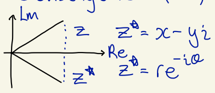
Complex Numbers $\theta = \theta + 2\pi = \theta + 4\pi$

Polar Form: $z = r(\cos\theta + i\sin\theta) = r(\mathrm{cis}\,\theta)$

Exponential Form: $z = re^{i\theta}$

Conjugate $\to (z^*)^* = z$



$z^* = x - yi$

$z^* = re^{-i\theta}$

if $z = \bar{z}$, z is REAL
if $z = -\bar{z}$, z is IMAG

$|z| = |\bar{z}|$ & $\arg(z) = -\arg(\bar{z})$

$(z_1)(z_2) = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

$z^n = r^n e^{in\theta} = r^n(\cos(n\theta) + i\sin(n\theta))$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)) = \frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$

$\frac{z_1}{z_2} = \frac{z_1}{z_2^*} \cdot \frac{z_2^*}{z_2^*}$

$|z-1| = |(a-1) + bi| = \sqrt{(a-1)^2 + (b)^2}$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ $\quad \arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$

$z^4 = -16i$

$z^4 = 16e^{i(\frac{-\pi}{2} + 2n\pi)}$

$z = 16^{\frac{1}{4}} e^{i(\frac{-\pi}{8} + \frac{1}{2}n\pi)}$

Vectors

Cartesian: $\frac{x-a_1}{v_1} = \frac{y-a_2}{v_3} = \frac{z-a_3}{v_3}$

Parametric: $x = a_1 + tv_1$
$y = a_2 + tv_2$
$z = a_3 + tv_3$

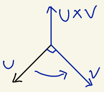
$(A\mu)\cdot(B\lambda) = (A+B)(\mu\cdot\lambda)$

Vector: $r = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} + t\begin{pmatrix}v_1\\v_2\\v_3\end{pmatrix}$, $t \in \mathbb{R}$

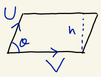
$(\mu\cdot N)(V) = V\cdot\mu + V\cdot\lambda$

Dot: $U\cdot V = \|U\|\|V\|\cos(\theta)$

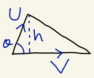
Cross: $U\times V = \|U\|\|V\|\sin(\theta)\,\hat{n}$



$\begin{pmatrix}U_1\\U_2\\U_3\end{pmatrix}\begin{pmatrix}V_1\\V_2\\V_3\end{pmatrix}$ $\begin{pmatrix}U_1\\U_2\\U_3\end{pmatrix}\begin{pmatrix}V_1\\V_2\\V_3\end{pmatrix}$ $\begin{pmatrix}U_1\\U_2\\U_3\end{pmatrix}\begin{pmatrix}V_1\\V_2\\V_3\end{pmatrix}$ $\begin{pmatrix}U_2V_3 - U_3V_2\\U_3V_1 - U_1V_3\\U_1V_2 - U_2V_1\end{pmatrix}$

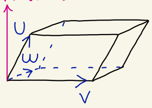


$A = \|V\|(h) = \|V\|\|U\|\sin\theta = \|U\times V\|$



$A = \frac{1}{2}\|U\times V\|$

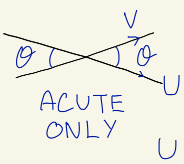
$n = W\times V$



1) Height $= U\cdot\hat{n} = U\cdot\frac{W\times V}{\|W\times V\|}$

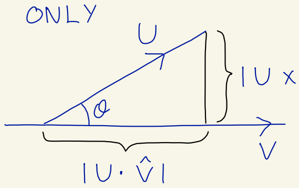
2) Base Area $= \|W\times V\|$

3) Volume $= U\cdot\frac{W\times V}{\cancel{\|W\times V\|}}\cancel{\|W\times V\|}$
$= U\cdot(W\times V)$



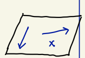
$\mathrm{abs}(V\cdot U) = \|V\|\|U\|\cos\theta$

$\theta = \cos^{-1}\left(\frac{V\cdot U}{\|V\|\|U\|}\right)$



$\underbrace{\quad}_{|U\cdot\hat{V}|}$ $\quad |U\times\hat{V}|$

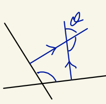
Cartesian: $ax + by + cz = d$ (Scalar)



Parametric: $x = s\begin{pmatrix}U_1\\U_2\\U_3\end{pmatrix} + t\begin{pmatrix}V_1\\V_2\\V_3\end{pmatrix} + \begin{pmatrix}A\\B\\C\end{pmatrix}$, $s, t \in \mathbb{R}$

Vector: $r\cdot n = d = \text{Point}\cdot n$

∡ Between Planes



$n_1\cdot n_2 = \|n_1\|\|n_2\|\cos\theta$

$\theta = \cos^{-1}\left(\frac{n_1\cdot n_2}{\|n_1\|\|n_2\|}\right)$

$\mu = 180° - \theta = 180° - \cos^{-1}\left(\frac{n_1\cdot n_2}{\|n_1\|\|n_2\|}\right)$

HAVE A NICE DAY

MATRICES $m\times n$

zero $\begin{bmatrix}0&0\\0&0\end{bmatrix}$ Diagonal $\begin{bmatrix}4&0\\0&3\end{bmatrix}$ Identity $\begin{bmatrix}1&0\\0&1\end{bmatrix}$

Triangular Upper $\begin{bmatrix}1&4\\0&3\end{bmatrix}$ lower $\begin{bmatrix}1&0\\3&4\end{bmatrix}$

$AB = \begin{bmatrix}A_{11}&A_{12}\\A_{21}&A_{22}\\A_{31}&A_{32}\\A_{41}&A_{42}\end{bmatrix}\begin{bmatrix}B_{11}&B_{12}&B_{13}&B_{14}\\B_{21}&B_{22}&B_{23}&B_{24}\end{bmatrix}$

$= \begin{bmatrix}(A_{11}B_{11} + A_{12}B_{21}) & (A_{11}B_{12} + A_{12}B_{22}) & \cdots\\(A_{21}B_{11} + A_{22}B_{21}) & &\end{bmatrix}$

TRANSPOSE
$3\times 2 \to 2\times 3$ $\quad A = \begin{bmatrix}1&2\\3&4\\5&6\end{bmatrix}$ $\quad A^T = \begin{bmatrix}1&3&5\\2&4&6\end{bmatrix}$ $\quad \boxed{A^{-5} = (A^5)^{-1}}$

Singular, det $= 0$

$\begin{bmatrix}0&0&0\\a&b&c\\d&e&f\end{bmatrix}\begin{bmatrix}1&1&1\\1&1&1\\a&b&c\end{bmatrix}\begin{bmatrix}1&2&a\\1&2&b\\1&2&c\end{bmatrix}\begin{bmatrix}a&b&c\\d&e&f\\2a-5d&2b-5e&2c-5f\end{bmatrix}$

Invertible

$AB = I$ & $BA = I$, $A^{-1} = B$ $\quad$ Adjugate

$A = \begin{bmatrix}a&b\\c&d\end{bmatrix}$ $\quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d&-b\\-c&a\end{bmatrix}$ $\quad \begin{bmatrix}+&-&+&-\\-&+&-&+\\+&-&+&-\\-&+&-&+\end{bmatrix}$

$B = \begin{bmatrix}a&0&0\\0&b&0\\0&0&c\end{bmatrix}$ $\quad B^{-1} = \begin{bmatrix}\frac{1}{a}&0&0\\0&\frac{1}{b}&0\\0&0&\frac{1}{c}\end{bmatrix}$

Determinant $m\times n$ $\quad$ Minor o matrix is det of sub-mat

$\begin{bmatrix}\cancel{a}&\cancel{b}&\cancel{c}&\cancel{d}\\\cancel{e}&f&g&h\\\cancel{i}&j&k&l\end{bmatrix}$ $\quad$ Cofactor$^{ij} = (-1)^{i+j}$ Minor$_{ij}$

$\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}\begin{matrix}a&b\\d&e\\g&h\end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{33}a_{21}a_{12}$

$\det\begin{bmatrix}1&0&0\\4&3&0\\2&5&7\end{bmatrix} = 1\times 3\times 7$

$\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}j\\k\\l\end{bmatrix}$ $\quad x = \dfrac{\begin{vmatrix}j&b&c\\k&e&f\\l&h&i\end{vmatrix}}{\begin{vmatrix}a&b&c\\d&e&f\\g&h&i\end{vmatrix}}$

Limits

If $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$, $\lim f(x)$ DNE

$\lim_{x\to c}$
1) $\frac{a}{b}$, $a, b \neq 0$
2) $\frac{a}{0}$, DNE
3) $\frac{0}{0}$, Factorize

Squeeze

$-1 \le \sin\theta \le 1$
$-1 \le \sin(\frac{1}{x^2}) \le 1$
$-x^2 \le x^2\sin(\frac{1}{x^2}) \le x^2$

To Infinity

When $f(a) = 0$

$\lim_{x\to a}\frac{1}{f(x)}$ DNE

$\lim_{x\to a}\frac{1}{f(x)} = +\infty$

$\lim_{x\to a}\frac{1}{f(x)} = -\infty$

$\lim_{x\to\infty} e^x = \infty$

$\lim_{x\to\infty} \ln x = \infty$

$\lim_{x\to 0} \ln x = -\infty$

$\lim_{x\to 0}\frac{\sin ax}{x} = a(1)$

$\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$

$\lim_{x\to 0}\frac{\tan x}{x} = 1$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\lim_{x\to a}$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$\lim_{x\to\infty}\sinh = +\infty$

$\lim_{x\to\infty}\cosh = +\infty$

$\lim_{x\to\infty}\tanh = 1 \} \div e^x$

$\lim_{x\to-\infty}\sinh = -\infty$

$\lim_{x\to-\infty}\cosh = +\infty$

$\lim_{x\to-\infty}\tanh = -1 \} \div e^{-x}$

$\lim_{x\to\infty} 2x^4 \cdot \frac{1}{x^4} = \frac{2}{1}$

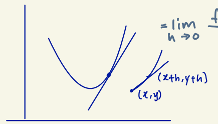
$\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \frac{\sqrt{\frac{2x^2+1}{x^2}}}{1}$

$\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{x}\cdot\frac{\frac{1}{x}}{\frac{1}{x}} = \frac{\sqrt{2x^2+1}}{-\sqrt{x^2}} = \frac{-\sqrt{\frac{2x^2+1}{x^2}}}{1}$

Domain = Input
Range = Output

$[a, b)$ — Included, Excluded

$f'(c) = \lim_{x\to c}\frac{f(x) - f(c)}{x - c} = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$



$= \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$ $\quad (y_2 - y_1)$

$f(x+h)$

Completing The Square

$$ax^2 - bx + C = 0$$

$$x^2 - \frac{b}{a}x = -\frac{c}{a}$$

$$\underbrace{x^2 - \frac{b}{a}x + \left(\frac{b}{2a}\right)^2}_{\text{Punch Calc}} = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

TRIGO RULES

Even Odd Identities

$$\sin(-\theta) = -\sin(\theta)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\tan(-\theta) = -\tan(\theta)$$

Pytho Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

SUM Difference IDS

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$
$$\cos(a \pm b) = \cos a \cos b \pm \sin a \sin b$$
$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$$

Double Angle

$$\sin(2a) = 2\sin a \cos a$$
$$\cos(2a) = \cos^2 a - \sin^2 a$$
$$= 2\cos^2 a - 1$$
$$= 1 - 2\sin^2 a$$

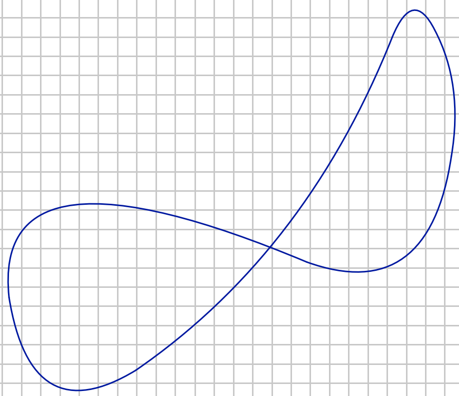
Cofunction

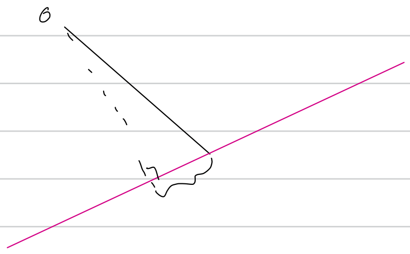
$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$
$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

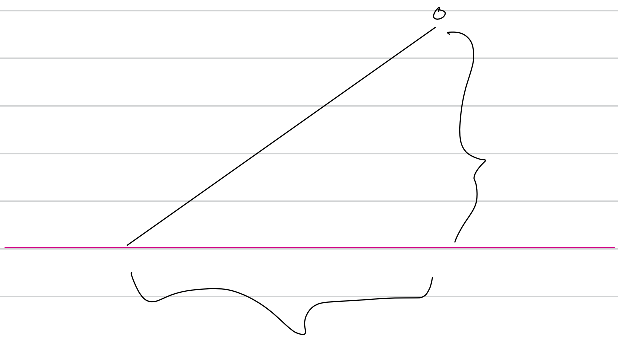
ln Rules

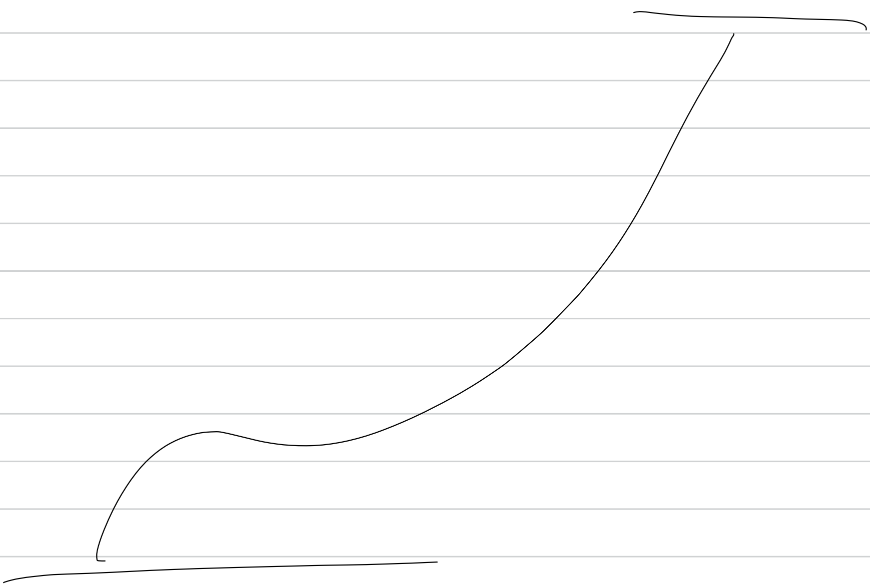
$$\ln(a) + \ln(b) = \ln(ab)$$
$$\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$$
$$-\ln(x) = \ln\left(\frac{1}{x}\right)$$
$$\ln(x^n) = n\ln(x)$$
$$\log_a b = \frac{\ln b}{\ln a}$$

$$\lim_{\infty} e^{-x} = 0$$









$\frac{1}{x}$ $\sqrt{0}$