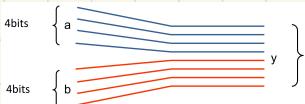
Instantiating
gate
primitives

```
wire and1out, and2out;
nor g3 (out, and1out, and2out);
and g2 (and2out, c, d);
and g1 (and1out, a, b);
```

```
assign out = ~((a&b) | (c&d));
Continuous assignment
```



Concatenation



```
wire [3:0] a, b;
wire [7:0] y;
...
assign y = {a, b};
```

Literal Assign

- <size>'<radix><value>**

- <size> is the width in bits
- <radix>: b for binary, o for octal, h for hex, d for decimal
- <value>: the number you want, with as many optional underscores as needed (for readability)

Examples:

- 4'b0000 (4 binary bits "0 0 0 0")
- 8'h4F (= 8'b01001111)
- 8'b0100_1111 (Same as above. Note the use of the underscore)
- 1'b1 (a single "1")

Parameter

```
module some_mod #(parameter SIZE=8, WIDTH=16)(
    input [SIZE-1:0] X, Y,
    output [WIDTH-1:0] Z);
```

- It is also possible to redefine a parameter

- Consider the following 8-bit module

```
module submod #(parameter SIZE=8)(
    input [SIZE-1:0] X, Y, output [SIZE-1:0] Z);
    // some statements in here
endmodule
```

- Then, note the change in the parameter in the submodule instantiation below

redefine

- Now using two 16-bit submodules

```
module top_mod #(parameter SIZE=16) (
    input [SIZE-1:0] a, b, c, output [SIZE-1:0] D, E);

    submod #(.SIZE(SIZE)) U1 (.X(a), .Y(b), .Z(D));
    submod #(.SIZE(SIZE)) U2 (.X(c), .Y(b), .Z(E));

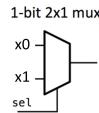
endmodule
```

In practice, it may be better to call them different names to avoid confusion

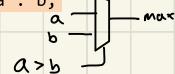
Conditional Assignment

```
assign y = sel ? x1 : x0; // a multiplexer
```

The signal **y** will be connected to **x1** if **sel** is 1, else it is connected to **x0**



```
assign max = (a>b) ? a : b;
```



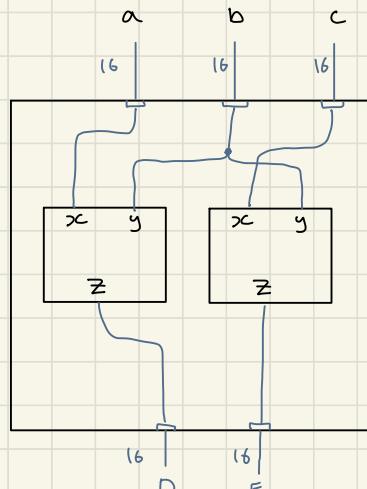
Replication

assign z = {4{c}} \rightarrow cccc

→ assigning literal to

1) Larger bus, MSB padded with 0

2) Smaller bus, MSB cut off



```

module adder #(parameter SIZE=32)(  
    input Cin,  
    input [SIZE-1:0] A, B,  
    output Cout,  
    output [SIZE-1:0] Sum);  
  
    assign {Cout, Sum} = A+B+Cin;  
  
endmodule

```

- Note:**
- How the concatenation operator handles the 33-bit result produced by A+B+Cin
 - The use of the arithmetic addition operator (+) to generate the sum.

Behavioural Modeling

Combinational Always Block

```

always @ (a, b)  
begin  
    x = a & b;  
    y = a | b;  
end

```

Sensitivity List
Procedural statements

The **always** keyword starts a block

The **sensitivity list** must contain the names of any signals that affect the output of the block

always @ * JUST USE *

Reg (Behaves like variable in code)

Signals you assign to from within an **always block**

- Must be declared as being of type: **reg**

reg is synonymous with **wire**, but these signals can be assigned to from inside an always block, **wires** cannot!

A **Wire** is simply a **connection**, it holds no value of its own

The **reg** type is more like a **variable** in programming, as we will see

Note: you *cannot* assign to a **reg** signal using an **assign statement**, or connect it to module instance outputs.

```

module temp (input a, b,  
             output out);  
  
    wire w1;  
  
    assign w1 = a & b;  
    assign out = w1;  
  
endmodule

```

```

module temp (input a, b,  
             output out);  
  
    reg w1;  
  
    always @ *  
        w1 = a & b;  
  
    assign out = w1;  
endmodule

```

```

module temp (input a, b,  
             output out);  
  
    reg w1;  
  
    always @ *  
        w1 = a & b;  
        out = w1;  
    assign out = w1;  
endmodule

```

I f intend assignment

Declare reg signals:
• You can set an initial value when you declare reg signals

```
reg a, b;
```

```
reg [3:8] x = 4'b0000;
```

You can also declare your module outputs as **reg** if you plan to assign to them directly from within an always block:

```
module sel_one (input [5:8] a, b, c, d,  
                input [1:8] sel,  
                output reg [5:8] sigsel);
```

If Else

```

always @ *  
begin  
    if (alarm == 1'b1)  
        begin  
            if (after_hours)  
                siren = 1'b0;  
            else  
                siren = 1'b1;  
            light = 1'b1;  
        end  
    end  
    else  
        begin  
            siren = 1'b0;  
            light = 1'b0;  
        end  
end

```

If only (ifelse), no need begin ;

Default for any exception

Case Statement

```

always @ *  
case (sel)  
    2'b00 : y = a;  
    2'b01 : y = b;  
    2'b10 : y = c;  
    default : y = 4'b1010;  
endcase

```

```

always @ *  
case (sel)  
    2'd0 : y = a;  
    2'd1 : begin  
        y = b;  
        z = c;  
    end  
    2'd2 : y = c;  
    default : y = 4'b1010;  
endcase

```

There is no need for a break in each branch (unlike some software languages)

```

module mux4 (output reg q,  
             input [3:0] d,  
             input [1:0] sel);  
  
    always @ * begin  
        case (sel)  
            2'b00 : q = d[0];  
            2'b01 : q = d[1];  
            2'b10 : q = d[2];  
            2'b11 : q = d[3];  
        endcase  
    end  
endmodule

```

1-bit 4x1 mux

Note the begin and end. These are not needed as a case is a single statement. But makes it easier to read!!

- Example: 3-to-8 decoder:

Input	Output
000	00000001
001	00000010
010	00000100
011	00001000
...	...
111	10000000

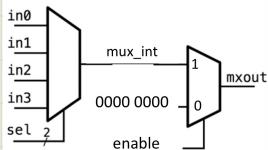
Reg 7

```

module decoder3_8(output reg[7:0] d_out,  
                   input [2:0] ival);  
  
    always @ *  
        case(ival)  
            3'b000 : d_out = 8'b00000001;  
            3'b001 : d_out = 8'b00000010;  
            3'b010 : d_out = 8'b00000100;  
            3'b011 : d_out = 8'b00001000;  
            3'b100 : d_out = 8'b00010000;  
            3'b101 : d_out = 8'b00010000;  
            3'b110 : d_out = 8'b00100000;  
            3'b111 : d_out = 8'b10000000;  
        endcase  
    endmodule

```

Example



```
module mux_4_32(output [7:0] mxout,
                  input [7:0] in3, in2,
                  input [7:0] in1, in0,
                  input [1:0] sel,
                  input enable);
  reg [7:0] mux_int;
  assign mxout = enable ? mux_int : 8'd0;
  always @ *
    begin
      case(sel)
        2'b00 : mux_int = in0;
        2'b01 : mux_int = in1;
        2'b10 : mux_int = in2;
        2'b11 : mux_int = in3;
      endcase
    end
endmodule
```

Always assign all values, or else the loop will "stove" a value and it will no longer be a combi circuit as output not 100% reliant on inputs provided

Latch

One way to fix this is to use a default assignment at the top of the **always** block:

```
always @*
begin
  y = x;
  if(valid) begin
    c = a | b;
    y = z;
  end
  else
    c = a;
end
```

The default is overwritten by any subsequent assignment
Assignments at the top of an **always** block are hence a great way of remembering to assign to all your signals

Statement Order

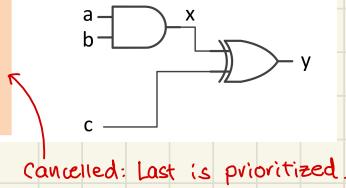
always @ *

begin

x = a & b;

y = x | c;

y = x ^ c;

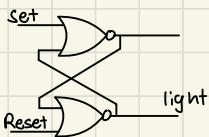


Cancelled: Last is prioritized.

Sequential Circuits

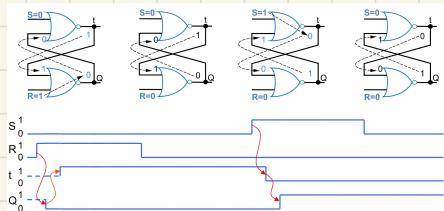
- Can store information

SR Latch



- Most basic to store

S	R	Light
1	0	1
0	1	0
0	0	Store



Horny

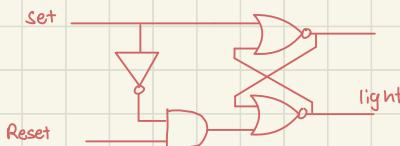
Iaozai

Hospital

* If both asserted & de-asserted

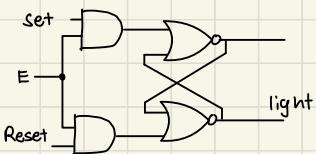
at the same time, the circuit will

oscillate and flicker



However these inputs will have a propagation delay as the set inverts & the AND output turns to zero

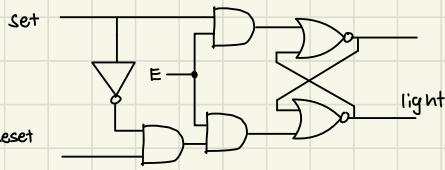
Enable SR latch (call button)



- Only works when $E=1$

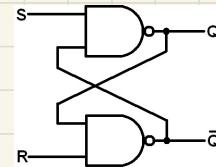
E	S	R	Q+	Function
0	X	X	Q	Store
1	0	0	Q	Store
1	0	1	0	Reset
1	1	0	1	Set
1	1	1	?	Undefined

Gate Enable / Level sensitive SR latch (Enable call button)

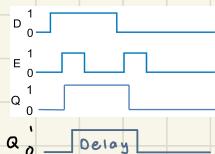
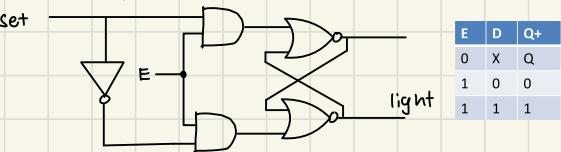


- Only works when $E=1$

E	S	R	Q+	Function
0	X	X	Q	Store
1	0	0	Q	Store
1	0	1	0	Reset
1	1	0	1	Set
1	1	1	?	Undefined



Transparent Enabled D-latch ("storage" w. enable)



Clock (↑) Clock

Frequency $\sim f$ (GHz, mHz)

Period \sim One cycle

$$f = \frac{1}{P}$$

Long Clock vs Short Clock

The long clock can penetrate circuit deeper

The short clock cannot, fails to turn on the circuit

However, Ryan's clock can touch the floor

Example: If clock ~ 5 kHz,

$$\text{Period} = \frac{1}{5 \times 10^3} = 2 \times 10^{-4}$$

milli

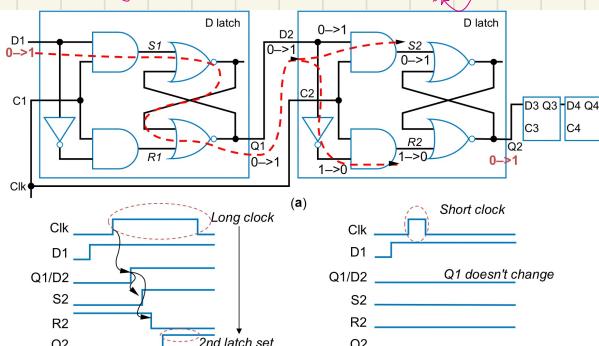
$$m = 1 \times 10^{-3}$$

micro

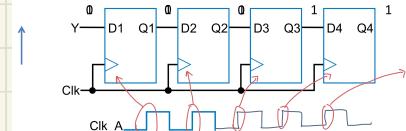
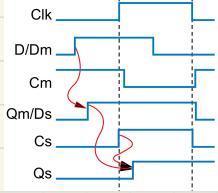
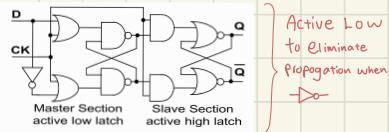
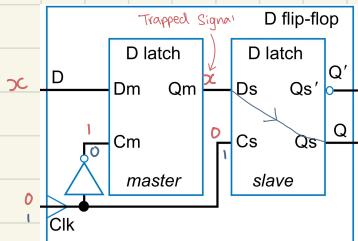
$$\mu = 1 \times 10^{-6}$$

nano

$$n = 1 \times 10^{-9}$$



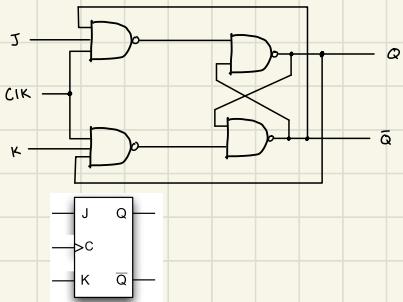
Edge Triggered D Flip Flop



* Positive edge trigger
Negative edge trigger

Basically only activate when edge trigger, store that value

J K Flip Flop



C I K	J	K	Q+
↑	0	0	Q
↑	0	1	0
↑	1	0	1
↑	1	1	Q'
X	X	X	Q

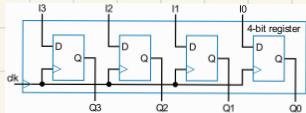
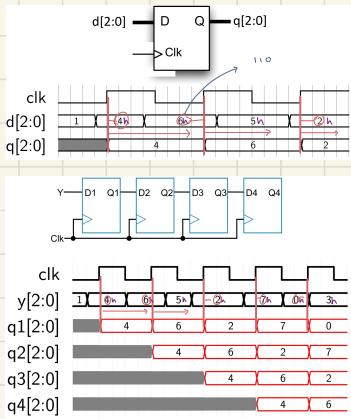
When both 0, Q stored

01, Q=0

10, Q=1

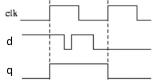
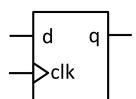
11, Q=flip Over

Register



Sequential Verilog

Flip flops



```
module simplereg (input d, clk,
                   output reg q);
  always@ (posedge clk)
    q <= d;
  end
endmodule
```

inside assignment
special endmodule

- This creates a 1-bit register/D flip-flop with input d and output q

for -ve, use negedge

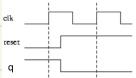
NON-blocking assignment

- Order does not matter
- Imp. for syncro. always block

Reset Asynchronous / Synchronous

- Reset anytime

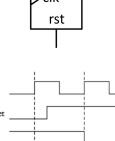
- For an asynchronous reset, we would need to add the reset signal to the sensitivity list:



```
module simplereg (input [7:0] d,
                   input clk, reset,
                   output reg [7:0] q);
  always@ (posedge clk or posedge reset)
    begin
      if (reset)
        q <= 8'b0000_0000;
      else
        q <= d;
    end
endmodule
```

- Reset on clock ✓

- A register with synchronous reset:



```
module simplereg (input [7:0] d,
                   input clk, rst,
                   output reg [7:0] q);
```

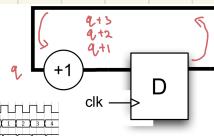
```
always@ (posedge clk)
begin
  if (rst) // same as (rst==1'b1)
    q <= 8'b0000_0000;
  else
    q <= d;
end
endmodule
```

Registers

- We can create multiple registers by including multiple assignments
- Each assignment in a synchronous always block results in a register

```
module multireg (input [7:0] a, b, c,
                  input clk, rst,
                  output reg [7:0] q, r, s);
  always@ (posedge clk)
  begin
    if (rst)
      q <= 8'b0000_0000;
      r <= 8'b0000_0000;
      s <= 8'b0000_0000;
    end else begin
      q <= a;
      r <= b;
      s <= c;
    end
  end
endmodule
```

Counter

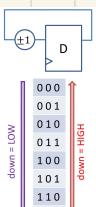


```
module simplecnt (input clk, rst,
                  output reg [3:0] q);
  always@ (posedge clk)
  begin
    if (rst)
      q <= 4'b0000;
    else
      q <= q + 1'b1;
  end
endmodule
```

At each rising edge, we pass through the incremented value of the current count

The data width can be any number of bits

Up & Down Counter



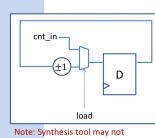
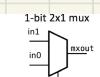
```
module simplecnt (input clk, rst, down,
                  output reg [3:0] q);
  always@ (posedge clk)
  begin
    if (rst)
      q <= 4'b0000;
    else
      if (down)
        q <= q - 1'b1;
      else
        q <= q + 1'b1;
    end
  end
endmodule
```

down = LOW	up = HIGH
000	
001	
010	
100	
101	
110	
111	

Input time

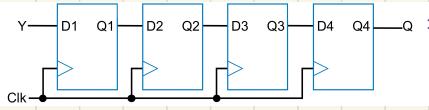
Synchronous Counters in Verilog

```
module simplecnt (input clk, rst,
                  input down, load,
                  input [3:0] cnt_in,
                  output reg [3:0] q);
  always@ (posedge clk)
  begin
    if (rst)
      q <= 4'b0000;
    else
      if (load)
        q <= cnt_in;
      else
        if (down)
          q <= q - 1'b1;
        else
          q <= q + 1'b1;
    end
  end
endmodule
```



Note: Synthesis tool may not instantiate a multiplexer

Shift Register



```
module shiftreg (input clk, y, output reg q);
  reg q1, q2, q3;
  always@(posedge clk)
    begin
      q1 <= y;
      q2 <= q1;
      q3 <= q2;
      q <= q3;
    end
endmodule
```

```
module shiftreg (input clk, y, output reg q_out);
  reg [3:1] q;
  always@(posedge clk)
    begin
      q[1] <= y;
      q[3:2] <= q[2:1];
      q_out <= q[3];
    end
endmodule
```

For many reg Shift on command

```
module shiftreg (input clk, y, output reg q);
  reg [4:0] q;
  always@(posedge clk)
    begin
      q[0] <= y;
      q[4:1] <= q[3:0];
      q <= q[4];
    end
endmodule
```

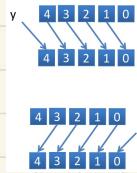


```
module shiftreg2 (input clk, y, output q_out);
  reg [4:0] q;
  always@(posedge clk)
    begin
      if (sh)
        q[0] <= y;
        q[4:1] <= q[3:0];
      end
    assign q_out = q[4];
  endmodule
```

Direction, Output word, Input word

We can also add the capability to shift in the other direction

When the *rt* input is high, the new sample is registered at the MSB, and the rest shift right

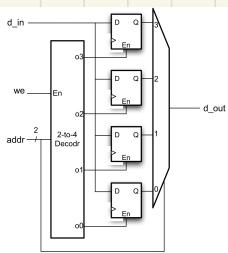


```
module shiftreg5 (input clk, y, sh, rt, ld,
  input [4:0] ld_val,
  output q_out,
  output [4:0] q_word);
  reg [4:0] q;
  always@(posedge clk)
    load value in
    begin
      if (ld) q <= ld_val;
      else if (sh) begin
        if (rt) begin
          q[4] <= y; q[3:0] <= q[4:1];
        end else begin
          q[0] <= y; q[4:1] <= q[3:0];
        end
      end
    end
  assign q_out = rt ? q[0] : q[4];
  assign q_word = q[4:0] <--> y;
endmodule
```

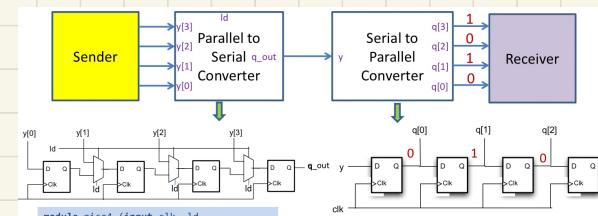
Memory

In order to **store** a value, we must assert *we* (write enable) and provide an address

The decoder outputs a 1 to enable the corresponding register to store whatever is on *d_in* at the clock edge



Sender to Receiver



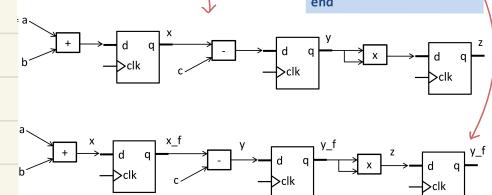
```
module piso4 (input clk, ld,
  input [3:0] y,
  output q_out);
  reg [3:0] q;
  always@(posedge clk)
    begin
      if (ld) q <= y;
      else begin
        q[0] <= y[0];
        q[3:1] <= q[2:0];
      end
    end
  assign q_out = q[3];
endmodule
```

```
module sipo4 (input clk, y,
  output reg [3:0] q);
  always@(posedge clk)
  begin
    q[0] <= y;
    q[3:1] <= q[2:0];
  end
endmodule
```

- Some prefer to code with the combinational and synchronous aspects separated

```
always@(posedge clk)
begin
  x_f <= x;
  y_f <= y;
  z_f <= z;
end
```

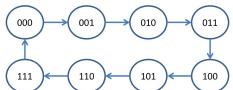
```
always@*
begin
  x = a + b;
  y = x_f - c;
  z = y_f * y_f;
end
```



States

Sequences and States

- We can use a figure to show these transitions:



000
001
010
011
100
101
110
111

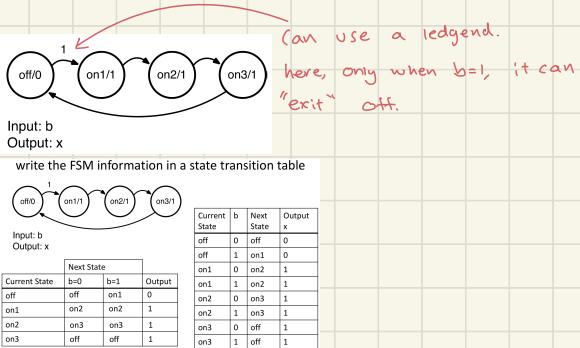
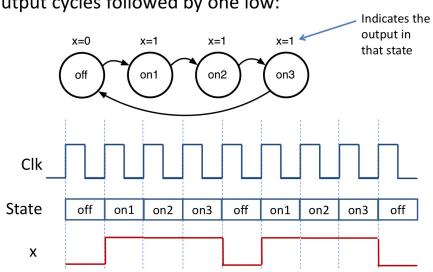
This is called *state transition diagram*

- Each node is a possible *state* of the system 11
 - It shows the movement from one state to the next
 - In this case, the states are simply labeled with their output values

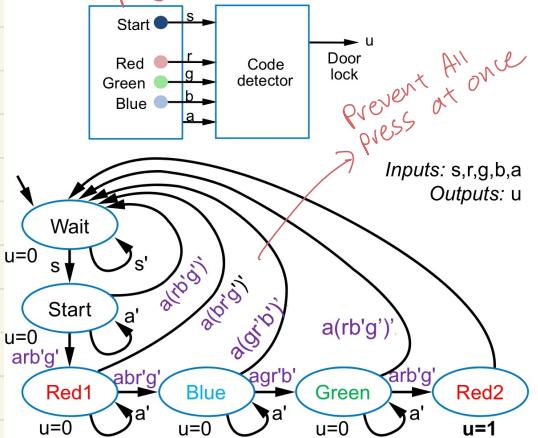
Finite State Machines

Example

An always-on state machine that produces three high output cycles followed by one low:

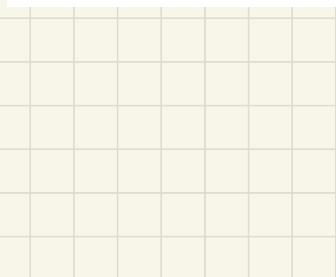


Example:



Another Example

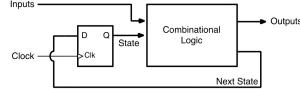
- We want to design an FSM for a vending machine, that accepts only \$1 and 50c coins, dispensing a drink that costs \$1.50, and change if necessary
- This can be a circuit with **two inputs**, **c100**, for a \$1 coin inserted and **c50** for a 50c coin
- We assume only 1 input is ever high, and it tells us what coin was inserted
- The FSM has **two outputs**, **vend**, to release a drink, and **change**, to give 50c of change



Basic Struc For Vlog

Mapping to Implementation

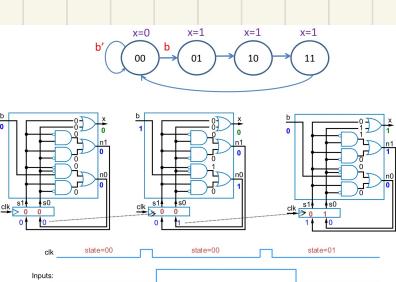
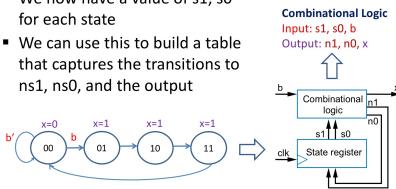
- Hence, the basic structure looks like this:



- The combinational logic the following based on the **current state** and **inputs**
 - Next State**
 - Outputs**
- We must turn the states into some binary representation, then we can use those values and the inputs to write the logic to produce the next state and outputs



- We now have a value of s_1, s_0 for each state
- We can use this to build a table that captures the transitions to ns_1, ns_0 , and the output



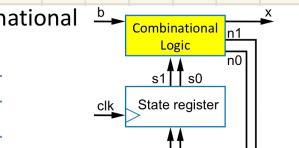
How many states are needed?



For m states, $\log_2 m$ bits needed

- Now determine the combinational logic:

	Inputs		Outputs			
	s_1	s_0	b	x	n_1	n_0
off	0	0	0	0	0	0
	0	0	1	0	0	1
on1	0	1	0	1	1	0
	0	1	1	1	1	0
on2	1	0	0	1	1	1
	1	0	1	1	1	1
on3	1	1	0	1	0	0
	1	1	1	1	0	0



$$x = s_1 + s_0 \quad (\text{note that } x=1 \text{ if } s_1=1 \text{ or } s_0=1)$$

$$n_1 = s_1' s_0 b' + s_1' s_0 b + s_1 s_0' b' + s_1 s_0' b$$

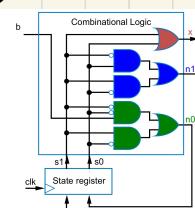
$$n_1 = s_1' s_0 + s_1 s_0'$$

$$n_0 = s_1' s_0' b + s_1 s_0$$

$$n_0 = s_1' s_0' b + s_1 s_0'$$



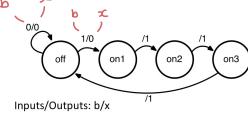
$$\begin{aligned} x &= s_1 + s_0 \\ n_1 &= s_1' s_0 + s_1 s_0' \\ n_0 &= s_1' s_0' b + s_1 s_0' \end{aligned}$$



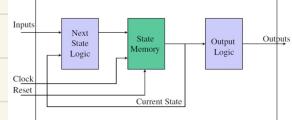
Mealy Machines

Mealy Machines

- For Mealy machines, we can't draw the outputs inside the state circles, so we add them to the arrows instead, after a slash
- The output written on an arrow tells us what the output should be during the emitting state, when the input values match those on the same arrow
- A previous example with outputs on arrows:



This is moore:
Output only based on states



```

module pulse3 (input b,
               input clk, rst,
               output x);

wire n1, n0; - next state
reg s1, s0; - current state

assign n1 = s1 ^ s0;
assign n0 = (~s1&~s0&b) | (s1&~s0);
assign x = s1 | s0;

always @ (posedge clk)
begin
    if(rst) begin
        s1 <= 1'b0;
        s0 <= 1'b0;
    end else begin
        s1 <= n1;
        s0 <= n0;
    end
end
endmodule

```

↓

*State Register
the assign on clock part*

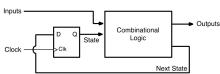
Parameter

Verilog Constants

- First we need to encode the states:
 - Wait = 000, Start = 001, Red1 = 010, Blue = 011, Green = 100, Red2 = 101
 - As we assign each state a binary value, we'd still rather not deal with binary numbers in our Verilog code.
 - We can use the Verilog parameter keyword to declare named constants:
- ```
parameter st1 = 2'b00, st2 = 2'b01, st3 = 2'b10;
```
- We can then use these names in place of the values anywhere in the code.

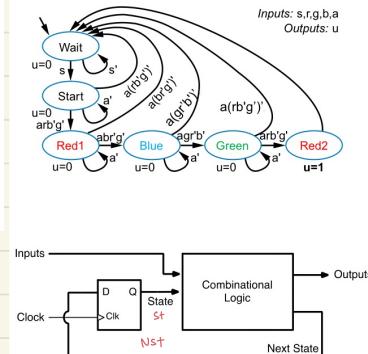
```
nst = st2;
```

- On the previous slide:
  - We declare two 3-bit reg signals (since both will be assigned from always blocks)
  - The output is high only when the current state is Red2 (101), so we can assign it directly
  - The state register process is always the same



- We still need to describe the combinational logic to determine the next state (based on current state and inputs)

## A More Complex Example



```

module seqdet (input s, r, g, b, a,
 input clk, rst,
 output u);

parameter waite=3'b000, start=3'b001, red1=3'b010,
blue=3'b011, green=3'b100, red2=3'b101;

reg [2:0] nst, st;

assign u = (st <= red2); → Logic to drive output

always @ (posedge clk)
begin
 if(rst)
 st <= waite;
 else
 st <= nst;
end

always @ *
begin
 If 'a', will maintain @
 if(a) → If 'a', will maintain @
 case(st)
 waite: if(s) nst = start;
 start: if(a)
 if(r&~b&g) nst = red1;
 else nst = waite;
 red1: if(a)
 if(b&~r&g) nst = blue;
 else nst = waite;
 blue: if(a)
 if(g&~r&b) nst = green;
 else nst = waite;
 green: if(a)
 if(r&g&~b) nst = red2;
 else nst = waite;
 red2: nst = waite;
 endcase
end
endmodule

```

*State register*

*Logic to determine next state*

- So the generalized FSM structure looks like this:

```

module seqdet (* FSM inputs */
 input clk, rst,
 output /* FSM outputs */);

parameter st1 = ... /* state names and assignments */

reg [2:0] nst, st; /* next and current state signal */

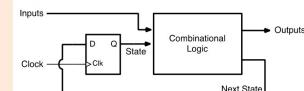
/* state register always block
 always the same structure
 st <= nst, with a reset to initial */

/* state transition always block
 generally uses a case statement
 assigning to nst based on st and inputs */

/* output assignment can be separate using
 assign or always block, or combined in
 state transition always block */

endmodule

```



# Recap (Structural Design)

## Ports

```
module (input in1, in2,
 output out3);
endmodule
```

wires  
 wire  
 wire thing;  
 wire [3:0] that;  
 assign thing = 1'b0  
 $\rightarrow$  Only can be assigned from  
 always @ \* block

## Gates

```
and al (out3, in1, in2);
xor xl (out4, in3, in4);
 ↴
 output first
- also, cannot assign to same output
```

wire manipulation  
 assign thing = in1[1]  
 Specific bit selection

## Ordered instantiation

```
module top (input [1:0] sel,
 input full,
 output [3:0] queue);
 decoder2to4(a(sel), enable(~full), one-hot(queue));

```

## Assign Statements

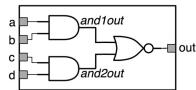
- We can use a range of operators:

- $\&&$  : and (bitwise is: &)
- $\|$  : or (bitwise is: |)
- $!$  : not (bitwise is: !)
- $\wedge$  : xor (bitwise is: ^)

| Operator | Name         |
|----------|--------------|
| $\sim$   | Bitwise NOT  |
| &        | Bitwise AND  |
|          | Bitwise OR   |
| $\wedge$ | Bitwise XOR  |
| $\sim\&$ | Bitwise NAND |
| $\sim $  | Bitwise NOR  |

- The and-or-inverter example previously shown:

- The **assign statement** allows the use of more complex operators and operands – more on this later



Assign  $y = x ? a : b;$

If  $x = 1, y = a$   
 $x = 0, y = b$

Instantiating  
gate  
primitives

```
wire and1out, and2out;
nor g3 (out, and1out, and2out);
and g2 (and2out, c, d);
and g1 (and1out, a, b);
```

```
assign out = ~((a&b) | (c&d));
Continuous assignment
```

# Recap ( Behavioural Design)

## Combinational Always Block

Basic

```
always @ *
```

```
begin
```

```
 x = a & b
```

```
 y = x ^ c
```

```
end
```

Case

```
always @ *
```

```
begin
```

```
 case (x)
```

```
 2'b00 : q = d[0];
```

```
 2'b01 : q = d[1];
```

```
 2'd2 : q = d[2];
```

```
 2'd3 : q = d[3];
```

```
 endcase
```

```
end
```

must always apply for  
ALL CASES or else  
a LATCH is produced.

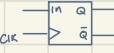
## Latches 3 Sequential Circuits

Latch



When clock = 1 (or 0, if -ve trigger)  
Q exactly matches input

Flip-Flop



On posedge CLK (or negedge CLK)  
Q will copy in. Once

```
reg q1, q2, q3
```

```
always @ (posedge clk)
```

```
begin
```

```
 if (rst)
```

```
 begin
```

```
 q1 <- 1'b0
```

```
 q2 <- 1'b0
```

```
 q3 <- 1'b0
```

```
 end
```

```
 else
```

```
 begin
```

```
 q1 <- y
```

```
 q2 <- q1
```

```
 q3 <- q2
```

```
 end
```

```
end
```

Using non-blocking  
assignment for CLK

