## NANYANG TECHNOLOGICAL UNIVERSITY

## MIDTERM II (CA2)

## MH1812 – Discrete Mathematics

March 2017	TIME ALLOWED: 40	TIME ALLOWED: 40 minutes	
Name:			
Matric. no.:	Tutor group:		

## INSTRUCTIONS TO CANDIDATES

- 1. DO NOT TURN OVER PAPER UNTIL INSTRUCTED.
- 2. This midterm paper contains FOUR (4) questions.
- 3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
- 4. Candidates can write anywhere on this midterm paper.
- 5. This **IS NOT** an **OPEN BOOK** exam.
- 6. Candidates should clearly explain their reasoning when answering each question.

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QUESTION 1. (25 marks)

Solve the following linear recurrences:

- (a)  $a_n = 7a_{n-1}$ , with initial condition  $a_1 = 5$ ;
- (b)  $b_n = 20b_{n-1} 51b_{n-2}$ , with initial conditions  $b_0 = 5$  and  $b_1 = 6$ .

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QUESTION 2.

(20 marks)

Prove that

$$\sum_{j=0}^{n} j! j = (n+1)! - 1 \qquad \forall n \in \mathbb{N}.$$

QUESTION 3.

(25 marks)

Let  $S = \{1, ..., n\}$  be a finite set and let  $\mathcal{P}(S)$  denote the power set of S. Set  $A = \{s \in \mathcal{P}(S) : |s| \text{ is even }\}$  and  $B = \{s \in \mathcal{P}(S) : |s| \text{ is odd }\}$ . Using the binomial theorem, or otherwise, prove that the cardinalities of A and B are equal, that is, prove that |A| = |B|.

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QUESTION 4.

(30 marks)

(a) Prove, for the sets A, B, C, D, that

$$(A \times B) \cap (C \times D) \subseteq (A \cap C) \times (B \cap D).$$

(b) Does equality hold? Justify your answer with either a proof or a counterexample.