Ejercicio 9. Resuelva utilizando Branch & Bound los siguientes problemas:

$$\begin{array}{lrcl} \max & z = 2x_1 + x_2 & & \\ s.a: & 2x_1 + 3x_2 & \leq & 19 \\ & 7x_1 + 3x_2 & \leq & 43 \\ & x_1, x_2 & \geq & 0 \\ & x_1, x_2 & \in & \mathbb{Z} \end{array}$$

→ Planteamos la relajación lineal

max $z = 2x_1 + x_2$

s.a: $2x_1 + 3x_2 \leq 19$

$7x_1 + 3x_2 \leq 43$

$x_1, x_2 \geq 0$

→ Gráficamente z_{op} / $(x_1, x_2) = (4.8, 3.1\hat{3})$

$z = 12,73$

$(x_1, x_2) = (4.8, 3,1\hat{3})$

$\bar{z} = -\infty$

$x_1 \geq 5$ / $x_1 \leq 4$

$z = 12,66$

$x_1, x_2 = (5, 2.6\hat{6})$

$\bar{z} = -\infty$

$z = 11,66$

$(x_1, x_2) = (4, 3,66)$

$\bar{z} = -\infty$

$x_2 \geq 3$ / $x_2 \leq 2$ / $x_2 \geq 4$ / $x_2 \leq 3$

infactible

$z = 12,57$

$(x_1, x_2) = (5,2857, 2)$

$\bar{z} = -\infty$

$z = 11$

$(x_1, x_2) = (3.5, 4)$

$\bar{z} = 12$

$z = 11$

$(x_1, x_2) = (4, 3)$

$\bar{z} = 12$

$x_1 \geq 6$ / $x_1 \leq 5$

NO tiene sentido seguir porque $z < \bar{z}$

$z = 12,33$

$(x_1, x_2) = 6, 0.33$

$\bar{z} = 12$

$z = 12$

$x_1, x_2 = (5, 2)$

$\bar{z} = 12$

$x_2 \geq 1$ / $x_2 = 0$

infactible

$z = 12$

$(x_1, x_2) = (6, 0)$

$\bar{z} = 12$

$$
\begin{aligned}
\min \quad & z = -10x_1 - 15x_2 \\
s.a: \quad & 8x_1 + 4x_2 \leq 40 \\
& 15x_1 + 30x_2 \leq 20 \\
& x_1, x_2 \geq 0 \\
& x_1, x_2 \in \mathbb{Z}
\end{aligned}
$$

→ Planteamos la relajación lineal y resolvemos graficamente

↳ Sería lo mismo que max $10x + 15y$ porque debemos considerar $-\nabla f$

$z = -13,\widehat{3}$

$(x_1, x_2) = (1,\widehat{3}, 0)$

$\overline{z} = +\infty$

$x_1 \leq 1$ ↙ ↘ $x_1 \geq 2$

$z = -12,49$ — infactible

$(x_1, x_2) = (1, 0,1\widehat{6})$

$x_2 \geq 1$ ↙ ↓ $x_2 \leq 0$

$z = +\infty$

infactible — $z = -10$

$(x_1, x_2) = (1, 0)$

$\overline{z} = -10$

$$
\begin{array}{lrcl}
\min & z = -4x_1 + 5x_2 & & \\
s.a: & 5x_1 - 2x_2 & \geq & -2 \\
& 2x_1 - 4x_2 & \leq & 5 \\
& x_1 & \leq & 5 \\
& x_2 & \leq & 4 \\
& x_1, x_2 & \geq & 0 \\
& x_1, x_2 & \in & \mathbb{Z}
\end{array}
$$

→ Planteamos la relajación lineal y resolvemos

$z = -13{,}75$

$(x_1, x_2) = (5, 1{,}25)$

$\overline{z} = +\infty$

$x_2 \geq 2$

$z = -10$

$(x_1, x_2) = (5, 2)$

$\overline{z} = -11$

$x_2 \leq 1$

$z = -13$

$(x_1, x_2) = (4{,}5, 1)$

$\overline{z} = +\infty$

$x_1 \leq 4$

$z = -12{,}25$

$(4, 0.75)$

$x_2 \leq 1$

$\overline{z} = +\infty$

$x_2 = 0$

infactible

$z = -11$

$(x_1, x_2) = (4, 1)$

$\overline{z} = -11$

$x_1 \geq 5$

infactible