Ma 221 - Differential Equations **Homework 1** **Due: Sep 13, 2021**

See the canvas assignment page for detailed instructions on submitting your work.

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1. Consider the first-order ODE, $\dfrac{dy}{dt} = y^2 - 4$.

   (a) Verify that $y = 2 \cdot \dfrac{1 - Ce^{4t}}{1 + Ce^{4t}}$ is a family of solutions to the ODE (C is any constant).

   (b) For each of the initial conditions, $y(0) = 0$ and $y(0) = 4$, answer the following questions:

   - find the (unique) solution, $y = \phi(t)$, satisfying the initial condition and determine its existence interval (a, b);
   - Evaluate the limit of the solution as t approaches the end points of the existence interval. That is, evaluate $\lim_{t\to a^+} \phi(t)$ and $\lim_{t\to b^-} \phi(t)$.

2. Solve the initial value problem (IVP) and determine the largest interval on which the solution is defined (the *existence interval*).

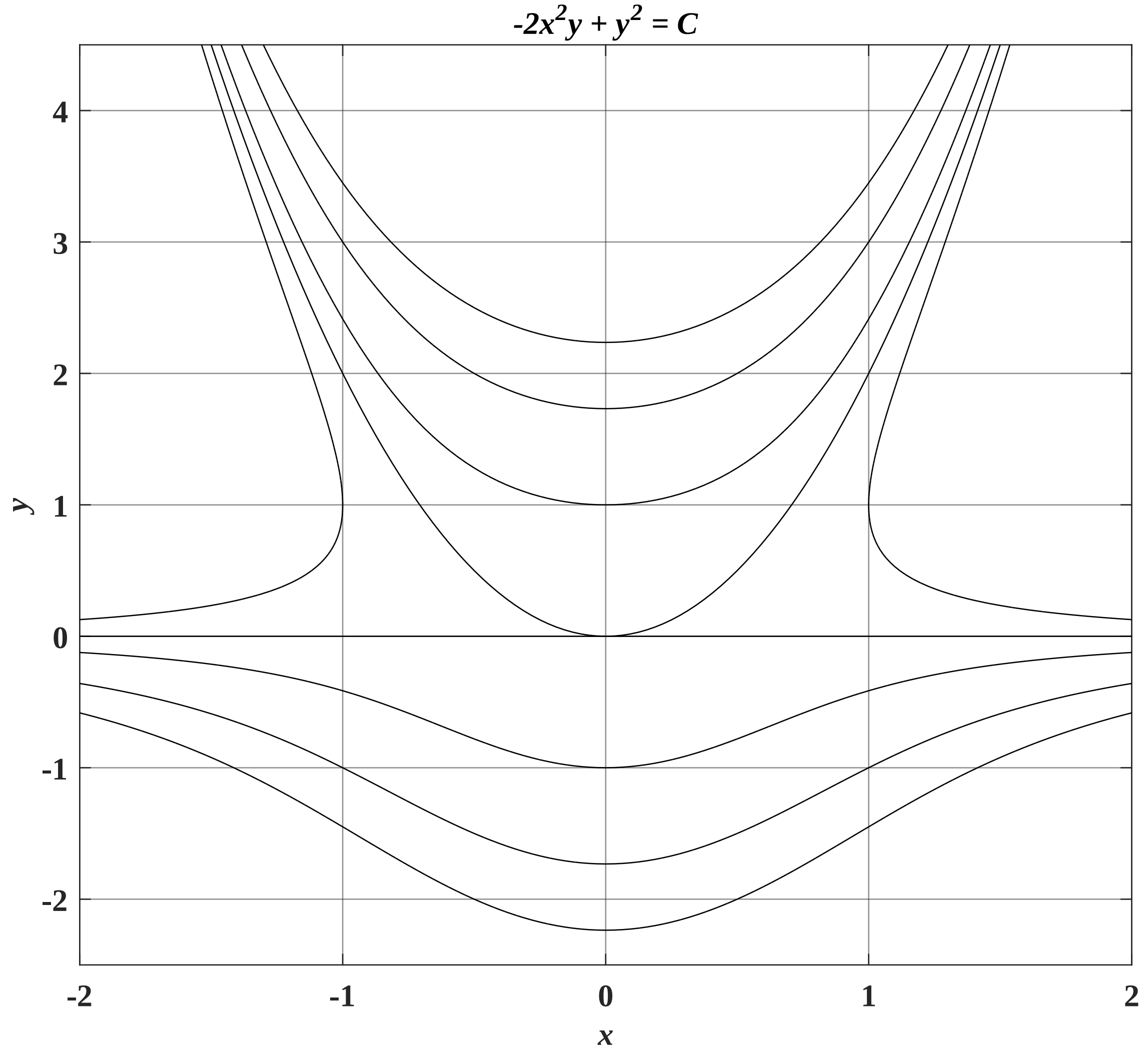
$$\frac{dy}{dt} = 3y^{4/3}, \quad y(0) = 1/8$$

3. Consider the first-order ODE, $(x^2 - y)\dfrac{dy}{dx} + 2xy = 0$ and the function $F(x, y) = -2x^2y + y^2$.

   (a) The figure on the following page shows a sample of level curves, $F(x, y) = C$. Label each of the curves with the appropriate (integer) value of C.

   (b) Verify that the equation $F(x, y) = C$ is an *implicit* solution to the differential equation, for any constant C.

   (c) Identify the curve in the figure that appears to be the solution to the ODE satisfying $y(1) = -1$ and express the solution in explicit form, $y = \phi(x)$.



$-2x^2y + y^2 = C$
4
3
2
1
0
-1
-2
y
-2
-1
0
1
2
x