Machine Learning

Neural Network

Dr. Shuang LIANG

Today’s Topics

- Neural Network Introduction
- Neural Network Structure
- How Neural Network Works
- Backpropagation

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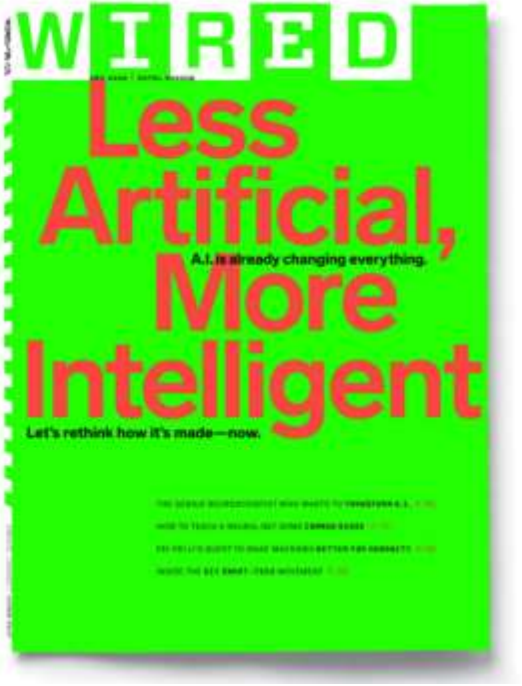
Neural networks are a hot topic



A bit of history

The **Mark I Perceptron** machine was the first implementation of the perceptron algorithm.

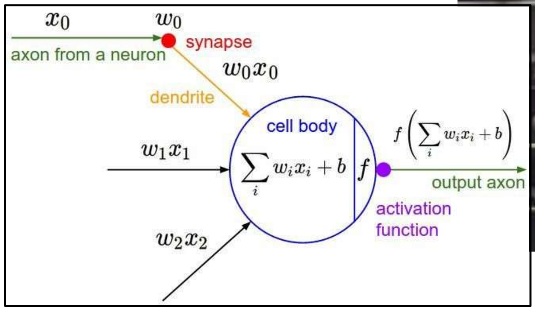
The machine was connected to a camera that used 20×20 cadmium sulfide photocells to produce a 400-pixel image.

recognized
letters of the alphabet

$$f(x) = \begin{cases} 1 & \text{if } w \cdot x + b > 0 \\ 0 & \text{otherwise} \end{cases}$$

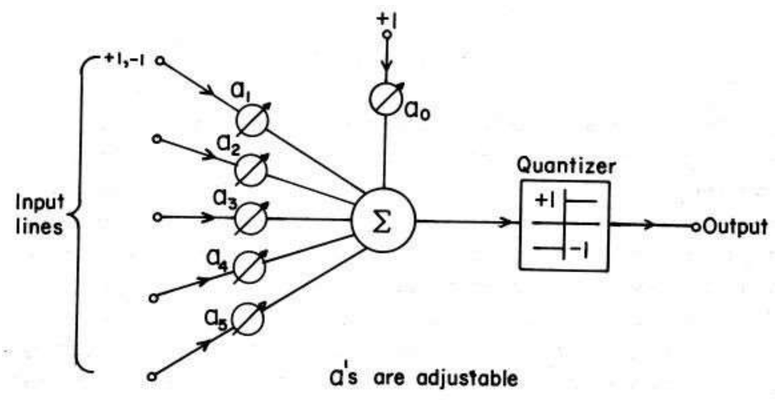
update rule:

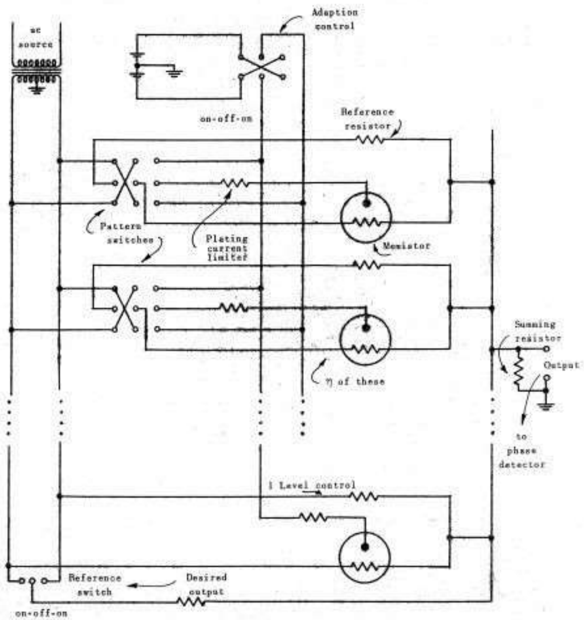
$$w_i(t+1) = w_i(t) + \alpha(d_j - y_j(t))x_{j,i},$$



Frank Rosenblatt, ~1957: Perceptron

A bit of history





Widrow and Hoff, ~1960: Adaline/Madaline

These figures are reproduced from Widrow 1960, Stanford Electronics Laboratories Technical Report with permission from Stanford University Special Collections.

A bit of history

$$\frac{\partial E_p}{\partial w_{ji}} = \frac{\partial E_p}{\partial o_{pj}} \frac{\partial o_{pj}}{\partial w_{ji}}$$

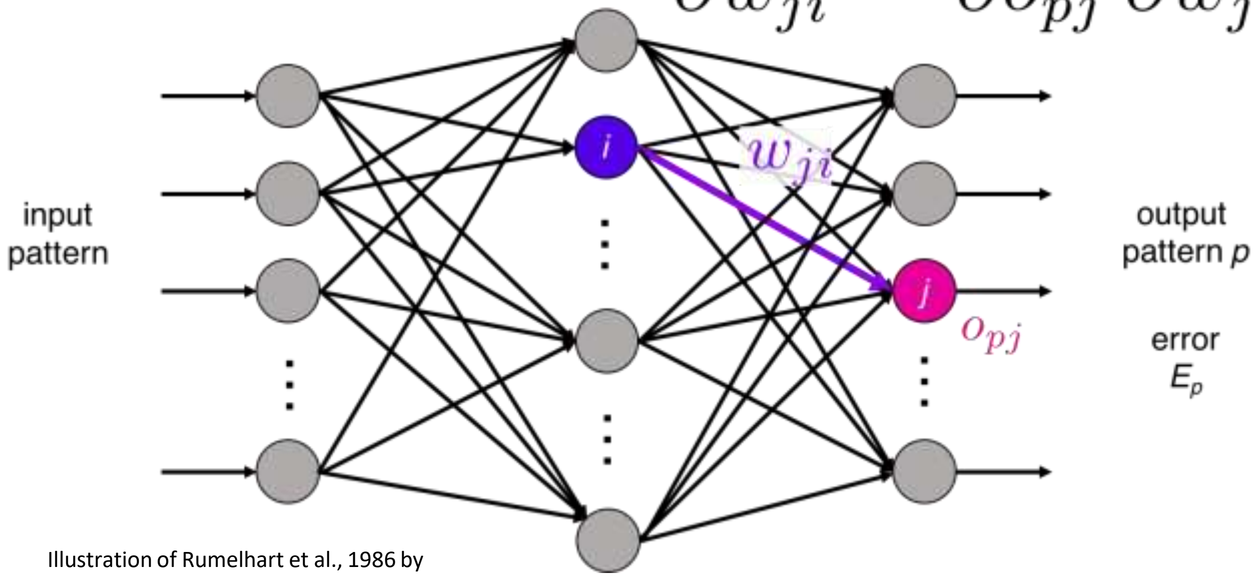


Illustration of Rumelhart et al., 1986 by Lane McIntosh, copyright CS231n 2017

recognizable math

Rumelhart et al., 1986: First time back-propagation became popular

A bit of history

[Hinton and Salakhutdinov 2006]

Reinvigorated research in Deep Learning

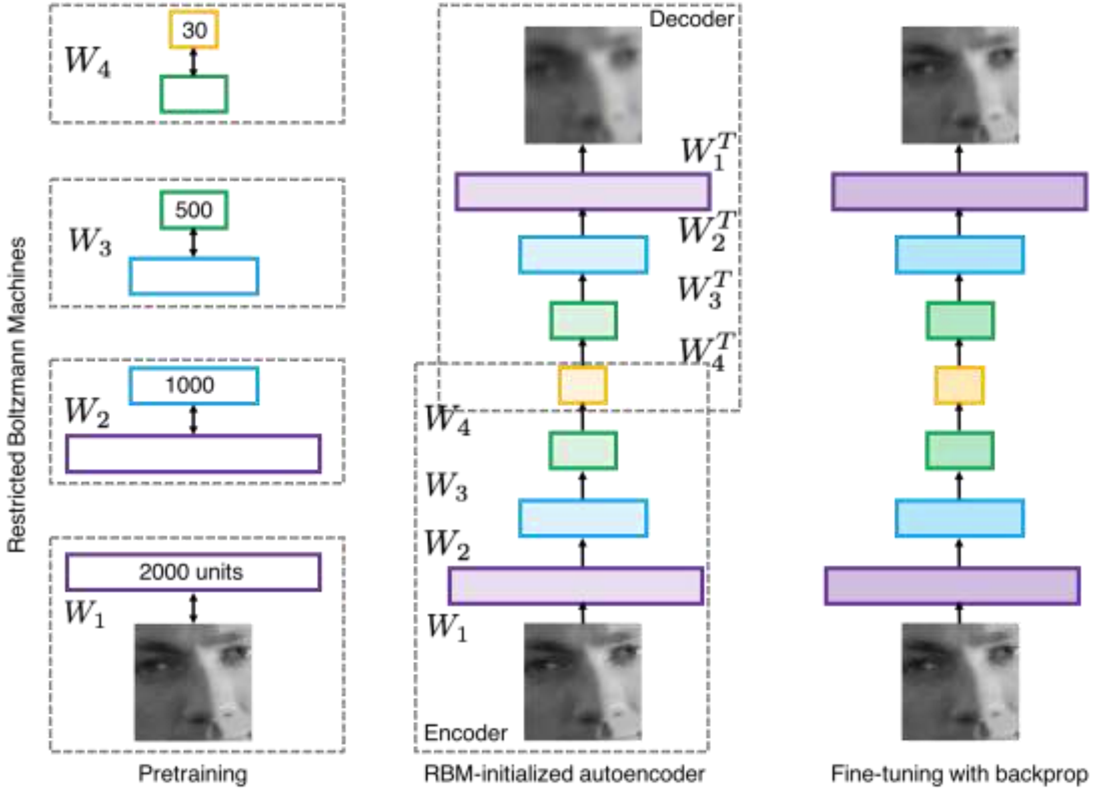


Illustration of Hinton and Salakhutdinov 2006 by Lane McIntosh, copyright CS231n 2017

A bit of history

First strong results

Acoustic Modeling using Deep Belief Networks
Abdel-rahman Mohamed, George Dahl, Geoffrey Hinton, 2010

Context-Dependent Pre-trained Deep Neural Networks for Large Vocabulary Speech Recognition
George Dahl, Dong Yu, Li Deng, Alex Acero, 2012

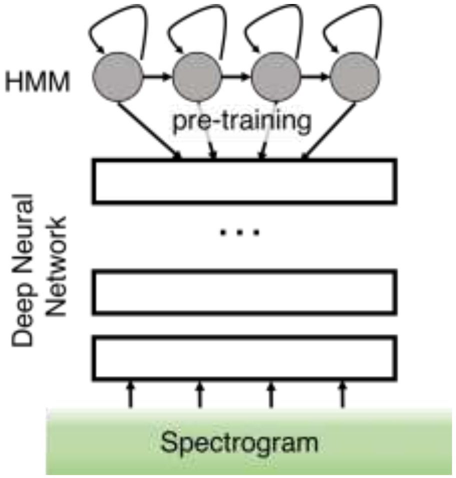
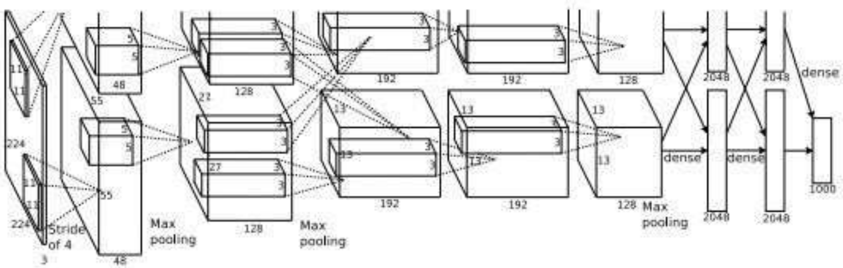
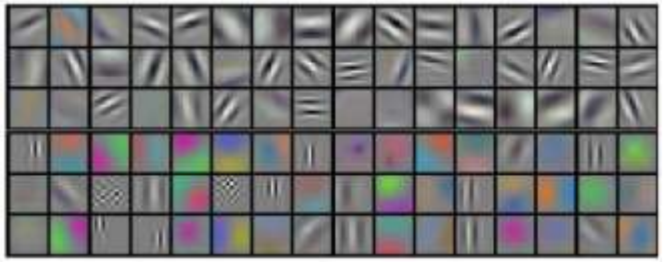


Illustration of Dahl et al. 2012 by Lane McIntosh, copyright CS231n 2017

Imagenet classification with deep convolutional neural networks
Alex Krizhevsky, Ilya Sutskever, Geoffrey E Hinton, 2012





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Ups and downs of Neural Networks

- 1958: Perceptron (linear model)
- 1969: Perceptron has limitation
- 1980s: Multi-layer perceptron
- 1986: Backpropagation
- 1989: 1 hidden layer is “good enough”, why deep?
- 2006: RBM initialization

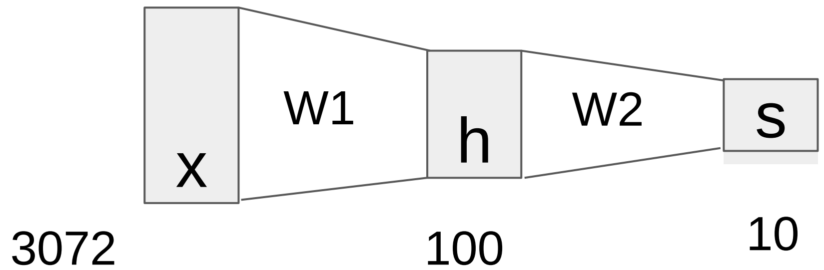
Ups and downs of Neural Networks

- 2009: GPU
- 2011: Start to be popular in speech recognition
- 2012: win ILSVRC image competition
- 2015.2: Image recognition surpassing human-level performance
- 2016.3: Alpha GO beats Lee Sedol
- 2016.10: Speech recognition system as good as humans
- Now: Transformer, BERT, Autopilot...

Neural Networks

(**Before**) Linear score function $f = Wx$

(**Now**) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$



Biological neuron

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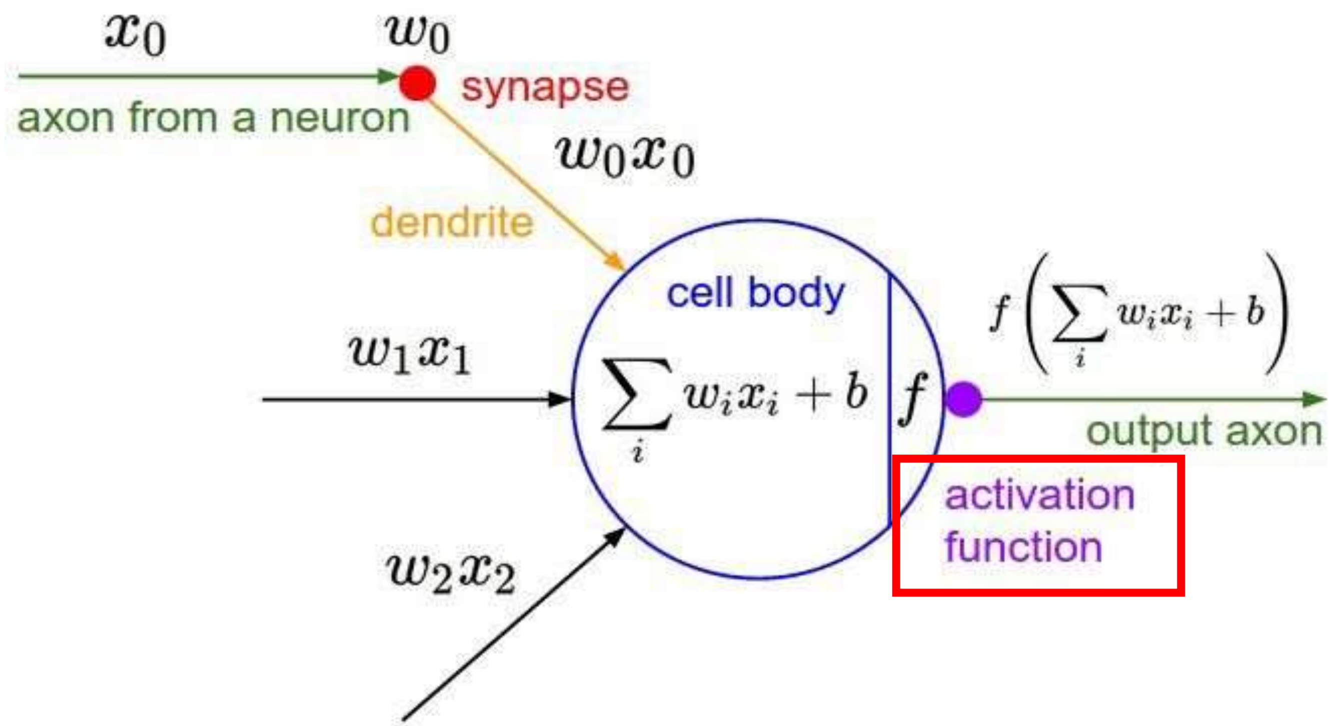
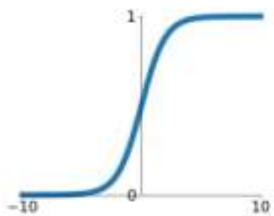
Artificial neuron



Activation Function

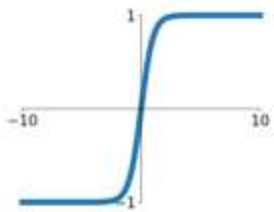
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



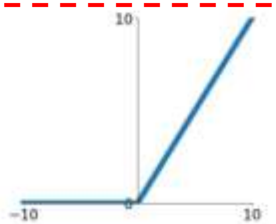
tanh

$\tanh(x)$



ReLU

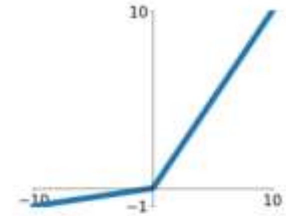
$\max(0, x)$



commonly used

Leaky ReLU

$\max(0.1x, x)$

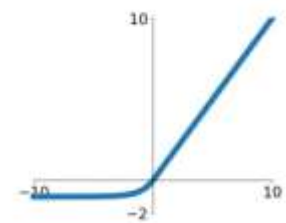


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

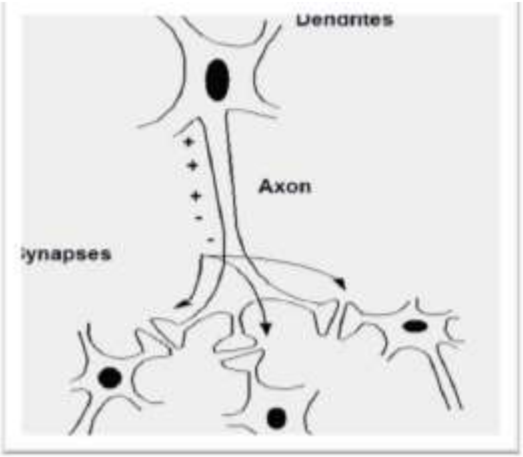
ELU

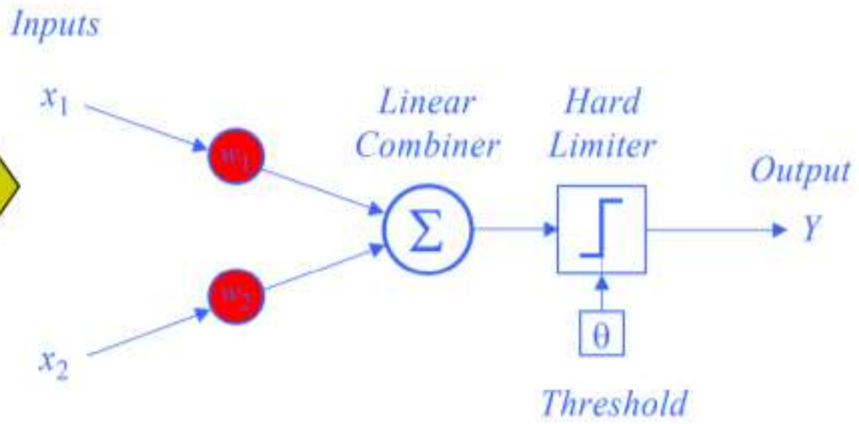
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Biological v.s. Artificial

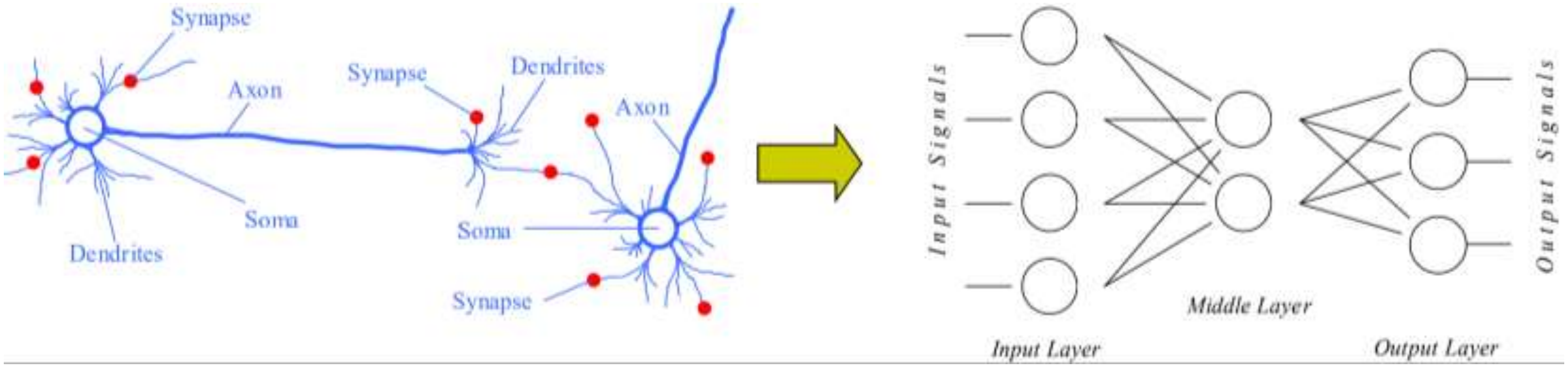
- From biological neuron to artificial neuron (perceptron)





Biological v.s. Artificial

- From biological neuron network to artificial neuron networks



Be very careful with your brain analogies!

- **Biological Neurons**
  - Many different types
  - Dendrites can perform complex non-linear computations
  - Synapses are not a single weight but a complex non-linear dynamical system
  - Rate code may not be adequate
- We can ignore whether the neural network actually simulates a biological neural network, and just think of a neural network as **a mathematical model with many parameters**

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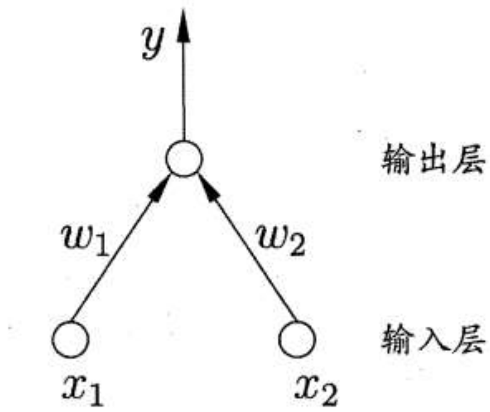
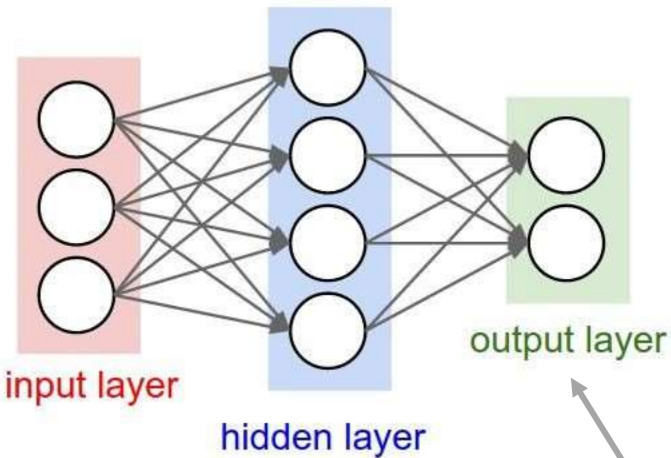
Perceptron

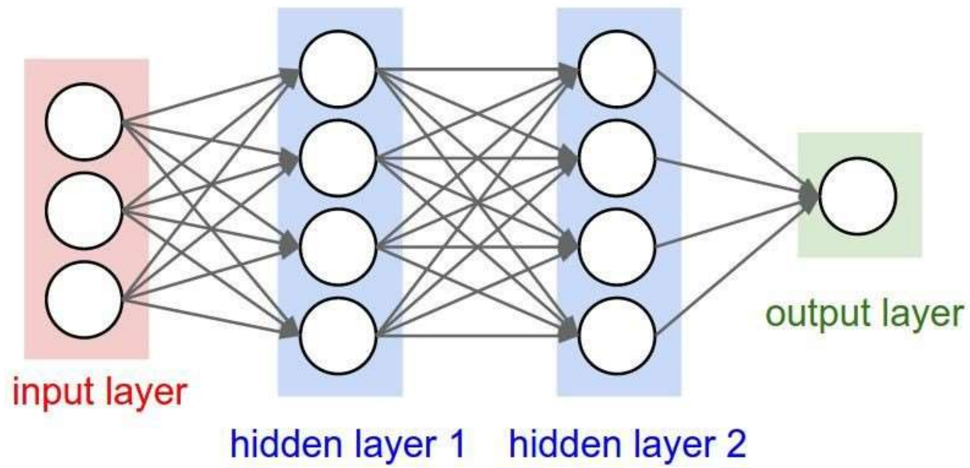
- It consists of two layers of neurons, the **input layer** and the **output layer**.



- Can realize logical AND, OR, NOT operation
- Only the neurons in the output layer perform activation function processing, and the learning ability is very limited
- Can't solve problems that are not linear separable, like XOR.

Multi-layer Network





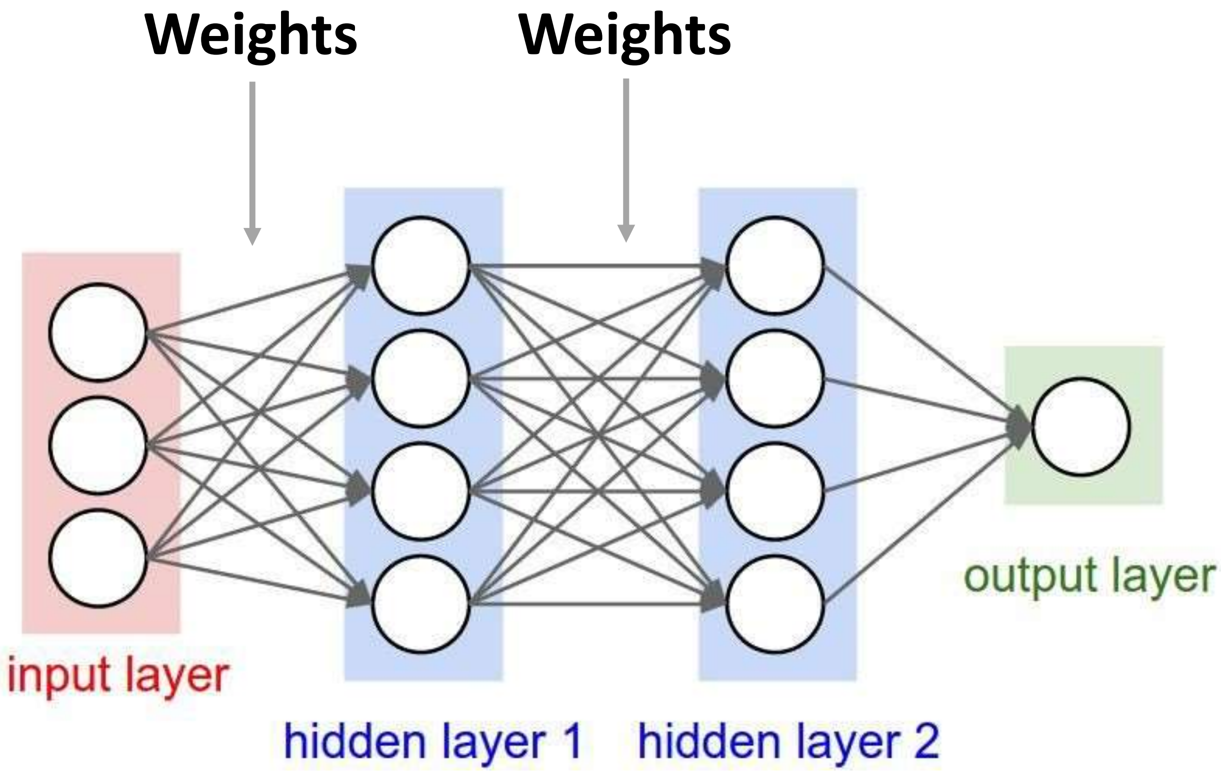
"2-layer Neural Net", or
"1-hidden-layer Neural Net"

"3-layer Neural Net", or
"2-hidden-layer Neural Net"

"Fully-connected" layers

Hidden layer and output layer neurons are ***functional neurons*** with activation functions

Multi-layer Network



The learning process of the neural network is to adjust the ***"connection weight"*** between neurons and the ***threshold*** of each functional neuron according to the training data

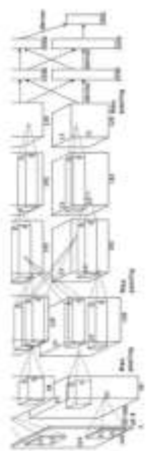
Deep Neural Network

Deep = Many hidden layers

Now the commonly used **ResNet** has reached **152** layers

8 layers

16.4%



AlexNet (2012)

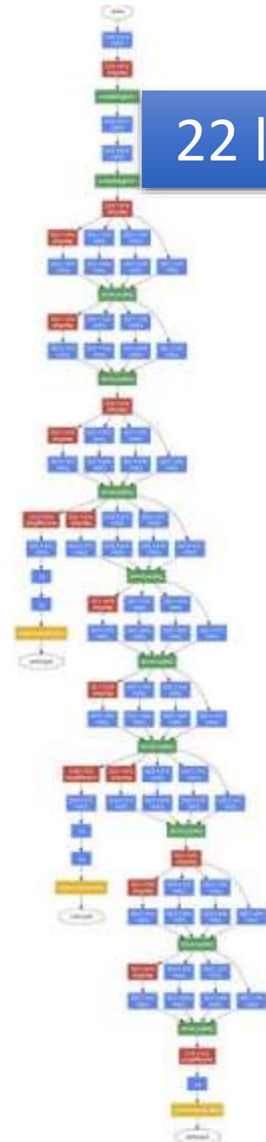
19 layers

7.3%

VGG (2014)

22 layers

6.7%



GoogleNet (2014)

http://cs231n.stanford.edu/slides/winter1516_lecture8.pdf

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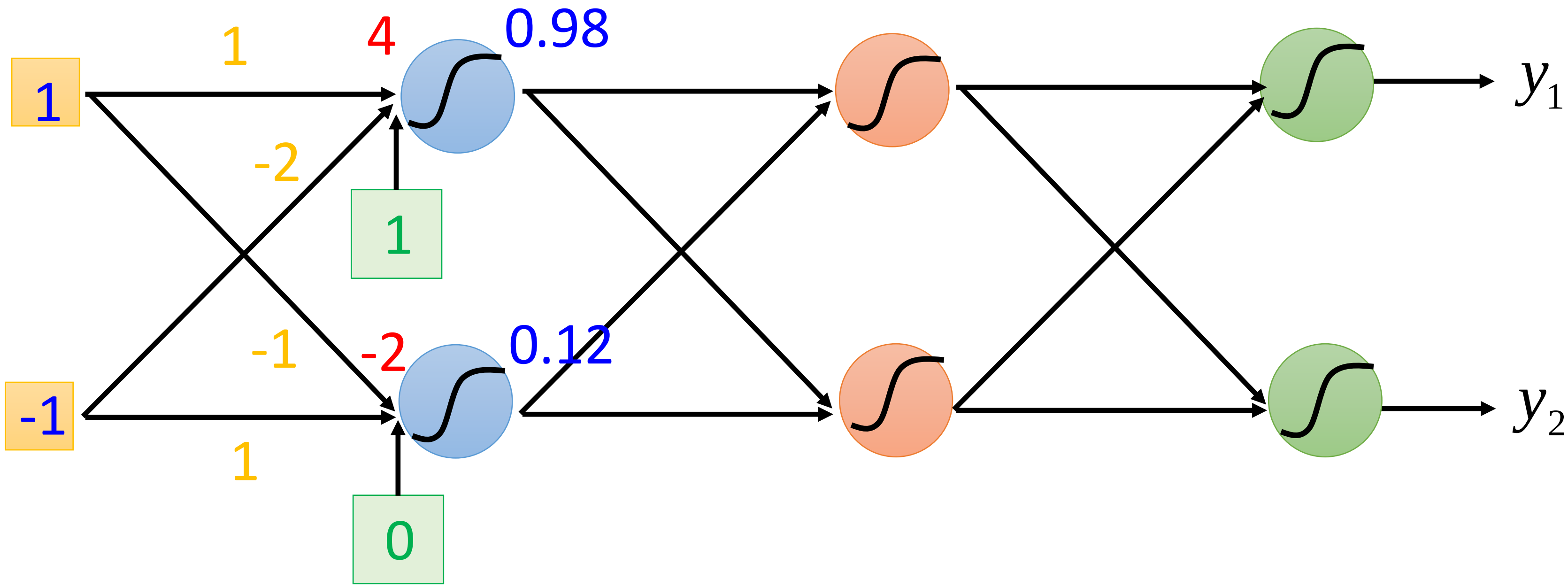
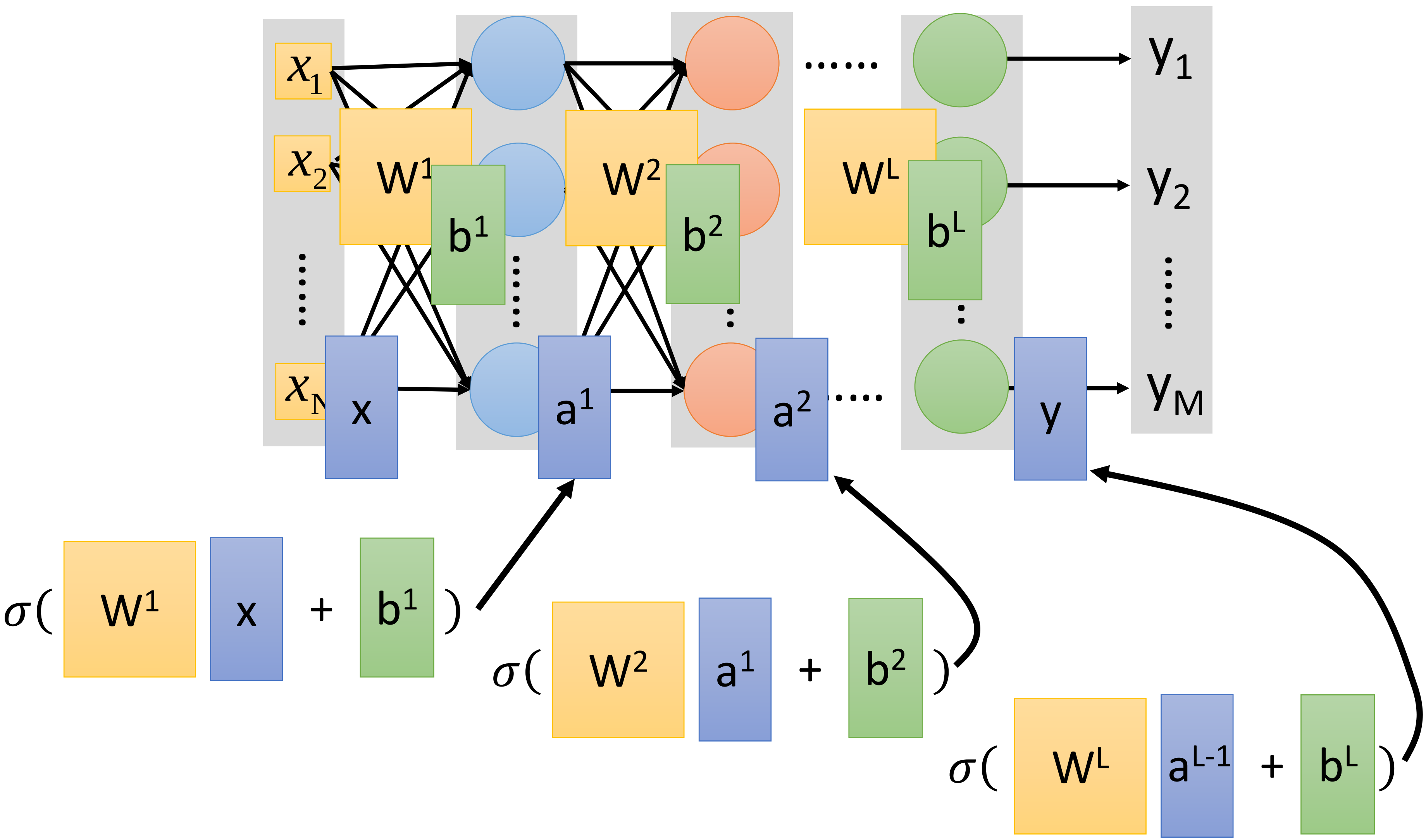
Matrix Operation

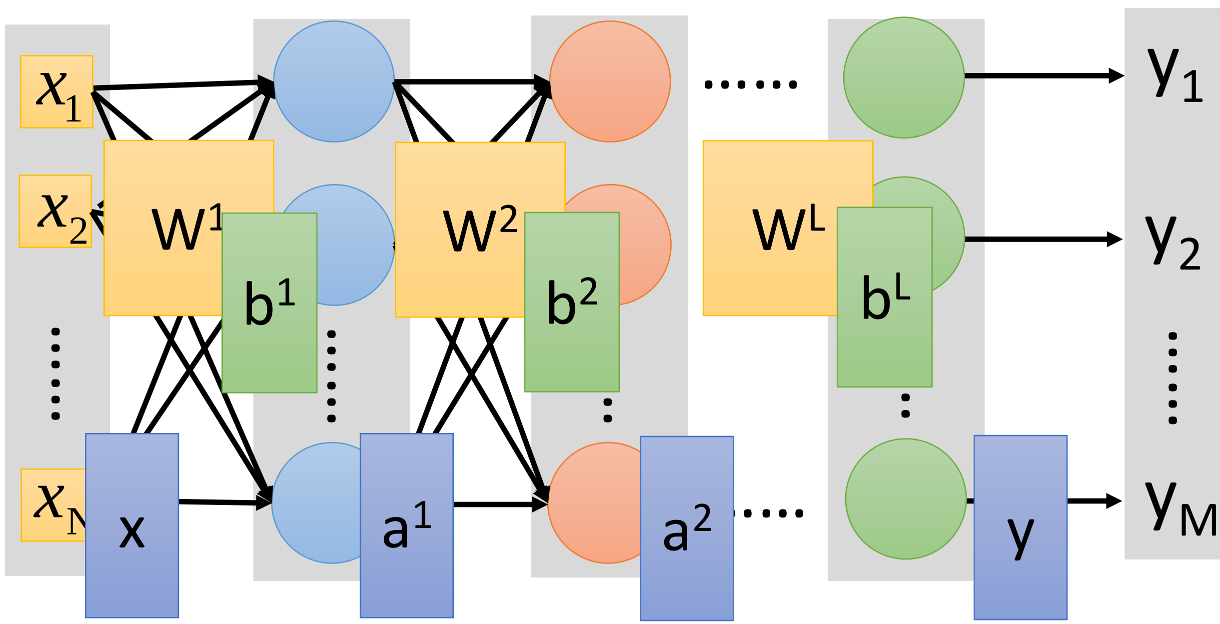


$$\sigma\Big(\underbrace{\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{\begin{bmatrix} 4 \\ -2 \end{bmatrix}}\Big) = \begin{bmatrix} 0.98 \\ 0.12 \end{bmatrix}$$

Function Nesting



Function Nesting

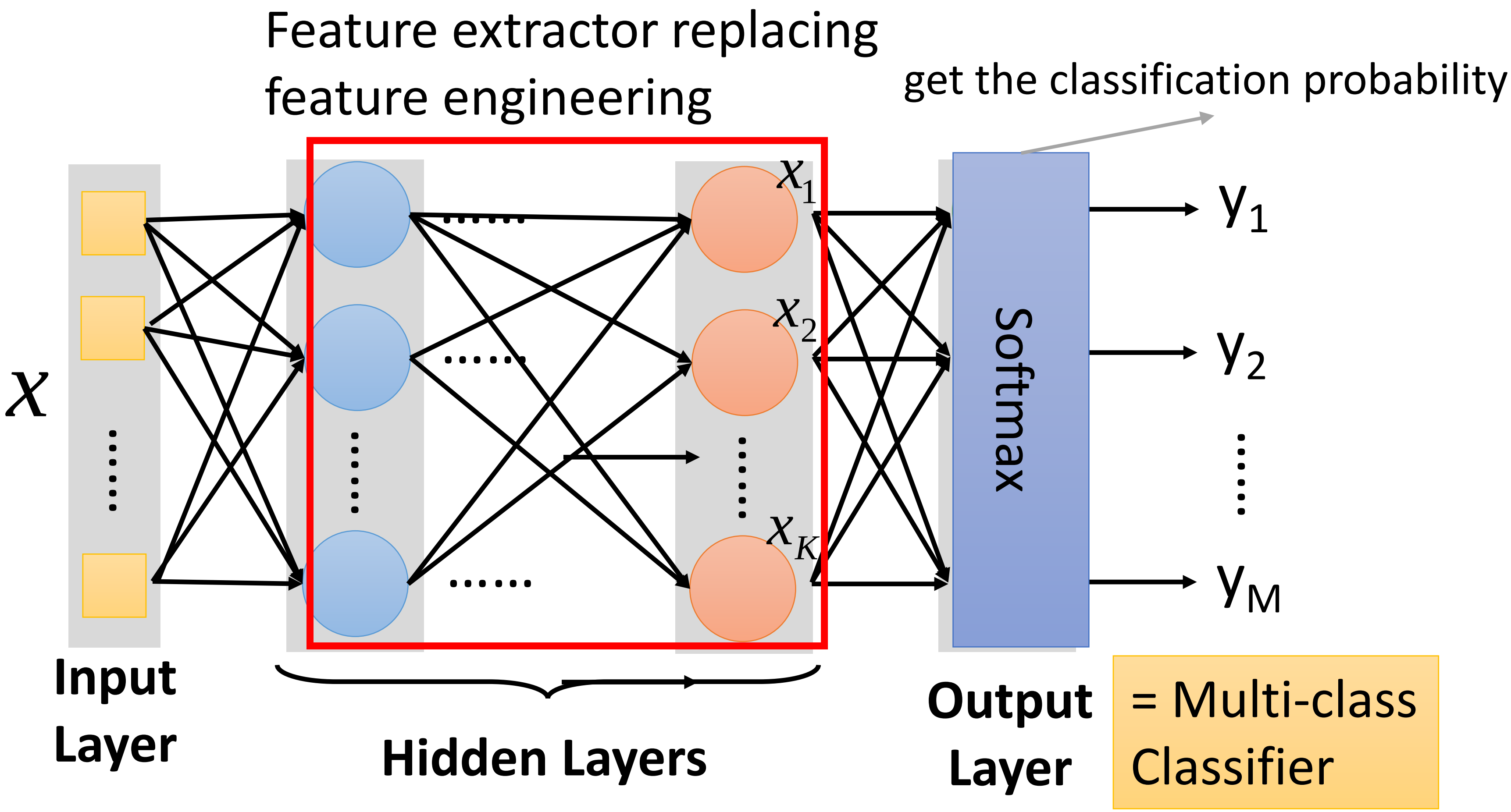


$y = f(x)$

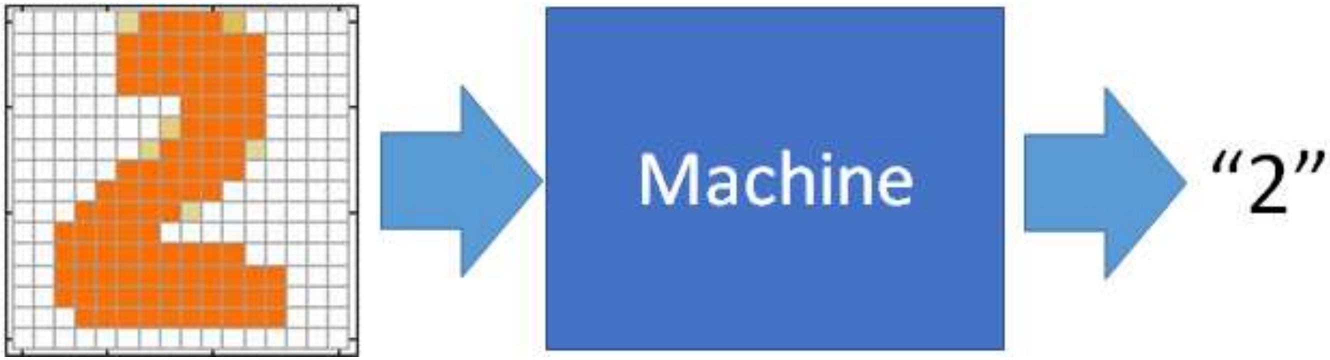
Using parallel computing techniques to speed up matrix operation

$$= \sigma(W^L \cdots \sigma(W^2 \, \sigma(W^1 x + b^1) + b^2) \cdots + b^L)$$

As a multi-class classifier

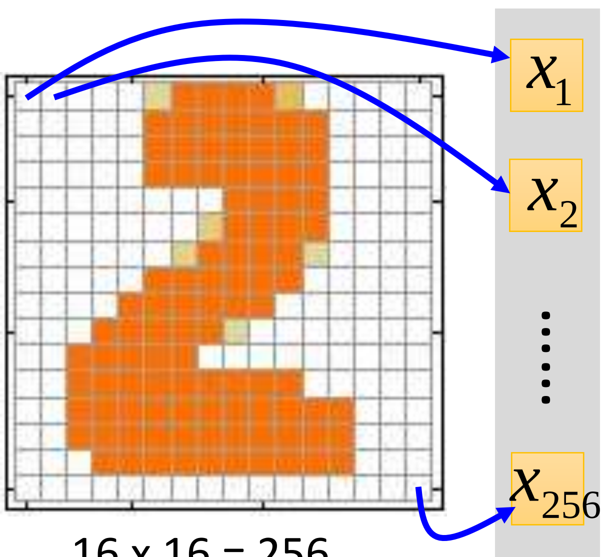


Case study
Handwriting Digit Recognition



Case study
Handwriting Digit Recognition

Input

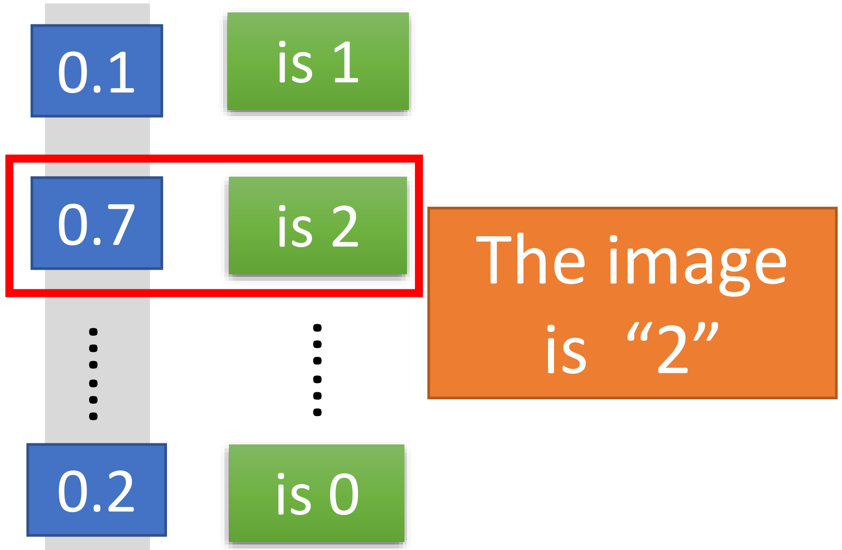


16 x 16 = 256

Ink → 1

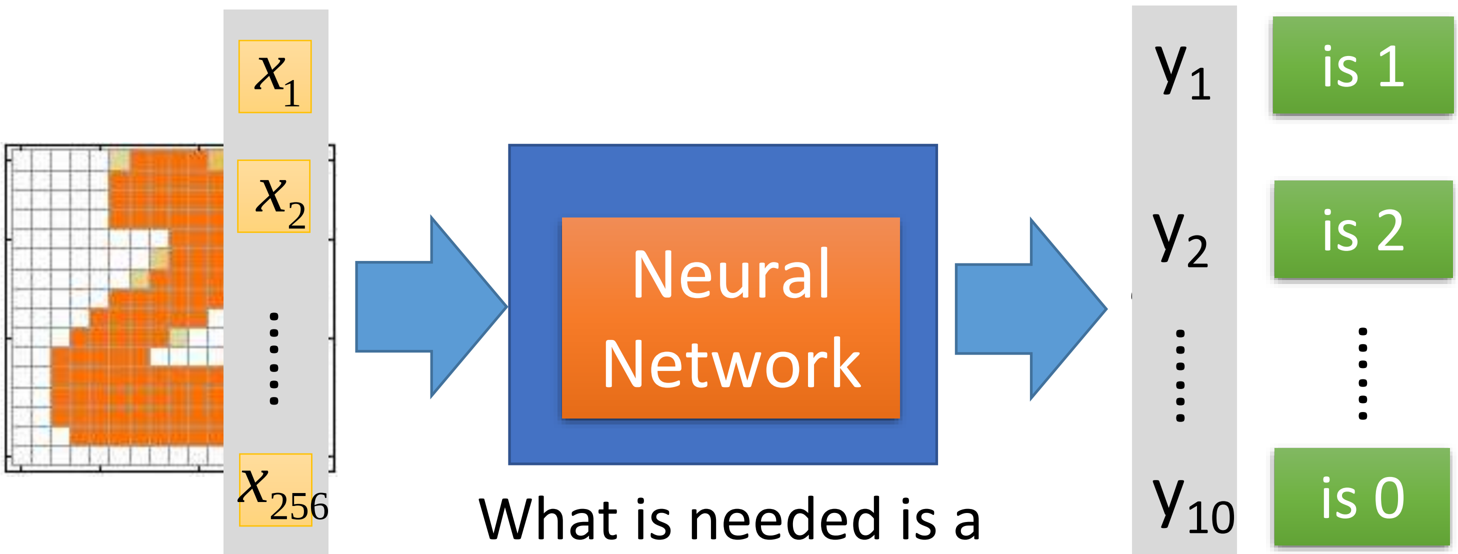
No ink → 0

Output



Each dimension represents the confidence of a digit.

Case study
Handwriting Digit Recognition

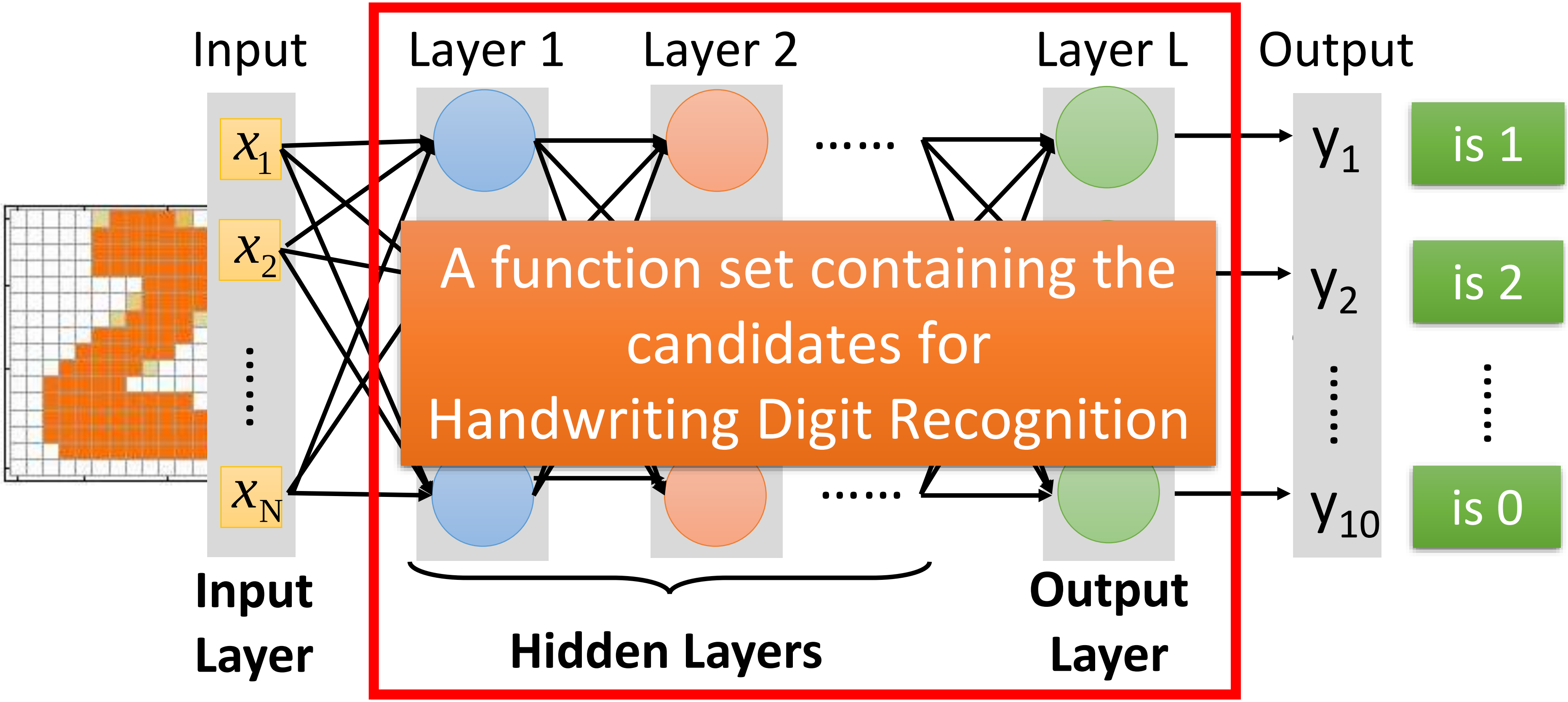


What is needed is a function ……

Input:
256-dim vector

output:
10-dim vector

Case study
Handwriting Digit Recognition

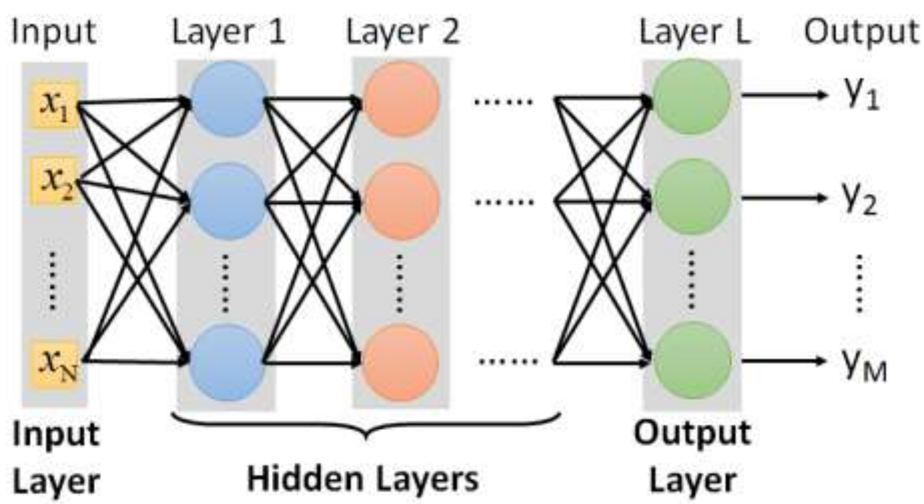


You need to decide the network structure to let a good function in your function set.

Case study
Handwriting Digit Recognition

- How many layers? How many neurons for each layer?

Trial and Error + Intuition



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Neural Network Optimization

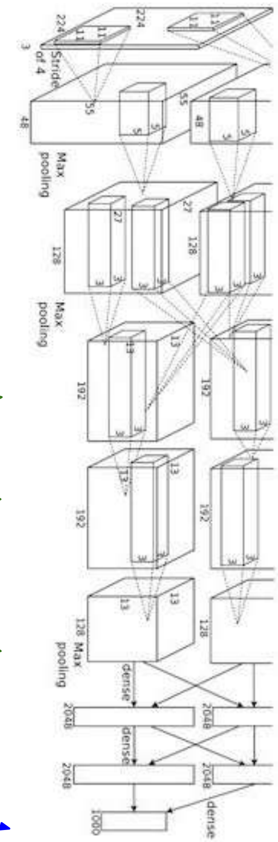
- We have already learned to optimize the learner using the gradient descent method
- Can neural networks also be optimized using gradient descent?

Case: CNN (AlexNet)

input image

weights

loss



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Case: Neural Turing Machine

input image

loss

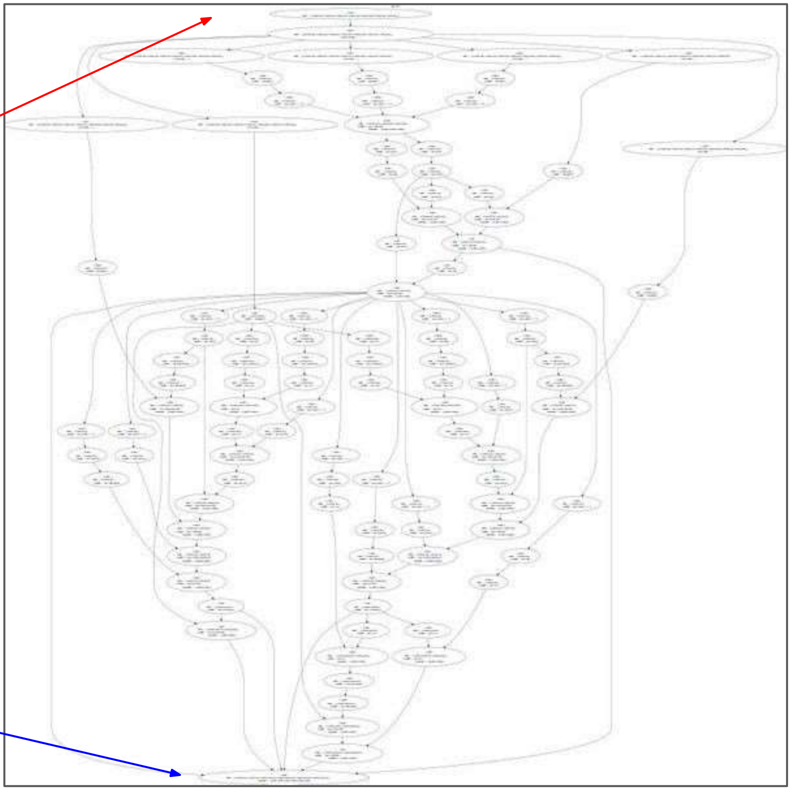


Figure reproduced with permission from a Twitter post by Andrej Karpathy.

Why we need BP

- If we use gradient descent directly

Network parameters $\theta = \{w_1, w_2, \cdots, b_1, b_2, \cdots\}$

Starting Parameters $\theta^0 \longrightarrow \theta^1 \longrightarrow \theta^2 \longrightarrow$ ……

$$\nabla L(\theta) = \begin{bmatrix} \partial L(\theta)/\partial w_1 \\ \partial L(\theta)/\partial w_2 \\ \vdots \\ \partial L(\theta)/\partial b_1 \\ \partial L(\theta)/\partial b_2 \\ \vdots \end{bmatrix}$$

$Compute\ \nabla L(\theta^0)$ $\qquad \theta^1 = \theta^0 - \eta \nabla L(\theta^0)$

$Compute\ \nabla L(\theta^1)$ $\qquad \theta^2 = \theta^1 - \eta \nabla L(\theta^1)$

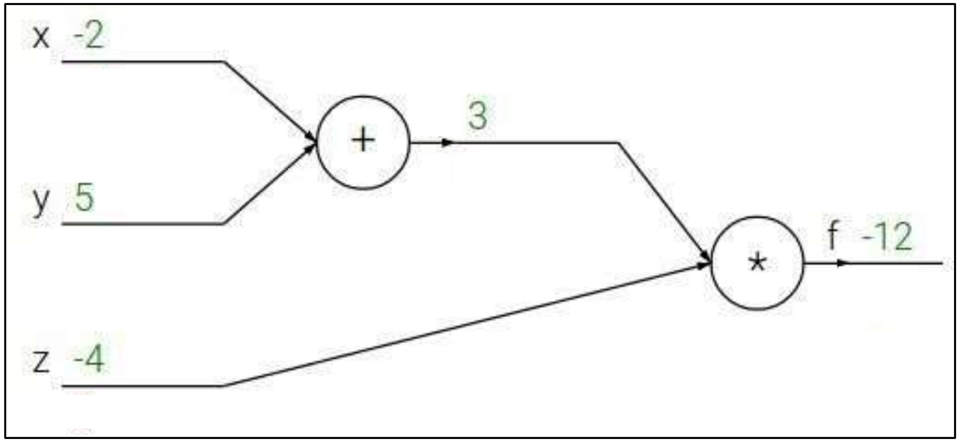
Millions of parameters ……

To compute the gradients efficiently, we use ***backpropagation***.

BP: A simple example

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



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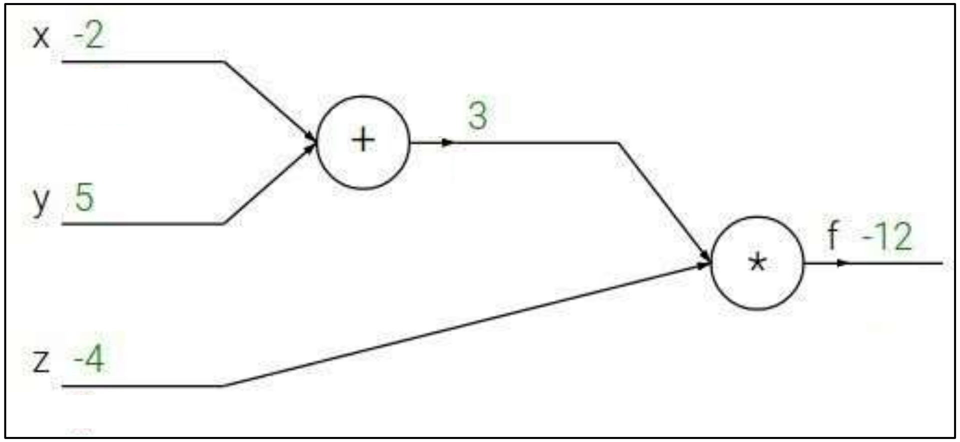

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

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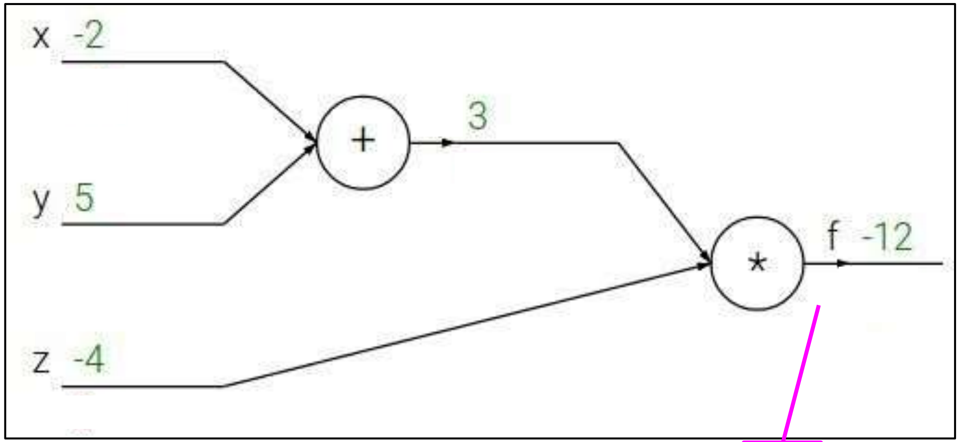

$\frac{\partial f}{\partial f}$

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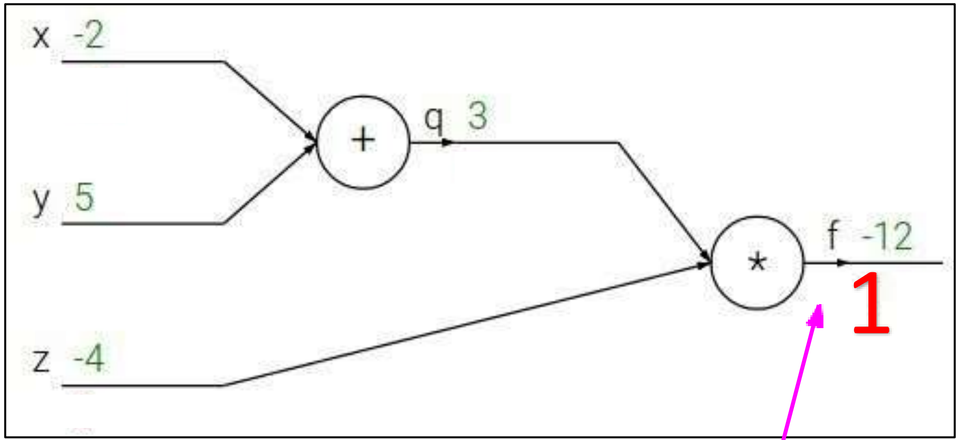

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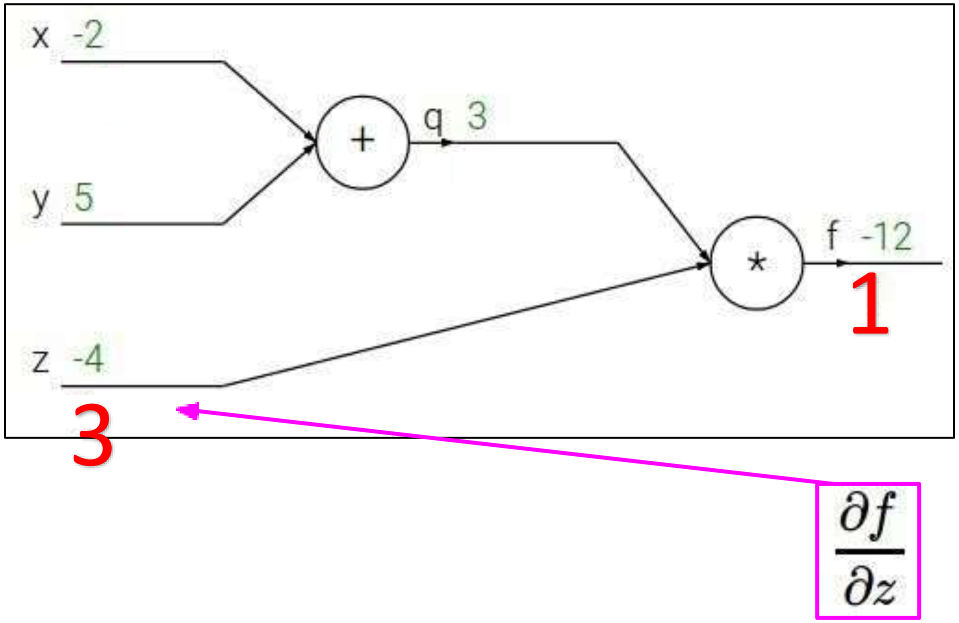

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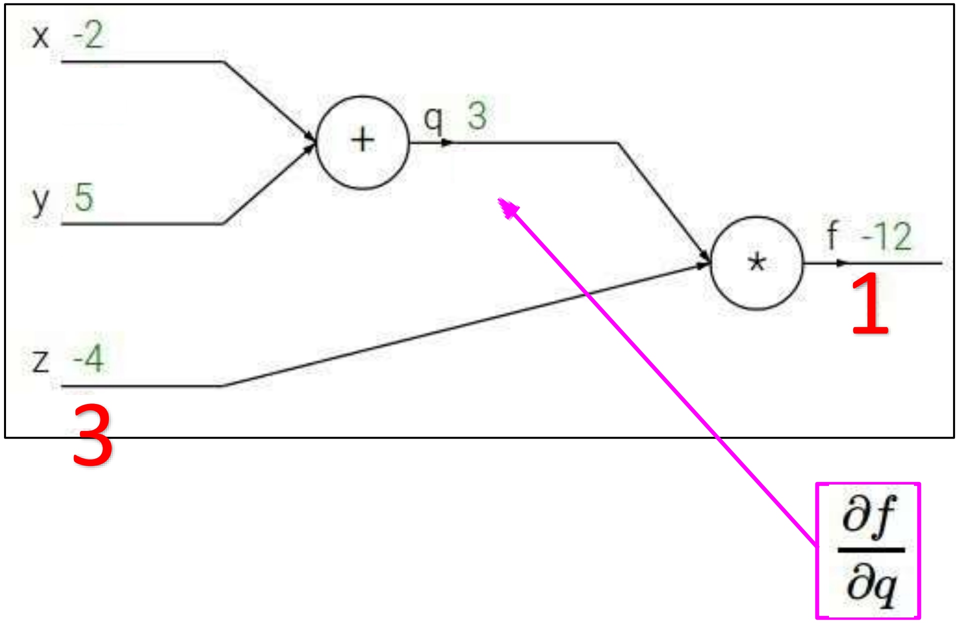

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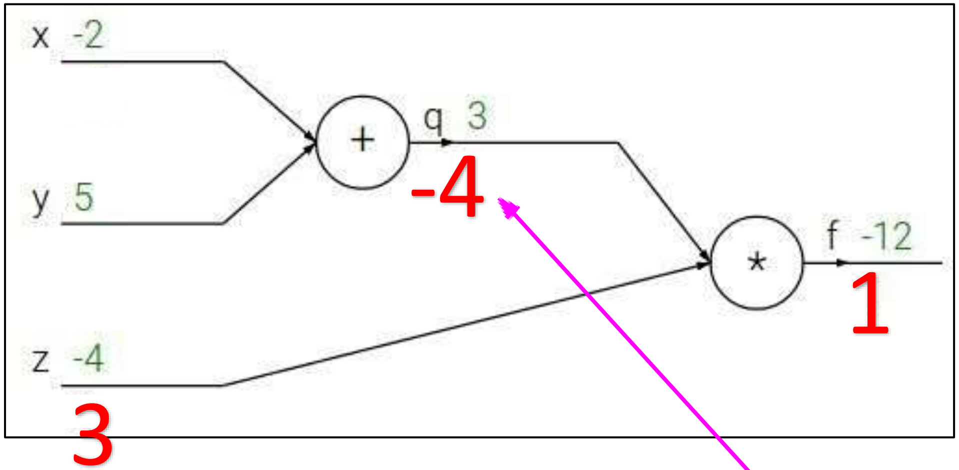

$\frac{\partial f}{\partial q}$

BP: A simple example

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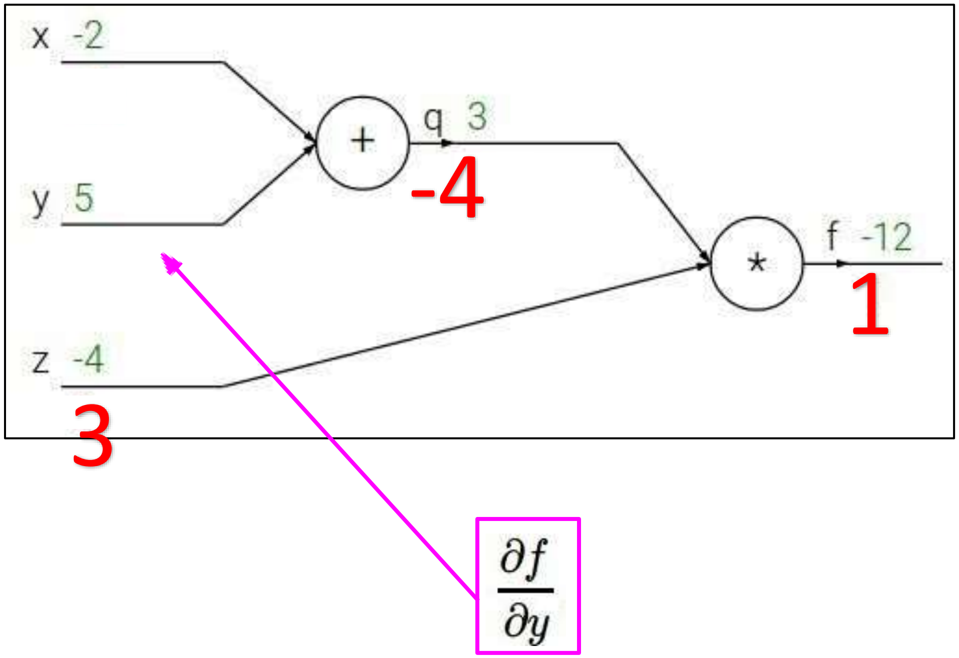

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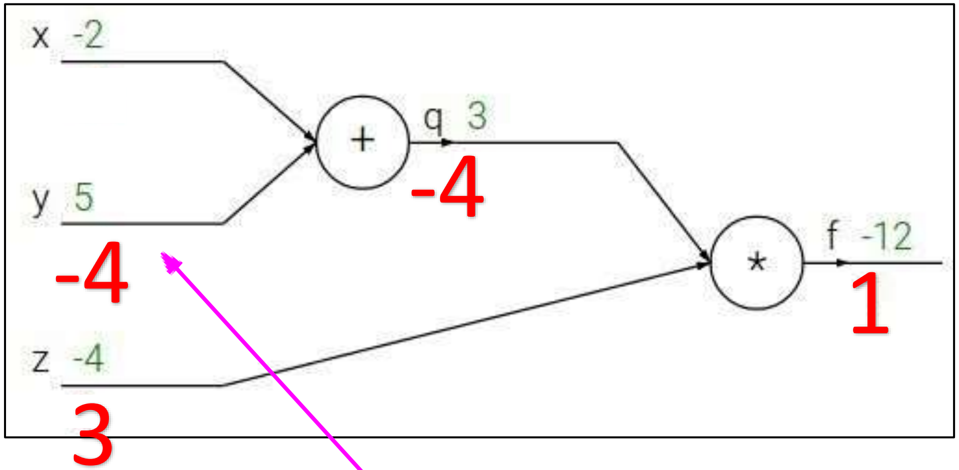
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



$\frac{\partial f}{\partial y}$

Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

BP: A simple example

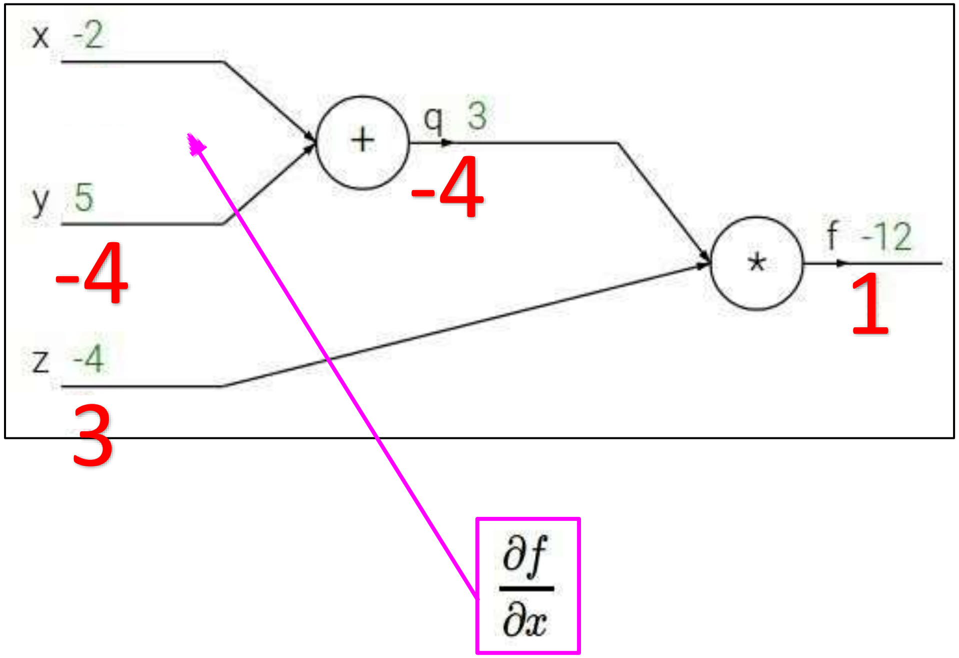
$$f(x,y,z) = (x+y)z$$

e.g. x = -2, y = 5, z = -4

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f = qz \qquad \frac{\partial f}{\partial q} = z, \frac{\partial f}{\partial z} = q$$

Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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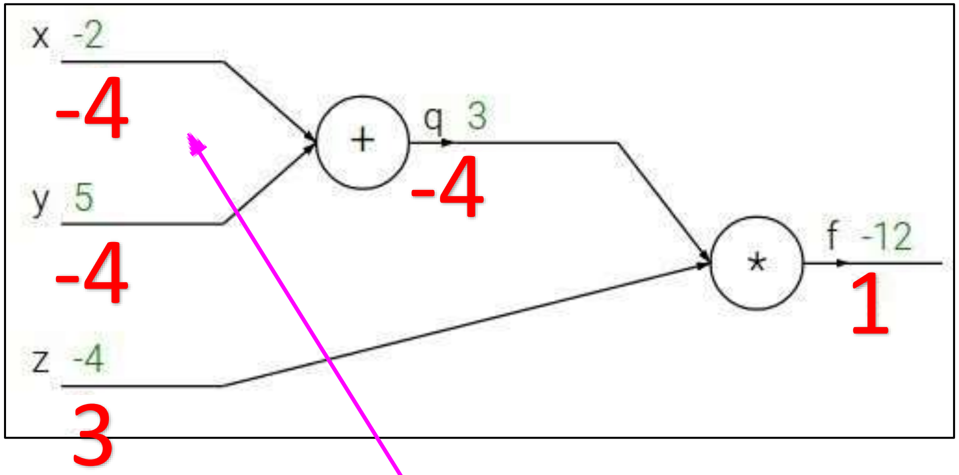
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$

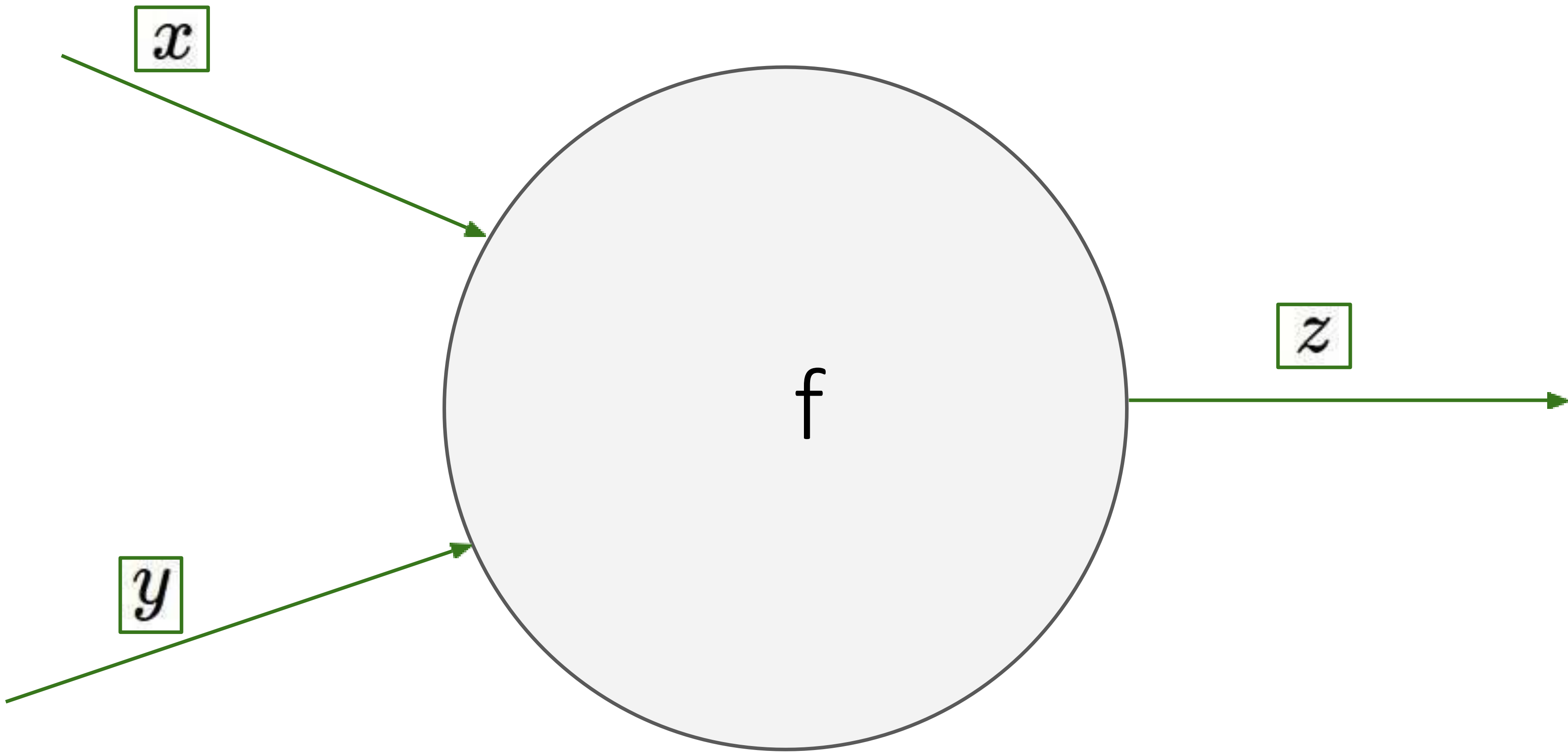


$\frac{\partial f}{\partial x}$

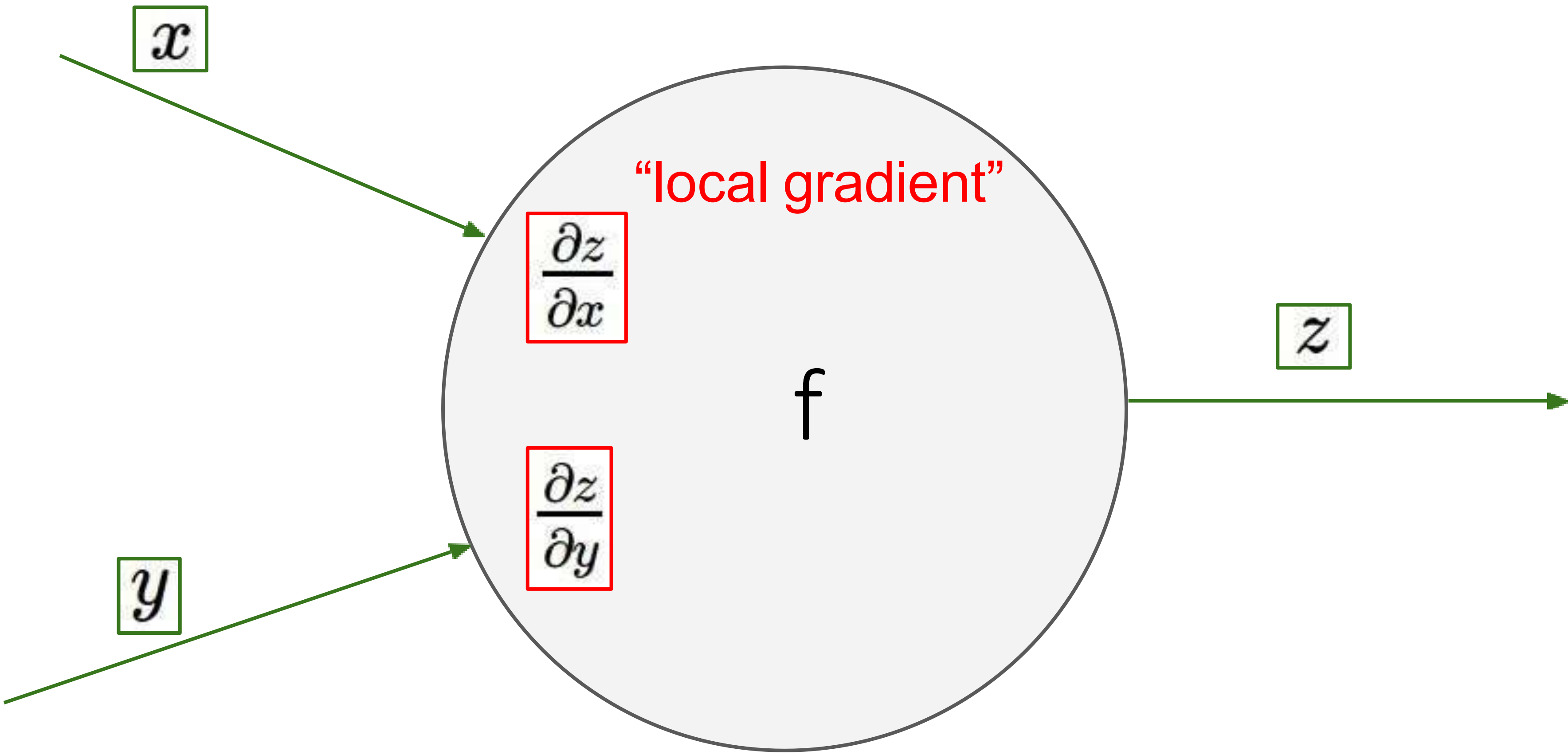
Chain rule:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$$

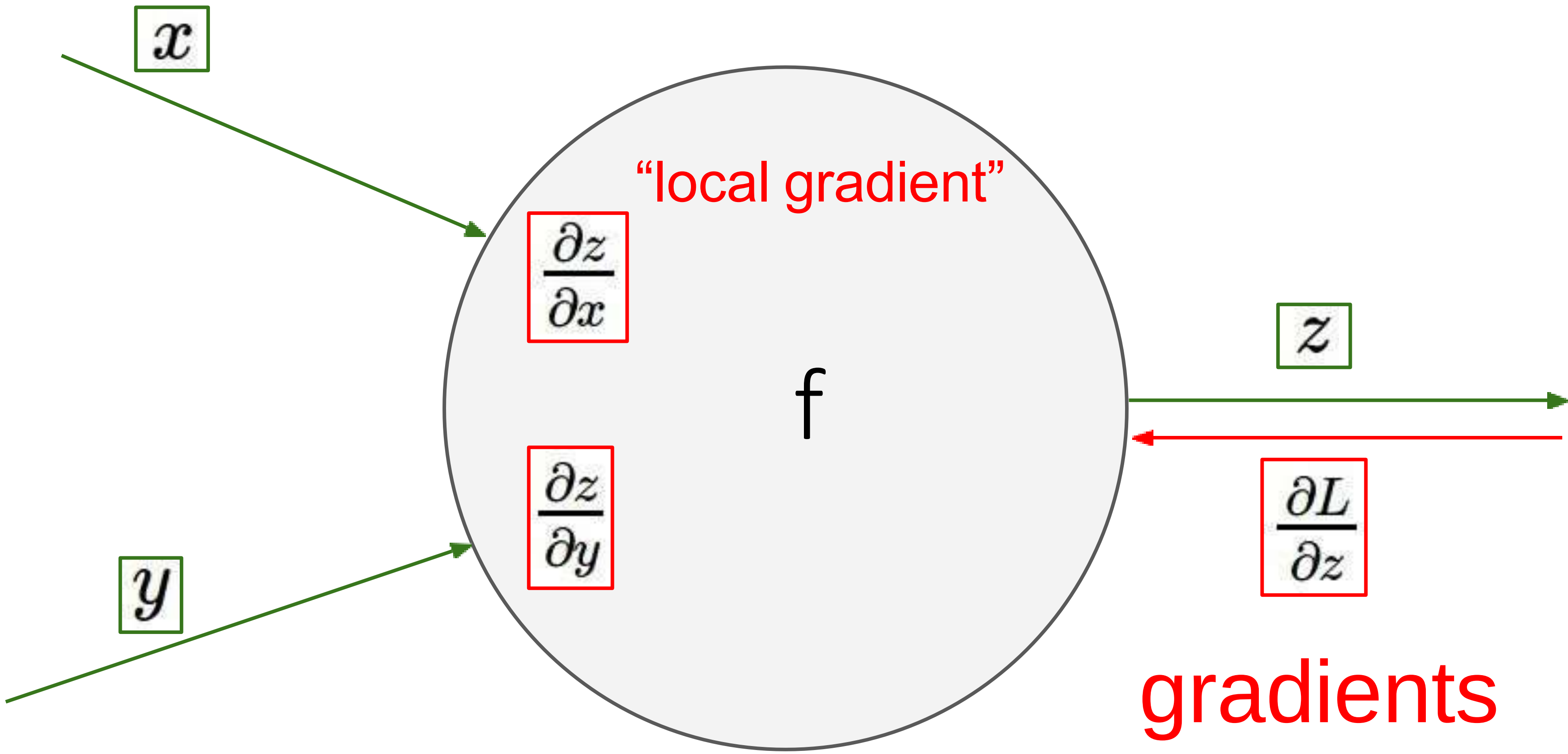
Backpropagation



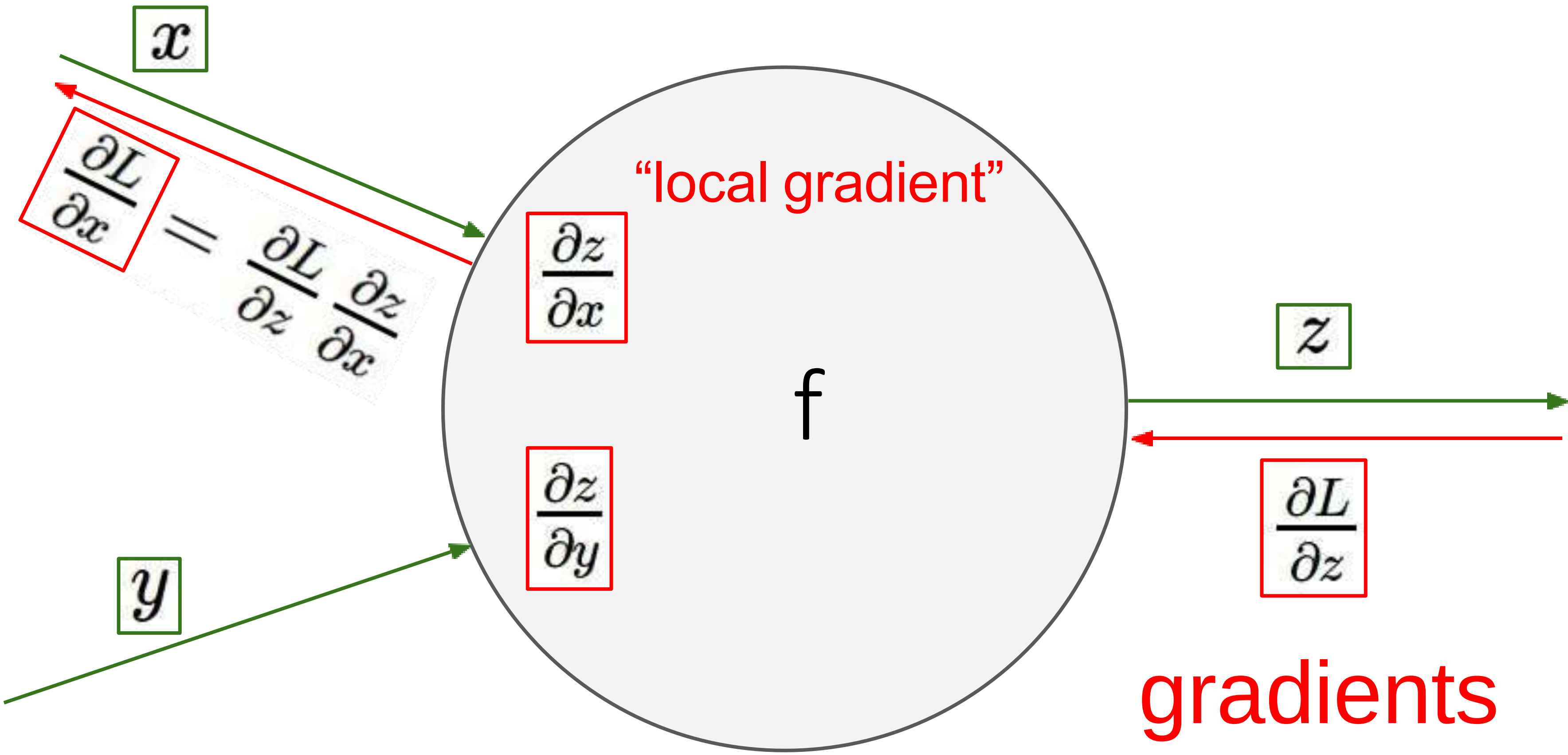
Backpropagation



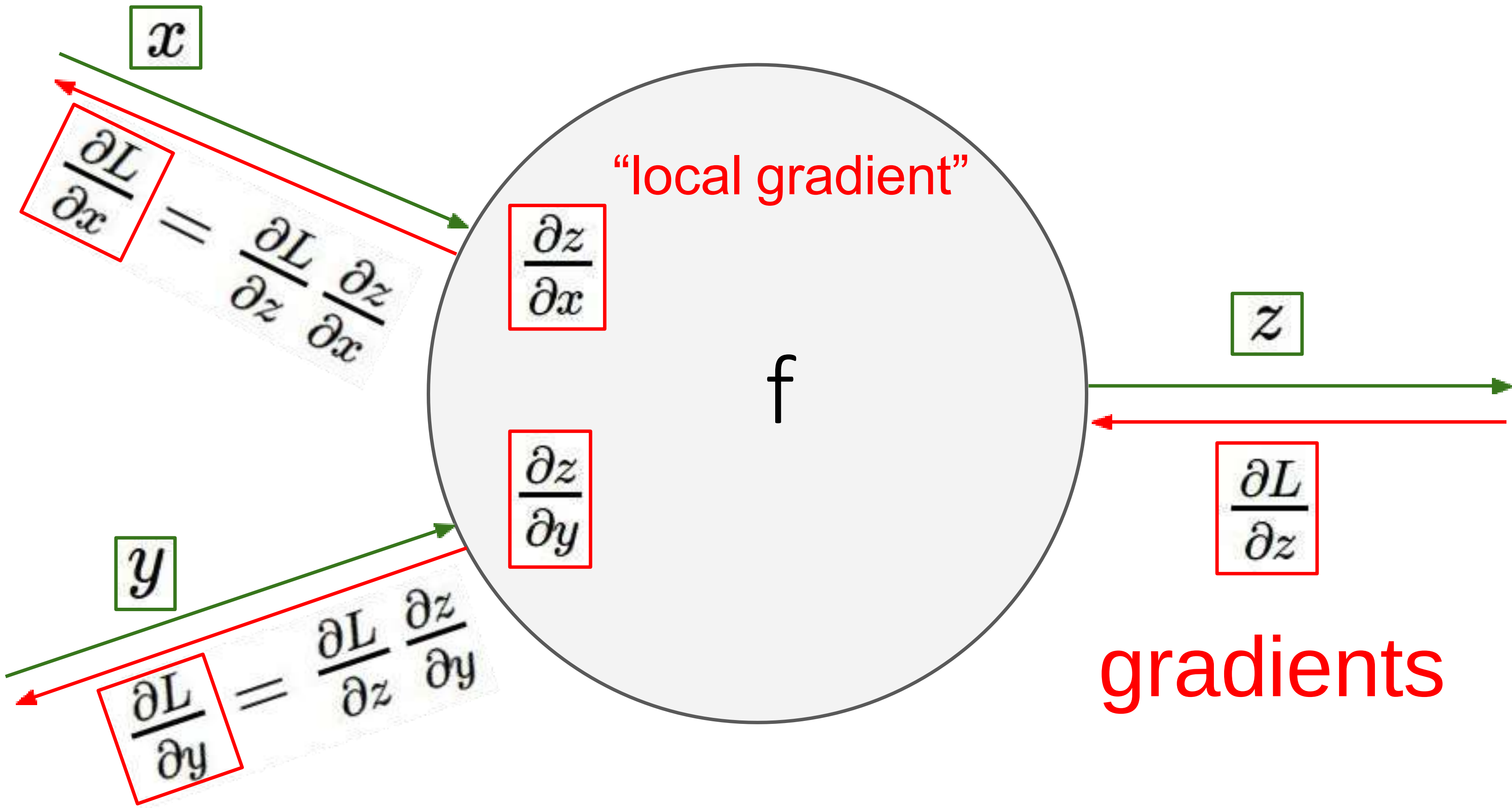
Backpropagation



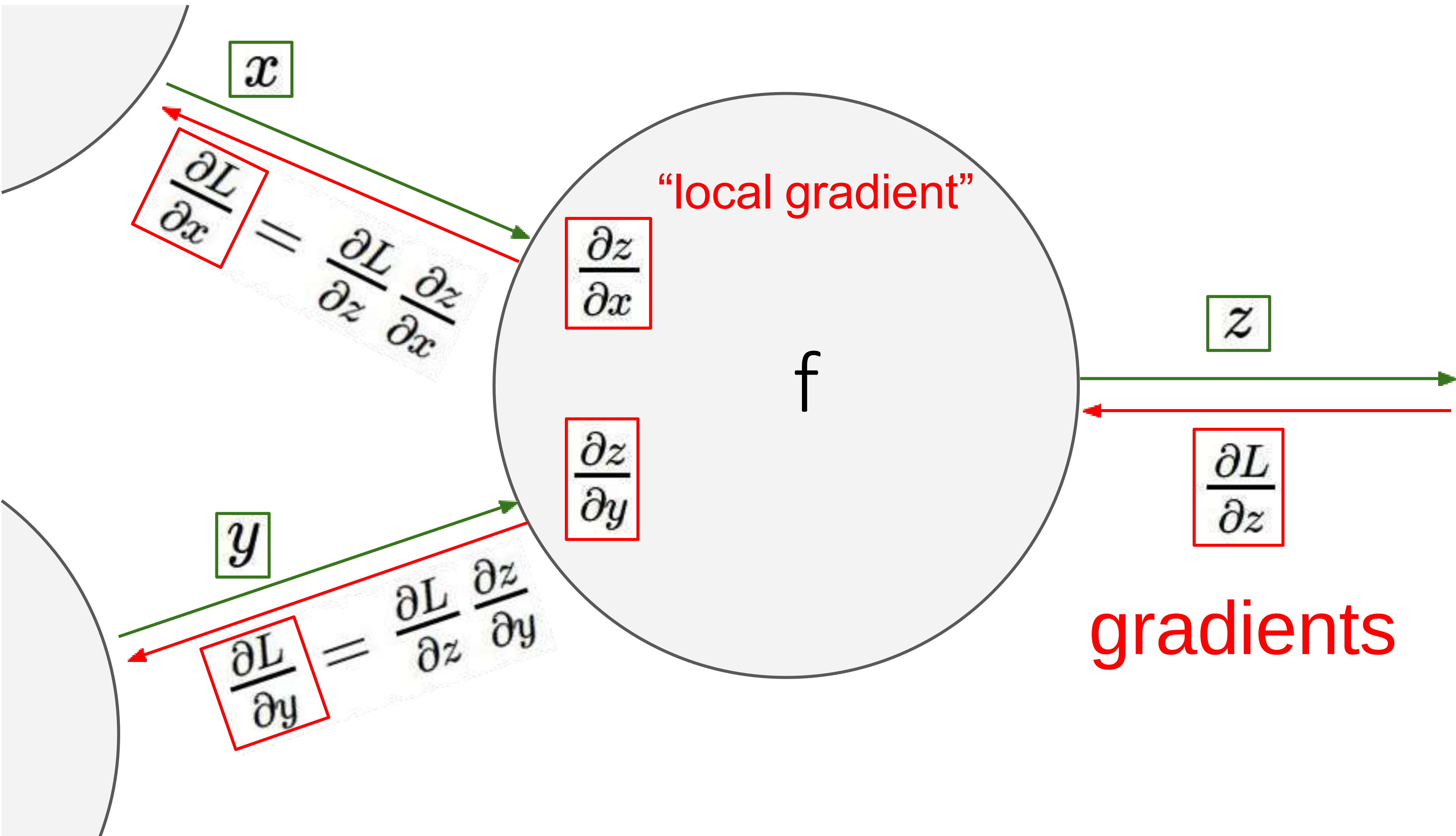
Backpropagation



Backpropagation

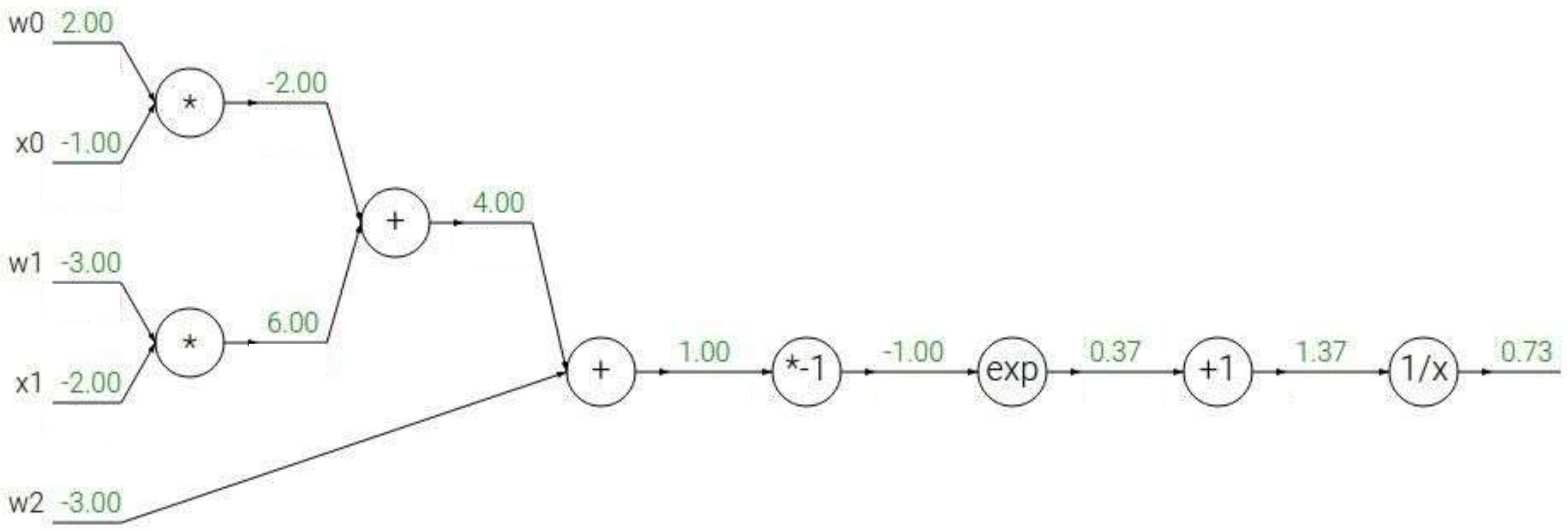


Backpropagation



BP: Another example

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$

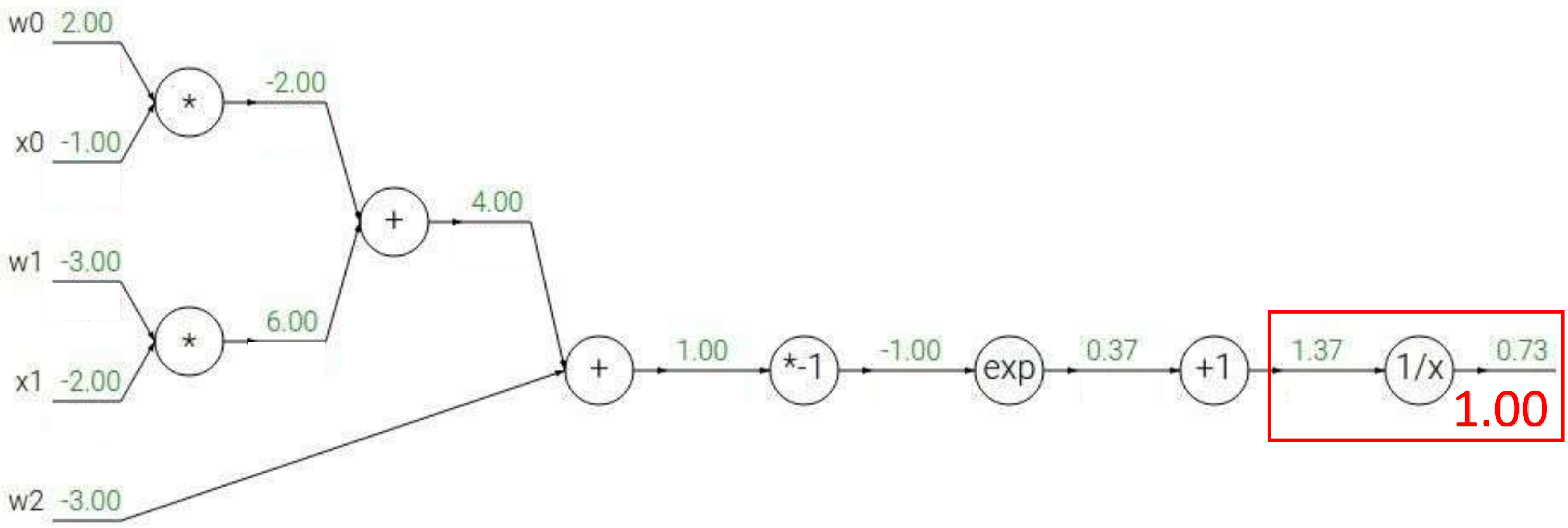
$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$

$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$

$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$

BP: Another example

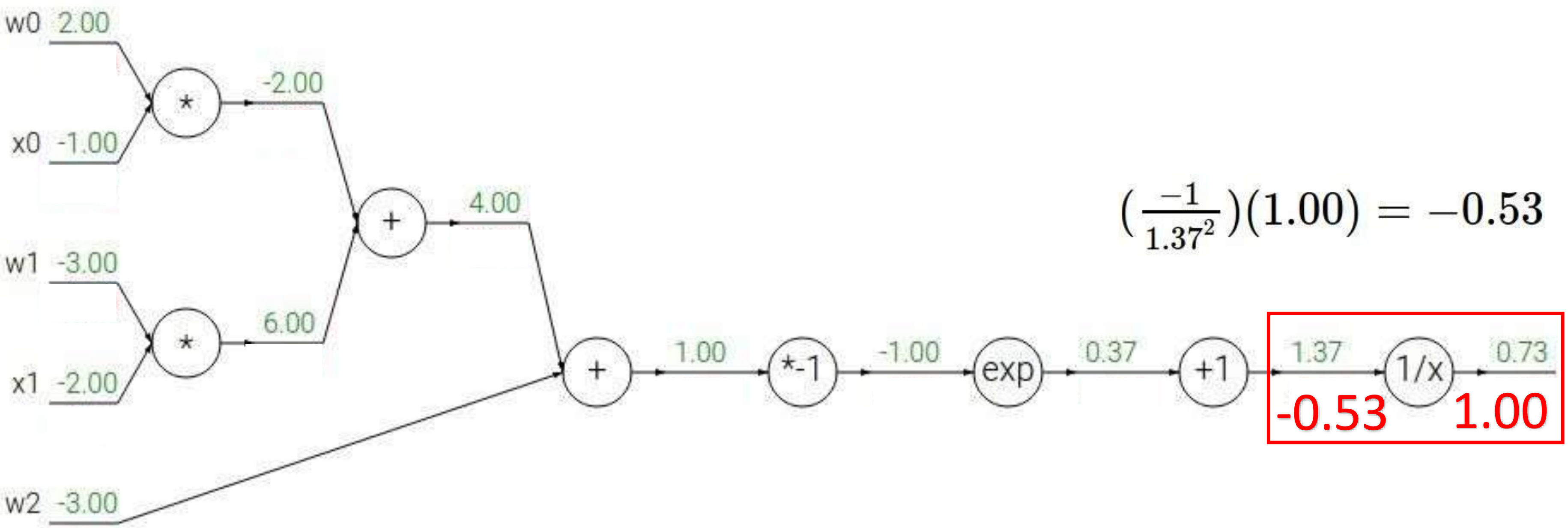
$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$f(x) = e^x$	$\rightarrow$	$\frac{df}{dx} = e^x$	$f(x) = \frac{1}{x}$	$\rightarrow$	$\frac{df}{dx} = -1/x^2$
$f_a(x) = ax$	$\rightarrow$	$\frac{df}{dx} = a$	$f_c(x) = c + x$	$\rightarrow$	$\frac{df}{dx} = 1$

BP: Another example

$$f(w,x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$\left(\frac{-1}{1.37^2}\right)(1.00) = -0.53$$

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

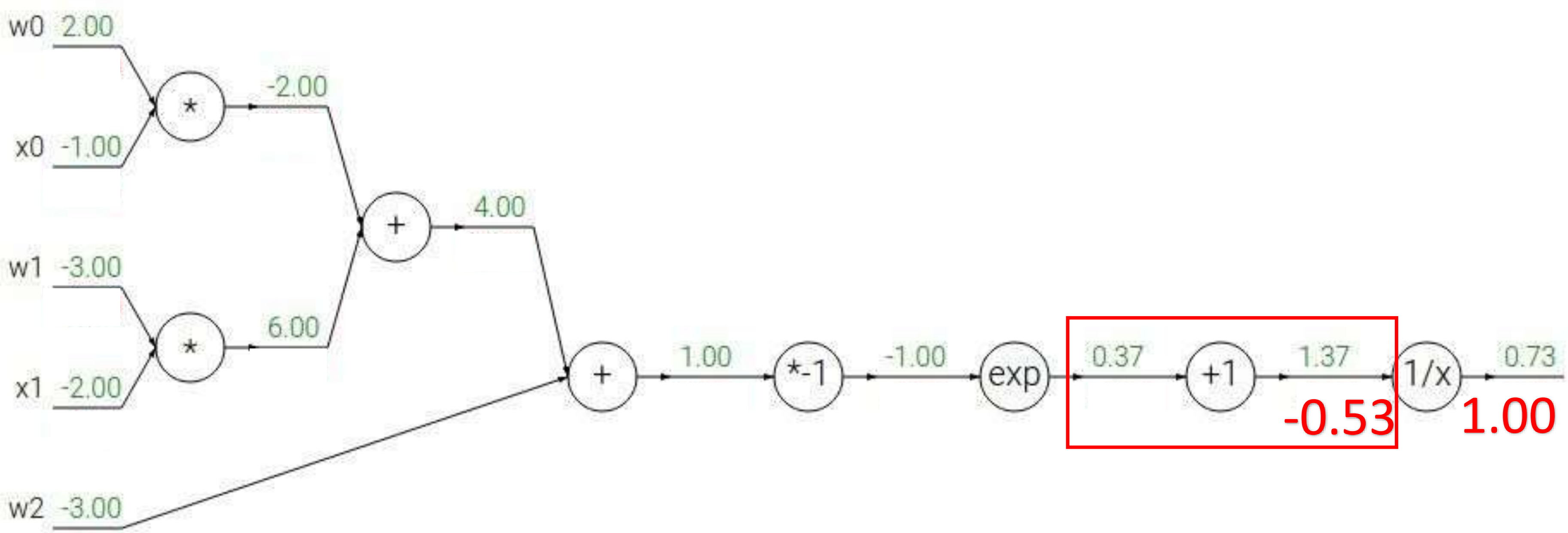
$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$\boxed{f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2}$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

BP: Another example

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

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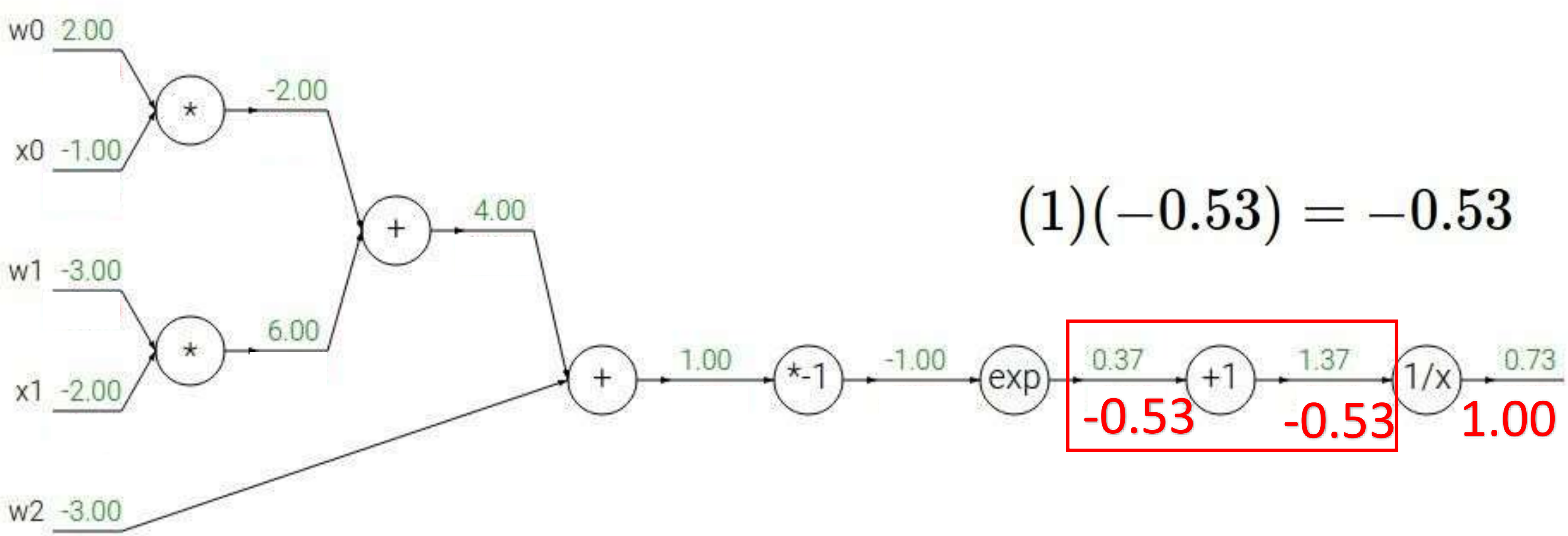


$$(1)(-0.53) = -0.53$$

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

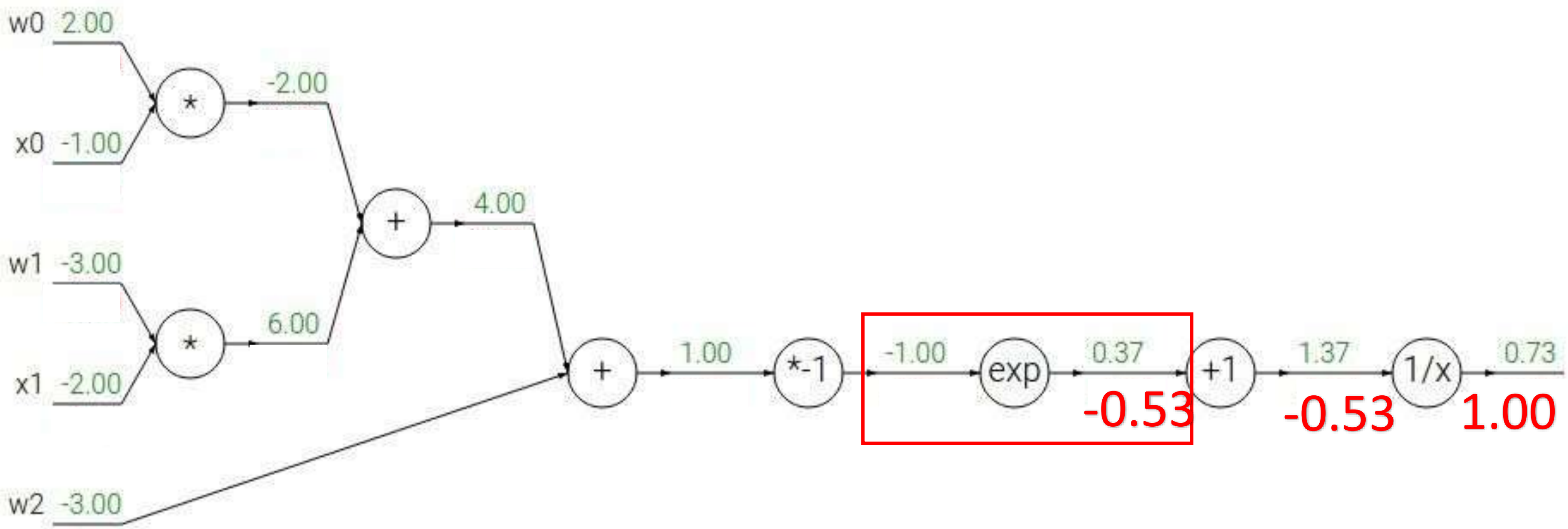
$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$\boxed{f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1}$$

BP: Another example

$$f(w,x)=\frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$\boxed{f(x)=e^x \quad \rightarrow \quad \frac{df}{dx}=e^x}$$

$$f_a(x)=ax \quad \rightarrow \quad \frac{df}{dx}=a$$

$$f(x)=\frac{1}{x} \quad \rightarrow \quad \frac{df}{dx}=-1/x^2$$

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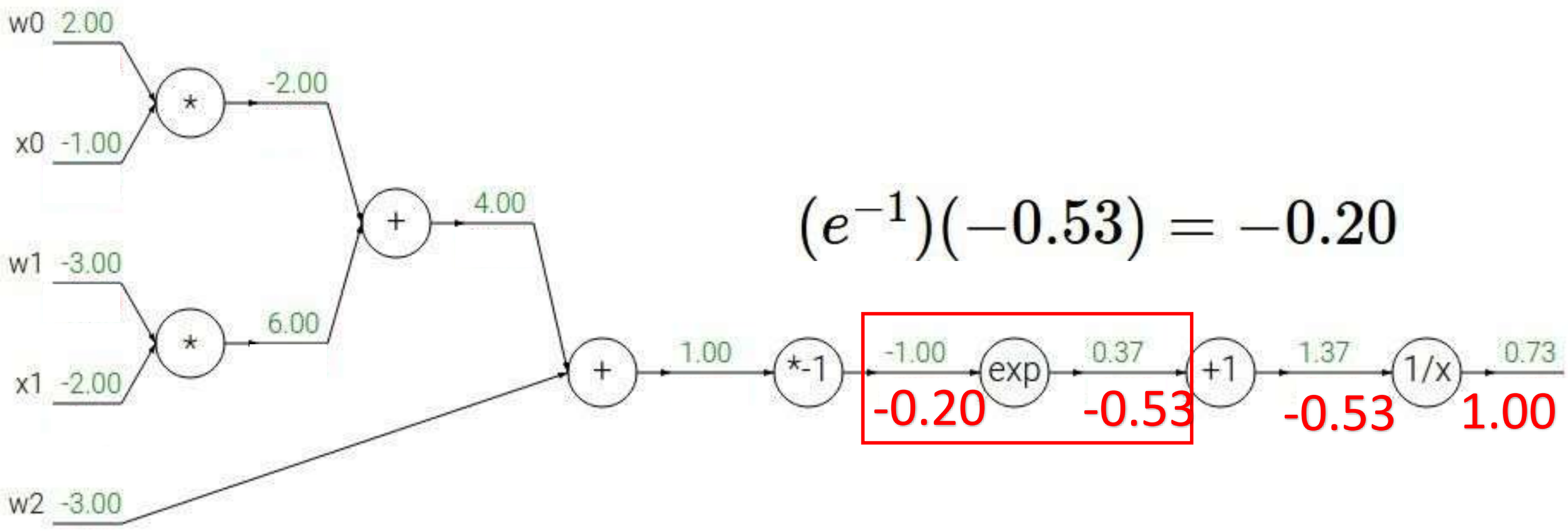
BP: Another example

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$(e^{-1})(-0.53) = -0.20$$

$$f(x) = e^x \rightarrow \frac{df}{dx} = e^x$$

$$f_a(x) = ax \rightarrow \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \rightarrow \frac{df}{dx} = -1/x^2$$

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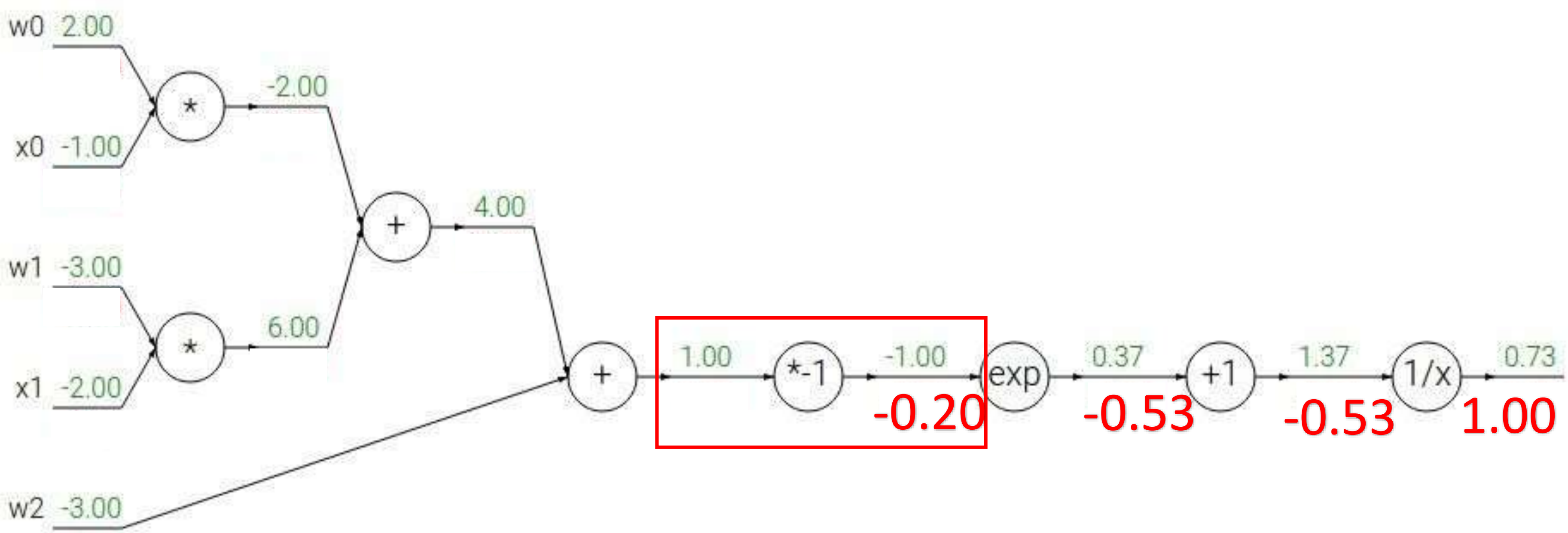

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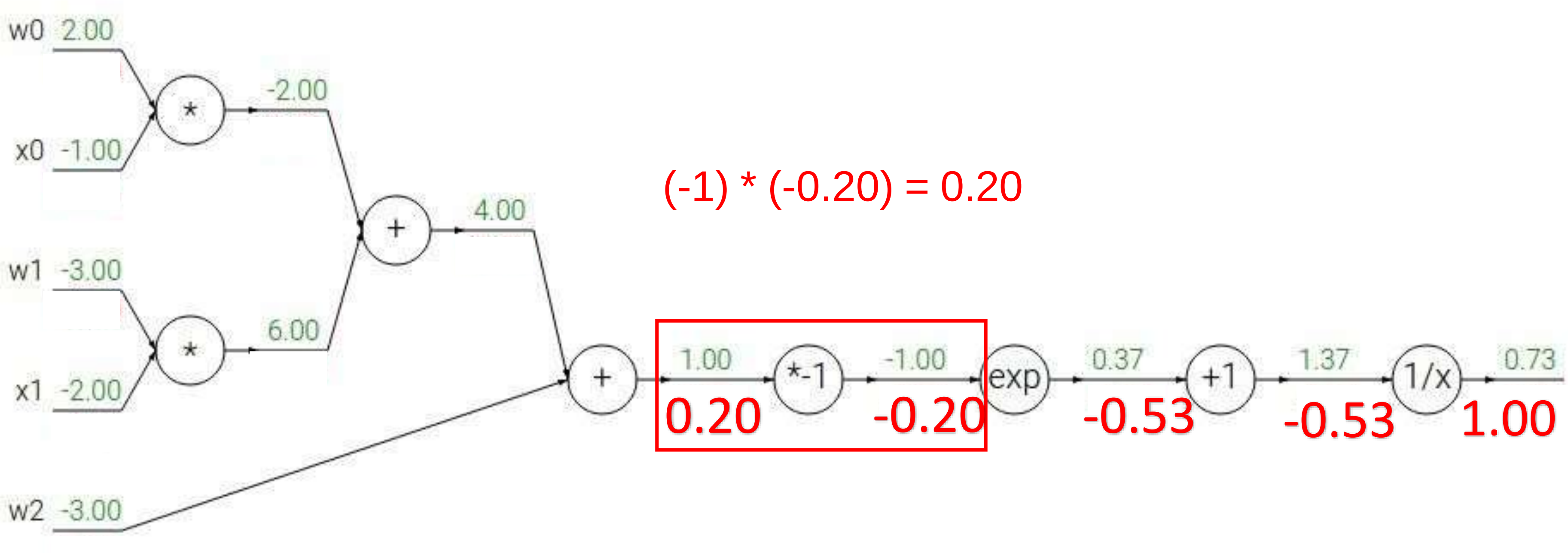

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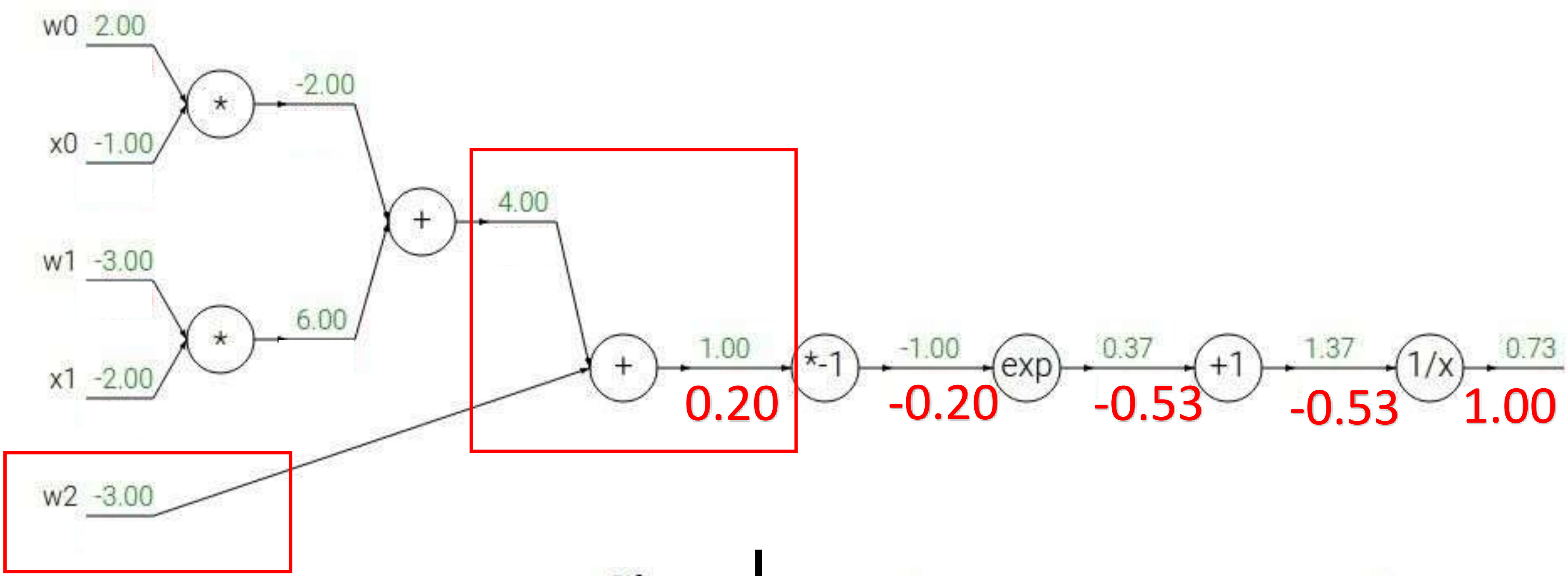


$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a \qquad f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

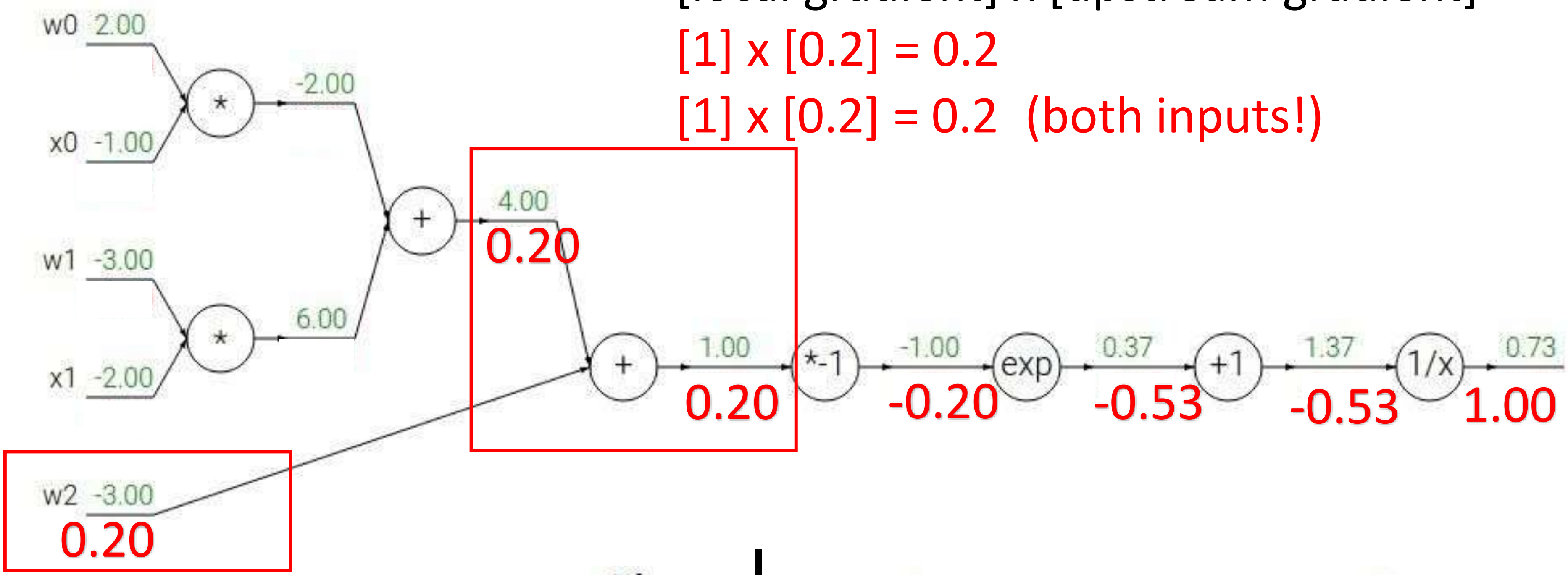
BP: Another example

$$f(w,x) = \frac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$

[local gradient] x [upstream gradient]

[1] x [0.2] = 0.2

[1] x [0.2] = 0.2 (both inputs!)



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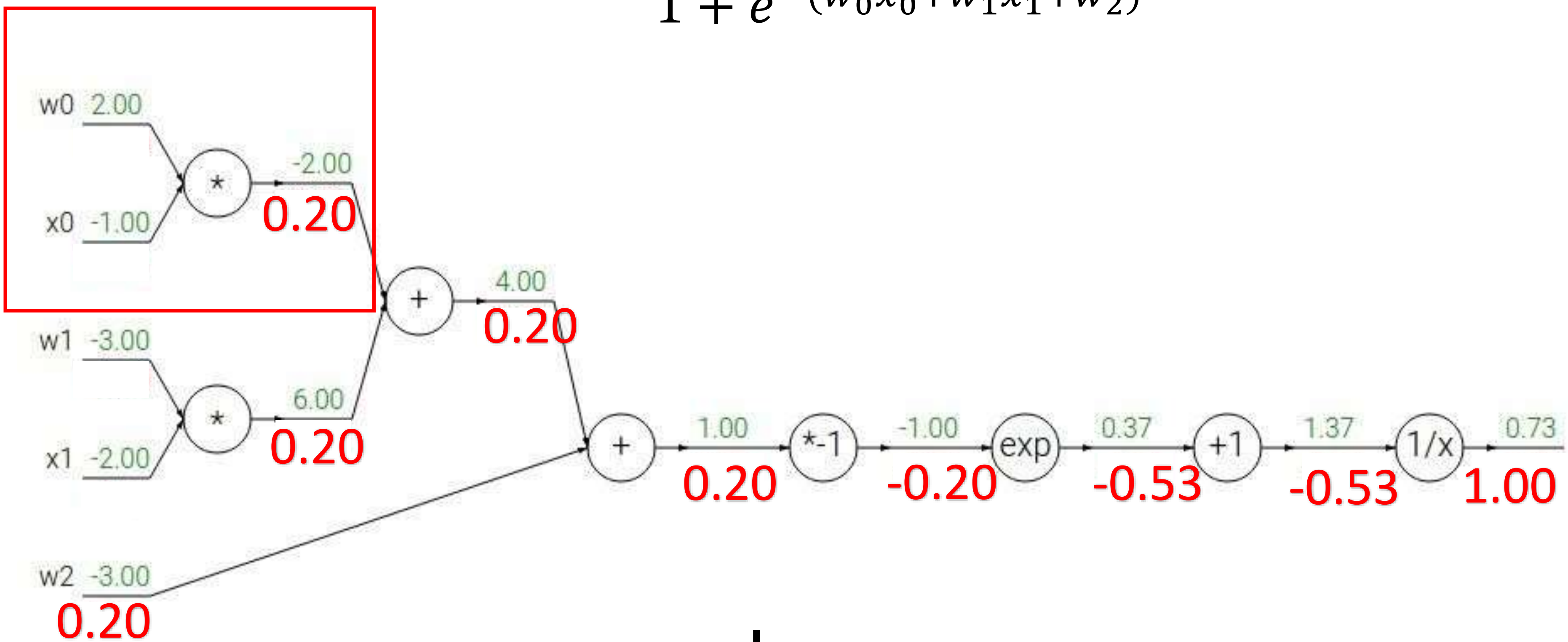

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$

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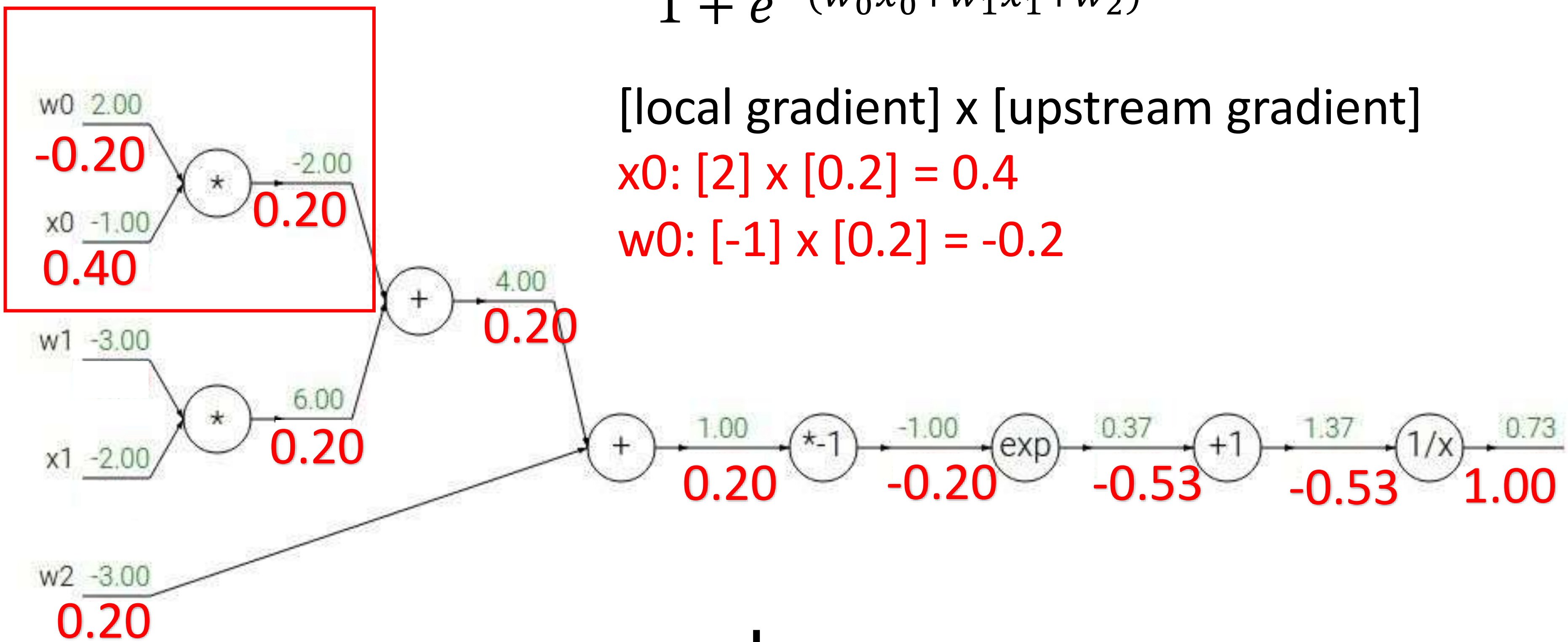
BP: Another example

$$f(w,x)=\frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

[local gradient] x [upstream gradient]

x0: [2] x [0.2] = 0.4

w0: [-1] x [0.2] = -0.2



$$f(x)=e^x \quad \rightarrow \quad \frac{df}{dx}=e^x$$

$$f(x)=\frac{1}{x} \quad \rightarrow \quad \frac{df}{dx}=-1/x^2$$

$$f_a(x)=ax \quad \rightarrow \quad \frac{df}{dx}=a$$

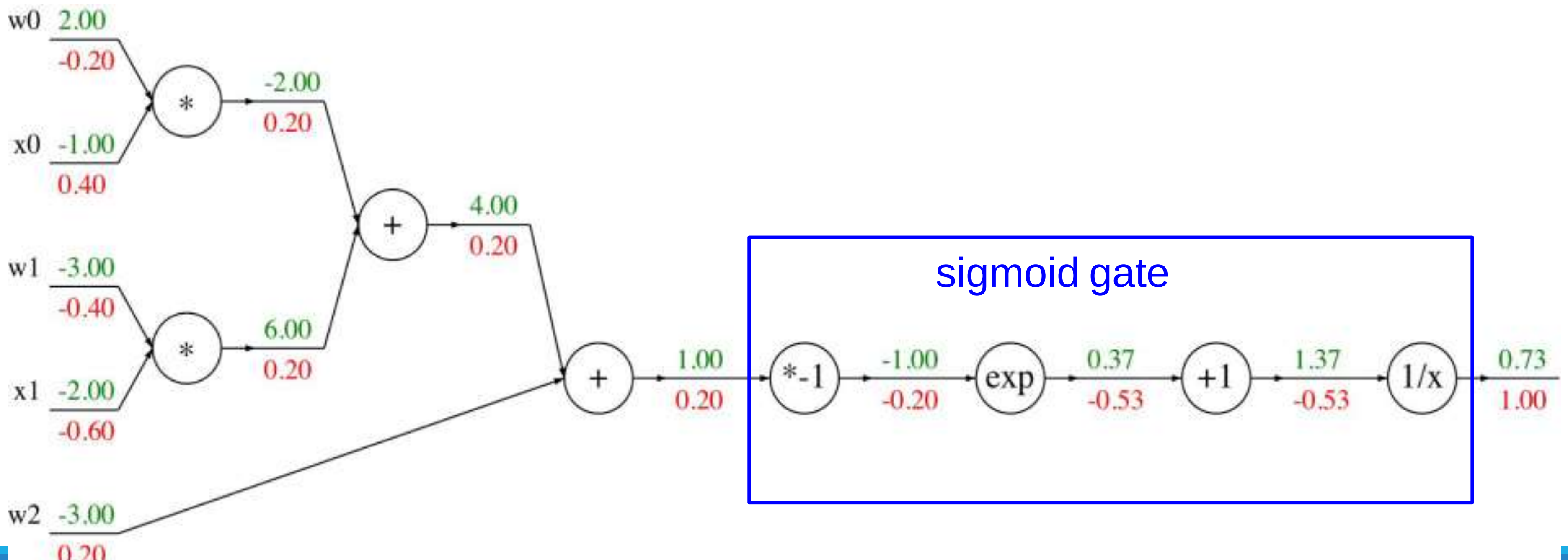
$$f_c(x)=c+x \quad \rightarrow \quad \frac{df}{dx}=1$$

BP: Another example

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$



BP: Another example

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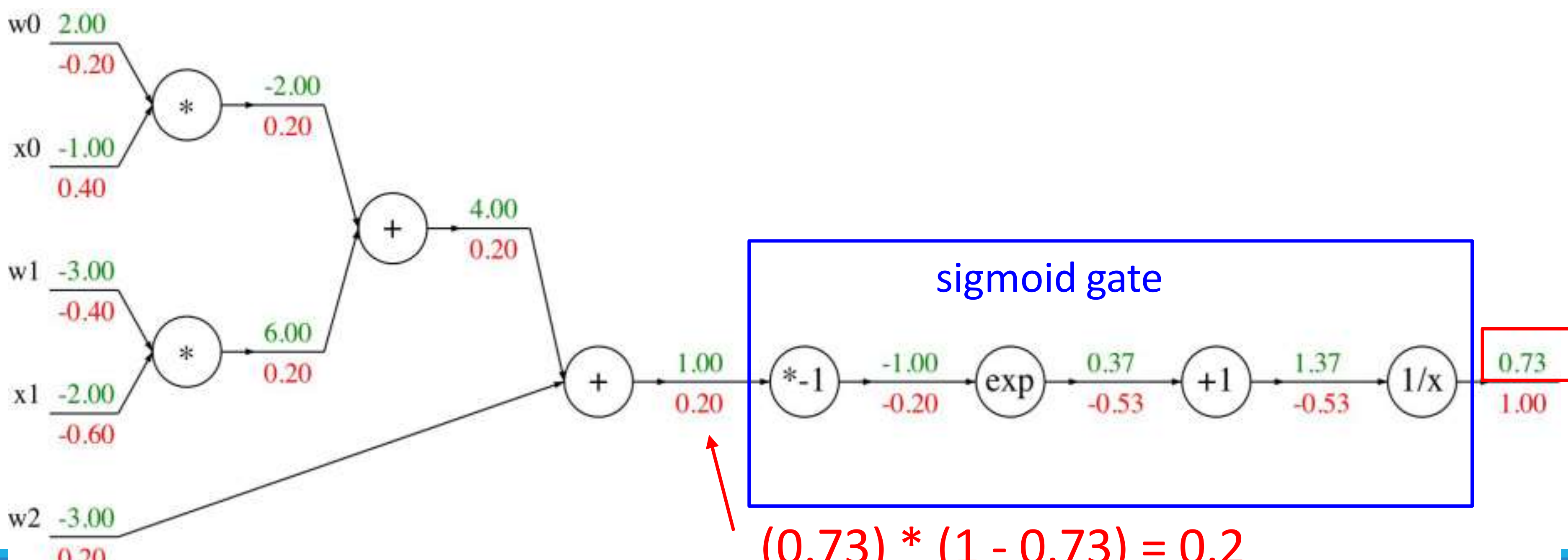

(0.73) * (1 - 0.73) = 0.2

Summary

- **Neuron**
  - Input and output
- **Neural Networks**
  - Perceptron
  - Multi-layer network
  - Deep neural networks
- **How Neural Network Works**
  - Calculation process
  - Work as a multi-class classifier
- **Backpropagation**

Thinking

- Are more layers in a neural network better?
- Why shouldn't we use linear function as activation function of neural network?