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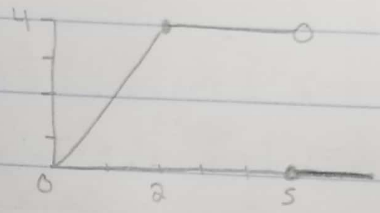
I pledge my honor that I have abided by the Stevens Honor System.

Aughd Bresli

HW5

1) $F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt$

Find $L\{f(t)\}$ where $f(t) = \begin{cases} 2t & 0 \le t < 2 \\ 4 & 2 \le t < 5 \\ 0 & 5 \le t < \infty \end{cases}$



$L\{f(t)\} = \int_0^2 2te^{-st}\,dt + \int_2^5 4e^{-st}\,dt + \int_5^\infty 0e^{-st}\,dt = 2\int_0^2 te^{-st}\,dt + 4\int_2^5 e^{-st}\,dt$

• $2\int_0^2 te^{-st}\,dt \to u = t \quad dv = e^{-st}\,dt \to 2\left(-\frac{1}{s}te^{-st}\Big]_0^2 + \int_0^2 \frac{1}{s}e^{-st}\,dt\right) \to 2\left(-\frac{1}{s}te^{-st}\Big]_0^2 - \frac{1}{s^2}e^{-st}\Big]_0^2\right)$

$du = dt \quad v = -\frac{1}{s}e^{-st}$

$\to 2\left(-\frac{1}{s}\left(2e^{-2s} - 0e^{-0s}\right) - \frac{1}{s^2}\left(e^{-2s} - e^{0s}\right)\right) \to \underline{-\frac{4}{s}e^{-2s} - \frac{2}{s^2}\left(e^{-2s} - 1\right)}$

• $4\int_2^5 e^{-st}\,dt \quad u = -st \quad du = -s\,dt \quad \left(-\frac{1}{s}\right) \quad (-s) \to -\frac{4}{s}e^{-st}\Big]_2^5 \to -\frac{4}{s}\left(e^{-5s} - e^{-2s}\right) = \underline{-\frac{4}{s}e^{-5s} + \frac{4}{s}e^{-2s}}$

$L\{f(t)\} = -\frac{4}{s}e^{-2s} + \frac{4}{s}e^{-2s} - \frac{2(e^{-2s} - 1)}{s^2} - \frac{4e^{-5s}}{s} = \boxed{\frac{2(1 - e^{-2s})}{s^2} - \frac{4e^{-5s}}{s}}$

2) Determine inverse Laplace Transforms

a) $f(t) = L^{-1}\left\{\frac{3}{s^5}\right\} = \frac{1}{8}L^{-1}\left\{\frac{24}{s^5}\right\} = \boxed{\frac{1}{8}t^4}$

b) $g(t) = L^{-1}\left\{\frac{-3s+4}{s^2+9}\right\} = -3L^{-1}\left\{\frac{s}{s^2+9}\right\} + \frac{4}{3}L^{-1}\left\{\frac{3}{s^2+9}\right\} = \boxed{-3\cos(3t) + \frac{4}{3}\sin(3t)}$

c) $h(t) = L^{-1}\left\{\frac{-3s+4}{s^2+4s+20}\right\} = L^{-1}\left\{\frac{-3(s+2)+10}{(s+2)^2+16}\right\} = -3L^{-1}\left\{\frac{s+2}{(s+2)^2+16}\right\} + \frac{10}{4}L^{-1}\left\{\frac{4}{(s+2)^2+16}\right\} = \boxed{-3e^{-2t}\cos(4t) + \frac{5}{2}e^{-2t}\sin(4t)}$

3) Determine $L^{-1}\{\}$

a) $f(t) = L^{-1}\left\{ \frac{3s^2+8s+9}{(s+2)^3} \right\} = L^{-1}\left\{ \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{2C}{(s+2)^3} \right\} = AL^{-1}\left\{ \frac{1}{s+2} \right\} + BL^{-1}\left\{ \frac{1}{(s+2)^2} \right\} + C\left\{ \frac{2}{(s+2)^3} \right\}$

$= Ae^{-2t} + Bte^{-2t} + Ct^2e^{-2t}$ Find ABC by $A(s+2)^2 + B(s+2) + 2C = 3s^2+8s+9$

$As^2 + 4As + 4A + Bs + 2B + 2C \rightarrow A = 3$, $4A + B = 8 \rightarrow B = -4$, $4A + 2B + 2C = 9 \rightarrow 2C = 5 \rightarrow C = \frac{5}{2}$

$\boxed{f(t) = 3e^{-2t} - 4te^{-2t} + \frac{5}{2}t^2e^{-2t}}$

b) $y(t) = L^{-1}\left\{ \frac{s^2+11s+20}{(s^2+4s+8)(s+1)^2} \right\} = L^{-1}\left\{ \frac{A(s+2)+2B}{s^2+4s+8} + \frac{C}{s+1} + \frac{D}{(s+1)^2} \right\} = AL^{-1}\left\{ \frac{s+2}{(s+2)^2+4} \right\} + BL^{-1}\left\{ \frac{2}{(s+2)^2+4} \right\} + CL^{-1}\left\{ \frac{1}{s+1} \right\} + DL^{-1}\left\{ \frac{1}{(s+1)^2} \right\}$

$= Ae^{-2t}\cos(2t) + Be^{-2t}\sin(2t) + Ce^{-t} + Dte^{-t}$

Find ABCD by $A(s+2)(s+1)^2 + 2B(s+1)^2 + C(s+1)(s^2+4s+8) + D(s^2+4s+8) = s^2+11s+20$

$As^3 + 4As^2 + 5As + 2A + 2Bs^2 + 4Bs + 2B + Cs^3 + 5Cs^2 + 12Cs + 8C + Ds^2 + 4Ds + 8D = s^2 + 11s + 20$

$A + C = 0 \rightarrow \underline{A = -C}$, $4A + 2B + 5C + D = 1 \rightarrow \underline{2B + C + D = 1}$, $5A + 4B + 12C + 4D = 11 \rightarrow \underline{4B + 7C + 4D = 11}$

$2A + 2B + 8C + 8D = 20 \rightarrow \underline{2B + 6C + 8D = 20} \rightarrow 5C + 7D = 19 \rightarrow 5C = 19 - 7D$ $5C + 2D = 9 \rightarrow$

$\rightarrow 19 - 7D + 2D = 9 \rightarrow 5D = 10 \rightarrow \underline{D = 2}$, $5C + 2D = 9 \rightarrow 5C = 5 \rightarrow \underline{C = 1}$, $A = -C \rightarrow \underline{A = -1}$

$2B + C + D = 1 \rightarrow 2B = -2 \rightarrow \underline{B = -1}$

$\boxed{f(t) = -e^{-2t}\cos(2t) - e^{-2t}\sin(2t) + e^{-t} + 2te^{-t}}$

4) Solve w/ $\mathcal{L}\{\}$, $y''(t) + 2y'(t) + 10y = 9e^{-t}$, $y(0) = 7$, $y'(0) = -1$

$$\mathcal{L}\{y''(t) + 2y'(t) + 10y\} = \mathcal{L}\{9e^{-t}\} \xrightarrow{y(s) = \mathcal{L}\{y(t)\}} (s^2y(s) - sy(0) - y'(0)) + 2(sy(s) - y'(0)) + 10y(s) = \frac{9}{s+1}$$

$$y(s)(s^2+2s+10) - 7s + 1 - 14 = \frac{9}{s+1} \rightarrow y(s)(s^2+2s+10) = \frac{9 + (7s+13)(s+1)}{(s+1)} \rightarrow y(s) = \frac{7s^2+20s+22}{(s+1)(s^2+2s+10)}$$

$$\mathcal{L}^{-1}\left\{\frac{7s^2+20s+22}{(s+1)(s^2+2s+10)}\right\} = \mathcal{L}^{-1}\left\{\frac{A}{s+1} + \frac{B(s+1)+3C}{s^2+2s+10}\right\} = A\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + B\mathcal{L}^{-1}\left\{\frac{s+1}{(s+1)^2+9}\right\} + C\mathcal{L}^{-1}\left\{\frac{3}{(s+1)^2+9}\right\}$$

$= Ae^{-t} + Be^{-t}\cos(3t) + Ce^{-t}\sin(3t)$ Find ABC by $A(s^2+2s+10) + B(s+1)^2 + 3C(s+1) = 7s^2+20s+22$

$As^2 + 2As + 10A + Bs^2 + 2Bs + B + 3Cs + 3C = 7s^2 + 20s + 22$

$A+B=7 \rightarrow B = 7-A$, $2A+2B+3C = 20 \rightarrow 2A+14-2A+3C = 20 \rightarrow 3C = 6 \rightarrow C=2$, $10A+B+3C = 22 \rightarrow$

$\rightarrow 10A + 7 - A + 6 = 22 \rightarrow 9A = 9 \rightarrow A = 1$, $B = 6$

$$\boxed{f(t) = e^{-t} + 6e^{-t}\cos(3t) + 2e^{-t}\sin(3t)}$$