Ma 221 - Differential Equations **Homework 10** **Due: Dec 6, 2021**

See the canvas assignment page for detailed instructions on submitting your work.

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1. Solve the initial value problem, $\left(1/x + 2y^2x\right)dx + \left(2yx^2 - \cos y\right)dy = 0$, with $y(1) = \pi$.

2. Consider the first-order ODE, $\left(2y^2 - 3xy\right)dx + \left(4xy - 3x^2\right)dy = 0$. Determine values of m and n such that $\mu(x, y) = x^n y^m$ works as an *integrating factor* for the ODE. Then solve the transformed equation.

3. A mathematical model for a falling chain in the absence of resistive forces is given by the ODE,

$$xv\frac{dv}{dx} + v^2 = gx.$$

Here $x > 0$ is the length of the chain hanging over the edge of the platform and v is the velocity of the chain. (x is in units of feet (ft), v is in ft/s, and $g = 32\,\mathrm{ft/s^2}$ is the gravitational acceleration.)

   (a) Find a one-parameter family of solutions using the method for Bernoulli equations.

   (b) With 3 feet of the chain hanging over the edge, the chain is falling at a rate of 2 ft/sec. Determine the speed of the falling chain at the point when its length is 6 feet.

4. Consider the following example of the *logistic* equation. This equation is used as a simple model for the growth rate of a single species population, $P(t)$, that includes competition for limited resources.

$$\frac{dP}{dt} = \frac{P(6 - P)}{12}$$

   (a) Use phase line analysis to classify the stability of the equilibrium solutions.

   (b) Use the method for Bernoulli equations to find the general solution for $P(t)$. Determine $\lim_{t\to+\infty} P(t)$ for positive initial conditions, $P(0) > 0$. Compare this with your phase line analysis in part (a).