NANYANG TECHNOLOGICAL UNIVERSITY

MIDTERM I (CA1)

MH1812 – Discrete Mathematics

February 2018 TIME ALLOWED: 40 minutes

Name:

Matric. no.: Tutor group:

INSTRUCTIONS TO CANDIDATES

1. **DO NOT TURN OVER PAPER UNTIL INSTRUCTED.**
2. This midterm paper contains **THREE (3)** questions.
3. Answer **ALL** questions. The marks for each question are indicated at the beginning of each question.
4. Candidates can write anywhere on this midterm paper.
5. This **IS NOT** an **OPEN BOOK** exam.
6. Candidates should clearly explain their reasoning when answering each question.

QUESTION 1. **(40 marks)**

(a) (10 marks) Which integer $a \in \{0, 1, 2, 3\}$ satisfies $a \equiv 2^{2018} \pmod 4$?

(b) (10 marks) Wednesday is two days after Monday. What day of the week is it 500 days after Tuesday?

(c) Decide whether or not the set S is closed under the operation Δ when

- $S = \{\text{odd integers}\}$ and Δ is addition. (10 marks)
- $S = \{\text{even integers}\}$ and Δ is division. (10 marks)

Briefly justify your answers.

a) $a \equiv 2^{2018} (\text{mod } 4)$

$a \equiv 4^{1009} (\text{mod } 4)$

$a \equiv 0 (\text{mod } 4)$ $a = 0$ //

b) let mon = 1 tue = 2 ... sun = 7

$502 \equiv 5 \text{ mod } 7$

$\therefore$ Friday //

c) for 3 & 5, $3 + 5 = 8$ ... $8 \neq$ odd

not closed under addition //

for 8 & 6, $8 \div 6 = 4/3$ ... not integer

not closed //

QUESTION 2. **(40 marks)**

(a) (20 marks) Prove or disprove the following statement:

$$(p \vee r) \rightarrow (p \wedge q) \equiv (p \rightarrow q) \wedge (r \rightarrow q).$$

(b) (20 marks) Decide whether or not the following argument is valid:

$$\begin{aligned} &p \vee q; \\ &\neg p \rightarrow r; \\ &\neg q \rightarrow r; \\ &r \vee p; \\ &\therefore r \end{aligned}$$

Briefly justify your answers.

a) For LHS = F, $(p \vee r)$ is T & $(p \wedge q)$ is F

$\therefore$ either

p	q	r	$(p \rightarrow q)$	$(r \rightarrow q)$
T	F	T	F	F
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T

for $p = F$, $q = T$, $r = T$, LHS = T.

$\therefore$ disproven.

b) If r is F,

$\neg p \Rightarrow r$ & $\neg q \Rightarrow r$ is T when p & q = T

$\therefore$ Counter example, $p = T$, $q = T$, $r = F$

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QUESTION 3. **(20 marks)**

(a) (10 marks) Consider the domain $\mathbb{Q}$ = {rational numbers} and the predicate $P(x,y)$ = "xy is an integer".

Determine the truth value of the statement:

$\forall x \in \mathbb{Q},\ \exists y \in \mathbb{Q},\ P(x,y)$.

(b) (10 marks) Let X and Y be domains, and let $P(x)$ and $Q(y)$ be predicates. Which of the following statements is the *negation* of the statement:

$\forall y \in Y,\ \exists x \in X,\ P(x) \to Q(y)$?

(i) $\forall y \in Y,\ \exists x \in X,\ \neg P(x) \wedge Q(y)$;

(ii) $\exists y \in Y,\ \exists x \in X,\ \neg P(x) \vee Q(y)$;

(iii) $\exists y \in Y,\ \forall x \in X,\ P(x) \wedge \neg Q(y)$; ✓

(iv) $\exists y \in Y,\ \forall x \in X,\ \neg P(x) \wedge \neg Q(y)$.

Briefly justify your answers.

a) let $x = \frac{a}{b}$, $a \in \mathbb{Z}$, $b \in \mathbb{Z}$

for any $x = \frac{a}{b}$

some $y = b$ makes $xy = a$, an integer. $\therefore$ T

b) $\neg(P(x) \to Q(y)) = \neg(\neg P(x) \vee Q(y)) = P(x) \wedge \neg Q(y)$

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