

Problem 1 [20pts] Draw $u = [4, 1]$, $w = [-2, 2]$ and $(u + w)$, $(u - w)$ in the plane.

$$u = [4, 1]$$

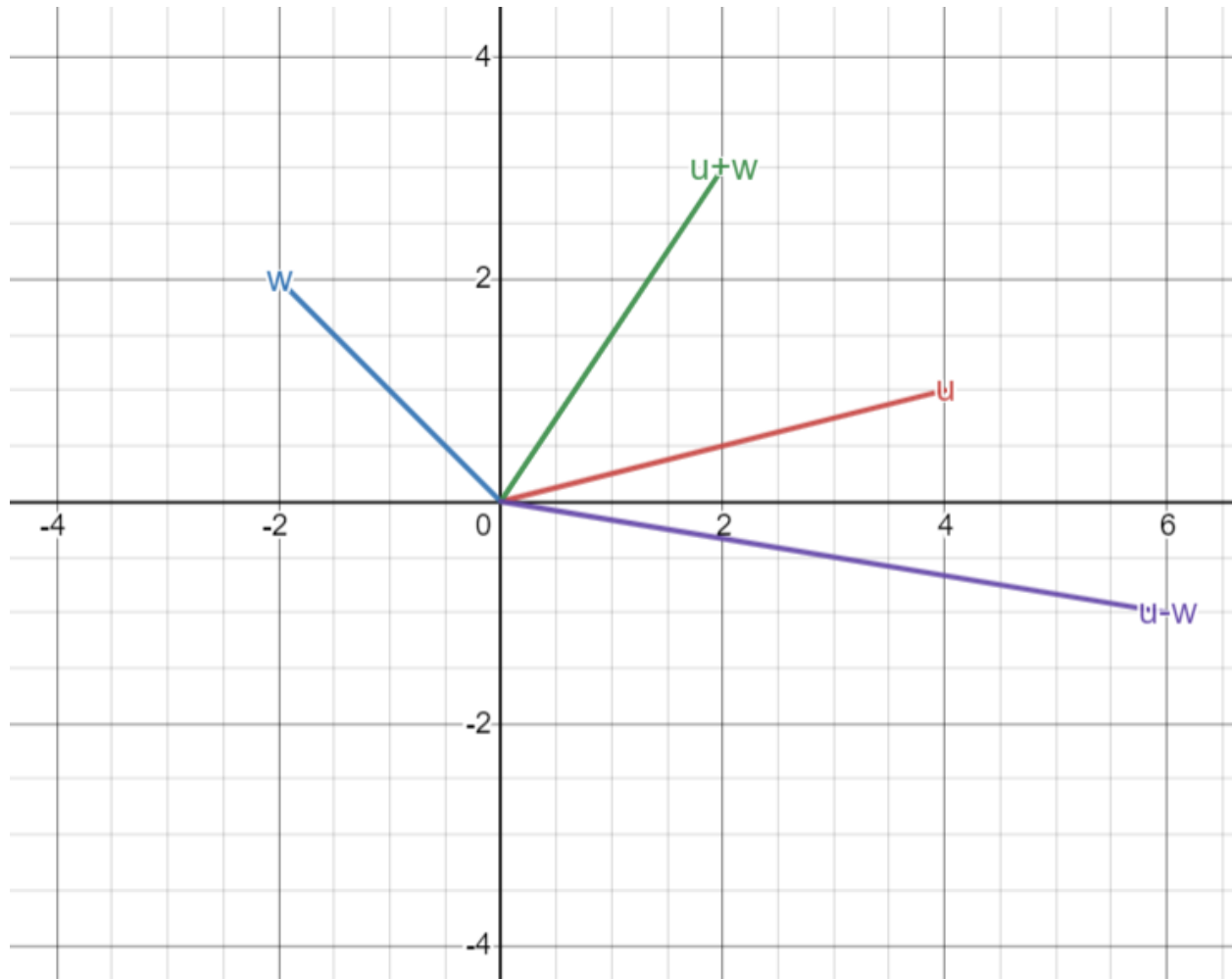
$$w = [-2, 2]$$

$$u+w = [4+(-2), 1+2]$$

$$= [2, 3]$$

$$u-w = [4-(-2), 1-2]$$

$$= [6, -1]$$



Problem 2 [20pts] Find vectors u and w such that $u + w = [4, 5, 6]$ and $u - w = [2, 5, 8]$.

$$u+w + u-w = [4,5,6] + [2,5,8]$$

$$2u = [6,10,14]$$

$$u = [3,5,7]$$

$$u + w = [4,5,6]$$

$$[3,5,7] + w = [4,5,6]$$

$$w = [4,5,6] - [3,5,7]$$

$$w = [1,0,-1]$$

$$u = [3,5,7]$$

$$w = [1,0,-1]$$

Problem 3 [20pts] Find two vectors u and w which are perpendicular to $[1, 0, 1]$ and to each other.

let $z = [1, 0, 1]$

check $u \cdot z = 0$

let $u = [0, 1, 0]$

let $w = u \times z$

check $w \cdot z = 0$

check $u \cdot w = 0$

$$u \cdot z = u_1 z_1 + u_2 z_2 + u_3 z_3 = 0$$

$$u_1 \cdot 1 + u_2 \cdot 0 + u_3 \cdot 1 = 0$$

$$\text{let } u_1 = 0, u_2 = 1, u_3 = 0$$

$$0 \cdot 1 + 1 \cdot 0 + 0 \cdot 1 = 0 + 0 + 0 = 0 \text{ GOOD}$$

$$u = [0, 1, 0]$$

$$w = u \times z = [u_2 z_3 - u_3 z_2, u_3 z_1 - u_1 z_3, u_1 z_2 - u_2 z_1]$$

$$= [1 \cdot 1 - 0 \cdot 0, 0 \cdot 1 - 0 \cdot 1, 0 \cdot 0 - 1 \cdot 1]$$

$$w = [1, 0, -1]$$

$$w \cdot z = w_1 z_1 + w_2 z_2 + w_3 z_3 = 0$$

$$1 \cdot 1 + 0 \cdot 0 + (-1) \cdot 1 = 0 \text{ GOOD}$$

$$u \cdot w = u_1 w_1 + u_2 w_2 + u_3 w_3 = 0$$

$$0 \cdot 1 + 1 \cdot 0 + 0 \cdot (-1) = 0 \text{ GOOD}$$

$$u = [0, 1, 0] \text{ and } w = [1, 0, -1]$$

Problem 4 [20pts] How long is the vector $u = [1, 1, 1, 1, 1]$?

$$||u|| = \sqrt{1^2+1^2+1^2+1^2+1^2} = \sqrt{5}$$

$$||u|| = \sqrt{5}$$

Problem 5 [20 pts] Consider the following system of equations:

$$2x + 3y + z = 8$$

$$4x + 7y + 5z = 20$$

$$-2y + 2z = 0$$

Apply Gauss Elimination in order to solve it.

$$2x + 3y + z = 8$$

$$4x + 7y + 5z = 20$$

$$-2y + 2z = 0$$

-> second = second - 2*first ->

$$2x + 3y + z = 8$$

$$\rightarrow 2x + 3y + z = 8$$

$$(4x - 4x + 7y - 6y + 5z - 2z = 20 - 16)$$

$$\rightarrow y + 3z = 4$$

$$-2y + 2z = 0 \rightarrow \text{divide by 2 to simplify} \rightarrow$$

$$-y + z = 0$$

-> third = third + second ->

$$2x + 3y + z = 8$$

$$\rightarrow 2x + 3y + z = 8$$

$$y + 3z = 4 \rightarrow y + 3z = 4$$

$$(-y + y + z + 3z = 0 + 4) \rightarrow 4z = 4$$

$$4z = 4 \rightarrow z = 1$$

$$y + 3(1) = 4 \rightarrow y = 4 - 3 \rightarrow y = 1$$

$$2x + 3(1) + (1) = 8 \rightarrow 2x + 4 = 8 \rightarrow 2x = 4 \rightarrow x = 2$$

$$x = 2, y = 1, \text{ and } z = 1$$