Aughdan Breslin

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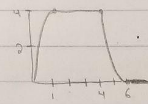
I pledge my honor that I have abided by the Stevens Honor System.

Aughdan Breslin

HW6

1) $g(t) = \begin{cases} -4t(t-2) & 0 \le t < 1 \\ 4 & 1 \le t < 4 \\ (t-6)^2 & 4 \le t < 6 \\ 0 & 6 \le t < \infty \end{cases}$



$$g(t) = -4t(t-2)\left(U(t) - U(t-1)\right) + 4\left(U(t-1) - U(t-4)\right) + (t-6)^2\left(U(t-4) - U(t-6)\right)$$

$$= (-4t^2+8t)U(t) + (4t^2-8t)U(t-1) + 4U(t-1) - 4U(t-4) + (t^2-12t+36)U(t-4) - (t-6)^2U(t-6)$$

$$= (-4t^2+8t)U(t) + 4(t^2-2t+1)U(t-1) + (t^2-12t+32)U(t-4) - (t-6)^2U(t-6)$$

$$= (-4t^2+8t)U(t) + 4(t-1)^2U(t-1) + (t-4)(t-4-4)U(t-4) - (t-6)^2U(t-6)$$

$$= -4t^2U(t) + 8tU(t) + 4(t-1)^2U(t-1) + (t-4)^2U(t-4) - 4(t-4)U(t-4) - (t-6)^2U(t-6)$$

$$\mathcal{L}\{g(t)\} = \mathcal{L}\{-4t^2U(t)\} + \mathcal{L}\{8tU(t)\} + \mathcal{L}\{4(t-1)^2U(t-1)\} + \mathcal{L}\{(t-4)^2U(t-4)\} + \mathcal{L}\{4(t-4)U(t-4)\} + \mathcal{L}\{-(t-6)^2U(t-6)\}$$

$$= \mathcal{L}\{-4t^2\}\cdot 1 + \mathcal{L}\{8t\}\cdot 1 + \mathcal{L}\{4((t+1)-1)^2\}e^{-s} + \mathcal{L}\{((t+4)-4)^2\}e^{-4s} + \mathcal{L}\{4((t+4)-4)\}e^{-4s} + \mathcal{L}\{-((t+6)-6)^2\}e^{-6s}$$

$$= -4\frac{2}{s^3} + 8\frac{1}{s^2} + 4\frac{2}{s^3}e^{-s} + \frac{2}{s^3}e^{-4s} + 4\frac{1}{s^2}e^{-4s} - \frac{2}{s^3}e^{-6s}$$

$$= \boxed{\frac{1}{s^3}\left(-8 + 8e^{-s} + 2e^{-4s} - 2e^{-6s}\right) + \frac{1}{s^2}\left(8 + 4e^{-4s}\right)}$$

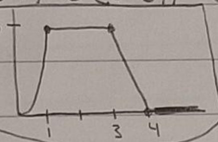
$5t^2 - 10t + 5 + 10t - 10 = 5t^2 - 5 \qquad 5t - 15 = 5t - 20 + 5$

2) $f(t) = 5t^2 U(t) - \left(5(t-1)^2 + 10(t-1)\right)U(t-1) - 5(t-3)U(t-3) + 5(t-4)U(t-4)$

$5t^2U(t) - 5t^2U(t-1) + 5U(t-1) - 5U(t-3) + (-5t+20)U(t-3) - (-5t+20)U(t-4)$

$5t^2\left(U(t) - U(t-1)\right) + 5\left(U(t-1) - U(t-3)\right) + (-5t+20)\left(U(t-3) - U(t-4)\right)$

$$\boxed{f(t) = \begin{cases} 5t^2, & 0 \le t < 1 \\ 5, & 1 \le t < 3 \\ -5t+20, & 3 \le t < 4 \\ 0, & 4 \le t \end{cases}}$$



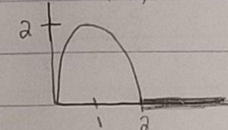
3) a) $G(s) = \frac{-4}{s^3} + \frac{4}{s^2} + \frac{4}{s^3}e^{-2s} + \frac{4}{s^2}e^{-2s} \rightarrow -2L^{-1}\left\{\frac{2}{s^3}\cdot 1\right\} + 4L^{-1}\left\{\frac{1}{s^2}\cdot 1\right\} + 2L^{-1}\left\{\frac{2}{s^3}e^{-2s}\right\} + 4L^{-1}\left\{\frac{1}{s^2}e^{-2s}\right\}$

$g(t) = (-2t^2 + 4t)U(t) + 2(t-2)^2U(t-2) + 4(t-2)U(t-2)$

$g(t) = -2t^2U(t) + 4tU(t) + 2t^2U(t-2) - 8tU(t-2) + 8U(t-2) + 4tU(t-2) - 8U(t-2)$

$g(t) = -2t^2\left(U(t) - U(t-2)\right) + 4t\left(U(t) - U(t-2)\right)$

$$\boxed{\begin{aligned} g(t) &= (-2t^2 + 4t)\left(U(t) - U(t-2)\right) \\ g(t) &= \begin{cases} -2t^2 + 4t, & 0 \le t < 2 \\ 0, & 2 \le t < \infty \end{cases} \end{aligned}}$$

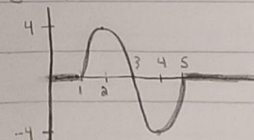


b) $G(s) = \frac{2\pi}{s^2 + \frac{\pi^2}{4}}e^{-s} - \frac{2\pi}{s^2 + \frac{\pi^2}{4}}e^{-5s} \rightarrow 4L^{-1}\left\{\frac{\frac{\pi}{2}}{s^2 + \frac{\pi^2}{4}}e^{-s}\right\} - 4L^{-1}\left\{\frac{\frac{\pi}{2}}{s^2 + \frac{\pi^2}{4}}e^{-5s}\right\}$

$g(t) = 4\sin\left(\frac{\pi}{2}(t-1)\right)U(t-1) - 4\sin\left(\frac{\pi}{2}(t-5)\right)U(t-5)$

$g(t) = -4\cos\left(\frac{\pi t}{2}\right)U(t-1) + 4\cos\left(\frac{\pi t}{2}\right)U(t-5)$

$$\boxed{\begin{aligned} g(t) &= -4\cos\left(\frac{\pi t}{2}\right)\left(U(t-1) - U(t-5)\right) \\ g(t) &= \begin{cases} 0, & 0 \le t < 1 \\ -4\cos\left(\frac{\pi t}{2}\right), & 1 \le t < 5 \\ 0, & 5 \le t < \infty \end{cases} \end{aligned}}$$



4) $y''(t) + y(t) = f(t), \quad y(0) = 0, \; y'(0) = 0 \qquad f(t) = \begin{cases} 1 & 0 \le t < \pi \\ -1 & \pi \le t < 2\pi \\ 0 & 2\pi \le t \end{cases}$

$L\{y''(t) + y(t)\} = L\{1(U(t) - U(t-\pi)) - 1(U(t-\pi) - U(t-2\pi))\}$

$s^2Y(s) - sy(0) - y'(0) + Y(s) = L\{U(t)\} + L\{-2U(t-\pi)\} + L\{U(t-2\pi)\}$

$Y(s)(s^2+1) = \frac{1}{s} - 2\frac{e^{-\pi s}}{s} + \frac{e^{-2\pi s}}{s} = \frac{e^{-2\pi s} - 2e^{-\pi s} + 1}{s}$

$Y(s) = \frac{e^{-2\pi s} - 2e^{-\pi s} + 1}{s(s^2+1)} \rightarrow \quad \cdot \; L^{-1}\left\{\frac{1}{s(s^2+1)} e^{-2\pi s}\right\} = L^{-1}\left\{\frac{A}{s} + \frac{Bs+C}{s^2+1}\right\} U(t-2\pi)$

$A L^{-1}\left\{\frac{1}{s}\right\} + B L^{-1}\left\{\frac{s}{s^2+1}\right\} + C L^{-1}\left\{\frac{1}{s^2+1}\right\} = A + B\cos(t) + C\sin(t)$ Find ABC

$A(s^2+1) + Bs^2 + Cs = 1 \rightarrow As^2 + A + Bs^2 + Cs = 1 \rightarrow A + B = 0,\; B = -A,\; \underline{C=0},\; \underline{A=1},\; \underline{B=-1}$

$\cdot \; (1 - \cos(t))U(t-2\pi) = (1-\cos(t))U(t-2\pi)$

$\cdot \; L^{-1}\left\{\frac{-2}{s(s^2+1)} e^{-\pi s}\right\} = L^{-1}\left\{\frac{A}{s} + \frac{Bs+C}{(s^2+1)}\right\} U(t-\pi)$

$A L^{-1}\left\{\frac{1}{s}\right\} + B L^{-1}\left\{\frac{s}{s^2+1}\right\} + C L^{-1}\left\{\frac{1}{s^2+1}\right\} = A + B\cos(t-\pi) + C\sin(t-\pi)$

$A(s^2+1) + Bs^2 + Cs = -2 \rightarrow As^2 + A + Bs^2 + Cs = -2 \rightarrow A+B=0,\; B=-A,\; \underline{C=0},\; \underline{A=-2},\; \underline{B=2}$

$\cdot \; (-2 + 2\cos(t-\pi))U(t-\pi) = (-2 - 2\cos(t))U(t-\pi)$

$\cdot \; L^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = L^{-1}\left\{\frac{A}{s} + \frac{Bs+C}{s^2+1}\right\} = A L^{-1}\left\{\frac{1}{s}\right\} + B L^{-1}\left\{\frac{s}{s^2+1}\right\} + C L^{-1}\left\{\frac{1}{s^2+1}\right\} = A + B\cos(t) + C\sin(t)$

$A(s^2+1) + Bs^2 + Cs = 1 \rightarrow As^2 + A + Bs^2 + Cs = 1 \rightarrow A+B=0,\; B=-A,\; \underline{C=0},\; \underline{A=1},\; \underline{B=-1}$

$\cdot \; (1 - \cos(t))U(t)$

$y(t) = (1-\cos(t))U(t-2\pi) + (-2+2\cos(t))U(t-\pi) + (1-\cos(t))U(t)$

$\boxed{y(t) = (1-\cos(t))(U(t) - 2U(t-\pi) + U(t-2\pi))}$