
See the canvas assignment page for detailed instructions on submitting your work.

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1. Draw the graph of the piecewise function, $g(t)$, and use the methods in 7.3.2 to find its Laplace transform $G(s) = \mathcal{L}\{g(t)\}$.

$$g(t) = \begin{cases} -4t(t-2), & 0 \le t < 1 \\ 4, & 1 \le t < 4 \\ (t-6)^2, & 4 \le t < 6 \\ 0, & 6 \le t \end{cases}$$

2. Consider the function $f(t)$ expressed in terms of unit step functions,

$$f(t) = 5t^2\,\mathcal{U}(t) - \Big(5(t-1)^2 + 10(t-1)\Big)\,\mathcal{U}(t-1) - 5(t-3)\,\mathcal{U}(t-3) + 5(t-4)\,\mathcal{U}(t-4)$$

Represent $f(t)$ in the usual format for a piecewise function and sketch its graph.

3. For each of the transforms $G(s)$,

- express the inverse transform, $g(t) = \mathcal{L}^{-1}\{G(s)\}$, in terms of unit step functions;
- represent $g(t)$ in the usual format for a piecewise function and sketch its graph.

(a) $G(s) = \dfrac{-4}{s^3} + \dfrac{4}{s^2} + \left(\dfrac{4}{s^3} + \dfrac{4}{s^2}\right)e^{-2s}$ (b) $G(s) = \dfrac{2\pi}{s^2 + \pi^2/4}e^{-s} - \dfrac{2\pi}{s^2 + \pi^2/4}e^{-5s}$

4. Use Laplace Transforms to solve the following initial value problem (IVP):

$$y''(t) + y(t) = f(t), \quad y(0) = 0,\ y'(0) = 0, \quad \text{where } f(t) = \begin{cases} 1, & 0 \le t < \pi \\ -1, & \pi \le t < 2\pi \\ 0, & 2\pi \le t \end{cases}$$