3. Sea E un espacio normado. Sean $(x_n)_{n\in\mathbb{N}} \subseteq E$ y $x_0 \in E$ tales que $\lim_{n\to\infty} x_n = x_0$. Probar que si definimos $(y_n)_{n\in\mathbb{N}} \subseteq E$ por

$$y_n = \frac{x_1 + x_2 + \cdots + x_n}{n},$$

entonces $\lim_{n\to\infty} y_n = x_0$.

Sean $(x_n)_{n\geq 1} \subseteq E$ y $x_0 \in E$ / $x_n \xrightarrow[n\to+\infty]{} x_0$ QVQ si

defino $(y_n)_{n\geq 1} = \dfrac{x_1 + \cdots + x_n}{n} \longrightarrow y_n \xrightarrow[n\to+\infty]{} x_0$

QVQ $\forall \varepsilon > 0\ \exists n_0 \in \mathbb{N} / \|y_n - x_0\| < \varepsilon\ \forall n \geq n_0$

Sea $\varepsilon > 0$ qvq $\exists n_0 \in \mathbb{N} / \|y_n - x_0\| < \varepsilon\ \forall n \geq n_0$

$$\|y_n - x_0\| = \left\| \frac{x_1 + \cdots + x_n}{n} - x_0 \right\| = \left\| \frac{\sum_{i=1}^{n} x_i}{n} - x_0 \right\| \leq$$

$$\left\| \frac{\max_{i\geq 1} x_i}{\cancel{n}} - x_0 \right\| = \left\| \max_{i\geq 1} x_i - x_0 \right\|$$

No me sirve X

Sé que $\forall \varepsilon > 0 \ \exists \tilde{n}_0 \in \mathbb{N} /$

$$\|x_n - x_0\| < \varepsilon \ \forall n \geq \tilde{n}_0$$

QVQ dado $\varepsilon > 0 \ \exists n_0 \in \mathbb{N} /$

$$\|y_n - x_0\| < \varepsilon$$

$$\left\| \frac{x_1 + \dots + x_n}{n} - x_0 \right\| =$$

Si tengo

$$\left\| \frac{x_1 + \dots + x_n - n x_0}{n} \right\| =$$

$n x_0$ puedo restandolo a cada término

$$\left\| \frac{x_1 - x_0 + (x_2 - x_0) + (x_3 - x_0) + \dots + (x_n - x_0)}{n} \right\|$$

$$\leq \left\| \frac{x_1 - x_0}{n} \right\| + \dots + \left\| \frac{x_n - x_0}{n} \right\|$$

1 Considero $n_0 = \tilde{n}_0$ tengo que $\forall n \geq n_0$

$$\leq \frac{1}{n}\sum_{i\geq 1}^{n_0}\|x_i-x_0\| + \frac{1}{n}\sum_{i\geq n_0}\|x_i-x_0\| \leq$$

$$\frac{n_0}{n}\cdot \max\|x_i-x_0\| + \frac{1}{n}(n-n_0)\cdot \max\|x_i-x_0\|$$

$$\leq \underbrace{\frac{n_0 \max\|x_i-x_0\|}{n}}_{<1} + \underbrace{\left(1-\frac{n_0}{n}\right)}_{<1}\varepsilon$$

$$\leq \varepsilon$$

Revisar!

Ese último paso está mal

$$\leq \underbrace{\frac{n_0 \max\|x_i-x_0\|}{n}}_{\text{si } n\to+\infty \to 0} + \underbrace{\left(1-\frac{n_0}{n}\right)}_{\text{si } n\to+\infty\to 1}\varepsilon \leq \varepsilon$$

Si $n\to+\infty$ $\quad \frac{n_0}{n}\to 0$