NANYANG TECHNOLOGICAL UNIVERSITY
SEMESTER I EXAMINATION 2019–2020
MH1810 – Mathematics 1

NOVEMBER 2019 TIME ALLOWED: 2 HOURS

Matriculation Number:

Seat Number:

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **EIGHT (8)** questions and comprises **TWENTY (20)** pages, including an Appendix.
2. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
3. This **IS NOT** an **OPEN BOOK** exam. However, a list of formulae is provided in the Appendix.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.
5. All your solutions should be written in this booklet within the space provided after each question. If you use an additional answer book, attach it to this booklet and hand them in at the end of the examination.

For examiners only

Questions	Marks
1 (10)	
2 (10)	
3 (10)	
4 (15)	

Questions	Marks
5 (15)	
6 (15)	
7 (15)	
8 (10)	

Total (100)	

QUESTION 1. **(10 Marks)**

Three points A, B, C and the vector $\overrightarrow{AB}$ are shown in Figure 1. Given that $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} -9 \\ -6 \\ -3 \end{pmatrix}$ and $\overrightarrow{OD} = \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}$.

(a) Draw the vector $\overrightarrow{CD}$ in Figure 1.

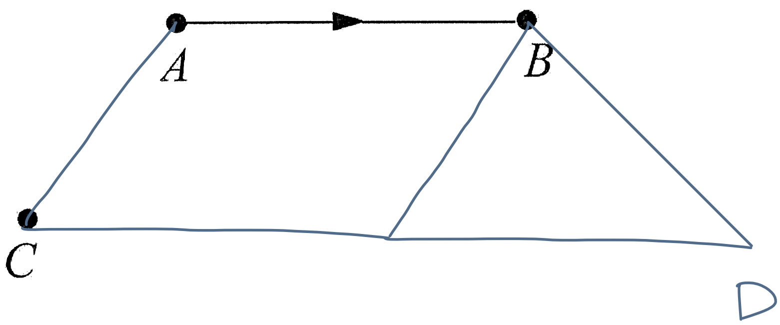


Figure 1.

AB =

Question 1 continues on Page 3.

(b) Find the area of quadrilateral $ABDC$. Express the answer in the form $m\sqrt{n}$, where m and n are integers to be determined.

QUESTION 2. **(10 Marks)**

(a) Consider the complex number

$$w = \frac{\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)^2 \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^3}{\left(\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}\right)^2}.$$

(i) Find the modulus and principal argument of w.

(ii) Use De Moivre's Theorem to show that w is a cube root of 1.

$$w = \frac{\left(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})\right)^2 \left(e^{i\frac{\pi}{3}}\right)^3}{\left(\cos(-\frac{\pi}{12}) + i\sin(-\frac{\pi}{12})\right)^2}$$

$$= \frac{\left(e^{i(-\frac{\pi}{2})}\right)\left(e^{i\pi}\right)}{e^{i(\frac{1}{6})\pi}} = e^{i\left(\frac{4\pi}{3}\right)}$$

Question 2 continues on Page 5.

(b) Find the value of the complex number z such that

$$z + |z + 12| = 6 + 6i.$$

$z = \_\_\_ + 6i$

Indicate on Figure 2 the complex numbers $-z$, iz, and z^*.

$$z + |z+12| = 6 + 6i$$

$$a + \cancel{6i} + \sqrt{(a+12)^2 + (6)^2} = 6 + \cancel{6i}$$

$$\sqrt{a^2 + 24a + 180} = 6 - a$$

$$a^2 + 24a + 180 = (6-a)^2$$

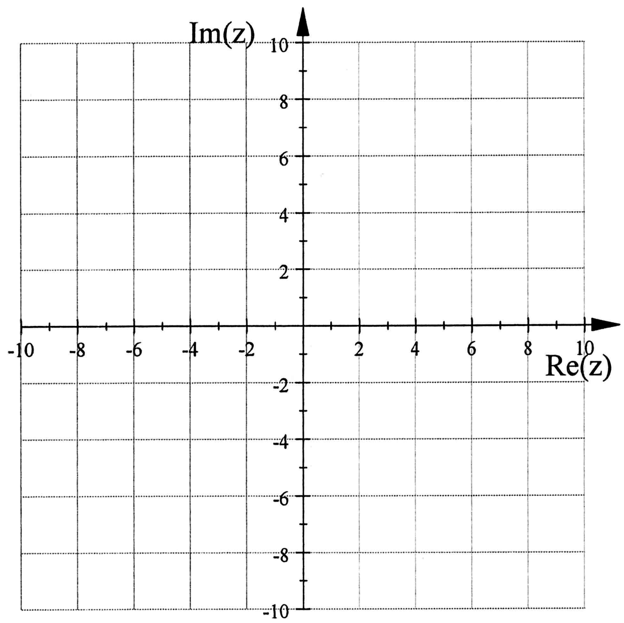


Figure 2

QUESTION 3. **(10 Marks)**

The graph of a cubic polynomial $f(x) = ax^3 + bx^2 + cx$ has turning points $(1, -7)$ and $(-2, 20)$.

x, y

(a) Show that a, b and c satisfy the following two equations.

$$12a - 4b + c = 0 \text{ and } 3a + 2b + c = 0.$$

Find another two equations relating a, b and c.

$f'(x) = 3ax^2 + 2bx + c$

$0 =$

Question 3 continues on Page 7.

(b) By using *cofactor expansion along the third row*, show that the determinant of the following matrix is 21.

$$\begin{pmatrix} 12 & -4 & 1 \\ 3 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$\det = 1(-4-2) - 1(12-3) + 1(24+12) = 21$

(c) By using Cramer's Rule, find the value of c.

$$f'(x) = \lim_{y \to x} \frac{f(x) - f(y)}{y - x} \qquad \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

QUESTION 4. **(15 Marks)**

(a) Let $f(x) = x^{2/3}$, using *the definition of derivative,* find $f'(x)$.

(b) Let

$$f(x) = \begin{cases} x^3 + x + 1 & \text{if} \quad x \geq 0 \\ a \sin x + b & \text{if} \quad x < 0 \end{cases},$$

where a and b are some constants. Find the values of a and b so that f is differentiable.

Question 4 continues on Page 9.

(c) A curve is defined implicitly by the equation

$$x^{2/3} + y^{2/3} = 4.$$

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$$

The point $P\left(x_0, y_0\right)$ lies on the curve.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y.

(ii) Show that the tangent of the curve at the point $P\left(x_0, y_0\right)$ is

$$\frac{x}{x_0^{1/3}} + \frac{y}{y_0^{1/3}} = 4.$$

$$\frac{dy}{dx} = \frac{-\cancel{\frac{2}{3}}x^{-\frac{1}{3}}}{\cancel{\frac{2}{3}}y^{-\frac{1}{3}}} = -\left(\frac{x}{y}\right)^{-\frac{1}{3}}$$

QUESTION 5 **(15 Marks)**

Find the following limits.

(a) $\displaystyle\lim_{x\to 1^+} \frac{(x^2-1)\,e^x}{\sqrt{x}-1}$

$$= \lim_{x\to 1^+} \frac{(x^2-1)\,e^x}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1} = \lim_{x\to 1^+} \frac{\cancel{(x-1)}(x+1)(\sqrt{x}+1)e^x}{\cancel{x-1}}$$

$$= \lim_{x\to 1} (x+1)(\sqrt{x}+1)e^x$$

(b) $\displaystyle\lim_{x\to 0} \frac{\sin x}{\sqrt{x+20}+\sin\frac{1}{x}}$

$$-1 \le \sin\frac{1}{x} \le 1$$

$$\sqrt{x+20}-1 \le \sin\frac{1}{x} \le \sqrt{x+20}+1$$

Question 5 continues on Page 11.

(c) $\lim_{x \to 0^+} x^{x^2}$

$$\lim_{x \to 0^+} e^{\ln x^{x^2}} = \lim_{x \to 0^+} e^{x^2 \ln x} = e^{\lim_{x \to 0^+} x^2 \ln x}$$

$$= e^{\lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}}}$$

QUESTION 6. **(15 Marks)**

(a) The volume of a metal cube is increasing at a rate of 10 $\mathrm{cm}^3/\mathrm{min}$. How fast is the surface area increasing when the length is 30 cm?

$V = l^3$ $\frac{dV}{dt} = 10$? $\frac{dA}{dt}$

$A = l^2$

Question 6 continues on Page 13.

(b) A boat left a jetty at 1:00 PM and travelled due north at 40 km/h. Another boat was travelling due west at 30 km/h and reached the same jetty at 2:00 PM. At what time were the two boats nearest to each other? Justify your answer.

QUESTION 7. **(15 Marks)**

Evaluate each of the following.

(a) $\int \frac{x+2}{\sqrt{7-2x^2+4x}}\,dx.$

(b) $\int \frac{1}{e^x+1}dx.$ $= \int \frac{1}{u}\left(\frac{1}{e^x}\right)du$

$= \int \frac{1}{u(u-1)}$

$u = e^x + 1$

$\frac{du}{dx} = e^x$

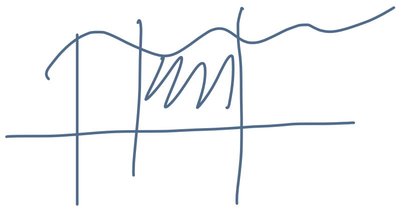
$dx = du\left(\frac{1}{e^x}\right)$

Question 7 continues on Page 15.

(c) $\displaystyle \lim_{n\to\infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{n+1}} + \frac{1}{\sqrt{n+2}} + \cdots + \frac{1}{\sqrt{n+n}} \right)$

QUESTION 8. **(10 Marks)**

The region R is enclosed by the lines $y = 0$, $x = 1$, $x = 2$ and the graph of $y = \frac{1}{4+x^2}$.



(a) Find the area of R.

$$\int_1^2 \frac{1}{2^2+x^2}\,dx = \left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right]_1^2$$

Question 8 continues on Page 17.

$$y = \frac{1}{4+(x+1)^2}$$

(b) Find the volume of the solid obtained when the region R is rotated about the line $x = 1$ by 2π radians.

END OF PAPER

Appendix

Numerical Methods.

- Linearization Formula:

$$L(x) = f(a) + f'(a)\,(x - a)$$

- Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\left[y_0 + 2\left(y_1 + y_2 + \cdots + y_{n-1}\right) + y_n\right]$$

- Simpson's Rule:

$$\int_a^b f(x)\,dx \approx S_n = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n\right],$$

where n is even.

Derivatives.

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \qquad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \mathrm{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\mathrm{csch}^2 x$$

$$\frac{d}{dx}(\mathrm{sech}\ x) = -\mathrm{sech} x \tanh x \qquad \frac{d}{dx}(\mathrm{csch}\ x) = -\mathrm{csch} x \coth x$$

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{x^2+1}}$$

Antiderivatives.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1 \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \tan x \sec x\,dx = \sec x + C \qquad \int \cot x \csc x\,dx = -\csc x + C$$

$$\int \tan x\,dx = \ln|\sec x| + C \qquad \int \cot x\,dx = \ln|\sin x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int a^x\,dx = \frac{a^x}{\ln a} + C,\ a > 0$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \qquad \int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C,\ |x| < |a| \qquad \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + C \qquad \int \frac{1}{\sqrt{x^2+a^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

MH1810 MATHEMATICS 1

Please read the following instructions carefully:

1. **Please do not turn over the question paper until you are told to do so. Disciplinary action may be taken against you if you do so.**

2. You are not allowed to leave the examination hall unless accompanied by an invigilator. You may raise your hand if you need to communicate with the invigilator.

3. Please write your Matriculation Number on the front of the answer book.

4. Please indicate clearly in the answer book (at the appropriate place) if you are continuing the answer to a question elsewhere in the book.