① Consider following augmented matrix of a linear system of equations. Determine if the linear syste is consistent:

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & 2 & 1 \end{array}\right)$$

A) Consistent B) Inconsistent C) Cannot be determine

② The following system of equations

$$ax + 3ay = 5a$$
$$2ax + 6ay = 10a$$

has

A) No solution B) one solution C) infinite solution.

D) 3 solutions

③ Consider set $\left\{ \begin{pmatrix} 2-2t \\ -4t \\ t \end{pmatrix} \mid t \in \mathbb{R} \right\} = V$

For every augmented matrix of a linear system determine if A is a set of solutions.

A) $\left(\begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 6 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$ B) $\left(\begin{array}{ccc|c} -2 & 0 & 2 & 1 \\ -4 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 \end{array}\right)$ C) $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & 4 & 0 \end{array}\right)$

D) $\left(\begin{array}{ccc|c} 2 & 2 & 0 & 1 \\ 1 & 0 & 0 & 4 \end{array}\right)$

④ Let $\{u, v, w\}$ be a linear independant set of vectors. Which of the following statements is true?

① Consider following augmented matrix of a linear system of equations. Determine if the linear syste is consistent:

$$\left(\begin{array}{ccc|c} 1 & 1 & -2 & 1 \\ 0 & 1 & -1 & -1 \\ 0 & -2 & 2 & 1 \end{array}\right)$$

$$\left[\begin{array}{cccc} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$ inconsistent ✓

② The following system of equations

$$ax + 3ay = 5a$$
$$2ax + 6ay = 10a$$

has

~~A)~~ No solution ~~B)~~ one solution C) infinite solution. ~~D)~~ 3 solutions

$$\left[\begin{array}{cc|c} 1 & 3 & 5 \\ 0 & 0 & 0 \end{array}\right]$$

③ Consider set $\left\{\begin{pmatrix} 2-2t \\ -4t \\ t \end{pmatrix} \mid t \in \mathbb{R}\right\} = V$

For every augmented matrix of a linear system determine if A is a set of solutions:

A) $\left(\begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$ B) $\left(\begin{array}{ccc|c} -2 & 0 & 2 & 1 \\ -4 & 0 & 0 & 2 \\ 1 & 0 & 0 & 1 \end{array}\right)$ C) $\left(\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & 4 & 0 \end{array}\right)$

D) $\left(\begin{array}{ccc|c} 2 & 2 & 0 & 1 \\ 1 & 0 & 0 & 4 \end{array}\right)$

$$V = t\begin{bmatrix} -2 \\ -4 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

④ Let $\{u, v, w\}$ be a linear independant set of vectors. Which of the following statements is true?

A) u is a linear combination of u and v.
B) $\{u, v, u+v\}$ is linear independant
C) $au + bu + cw = 0$ for some non-zero scalars a, b, c
D) $\{u, v, u+v+w\}$ is linear independant

D ✓ is true, as $u, v, u+v+w$

now w ?

∴ independant

⑤ About every matrix determine if it is invertible

A) ✓ $\left(\begin{array}{ccc} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{array}\right)$ ~~B)~~ $\left(\begin{array}{cc} 1 & 2 \\ 3 & 1 \\ 0 & 0 \end{array}\right)$ ~~C)~~ $\left(\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 5 \end{array}\right)$ det = 0

D) ✓ $\left(\begin{array}{ccc} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{array}\right)$

⑥ Which of the following sets of vectors are 1) linear independant 2) span $\mathbb{R}^3$.

A) $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}$ B) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ C) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$

D) $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ E) $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$

$$\begin{matrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{matrix} \qquad \begin{matrix} 1 & 1 \\ 2 & 0 \\ 0 & 0 \end{matrix}$$

does not span $\mathbb{R}^3$. only span $\mathbb{R}^2$

	A	B	C	D	E
independent:	~~✗~~ ✓	~~✗~~ ✓	~~✗~~ ✓	✓	✓
span $\mathbb{R}^3$:	~~✗~~ ✓	~~✗~~ ✓	✓	✗	✓

⑦ Choose matrix product of AB if it is defined.

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$$

C) $\begin{pmatrix} 3 & 2 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ ~~D)~~ AB is undefined ~~E)~~ $\begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 0 \end{pmatrix}$

F) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

$2 \times 2 \,/\, 2 \times 3 = 2 \times 3$

A) u is a linear combination of u and v.

B) $\{u, v, u+v\}$ is linear independant

C) $au + bu + cw = 0$ for some non-zero scalars a, b, c

D) $\{u, v, u+v+w\}$ is linear independant

(5) About every matrix determine if it is invertible

A) $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 0 \end{pmatrix}$ C) $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 5 \end{pmatrix}$

D) $\begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 1 \\ 0 & 2 & 3 \end{pmatrix}$

(6) Which of the following sets of vectors are 1) linear independant 2) span $\mathbb{R}^3$.

A) $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix}$ B) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ C) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}$

D) $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ E) $\begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 10 \end{pmatrix}$

(7) Choose matrix product of AB if it is defined.

$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$

C) $\begin{pmatrix} 3 & 2 & 1 \\ -3 & -2 & -1 \end{pmatrix}$ D) AB is undefined E) $\begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 3 & 0 \end{pmatrix}$

F) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

Answers:

1. B)
2. C)
3. C)
4. D)
5.

A	B	C	D
Yes	No	No	Yes

6. LD = linear dependant
   LI = linear independant
   S = span $\mathbb{R}^3$
   NS = not Span $\mathbb{R}^3$

A	B	C	D	E
LD	LD	LD	LI	LI
NS	NS	S	NS	S

7. F)