CS583 HW1

Aughdon Breslin

February 2022

1 $x = [5, -3, -1, 2]^T$

1. the squared L2-norm of x:

$$||x||_2 = \sqrt{|5|^2 + |-3|^2 + |-1|^2 + |2|^2} = \sqrt{25 + 9 + 1 + 4} = \sqrt{39}$$
$$||x||_2^2 = 39$$

2. the L1-norm of x:

$$||x||_1 = |5| + |-3| + |-1| + |2| = 5 + 3 + 1 + 2 = 11$$

3. the inner product of x and a, where $a = [4, -2, 6, -1]^T$

$$a^T x = (5)(4) + (-3)(-2) + (-1)(6) + (2)(-1) = 20 + 6 - 6 - 2 = 18$$

2 $A = \begin{bmatrix} 6 & 1 & -2 \\ -5 & 7 & 9 \end{bmatrix}$ and $b = \begin{bmatrix} -4 \\ 5 \\ 2 \end{bmatrix}$

1. the matrix-vector product:

$$Ab = \begin{bmatrix} (6)(-4) + (1)(5) + (-2)(2) \\ (-5)(-4) + (7)(5) + (9)(2) \end{bmatrix} = \begin{bmatrix} -23 \\ 73 \end{bmatrix}$$

2. the matrix-matrix product:

$$AA^T = \begin{bmatrix} (6)(6) + (1)(1) + (-2)(-2) & (6)(-5) + (1)(7) + (-2)(9) \\ (-5)(6) + (7)(1) + (9)(-2) & (-5)(-5) + (7)(7) + (9)(9) \end{bmatrix} = \begin{bmatrix} 41 & -41 \\ -41 & 155 \end{bmatrix}$$

3 Let $x = [x1, x2, x3]$ and $y = \frac{x_1^2}{2} + \ln x_2 - \frac{x_1}{x_3}$. Calculate $\frac{dy}{dx}$ at $x = [9, 1, \frac{1}{2}]$.

$$\frac{dy}{dx} = \begin{bmatrix} \frac{dy}{dx_1} \\ \frac{dy}{dx_2} \\ \frac{dy}{dx_3} \end{bmatrix} = \begin{bmatrix} x_1 - \frac{1}{x_3} \\ \frac{1}{x_2} \\ \frac{x_1}{x_3^2} \end{bmatrix} = \begin{bmatrix} 9 - \frac{1}{1/2} \\ \frac{1}{1} \\ \frac{9}{(1/2)^2} \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 36 \end{bmatrix}$$

4 X is an $n \times d$ matrix, y is an $n \times 1$ vector, and w is a $d \times 1$ vector. Let $f(w) = ||Xw - y||_2^2 + \lambda||w||_2^2$. Calculate $\frac{\partial f(w)}{\partial w}$.

$$\frac{\partial f(w)}{\partial w} = \frac{\partial ||Xw - y||_2^2}{\partial w} + \frac{\partial \lambda ||w||_2^2}{\partial w}$$

$$\frac{\partial ||Xw - y||_2^2}{\partial w} = 2(X^T Xw - X^T y)$$

$$\frac{\partial \lambda ||w||_2^2}{\partial w} = 2\lambda w$$

$$\frac{\partial f(w)}{\partial w} = 2(X^T Xw - X^T y) + 2\lambda w = 0$$

$$(X^T X + \lambda)w = X^T y$$

$$w^* = \frac{X^T y}{X^T X + \lambda}$$