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I pledge my honor that I have abided by the Stevens Honor System.

HW2

1) Find gen solution to $\frac{du}{dx} - \frac{2}{x}u = 4x$; $x>0$ → $\mu = e^{\int \frac{-2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2}$

$\int \frac{d}{dx}(\mu u) = \int \mu 4x dx$ → $x^{-2}u = \int \frac{4}{x}dx$ → $x^{-2}u = 4\ln x + c$ → $\boxed{u = x^2 \ln x^4 + cx^2}$

Which part satisfies $\frac{du}{dx} - \frac{2}{x}u = 0$? Plug in cx^2 → $2cx - \frac{2}{x}cx^2$ → $2cx - 2cx = 0$ ✓ $\boxed{cx^2}$

2) $L[y] = x^2y'' + xy' + 4y = 0$; $x>0$ $y(1)=2$ $y'(1)=3$

a) Verify $y_1 = \cos(2\ln x)$, $y_2 = \sin(2\ln x)$ form a Fund. Set of solutions on $0<x<\infty$

· Show $L[y_1]=0$ → $y_1' = \frac{-2}{x}\sin(2\ln x)$ → $y_1'' = \frac{2}{x^2}\sin(2\ln x) - \frac{2}{x}\cdot\frac{2}{x}\cos(2\ln x)$ →

$y_1'' = \frac{2}{x^2}(\sin(2\ln x) - 2\cos(2\ln x))$ Plug in.

$x^2 \cdot \frac{2}{x^2}(\sin(2\ln x) - 2\cos(2\ln x)) + x\cdot\frac{-2}{x}\sin(2\ln x) + 4\cos(2\ln x) = 0$ →

$2\sin(2\ln x) - 4\cos(2\ln x) - 2\sin(2\ln x) + 4\cos(2\ln x) = 0$ → $\boxed{0=0}$ ✓

· $W[y_1, y_2] = \begin{vmatrix} \cos(2\ln x) & \sin(2\ln x) \\ \frac{-2}{x}\sin(2\ln x) & \frac{2}{x}\cos(2\ln x) \end{vmatrix} = \frac{2}{x}\cos^2(\ln x) + \frac{2}{x}\sin^2(\ln x)$

$= \frac{2}{x}(\cos^2(\ln x) + \sin^2(\ln x)) = \frac{2}{x} \neq 0$ on $(0,\infty)$

b) $x^2y'' + xy' + 4y = 0$; $y(1)=2$, $y'(1)=3$, $y = c_1\cos(2\ln x) + c_2\sin(2\ln x)$

$y = c_1\cos(2\ln x) + c_2\sin(2\ln x)$; $y(1)=2$	$y' = \frac{-2c_1}{x}\sin(2\ln x) + \frac{2c_2}{x}\cos(2\ln x)$; $y'(1)=3$
$2 = c_1\cos(0) + c_2\sin(0)$	$3 = -2c_1\sin(0) + 2c_2\cos(0)$
$2 = c_1$ → $c_1 = 2$	$3 = 2c_2$ → $c_2 = \frac{3}{2}$

$$\boxed{y = 2\cos(2\ln x) + \frac{3}{2}\sin(2\ln x)}$$

3) $x^2\frac{dy}{dx} - y + 2xy = \frac{1}{x^2}$; $x > 0 \rightarrow x^2\frac{dy}{dx} + (2x-1)y = \frac{1}{x^2} \rightarrow \frac{dy}{dx} + \frac{2x-1}{x^2}y = \frac{1}{x^4}$

Find gen solution $\mu = e^{\int P(x)dx} = e^{\int(\frac{2}{x} - \frac{1}{x^2})dx} = e^{2\ln x + \frac{1}{x}} = e^{\frac{1}{x}}e^{\ln x^2} = e^{\frac{1}{x}}x^2$

$\frac{d}{dx}(\mu y) = \mu\frac{1}{x^4} \rightarrow \int\frac{d}{dx}(\mu y) = \int e^{\frac{1}{x}}x^2\frac{1}{x^4}dx \rightarrow e^{\frac{1}{x}}x^2 y = -\int e^u du \xrightarrow{u=\frac{1}{x}} e^{\frac{1}{x}}x^2 y = -e^{\frac{1}{x}} + C$

$y = \frac{-1}{x^2} + \frac{C}{e^{1/x}x^2} \rightarrow \boxed{y = \frac{C}{e^{1/x}x^2} - \frac{1}{x^2}}$ $\quad x \neq 0,\ x > 0 \rightarrow (0, \infty)$ $\quad y = \frac{C}{e^{1/x}x^2} - \frac{1}{x^2}\left(\frac{e^{1/x}}{e^{1/x}}\right)$

b) $\lim\limits_{x\to 0^+}\frac{C - e^{1/x}}{e^{1/x}x^2} \xrightarrow{LHR} \lim\limits_{x\to 0^+}\frac{e^{1/x}\frac{1}{x^2}}{e^{1/x}(2x-1)} \rightarrow \lim\limits_{x\to 0^+}\frac{\frac{1}{x^2}}{2x-1} \rightarrow \lim\limits_{x\to 0^+}\frac{1}{2x^3 - x^2} \rightarrow \boxed{-\infty}$

$\lim\limits_{x\to\infty}\frac{C - e^{1/x}}{e^{1/x}x^2} \rightarrow \frac{C - e^0}{e^0\,\infty} \rightarrow \frac{C}{\infty} \rightarrow \boxed{0}$