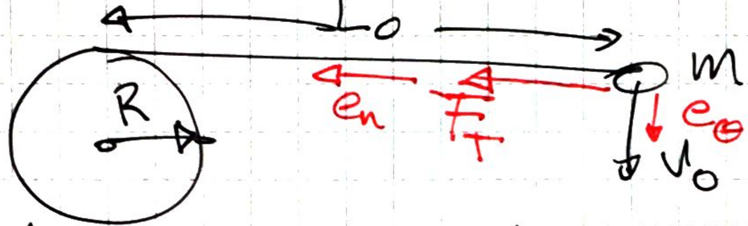
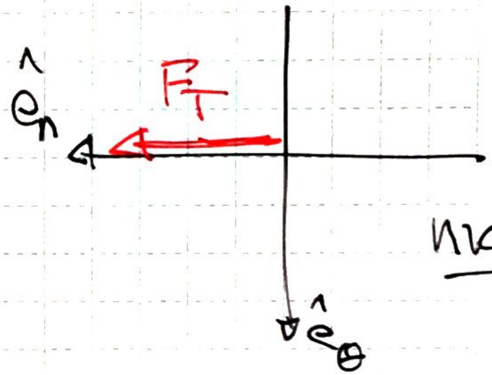
Given:





nice!

Req'd Find expression $\theta(t)$

Assump No gravity, F_T is only force $\Rightarrow a_t = 0$

Estimate I can only do proportional reasoning w/ no #'s
$\theta(t) \uparrow$ as $v_0 \uparrow$ so v_o should be in the numerator.
what matters is the relationship between L_o & R.
which suggest L_o/R is in there somewhere
when L_o is big I would expect θ to change slowly?
– no I take that back. Wow – I'm not sure!

Strategy: Set components of Forces in $n/t = m\,a(n,t)$

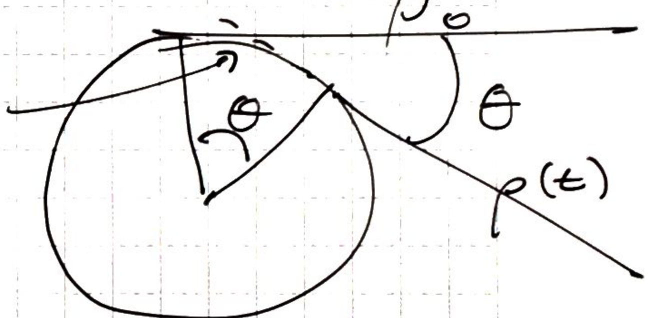
Soln: $F_T = F_n e_n \Rightarrow F_t = 0 \Rightarrow a_t = 0 \Rightarrow v_0 = \text{cnst}.$

(watch confusion between T & t)!

$a_n = \frac{F_T}{m}$ $\quad a_{n/t} = \rho\omega^2 \hat{e}_n + \underbrace{\alpha_t}_{0} \hat{e}_t = \rho\omega^2 = \frac{v^2}{\rho}$

$v_0 = \rho\omega$ $\quad \rho$ depends on θ

$\Rightarrow \rho(t) = L_0 - R\theta(t)$ $\quad R\theta$



$\Rightarrow \omega = \frac{V_0}{\rho} = \frac{V_0}{L_0 - R\theta(t)} = \omega(\theta) = \frac{d\theta}{dt}$

$\Rightarrow \int_0^t dt = \frac{d\theta}{\omega(\theta)} = \int_{\theta=0}^{\theta(t)} \left(\frac{L_0 - R\theta}{V_0}\right) d\theta$

$t\Big|_0^t = \left[\frac{L_0}{V_0}\theta - \frac{R\theta^2}{2V_0}\right]\Big|_0^{\theta(t)} \Rightarrow t = \frac{L_0}{V_0}\theta - \frac{R}{2V_0}\theta^2$

$\Rightarrow \theta^2 - \frac{2\cancel{V_0}}{R}\frac{L_0}{\cancel{V_0}}\theta + \frac{2V_0 t}{R} = 0$ $\quad$ Quadratic $a = 1$ $\quad b = -\frac{2L_0}{R}$ $\quad c = \frac{2V_0 t}{R}$

$$\theta = \frac{\frac{\cancel{2}L_0}{R} \pm \sqrt{\cancel{4}\left(\frac{L_0}{R}\right)^2 - \cancel{4}\frac{2V_0 t}{R}}}{\cancel{2}}$$

<u>Soln: Cont</u>

$$\theta(t) = \frac{L_o}{R} \pm \sqrt{\left(\frac{L_o}{R}\right)^2 - \frac{2v_o t}{R}}$$

$\theta(t=0) = 0 = \frac{L_o}{R} \pm \sqrt{\left(\frac{L_o}{R}\right)^2}$? $\Rightarrow$ ($-$) sign is correct.!

$$\boxed{\theta(t) = \frac{L_o}{R} - \sqrt{\left(\frac{L_o}{R}\right)^2 - \frac{2v_o t}{R}}}$$ whew!

<u>Discussion:</u> I like it in general, the $\frac{L_o}{R}$ feels good. I am bothered by the $\frac{1}{R}$ dependence I can't quite get that to make sense in my head.