MA 573 - Linear Algebra

Homework 7

Problem 1 [20 pts]
Orthogonally diagonalize the matrix A, i.e. $A = PDP^T$, where P is orthogonal.

$$A = \begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$$

Problem 2 [20 pts]

Consider the matrix $A = \begin{bmatrix} 2 & \text{b} \\ 1 & 0 \end{bmatrix}$. Find a value of b that makes:

- $A = QDQ^T$ possible, i.e. orthogonal diagonalization possible.
- $A = SDS^{-1}$ impossible.
- A^{-1} impossible.

Problem 3 [20 pts]
In the Cholesky factorization $A = C^T C$, with $C^T = L\sqrt{D}$. Find C upper triangular for

$$A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 8 \end{bmatrix}$$

Problem 4 [20 pts]

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$. Show that A, B are similar and find M such that $A = MBM^{-1}$.

Problem 5 [20 pts]
Find the Singular Value Decomposition (SVD) of the following rectangular matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$