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"I pledge my honor that I have abided by the Stevens Honor System."

Problem 1

Use the pumping lemma to show that the following languages are not regular

a) $\{ 0^i 1^j : i < j \}$

Let's make $A = \{ 0^i 1^j : i < j \}$ and assume A is a regular language.
By the pumping lemma, A has a pumping length p.

Let's say $S = 0^p 1^{p+1}$ where $S = xyz$
$S \in A$, $|y| > 0$, $|xy| \le p$
y must consist of some number of 0s. $y = 0^k$ where $1 <= k <= p$.

If k = 1, $xyyz = 0^p 0^1 1^{p+1} = 0^{p+1} 1^{p+1}$. In this string, i = j, which contradicts the rule that i < j. If k > 1, i will be greater than j, which contradicts the rule that i < j. Thus, for any k such that 1 <= k <= p, xyyz will violate the ruleset of A.

This violates the Pumping Lemma; therefore $\{ 0^i 1^j : i < j \}$ is not a regular language.

b) $\{ 0^i 1^j : i > j \}$

Proof that $\{ 0^i 1^j : i > j \}$ is non-regular

Let's assume that $\{ 0^i 1^j : i > j \}$ is regular, and $A = \{ 0^i 1^j : i > j \}$.
By the Pumping Lemma, A has a pumping length, p.

Let $s = 0^{p+1} 1^p \in A$, p+1 -> 0...0 1...1 <- p
$s = xyz$, $|y| > 0$, $|xy| <= p$
y must consist of some number of 0s. $y = 0^k$ where $1 <= k <= p$.
$xz = 0^{p+1-k} 1^p$.

If k = 1, $xz = 0^p 1^p$, which means i = j, thus this does not belong to the language A. If k > 1, i < j, thus this still does not belong to the language A. For any k where 1 <= k <= p, xz does not follow the rule set of the language.

This violates the Pumping Lemma. Therefore $\{ 0^i 1^j : i > j \}$ is not a regular language.

Problem 2

Prove that the language $B = \{ 0^i 1^j : i \neq j \}$ is not regular. Do not use the pumping lemma. Instead, express B as the result of regular operations between the nonregular language $\{ 0^i 1^i : i \geq 0 \}$ and a regular language.

Assume B is regular.
Let c represent complement

$\{ 0^i 1^i : i \geq 0 \}$ is equal to the regular expression $B^c \cap 0^*1^*$ where B^c is the language of strings that are not in B. Strings that are not in the language B are strings in which i=j (represented by i) and intersecting it with expression 0^*1^* ensures that the string has the correct order of numbers (only 0s followed by only 1s).

If B were regular then B^c would be regular due to the closure of regular languages under complement. $B^c \cap 0^*1^*$ would also be regular due to the closure of regular languages under intersection (both B^c and 0^*1^* are regular languages). However, since we know that $\{ 0^i 1^i : i \geq 0 \}$ is not regular there is a contradiction, and thus B cannot be regular.

Problem 3
The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or greater. If p is a pumping length for regular language A, then so is any length $p' \geq p$. The minimum pumping length for A is the smallest p that is a pumping length of A.

For example, the pumping length of 01* cannot be 1 because the string s = 0 of length 1 cannot be pumped to give another string in the language. But any string of length 2 or more can be pumped by choosing x = 0, y = 1, and z to be the rest of the string.

What is the minimum pumping length for each of the following languages? Justify your answer in each case.

1. 0001*
Answer: 4
Explanation: Every string in the language cannot be pumped with a pumping length less than 4 because y would consist of only 0s, completely omitting all the strings in the language with 1s. In order to pump the language, the string s would have to consist of x = 000, y = 1, and z to be the rest of the string.

2. 0*1*
Answer: 1
Explanation: The smallest string you can make with this language is the empty string, but y cannot equal 0 so 1 would be the minimum pumping length. This way you can make x = ε, y = 0 or 1, and z = the rest of the string. Other strings in the language would require higher pumping lengths.

3. 0*1*0*1* ∪ 10*1
Answer: 1
Explanation: Similar to #2, the smallest string you can make is the empty string, so the minimum pumping length must be 1. This way you can make x = ε, y = 0 or 1, and z = the rest of the string. Other strings in the language would require higher pumping lengths.

4. (01)*

Answer: 2

Explanation: Every string in the language can be pumped given that y = 01 because every string in the language is a chain of '01's and if you remove y from xyz you get the empty string, which is also a member of the language. Y cannot be just 0 or just 1, since that would break the ruleset if pumped. You would make x = **ε**, y = 01, and z = the rest of the string.

5. 1* 01* 01*

Answer: 1

Explanation: Every string in this language can be pumped given that x = **ε**, y = 1, and z = the rest of the string. If you pump y up or down the string will exist in the given language.

Problem 4

a) Show that the language L = { $a^i b^j c^k$: i, j, k ≥ 0 and i = 1 → j = k } satisfies the three conditions of the pumping lemma. Hint: set the pumping threshold to 2 and argue that every string in L can be divided into three parts to satisfy the conditions of the pumping lemma.

When i = 0:
s = xyz, |y| > 0, |xy| ≤ p
s ∈ L if broken up as follows:
x = ε, y = b, z = c^p where p ≥ 2

When i = 1:
s = xyz, |y| > 0, |xy| ≤ p
s ∈ L if broken up as follows:
x = ε, y = a, z = $b^p c^p$ where p ≥ 2

When i > 1:
s = xyz, |y| > 0, |xy| ≤ p
Let's say there is some variable u and v where 2 ≤ u ≤ p-1 and 2 ≤ v ≤ p-1
s ∈ L if broken up as follows:
x = a^u, y = b, z = c^v

b) Prove that L is not regular. Note that L = b*c* U aaa*b*c* U {ab^ic^i : i ≥ 0}, and use the fact that regular languages are closed under complement and difference.

Assume L is regular.
<u>Proof that {ab^ic^i : i ≥ 0} is non-regular</u>
Let's assume that {ab^ic^i : i ≥ 0} is regular, and A = {ab^ic^i : i ≥ 0}.
By the Pumping Lemma, A has a pumping length, p.

Let s = ab^pc^p ∈ A,
s = xyz, |y| > 0, |xy| <= p

y must consist of some number of bs. y = b^k where 1 <= k <= p

> If y included the first a as well, it would pump a's into the rest of the string, and would break out very quickly.

For any k where 1 <= k <= p, $xz = ab^{p-k}c^p$ and thus does not follow the rule set.

This violates the Pumping Lemma. Therefore $\{ ab^ic^i : i \geq 0\}$ is not a regular language.

<u>L = b*c* U aaa*b*c* U $\{ab^ic^i : i \geq 0\}$</u>
Since b*c*, aaa*b*c*, and $\{ab^ic^i : i \geq 0\}$ are mutually disjoint, we can manipulate the expression

> L = b*c* U aaa*b*c* U $\{ab^ic^i : i \geq 0\}$ to be
> L - b*c* - aaa*b*c* = $\{ab^ic^i : i \geq 0\}$.

Since we assume L is regular, and b*c*, aaa*b*c* are both regular languages, so is L - b*c* - aaa*b*c* under closure of difference. However, that is equivalent to a non-regular language which is a contradiction. Therefore, L must be non-regular.

c) Explain why parts (a) and (b) do not contradict the pumping lemma.
Parts (a) and (b) do not contradict the pumping lemma because the pumping lemma states:

> If A is a regular language, then there is a number p (the pumping length) where if s is any string in A of length at least p,then s may be divided into three pieces, s=xyz, satisfying the following conditions: 1.For each i ≥0, $xy^iz \in A$, 2. |y| >0, and 3. |xy| ≤ p.

The pumping lemma shows that if a regular language cannot be pumped then it is not a regular language. However it does not say anything about whether there exists non-regular languages that can be pumped.