CX1107 Data Structures and Algorithms

Trees

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So Far ...

Dynamic Memory Management

- **`#include <stdlib.h>`**
- **`malloc()`**
- **`free()`**

```
struct _listnode
{
    int item;
    struct _listnode *next;
};
typedef struct _listnode ListNode;
```

1. Display: **`printList()`**
2. Search: **`findNode()`**
3. Insert: **`insertNode()`**
4. Delete: **`removeNode()`**
5. Size: **`sizeList()`**

Linked List vs Array

1. **Display: Both are similar**
2. **Search: Array is better**
3. **Insert and Delete: Linked List is more flexible**
4. **Size: Array is better**

1. Display: printList()
2. Search: findNode()
3. Insert: insertNode()
4. Delete: removeNode()
5. Size: sizeList()

…

```
void printList(ListNode *cur){
    while (cur != NULL){
        printf("%d\n", cur->item);
        cur = cur->next;
    }
}
```

```
int sizeList(ListNode *head){
    int count = 0;
    while (head != NULL){
      count++;
      head = head->next;
    }
    return count;
}
```

```
ListNode *findNode(ListNode* cur, int i){
   if (cur==NULL || i<0)
      return NULL;
   while(i>0){
      cur=cur->next;
      if (cur==NULL)
         return NULL;
      i--;
   }
   return cur;
}
```

```
int insertNode(ListNode **ptrHead, int i, int item){
    ListNode  *pre, *newNode;
    if (i == 0){
        newNode = malloc(sizeof(ListNode));
        newNode->item = item;
        newNode->next = *ptrHead;
        *ptrHead = newNode;
        return 1;
    }
    else if ((pre = findNode(*ptrHead, i-1)) != NULL){
        newNode = malloc(sizeof(ListNode));
        newNode->item = item;
        newNode->next = pre->next;
        pre->next = newNode;
        return 1;
    }
    return 0;
}
```

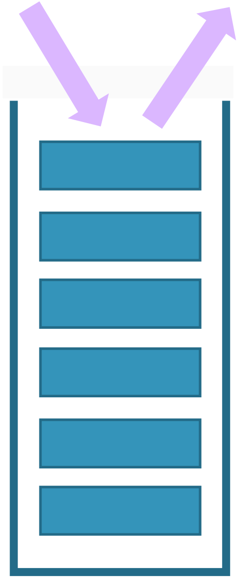
Stacks and Queues

```
typedef struct _linkedlist{
     int size;
     ListNode *head;
} LinkedList;
```

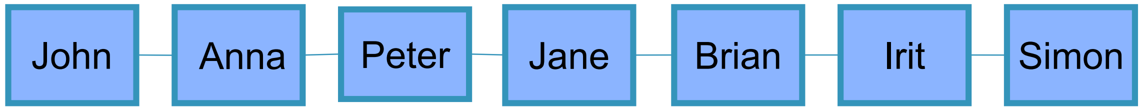
1. **Variations of the linked list**
   - **Doubly Linked List**
   - **Circular Linked List**
   - **Circular Doubly Linked List**
2. **Stacks and Queues**

`All these dynamic data structures are linear data structures`

LIFO

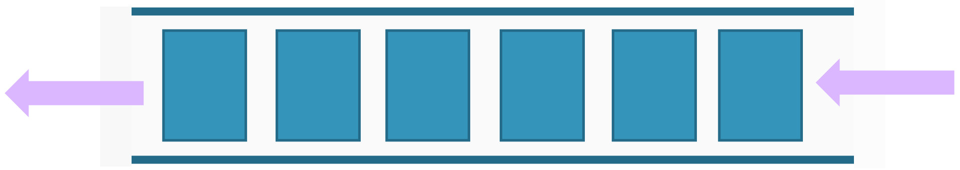


Stack



Linked List

Queue FIFO



```
typedef ListNode StackNode;

typedef LinkedList Stack;
```

Stack

1. Retrieve: `peek()`
2. Insert: `push()`
3. Delete: `pop()`
4. Size: `isEmptyStack()`

```
typedef ListNode QueueNode;
typedef struct _queue{
   int size;
   ListNode *head;
   ListNode *tail;
} Queue;
```

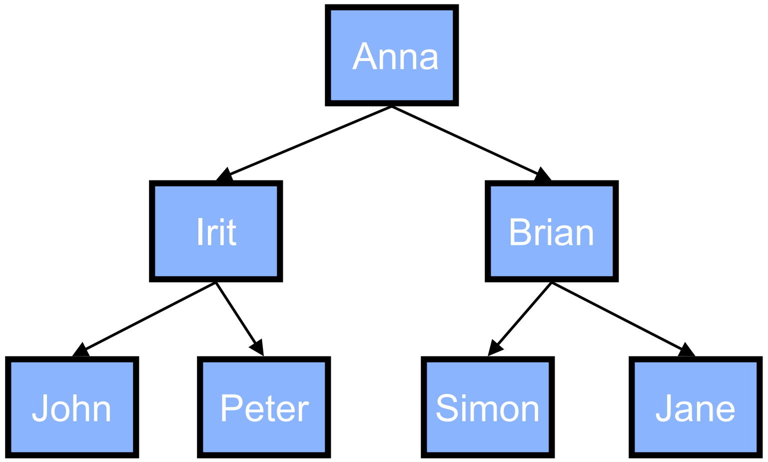
Queue

1. Retrieve: `getFront()`
2. Insert: `enqueue()`
3. Delete: `dequeue()`
4. Size: `isEmptyQueue()`

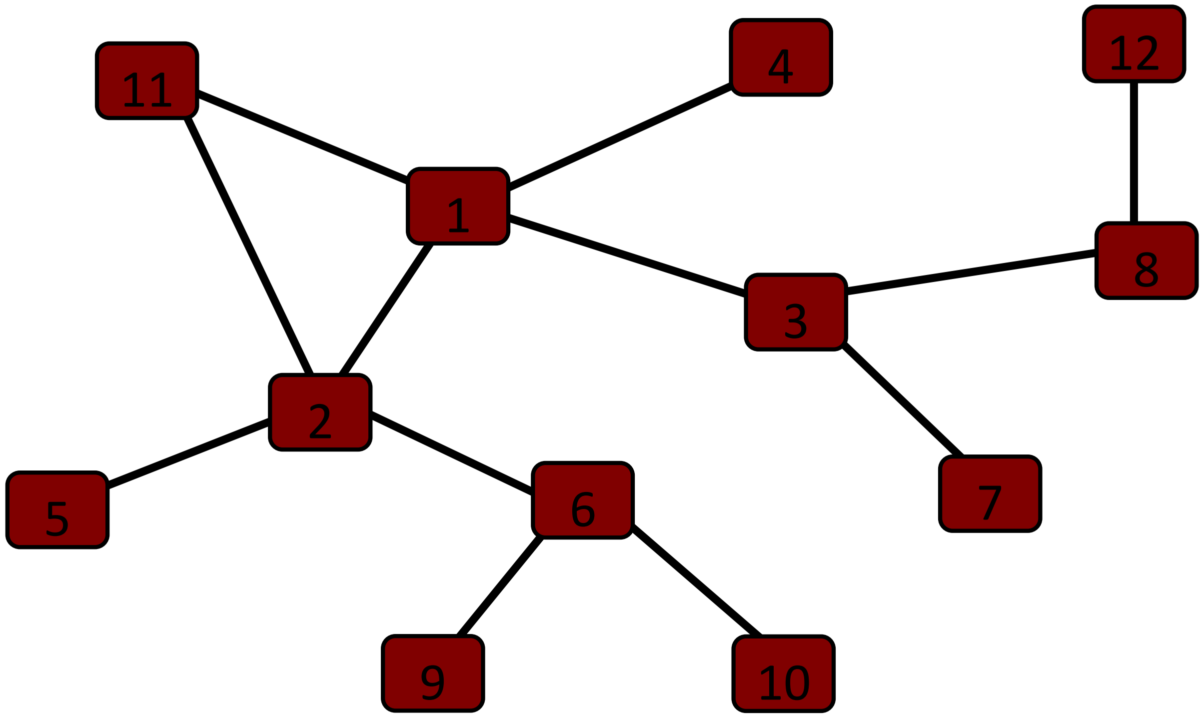
Overview

1. **What are trees?**
2. **Why do you need a tree?**
3. **How to create a tree?**
4. **How to use the tree?**

What are trees?

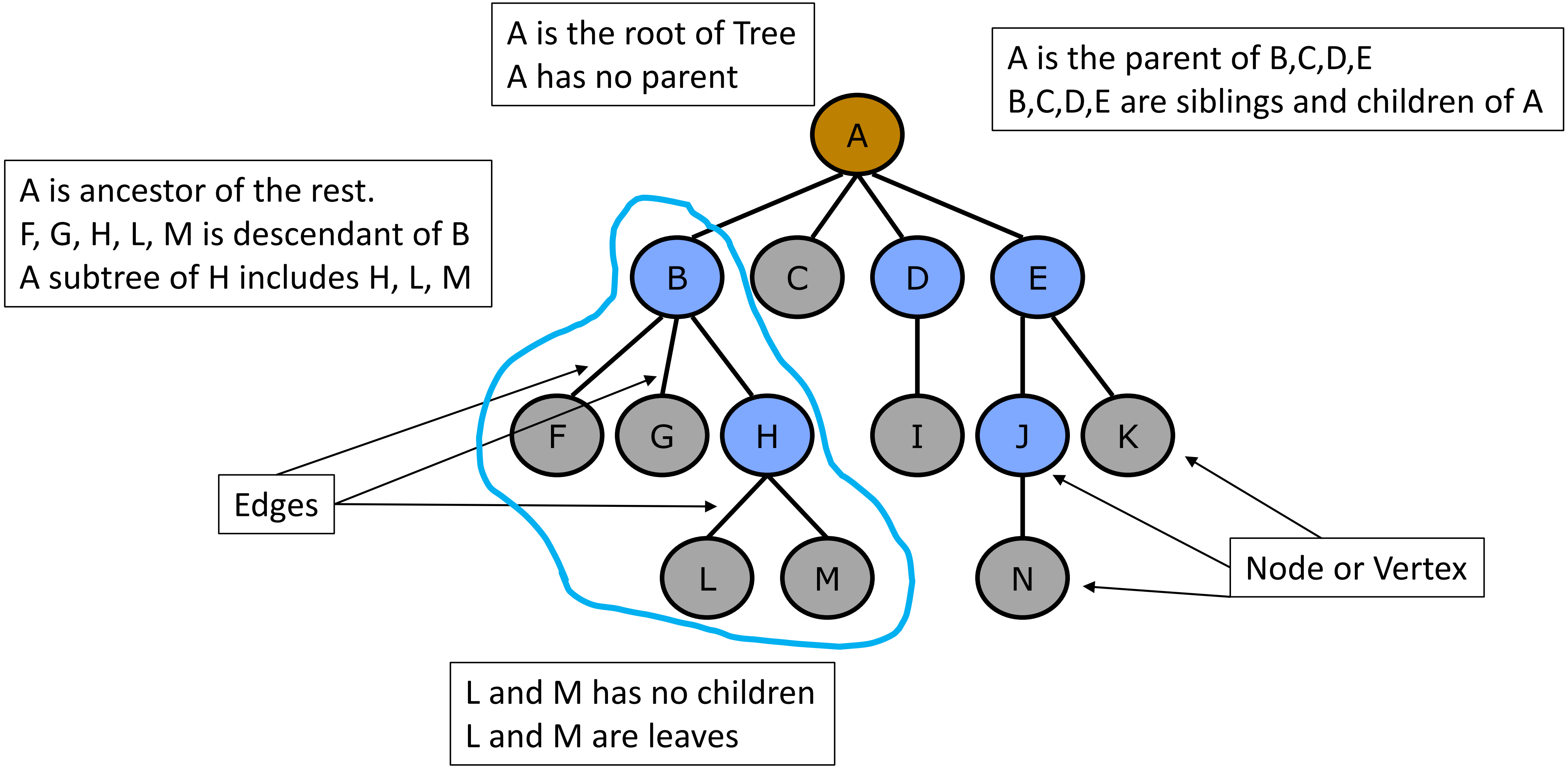


Tree



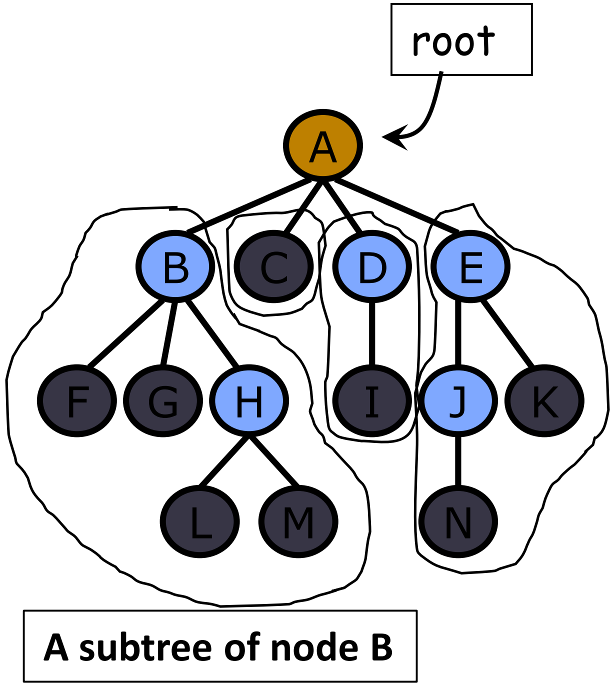
Graph

Terminology In Tree



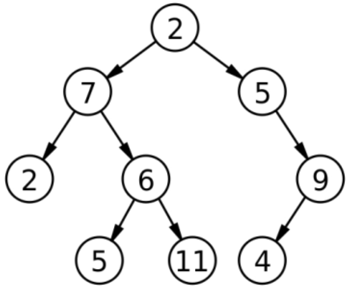
Tree Data Structure

- Similar to family tree concept
- One special node: root
- Each node can has many children
  A has four children: B, C, D, E
- Each node (except the root) has a parent node
- **A is the parent of B, C, D, E**
- Other children of your parent are your siblings
- **B, C, D and E are siblings**
- **Subtree**: Any node in the tree together with all of its descendants for a subtree.

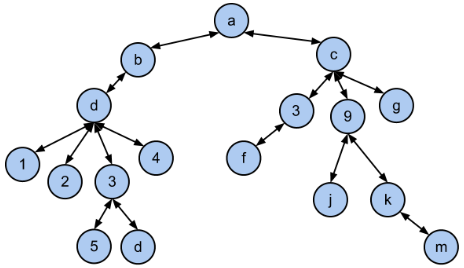


Tree Data Structure

- Tree data structure looks like… a tree: root, branches, leaves
  - Only one root node which has no parent
  - Each node branches out to some number of nodes
    - For binary tree, each node has up to two children (left and right child)
  - Each node has only one “parent” node – the node pointing to it (except the root node)



Binary Tree

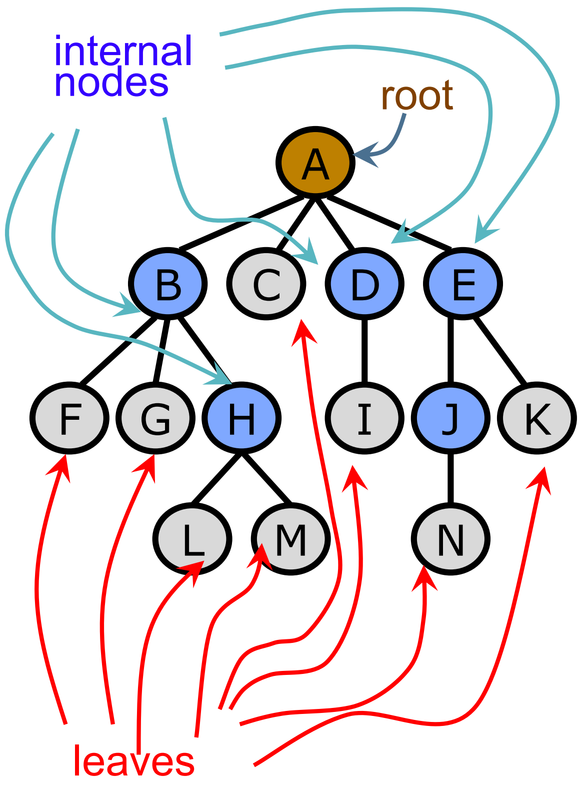


General Tree

- General tree
  - Each node can have links to any number of other nodes

Tree Data Structure

- A tree is composed of nodes
- Types of nodes
  - Root: only one in a tree, has no parent.
  - Internal node(non-leaf): Nodes with children are called internal nodes
  - Leaf (External Node): nodes without children are called leaves



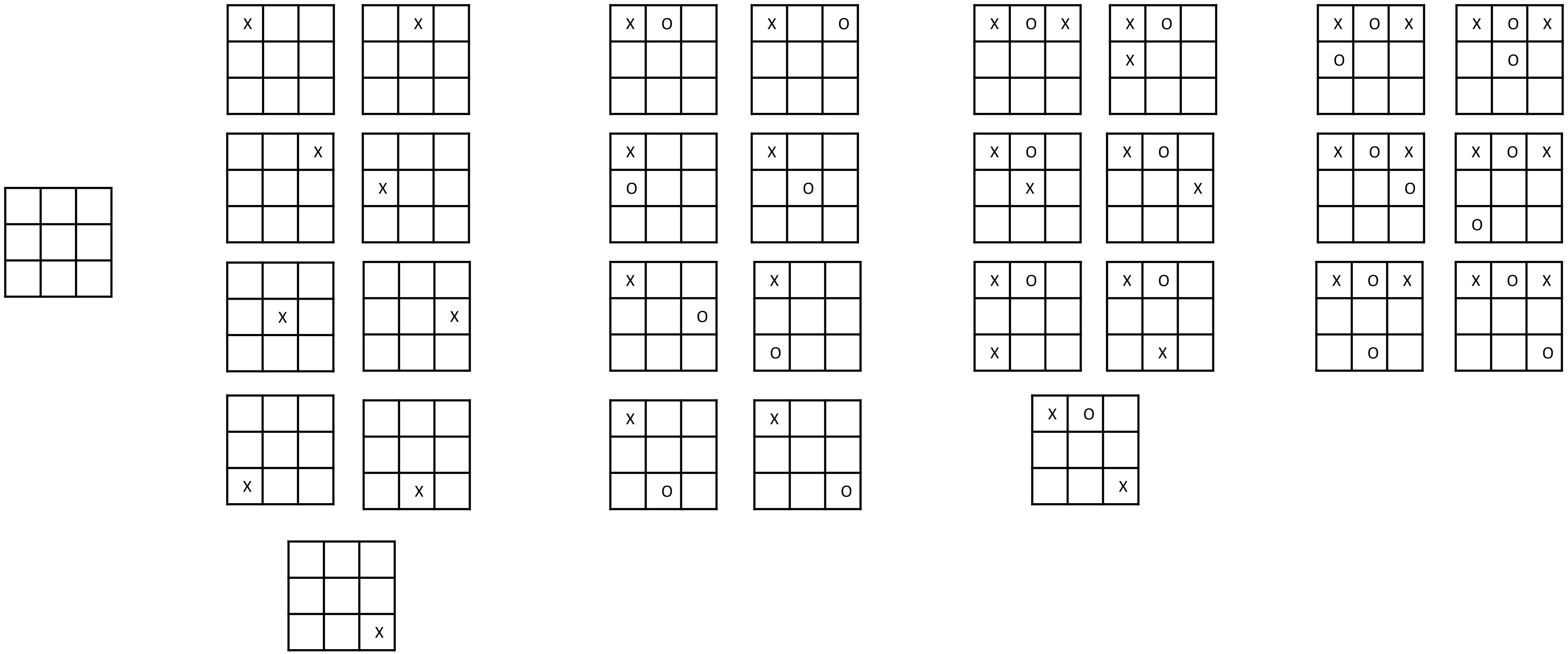
1. What are trees?
2. **Why do you need a tree?**
3. How to create a tree?
4. How to use the tree?

Why Trees?

- Model layouts with hierarchical relationships between items
  - Chain of command in the army
  - Personnel structure in a company
- Optimization problems – Huffman coding (a lossless data compression algorithm. It assigns variable-length codes to input characters based on the usage frequency)
- Permutation, Searching Problems –
  - Eight Queens Problem
  - Gaming eg. Sudoku, Tic-tac-toe

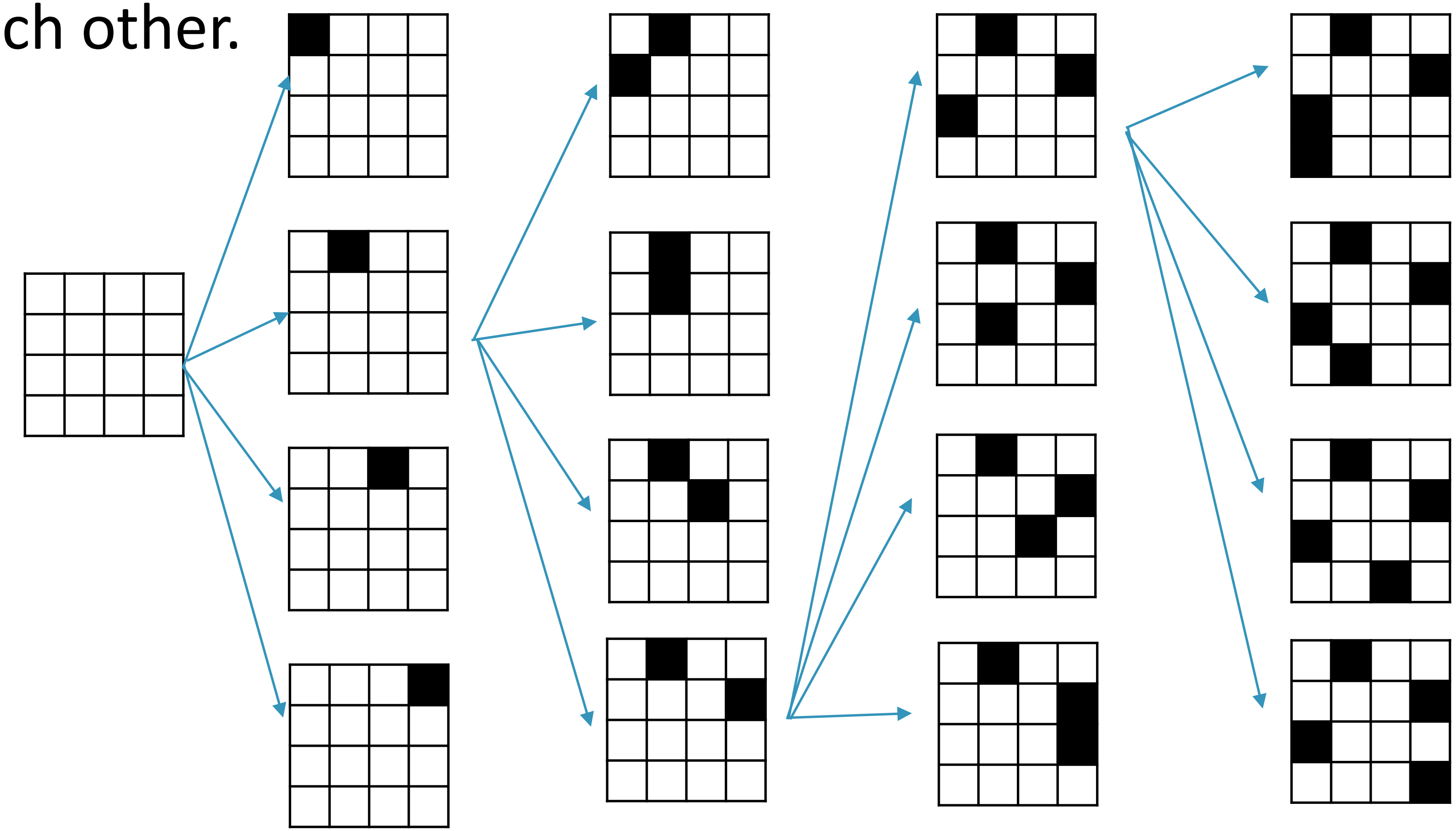
Tic-Tac-Toe

Tic-tac-toe aka noughts and crosses is a paper and pencil game for two players, who take turns marking the spaces in a 3x3 grid. The player who succeeds in placing three of their marks in a horizontal, vertical or diagonal row wins the game

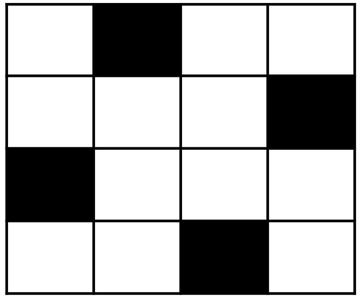


Four Queens Puzzle

Place four queens on a 4x4 chessboard so that no two queens can capture each other.

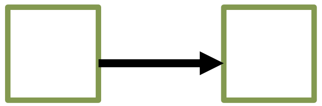


- Each node has 4 children
- The height of the tree is 5

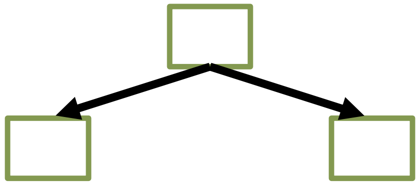


1. What are trees?
2. Why do you need a tree?
3. **How to create a tree?**
4. How to use the tree?

Binary Tree Structure

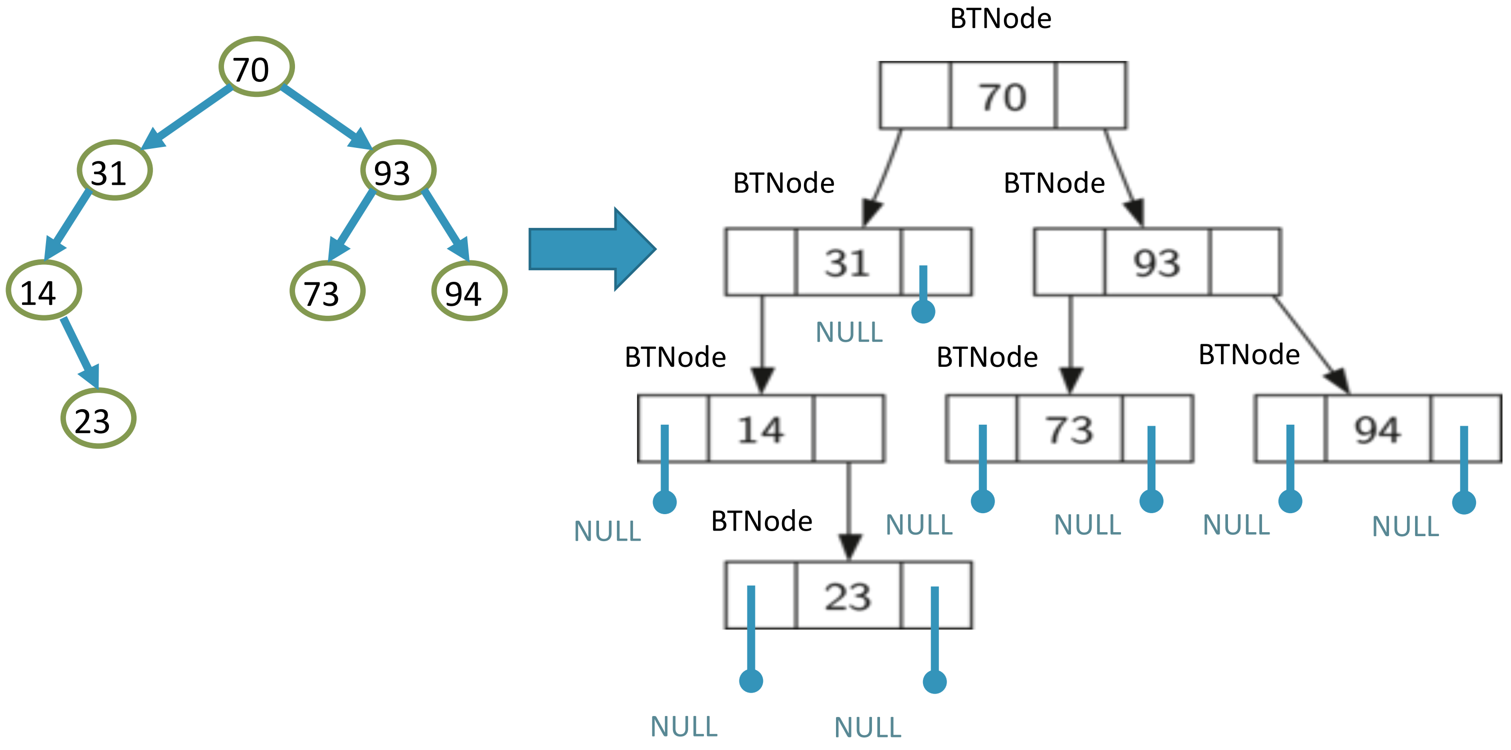


```
typedef struct _listnode{
    int item;
    struct _listnode *next;
}ListNode;
```

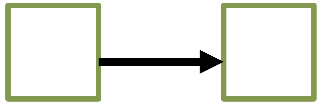


```
typedef struct _btnode{
    int item;
    struct _btnode *left;
    struct _btnode *right;
} BTNode;
```

Example Binary Tree



Tree Traversal Problems

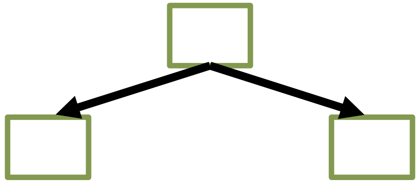


```
typedef struct _listnode{
   int item;
   struct _listnode *next;
}ListNode;
```

Interface Functions

1. Display: printList()
2. Search: findNode()
3. Insert: insertNode()
4. Delete: removeNode()
5. Size: sizeList()

…



```
typedef struct _btnode{
    int item;
    struct _btnode *left;
    struct _btnode *right;
} BTNode;
```

Traversal Problem:
How to systematically travel each node in the tree?

1. What are trees?
2. Why do you need a tree?
3. How to create a tree?
4. **How to use the tree?**
    - **Binary Tree Traversal**

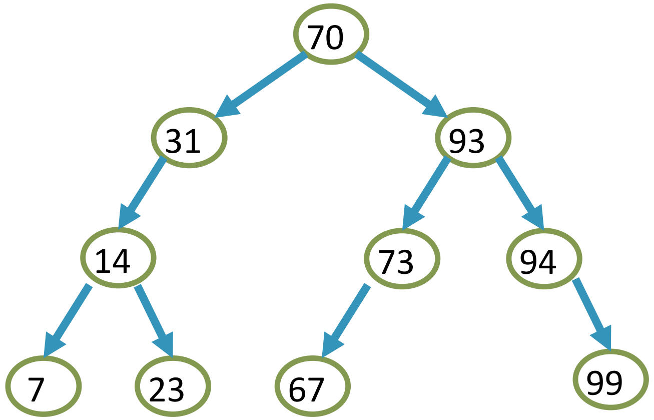
Binary Tree Traversal

```
typedef struct _btnode{
    int item;
    struct _btnode *left;
    struct _btnode *right;
} BTNode;
```

Given a binary tree,

how do you systematically visit every nodes once only?

- **Print the contents of a tree**
- **Search a node**
- **Find the size of a tree**



- **Insert a node**
- **Remove a node**

Binary Tree Traversal

```
typedef struct _btnode{
    int item;
    struct _btnode *left;
    struct _btnode *right;
} BTNode;
```

Traversal Problem:

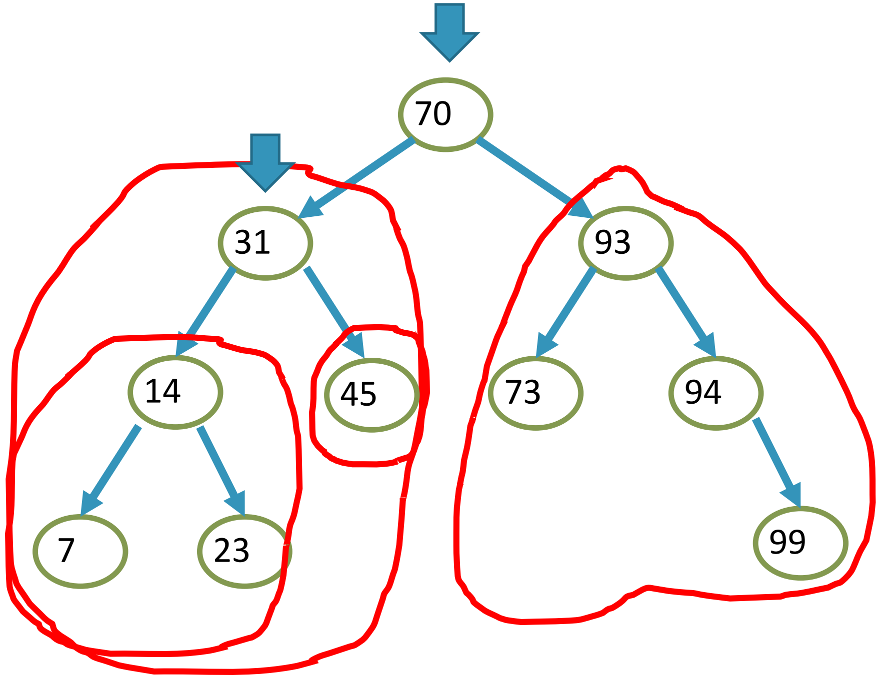
- **Visit root + right subtree + left subtree**
- **Each subtree repeat the same procedure**

  visit root + right subtree + left subtree

- **Until reach the leave**

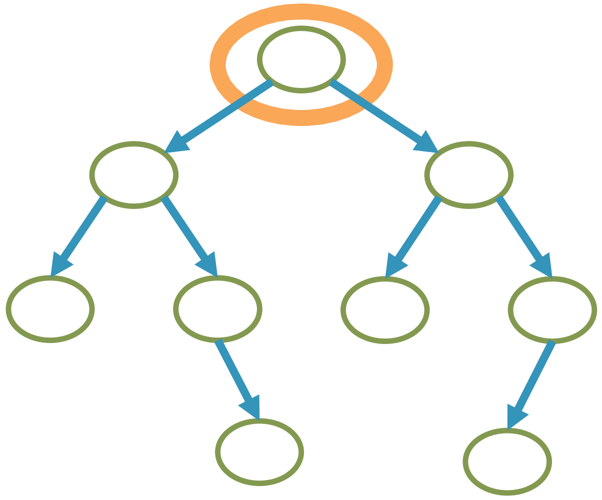
  visit the root (leaf only)

- **It is a recursive problem**



Pseudocode of Binary Tree Traversal

```
TreeTraversal(Node N):
    Visit N;
    If (N has left child)
        TreeTraversal(LeftChild);
    If (N has right child)
        TreeTraversal(RightChild);
    Return; // return to parent
```



This traversal approach is known as pre-order depth first traversal.

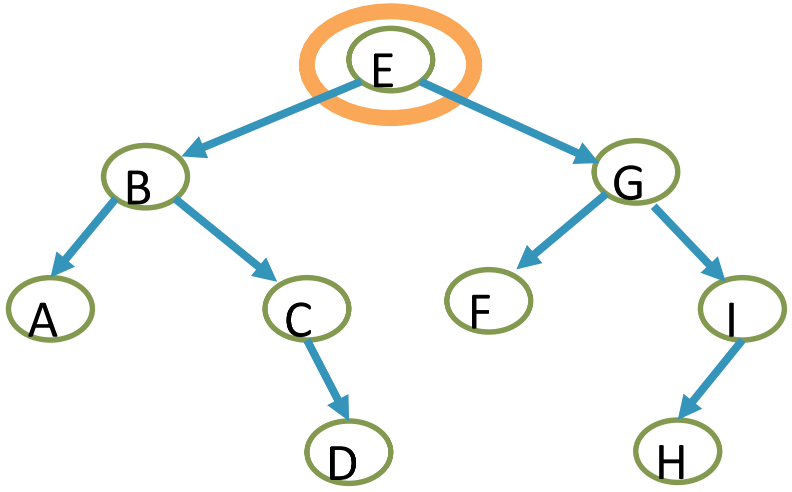
Traversal Approaches on A Binary Tree

- Depth-First Traversal: From the root of a tree, it explores as far as possible. Then it will do the backtracking. There are three traversal orders:
  - Pre-order
  - In-order
  - Post-order

- Breadth-First Traversal: From the root of a tree, it explores each node in level by level.

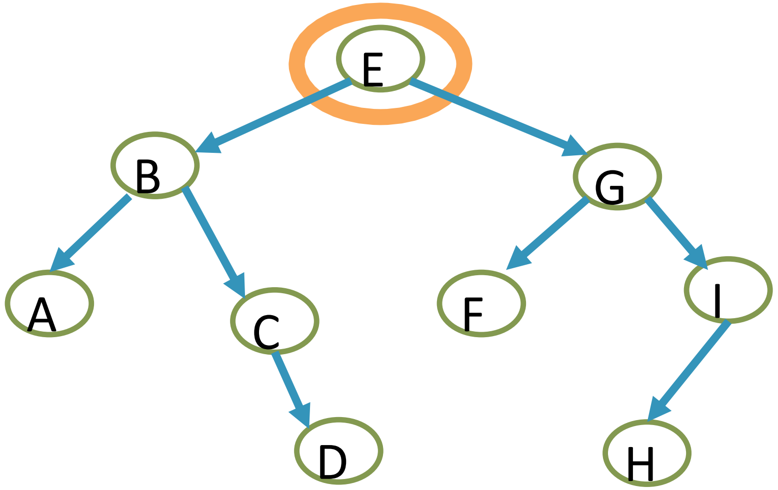
Depth First Traversal: Pre-Order

- **Pre-order**
  - **Process the current node's data**
  - **Visit the left child subtree**
  - **Visit the right child subtree**
- In-order
- Post-order



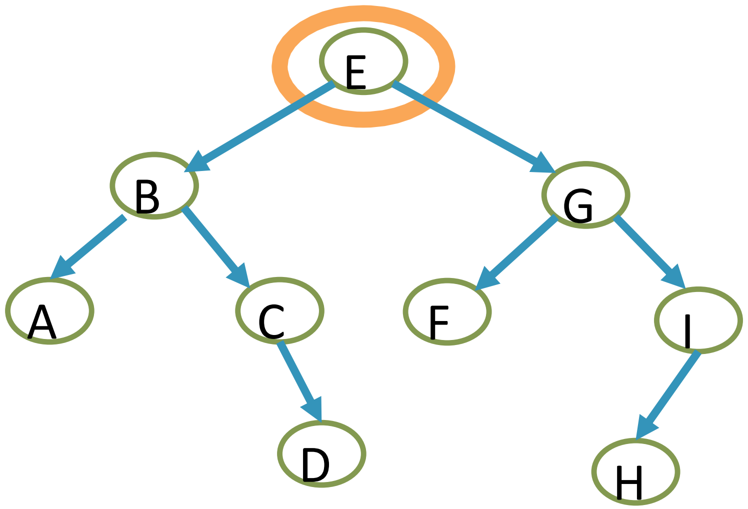
Depth First Traversal: In-Order

- Pre-order
  - Process the current node's data
  - Visit the left child subtree
  - Visit the right child subtree
- **In-order**
  - **Visit the left child subtree**
  - **Process the current node's data**
  - **Visit the right child subtree**
- Post-order



Depth First Traversal: Post-Order

- Pre-order
  - Process the current node's data
  - Visit the left child subtree
  - Visit the right child subtree
- In-order
  - Visit the left child subtree
  - Process the current node's data
  - Visit the right child subtree
- **Post-order**
  - **Visit the left child subtree**
  - **Visit the right child subtree**
  - **Process the current node's data**



Traversal Approaches on A Binary Tree

- Depth-First Traversal: From the root of a tree, it explores as far as possible. Then it will do the backtracking. There are three traversal orders:
  - Pre-order
  - In-order
  - Post-order

- **Breadth-First Traversal**: From the root of a tree, it explores each node in **level by level**.

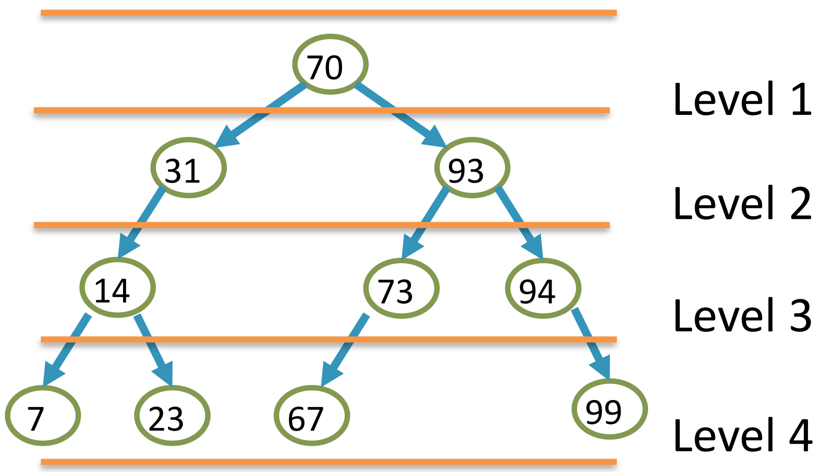
Breadth-First Traversal: Level-by-level

Level-By-Level Traversal:

Visit the root (Level 1)

Visit children of the root (Level 2)

Visit grandchildren of the root (Level3) ...



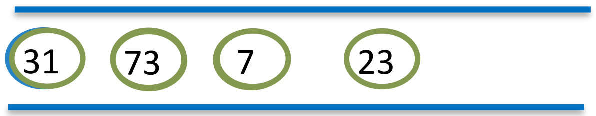
How?

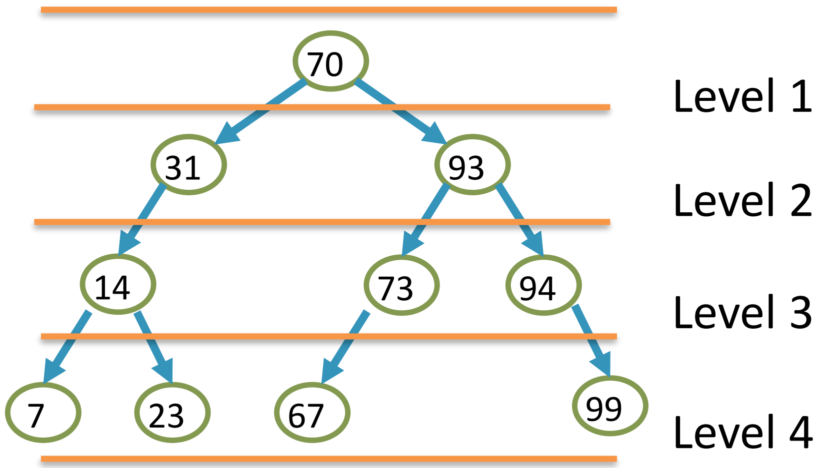
- Visiting the node
- Remember all its children
    - Use a queue (FIFO structure)

Breadth-First Traversal: Level-by-level

Level-By-Level Traversal:

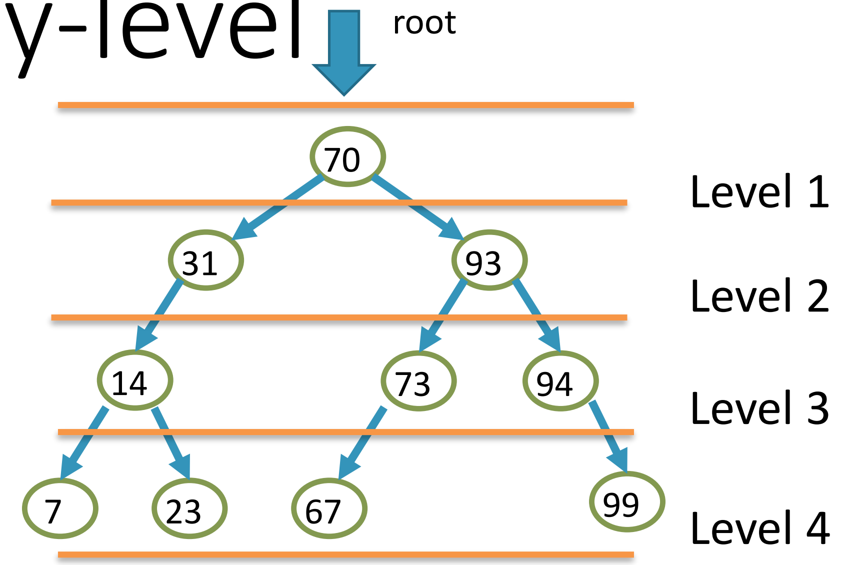
- Visiting the node
- Remember all its children
  - Use a queue (FIFO structure)





1. Enqueue the current node
2. Dequeue a node
3. Enqueue its children if it is available
4. Repeat Step 2 until the queue is empty

Breadth-First Traversal: Level-by-level



Level-By-Level Traversal:

1. Enqueue the current node

2. Dequeue a node

3. Enqueue its children if it is available

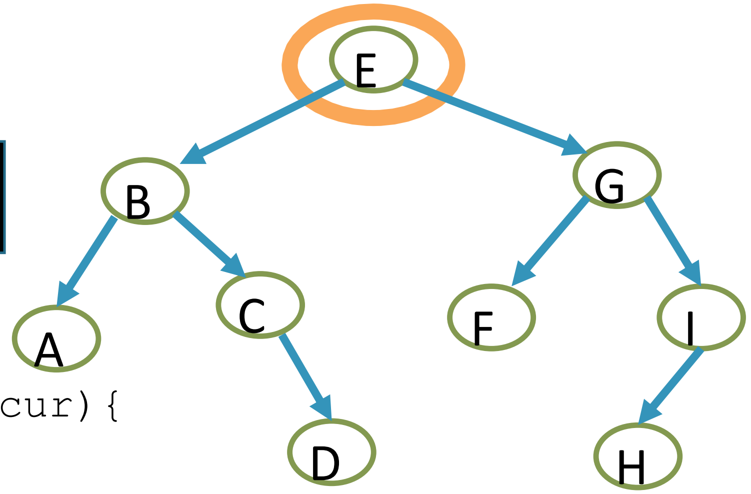
4. Repeat Step 2 until the queue is empty

```
void BFT(BTNode *root){
    Queue *q;
    BTNode* node;
    if(root){
        enqueue(q,root); //data type of item in queue is BTNode*
        while(!isEmptyQueue(*q)){
            node = getFront(*q);dequeue(q);
            if(node->left) enqueue(q,node->left);
            if(node->right) enqueue(q,node->right);
        }
    }
}
```

Tree Traversal Pre-order: Print

Output:

E B A C D G F I H

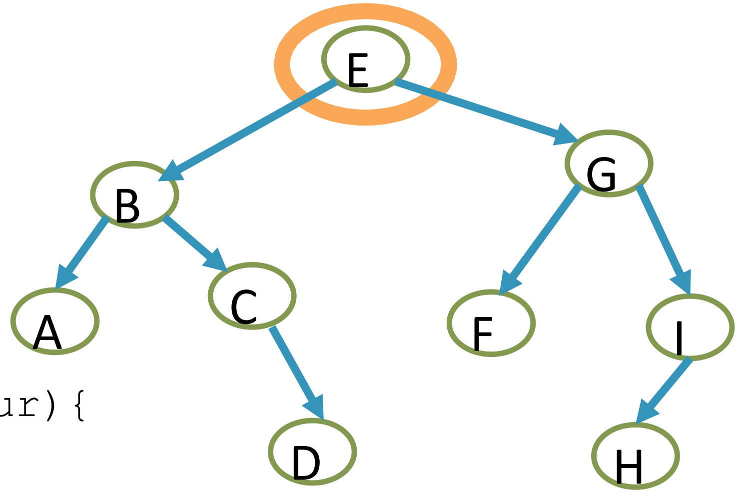



```
void TreeTraversal_pre(BTNode *cur){
    if (cur == NULL)
        return;

    printf("%c  ",cur->item);

    TreeTraversal_pre(cur->left); //Visit the left child node
    TreeTraversal_pre(cur->right);//Visit the right child node
}
```

Tree Traversal In-order: print

Output:

A B C D E F G H I

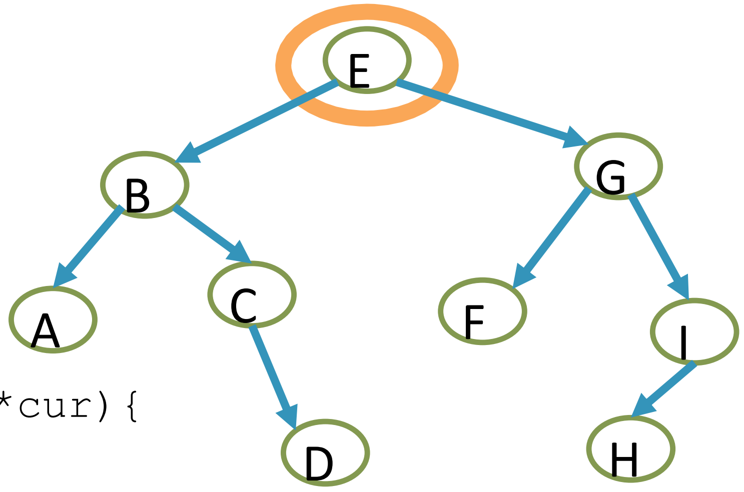



```
void TreeTraversal_in(BTNode *cur){
    if (cur == NULL)
        return;

    TreeTraversal_in(cur->left); //Visit the left child node
    printf("%c  ",cur->item);
    TreeTraversal_in(cur->right);//Visit the right child node
}
```

Tree Traversal Post-order: print

Output:

A D C B F H I G E




```
void TreeTraversal_post(BTNode *cur){
    if (cur == NULL)
        return;

    TreeTraversal_post(cur->left); //Visit the left child node
    TreeTraversal_post(cur->right);//Visit the right child node
    printf("%c  ",cur->item);
}
```

Pre-Order Traversal

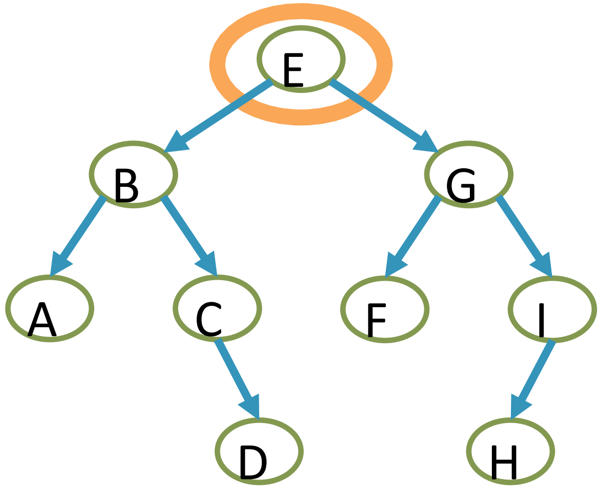
E B A C D G F I H

In-Order Traversal

A B C D E F G H I

Post-Order Traversal

A D C B F H I G E



Count Nodes in a Binary Tree (SIZE)

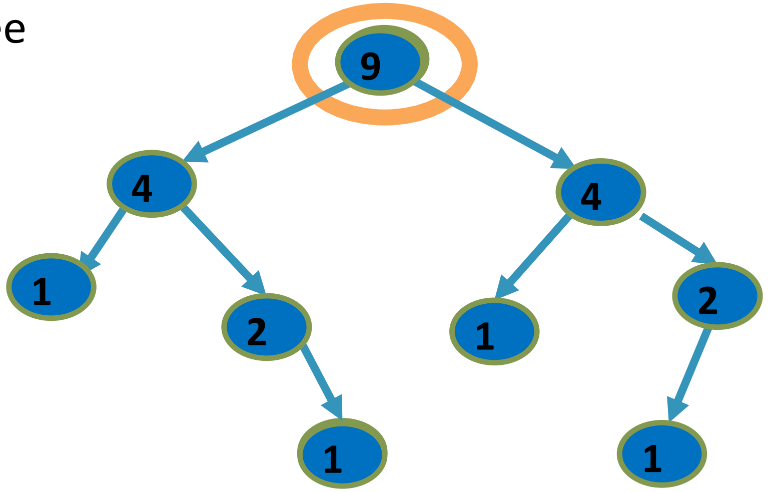
- Recursive definition:
  - Number of nodes in a tree

    = 1

    + number of nodes in left subtree

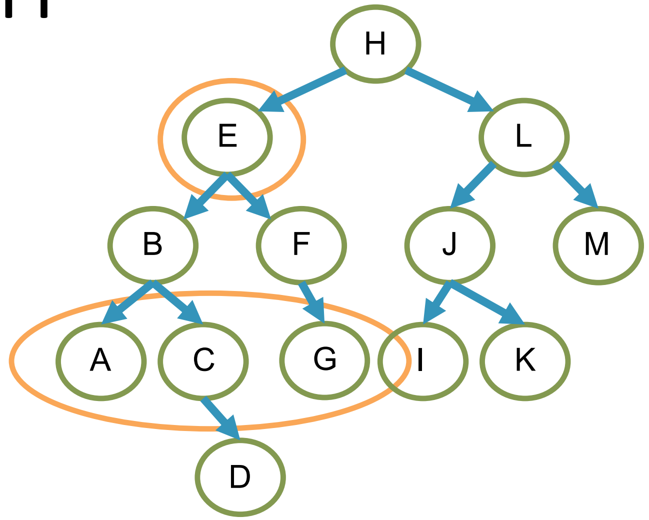
    + number of nodes in right subtree
- Each node returns the number of nodes in its subtree

```
int countNode(BTNode *cur){
    if (cur == NULL)
        return 0;

    return (countNode(cur->left)
          + countNode(cur->right)
          + 1);
}
```



Find the k-level Grandchildren

- Given a node X, find all the nodes that are X's grandchildren
- Given node E, we should return grandchild nodes A, C, and G
- What if we want to find k-level grandchildren?
  - **Need a way to keep track of how many levels down we've gone**




```
void findgrandchildren(BTNode *cur, int c){
    if (cur == NULL) return;
    if (c == k){
        printf("%d ", cur->item);
        return;
    }
    if (c < k){
        findgrandchildren(cur->left, c+1);
        findgrandchildren(cur->right, c+1);
}
```

2-level grandchildren

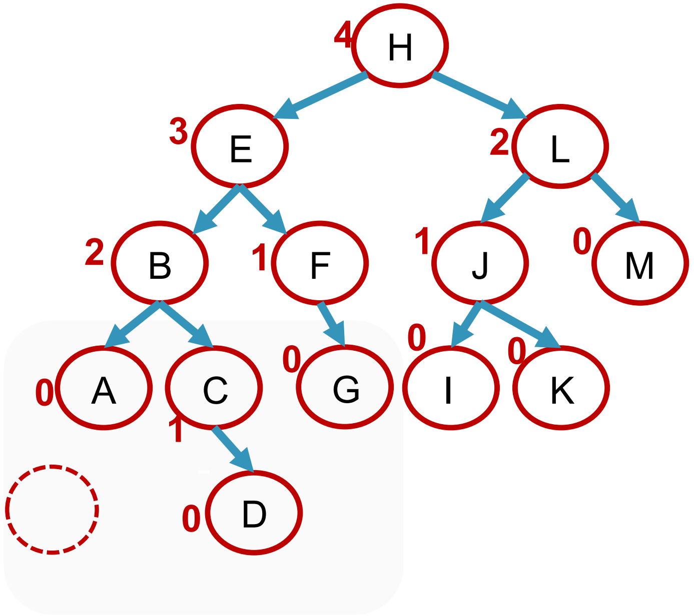
X->left->left
X->left->right
X->right->left
X->right->right

Calculate Height of Every Node

We want each node to report its height

- Leaf node must report 0
- At "null" condition, must report -1

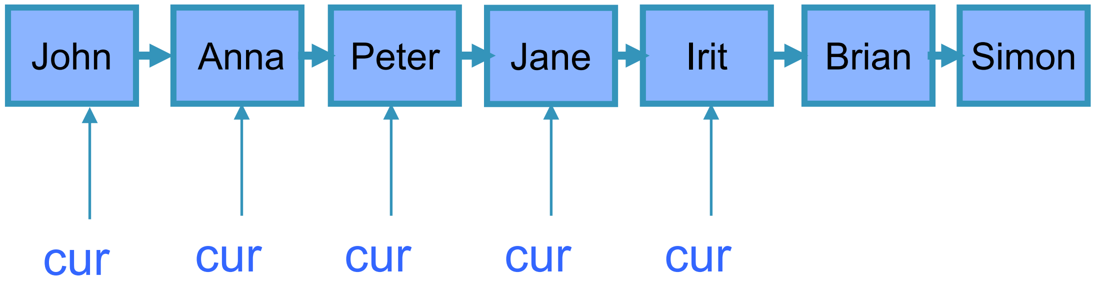
```
int TreeTraversal(BTNode *cur){
    if(cur == NULL)
        return -1;
    int l = TreeTraversal(cur->left);
    int r = TreeTraversal(cur->right);
    int c = max (l, r) + 1;
    return c;
}
```



Sequential Search by a linked list/ array

inefficient

- Given a linked list of names, how do we check whether a given name(e.g., Irit) is in the list?




```
while (cur!=NULL) {
        if cur->item == "Irit"
            found and stop searching;
        else
            cur = cur->next; }
```

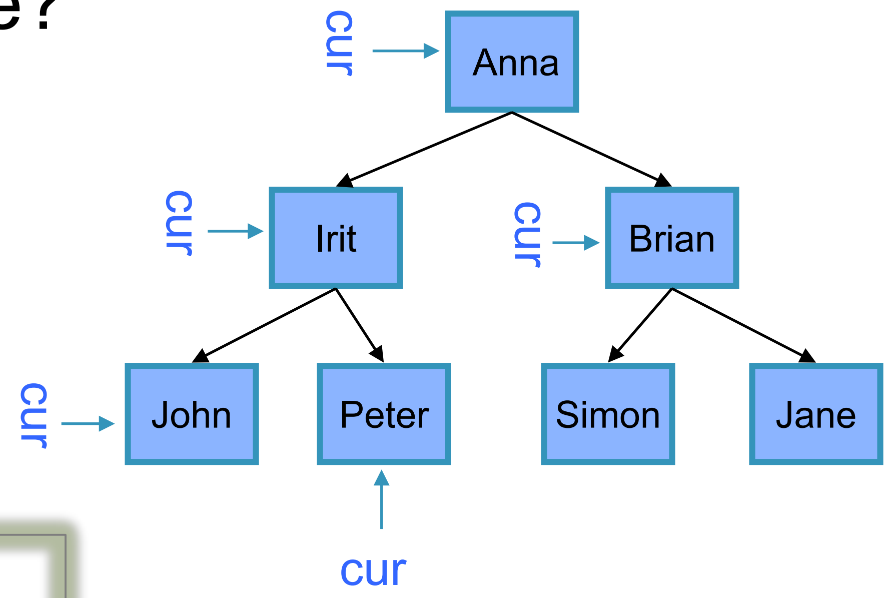
How many nodes are visited during search?
--best case: 1 node (John) => Θ(1)
--worst case: 7 nodes (Simon) => Θ(n)
--avg. case: (1+2+3+…+7)/7=4 nodes => Θ(n)

Sequential Search by a binary tree

inefficient

- Given a binary tree of names, how do we check whether a given name(e.g., Brian) is in the tree?

- **Use the TreeTraversal (Pre-order) template, to check every node**




```
TreeTraversal(Node N)
 if N==NULL return;
 if N.item=given_name return;
 TreeTraversal(LeftChild);
 TreeTraversal(RightChild);
 Return;
```

How many nodes are visited during search?
--best case: 1 node (John) => Θ(1)
--worst case: 7 nodes (Simon) => Θ(n)
--avg. case: (1+2+3+…+7)/7=4 nodes => Θ(n)

Summary

- The difference between linked lists and tree structures (linear and non-linear data structures)
- Overview of Tree
- Tree Traversal
  - Depth-First Traversal
    - Pre-order Traversal
    - In-order Traversal
    - Post-order Traversal
  - Breadth-First Traversal: Level-by-level traversal
- Examples

Make sure that you know the difference among them

SC1007
Data Structures and Algorithms

Binary Search Tree

College of Engineering

School of Computer Science and Engineering

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Lecturer

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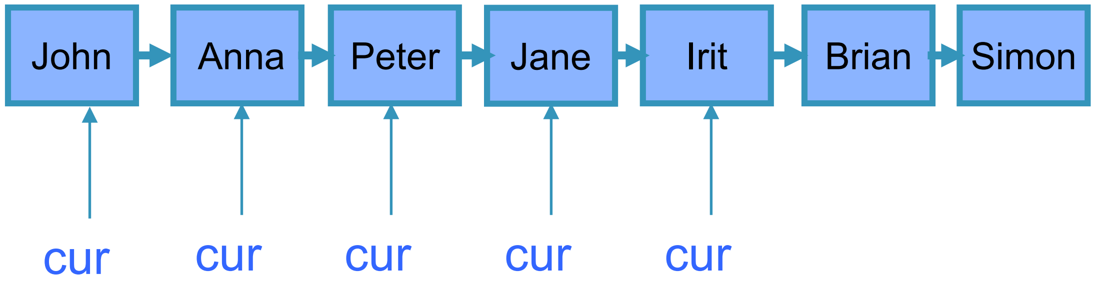
Linear Search by a linked list/ array

inefficient

- Given a linked list of names, how do we check whether a given name(e.g., Irit) is in the list?




```
while (cur!=NULL) {
        if cur->item == "Irit"
            found and stop searching;
        else
            cur = cur->next; }
```

How many nodes are visited during search?
--best case: 1 node (John) => Θ(1)
--worst case: 7 nodes (Simon) => Θ(n)
--avg. case: (1+2+3+…+7)/7=4 nodes => Θ(n)

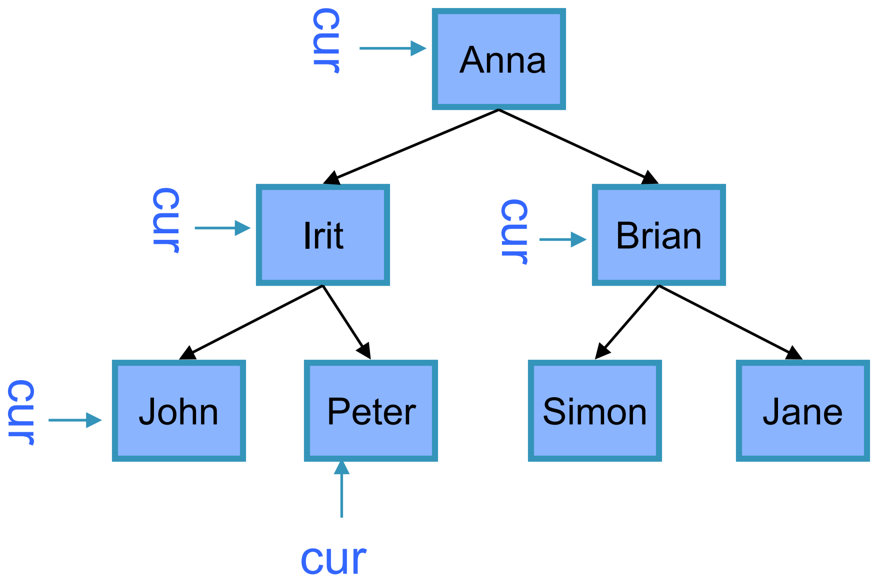
Linear Search by a binary tree

inefficient

- Given a binary tree of names, how do we check whether a given name(e.g., Brian) is in the tree?

- **Use the TreeTraversal (Pre-order) template, to check every node**

How do we insert data into the binary tree?



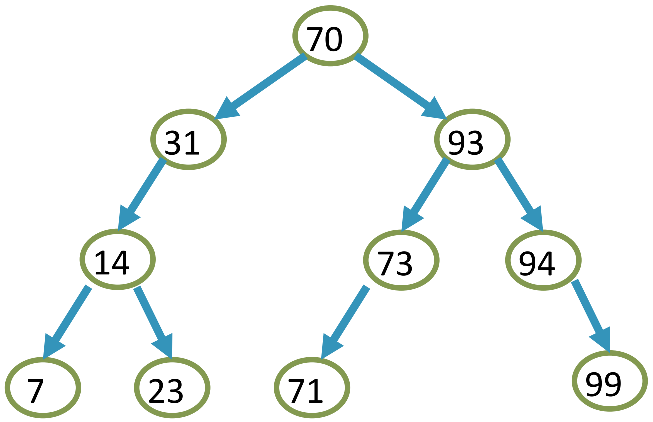

```
TreeTraversal(Node N)
 if N==NULL return;
 if N.item=given_name return;
 TreeTraversal(LeftChild);
 TreeTraversal(RightChild);
 Return;
```

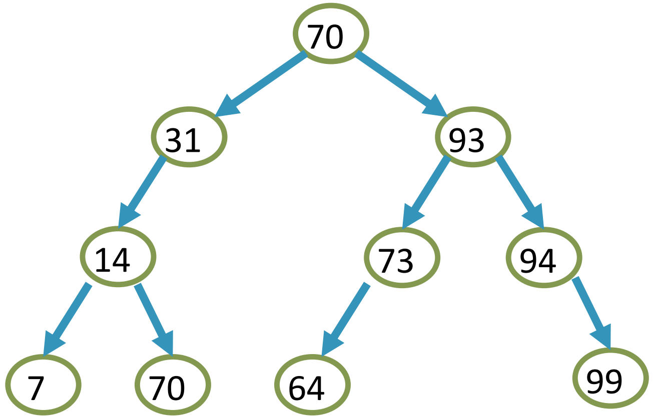
How many nodes are visited during search?
--best case: 1 node (John) => Θ(1)
--worst case: 7 nodes (Simon) => Θ(n)
--avg. case: (1+2+3+…+7)/7=4 nodes => Θ(n)

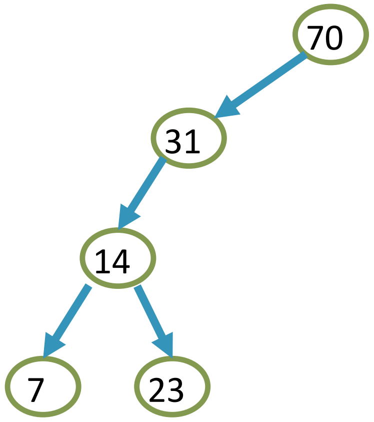
Binary Search Tree

If the given binary tree is a binary search tree (BST), then each node in the tree satisfies the following properties:

1. **Node's value is greater than all values in its left subtree.**
2. **Node's value is less than all values in its right subtree.**
3. **Both subtrees of the node are also binary search trees.**







Binary Search Tree: Search

- The approach is a decrease-and-conquer approach
- A problem is divided into two smaller and similar sub-problem, one of which does not even have to be solved
- The method uses the information of the order to reduce the search space.

```
BTNode* findBSTNode(BTNode *cur, char c){
    if (cur == NULL) {
       printf("can't find!");
       return cur;
    }

    if(c==cur->item){
       printf("Found!\n");
       return cur;
    }

    if(c<cur->item)
       return findBSTNode(cur->left,c);
    else
       return findBSTNode(cur->right,c);
}
```

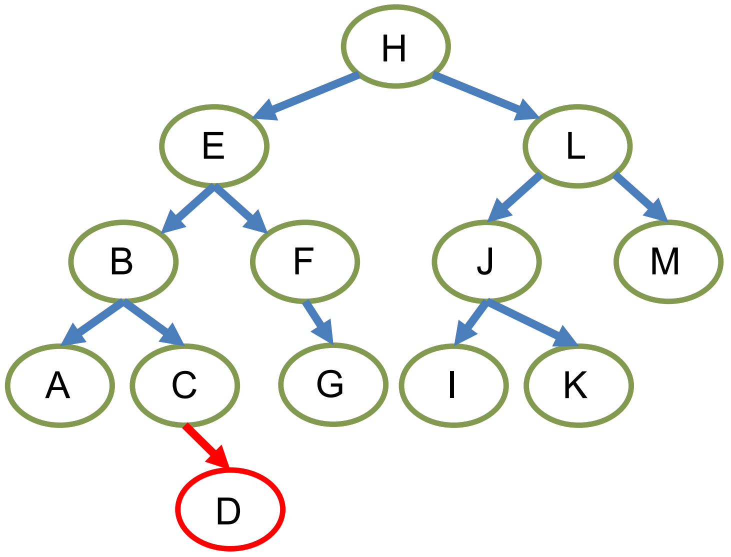
```
void TreeTraversal_pre(BTNode *cur){
    if (cur == NULL) return;

    printf("%c  ",cur->item);

    TreeTraversal_pre(cur->left);
    TreeTraversal_pre(cur->right);
}
```

Binary Search Tree: Insertion

- **After insert a node, the BST must remain as a BST**
- **A duplicate node is not allowed for insertion**
- **A unique position of the given BST will be given to the node**

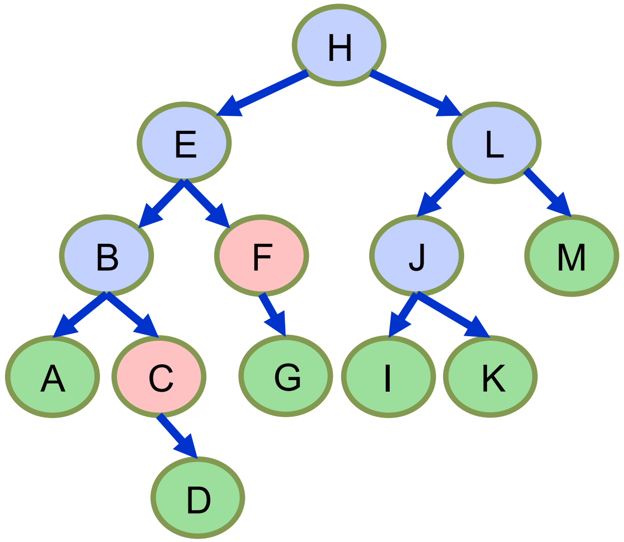



```
BTNode* insertBSTNode(BTNode* cur, char c)
{
    if (cur == NULL){
     BTNode* node = (BTNode*) malloc(sizeof(BTNode));
     node->item = c;
     node->left = node->right = NULL;
     return node;
     }
    if (c < cur->item)
        cur->left  = insertBSTNode (cur->left, c);
    else if (c > cur->item)
        cur->right = insertBSTNode (cur->right, c);

    return cur;
}
```

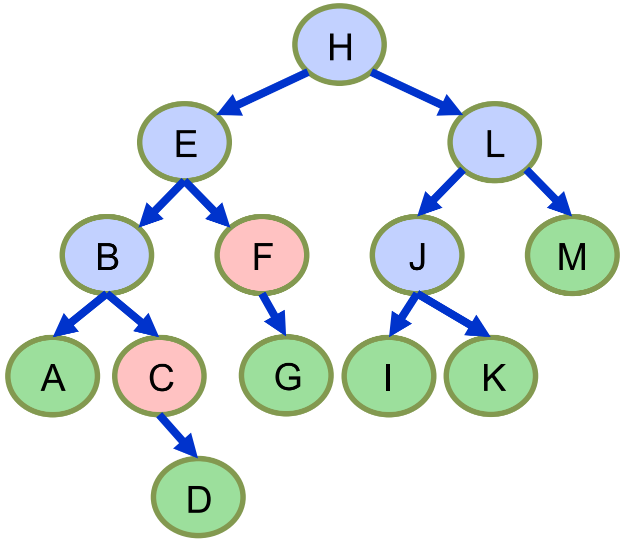
Binary Search Tree: Deletion

- **After remove a node X, the BST must remain as a BST**
- **Deletion operation on a BST is a bit tricky**
- **Three cases to be considered:**
  1. **X has no children**
  2. **X has one child**
  3. **X has two children**



Binary Search Tree: Deletion

- **After remove a node X, the BST must remain as a BST**
- **Deletion operation on a BST is a bit tricky**
- **Three cases to be considered:**
  1. **X has no children**
     - Remove **X**

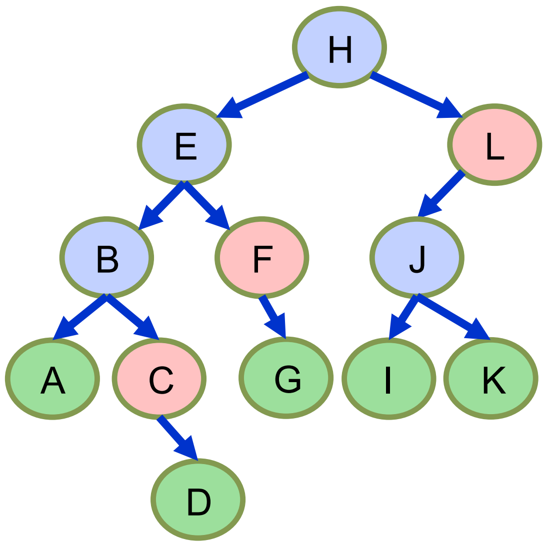


Binary Search Tree: Deletion

- **After remove a node X, the BST must remain as a BST**
- **Deletion operation on a BST is a bit tricky**
- **Three cases to be considered:**

  2. X has one child

  - Replace X with Y
  - Remove X

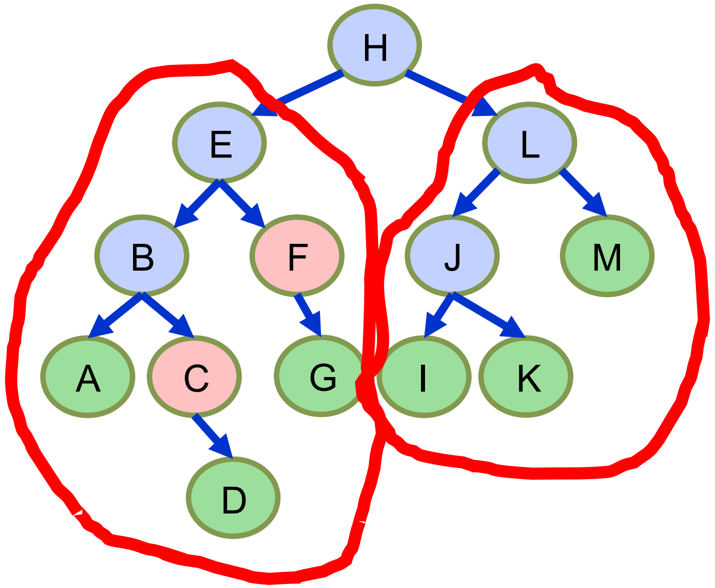


Binary Search Tree: Deletion

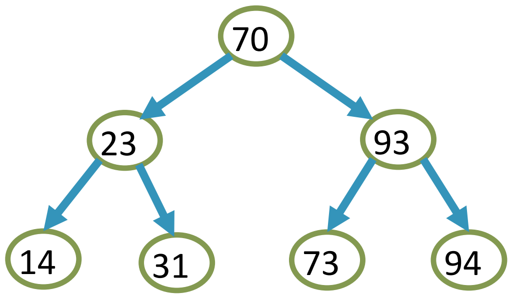
- **After remove a node X, the BST must remain as a BST**
- **Deletion operation on a BST is a bit tricky**
- **Three cases to be considered:**

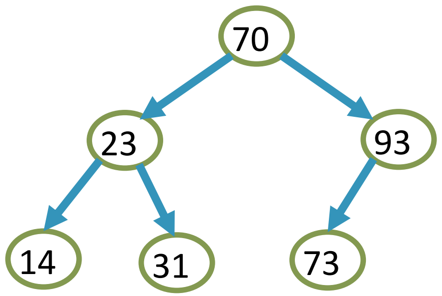
  3. X has two children

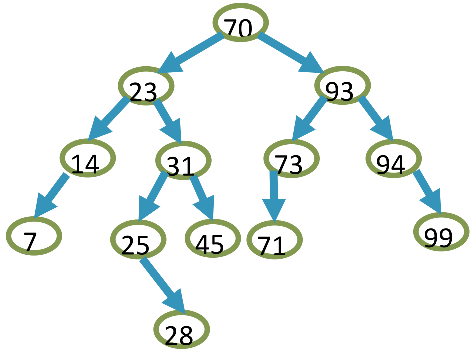
  - **Swap x with successor**
    - the (largest) rightmost node in left subtree
    - the (smallest) leftmost node in right subtree
  - **Perform case 1 or 2 to remove it**



Terminology

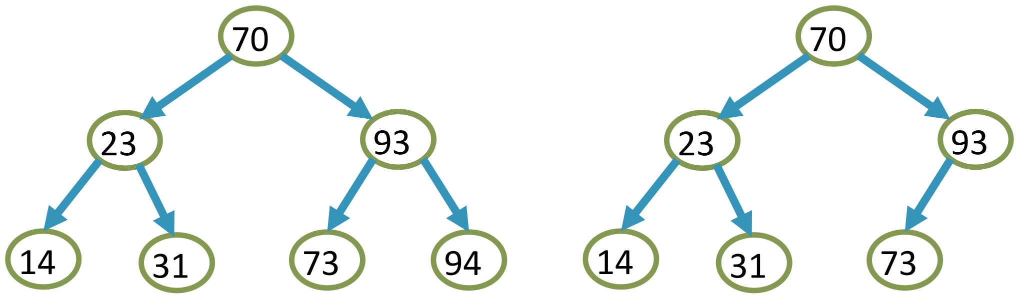






- The Height of a tree: The number of **edges** on the longest path from the root to a leaf
- The Depth/Level of a node: The number of edges from the node to the root of its tree.
- Empty Binary Tree: A binary tree with no nodes. It is still considered as a tree.
- Full Binary Tree: A binary tree of height *H* with no missing nodes. All leaves are at level *H* and all other nodes each have two children
- Complete Binary Tree: A binary tree of height *H* that is full to level *H-1* and has level *H* filled in from left to right
- Balanced Binary Tree: A binary tree in which the left and right subtrees of any node have heights that differ by at most 1

Terminology



- The Height of a tree: The number of **edges** on the longest path from the root to a leaf
- The Depth/Level of a node: The number of edges from the node to the root of its tree.

For a complete binary tree with height *H*, we have:

$$\mathbf{2^H\text{-}1 < \textit{n} \leq 2^{H+1} - 1}$$

where *n* is an integer and the size of the tree

$$\mathbf{2^H \leq \textit{n} < 2^{H+1}} \qquad \textbf{(eg. } 7<n \leq 15 \equiv 8 \leq n<16\textbf{)}$$

$$\mathbf{H \leq \log_2 n < H+1}$$

If H is an integer, H+1 must be the next integer.

Minimal Height = $\lfloor \log_2 n \rfloor$

Binary Search – Worst Case Time complexity

- Assumed that it is a complete binary tree

```
BTNode* findBSTNode(BTNode *cur, char c){
    if (cur == NULL) {
       printf("can't find!");
       return cur;
    }

    if(c==cur->item){
       printf("Found!\n");
       return cur;
    }

    if(c<cur->item)
       return findBSTNode(cur->left,c);
    else
       return findBSTNode(cur->right,c);
}
```

T(n)

Constant c

$\boldsymbol{T((n-1)/2)}$

$\boldsymbol{T((n-1)/2)}$

$$T(n) = T\left(\frac{n-1}{2}\right) + c$$

$$= T\left(\frac{\left(\frac{n-1}{2}\right)-1}{2}\right) + 2c = T\left(\frac{n-1-2}{2^2}\right) + 2c$$

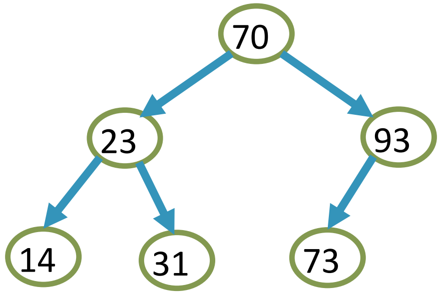
$$= T\left(\frac{\frac{n-1-2}{2^2}-1}{2}\right) + 3c = T\left(\frac{n-1-2-2^2}{2^3}\right) + 3c$$

…

$$= T\left(\frac{n-(1+2\ldots+2^{k-2}+2^{k-1})}{2^k}\right) + kc$$

$$= T\left(\frac{n-2^k+1}{2^k}\right) + kc$$

Binary Search – Worst Case Time complexity




```
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       return cur;
    }

    if(c<cur->item)
       return findBSTNode(cur->left,c);
    else
       return findBSTNode(cur->right,c);
}
```

T(n)

Constant c

$\boldsymbol{T((n-1)/2)}$

$\boldsymbol{T((n-1)/2)}$

- Assumed that it is a complete binary tree

$$= T\left(\frac{n - 2^k + 1}{2^k}\right) + kc$$

$$0 < \frac{n - 2^k + 1}{2^k} \leq 1$$

$$0 < \frac{n + 1}{2^k} - 1 \leq 1$$

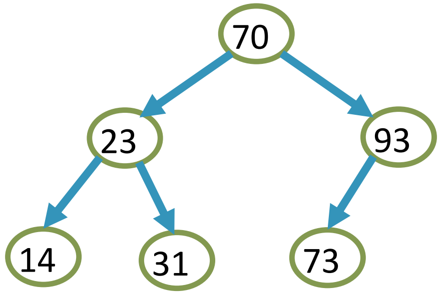
$$1 < \frac{n + 1}{2^k} \leq 2$$

$$2^k < n + 1 \leq 2^{k+1}$$

$$k < \log_2(n + 1) \leq k + 1$$

$$\lceil \log_2(n + 1) \rceil = k + 1$$

Binary Search – Worst Case Time complexity



- Assumed that it is a complete binary tree

```
BTNode* findBSTNode(BTNode *cur, char c){
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       printf("Found!\n");
       return cur;
    }

    if(c<cur->item)
       return findBSTNode(cur->left,c);
    else
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}
```

T(n)

Constant c

$\boldsymbol{T((n-1)/2)}$

$\boldsymbol{T((n-1)/2)}$

$$= T\left(\frac{n - 2^k + 1}{2^k}\right) + kc$$

$$\lceil \log_2(n+1) \rceil = k + 1$$

$$\lfloor \log_2 n \rfloor + 1 = \mathrm{k} + 1$$

$$k = \lfloor \log_2 n \rfloor$$

$$= c + kc$$

$$= (\lfloor \log_2 n \rfloor + 1)c$$

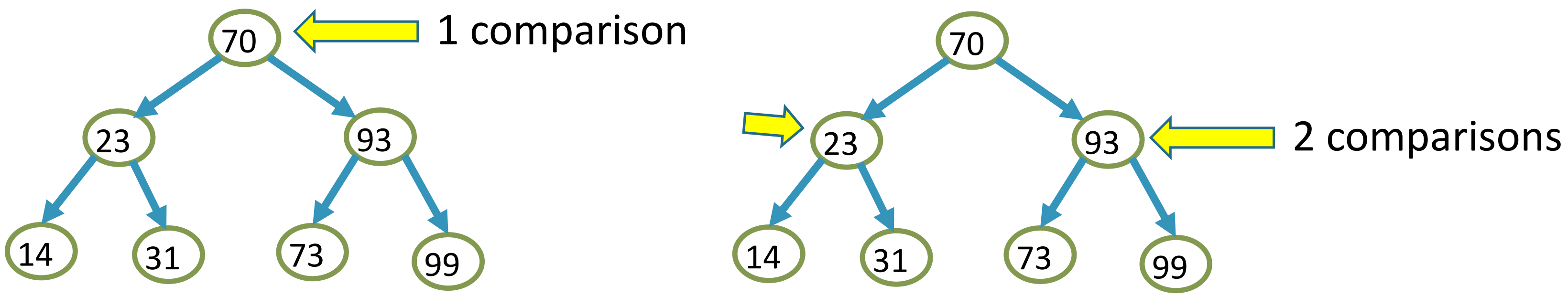
$$= \Theta(\log_2 n)$$

Binary Search – Average Case Time Complexity

- $A_s(n)$: # of comparisons for successful search
- $A_f(n)$: # of comparisons for unsuccessful search (worst case): $\Theta(\log_2 n)$

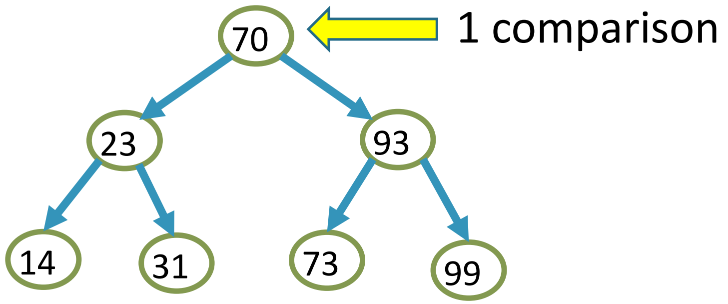
$$A(n) = qA_s(n) + (1-q)A_f(n)$$

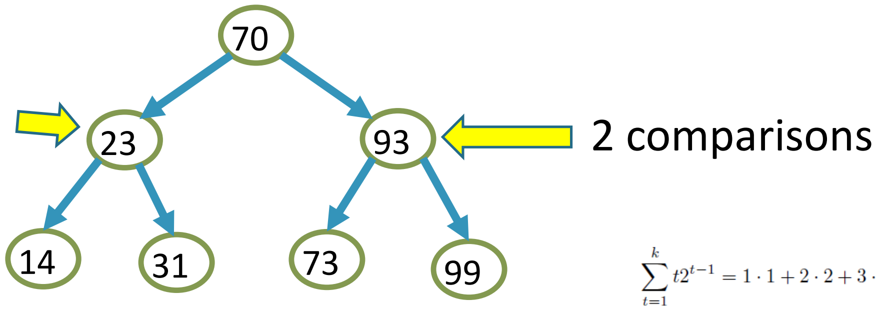
For $A_s(n)$, we assume $n = 2^k - 1$ first



Binary Search – Average Case Time Complexity

$$A(n) = qA_s(n) + (1-q)A_f(n)$$





- For $A_s(n)$, we assume $n = 2^k - 1$ first
- We can observe that:
  - 1 position requires 1 comparison
  - 2 positions requires 2 comparisons
  - 4 positions requires 3 comparisons
  - … …
  - 2^{t-1} positions requires t comparisons
- $n=2^k-1$, we have

$$A_s(n) = \frac{1}{n}\sum_{t=1}^{k} t2^{t-1}$$
$$= \frac{(k-1)2^k + 1}{n}$$
$$= \frac{[\log_2(n+1) - 1](n+1) + 1}{n}$$
$$= \log_2(n+1) - 1 + \frac{\log_2(n+1)}{n}$$

$$\sum_{t=1}^{k} t2^{t-1} = 1\cdot 1 + 2\cdot 2 + 3\cdot 4 + 4\cdot 8 + \ldots + k\cdot 2^{k-1}$$
$$2\sum_{t=1}^{k} t2^{t-1} = \quad 1\cdot 2 + 2\cdot 4 + 3\cdot 8 + \ldots + (k-1)\cdot 2^{k-1} + k\cdot 2^k$$
$$(2-1)\sum_{t=1}^{k} t2^{t-1} = -1\cdot 1 - 1\cdot 2 - 1\cdot 4 - 1\cdot 8 - \ldots - 1\cdot 2^{k-1} + k\cdot 2^k \quad \triangleright \text{eq. 2 - eq. 1}$$
$$\sum_{t=1}^{k} t2^{t-1} = -2^k + 1 + k\cdot 2^k \quad \triangleright \text{geometric series}$$
$$= 2^k(k-1) + 1$$

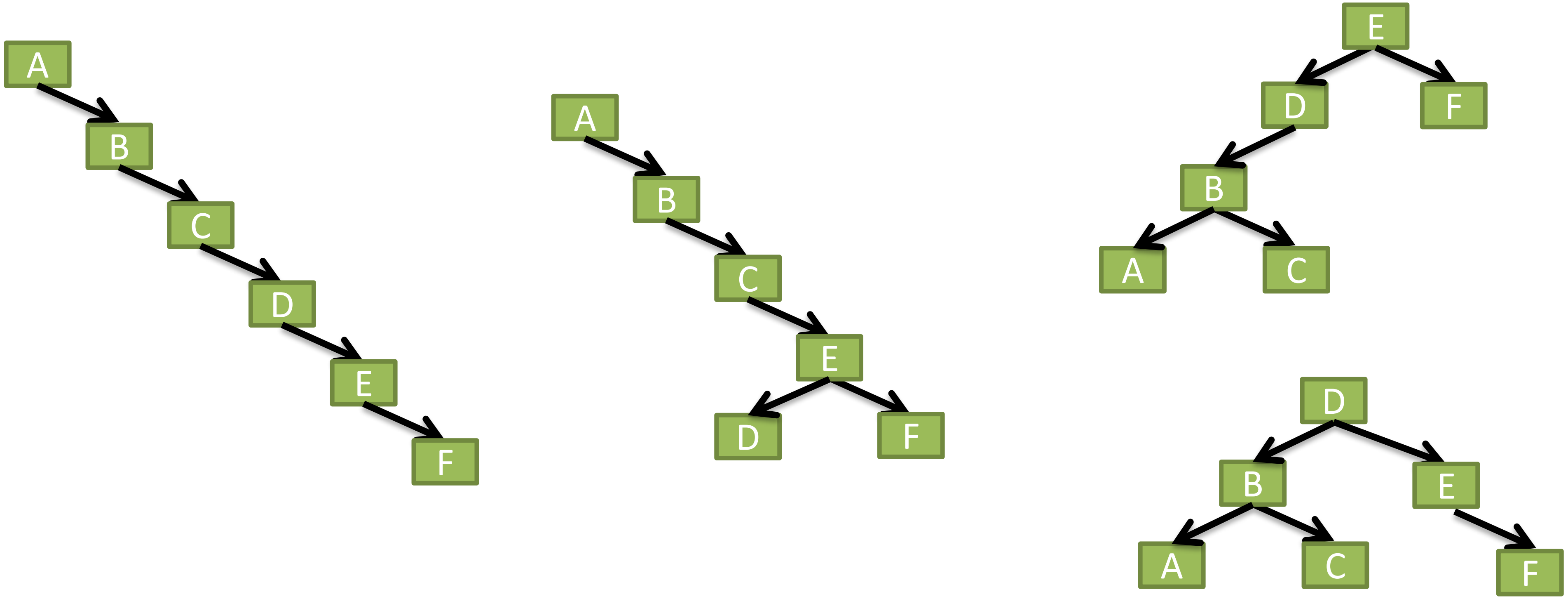
Binary Search – Average Case Time Complexity

- The time complexity is

$$\begin{aligned}
A_q(n) &= qA_s(n) + (1-q)A_f(n) \\
&= q[\log_2(n+1) - 1 + \frac{\log_2(n+1)}{n}] + (1-q)(\log_2(n+1)) \\
&= \log_2(n+1) - q + q\frac{\log_2(n+1)}{n} \\
&= \Theta(log_2(n))
\end{aligned}$$

- Binary search does approximately $\log_2(n+1)$ comparisons on average for n entries.
  - q is probability which is always ≤ 1
  - $\frac{\log_2(n+1)}{n}$ is very small especially when n >> 1

The ‘Good’ and ‘Bad’ Binary Search Trees



Summary

- Linear Search VS Binary Search
- Binary Search Tree: Search, Insertion and Deletion
- Time Complexity of Binary Search
- The importance of having a good tree structure

- Next Lecture
  - Tree Balancing