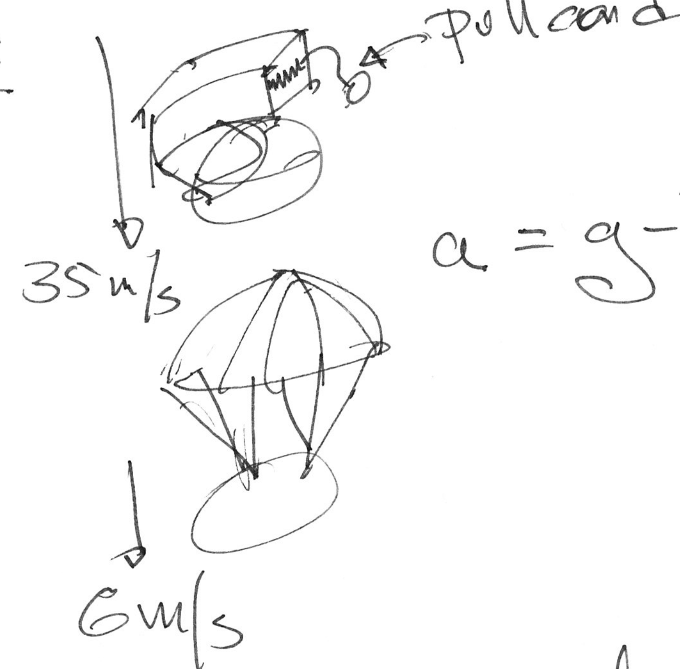
Given:



$a = g - Dv^2 \leftarrow a = a(v)$

Req'd: units of D, mag of D, max value of Dv^2, v @ 4.5 m after 'chute opens.

Assumptions: linear (vertical motion), down is + !

Strategy: $a(v) = \frac{dv}{dt} \rightarrow dt = \frac{dv}{a(v)}$ then integrate

at terminal velocity $a = 0$ which sets limits on D.

Estimate: a will be max when $v = 35$ m/s

$a = 0$ @ $v = 6$ m/s $\Rightarrow g - Dv^2 = 0 = 10 - D(36) = 0$

$\Rightarrow D \cong 1/4 \quad a_{max} = 10 - \frac{1}{4}(35)^2 = 10 - \frac{1}{4}(1200) = -290$ m/s²

$\Rightarrow$ will shed 30 m/s in about .1 s after traveling

< 3.5 m. $\Rightarrow$ I expect the object to already be

@ terminal velocity after 4.5 m.

Soln: Units: $a(v) = g - Dv^2 \quad \frac{m}{s^2} = g\left(\frac{m}{s^2}\right) - \left(\frac{D}{\frac{1}{m}}\right)\left(\frac{m^2}{s^2}\right)$

$\Rightarrow \boxed{D \text{ has units of } \frac{1}{m}}$

@ 6 m/s $= v \quad a = 0$

$0 = 9.8 \text{ m/s}^2 - D\left(6 \frac{m}{s}\right)^2 = 9.8 - 36D$

$36D = 9.8 \Rightarrow D = \frac{9.8}{36} = \boxed{.0272 \frac{1}{m} = D}$

Soln: (cont) $a(v) = \frac{dv}{dt} \Rightarrow dt = \frac{dv}{a(v)}$

$$\Rightarrow \int_{t=0}^{t} dt = \int_{v(t=0)}^{v(t)} \frac{dv}{g - Dv^2} = \left. \frac{\tan^{-1}\left(\sqrt{\frac{D}{g}}\, v\right)}{\sqrt{g}\sqrt{D}} \right|_{v(t=0)}^{v(t)} \leftarrow \text{ugly!}$$

let me stop and rethink... let me return to finding max drag term Dv^2

$$D v_{max}^2 = .0272\left(\frac{1}{m}\right)\left(35\,\frac{m}{s}\right)^2 = \boxed{333\,\frac{m}{s^2} \leftarrow \text{wow!}}$$

rethink: last question asks me to find v when s = 4.5 m
$\Rightarrow$ I'm looking for a relationship between s & v given a(v)

$$a(v) = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} \Rightarrow ds = \frac{v\,dv}{a(v)} \leftarrow \text{relates } s, v$$

$$\int_{s=0}^{s} ds = \int_{v(s=0)}^{v} \frac{v\,dv}{a(v)} = \int_{35\,m/s}^{v} \frac{v\,dv}{g - Dv^2} = \left. \frac{\ln(g - Dv^2)}{2(-D)} \right|_{35\,m/s}^{v}$$

(Wolfram α)

Soln: cont

$$s = \frac{1}{-2D}\left[\ln(g - Dv^2) - \underbrace{\ln\left(g - D(35\,m/s)^2\right)}_{-323}\right] = -\frac{1}{2D}\ln\left[\frac{g - Dv^2}{-323}\right]$$

when $s = 4.5\,m$

$$-2Ds = -2(.272\,\tfrac{1}{m})(4.5\,m) = -2.45$$

$$-2.45 = \ln\left(\frac{g - Dv^2}{-323}\right) \Rightarrow e^{-2.45} = \frac{g - Dv^2}{-323} \Rightarrow .0863 = \frac{g - Dv^2}{-323\,m/s^2}$$

$$-89.3\,m/s^2 = g - Dv^2 \Rightarrow Dv^2 = g + 89.3 = 99.3\,m/s^2$$

$$v^2 = \frac{99.3\,m/s^2}{.272\,1/m} = 365\,m^2/s^2 \Rightarrow v = \sqrt{365}\,m/s = \boxed{19.1\,m/s}$$

Discussion: I originally made a mistake (lost a (−) sign) in copying down the integral. (I had $\frac{1}{2D}$ instead of $\frac{1}{-2D}$).

My estimate for D was appropriate so thats good.

I estimated that $v|_{4.5m}$ would already be @ 6 m/s but I was over enthusiastic. I used $a_{max} = 333\,m/s$. If I had taken the $a_{ave} = \frac{333 + 0\,m/s^2}{2} = 160\,m/s^2$ I would have predicte rough .2s to shed 30 m/s which would cover about 7 m — live & learn!