Ma 221 - Differential Equations **Homework 2** **Due: Sep 17, 2021**

See the canvas assignment page for detailed instructions on submitting your work.

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1. Find the general solution to the differential equation, $\dfrac{du}{dx} - \dfrac{2}{x}u = 4x$, for $x > 0$.

   Which part of your solution satisfies the equation, $\dfrac{du}{dx} - \dfrac{2}{x}u = 0$?

2. Consider the initial value problem, $L[y] = x^2y'' + xy' + 4y = 0, \quad x > 0, \quad y(1) = 2,\ y'(1) = 3.$

   (a) Verify that $y_1(x) = \cos(2\ln x)$ and $y_2(x) = \sin(2\ln x)$ form a *fundamental set* of solutions on the interval $0 < x < \infty$.

   - It is enough to verify either $L[y_1] = 0$ or $L[y_2] = 0$. Not necessary to show both cases; the calculations are essentially the same.
   - Use the Wronskian to verify the linear independence of y_1 and y_2 on $(0, \infty)$.

   (b) Determine the unique solution to the initial value problem.

3. Consider the first-order ODE, $x^2\dfrac{dy}{dx} = y - 2xy + \dfrac{1}{x^2}$, for $x > 0$.

   (a) Find a one-parameter family of solutions (the *general solution*) to the ODE, expressed in explicit form, $y = \phi(x)$. Confirm that the existence interval is $(0, \infty)$ for any choice of the free parameter.

   (b) Evaluate the limits, $\lim_{x\to 0^+} \phi(x)$ and $\lim_{x\to\infty} \phi(x)$.