12. Sea $A \subseteq \mathbb{R}$. Probar que $A \in \mathcal{M}$ si y sólo si para todo $\varepsilon > 0$ existen conjuntos G abierto y F cerrado tales que $F \subseteq A \subseteq G$ y $\mu(G \setminus F) < \varepsilon$.

$\rightarrow$) Sea $A \in \mathcal{M}$ QVQ $\forall \varepsilon > 0$ $\exists F$ cerrado y G abierto / $F \subseteq A \subseteq G$ y $\mu(G \setminus F) < \varepsilon$

Sea ε fijo $\xrightarrow{A \in \mathcal{M}}$ $\exists U \supseteq A$, abierto / $\mu(U \setminus A) < \varepsilon / 3$

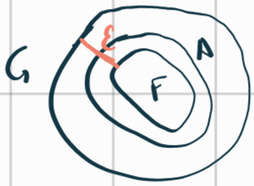
$\exists F \subseteq A$, cerrado / $\mu(A \setminus F) < \varepsilon / 3$

$\rightarrow$ $G \setminus F = G \setminus A \cup A \setminus F$ $\rightarrow$ a chequear

$\rightarrow$ $\mu(G \setminus F) = \mu(G \setminus A \cup A \setminus F) \leq \mu(G \setminus A) + \mu(A \setminus F) = 2\varepsilon/3 < \varepsilon$

$\leftarrow$)

$\forall \varepsilon \geq 0$ $\exists F$ cerrado y G abierto / $F \subseteq A \subseteq G$ y $\mu(G \setminus F) < \varepsilon$ QVQ $A \in \mathcal{M}$



Capaz si uso el 11 puedo resolverlo

Sea $\varepsilon = 1$ $\exists F_1 \subseteq A \subseteq G_1$, F_1 c y G_1 A / $\mu(G_1 \setminus F_1) < 1$

$\varepsilon = 1/2$ $\exists F_2 \subseteq A \subseteq G_2$, F_2 c $F_2 \supseteq F_1$ y G_2 A $G_2 \subseteq G_1$ / $\mu(G_2 \setminus F_2) < 1/2$

$\vdots$

$\varepsilon = 1/n$, $\exists F_n \subseteq A \subseteq G_n$, F_n C $F_n \supseteq F_{n-1}$, G_n A $G_n \subseteq G_{n-1}$ / $\mu(G_n \setminus F_n) < 1/n$

$\to \exists (F_n)_{n\geq 1}$ con F cerrado $\forall n\in\mathbb{N}$ y $(G_n)_{n\geq 1}$ con G abierto $\forall n\in\mathbb{N}$ / $(F_n)_{n\geq 1}\subseteq A\subseteq (G_n)_{n\geq 1}$

$\to \mu\left(\bigcap_{n\geq 1} G_n \setminus \bigcup F_n\right) < 1/n$

Pero $G\setminus F = G\setminus A \cup A\setminus F \to \mu\left(\bigcap_{n\geq 1} G_n \setminus \bigcup_{n\geq 1} F_n\right) = \mu\left(\left(\bigcap G_n\setminus A\right)\cup\left(A\setminus \bigcup F_n\right)\right) < 1/n$

$\mu(A\cup B) = \mu(A) + \mu(B\setminus A)$

$$\to \mu\left(\left(\bigcap G_n\setminus A\right)\cup\left(A\setminus \bigcup F_n\right)\right) = \mu\left(\bigcap G_n\setminus A\right) + \mu\left(\overbrace{\left(A\setminus\bigcup F_n\right)}^{=\mu(A\setminus \bigcup F_n)}\setminus\underbrace{\left(\bigcap G_n\setminus A\right)}_{\not\subset A}\right)$$

$$= \mu\left(\bigcap G_n\setminus A\right) + \mu\left(A\setminus\bigcup F_n\right) < 1/n$$

$\longrightarrow \mu\left(A\setminus\bigcup F_n\right) < 1/n \to 0 \quad \to$ medible

$\to \quad A = \bigcup F_n \cup \left(A\setminus \bigcup F_n\right) \to A\in\mathcal{M}$