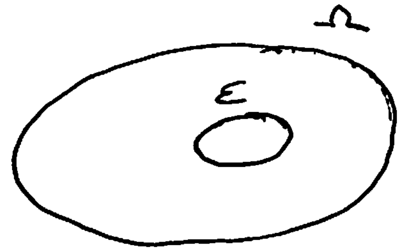
Probabilities and Counting

Terminology

- P → probability
- $\mathcal{E}$ → event
- n → the number or count
- Ω → sample space

Diagram - Venn



$$P(\mathcal{E}) = \frac{n(\mathcal{E})}{n(\Omega)} \rightarrow 0 \le P(\mathcal{E}) \le 1$$

Ways to Count

$$N! = \prod_{i=1}^{N} i = 1 \cdot 2 \cdot 3 \cdots N$$

$$\log N! = \log \prod_{i=1}^{N} i = \sum_{i=1}^{N} \log i$$

$$N! = e^{\log N!} = e^{\sum_{i=1}^{N} \log i}$$

Let P = permutation → ordered

C = combination → unordered

r = with repetion allowed

Define

$${}_N P_M^r = N^M$$

$${}_N P_M = \frac{N!}{(N-M)!}$$

↑ Permutations with and without repetions

$${}_N C_M^r = \frac{(N+M-1)!}{(N-1)!\,M!}$$

$${}_N C_M = \frac{N!}{(N-M)!\,M!}$$

↑ Combinations with and without repetions

Example Problems

1) Calculate how many pin numbers are possible using 3 digits of base 7?

$${}_N P_M^r \quad \text{where } N=7 \text{ and } M=3$$

$$= 7^3 = 343$$

2) Using 1) how many pin numbers have no duplicate digits?

$${}_N P_M = \frac{7!}{4!} = 210$$

3) What is the probability of receiving a pin number without duplicates

$$P(E) = \frac{n(e)}{n(s)} = \frac{{}_N P_M}{{}_N P_M^r} = \frac{210}{343} = 61\%$$

9/14/12

4) If you have 3 gallons of ice cream. A gallon each of chocolate, Vanilla, strawberry How many different triple cones are possible where order doesn't matter? List the combinations?

$$_{N}C_{m}^{r} = \frac{5!}{2!\,3!} = 10$$

CVS, CCV, CCS, VVC, VVS, SSC, SSV,
CCC, VVV, SSS

5) Powerball uses 5 numbers of 69 in any order and 1 number of 26
Note! See Program Written for Solution

a) How many different ways are there to pull a number from these arrangeat.

$$n(\Omega) = {}_{69}C_5 \cdot {}_{26}C_1$$

b) What is the probility or odds of winning

$$P(E) = \frac{n(E)}{n(\Omega)} = \frac{{}_{5}C_5 \cdot {}_{1}C_1}{{}_{69}C_5 \cdot {}_{26}C_1}$$

or odds $\longrightarrow 1 : \frac{{}_{69}C_5\,{}_{26}C_1}{{}_{5}C_5\,{}_{1}C_1}$

c) What is the probability or odds of 4 Numbers only

$$P(E) = \frac{{}_{69}C_4\,{}_{4}C_1\,{}_{25}C_1}{{}_{69}C_5\,{}_{26}C_1} \rightarrow 1 : \frac{{}_{69}C_5\,{}_{26}C_1}{{}_{69}C_4\,{}_{64}C_1\,{}_{25}C_1}$$