Aughdon Brelin

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I pledge my honor that I have abided by the Stevens Honor System.

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HW8

1) Consider $f(x) = 1-(x-1)^2$ defined on $0<x<2$. $a_0 = \frac{2}{L}\int_0^L f(x)\,dx$

a) cos: $f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$ $[0,L]$ where $a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

$a_0 = \frac{2}{2}\int_0^2 \left(1-(x^2-2x+1)\right)dx = \int_0^2 -x^2+2x\,dx = \frac{-1}{3}(2^3-0^3) + \frac{2}{2}(2^2-0^2) = \frac{-8}{3}+4 = \boxed{\frac{4}{3}}$

$a_n = \frac{2}{2}\int_0^2 (-x^2+2x)\cos\left(\frac{n\pi x}{2}\right)dx = \int_0^2 -x^2\cos\left(\frac{n\pi x}{2}\right)dx + \int_0^2 2x\cos\left(\frac{n\pi x}{2}\right)dx$

• $-\int_0^2 x^2\cos\left(\frac{n\pi x}{2}\right)dx \to$ $u=x^2$, $du=2x\,dx$, $dv=\cos\left(\frac{n\pi x}{2}\right)dx$, $v=\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)$ $\to -\left(x^2\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right) - \int\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)2x\,dx\right)$

$\to \frac{-2x^2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\Big]_0^2 + \frac{4}{n\pi}\int_0^2 x\sin\left(\frac{n\pi x}{2}\right)dx \to$ $u=x$, $du=dx$, $dv=\sin\left(\frac{n\pi x}{2}\right)dx$, $v=\frac{-2}{n\pi}\cos\left(\frac{n\pi x}{2}\right)$ $\to \frac{4}{n\pi}\left(\frac{-2x}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\Big]_0^2 - \frac{-2}{n\pi}\int_0^2\cos\left(\frac{n\pi x}{2}\right)dx\right.$

$\to \frac{-2(4)}{n\pi}\sin(n\pi) - 0 + \frac{4}{n\pi}\left(\frac{-4}{n\pi}\cos(n\pi) - 0 + \frac{2}{n\pi}\left(\frac{2}{n\pi}\sin(n\pi) - 0\right)\right.$

$\to \frac{-16}{n^2\pi^2}(-1)^n$

• $2\int_0^2 x\cos\left(\frac{n\pi x}{2}\right)dx \to$ $u=x$, $du=dx$, $dv=\cos\left(\frac{n\pi x}{2}\right)dx$, $v=\frac{2}{n\pi}\sin\left(\frac{n\pi x}{2}\right)$ $\to 2\left(\frac{2x}{n\pi}\sin\left(\frac{n\pi x}{2}\right)\Big]_0^2 - \frac{2}{n\pi}\int_0^2\sin\left(\frac{n\pi x}{2}\right)dx\right)$

$\to 2\left(\frac{4}{n\pi}\sin(n\pi) - 0 - \frac{2}{n\pi}\left(\frac{-2}{n\pi}\cos(n\pi) + \frac{+2}{n\pi}(1)\right)\right)$

$\to \frac{8}{n^2\pi^2}(-1)^n - \frac{8}{n^2\pi^2} = \frac{8}{n^2\pi^2}\left((-1)^n - 1\right)$

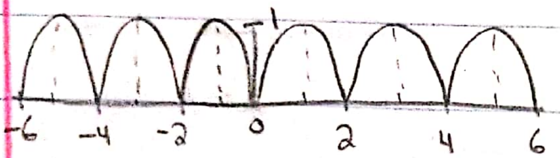
$a_n = \boxed{\frac{-8(-1)^n - 8}{n^2\pi^2}}$

$\boxed{\text{cos: } f(x) \sim \frac{2}{3} + \sum_{n=1}^{\infty}\left(\frac{-8(-1)^n-8}{n^2\pi^2}\right)\cos\left(\frac{n\pi x}{2}\right)}$

$f_4(x) \sim \frac{2}{3} + 0 + \frac{-16}{4\pi^2}\cos(\pi x) + 0 + \frac{-16}{16\pi^2}\cos(2\pi x) + 0 + \frac{-16}{36\pi^2}\cos(3\pi x)$

n=0 n=1 n=2 n=3 n=4 n=5 n=6

$\boxed{f_4(x) \sim \frac{2}{3} - \frac{4}{\pi^2}\cos(\pi x) - \frac{1}{\pi^2}\cos(2\pi x) - \frac{4}{9\pi^2}\cos(3\pi x)}$

b) 

c) $T = \frac{2\pi}{\omega}$ $\qquad \omega = \pi, 2\pi, 3\pi \quad T = 2, 1, \frac{2}{3}$

$\frac{T_1}{T_2} = 2 \qquad \frac{T_1}{T_3} = 3 \qquad \frac{1\,2\,3\ldots}{1\,2\,3\ldots}$ LCM = 1

$T_0 = LCM^{*}\, T_1 = 1^{*} 2 = 2 \qquad \omega_0 = \frac{2\pi}{T_0} = \pi \qquad \boxed{T_0 = 2 \quad \omega_0 = \pi}$

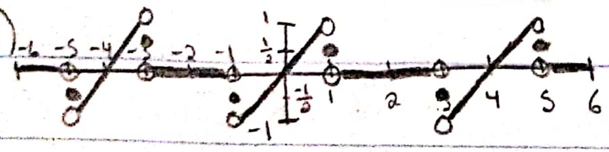
2) $f(x) = \begin{cases} x & 0<x<1 \\ 0 & 1 \le x < 2 \end{cases}$ defined on $0<x<2$

a) sin: $f(x) \sim \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right) \qquad b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

$b_n = \frac{2}{2}\int_0^1 x\sin\left(\frac{n\pi x}{2}\right)dx \rightarrow \begin{matrix} u = x \\ du = dx \end{matrix} \quad \begin{matrix} dv = \sin\left(\frac{n\pi x}{2}\right)dx \\ v = \frac{-2}{n\pi}\cos\left(\frac{n\pi x}{2}\right) \end{matrix} \rightarrow \frac{-2x}{n\pi}\cos\left(\frac{n\pi x}{2}\right)\Big]_0^1 + \frac{2}{n\pi}\int_0^1 \cos\left(\frac{n\pi x}{2}\right)dx$

$= \frac{-2}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - 0\right) + \frac{2}{n\pi}\left(\frac{2}{n\pi}\left(\sin\left(\frac{n\pi}{2}\right) - 0\right)\right) = \frac{-2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$

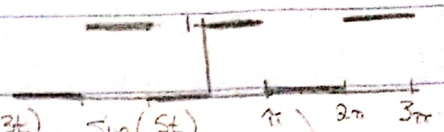
$$\boxed{f(x) \sim \sum_{n=1}^{\infty}\left(\frac{-2}{n\pi}\cos\left(\frac{n\pi}{2}\right) + \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)\right)\sin\left(\frac{n\pi x}{2}\right)}$$

b) 

At jump discontinuities: for $y>0 \rightarrow y = \frac{1}{2}$

for $y<0 \rightarrow y = \frac{-1}{2}$

3) $L[y] = y'' + 2y = f(t)$ where $f(t) = \begin{cases} 0 & -\pi < x < 0 \\ 1 & 0 < x < \pi \end{cases}$



$f(t) \sim \frac{1}{2} + \sum_{k=1}^{\infty} \frac{2}{(2k-1)\pi} \sin((2k-1)t) = \frac{1}{2} + \frac{2}{\pi}\left(\sin(t) + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5} + \dots\right)$

a) $f_0(t) = \frac{1}{2}$ $\quad f_1(t) = \frac{2}{\pi}\sin(t)$ $\quad f_2(t) = \frac{2}{3\pi}\sin(3t), \dots$

- $k = 0 \to y'' + 2y = f_0(t) \to y'' + 2y = \frac{1}{2} \to y_p = A$ $\quad g(x) = \frac{1}{2}$ $\quad y_p' = y_p'' = 0 \to$

$\to 0 + 2A = \frac{1}{2} \to A = \frac{1}{4} \to \boxed{y_0 = \frac{1}{4}}$

- $k = 1 \to y'' + 2y = f_1(t) \to y'' + 2y = \frac{2}{\pi}\sin(t) \to$

$y_p = A\cos(t) + B\sin(t)$ $\quad g(x) = \frac{2}{\pi}\sin(t)$

$y_p' = -A\sin(t) + B\cos(t) \to (-A + 2A)\cos(t) + (-B + 2B)\sin(t) = \frac{2}{\pi}\sin(t)$

$y_p'' = -A\cos(t) - B\sin(t)$ $\quad A = 0 \quad B = \frac{2}{\pi}$ $\quad \boxed{y_1 = \frac{2}{\pi}\sin(t)}$

- $k = 2 \to y'' + 2y = f_2(t) \to y'' + 2y = \frac{2}{3\pi}\sin(3t)$

$y_p = A\cos(3t) + B\sin(3t)$ $\quad g(x) = \frac{2}{3\pi}\sin(3t)$

$y_p'' = -9A\cos(3t) - 9B\sin(3t) \to (-9A + 2A)\cos(3t) + (-9B + 2B)\sin(3t) = \frac{2}{3\pi}\sin(3t)$

$A = 0 \quad B = \frac{-2}{21\pi}$ $\quad \boxed{y_2 = \frac{-2}{21\pi}\sin(3t)}$

- $k = 3 \to y'' + 2y = f_3(t) \to y'' + 2y = \frac{2}{5\pi}\sin(5t)$

$y_p = A\cos(5t) + B\sin(5t)$ $\quad g(x) = \frac{2}{5\pi}\sin(5t)$

$y_p'' = -25A\cos(5t) - 25B\sin(5t) \to (-25 + 2)A\cos(5t) + (-25 + 2)B\sin(5t) = \frac{2}{5\pi}\sin(5t)$

$A = 0 \quad B = \frac{-2}{115\pi}$ $\quad \boxed{y_3 = \frac{-2}{115\pi}\sin(5t)}$

$y_p(t) = y_0 + y_1 + y_2 + y_3 + \dots$

$\boxed{y_p(t) = \frac{1}{4} + \frac{2}{\pi}\sin(t) - \frac{2}{21\pi}\sin(3t) - \frac{2}{115\pi}\sin(5t) + \dots}$

1st term: $\frac{1}{4}$ $\qquad$ 3rd term: $\frac{-2}{21\pi}\sin(3t)$

2nd term: $\frac{2}{\pi}\sin(t)$ $\qquad$ 4th term: $\frac{-2}{115\pi}\sin(5t)$