Analysis of Supernova Data

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1 Abstract

This analysis combined data collected from multiple photmetric and spectroscopic observations and tecniques from decades of scientific papers[4][3] to create code that estimates the distance to the SN1987a-like supernova, SN2018hna. The analysis concludes that the derived distance, between 13-15 Mpcs, is within acceptable range of other estimates and confirms the accuracy of the data and method.

2 Introduction

The distance between us and the stars above was a mystery to ancient astronomers. Not knowing the true unfathomable distance, they assumed the stars were just points of light in a cosmic dome that surrounded and rotated around the earth, the "center of the universe". It took centuries of free thinkers to finally accept that we arent the center of the universe, and that stars are in fact just like our own sun, just very far away. It took longer still to determine how far away those stars actually were. Today, we have developed many techniques to solve this very problem. The techniqe I will be using in this paper to determine the distance to a supernova is called the Expanding Photosphere Method (EPM). It involves the expanding envelope of gas around the supernova, a calculated speed of the expansion, the temperature of the gas, and various spectral and photometric data.

3 Procedure

To start off, we have to talk about what becomes of a star during a Type - IIp supernova. When the star explodes, an envelope of gas expands in all directions at a high rate of speed. The speed of the expanding gas can be determined by looking at the spectrum of the light being emitted by the gas and comparing it to another source to see how blueshifted it is. This is done using spectroscopy. For the Expanding Photosphere Method to work, we have to assume that the expanding photosphere of gas is thin and uniform, meaning that it expands

evenly in all directions away from the star.
The equation we will use to determine the distance is called the angular radius.

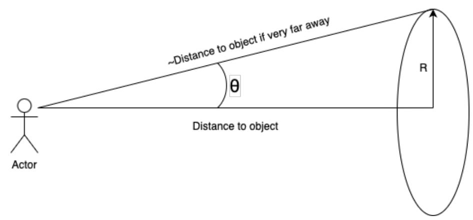

Figure 1: A diagram of angular radius.

In the above figure 3 there is a disk, a person, and a right triangle between them. Lets assume for the sake of explination that the disk is the Sun and the person is on Earth. The distance between the person and the Sun is d and the radius of the Sun is R. The angle θ created by the right triangle formed is called the angular radius. We can write this relationship as:

$$\tan\theta = \frac{\mathrm{R}}{\mathrm{d}} \tag{1}$$

The next thing we must take into account is the fact that the supernova that we are looking at is incredibly far away. This means the angular radius θ approaches 0. Using the small angle approximation, $\tan\theta$ becomes θ, and the equation 1 becomes:

$$\theta = \frac{\mathrm{R}}{\mathrm{d}} \tag{2}$$

Because the photosphere is expanding, the radius R is increasing over time with a measurable velocity. This allows us to set R to:

$$R = v(t - t_0) \tag{3}$$

Where v is the velocity of the photosphere at that time, t is the time since explosion, and t_0 is the time of explosion.

Next, since the supernova is very far away, we have to account for time dialation. This can be accomplished by using a constant in the following way:

$$d = d(1 + z) \tag{4}$$

Where d is the distance to the supernova and z is the time dialation constant.

This gives us:

$$\theta = \frac{v(t - t_0)}{d(1 + z)} \tag{5}$$

We rearrange this equation to get it into slope intercept form:

$$t = \frac{\theta}{v}(1 + z)d + t_0 \tag{6}$$

Where $(1 + z)d$ is the slope and t_0 is the intercept. With this function 6 we can graph data points (in $(\frac{\theta}{v}, t)$ form) and find the line of best fit to determine the distance. The graphed data will look like figure 5.

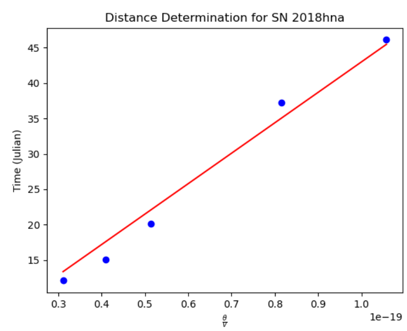

Figure 2: The graph of data points in blue and line of best fit in red.

Depending on the method of determining the angular radius θ, you may have to make multiple distances like figure 5. I will speak more about this in the data section.

4 Data

To determine the distance to the supernova, we need to know a few important things for each day/data point of observation:
We need:

v: The velocity of the expanding gas, Units: km/s.
t: The time since explosion, Units: Julian Date.
t_0: The time of explosion, Units: Julian Date.
z: The time dialation factor, Unitless.
θ: The angular radius, Units: Degrees.

As seen in the figure 5, we have 5 data points. For each of these data points, we have a t, v, and θ.

First we start with velocity v. This is determined by looking at the spectrum of the expanding photosphere and seeing how much it is blueshifted compared to a source nearby. This will tell us how fast the expanding gas is moving towards us, thus giving us a velocity.

Next we will find the time t, which is as easy as using a device that tells you the julian date[1] of the time of observation.

t_0 is a bit harder to find because we dont always see exactly when a supernova happens, often we find out a few dats after the fact. So for this analysis we used two methods.
Method 1:
We calculate the line of best fit normally, which tells us the distance d and the time of explosion t_0.
Method 2: We calculate the line of best fit but set the y-intercept, t_0, to zero, and calculated the distance.

The time dialation factor z is calculated using spectroscopy, looking at spectrums of the light and determing how much time will be dialated from traveling from the direction of the supernova. This is something that needs to be looked up. We did not calculate this ourselves.

[1] Julian date is the number of days (and fractions of days) since 12:00 PM November 24, 4714 BC.

Now, we have the angular radius θ. Determinig the angular radius is the most difficult part of this project, and requires a large amount of data. To start off, we will look at the equation for calculating θ, which is derived in this paper by Mitchell [4].

$$\log\theta = -\log\zeta_{\lambda^{'}} - 0.2(m_\lambda - A_\lambda - b_{\lambda^{'}}) + 0.5\log{(1+z)} \tag{7}$$

Or with θ isolated,

$$\theta = 10^{-\log\zeta_{\lambda^{'}} - 0.2(m_\lambda - A_\lambda - b_{\lambda^{'}}) + 0.5\log{(1+z)}} \tag{8}$$

To break this equation down, here are the component parts:
m_λ: The magnitude of the filter used. We will be using the B and V filter separately, so we will have two different distances.

A_λ: The extinction factor. This value accounts for any interference from gas in the space between the observed object and the observer. This includes the athmosphere.

z: The time dialation factor. This is explained more above.

$\zeta_{\lambda^{'}}$: This is the dilution factor. This helps us get more accurate numbers and accounts for any sort of distortion effects. It is a temperature-dependant polynmial with the form of this 9 equation below and with constants a_i pulled from the paper by Hamuy [2] and are specific for the filter used for observation, in our case we used the B and V filter so we will be using the B-V values.

$$\zeta_{\lambda^{'}}(T) = \sum_{i=0}^{2} a_i(\frac{10^4K}{T})^i, a_i = (0.7557, -0.8997, 0.5199) \tag{9}$$

$b_{\lambda^{'}}$: This is the synthetic magnitude at the wavelength λ. It is also a temperature-dependant polynmial with the form below. The values for $c_{i,\lambda}$ were also pulled from the paper by Hamuy [2], which were modified from Estmans [1] calculations. The values for $c_{i,\lambda}$ depend on the filter (wavelength of light) used for the calculation and will depend on which way you measure temperature, which will be mentioned below.

$$b_{\lambda^{'}}(T) = \sum_{i=0}^{5} c_{i,\lambda}(\frac{10^4K}{T})^i \tag{10}$$

The last thing needed is to determine the temperature of the expanding gas at a certain time. Though there are many different methods that can be used, we will be using the one called Color Temperature. This method is defined in the paper by Matthews and Sandage [3].

We use two filters, in our case the B and V filters. We find the B-V values and their corresponding temperature values listed in the Matthews and Sandage [3] paper. We use the equation below to calculate the line of best fit, which gives our K and C values (slope and intercept of line of best fit).

$$\frac{10000k}{T} = K(B - V) + C \tag{11}$$

When finding the line of best fit of this equation 4, you get a K of 1.58 and a C of 0.72. It is suggested by Matthews and Sandage [3] that a C of 0.48 should be used for a Type-II supernova observation.

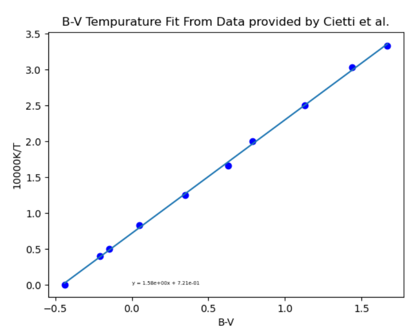

Figure 3: The temperature fit for B-V filters.

When we rearrange equation 11 to isolate temperature T, we get:

$$T = \frac{10000}{K(B - V) + C} \tag{12}$$

Now we correct for any errors in the B and V values:

$$T = \frac{10000}{K((B - B_{\text{correction}}) - (V - V_{\text{correction}})) + C} \tag{13}$$

When we plug in our values for B and V, the B and V correction factors, and the K and suggested C values into the temperature equation 13 above, we get the temperature of the expanding gas during the observation time.

Now that we have all the data nessisary, we can calculate the distance.

5 Analysis

When compiling all this data, it is recomended to use python to make repeating your work easier and to keep track of the values of variables.

Because we are using the temperature determination method of color temperature, we have two distinct distance values, the B-Filter distance and the V-Filter distance. We will also be determining the line of best fit in two different ways, one way with a set y-intercept of 0 and one without a set y-intercept, giving us another set of B and V filter distances. Here is the graph of my data again,

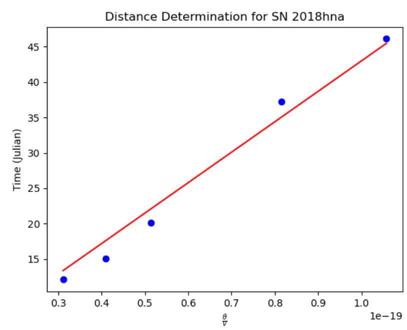

Figure 4: The graph of data points in blue and line of best fit in red.

with the line of best fit and all the data points available. This particular graph is of the V Filter with a Y-Intercept of 0.

Here is a table of the distances calculated with the data collected in previous sections.

Table 1: Distances

Y-Intercept	Filter Used	Distance (Mpc)
Standard	B Filter	16.37
	V Filter	15.33
Forced Zero	B Filter	14.88
	V Filter	13.93

You can see from the determined distances in the table1 above that the B Filter generally had a greater distance than the V Filter, and the forced zero y-intercept reduced the distance as well. These numbers match what other papers have found for this same supernova, so we are confident in the accuracy of our answers.

References

[1] Ronald G. Eastman, Brian P. Schmidt, and Robert Kirshner. The Atmospheres of Type II Supernovae and the Expanding Photosphere Method. *The Astrophysical Journal*, 466:911, August 1996. ADS Bibcode: 1996ApJ...466..911E.

[2] Mario Hamuy, Philip Pinto, Jose Maza, Nicholas Suntzeff, M. Phillips, Ronald Eastman, R. Smith, Christopher Corbally, D. Burstein, Yong Li, Valentin Ivanov, Amaya Moro-Martin, L. Strolger, Ramoj Souza, S. Anjos, Elizabeth Green, T. Pickering, Luis Gonzalez, Roberto Antezana, and RA Schommer. The Distance to SN 1999em from the Expanding Photosphere Method. *The Astrophysical Journal*, 558, April 2001.

[3] Thomas A. Matthews and Allan R. Sandage. Optical Identification of 3C 48, 3C 196, and 3C 286 with Stellar Objects. *The Astrophysical Journal*, 138:30, July 1963. ADS Bibcode: 1963ApJ...138...30M.

[4] R. C. Mitchell, B. Didier, S. Ganesh, K. Acharya, R. Khadka, and B. Silwal. Locating Type II-P Supernovae Using the Expanding Photosphere Method. I. Comparing Distances from Different Line Velocities. *The Astrophysical Journal*, 942:38, January 2023. ADS Bibcode: 2023ApJ...942...38M.