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I pledge my honor that I have abided by the Stevens Honor System.

HW1

1) $\frac{dy}{dt} = y^2 - 4$

c) $y = 2\frac{1-ce^{4t}}{1+ce^{4t}}$ $\quad \frac{dy}{dt} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} = 2\left(\frac{-4ce^{4t}(1+ce^{4t}) - (1-ce^{4t})4ce^{4t}}{(1+ce^{4t})^2}\right)$

$= 2\left(\frac{-4ce^{4t} - 4c^2e^{8t} - 4ce^{4t} + 4c^2e^{8t}}{1+2ce^{4t}+c^2e^{8t}}\right) = 2\left(\frac{-8ce^{4t}}{c^2e^{8t}+2ce^{4t}+1}\right) = \frac{-16ce^{4t}}{c^2e^{8t}+2ce^{4t}+1}$

$y^2 - 4 \rightarrow \left(2\frac{1-ce^{4t}}{1+ce^{4t}}\right)^2 - 4 \rightarrow 4\frac{c^2e^{8t}-2ce^{4t}+1}{c^2e^{8t}+2ce^{4t}+1} - 4\frac{c^2e^{8t}+2ce^{4t}+1}{c^2e^{8t}+2ce^{4t}+1} \rightarrow \frac{4c^2e^{8t}-8ce^{4t}+4-4c^2e^{8t}-8ce^{4t}-4}{c^2e^{8t}+2ce^{4t}+1}$

$\rightarrow \frac{-16ce^{4t}}{c^2e^{8t}+2ce^{4t}+1}$ $\quad \frac{dy}{dt} = y^2 - 4 \rightarrow \frac{-16ce^{4t}}{c^2e^{8t}+2ce^{4t}+1} = \frac{-16ce^{4t}}{c^2e^{8t}+2ce^{4t}+1}$ ✓

b) $y = 2\frac{1-ce^{4t}}{1+ce^{4t}}$; $y(0) = 0 \rightarrow 0 = 2\frac{1-ce^{0}}{1+ce^{0}} = 2\frac{1-c}{1+c} = 0 \rightarrow 1-c = 0$

$\rightarrow c = 1$, $c \neq -1$ $\quad (-\infty,-1) \cup (-1,+\infty)$ $\quad \boxed{I:(-1,+\infty)}$

$\lim_{t\to -1} 2\frac{1-e^{-4}}{1+e^{-4}} = \boxed{\frac{2-2e^{-4}}{1+e^{-4}}}$ $\qquad \lim_{t\to+\infty} 2\frac{1-e^{4t}}{1+e^{4t}} \xrightarrow{LHR} \lim_{t\to+\infty} 2\frac{-4e^{4t}}{4e^{4t}} = \boxed{-2}$

$y = 2\frac{1-ce^{4t}}{1+ce^{4t}}$; $y(0) = 4 \rightarrow 4 = 2\left(\frac{1-ce^{0}}{1+ce^{0}}\right) \rightarrow 4 = 2\left(\frac{1-c}{1+c}\right) \rightarrow 2 = \left(\frac{1-c}{1+c}\right)$

$\rightarrow 2+2c = 1-c \rightarrow 1 = -3c \rightarrow c = \frac{1}{3}$ $\quad c \neq -1$ $\quad \boxed{I:(-1,+\infty)}$

$\lim_{t\to -1} 2\frac{1+\frac{1}{3}e^{-4}}{1-\frac{1}{3}e^{-4}} = \boxed{\frac{2+\frac{2}{3}e^{-4}}{1-\frac{1}{3}e^{-4}}}$ $\qquad \lim_{t\to+\infty} 2\frac{1+\frac{1}{3}e^{4t}}{1-\frac{1}{3}e^{4t}} \underset{LHR}{\Rightarrow} \lim_{t\to+\infty} 2\frac{\frac{4}{3}e^{4t}}{-\frac{4}{3}e^{4t}} = \boxed{-2}$

2) $\frac{dy}{dt} = 3y^{4/3}$; $y(0) = \frac{1}{8} \rightarrow \int \frac{1}{3y^{4/3}}dy = \int dt \rightarrow \int \frac{1}{3}y^{-4/3}dy = \int dt \rightarrow$

$\rightarrow -y^{-1/3} + c = t \rightarrow \boxed{-y^{-1/3} - t + c = 0}$

$y(0) = \frac{1}{8} \rightarrow -\left(\frac{1}{8}\right)^{-1/3} - 0 + c = 0 \rightarrow c = 2$

$-y^{-1/3} - t + 2 = 0 \rightarrow y^{-1/3} = -t+2 \rightarrow y = (-t+2)^{-1/3}$ $\quad t \neq -2$ $\quad \boxed{I:(-\infty,2)}$

3) $(x^2-y)\frac{dy}{dx}+2xy=0 \qquad F(x,y)=-2x^2y+y^2$

c) On graph

b) $-2x^2y+y^2=C \qquad \frac{d}{dx}(-2x^2y+y^2)=\frac{d}{dx}C=0$

$-2(2xy+x^2\frac{dy}{dx})+2y\frac{dy}{dx}=0 \rightarrow -4xy-2x^2\frac{dy}{dx}+2y\frac{dy}{dx}=0$

$\rightarrow -4xy=\frac{dy}{dx}(2x^2-2y) \rightarrow \frac{dy}{dx}=\frac{-4xy}{2x^2-2y} \rightarrow \frac{dy}{dx}=\frac{-2xy}{2x^2-2y}$

$(x^2-y)(\frac{-2xy}{x^2-y})+2xy=0 \rightarrow -2xy+2xy=0 \rightarrow 0=0 \checkmark$

c) $y(1)=-1, \; -2x^2y+y^2=3 \rightarrow y^2-2x^2y-3=0 \rightarrow y=\frac{-(-2x^2)\pm\sqrt{(-2x^2)^2-4(1)(3)}}{2(1)} \rightarrow$

$\rightarrow y=\frac{2x^2\pm\sqrt{4x^4+12}}{2} \rightarrow y=\frac{2x^2\pm 2\sqrt{x^4+3}}{2} \rightarrow \boxed{y=x^2\pm\sqrt{x^4+3}}$

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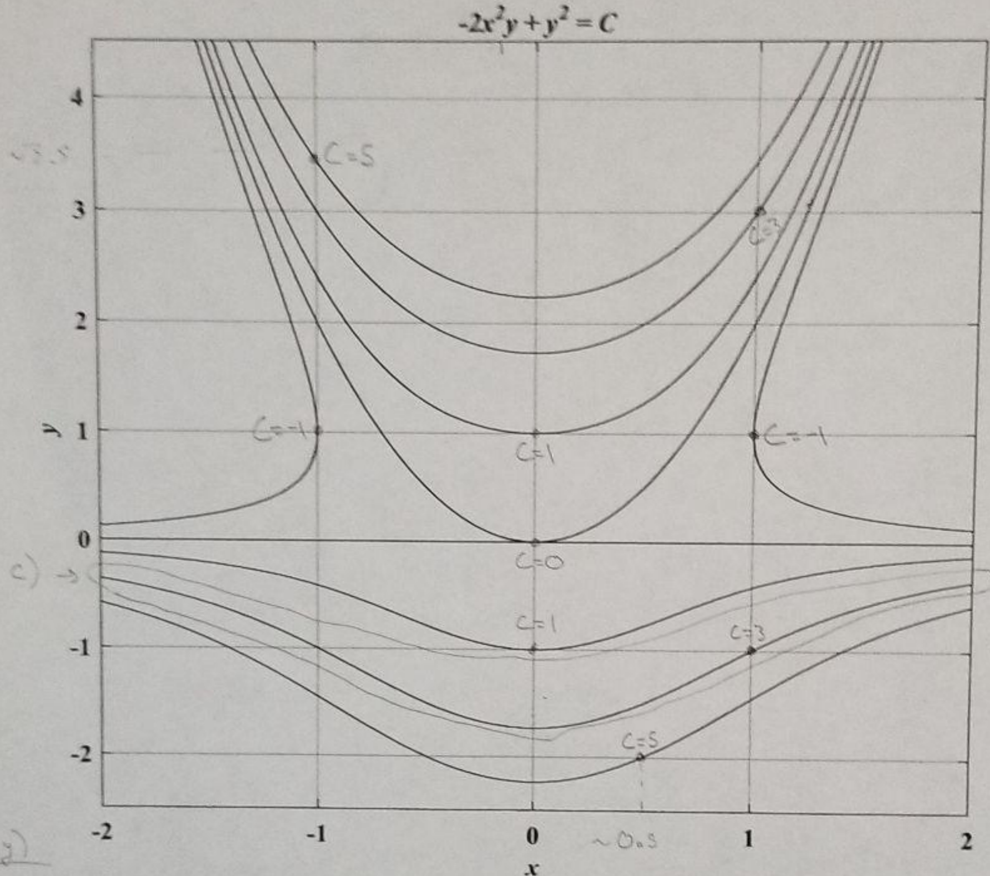

$\underline{C(x,y)}$

$C(0,0) = 0$

$C(0,1) = (1^2) = 1$

$C(1,1) = -2(1)^2(1) + (1)^2 = -1$ $\quad$ $C(-1,1) = -2(-1)^2(1) + (1)^2 = -1$

$C(1,3) = -2(1)^2(3) + (3)^2 = 3$

$C(-1,\frac{7}{2}) = -2(-1)^2(\frac{7}{2}) + (\frac{7}{2})^2 = \frac{21}{4} \approx 5$

$C(0,-1) = -2(0)^2(-1) + (-1)^2 = 1$

$C(1,-1) = -2(1)^2(-1) + (-1)^2 = 3$

$C(\frac{1}{2},-2) = -2(\frac{1}{2})^2(-2) + (-2)^2 = 5$