Image Processing and Computer Graphics

Rasterization

Matthias Teschner

Computer Science Department
University of Freiburg

Albert-Ludwigs-Universität Freiburg

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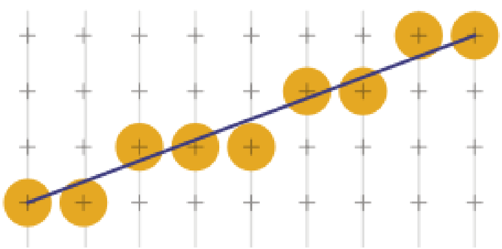
Motivation

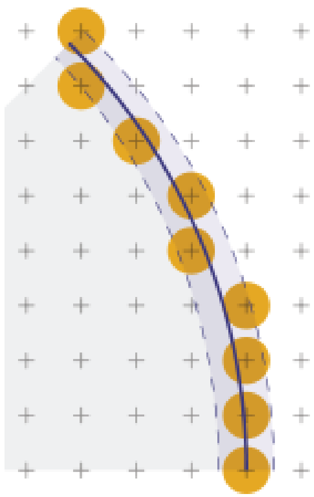
- rasterization is
  - the transformation of geometric primitives (line segments, circles, polygons) into a raster image representation, i.e. pixel positions
  - the estimation of an appropriate set of pixel positions to represent a geometric primitive
- the rendering pipeline
  - processes vertices (transformations and lighting)
  - assembles primitives from vertices in window space and topology information
  - rasterizes primitives, i.e. converts primitives to fragments with interpolated attributes
  - processes fragments, updates the framebuffer

Motivation

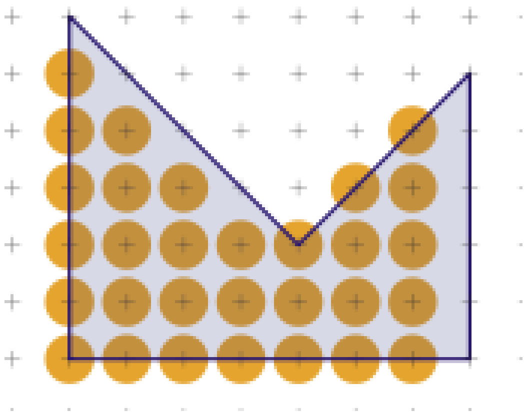
- computation of pixel positions that represent a primitive



Line (segment) rasterization



Circle rasterization



Polygon rasterization

[Wikipedia: Rasterung von Linien, Rasterung von Polygonen, Rasterung von Kreisen]

Outline

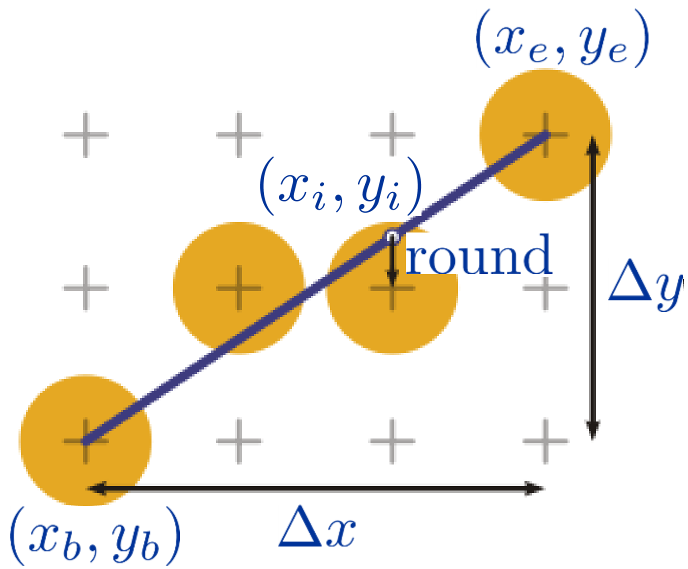
- lines
- circles
- polygons

General Setting

- components of start and end point of a line are integer values $\mathbf{p}_b = (x_b, y_b)$ $\mathbf{p}_e = (x_e, y_e)$
- lines are represented as $y = mx + b$ or $F(x, y) = ax + by + c = 0$
- algorithms are often restricted to $0 \leq m \leq 1$
  - arbitrary lines are handled by employing symmetries
- algorithms consist of an initialization step and a loop
  - efficiency of a particular algorithm depends on the line length

A Simple Algorithm

- $y = \frac{\Delta y}{\Delta x}(x - x_b) + y_b = \frac{y_e - y_b}{x_e - x_b}(x - x_b) + y_b$



- for each $x_b \le x_i \le x_e$
  compute $y_i = \mathrm{round}(y(x_i))$
  set $\mathbf{p}_i = (x_i, y_i)$

- efficient incremental update

$$y(x_{i+1}) - y(x_i) = m(x_{i+1} - x_b) + y_b - (m(x_i - x_b) + y_b) = m(x_{i+1} - x_i) = m$$

$$y(x_{i+1}) = y(x_i) + m$$

[Wikipedia: Rasterung von Linien]

Generalization

- $-1 \leq m \leq 1$
  - $x_b < x_e$: increment x_i , compute $(x_i, \text{round}(y(x_i)))$
  - $x_e < x_b$: decrement x_i , compute $(x_i, \text{round}(y(x_i)))$
- $m > 1$ or $m < -1$
  - $y_b < y_e$: increment y_i , compute $(\text{round}(x(y_i)), y_i)$
  - $y_e < y_b$: decrement y_i , compute $(\text{round}(x(y_i)), y_i)$

Bresenham Algorithm (Midpoint Algorithm)

- explicit form of a line
  $y = \frac{y_e - y_b}{x_e - x_b}(x - x_b) + y_b$
- implicit form of a line
  $0 = \frac{\Delta y}{\Delta x} x - y + y_b - \frac{\Delta y}{\Delta x} x_b = \Delta y \cdot x - \Delta x \cdot y + \Delta x \cdot y_b - \Delta y \cdot x_b$
- implicit form of a line
  - for all points (x, y) on a line
    $F(x, y) = \Delta y \cdot x - \Delta x \cdot y + \Delta x \cdot y_b - \Delta y \cdot x_b = 0$
  - all points with $F(x, y) > 0$ are on one side of the line
  - all points with $F(x, y) < 0$ are on the other side

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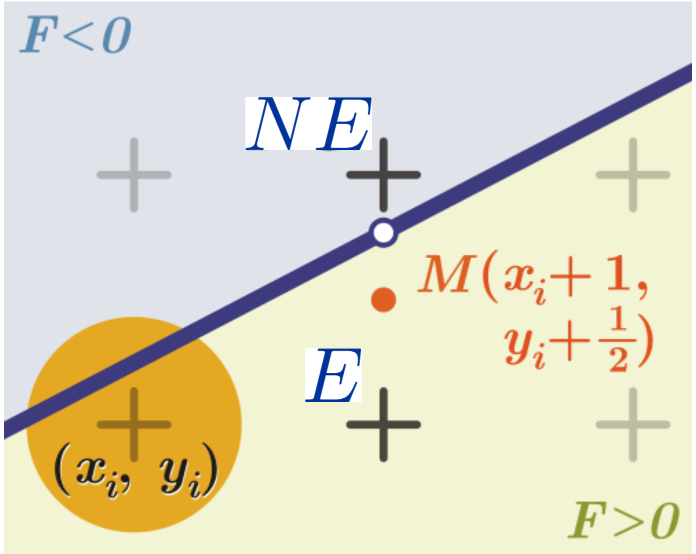
Bresenham Algorithm

- for incremented values of x, the algorithm decides whether to increment y or not
- based on the current pixel (x_i, y_i), the algorithm decides whether to choose (x_i+1, y_i) or (x_i+1, y_i+1) (E east, NE north east)
- F is evaluated at the next midpoint $F(x_i+1, y_i+\frac{1}{2})$
- $F(x_i+1, y_i+\frac{1}{2}) > 0 \Rightarrow$ choose NE
- $F(x_i+1, y_i+\frac{1}{2}) \leq 0 \Rightarrow$ choose E



[Wikipedia: Rasterung von Linien]

Incremental Update of the Decision Variable

- decision variable $d_i = F(x_i + 1, y_i + \frac{1}{2})$
- incremental update from d_i to d_{i+1} depending on d_i
- $d_i > 0 \Rightarrow$ choose NE, $d_{i+1} = F(x_i + 2, y_i + 1 + \frac{1}{2})$
- $d_i \leq 0 \Rightarrow$ choose E, $d_{i+1} = F(x_i + 2, y_i + \frac{1}{2})$
- in case of $d_i > 0$:

$$\Delta_{NE} = d_{i+1} - d_i = \Delta y \cdot (x_i + 2) - \Delta x \cdot (y_i + \tfrac{3}{2}) + c - (\Delta y \cdot (x_i + 1) - \Delta x \cdot (y_i + \tfrac{1}{2}) + c)$$

$$\Delta_{NE} = \Delta y - \Delta x$$

- in case of $d_i \leq 0$:

$$\Delta_E = d_{i+1} - d_i = \Delta y \cdot (x_i + 2) - \Delta x \cdot (y_i + \tfrac{1}{2}) + c - (\Delta y \cdot (x_i + 1) - \Delta x \cdot (y_i + \tfrac{1}{2}) + c)$$

$$\Delta_E = \Delta y$$

Bresenham Algorithm
Initialization

- for start point $\mathbf{p}_b = (x_b, y_b)$,
  decision variable d_1 can be initialized as

$$
\begin{aligned}
d_1 &= F(x_b + 1, y_b + \tfrac{1}{2}) = \Delta y \cdot (x_b + 1) - \Delta x \cdot (y_b + \tfrac{1}{2}) + c \\
&= \Delta y \cdot x_b - \Delta x \cdot y_b + c + \Delta y - \tfrac{1}{2}\Delta x \\
&= F(x_b, y_b) + \Delta y - \tfrac{1}{2}\Delta x \\
&= \Delta y - \tfrac{1}{2}\Delta x
\end{aligned}
$$

- floating-point arithmetic is avoided by
  considering 2·F(x,y): $d_1 = 2\Delta y - \Delta x$

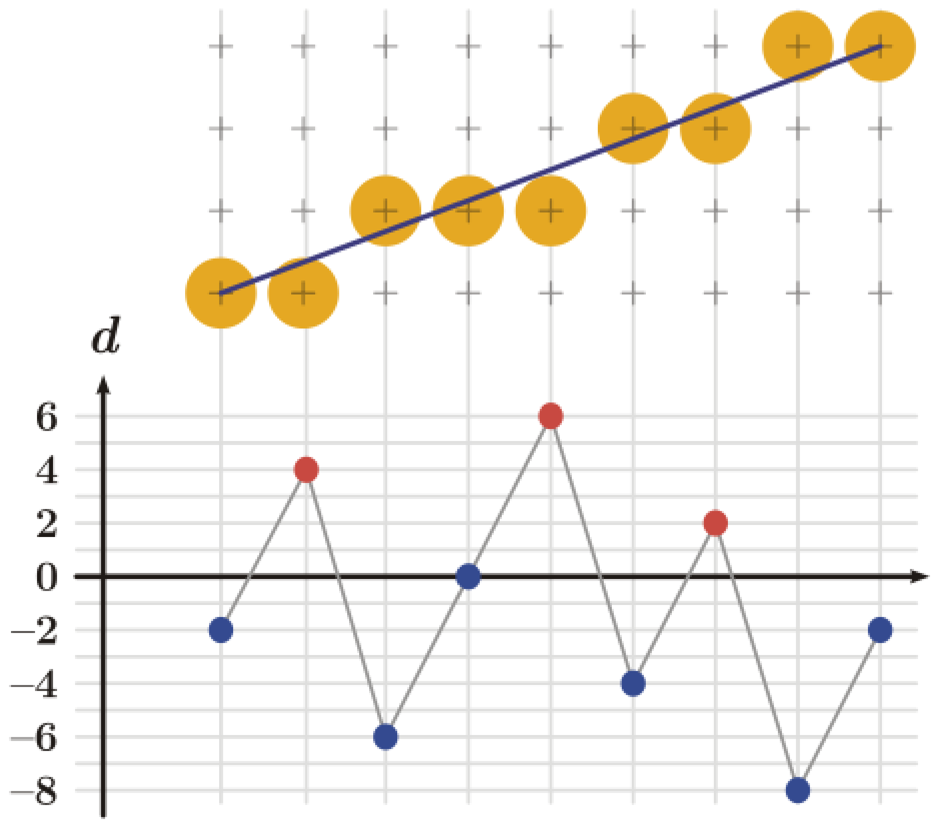
$$\Delta_E = 2\Delta y$$

$$\Delta_{NE} = 2\Delta y - 2\Delta x$$

Bresenham Algorithm Implementation

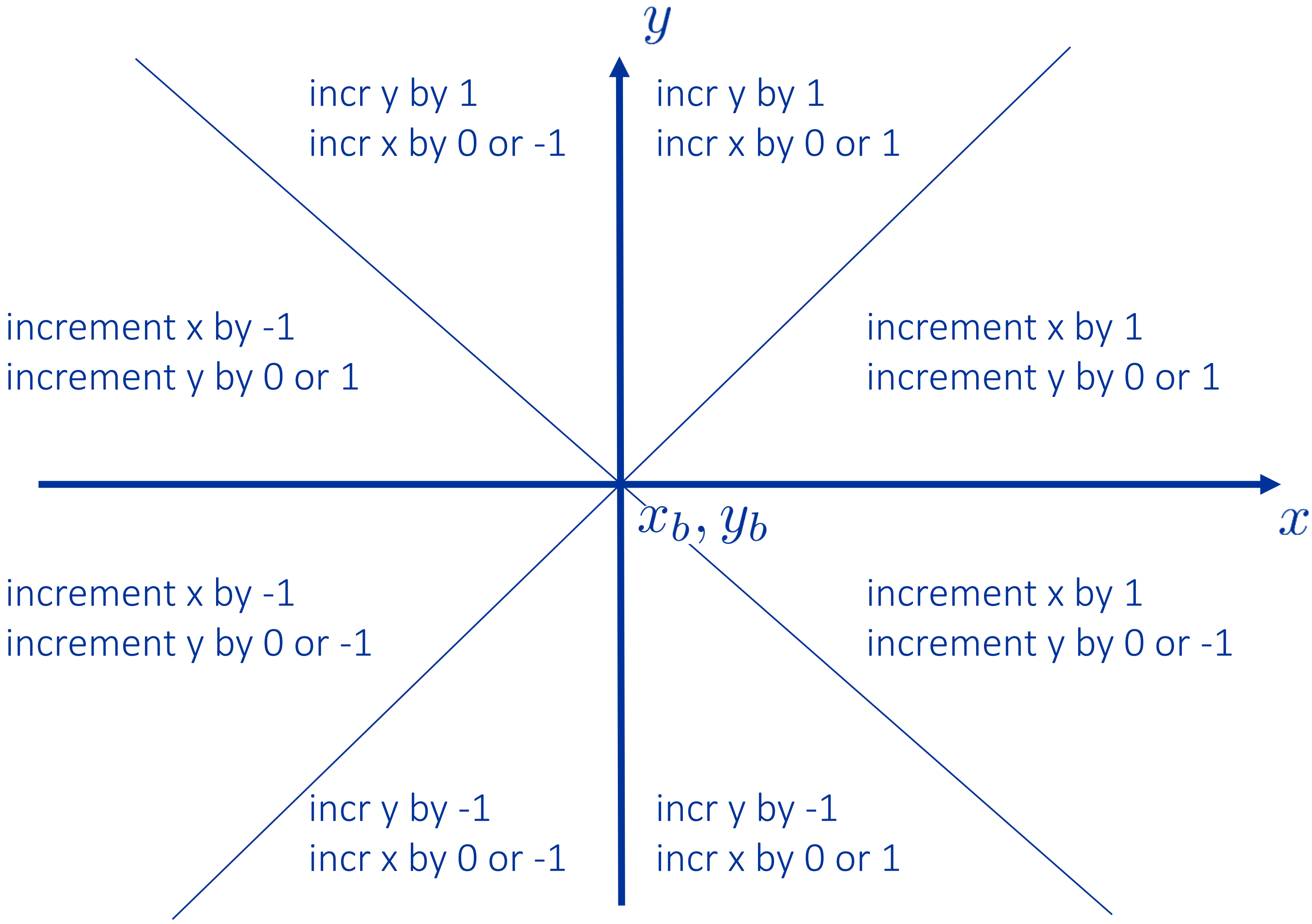
```
void BresenhamLine(int xb, int yb, int xe, int ye) {

    int dx, dy, incE, incNE, d, x, y;

    dx = xe - xb; dy = ye - yb;
    d = 2*dy - dx;
    incE = 2*dy;
    incNE = 2*(dy - dx);
    x = xb; y = yb;
    WritePixel(x, y);      /* write start pixel */
    while (x < xe) {
        x++;
        if (d <= 0)     /* choose E */
            d += incE;
        else {
            d += incNE; /* choose NE */
            y++;
        }
        WritePixel(x, y);
    }
}
```

Bresenham Algorithm
Decision Variable



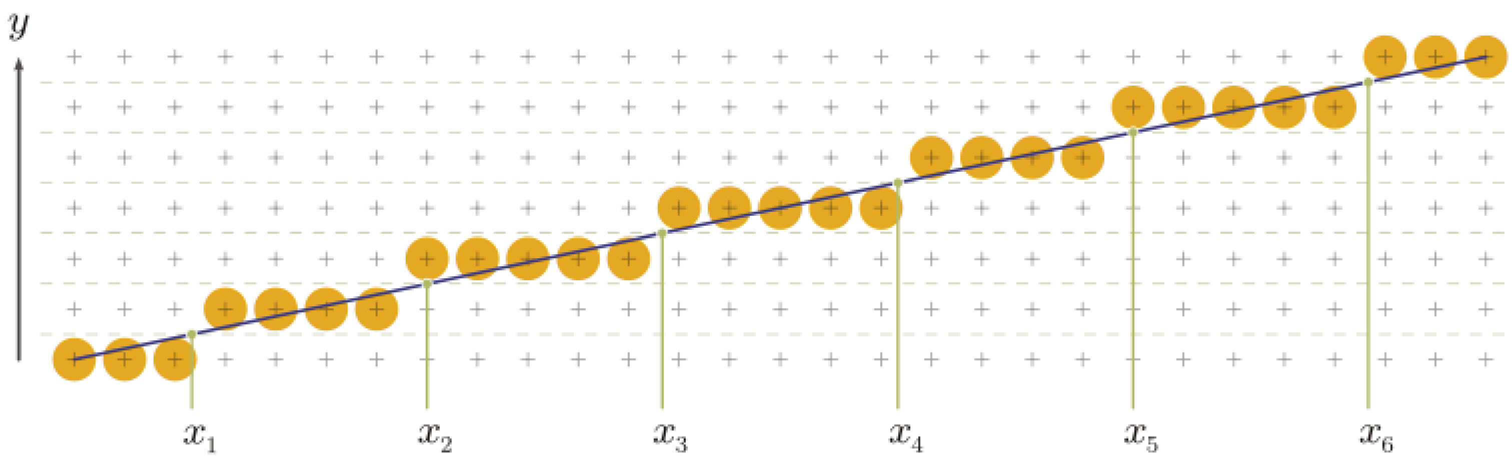
[Wikipedia: Rasterung von Linien]

Generalization



Run Length Slices

- estimate x-values where the y-value is incremented



- x_i is the (floating-point) intersection of the line with the line defined by (x_b, y_b+i+0.5) and (x_e, y_b+i+0.5)
- increment y, compute x_i,
  draw pixels with the same y-value up to $\lfloor x_i \rfloor$

[Wikipedia: Rasterung von Linien]

Run Length Slices

- line: $y = \frac{\Delta y}{\Delta x}(x - x_b) + y_b$

  $x = \frac{\Delta x}{\Delta y}(y - y_b) + x_b$

- x-components of the intersection at $y = y_b + i + \frac{1}{2}$:

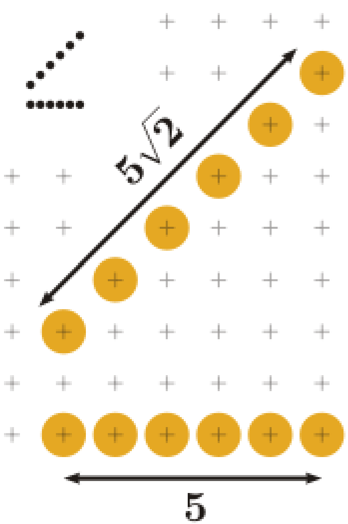
  $x_i = \frac{\Delta x}{\Delta y}(y_b + i + \frac{1}{2} - y_b) + x_b$

- differential update using $x_{i+1} - x_i = \frac{\Delta x}{\Delta y}$

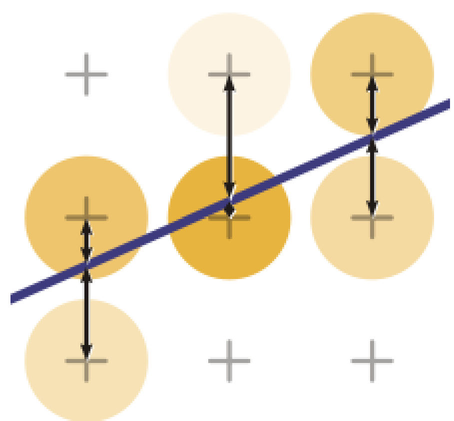
- initialization: $x_1 = \frac{3\Delta x}{2\Delta y} + x_b$
- loop: $x_{i+1} = x_i + \frac{\Delta x}{\Delta y}$

Issues / Limitations

- aliasing
  - stair-case artifacts, varying line intensity
- clipping
  - artifacts due to round-off of clipped end points



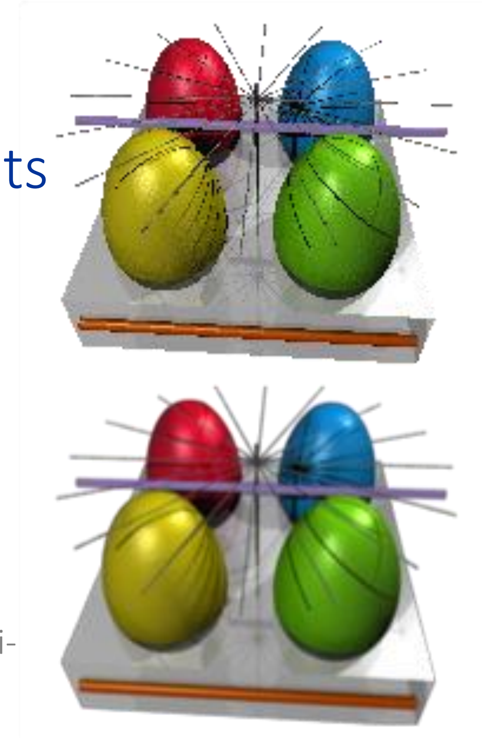
same number of pixels for lines with different length



aliasing can be addressed by rendering thick lines with varying pixel intensities

no anti-aliasing

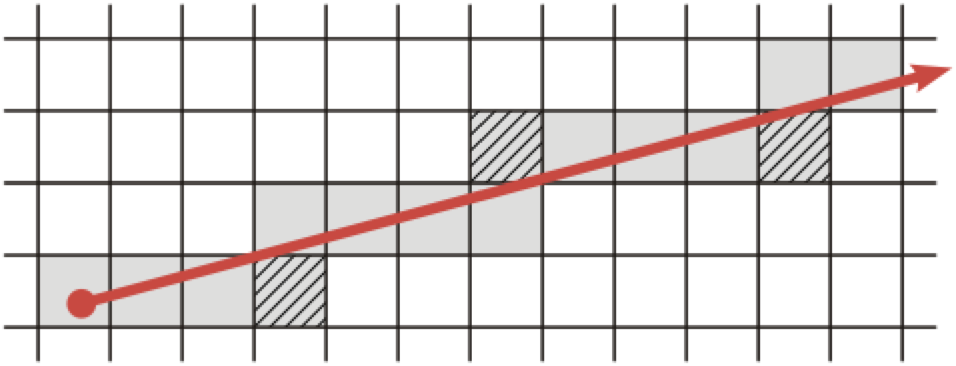
with anti-aliasing



[Wikipedia: Antialiasing, Rasterung von Linien]

Summary - Lines

- line rasterization algorithms are usually described for a subset of lines and generalized using symmetries
- incremental updates are often employed
- Bresenham avoids floating-point arithmetic
- improved algorithms address aliasing / clipping artifacts
- note that the algorithms do not compute all pixels that are intersected by the line



[Wikipedia: Rasterung von Linien]

Outline

- lines
- circles
- polygons

General Setting

- circle with center at (0,0) and radius r
- implicit representation
  $F(x, y) = x^2 + y^2 - r^2 = 0$
- algorithms compute only one eighth of a circle
  - if (x, y) is on the circle, then (±x, ±y) and (± y, ± x) are on the circle

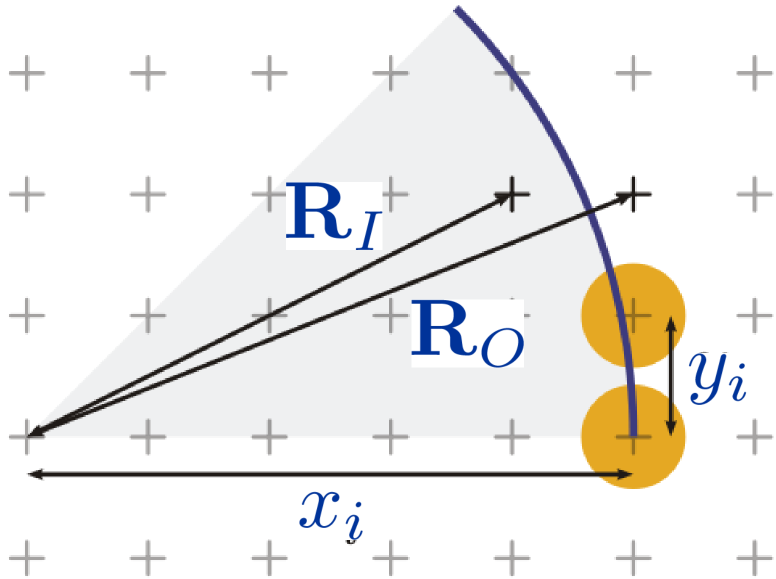
Metzger Algorithm

- if (x_i, y_i) is on the circle, the algorithm decides whether $\mathbf{R}_O = (x_i, y_i + 1)$ or $\mathbf{R}_I = (x_i - 1, y_i + 1)$ is the next point on the circle
- the point with the shortest distance to the circle is chosen

$$d_I = r - \|\mathbf{R}_I\| = r - \sqrt{(x_i - 1)^2 + (y_i + 1)^2}$$

$$d_O = \|\mathbf{R}_O\| - r = \sqrt{x_i^2 + (y_i + 1)^2} - r$$

- if $d_I \leq d_O \Rightarrow \mathbf{R}_I$
- if $d_I > d_O \Rightarrow \mathbf{R}_O$



[Wikipedia: Rasterung von Kreisen]

Horn Algorithm

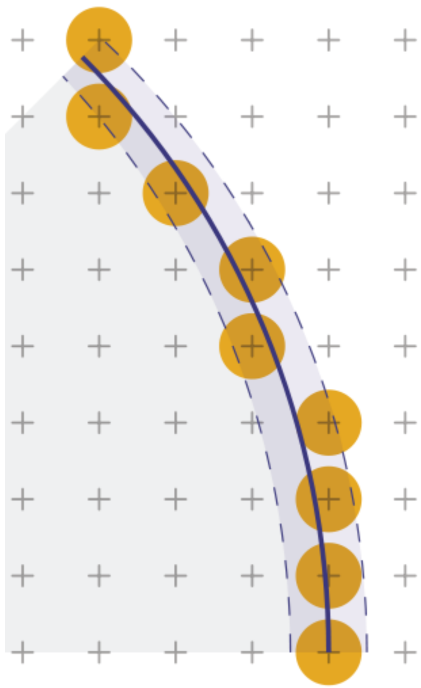
- the algorithm checks whether $(x_i - \frac{1}{2}, y_i + 1)$ is outside
  - if so, it chooses $(x_i - 1, y_i + 1)$
  - if not, it chooses $(x_i, y_i + 1)$
- decision variable

  $d_i = (x_i - \frac{1}{2})^2 + y_i^2 - r^2$
- incremental update
- if $d_i < 0 \Rightarrow (x_{i+1}, y_{i+1}) = (x_i, y_i + 1)$

  $d_{i+1} = (x_i - \frac{1}{2})^2 + (y_i + 1)^2 - r^2$

  $d_{i+1} = d_i + 2y_i + 1$
- if $d_i \geq 0 \Rightarrow (x_{i+1}, y_{i+1}) = (x_i - 1, y_i + 1)$

  $d_{i+1} = d_i + 2y_i + 1 - 2x_i + 2$



[Wikipedia: Rasterung von Kreisen]

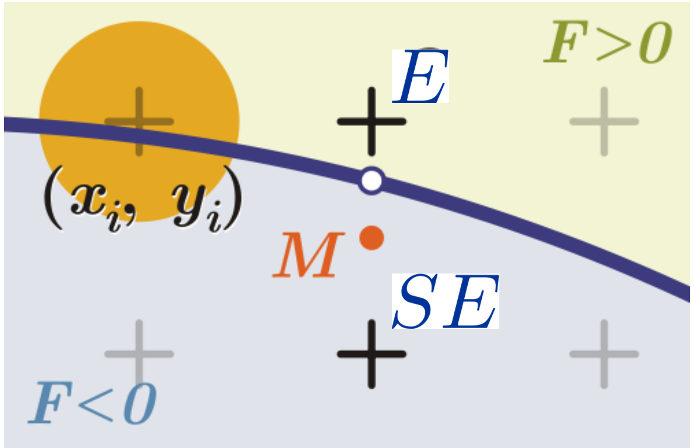
Horn Algorithm Implementation

```
void HornCircle(int r) {

    int d, x, y;

    d = -r;
    x = r;
    y = 0;

    while (y < x) {
        WritePixel(x, y);  /* and symmetric pixels */
        d += 2*y + 1;
        y += 1;
        if (d >= 0) {
            d += -2*x + 2;
            x += -1;
        }
    }
}
```

Bresenham Algorithm (Midpoint Algorithm)

- $F(x,y) = x^2 + y^2 - r^2 = 0 \Rightarrow$ (x,y) is on the circle
- based on the current pixel (x_i, y_i), the algorithm decides whether to choose $(x_i + 1, y_i)$ or $(x_i + 1, y_i - 1)$ (E east, SE southeast)
- F is evaluated at the next midpoint

  $F(x_i + 1, y_i - \frac{1}{2})$
- $F(x_i + 1, y_i - \frac{1}{2}) \geq 0 \Rightarrow$ choose SE
- $F(x_i + 1, y_i - \frac{1}{2}) < 0 \Rightarrow$ choose E



[Wikipedia: Rasterung von Kreisen]

Incremental Update of the Decision Variable

- decision variable $d_i = F(x_i + 1, y_i - \frac{1}{2})$
- incremental update from d_i to d_{i+1} depending on d_i
- $d_i \geq 0 \Rightarrow$ choose SE, $d_{i+1} = F(x_i + 2, y_i - 1 - \frac{1}{2})$
- $d_i < 0 \Rightarrow$ choose E, $d_{i+1} = F(x_i + 2, y_i - \frac{1}{2})$

- in case of $d_i \geq 0$:
  $\Delta_{SE} = 2x_i - 2y_i + 5$
- in case of $d_i < 0$:
  $\Delta_E = 2x_i + 3$

Incremental Update of the Increments

- four patterns of a set of three adjacent points
  - (1) $(x_i, y_i), (x_i + 1, y_i), (x_i + 2, y_i)$
  - (2) $(x_i, y_i), (x_i + 1, y_i), (x_i + 2, y_i - 1)$
  - (3) $(x_i, y_i), (x_i + 1, y_i - 1), (x_i + 2, y_i - 1)$
  - (4) $(x_i, y_i), (x_i + 1, y_i - 1), (x_i + 2, y_i - 2)$
- increments $\Delta_{E,i} = 2x_i + 3 \quad \Delta_{SE,i} = 2x_i - 2y_i + 5$
  - if the algorithm moves towards E
  - (1) $\Delta_{E,i+1} = 2(x_i + 1) + 3 \Rightarrow \Delta_{E,i+1} = \Delta_{E,i} + 2$
  - (2) $\Delta_{SE,i+1} = 2(x_i + 1) - 2y_i + 5 \Rightarrow \Delta_{SE,i+1} = \Delta_{SE,i} + 2$
  - if the algorithms moves towards SE
  - (3) $\Delta_{E,i+1} = 2(x_i + 1) + 3 \Rightarrow \Delta_{E,i+1} = \Delta_{E,i} + 2$
  - (4) $\Delta_{SE,i+1} = 2(x_i + 1) - 2(y_i - 1) + 5 \Rightarrow \Delta_{SE,i+1} = \Delta_{SE,i} + 4$

Incremental Update of Increments

- point (x_i, y_i) is on the circle
- if next point is E,
  - $d_i = d_i + \Delta_{E,i}$
  - $\Delta_{E,i} = \Delta_{E,i} + 2$
  - $\Delta_{SE,i} = \Delta_{SE,i} + 2$
- if next point is SE,
  - $d_i = d_i + \Delta_{SE,i}$
  - $\Delta_{E,i} = \Delta_{E,i} + 2$
  - $\Delta_{SE,i} = \Delta_{SE,i} + 4$

Bresenham Algorithm
Initialization

- at point (0, r)
  $d_1 = F(0+1, r-\frac{1}{2}) = 1 + (r-\frac{1}{2})^2 - r^2 = \frac{5}{4} - r$
  $\Delta_{SE} = -2r + 5$
  $\Delta_E = 3$

- as d is incremented only by integer values,
  $d_1 = 1 - r$

Bresenham Algorithm Implementation

```
void BresenhamCircle (int r) {
    int x, y, d, deltaE, deltaSE;

    x = 0; y = r; d = 1 - r; deltaE = 3; deltaSE = -2*r + 5;

    WritePixel(x, y);                  /* and symmetric points */
    while (y > x) {
        if (d < 0) {               /* choose E */
            d += deltaE;
            deltaE += 2;
            deltaSE += 2;
        }
        else {                 /* choose SE */
            d += deltaSE;
            deltaE += 2;
            deltaSE += 4;
            y--;
        }
        x++;
        WritePixel(x, y);              /* and symmetric points */
}}
```

Summary - Circles

- circle rasterization algorithms are usually described for one eighth of a circle and generalized using symmetries
- incremental updates are often employed
- floating-point arithmetic is avoided

Outline

- lines
- circles
- polygons

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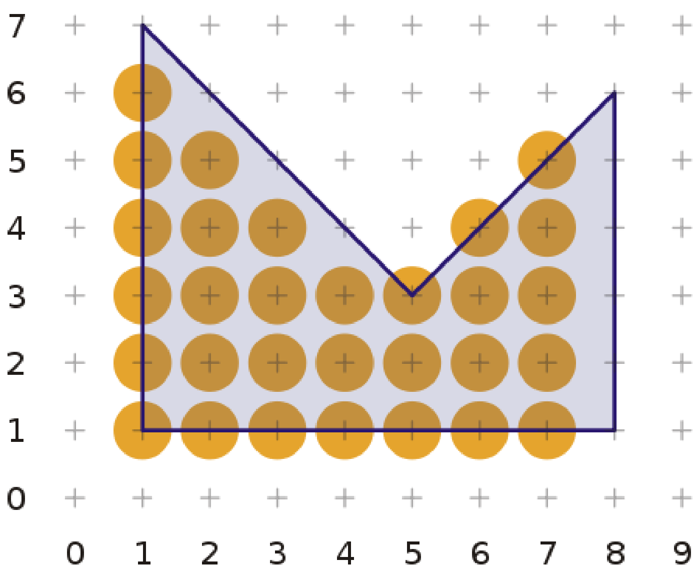
General Setting

- a polygon is defined by edges
- the polygon should be closed to allow inside / outside classification
- rasterization estimates all pixel positions inside a polygon
- in general simple, but
  - if adjacent polygons share an edge, each pixel on the edge should belong to exactly one polygon
  - no pixel along the edge should be missed
  - no pixel along the edge should be rasterized twice

Edge List Algorithms

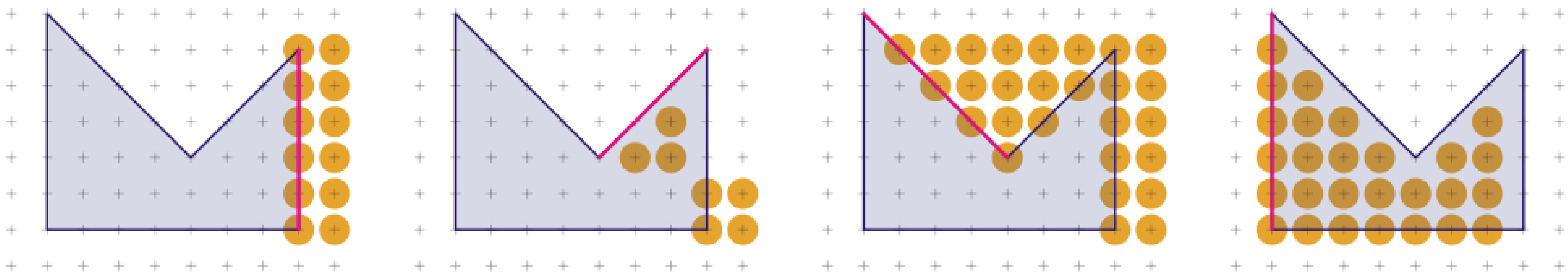
- compute intersections of non-horizontal polygon edges with lines (scanlines)
- intersections are computed for $y = y_i + 0.5$
- fill pixel positions in-between two intersection points
  - scan from left to right
  - enter the polygon at the first intersection, leave the polygon at the next intersection



[Wikipedia: Rasterung von Polygonen]

Edge Fill Algorithms

- for each polygon edge
  - process all scanlines intersected by the edge
  - invert all pixels with an x-component larger than the intersection point



[Wikipedia: Rasterung von Polygonen]

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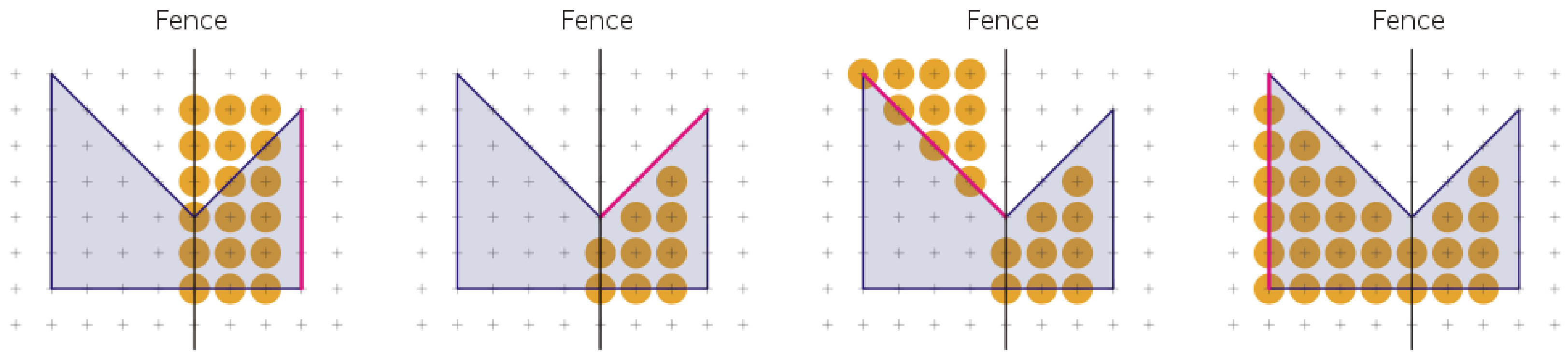
Fence Fill Algorithm

- for each polygon edge
  - process all scanlines intersected by the edge
  - if $x_{intersection} \geq x_{fence}$ invert all pixels with $x_{fence} \leq x_{pixel} < x_{intersection}$
  - if $x_{intersection} < x_{fence}$ invert all pixels with $x_{intersection} \leq x_{pixel} < x_{fence}$



[Wikipedia: Rasterung von Polygonen]

Summary - Polygons

- polygon rasterization algorithms work for closed polygons
  - inside / outside classification
- rasterization estimates all pixel positions inside a polygon
- processing of edges has to consider that pixels on shared edges should be rasterized exactly once

Summary

- vertices in window space and topology information are used to assemble primitives
- rasterization converts primitives to fragments with interpolated attributes rasterization of lines
  - rasterization of lines
  - rasterization of circles
  - rasterization of polygons
- rasterized pixel positions with interpolated attributes are further processed in the rendering pipeline

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