
See the canvas assignment page for detailed instructions on submitting your work.

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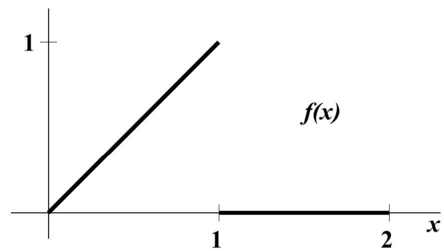
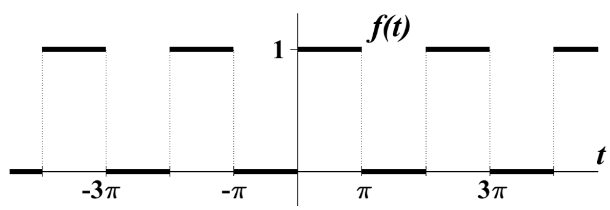
1. Consider the function $f(x) = 1 - (x-1)^2$ defined on the interval $0 < x < 2$.
   (a) Derive a general expression for the coefficients in the Fourier Cosine series for $f(x)$. Then write out the Fourier series through the *first four nonzero terms*.
   (b) Graph the extension of $f(x)$ on the interval $(-6, 6)$ that represents the pointwise convergence of the Cosine series in (a). At jump discontinuities, identify the value to which the series converges.
   (c) Identify the fundamental frequency and period of the Fourier Series.

2. Consider the function $f(x)$ defined on $0 < x < 2$ (*see graph*).
   (a) Derive a general expression for the coefficients in the Fourier Sine series, $f(x) \sim \sum_{n=1}^{\infty} b_n \sin(n\pi x/2)$.
   (b) Graph the extension of $f(x)$ on the interval $(-6, 6)$ that represents the pointwise convergence of the Sine series. Identify the limit of the series at jump discontinuities.



3. **Linear Oscillator with Periodic Forcing.**

   Consider the second-order differential equation, $L[y] = y'' + 2y = f(t)$, where $f(t)$ is the periodic square wave in the figure.



   The Fourier trigonometric series for $f(t)$ is given by,

$$f(t) \sim \frac{1}{2} + \sum_{k=1}^{\infty} \frac{2}{(2k-1)\pi} \sin\big((2k-1)t\big) = \frac{1}{2} + \frac{2}{\pi}\left(\sin(t) + \frac{\sin(3t)}{3} + \frac{\sin(5t)}{5} + \cdots\right).$$

   (a) Derive a Fourier series representation for a *particular solution* to $L[y] = f(t)$, as follows:
   - For $k = 0, 1, 2, \ldots$, find a particular solution $y_k(t)$ satisfying $y_k'' + 2y_k = f_k(t)$, where $f_k(t)$ is the k^{th} term in the Fourier series for $f(t)$.
   - Sum over all $y_k(t)$ to get the solution $y_p(t)$.

   (b) Write out the *first four terms* in the Fourier series for $y_p(t)$.