Bruce Emerson Sample Prob (Angular) ENGR212 1/

Given $\omega_0 = 200$ rpm and $\alpha = -0.014\omega$

Req'd $v\big|_{t=30s}$ & total revolutions to stop.

Assump 1 axis of rotation (linear problem)

Strategy: treat it like $a(v)$ $\alpha(\omega) = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \omega\frac{d\omega}{d\theta}$
and integrate

Estimate 200 rpm $= \frac{200}{60}$ rps $\simeq 3$ rps $\cong 18$ rad/s $= \omega_0$

$\alpha \Rightarrow$ loses $.01$ ~~rad~~/ω $\Rightarrow .2$ rad/s $= \alpha_{peak}$ at begining

estimat $\alpha_{ave} = \frac{1}{2}\alpha_{peak} = -.1$ rad/s $\Rightarrow$ $\boxed{180s \text{ to lose } 18 \text{ rad/s}}$

$\Rightarrow$ rev to stop < 600 rev $= 3m \cdot 200$ rpm 3 min

$\Rightarrow 30s = 1/6$th $\Rightarrow \omega_{30} \cong 15$ rad/s

expect $\approx$ 300 rev to stop

Soln: $\alpha(\omega) = \omega\frac{d\omega}{d\theta} \Rightarrow d\theta = \frac{\omega\, d\omega}{\alpha(\omega)}$

$\alpha(\text{rad}/s^2) = -0.014\ \omega(\frac{\text{rad}}{s}) \Rightarrow 0.014$ has units of $1/s$

Soln: cont.

relates θ, ω →

$$\int_0^{\theta_f} d\theta = \int_{\omega_0}^{0} \frac{\cancel{\omega}\, d\omega}{-0.014\cancel{\omega}} \Rightarrow \theta_f = \frac{1}{-0.014}\int_{\omega_0}^{0} d\omega$$

$$\theta_f = \frac{1}{-0.14}\,\omega\Big|_{\omega_0}^{0} = \frac{\cancel{-}\omega_0 \text{ rad}/\cancel{s}}{\cancel{-}0.014\, 1/\cancel{s}} = \frac{\omega_0}{0.014}\text{ rad.}$$

$$\omega_0 = 200\, r/\text{min}\ \frac{1\text{ min}}{60\text{ s}} = 3.33\text{ rev}/s = 3.33\text{ rev}/s \cdot \frac{2\pi\text{ rad}}{\text{rev}} = 6.67\pi\text{ rad}/s$$

$$\theta_f = \frac{6.67\pi\text{ rad}/\cancel{s}}{0.014\, 1/\cancel{s}} = 1496\text{ rad} = \frac{1496\,\cancel{\text{rad}}}{2\pi\,\cancel{\text{rad}}/\text{rev}} = \boxed{238\text{ revolutions}}$$

now I need $\omega(t)$! $\quad \alpha(\omega) = \frac{d\omega}{dt} \Rightarrow dt = \frac{d\omega}{\alpha(\omega)}$

$$\int_0^{30\text{ s}} dt = \int_{\omega_0}^{\omega(30\text{ s})} \frac{d\omega}{-0.014\,\omega} \Rightarrow 30\text{ s} = \frac{1}{-0.014\frac{1}{s}} \ln\omega\Big|_{\omega_0}^{\omega(t=30\text{ s})}$$

$$\Rightarrow 30\cancel{s}(-0.014\, 1/\cancel{s}) = \ln\omega_{30} - \ln\omega_0 = \ln\left(\frac{\omega_{30}}{\omega_0}\right)$$

$$\Rightarrow e^{-0.42} = \frac{\omega_{30}}{\omega_0} = .657 \Rightarrow \omega_{30} = .657\,(6.67\pi\text{ rad}/s)$$

$$\boxed{= 4.38\pi\text{ rad}/s}$$

Soln: cont $\boxed{\theta_f = 238 \text{ rev}}$ $\boxed{\omega\Big|_{t=30} = 13.8 \text{ rad/s}}$

Discussion! Both answers agree well w/ estimates. Had a small calculator issue at one point that had the drill speeding up which was clearly wrong.

I need to articulate the relationships between the variables that I need as I choose tools for my soln. Perhaps I solved this problem "backwards" but I got there.