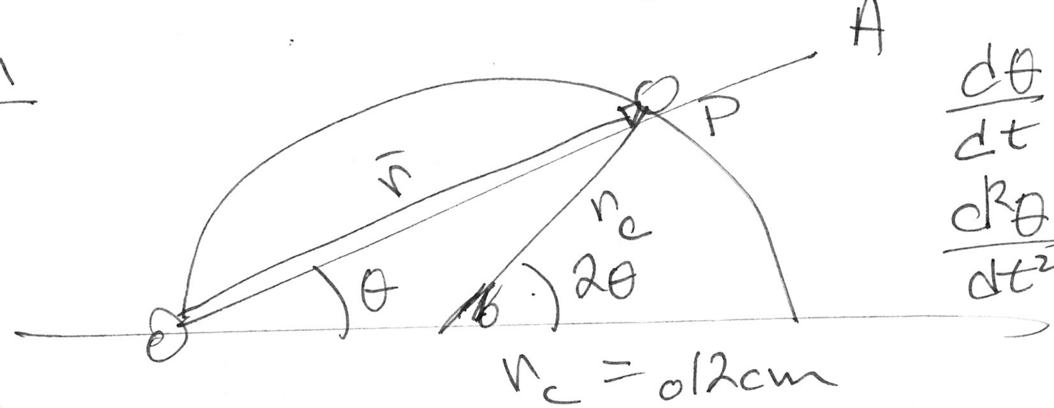
Given



$\frac{d\theta}{dt} = .4 \text{ rad/s} = \omega$
$\frac{d^2\theta}{dt^2} = .8 \text{ rad/s}^2 = \alpha$
when $\theta = 30^\circ = \pi/6$

$r = 2 r_c \cos\theta$

Req'd: $\bar{a}$ when $\theta = 30^\circ = \pi/6$, components

Assump: no friction,

Strategy: take horrifying expression for $\bar{a}_{polar}$ and try to figure out the terms.

$\bar{a}_{polar} = a_r \hat{e}_r + a_\theta \hat{e}_\theta \quad a_r = \frac{d^2 r}{dt^2} - r\omega^2 \quad a_\theta = r\alpha + 2\frac{dr}{dt}\omega$.

Estimate: let me roughly figure out what I don't know - they all turn out to be derivatives of r - hmm.

$r\Big|_{\theta=30^\circ} = 2(.12)\frac{\sqrt{3}}{2} = .2 \text{ m/s}$

$\frac{dr}{dt} = ?$ ~~2~~ ~~7~~s to go 180°

$\Rightarrow$ ~~3~~s to go 90°

where $\sqrt{2}\, r_c < r < 2 r_c$ ($.8\text{m}$, $.24\text{ m}$)

$\Rightarrow\Rightarrow \frac{dr}{dt} \approx$ ~~.1 m/s~~ $= .01 \text{ m/s}$

Estimate: $V_{max} = r\omega = .05\,m/s$ changes very little over 1st 3s
$.12(.4)$

$\Delta v \sim .02\,m/s$ over 3s $\cong \frac{d^2r}{dt^2} < .001\,m/s^2$??

All small #'s !

✓ = known

Soln: $a_r = \frac{d^2r}{dt^2} - r\omega^2$ (@) (*)

$a_\theta = r\alpha + 2\frac{dr}{dt}\omega$ (*) (!)

$r\Big|_{\theta=30°} = 2(.12m)\cos(\pi/6)$

$= 2(.12)\frac{\sqrt{3}}{2} = .21\,m$ (*)

$\frac{dr}{dt} = 2r_c\frac{d}{dt}(\cos(\theta))$

$= 2r_c\left(-\sin(\theta)\frac{d\theta}{dt}\right)$

$= -2r_c\omega\sin\theta$

$\frac{dr}{dt}\Big|_{\theta=30°} = -2(.12m)(.4\,\frac{rad}{s})\sin(30°)$ [$\sin(30°) = 1/2$]

$= -4.8\cdot10^{-2}\,m/s$ (!)

$\frac{d^2r}{dt^2} = \frac{d}{dt}(-2r_c\omega\sin\theta)$

$= -2r_c\frac{d}{dt}(\omega\sin\theta)$

$= -2r_c(\alpha\sin\theta + \omega^2\cos\theta)$

$\frac{d^2r}{dt^2}\Big|_{\theta=30°} = -2(.12m)\left[.8\frac{rad}{s^2}\sin30° + (.4\,rad/s)^2\cos30°\right]$

$= -2(.12m)\underbrace{\left[.8\frac{rad}{s^2}\frac{1}{2} + (.4\frac{rad}{s})^2\frac{\sqrt{3}}{2}\right]}_{.522\,rad/s^2}$ [.277]

$= -6.27\cdot10^{-2}\,\frac{m}{s^2}$ (@)

Soln: cont

$$a_r = -6.27\cdot 10^{-2}\, m/s^2 - .21\, m\left(.4\frac{rad}{s}\right)^2$$

$$= -6.27\cdot 10^{-2} - 3.36\cdot 10^{-2}\, m/s^2$$

$$a_r = -9.63\cdot 10^{-2}\, m/s^2$$

$$a_\theta = (.21\, m)\left(-.8\frac{rad}{s^2}\right) + 2\left(-4.8\cdot 10^{-2}\, m/s\right)\left(.4\frac{rad}{s}\right)$$

$$= .168\frac{m}{s^2} - .0384\, m/s^2$$

$$a_\theta = .13\frac{m}{s^2}$$

$$\boxed{\begin{aligned} \vec{a} &= -.0963\frac{m}{s^2}\hat{e}_r + .13\frac{m}{s^2}\hat{e}_\theta \\ |\vec{a}| &= \sqrt{.096^2 + .13^2} = .16\frac{m}{s^2} \end{aligned}}$$

169
92.7

Discussion: r & $\frac{dr}{dt}$ seem to match estimates well. I'm not surprised that $|\vec{a}|$ is bigger than I thought
I found it hard to picture ΔV. Maybe next time. I should have noted that I would have expected $a_r < 0$ because radius in shrinking