Aughdon Breslin

Patrick Miller

I pledge my honor that I have abided by the Stevens Honor System.

Aughdon B

Homework 10

1) $\underbrace{(x^{-1} + 2xy^2)}_{M}dx + \underbrace{(2x^2y - \cos y)}_{N}dy = 0$, $y(1) = \pi$

Check $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \to 4xy = 4xy$ ✓ Exact ODE

$\frac{\partial F}{\partial x} = M \to F = \int M dx \to F = \ln x + x^2y^2 + g(y)$

$N = 2x^2y - \cos y = \frac{\partial F}{\partial y} = 2x^2y + g'(y)$ $\int g'(y) = \int -\cos y \to g(y) = -\sin y$

$F = \ln x + x^2y^2 - \sin y = C$

$F(1, \pi) = \ln(1) + 1^2\pi^2 - \sin\pi = C \to \pi^2 - \sin\pi = C$

$\boxed{\ln x + x^2y^2 - \sin y = \pi^2}$

2) $\underbrace{(2y^2 - 3xy)}_{M}dx + \underbrace{(4xy - 3x^2)}_{N}dy = 0$

Check $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \to 4y - 3x = 4y - 6x$ X Non-exact ODE

$\underbrace{(2x^n y^{m+2} - 3x^{n+1}y^{m+1})}_{M}dx + \underbrace{(4x^{n+1}y^{m+1} - 3x^{n+2}y^m)}_{N}dy$

Check $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \to 2(m+2)x^n y^{m+1} - 3(m+1)x^{n+1}y^m = 4(n+1)x^n y^{m+1} - 3(n+2)x^{n+1}y^m$

$2m + 4 = 4n + 4 \to m = 2n \to m = 2$ $-3m - 3 = -3n - 6 \to -3(2n) - 3 = -3n - 6 \to -6n - 3 = -3n - 6$

$-3n = -3 \to n = 1$

$\mu = x^1y^2 \to (2xy^4 - 3x^2y^3)dx + (4x^2y^3 - 3x^3y^2)dy$

Check $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \to 8xy^3 - 9x^2y^2 = 8xy^3 - 9x^2y^2$ ✓ Exact ODE

$\frac{\partial F}{\partial x} = M \to F = \int M dx = \int 2xy^4 - 3x^2y^3 dx \to x^2y^4 - x^3y^3 + g(y)$

$N = 4x^2y^3 - 3x^3y^2 = \frac{\partial F}{\partial y} = 4x^2y^3 - 3x^3y^2 + g'(y) \to g'(y) = 0 \to g(y) = c$

$\boxed{F(x,y) = x^2y^4 - x^3y^3 + c}$

$$\frac{dy}{dx} + P(x)y = f(x)y^n$$

3) $xv\frac{dv}{dx} + v^2 = 32x \rightarrow \frac{dv}{dx} + \frac{1}{x}v = 32v^{-1}$ $\quad n=-1 \quad w = v^{1--1} = v^2 \quad \frac{dw}{dx} = 2v\frac{dv}{dx}$

$v\frac{dv}{dx} + \frac{1}{x}v^2 = 32 \rightarrow \frac{1}{2}\frac{dw}{dx} + \frac{1}{x}w = 32 \rightarrow \frac{dw}{dx} + \frac{2}{x}w = 64 \quad \mu = e^{\int P(x)dx} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

$\frac{d}{dx}(x^2 w) = 64x^2 \rightarrow x^2 w = \int 64x^2 dx \rightarrow x^2 w = \frac{64}{3}x^3 + C \rightarrow w = \frac{64}{3}x + \frac{C}{x^2}$

$v^2 = \frac{64}{3}x + \frac{C}{x^2} \rightarrow \boxed{v = \sqrt{\frac{64}{3}x + \frac{C}{x^2}}}$

b) $v(3) = 2 \quad v(6) = ? \rightarrow 2^2 = \frac{64}{3}(3) + \frac{C}{3^2} \rightarrow 4 = 64 + \frac{C}{9} \rightarrow -60 = \frac{C}{9} \rightarrow C = -540$

$v = \sqrt{\frac{64}{3}x - \frac{540}{x^2}} \rightarrow v = \sqrt{128 - 15} \rightarrow \boxed{v = \sqrt{113}}$

4) $\frac{dP}{dt} = \frac{P(6-P)}{12}$

a) $\frac{6P - P^2}{12} = 0 \rightarrow 6P - P^2 = 0 \rightarrow P = 0, 6$

$\boxed{\text{stable: } 6 \quad \text{unstable: } 0}$

b) $\frac{dP}{dt} = \frac{1}{2}P - \frac{1}{12}P^2 \rightarrow \frac{dP}{dt} - \frac{1}{2}P = \frac{-1}{12}P^2$ $\quad n=2 \quad w = P^{1-2} = P^{-1} \quad \frac{dw}{dt} = -P^{-2}\frac{dP}{dt}$

$P^{-2}\frac{dP}{dt} - \frac{1}{2}P^{-1} = \frac{-1}{12} \rightarrow -\frac{dw}{dt} - \frac{1}{2}w = \frac{-1}{12} \rightarrow \frac{dw}{dt} + \frac{1}{2}w = \frac{1}{12} \rightarrow \mu = e^{\int P(t)dt} = e^{\int \frac{1}{2}dt} = e^{\frac{1}{2}t}$

$\frac{d}{dt}(e^{\frac{1}{2}t}w) = \frac{1}{12}e^{\frac{1}{2}t} \rightarrow e^{\frac{1}{2}t}w = 2\int \frac{1}{12}e^{\frac{1}{2}t}\frac{1}{2}dt \rightarrow e^{\frac{1}{2}t}w = \frac{1}{6}e^{\frac{1}{2}t} + C \rightarrow w = \frac{e^{\frac{1}{2}t} + C}{6e^{\frac{1}{2}t}}$

$P^{-1} = \frac{e^{\frac{1}{2}t} + C}{6e^{\frac{1}{2}t}} \rightarrow P = \frac{6e^{\frac{1}{2}t}}{e^{\frac{1}{2}t} + C}$