Nanyang Technological University

MH1810 MATHEMATICS 1

Mock Midterm Test (10 Questions)

Name: Matric Number: Tutorial Group:

This mock test only shows the format of questions that will be asked in the Midterm (which is longer with 20 questions/50 mins). It does not reflect the level of difficulty, nor suggest the type of questions to be asked.

Instructions: Write the answers in the boxes provided. You are not required to write down the working. Only answers in the boxes will be marked.

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1. Find the complex number z satisfying $|z| = 1$ and $\mathrm{Re}(z) = \mathrm{Im}(z) > 0$.

   Answer : $\boxed{\dfrac{\sqrt{2}}{2} + i\,\dfrac{\sqrt{2}}{2}}$

   $\dfrac{\sqrt{2}}{2}$ $1^2 = a^2 + b^2$ C_{ah} $\angle 45$ △

2. Find the smallest positive integer n such that $(-1+i)^n$ a real number.

   Answer : $\boxed{4}$

   $(-1+i)(-1+i)$

3. For any real numbers n and m, consider
$$\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ n \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 2 \\ m+2n-4 \\ m \end{pmatrix}.$$

   If $\mathbf{u}$ is parallel to $\mathbf{v}$, find n and m.

   Answer : $n = \boxed{2}$ and $m = \boxed{4}$

   $4 = m + 2n - 4$
   $2n = m$
   $4 + 4 = 2n + 2n$
   $n = 2$
   $m = 4$

4. Find the distance between the planes $\mathbf{r} \cdot (0, 3, -4) = -10$ and $\mathbf{r} \cdot (0, -6, 8) = -10$.

   Answer: $\boxed{3}$

   $r_1 \cdot \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} = -10$ $r_2 \cdot \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix} = -10$

   $r_1 = \begin{pmatrix} 0 \\ 0 \\ \frac{10}{4} \end{pmatrix}$ $r_2 = \begin{pmatrix} 0 \\ \frac{10}{6} \\ 5 \end{pmatrix}$

   $r_1 \cdot r_2 = \begin{pmatrix} 0 \\ \frac{10}{6} \\ -\frac{10}{4} \end{pmatrix}$

   $\left| \begin{pmatrix} 0 \\ \frac{10}{6} \\ -\frac{10}{4} \end{pmatrix} \cdot \frac{1}{\sqrt{25}} \begin{pmatrix} 0 \\ 3 \\ -4 \end{pmatrix} \right| = \frac{1}{\sqrt{25}}(15)$

   $= \frac{15}{\sqrt{25}}$

   $= 3$

5. Find x and y such that

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}.$$

Answer : $x =$ [4] and $y =$ [2]

6. Find the determinant of the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{pmatrix}.$$

Answer : [-8]

7. Evaluate the limit

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2 + \sqrt{x - 2}}.$$

Answer:

8. Evaluate the limit

$$\lim_{x \to 0} \sin^2(x) \sin\left(\frac{1}{x^2}\right).$$

Answer :

9. Find the largest possible number a such that the function

$$f(x) = \frac{x + \sin(x)}{a + \sin(x)}$$

is <u>not continuous.</u>

Answer : [1]

10. Find the range of the function $f(x) = \sin(g(x))$ if g is a continuous function and the range of g is $[0, \pi]$.

Answer : [$[0, 1]$]

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