Image Processing and Computer Graphics

Image Processing

Class 6

Motion estimation

Motion estimation

- Given a sequence of images $I(x, y, t)$, what is the motion of each pixel between subsequent frames?

Hamburg taxi sequence

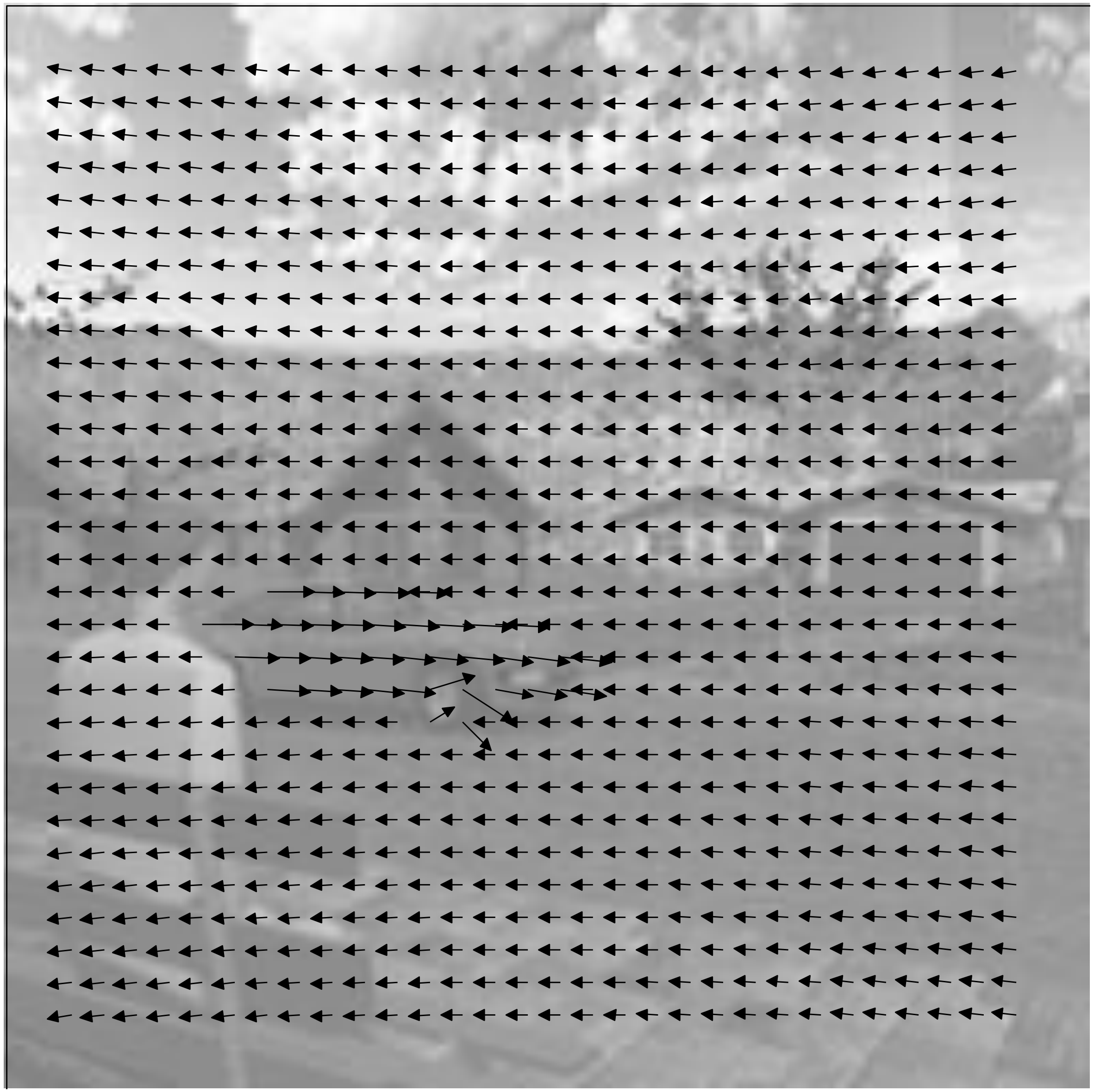
Sequence from VSB100 dataset

- Formally, we seek a vector field $(u, v)(x, y, t)$ that transforms the second image into the first one.

- This vector field is called **optical flow**. It individually moves each point (x, y) at time t such that it fits the point at time $t + 1$.

Motion estimation

Street sequence

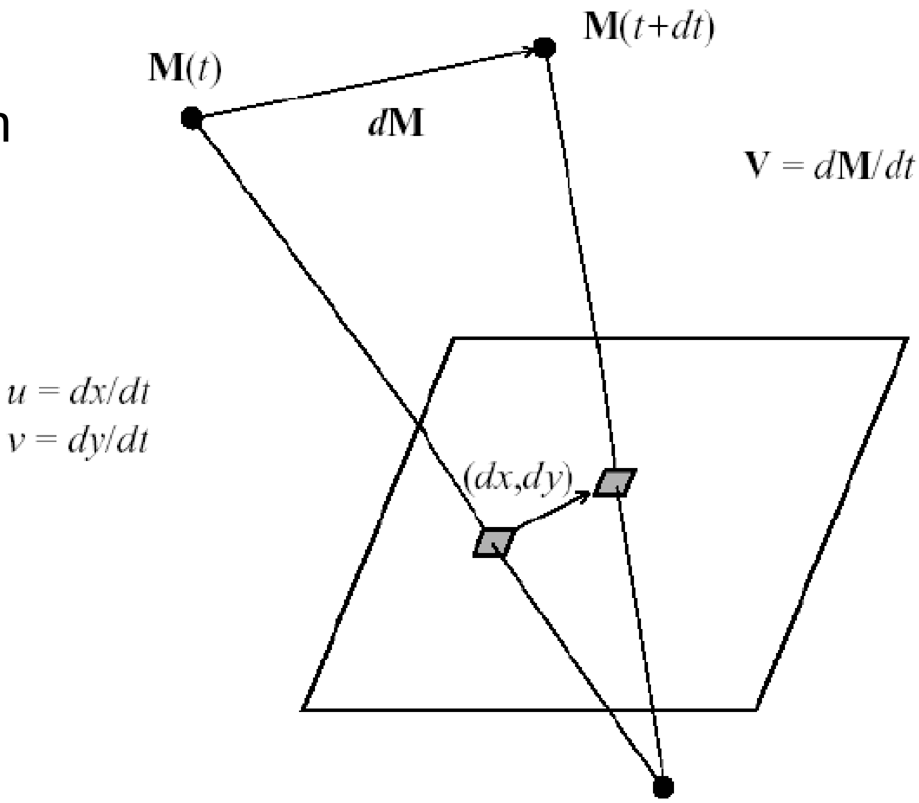


Importance of motion for vision

- Optical flow provides important information for many other tasks in vision.
- It can be used for detection or segmentation of objects by comparing the object motion and the background motion → **motion segmentation**
- It can be used for estimating the 3D motion of objects, the so-called **scene flow**. It is important, e.g., for catching a fly or for avoiding collisions.
- Optical flow can also help estimate the 3D structure of objects. This is called **structure from motion**.
  - Moving your head, the image of near objects moves faster than the image of distant objects (motion parallax).
  - For many animals, motion is the only way to see 3D.

3D motion, 2D motion, optical flow

- There is a difference between 3D motion (in the scene) and 2D motion (in the image).
- Moreover, there can be a difference between the true motion in the image and the apparent motion.
- How comes that there is a difference between the true and the apparent motion?

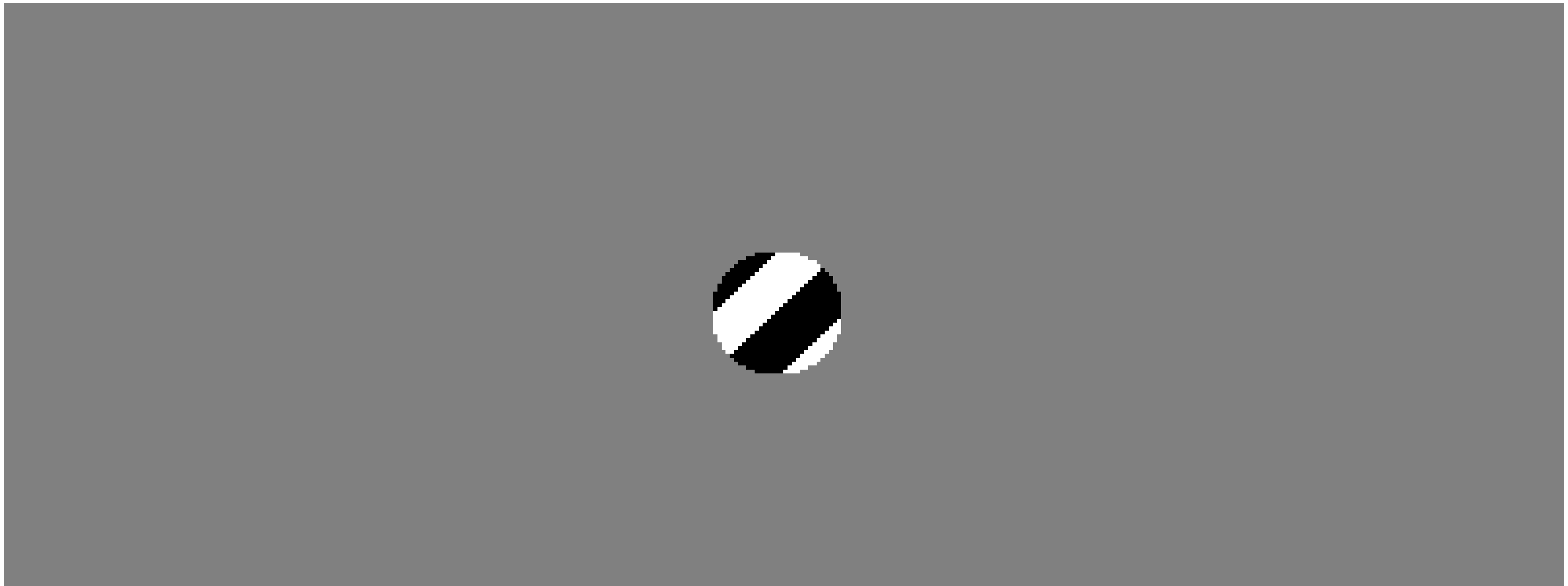


Author: Martial Hebert

Ambiguities – the aperture problem

- The optical flow is usually not uniquely determined from the image data alone. A black pixel potentially could have moved to any black pixel in the other frame.

- Prior assumptions (e.g. that neighboring pixels move the same way) are needed for a unique solution. The neighborhood (aperture) determines the solution. Therefore, the ambiguity is also called the **aperture problem**.



Source: Wikipedia

Optic flow constraint

- The most common assumption for optical flow estimation is that the gray value of a moving pixel stays constant over time.

$$I(x+u, y+v, t+1) - I(x, y, t) = 0$$

- This constraint is nonlinear in the unknown flow, which makes estimation difficult.

- To ease optical flow estimation, we can linearize the first expression with a Taylor expansion:

x-derivative of the image

$$I(x+u, y+v, t+1) = I(x, y, t) + I_x u + I_y v + I_t + O(u^2, v^2)$$

- This leads to the (linearized) **optic flow constraint**:

$$I_x u + I_y v + I_t = 0$$

- This linearization is only sufficiently precise, if the images are smooth (locally linear) and the flow is small.

Lucas-Kanade method

- Optic flow constraint: $I_x u + I_y v + I_t = 0$

- We see the aperture problem: regarding a single pixel, we cannot uniquely determine its flow vector. There is only one equation, but two unknowns.

- We need a second assumption to obtain a unique solution.

- The **Lucas-Kanade method** assumes that the motion is constant within a local neighborhood.

- We then obtain multiple equations for the two unknowns, i.e., an over-determined system of equations:

$$I_x(x', y', t)u + I_y(x', y', t)v + I_t(x', y', t) = 0 \quad \forall x', y' \in \mathcal{R}(x, y)$$

Lucas-Kanade method

- In general, such an over-determined system has not a solution anymore.

- Instead we seek a solution that minimizes the total squared error:

$$\operatorname*{argmin}_{u,v} \sum_{x,y\in\mathcal{R}} (I_x(x,y)u + I_y(x,y)v + I_t(x,y))^2$$

- The necessary condition for a minimum is:

$$\frac{\partial E}{\partial u} = 2 \sum_{x,y\in\mathcal{R}} (I_x(x,y)u + I_y(x,y)v + I_t(x,y))I_x(x,y) = 0$$

$$\frac{\partial E}{\partial v} = 2 \sum_{x,y\in\mathcal{R}} (I_x(x,y)u + I_y(x,y)v + I_t(x,y))I_y(x,y) = 0$$

- This leads to a linear system at each pixel

$$\begin{pmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -\sum I_x I_t \\ -\sum I_y I_t \end{pmatrix}$$

Lucas-Kanade method

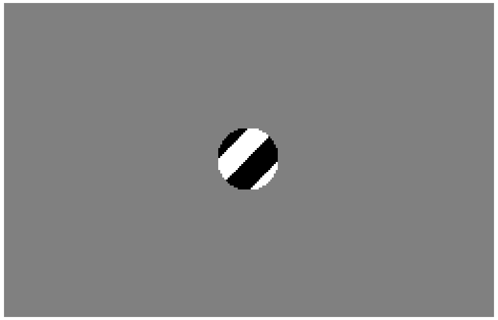
- Instead of a uniformly weighted, box-shaped window, we can also use a Gaussian window.

- The sums weighted by the window function come down to a convolution:

$$\begin{pmatrix} K_\rho * I_x^2 & K_\rho * I_x I_y \\ K_\rho * I_x I_y & K_\rho * I_y^2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -K_\rho * I_x I_t \\ -K_\rho * I_y I_t \end{pmatrix}$$

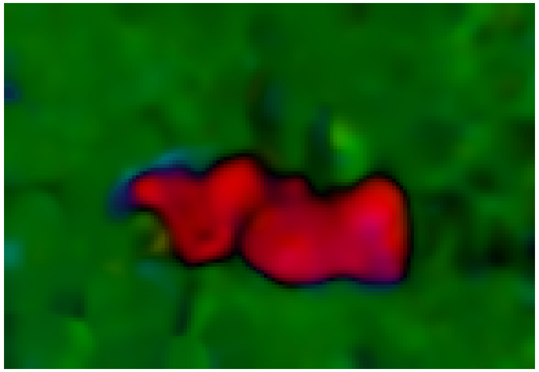
- A unique solution will be obtained only if the system is not singular.

- What does this mean?
  - The local neighborhood must contain non-parallel gradients.
  - Otherwise the aperture problem is still present.

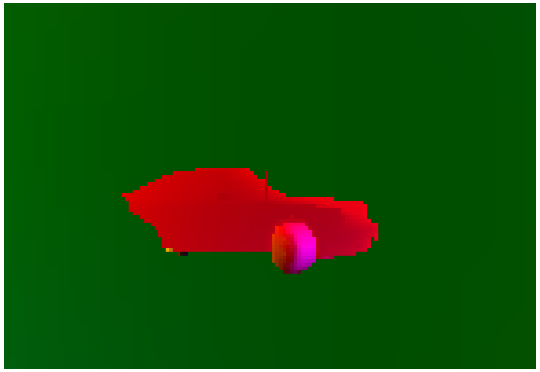


Lucas-Kanade method: example

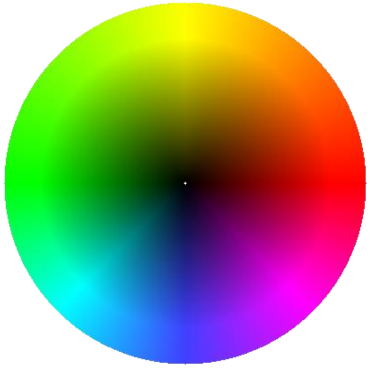
Input sequence



Optical flow estimated with Lucas-Kanade method



Correct flow field



- Drawbacks of the Lucas-Kanade method:
  - The assumption of locally constant motion often is not realistic (e.g. at motion boundaries).
  - There are no direct constraints on the smoothness of the resulting flow field.

- Advantages: fast and simple

From local to global optic flow estimation

- The Lucas-Kanade method is a “parametric” or “local” method
  - It assumes a parametric motion model.
  - The parameters are estimated from data in a local neighborhood of each point.
  - ➔ Point estimates are computed independently.

- In contrast, global methods have a global dependency between points due to a global smoothness assumption.

- A global solution is usually derived with variational methods.

- Most basic variational model: Horn-Schunck method

- This model can be further extended to yield highly accurate estimates in many practical situations.

Horn-Schunck model

- The first assumption is the same as in the Lucas-Kanade method: gray value constancy leading to the optic flow constraint.

$$I_x u + I_y v + I_z = 0$$

- The second assumption is different. The Horn-Schunck model assumes global smoothness of the flow field:

$$|\nabla u|^2 + |\nabla v|^2 \to \min$$

- Both assumptions can be combined in an energy functional:

$$E(u,v) = \int_\Omega (I_x u + I_y v + I_z)^2 + \alpha(|\nabla u|^2 + |\nabla v|^2)\, dx dy$$

- The sought optical flow is the minimizer of this energy.

- Note the similarities to the denoising model from the previous class.

Calculus of variation

- We can find the minimizer of this functional with the calculus of variation.

- The energy is convex → we get a unique global optimum.

- With the Gâteaux derivatives

$$\frac{d}{d\epsilon}E(u(x)+\epsilon h(x), v(x))\Big|_{\epsilon=0} = 0 \qquad \forall h(x)$$

$$\frac{d}{d\epsilon}E(u(x), v(x)+\epsilon h(x))\Big|_{\epsilon=0} = 0 \qquad \forall h(x)$$

we obtain the Euler-Lagrange equations

$$(I_x u + I_y v + I_z)I_x - \alpha \Delta u = 0$$

$$(I_x u + I_y v + I_z)I_y - \alpha \Delta v = 0$$

Laplacian: $\Delta v = v_{xx} + v_{yy}$

Linear system of equations

- The discretized versions of these equations read

$$\frac{u_{i+1,j}+u_{i-1,j}+u_{i,j+1}+u_{i,j-1}-4u_{i,j}}{h^2}-\frac{1}{\alpha}(I_x u_{i,j}+I_y v_{i,j}+I_z)I_x=0$$

$$\frac{v_{i+1,j}+v_{i-1,j}+v_{i,j+1}+v_{i,j-1}-4v_{i,j}}{h^2}-\frac{1}{\alpha}(I_x u_{i,j}+I_y v_{i,j}+I_z)I_y=0$$

- The equations are linear in (u,v). Thus we obtain a large linear system to solve (as in the denoising case).

- This system can be written in matrix-vector notation: $A^h w = b$, where $w := (u,v)$.

- As in the denoising case, the system matrix is sparse with only few diagonals different from zero.

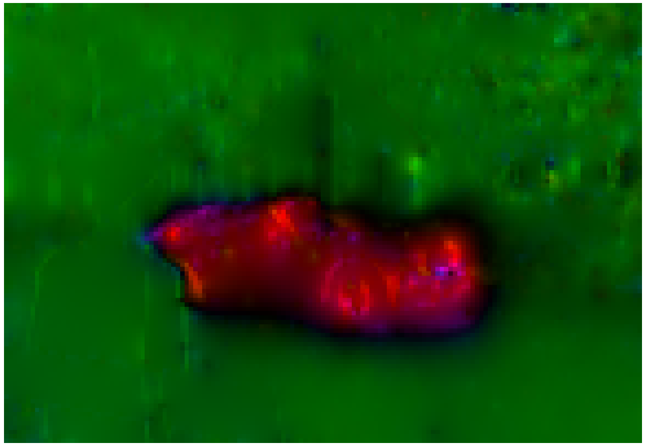
Linear system of equations

$$A^h = \left(\begin{array}{ccccc|ccccc}
d & & & & & d & & & & \\
 & d & & & & & d & & & \\
 & & d & & & & & d & & \\
 & & & d & & & & & d & \\
 & & & & d & & & & & d \\
\hline
d & & & & & d & & & & \\
 & d & & & & & d & & & \\
 & & d & & & & & d & & \\
 & & & d & & & & & d & \\
 & & & & d & & & & & d
\end{array}\right) + \left(\begin{array}{ccccc|ccccc}
s & s & & s & & & & & & \\
s & s & s & & s & & & & & \\
 & s & s & s & & & & & & \\
s & & s & s & s & & & & & \\
 & s & & s & s & & & & & \\
\hline
 & & & & & s & s & & s & \\
 & & & & & s & s & s & & s \\
 & & & & & & s & s & s & \\
 & & & & & s & & s & s & s \\
 & & & & & & s & & s & s
\end{array}\right)$$

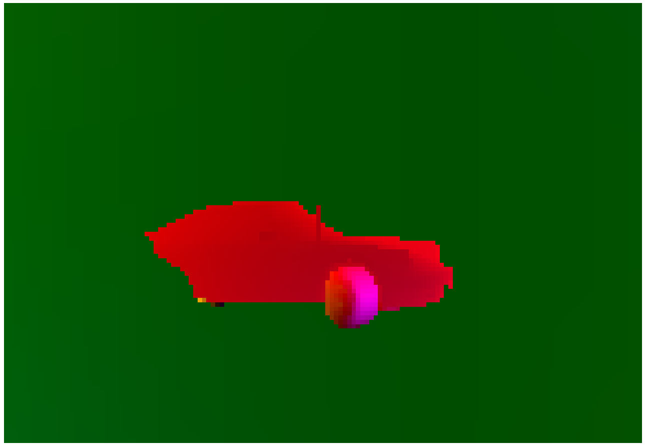
- The blocks structure is due to the coupling of u and v .
- The data term contributes only to block main diagonals.
- The smoothness term has four additional block off-diagonals (coupling of neighboring pixels).
- We can solve this linear system with the linear solvers from class 4 (Gauss-Seidel, SOR, Multigrid).

Results with Horn-Schunck method

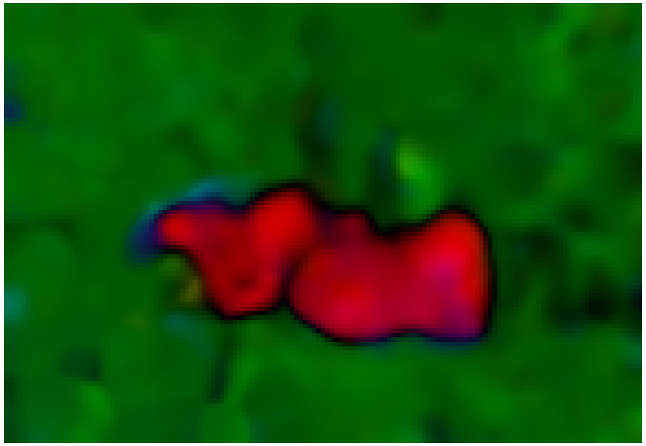
Input sequence



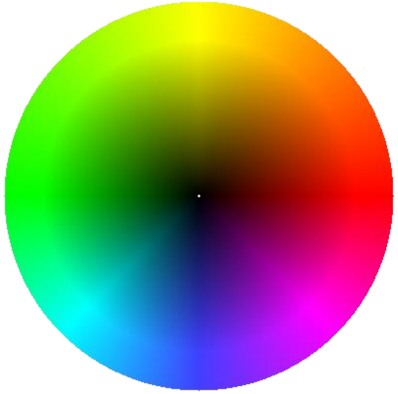
Horn-Schunck (10.4°)



Ground truth



Lucas-Kanade (10.54°)



Average angular error

- An established measure of how accurately the optical flow has been estimated is the **average angular error**.
- This requires, of course, knowledge about the true motion field, the so-called **ground truth**. For some synthetic sequences such ground truth data is available.
- The average angular error measures the 3D angle between the correct and the estimated flow vectors:

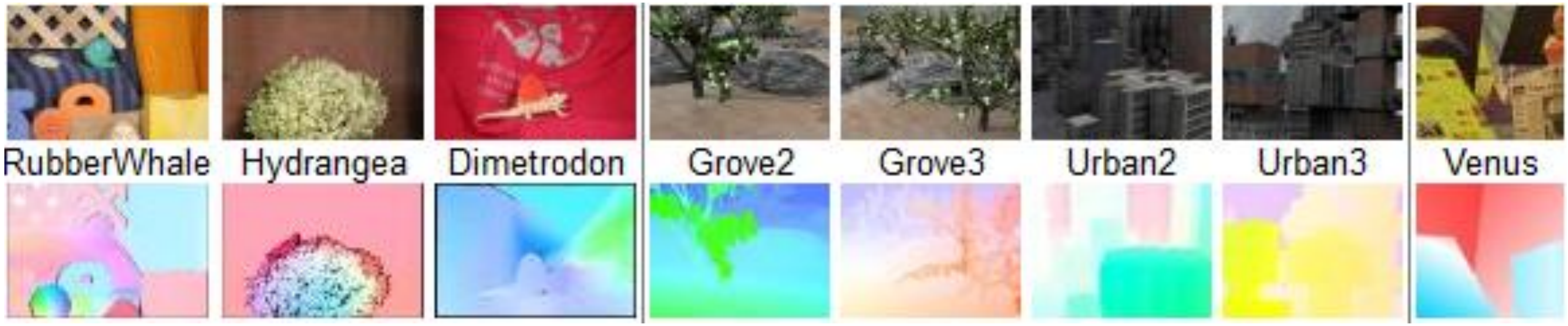
$$\text{AAE} := \frac{1}{n}\sum_{i=1}^{n} \arccos\left(\frac{(u_c)_i u_i + (v_c)_i v_i + 1}{\sqrt{((u_c)_i^2 + (v_c)_i^2 + 1)(u_i^2 + v_i^2 + 1)}}\right)$$

- Note that this also captures errors in the length of the 2D flow vector.
- An alternative measure is the **end point error**:

$$\text{EPE} := \frac{1}{n}\sum_{i=1}^{n} \sqrt{(u_i - (u_c)_i)^2 + (v_i - (v_c)_i)^2}$$

Middlebury flow benchmark

- Available at http://vision.middlebury.edu/flow/



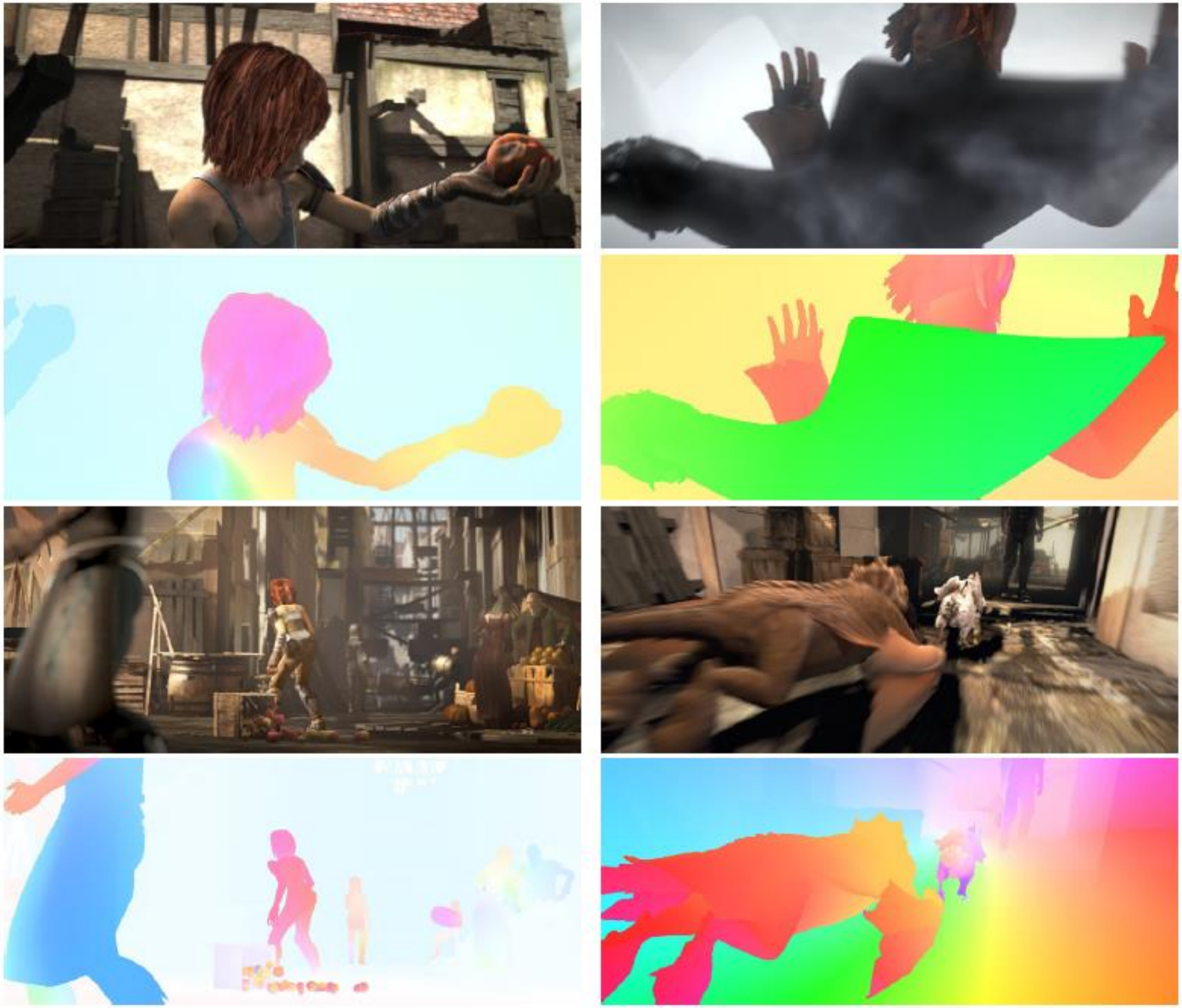
8 training sequences with ground truth (for parameter tuning)

8 test sequences with ground truth (for evaluation)

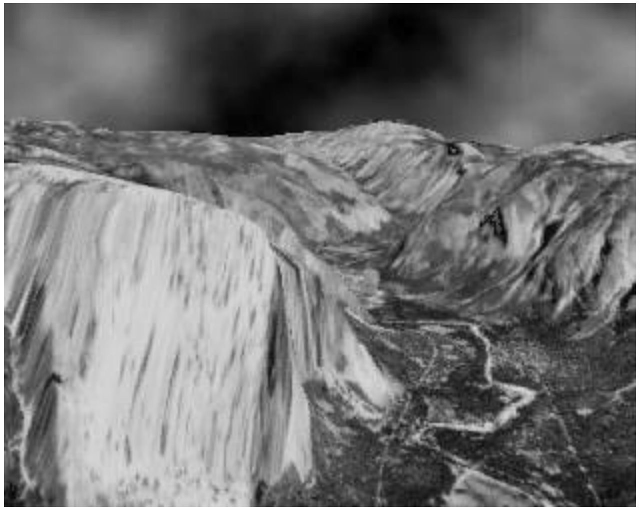
Too few test data → overfitting

Sintel – an open source movie

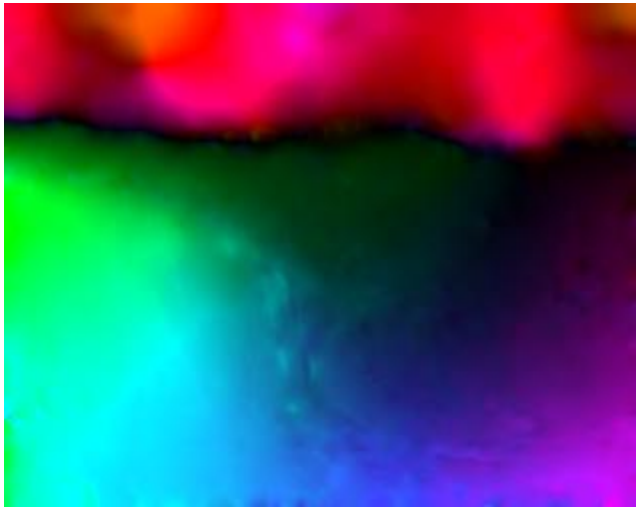
Sample frames and optical flow ground truth (Butler et al. ECCV 2012)



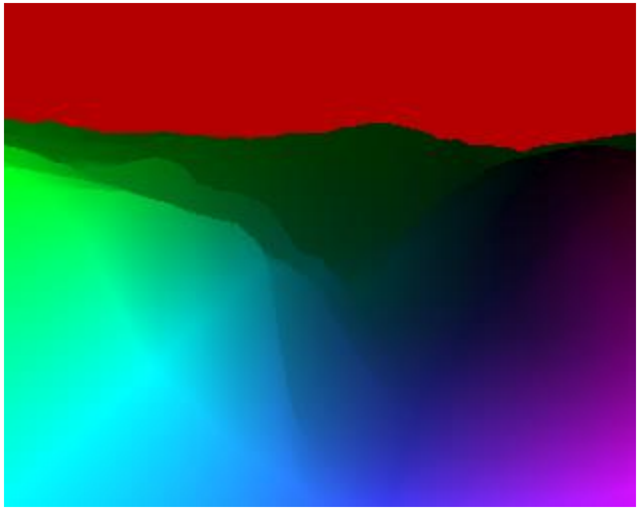
Results with Horn-Schunck method



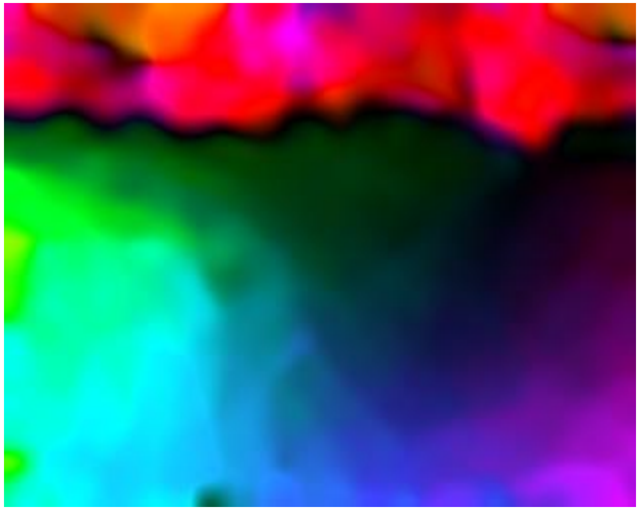
Input sequence



Horn-Schunck (7.17°)



Ground truth

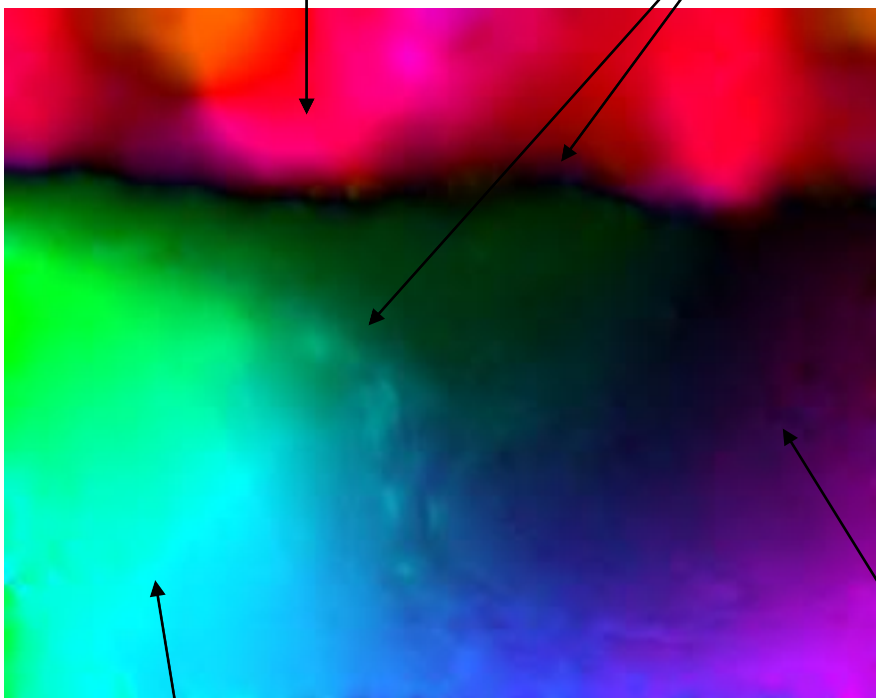


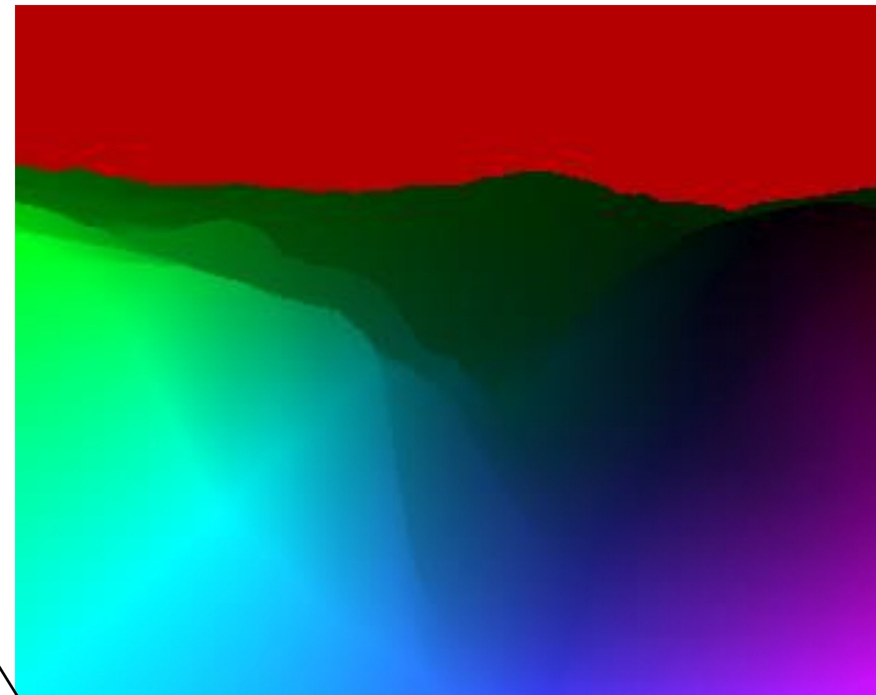
Lucas-Kanade (8.78°)

Limitations of the Horn-Schunck method

Cloud region not well estimated due to illumination changes

Motion discontinuities blurred





Underestimation of large displacements

Outliers due to non-Gaussian noise or occlusion

Tackling motion discontinuities and occlusions

- As in the denoising case we can consider a **robust function** applied to the smoothness term to allow for motion discontinuities, for instance:

$$\Psi(s^2) = \sqrt{s^2 + \epsilon^2}$$

- Occlusions can be approached by such a robust function applied to the data term. The energy then reads (Mémin-Pérez 1998):

$$E(u, v) = \int_\Omega \Psi\left((I_x u + I_y v + I_z)^2\right) + \alpha \Psi\left(|\nabla u|^2 + |\nabla v|^2\right) dx dy$$

- We obtain the Euler-Lagrange equations:

$$\Psi'\left((I_x u + I_y v + I_z)^2\right)(I_x u + I_y v + I_z) I_x - \alpha \operatorname{div}\left(\Psi'(|\nabla u|^2 + |\nabla v|^2)\nabla u\right) = 0$$
$$\Psi'\left((I_x u + I_y v + I_z)^2\right)(I_x u + I_y v + I_z) I_y - \alpha \operatorname{div}\left(\Psi'(|\nabla u|^2 + |\nabla v|^2)\nabla v\right) = 0$$

- This nonlinear system can be solved with **lagged nonlinearity**.

Illumination changes

“Schefflera” from Middlebury benchmark

Two frames from a Miss Marple movie

Typically caused by:

- Shadows
- light source flickering
- self-adaptive cameras
- different viewing angles and non-Lambertian surfaces

Large motion

- So far, we linearized the constancy assumptions.

$$I(x+u, y+v, z+1) - I(x, y, z) = 0 \quad \rightarrow \quad I_x u + I_y v + I_z = 0$$

- This linearization is only valid for very small displacements (actually subpixel displacements).

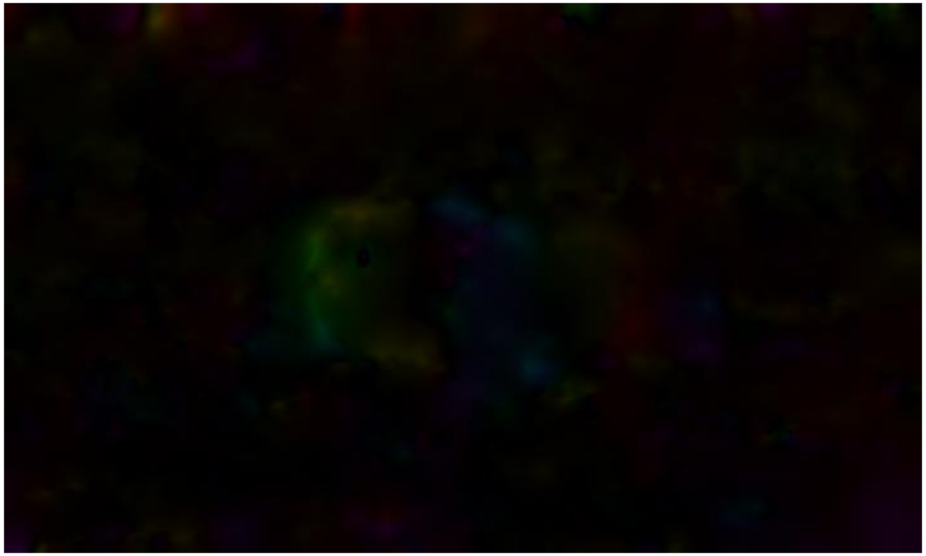
- To tackle displacements larger than ~ 4 pixels we must refrain from such a linearization.

- This raises several difficulties:
  - The energy becomes non-convex, i.e., we can only find local minima.
  - The Euler-Lagrange equations get highly nonlinear.

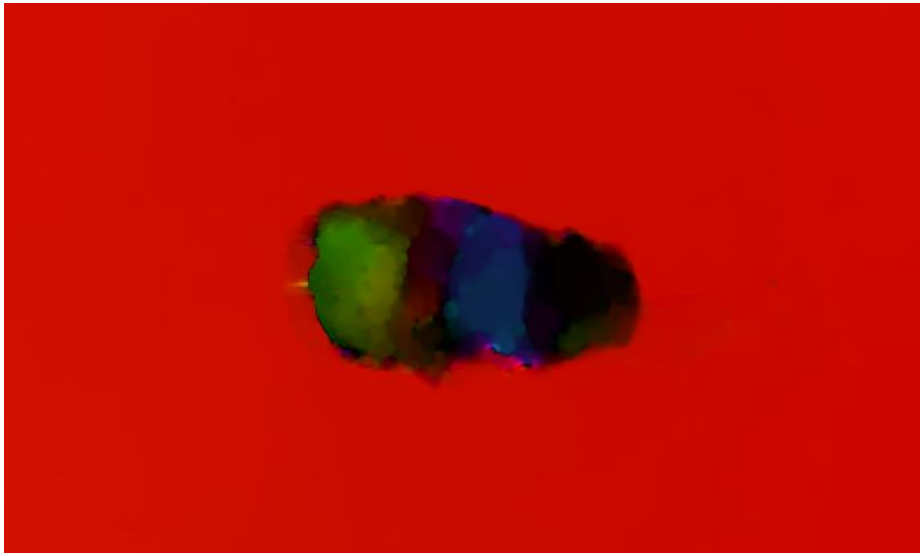
- These problems can be handled with the Gauss-Newton method and a coarse-to-fine approach working on an image pyramid (Brox et al. 2004).

- Details in the Computer Vision course.

Large motion



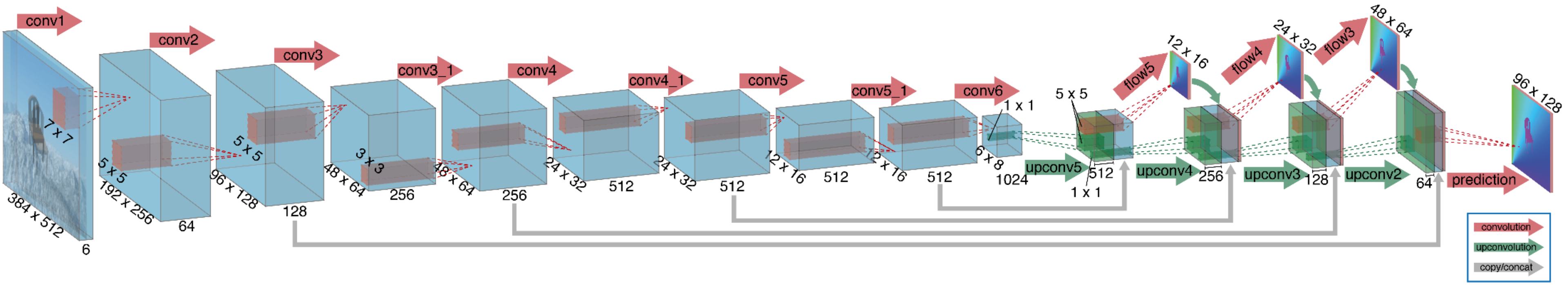
Horn&Schunck

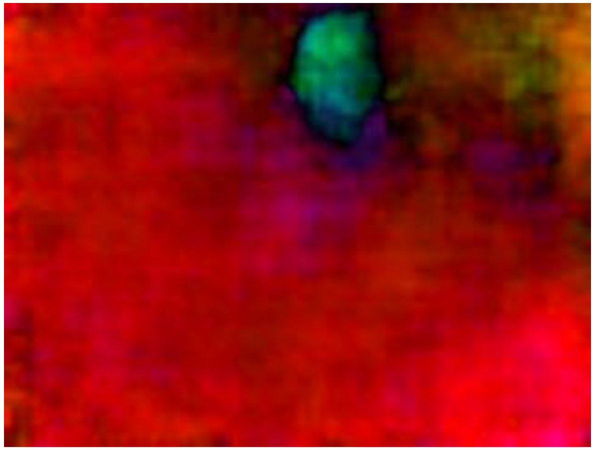


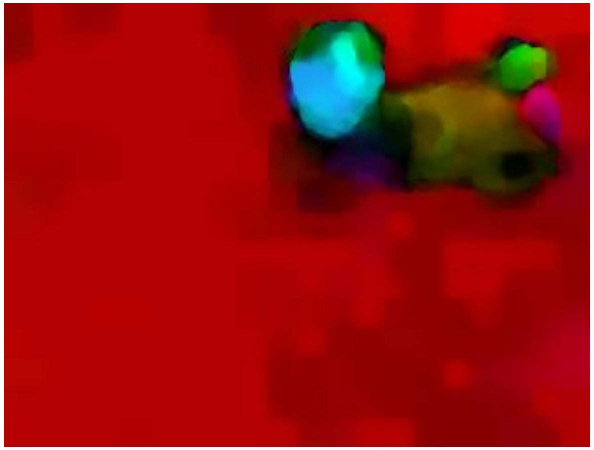
Brox et al. 2004

Results: zoom into a scene

FlowNet: Learning to estimate optical flow (Dosovitskiy et al. 2015)



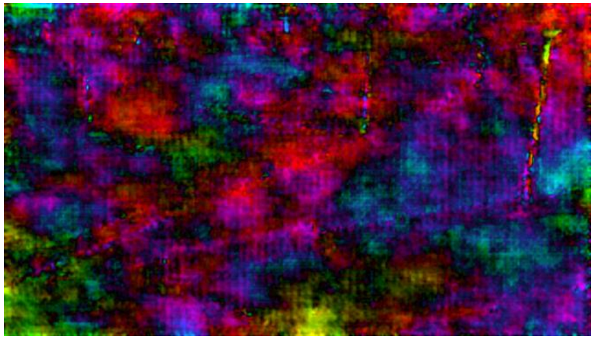




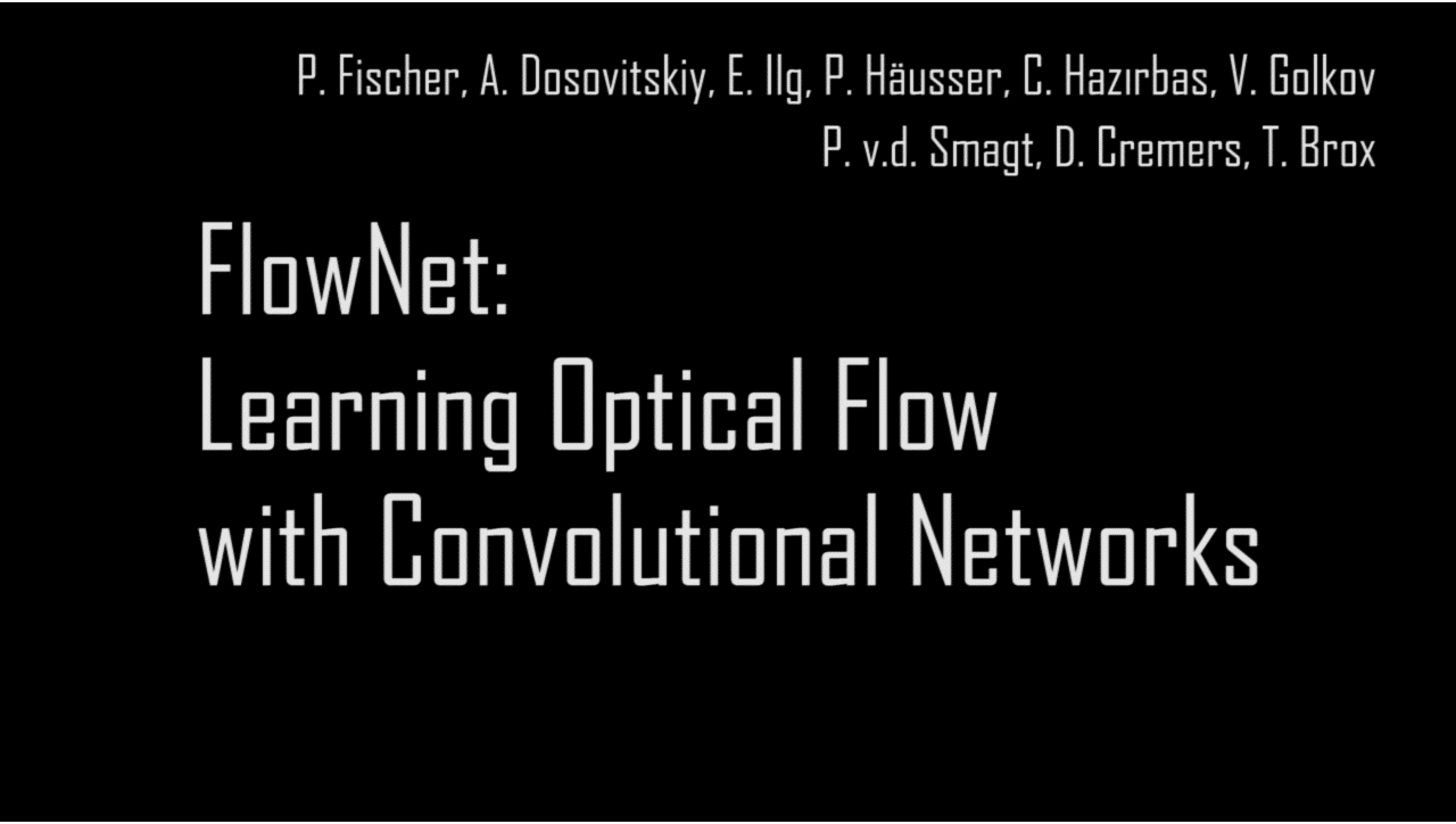
Input

FlowNet

Variational method



FlowNet


P. Fischer, A. Dosovitskiy, E. Ilg, P. Häusser, C. Hazırbas, V. Golkov
P. v.d. Smagt, D. Cremers, T. Brox
FlowNet:
Learning Optical Flow
with Convolutional Networks

Summary

- The optical flow is the apparent motion we can measure in the image plane.
- Optical flow estimation is usually based on the <u>gray value constancy assumption</u>.
- Local methods like the Lucas-Kanade method estimate motion parameters independently for each pixel by considering a <u>local neighborhood</u>.
- Global methods rather assume <u>smoothness of the flow field</u> and estimate the whole displacement field in one piece. The solution is found by variational techniques.
- Variational methods are still the state of the art in motion estimation
- This could change next week.

FlowNet 2.0: Preliminary numbers on Sintel.train.clean

	EPE	runtime
Variational method (Revaud et al. 2015)	2.4	15-20s
FlowNet 1.0 (Dosovitskiy et al. 2015)	4.5	80ms (12 fps)
FlowNet 2.0	**2.1**	90ms (11 fps)
FlowNet 2.0	2.6	28ms (36 fps)
FlowNet 2.0	4.5	**9ms (110 fps)**

References

- B. D. Lucas, T. Kanade: An iterative image registration technique with an application to stereo vision, *Proc. International Joint Conference on Artificial Intelligence*, pp. 674-679, 1981.

- B. Horn, B. Schunck: Determining optical flow, *Artificial Intelligence*, 17:185-203, 1981.

- E. Mémin, P. Pérez: Dense estimation and object-based segmentation of the optical flow with robust techniques, *IEEE Transactions on Image Processing,* 7(5):703-719, 1998.

- T. Brox, A. Bruhn, N. Papenberg, J. Weickert: Highly accurate optical flow estimation based on a theory for warping, *European Conference on Computer Vision*, Springer LNCS 3024, pages 25-36, 2004.

- D. Butler, J Wulff, G. Stanley, M. Black: A naturalistic open source movie for optical flow evaluation. *European Confernce on Computer Vision*, 2012.

- A. Dosovitskiy, P. Fischer, E. Ilg, P. Häusser, C. Hazirbas, V. Golkov, P. van der Smagt, D. Cremers, T. Brox: FlowNet: learning optical flow with convolutional networks, *International Conference on Computer Vision*, 2015.

Programming and theoretical assignment

- Implement the Lucas-Kanade method for optical flow estimation. For your convenience, make use of the convolution function `NFilter::filter` and the predefined filter masks `CSmooth` and `CDerivative` in `CFilter.h`.
  Try your implementation with the Street sequence. For visualization you can make use of the function `flowToImage`.
  Adjust the size of the Gaussian neighborhood and play with this parameter.
  Presmooth the input images before computing the gradients. Play with the amount of smoothing. Can you explain why the results get better with some presmoothing?

- As an exercise for the exam, derive the Euler-Lagrange equations of the Horn and Schunck functional using Gateaux derivatives.

- Implement the basic Horn-Schunck method. You can implement it via any linear solver discussed in class 4. The SOR solver is recommended. Find a regularization parameter that gives good results. Compare the Horn-Schunck results to the Lucas-Kanade results.

- Extra assignment (optional): Extend your Horn-Schunck method by an edge preserving penalizer.