NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM II (CA2)

MH1812 – Discrete Mathematics

April 2018 TIME ALLOWED: 40 minutes

Name:			
Matric. no.:		Tutor group:	

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1. **(30 marks)**

Solve the following linear recurrences, that is, write a_n and b_n in terms of n:

(a) $a_n = 10a_{n-1} - 21a_{n-2}$ for $n \geqslant 2$, with initial conditions $a_0 = 3$, $a_1 = 5$;

(b) $b_n = b_{n-1} + 2$ for $n \geqslant 1$, with initial condition $b_0 = 2$.

Justify your answers.

a) $x^2 = 10x - 21$

$x = 7,\ x = 3.$

$a_n = U(7)^n + V(3)^n$

$a_0 = 3 = U + V$

$a_1 = 5 = U(7) + V(3)$

$\therefore U = -1 \quad V = 4$

$a_n = -(7)^n + 4(3)^n$

b) $b_n = b_{n-1} + 2$

$= (b_{n-2} + 2) + 2 = b_{n-2} + 4$

$= (b_{n-3} + 2) + 4 = b_{n-3} + (3)(2)$

$= b_{n-k} + k(2)$ for $0 \leq k \leq n$

$= b_0 + n(2)$

$= 2 + 2n$

Let $P(x): b_x = 2 + 2x$

Basic, $P(0) = 2 + 2(0) = 2$, T

Let $P(x)$ be T

inductive, $P(n+1) = b_n + 2$

$= 2 + 2n + 2$

$= 2 + 2(n+1)$

$\therefore$ True!

QUESTION 2. **(30 marks)**

(a) Prove that

$$\sum_{j=1}^{n} j(3j-1) = n^2(n+1), \qquad \forall n \in \mathbb{N}.$$

(b) Let $A = \{0, 1\}$ and $B = \{4, 5\}$.

(i) Write out all elements of the set $A \times B$.

(ii) What is the cardinality of the power set of $A \times B$?

a) Let $P(n)$ be $\sum_{j=1}^{n} j(3j-1) = n^2(n+1)$

Base case $P(1)$: $\sum_{j=1}^{1} j(3j-1) = 1^2(1+1)$

$1(3-1) = 2$

$2 = 2$

$(n+1)^2((n+1)+1)$

Let $P(n)$ be T

inductive step $P(n+1) = \sum_{j=1}^{n+1} j(3j-1)$

$= \sum_{j=1}^{n} j(3j-1) + (n+1)(3(n+1)-1)$

$= n^2(n+1) + (n+1)(3n+3-1)$

$= (n+1)(n^2 + 3n + 2)$

$= (n+1)(n+1)(n+2)$

$= (n+1)^2((n+1)+1)$

b) i) $\{\{0,4\}, \{0,5\}, \{1,4\}, (1,5)\}$

ii) $2^4 = 16$

QUESTION 3. **(40 marks)** a(i)

Prove LHS

$x \in (\overline{A \cap B}) \wedge C$

$x \in (\overline{A \cap B}) \wedge x \in C$

$x \notin (A \cap B) \wedge x \in C$

$x \notin A \wedge x \notin B \wedge x \in C$

$x \in C \wedge x \notin A \quad \vee \quad x \in C \wedge x \notin B$

(a) Let A, B, and C be sets.

(i) Prove that $(\overline{A \cap B}) \cap C = (C - A) \cup (C - B)$;

(ii) Is $(C - A) \cup (C - B) = C$? If yes, prove it, if no, give a counterexample.

(b) Let $S = \{3a + 6b \,|\, a, b \in \mathbb{Z}\}$.

(i) Show that $S \subseteq \mathbb{Z}$;

(ii) Is $S = \mathbb{Z}$? If yes, prove it, if no, give a counterexample.

take $x \in S$, we want to show $x \in \mathbb{Z}$

$\therefore x \in 3a + 3b$

$\mathbb{Z}$ close under addn