Computer Vision: Assignment #3

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Problem 1

Solution: Let's prove that $L(\mathbf{h})$ is strictly convex by analyzing its Hessian matrix step by step.
1) According to the hint, to prove that $L(\mathbf{h})$ is strictly convex, we need to show that its Hessian matrix is positive definite.
2) First, let's calculate the first derivative of $L(\mathbf{h})$:

$$\nabla L(\mathbf{h}) = \mathbf{J}(\mathbf{x})^T \mathbf{f}(\mathbf{x}) + \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x})\mathbf{h} + \mu \mathbf{h}$$

3) Then, we calculate the second derivative (Hessian matrix):

$$\nabla^2 L(\mathbf{h}) = \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x}) + \mu \mathbf{I}$$

where $\mathbf{I}$ is the identity matrix.
4) Now, we need to prove that

$$\mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x}) + \mu \mathbf{I}$$

is positive definite.
5) For any non-zero vector $\mathbf{v}$, consider the quadratic form:

$$\begin{aligned} \mathbf{v}^T [\mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x}) + \mu \mathbf{I}]\mathbf{v} &= \mathbf{v}^T \mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x})\mathbf{v} + \mu \mathbf{v}^T \mathbf{v} \\ &= \|\mathbf{J}(\mathbf{x})\mathbf{v}\|^2 + \mu \|\mathbf{v}\|^2 \end{aligned}$$

6) Observe that:

- $\|\mathbf{J}(\mathbf{x})\mathbf{v}\|^2 \geq 0$ (as it's a squared norm)
- $\mu \|\mathbf{v}\|^2 > 0$ (since $\mu > 0$ and $\mathbf{v}$ is non-zero)

7) Therefore:

$$\mathbf{v}^T [\mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x}) + \mu \mathbf{I}]\mathbf{v} > 0$$

holds for any non-zero vector $\mathbf{v}$.
Conclusion: We have proven that the Hessian matrix of $L(\mathbf{h})$ is positive definite, therefore $L(\mathbf{h})$ is strictly convex. This result is significant because it guarantees that the subproblem solved in each iteration of the Levenberg-Marquardt algorithm has a unique minimum.
Additional Note: This result also explains why the damping coefficient $\mu > 0$ is crucial in the Levenberg-Marquardt algorithm: it ensures strict convexity of the local approximation model even when $\mathbf{J}(\mathbf{x})^T \mathbf{J}(\mathbf{x})$ is not positive definite.

Problem 2

The result video is in the folder problem2

Problem 3

Lab Name

3D Model Scanning and Data Processing

Problem 3 continued on next page...

Objectives

1. Understand the representation formats and storage formats of 3D model data
2. Master the usage of handheld 3D surface scanning system Creaform Go Scan 3D hardware and software
3. Master common geometric editing software (such as GeoMagic Studio) operations and perform basic editing on raw 3D scan data

Equipment

1. Hardware: Creaform Go Scan 3D scanner
2. Software: Creaform Go Scan 3D scanning software
3. Software: Geomagic Studio 2013 (64 bit)

Lab Content

3D Scanner Data Acquisition Process

Scanning Preparation

1. Ensure proper equipment connection and power supply
2. Check lighting conditions in scanning environment to avoid strong light interference
3. Adjust position of scanning object to ensure stability

Scanning Operation

1. Launch Creaform Go Scan 3D software, click ”New Session” → ”Scan” to begin scanning
2. Hold the 3D scanner and move slowly until facial 3D model scanning is complete

Notes

- During scanning, front red light indicates distance too close
- Rear red light indicates distance too far
- Under normal conditions, only middle green light should be on
- If both front and rear red lights are on, tracking has failed. Slightly adjust position to try re-tracking
- If tracking failure persists, must create new session and re-scan

Problem 3 continued on next page...

3D Model Post-Processing

Data Import and Pre-processing

1. Import raw scan data into Geomagic Studio 2013

2. Perform initial inspection of imported data to identify issues:

    - Incomplete model
    - Presence of holes
    - Irregular mesh structure

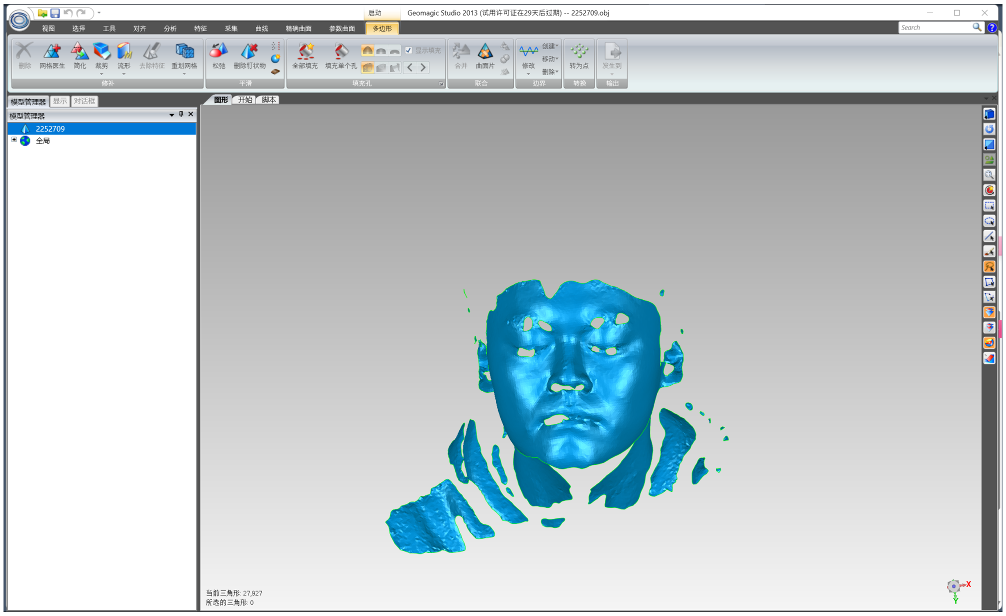

Image 1: Origin Image

Model Repair Process

(1) Polygon Fill Single Holes

- Use ”Fill Single Holes” function under ”Polygon” menu
- Select small hole areas for filling
- Software automatically performs intelligent filling based on surrounding mesh features
- Check repair effect after filling to ensure natural transition with existing surface

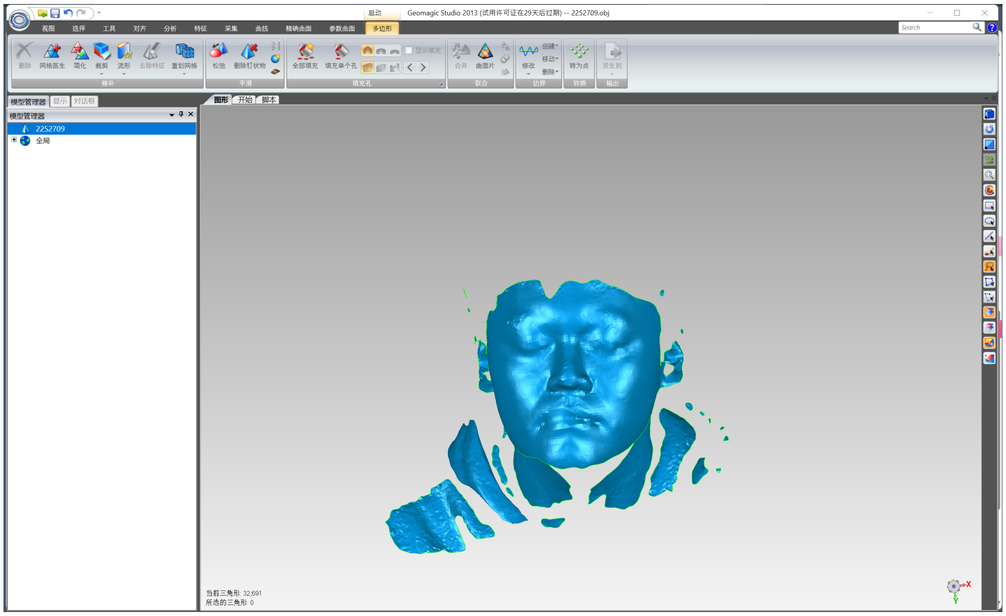

Image 2: Processed Image

(2) Polygon Bridge Processing

- Use ”Polygon-Bridge” function for larger holes
- Create bridge structures at suitable positions along hole edges
- Split large holes into several smaller holes through bridging
- Ensure bridge positions and directions align with existing surface contours

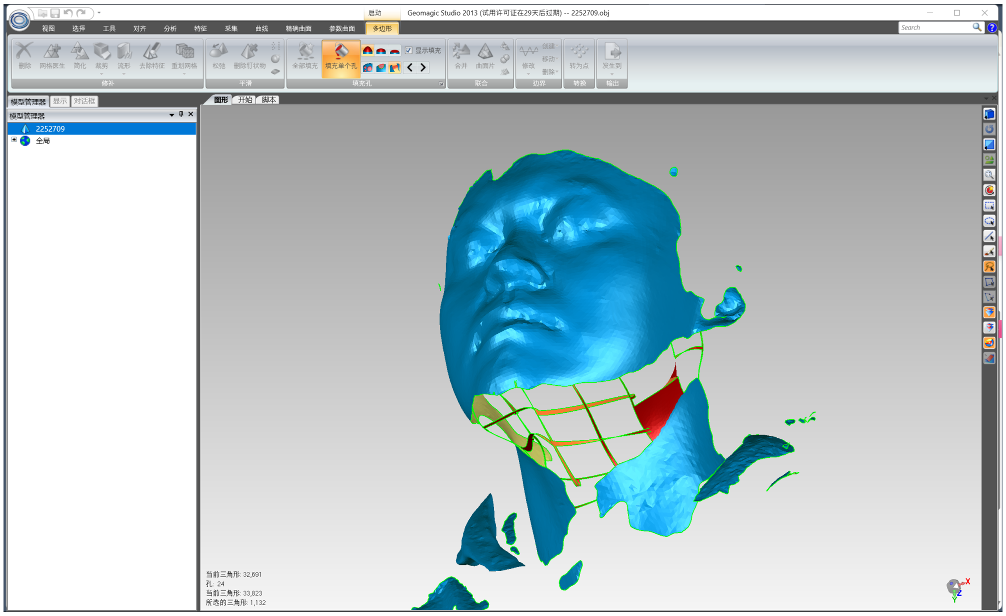

Image 3: Processed Image

Problem 3 continued on next page...

(3) Secondary Hole Filling

- Re-use ”Polygon-Fill Single Holes” function
- Process small holes formed after bridging
- Fill holes one by one, ensuring smooth surface transitions

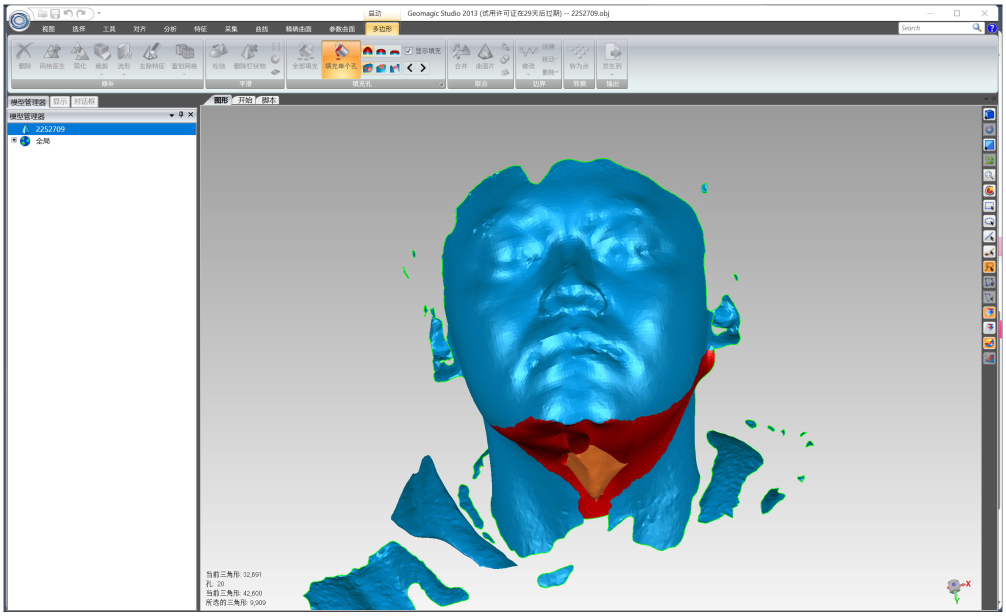

Image 4: Processed Image

(4) Mesh Doctor Repair

- Use ”Mesh Doctor” tool for final repairs
- Process remaining small defects and irregular areas
- Optimize mesh structure to improve model quality
- Check and correct potential mesh overlaps or intersections

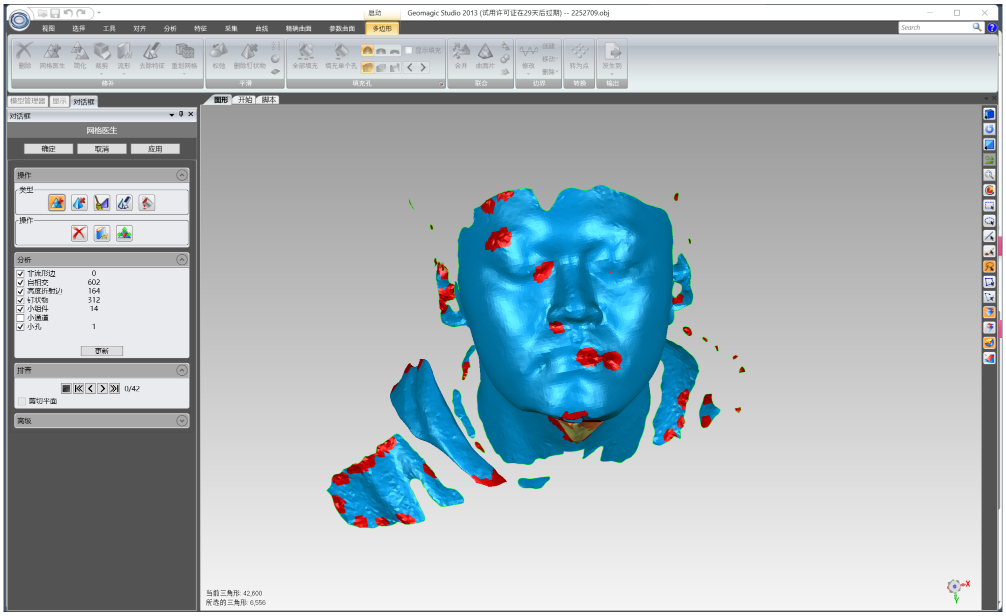

Image 5: Processed Image

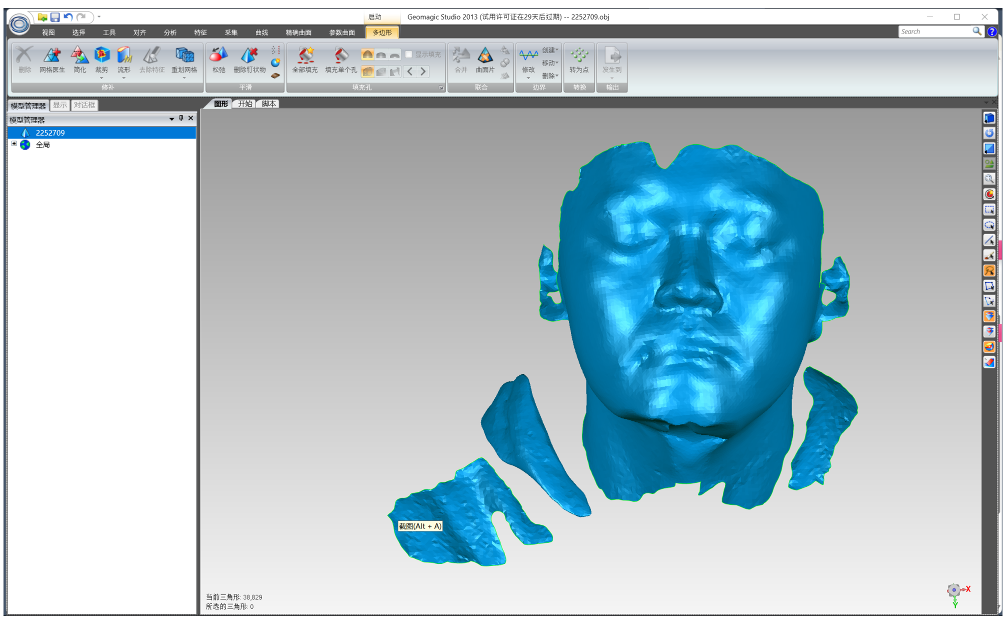

Image 6: Final Image

Processing Effect Verification

- Evaluate processing effect through surface continuity and smoothness
- Check natural transition between filled areas and original scan areas
- Confirm overall model completeness and accuracy
- Perform local detail adjustments if necessary

Problem 3 continued on next page...

Important Notes

- Maintain original surface geometric features when filling holes
- Consider overall model morphology during bridging operations
- Avoid over-processing that could distort the model
- Regularly save intermediate results during processing

Results

1. Successfully completed facial 3D model scanning
2. Repaired model defects through systematic post-processing
3. Obtained complete, smooth 3D facial model

Conclusion

1. Mastered basic operation of Creaform Go Scan 3D scanner
2. Learned 3D model data processing techniques and methods
3. Understood repair strategies for different types of defects
4. Gained practical experience in 3D model processing