MA 573 - Linear Algebra

Homework 5

Problem 1 [20pts] Find the line $y = C + Dx$ that best fits the data $(x, y) = \{(-2, 4), (-1, 2), (0, -1), (1, 0), (2, 0)\}$.

Problem 2 [20pts] Use the Gram-Schmidt method to find orthonormal vectors A, B, C from $a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ and $c = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$.

Problem 3 [20pts] Suppose Q_1, Q_2 are square $n \times n$ matrices that are orthonormal. Show that their product $Q_1 Q_2$ is an orthonormal square matrix.

Problem 4 [20pts] Let A, B, C, D be 2×2 matrices. Does the following equality always hold? (If yes prove why, if not find a counterexample)

$$det(\begin{bmatrix} \text{A} & \text{B} \\ \text{C} & \text{D} \end{bmatrix}) = det(A) \cdot det(D) - det(C) \cdot det(B)$$

Problem 5 [20pts] Reduce $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ to U and find the determinant of A as a product of pivots.