Given: $a = 1.0 - 4s^2$ in m/s^2 $\quad v\big|_{s=0} = 2.0\ m/s$

Req'd: s when $v = 0$
v when $s = 0.5m$.

Assumptions: linear motion.

Strategy: Check units, $a(s) = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = \frac{dv}{ds}v$, separate and integrate.

Estimate: when $s = 0$ $a = 1\ m/s^2 \Rightarrow$ speeding up from $v = 2m/s$
when $s = 1$ $a = -3\ m/s^2$ slowing down
this suggests $v = 0$ before $s = 1$
@ $s = 1/2$ $a = 0 \Rightarrow$ maximum velocity $> 2\ m/s < 3\ m/s$

Soln:

$$a(s) = v\frac{dv}{ds} \Rightarrow \quad a(s)\,ds = v\,dv \qquad \int_{s=0}^{s} a(s)\,ds = \int_{v(s=0)}^{v(s)} v\,dv$$

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Soln: cont

$$\int_0^s (1-4s^2)\,ds = \int_{v=2\,m/s}^{v} v\,dv = \frac{V^2}{2}\Big|_{2\,m/s}^{v}$$

Quick units check:

$$a\left(\frac{m}{s^2}\right) = 1 - 4s^2 = \left(\frac{m}{s^2}\right) - \left(\frac{1}{m\cdot s^2}\right)(m^2) \Rightarrow 4 \text{ is in } \frac{1}{m\cdot s^2}$$

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$$1s - \frac{4s^3}{3}\Big|_0^s = \frac{V^2}{2}\Big|_{2\,m/s}^{v} \Rightarrow 1\frac{m}{s^2}s - 1.33\frac{1}{m\cdot s^2}s^3 = \frac{v^2}{2} - \frac{4\,m^2/s^2}{2}$$

$$\boxed{s - 1.33s^3 = \frac{1}{2}(v^2-4)}$$

when v = 0: $\Rightarrow$ $\cancel{1-s^3}$ $s - 1.33s^3 = -2$ ← plot to find answer

From Wolfram soln is approx $\boxed{1.35\,m = s}$ → better 1.36 using notebook transcendental Soln.

when s = 0.5 m $\quad .5 - 1.33\frac{1}{8} = \frac{1}{2}(v^2-4) \Rightarrow \underbrace{2\left(\frac{1}{2} - 1.33\frac{1}{8}\right)}_{2/3 = .67} = v^2 - 4$

$4/3$

$\Rightarrow \cancel{\frac{2}{3}\frac{m^2}{s^2} = v^2 \Rightarrow v = .82\,m/s}$ $\Rightarrow 4.67\,m^2/s^2 = v^2$

Soln: cont

$$4.67\,\mathrm{m^2/s^2} = v^2 \Rightarrow \boxed{v\big|_{s=.5} = 2.16\ \mathrm{m/s^2}}$$

Discussion

Again, I lost a (−) sign when I was solving for v = 0 and got a odd answer. Went back to check and found missing (−) sign – reinforces the point of estimation!

I'm happy that s is close to 1 for v = 0. I expected s < 1 but my sense of cubic behavior is fuzzy.

Estimation of $v\big|_{s=.5}$ landed quite close. Again, in doing soln I lost the −4 for a moment and the estimation provoked me to go back and find my error.