CSCI 4041, Spring 2019, Quiz 5 (30 minutes, 20 points)

Name:

x500:

Discussion Start Time (**circle one**): 3:35 4:40 5:45 6:50 7:55 other:

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1. (1 points each) True/False - Circle one. Note that when asking about the properties of an algorithm, we specifically mean the version of that algorithm discussed in lecture.

   True **False** Huffman Codes assign longer encodings to high frequency characters.
   In order to minimize the overall encoding length, Huffman Codes aim to assign smaller encodings to the highest frequency characters.

   True False Dynamic Programming algorithms generally run faster than their brute-force, recursive counterparts.
   The point of Dynamic Programming algorithms is that they trade off higher memory usage for faster runtimes.

   True **False** If a problem can be solved optimally by either Dynamic Programming or a Greedy Algorithm, the Dynamic Programming solution will generally use more memory.
   Dynamic Programming algorithms must store the solutions to previously computed subproblems to avoid recomputing them. Greedy algorithms don't do this: they just make a choice about which subproblem is locally optimal and ignore all others.

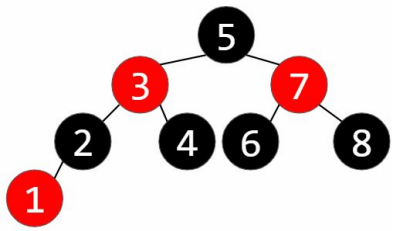
   True **False** For the activity selection problem, choosing the activity with the shortest duration is an optimal greedy choice.
   If you have one short activity that overlaps with two long activities, and the two long activities don't overlap, then choosing the two long activities is optimal

   True **False** An adjacency-matrix is preferable to an adjacency-list for representing a sparse graph.
   An adjacency matrix wastes a lot of memory compared to an adjacency list for a sparse graph scenario, since an adjacency list requires a bit of memory for every possible connection not present, but an adjacency list does not have to store anything for that.

2. (5 points) Define the imbalance factor of a Red-Black Tree or B-Tree with distinct keys to be the absolute difference between the number of keys that are less than any key in the root node and the number of keys greater than any key in the root node. Draw the following:
   a. An 8-key Red-Black Tree with the minimum imbalance factor
   b. An 8-key Red-Black Tree with the maximum imbalance factor
   c. An 8-key B-Tree with t = 3 that has the minimum imbalance factor

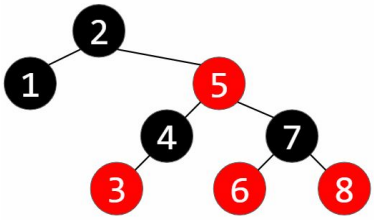
d. An 8-key B-Tree with t = 3 that has the maximum imbalance factor

a.



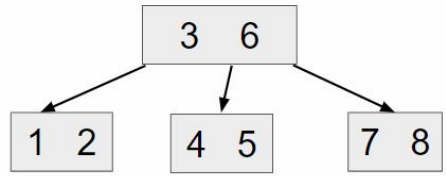
(Imbalance: 4 - 3 = 1)

b.



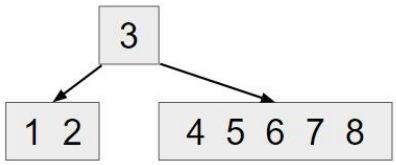
(Imbalance: 6 - 1 = 5)

c.



(Imbalance: 2 - 2 = 0)

d.

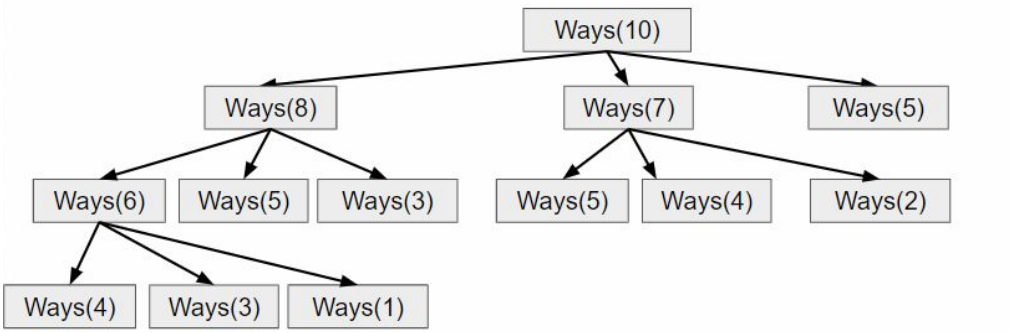


(Imbalance: 5 - 2 = 3)

3. You are playing a board game on an infinitely long board in which each turn, you either move 2, 3, or 5 spaces forward, depending on a die roll. You want to know how many different ways there are to reach the n^{th} space on the board (here moving forward 3 and then 5 is counted a different 'way' to reach the 8^{th} space than moving 5 and then 3). Here is a recursive function to solve the problem (assume indexing starting at 1):

```
Ways(n)
      baseCases = [0,1,1,1,3]
      if n <= 5
            return baseCases[n]
      else
            return Ways(n-2) + Ways(n-3) + Ways(n-5)
```

a. (2 points) Draw the recursion tree for Ways(10).



b. (2 point) What is the height and maximum branching factor of the recursion tree for Ways(n), in terms of n? Assume n > 5.

Height: ceiling((n - 5)/2)
Max Branching factor: 3

c. (3 points) Write a dynamic programming algorithm in pseudocode, Python, or Java to solve the above problem in Theta(n) time.

```
def Ways(n):
      ans = [0,0,1,1,1,3] + [0]*(n-5)
      for i in range(6,n):
            ans[i] = ans[i-2] + ans[i-3] + ans[i-5]
      return ans[n]
```

d. (3 points) Argue that this problem has both the overlapping subproblems and (optimal) substructure properties.

This problem has the substructure property, because to solve for Ways(n), we must solve smaller versions of the same problem, Ways(n-2), Ways(n-3), and Ways(n-5).

This problem has the overlapping subproblems property, because the brute-force, recursive version algorithm for the problem solves the same problem at multiple different places in the recursion tree. For example, in solving Ways(10), Ways(5) is called 3 times.