
See the canvas assignment page for detailed instructions on submitting your work.

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1. Find the general solution to the linear equation, $L[y] = y'' - 4y' + 3y = 2te^t$.
   Use the method of ***undetermined coefficients*** *for finding a particular solution.*

2. Find the general solution to the linear ODE, $L[y] = x^2 y''(x) + xy'(x) - y(x) = \dfrac{4 \ln x}{x^3}$.

3. A mass-spring system is suspended vertically with a mass of M kg, a damping force that is 4 times the instantaneous velocity, and a spring constant of 8 N/m. The system is driven by a periodic external force, $f(t) = 5\sin(4t)$ N. Let $y(t)$ denote the vertical displacement of the mass from its equilibrium position oriented so that y is increasing in the downward direction. (*i.e.*, $y > 0$ corresponds to the spring being stretched.) At $t = 0$, the mass is released at a position 0.25 m above the equilibrium point with a downward velocity of 2 m/sec.

   (a) Set up (but do not solve) the initial value problem (IVP) that represents the mass-spring described above.

   (b) Determine the values of mass $M > 0$ for which the system is (*i*) underdamped, (*ii*) critically damped, and (*iii*) overdamped.

4. Consider the following initial value problem (IVP) for a linear oscillator model,

$$L[y] = y'' + 4y' + 4y = 13\cos(3t), \qquad y(0) = -1.0,\ y'(0) = 2$$

   (a) Solve for the unique solution to the IVP.

   (b) Represent the *steady-state solution* in the form, $y_p(t) = A\sin(3t - \phi)$. (Determine A and ϕ.)