
See the canvas assignment page for detailed instructions on submitting your work.

1. **Laplace Transforms from the definition:** $F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st} dt$

Sketch the graph of the function $f(t)$ and calculate its Laplace Transform directly from the definition as an improper integral. *Be sure to simplify (clean up) your final answer.*

$$f(t) = \begin{cases} 2t, & 0 \leq t < 2 \\ 4, & 2 \leq t < 5 \\ 0, & 5 \leq t < \infty \end{cases}$$

2. Determine the inverse Laplace transforms.

(a) $f(t) = \mathcal{L}^{-1} \left\{ \frac{3}{s^5} \right\}$

(b) $g(t) = \mathcal{L}^{-1} \left\{ \frac{-3s + 4}{s^2 + 9} \right\}$

(c) $h(t) = \mathcal{L}^{-1} \left\{ \frac{-3s + 4}{s^2 + 4s + 20} \right\}$

3. Determine the inverse Laplace transforms.

(a) $f(t) = \mathcal{L}^{-1} \left\{ \frac{3s^2 + 8s + 9}{(s + 2)^3} \right\}$

(b) $y(t) = \mathcal{L}^{-1} \left\{ \frac{s^2 + 11s + 20}{(s^2 + 4s + 8)(s + 1)^2} \right\}.$

4. Use the Laplace transform to solve the following initial value problem for $y(t)$.

$$y''(t) + 2y'(t) + 10y = 9e^{-t}, \quad y(0) = 7, \quad y'(0) = -1.$$