

CSCI 4041, Spring 2019, Quiz 1 (30 minutes, 20 points)

Name:

x500:

Discussion Section Start Time:

1. (1 point each) True/False - Circle one:

True False If $f(n) = n^2 - 8n$, then $f(n)$ is $O(n^3)$.

True False $f(n) = \Theta(g(n))$ implies $g(n) = O(f(n))$.

True False Bubble Sort is stable.

True False Insertion Sort has a $\Theta(n^2)$ best-case runtime.

True False Merge Sort is in place.

2. (4 points) Find a tight (big- Θ) bound on the asymptotic runtime of the following Python function, in terms of n , the number of elements in the input list `in_list`. Then give a short (1-2 sentence) justification of your answer. You may assume that `.append()` is a $\Theta(1)$ operation.

```
def mystery(in_list):
    prev_list = [0]
    for val1 in in_list:
        new_list = []
        for val2 in prev_list:
            new_list.append(val2)
            new_list.append(val1 + val2)
        prev_list = new_list
    return new_list
```

3. (5 points) Find a non-decreasing function such that $f(n)$ is not $O(f(n/2))$, and then prove it using the definition of big-O:

$$O(g(n)) = \{ f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0 \}$$

4. (6 points, 2 per part) The following algorithm replaces all instances of the number 2 in an array A with the number 3.

```
TwoToThree(A)
  for i = 1 to A.length:
    if A[i] == 2
      A[i] = 3
```

We can use the following loop invariant to prove TwoToThree is correct:

At the start of each iteration of the for loop, A consists of its original contents, except that any 2's occurring in the subarray $A[1 \dots i-1]$ have been replaced by 3's.

- State, but do not prove, the Initialization property for the above invariant (that is, what must we prove to show that the invariant holds for Initialization)?
- State, but do not prove, the Maintenance property for the above invariant (that is, what must we prove to show that the invariant holds for Maintenance)?
- State the Termination property for the above invariant (what does the invariant say if it holds true after the loop terminates)?