

See the canvas assignment page for detailed instructions on submitting your work.

1. **Separation of Variables for PDEs.** Use separation of variables to find a product solution, $u(x, y) = X(x) \cdot Y(y)$, for the following first-order PDE:

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} + u$$

2. **Boundary Value Problem with Nonhomogeneous ODE.**

For each choice of $g(x)$ listed below, find *all solutions* to the following boundary value problem.

$$4y'' + \pi^2 y = g(x) \quad \text{for } 0 < x < 2, \quad y'(0) = 0, \quad y'(2) = 0.$$

(a) $g(x) = 0$

(b) $g(x) = x$

3. **An Eigenvalue Problem.**

Consider the following *eigenvalue problem*,

$$y'' + (\lambda - 1)y = 0, \quad \text{for } 0 < x < \pi, \quad y(0) = 0, \quad y'(\pi) = 0.$$

- (a) Identify the values of λ for which the ODE's auxiliary equation has (i) complex-valued roots, (ii) repeated roots, and (iii) distinct real-valued roots.
- (b) Find all eigenvalues and eigenfunctions for case (i) as defined in part (a).
- (c) Verify that there are no eigenvalues for cases (ii) and (iii) as defined in part (a).