NANYANG TECHNOLOGICAL UNIVERSITY

SEMESTER 1 EXAMINATION 2020-2021

MH1812 - DISCRETE MATHEMATICS

December 2020 TIME ALLOWED: 2 HOURS

INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **FIVE (5)** questions and comprises **THREE (3)** printed pages.

2. Answer **ALL** questions. The marks for each question are indicated at the end of each question.

3. Answer each question beginning on a **FRESH** page of the answer book.

4. This **IS NOT** an **OPEN BOOK** exam.

5. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(20 marks)**

For a finite set A of real numbers, we define $\Pi(A)$ to be the product of all elements in A. For example, $\Pi(\{-2, 3, \pi, 5\}) = (-2) \cdot 3 \cdot \pi \cdot 5 = -30\pi$. Additionally we define $\Pi(\emptyset) = 1$.

(a) Define $\Sigma = \{\Pi(A) \bmod 12 \mid A \subseteq \{1, \ldots, 10\}\}$. Determine whether Σ is closed under "addition modulo 12". Justify your answer. **(5 marks)**

(b) Find the number of subsets $A \subseteq \{1, \ldots, 100\}$ such that $\Pi(A)$ is <u>not</u> divisible by 5. Justify your answer. **(5 marks)**

(c) Find the number of subsets $A \subseteq \{1, \ldots, 100\}$ such that $\Pi(A)$ is <u>not</u> divisible by 8. Justify your answer. **(10 marks)**

QUESTION 2. **(20 marks)**

On a set $S = \{a, b, c, d, e\}$ we define a relation $R = \{(a, a), (a, b), (b, c), (d, e)\}$.

(a) What is the transitive closure of R ? **(6 marks)**

(b) What is the smallest equivalence relation containing R ? **(7 marks)**

(c) What is the smallest partial order containing R ? **(7 marks)**

QUESTION 3. **(10 marks)**

Show that

$$\frac{n}{2} < 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2^n - 1} < n$$

for all integers $n \geq 2$.

QUESTION 4. **(30 marks)**

(a) How many surjective functions are there from set A to B, where $|A| = 5$ and $|B| = 3$? Justify your answer. **(10 marks)**

(b) How many surjective functions are there from set $A = \{1, 2, \ldots, m\}$ to $B = \{1, 2, \ldots, n\}$ with positive integers $m \geq n$, such that $f(1) \leq f(2) \leq \cdots \leq f(m)$? Justify your answer. **(10 marks)**

(c) For an injective function $f : D \rightarrow R$, prove or disprove $f(A \cap B) = f(A) \cap f(B)$, where $A, B \subseteq D$ and $f(X)$ is defined as $f(X) = \{f(x) \mid x \in X\}$ for any $X \subseteq D$. **(10 marks)**

QUESTION 5. **(20 marks)**

A quinary string is a string whose characters are 0, 1, 2, 3 or 4. It is clear that there are 5^n quinary strings of length n for integers $n \geq 1$.

For each integer $n \geq 1$, let a_n be the number of quinary strings of length n that do not contain adjacent 2s. Find an explicit formula for a_n.

END OF PAPER