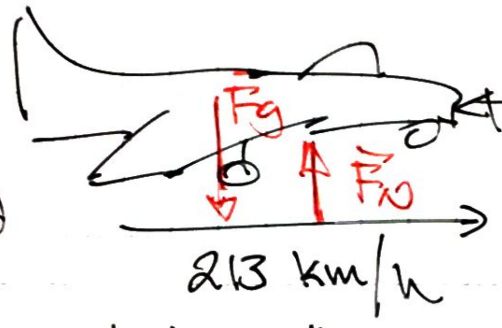
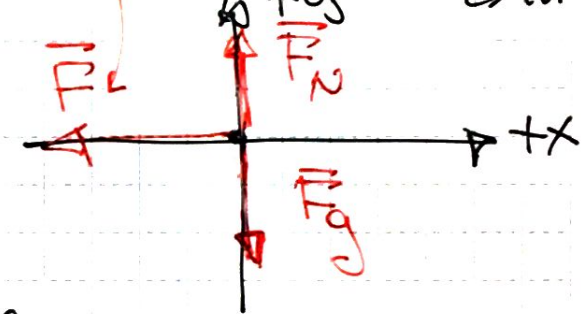
Given:

$m = 18\cdot10^{3}$ kg



$\vec{F} = 80k + 2.5v^2$ N
v in m/s



Req'd "landing distance"

Assump: no other forces, flat landing

Strategy: find a from Newton's 2nd then do kinematics
– expect $a = a(v)$ ← particular tools from CH 13

Estimate: $213\ \text{km/h} = \dfrac{213\cdot10^{3}\ \text{m}}{3.6\cdot10^{3}\ \text{s}} \cong 50\ \text{m/s}$

$\Rightarrow$ initial drag force $= 80k + 2.5(50)^2 \cong 87k$

$\Rightarrow$ v^2 term is not a big driver

take $F_{ave} = 80k = 18\cdot10^{3}\ \text{kg}\ a \Rightarrow a \cong 4\ \text{m/s} \Rightarrow$ 12 s to stop.

12 s @ 25 m/s $\left(v_{ave} = \dfrac{50+0}{2}\right) \cong$ 300 m $= s$.

this should be a low side estimate

Soln: $213\ \text{km/h} = \dfrac{213 \cdot 10^3\ \text{m}}{3.6 \cdot 10^3\ \text{s}} = 59.2\ \text{m/s}$

$$\vec{F}_{net_x} = m\bar{a}_x$$

$$-(80k + 2.5v^2) = m\bar{a}_x$$

$$\frac{-(80k + 2.5v^2)}{18 \cdot 10^3\ \text{kg}} = \bar{a}_x$$

$$-4.44\ \text{m/s}^2 - \frac{.139\, v^2}{10^3} = a_x$$

$$\underline{-c - dv^2 = a_x}$$

$$\int_{s_0=0}^{s_f=?} ds = \int_{v_0}^{v_f=0} \frac{v\,dv}{a(v)}$$

(from eqn sheet!)

$$s_f - 0 = \int_{59.2\ \text{m/s}}^{0} \frac{v\,dv}{-c - dv^2}$$

$$u = v^2 \Rightarrow du = 2v\,dv$$

$$\Rightarrow \int \frac{v\,dv}{-c - dv^2} = \int \frac{\frac{1}{2}\,du}{-c - \text{"}d\text{"}u}$$

↳ bad choice of constant name

Note: Never use "d" as a label in problems with integration — silly!

$$p = -c - \text{"}d\text{"}u$$

$$dp = -\text{"}d\text{"}\,du \Rightarrow du = \frac{dp}{-\text{"}d\text{"}}$$

$$\int \frac{\frac{1}{2}\frac{dp}{(-\text{"}d\text{"})}}{p} = \frac{-1}{2\text{"}d\text{"}} \int \frac{dp}{p} = \underline{\underline{\frac{-1}{2\text{"}d\text{"}} \ln p}}$$

<u>Soln:</u> (still working out integral)

$$\int \frac{v\,dv}{a(v)} = \frac{-1}{2\text{"}d\text{"}} \ln\left(-c - \text{"}d\text{"}v\right)\Big|_{v_0}^{v_f} = -\frac{1}{2\text{"}d\text{"}} \ln\left(-c - \text{"}d\text{"}v^2\right)\Big|_{v_0}^{v_f}$$

$$= -\frac{1}{2\text{"}d\text{"}}\left[\ln\left(-c - \text{"}d\text{"}\underbrace{v_f^2}_{\to 0}\right) - \ln\left(-c - \text{"}d\text{"}v_0^2\right)\right]$$

$$S_f = -\frac{1}{2\text{"}d\text{"}} \ln\left[\frac{-c}{-c - d v_0^2}\right] = -\frac{1}{2\left(\frac{.139}{10^3}\right)} \ln\left[\frac{4.44\ m/s^2}{4.44 + \left(\frac{.139}{10^3}\right)(59.2\ m/s)^2}\right]$$

$$S_f = -\frac{10^3}{.278} \ln\left[\frac{4.44}{4.44 + .487}\right]$$

units all cancel (as they must for arg of ln!!)

$$= +\frac{10^3}{.278}\left[+.104\right] = \boxed{374\,m = S_f}$$ sweet!

Discussion: Watch out for labels like "d" (silly). Estimate matches w/ lower limit at 300 m. Seems quite long at 4 football pitches but thats 1200' or so which makes sense for an airport. Not an aircraft carrier!