CE/CZ1104 & SC1004 (Semester 2 – AY 21-22): Take home test 1: Version L

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1. (2 points) Find basis of $Span\left\langle \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 8 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \right\rangle$.

2. (3 points) Find LU factorization of $A = \begin{pmatrix} 2 & -3 & 3 \\ 1 & 4 & -1 \\ 1 & -1 & 4 \end{pmatrix}$.

3. (5 points) Let V be a vector space of all symmetric matrices of the size 2×2. Choose a basis in this space and find the matrix of the linear operator L with respect to this basis if $L(A) = \begin{pmatrix} -4 & 2 \\ 1 & -1 \end{pmatrix} \cdot A \cdot \begin{pmatrix} -4 & 1 \\ 2 & -1 \end{pmatrix}$. Find the range and the kernel of the linear operator L. Find the Null space, the column space and the rank of the matrix of the operator L.

1. (2 points) Find basis of $Span\left\langle \begin{pmatrix}4\\1\\1\end{pmatrix}, \begin{pmatrix}2\\1\\2\end{pmatrix}, \begin{pmatrix}8\\2\\2\end{pmatrix}, \begin{pmatrix}-1\\-1\\1\end{pmatrix} \right\rangle$.

$$\begin{bmatrix} 4 & 2 & 8 & -1 \\ 1 & 1 & 2 & -1 \\ 1 & 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ The basis is $\left\{ \begin{bmatrix}4\\1\\1\end{bmatrix}, \begin{bmatrix}2\\1\\2\end{bmatrix}, \begin{bmatrix}-1\\-1\\1\end{bmatrix} \right\}$ in $\mathbb{R}^3$

2. (3 points) Find LU factorization of $A = \begin{pmatrix} 2 & -3 & 3 \\ 1 & 4 & -1 \\ 1 & -1 & 4 \end{pmatrix}$.

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 4 & -1 \\ 1 & -1 & 4 \end{bmatrix} \xrightarrow[L_{13}:\ R_3 = R_3 + (-\frac{1}{2})R_1]{L_{12}:\ R_2 = R_2 + (-\frac{1}{2})R_1} \begin{bmatrix} 2 & -3 & 3 \\ 0 & 11/2 & -5/2 \\ 0 & 1/2 & 5/2 \end{bmatrix} \xrightarrow{L_{23}:\ R_3 = R_3 + (-\frac{1}{11})R_2} \begin{bmatrix} 2 & -3 & 3 \\ 0 & 11/2 & -5/2 \\ 0 & 0 & 30/11 \end{bmatrix} = U$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 1/11 & 1 \end{bmatrix}$$

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 1/2 & 1/11 & 1 \end{bmatrix} \begin{bmatrix} 2 & -3 & 3 \\ 0 & 11/2 & -5/2 \\ 0 & 0 & 30/11 \end{bmatrix}$$

3. (5 points) Let V be a vector space of all symmetric matrices of the size 2×2. Choose a basis in this space and find the matrix of the linear operator L with respect to this basis if $L(A) = \begin{pmatrix} -4 & 2 \\ 1 & -1 \end{pmatrix} \cdot A \cdot \begin{pmatrix} -4 & 1 \\ 2 & -1 \end{pmatrix}$. Find the range and the kernel of the linear operator L. Find the Null space, the column space and the rank of the matrix of the operator L.

Symetrical = $A = A^T$, $\rightsquigarrow \begin{bmatrix} a & c \\ c & b \end{bmatrix}$

Look at how A transforms $L(A)$ for basis $\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}, \begin{bmatrix}0&1\\1&0\end{bmatrix}$

$$L\left(\begin{bmatrix}1&0\\0&0\end{bmatrix}\right) = \begin{bmatrix}-4&2\\1&-1\end{bmatrix}\begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}-4&1\\2&-1\end{bmatrix} = \begin{bmatrix}16&-4\\-4&1\end{bmatrix} \Rightarrow 16\begin{bmatrix}1&0\\0&0\end{bmatrix} - 4\begin{bmatrix}0&0\\0&1\end{bmatrix} + \begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$$L\left(\begin{bmatrix}0&0\\0&1\end{bmatrix}\right) = \begin{bmatrix}-4&2\\1&-1\end{bmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}-4&1\\2&-1\end{bmatrix} = \begin{bmatrix}4&-2\\-2&1\end{bmatrix} \Rightarrow 4\begin{bmatrix}1&0\\0&0\end{bmatrix} - 2\begin{bmatrix}0&0\\0&1\end{bmatrix} + \begin{bmatrix}0&1\\1&0\end{bmatrix}$$

$$L\left(\begin{bmatrix}0&1\\1&0\end{bmatrix}\right) = \begin{bmatrix}-4&2\\1&-1\end{bmatrix}\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}-4&1\\2&-1\end{bmatrix} = \begin{bmatrix}-16&6\\6&-2\end{bmatrix} \Rightarrow -16\begin{bmatrix}1&0\\0&0\end{bmatrix} + 6\begin{bmatrix}0&0\\0&1\end{bmatrix} - 2\begin{bmatrix}0&1\\1&0\end{bmatrix}$$

Matrix of linear transformation $= \begin{bmatrix} 16 & 4 & -16 \\ 1 & 1 & -2 \\ -4 & -2 & 6 \end{bmatrix}$

$$\begin{bmatrix} 16 & 4 & -16 \\ 1 & 1 & -2 \\ -4 & -2 & 6 \end{bmatrix} \xrightarrow[L_{13}:\ R_3 = R_3 + (\frac{1}{4})R_1]{L_{12}:\ R_2 = R_2 + (-\frac{1}{16})R_1} \begin{bmatrix} 16 & 4 & -16 \\ 0 & 3/4 & -1 \\ 0 & -1 & 2 \end{bmatrix} \xrightarrow{L_{23}:\ R_3 = R_3 + (\frac{4}{3})R_2} \begin{bmatrix} 16 & 4 & -16 \\ 0 & 3/4 & -1 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}$$

$\therefore$ Range (L) = Vector Space V, Kernel $(L) = \left\{ \begin{bmatrix}0&0\\0&0\end{bmatrix} \right\}$

$C(L) = \text{Span}\left\{ \begin{bmatrix}16\\1\\-4\end{bmatrix}, \begin{bmatrix}4\\1\\-2\end{bmatrix}, \begin{bmatrix}-16\\-2\\6\end{bmatrix} \right\}$, $N(L) = \{\vec{0}\}$, Rank$(L) = 3$