MA 573 - Linear Algebra

Homework 4

Problem 1 [20pts] Project the vector $b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ onto the line through $a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Check that the "error" $e = b - p$ is perpendicular to a.

Problem 2 [20pts] Project the vector $b = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ onto the column space of the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$. Find the "error" $e = b - p$ and check that it is perpendicular to the columns of A.

Problem 3 [20 pts]
Find a basis for each of the four fundamental subspaces (column, null, row, left null) associated with the following matrix:

$$A = \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 & 6 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Problem 4 [20 pts]

Let $u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $u_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $u_1 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$. Show that u_1, u_2, u_3 are independent but u_1, u_2, u_3, u_4 are dependent.

Problem 5 [20 pts]

Suppose P is the subspace of $\mathbb{R}^4$ that consists of vectors $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ that satisfy $x_1 + x_2 + x_3 + x_4 = 0$. Find a basis for the perpendicular complement $P^{\perp}$ of P.