

Team 3 Final Project

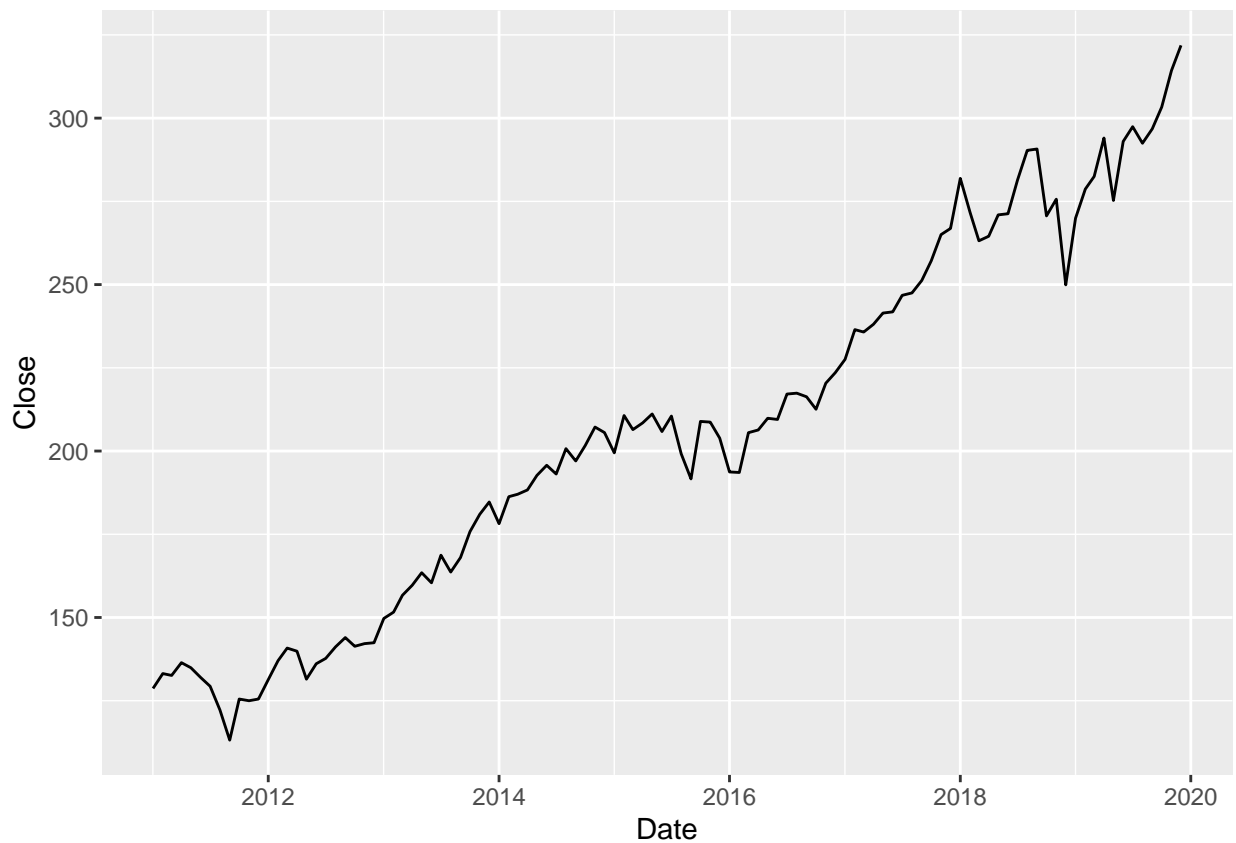
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10/08/2020

Contents

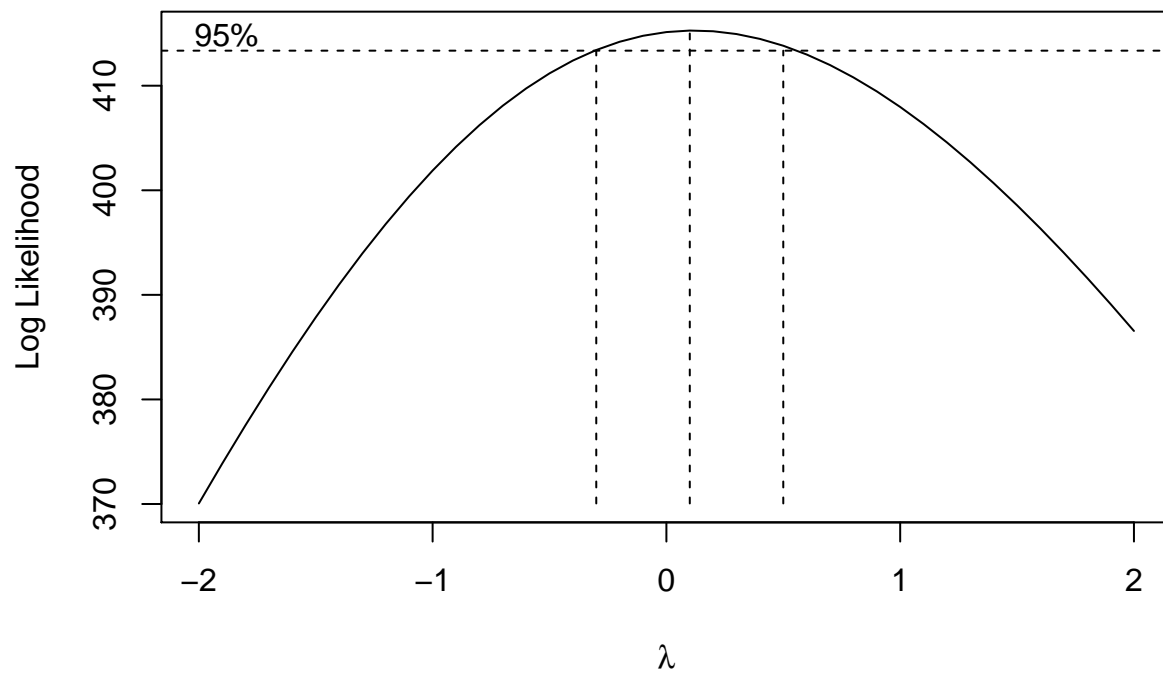
```
SandP <- SandP %>%  
  mutate(Date = as.Date(Date)) %>%  
  select(Date, Close)  
SandP2 <- SandP %>%  
  filter(Date > "2020-01-01")  
SandP <- SandP %>%  
  filter(Date < "2020-01-01")  
  
SandPTS <- xts(SandP$Close, order.by = as.POSIXct(SandP$Date))
```

```
ggplot(SandP, aes(x = Date, y = Close)) + geom_line()
```



The plot looks like it may have some nonconstant variance.

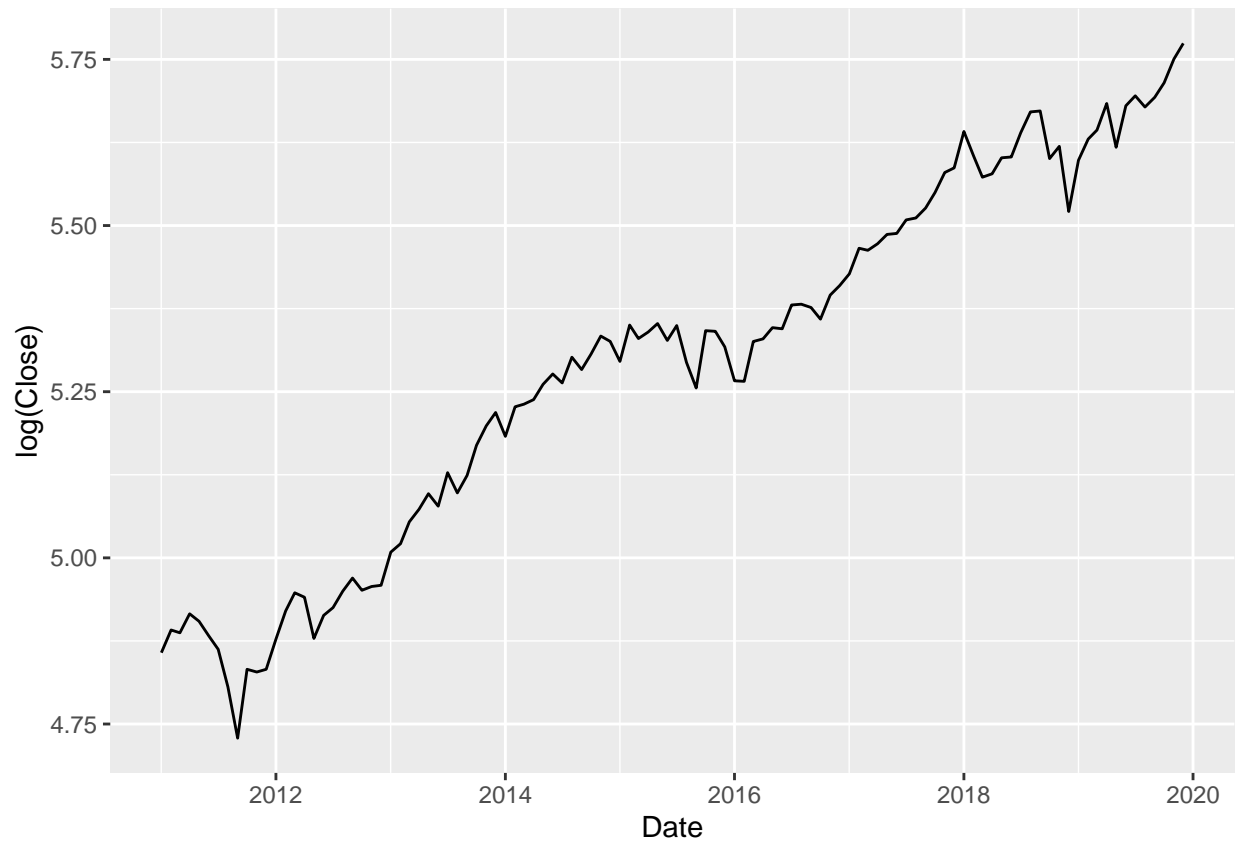
```
BoxCox.ar(SandPTS)$ci
```



```
[1] -0.3  0.5
```

The box-cox test contains zero, therefore we will use a log transformation.

```
ggplot(SandP, aes(x = Date, y = log(Close))) + geom_line()
```



The time series plot looks better after the log transformation.

```
SandPTS <- ts(log(SandPTS))
adf.test(SandPTS)
```

Augmented Dickey-Fuller Test

```
data: SandPTS
Dickey-Fuller = -2.2506, Lag order = 4, p-value = 0.4727
alternative hypothesis: stationary
```

```
SandPdiff <- diff(SandPTS)
```

The dickey-fuller test indicates that the base model is not stationary.

```
adf.test(SandPdiff)
```

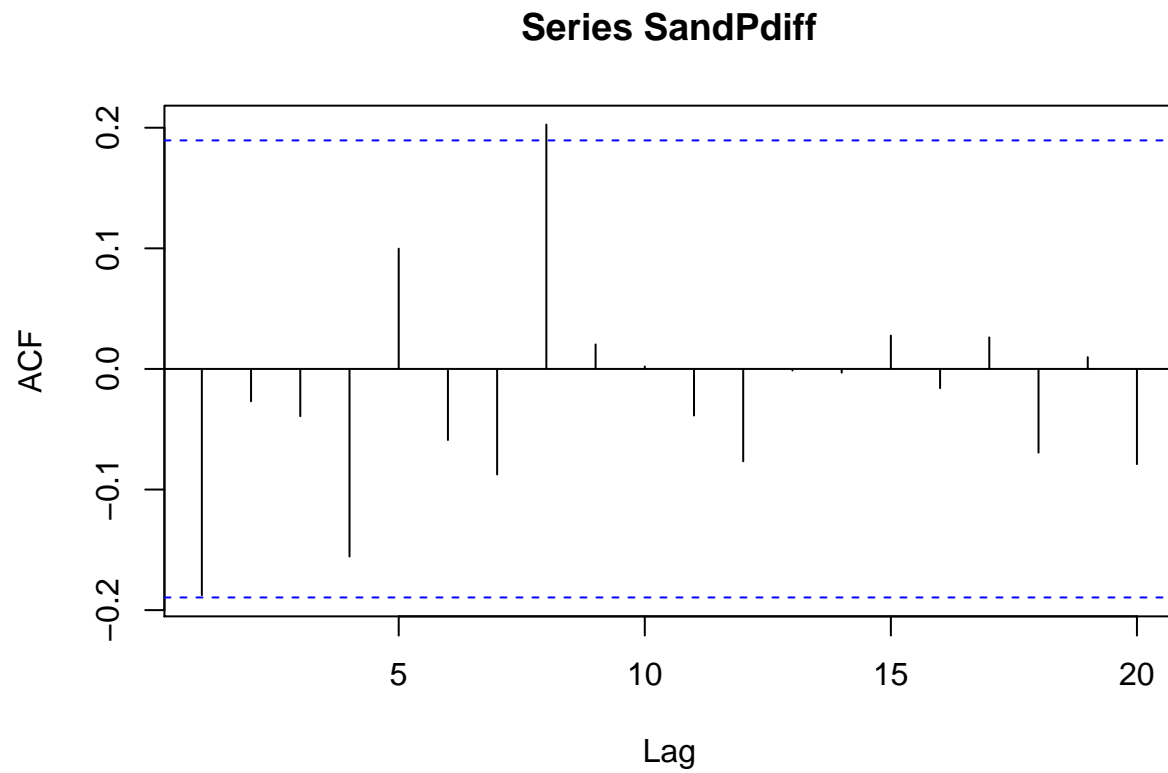
Warning in adf.test(SandPdiff): p-value smaller than printed p-value

Augmented Dickey-Fuller Test

```
data: SandPdiff
Dickey-Fuller = -5.2713, Lag order = 4, p-value = 0.01
alternative hypothesis: stationary
```

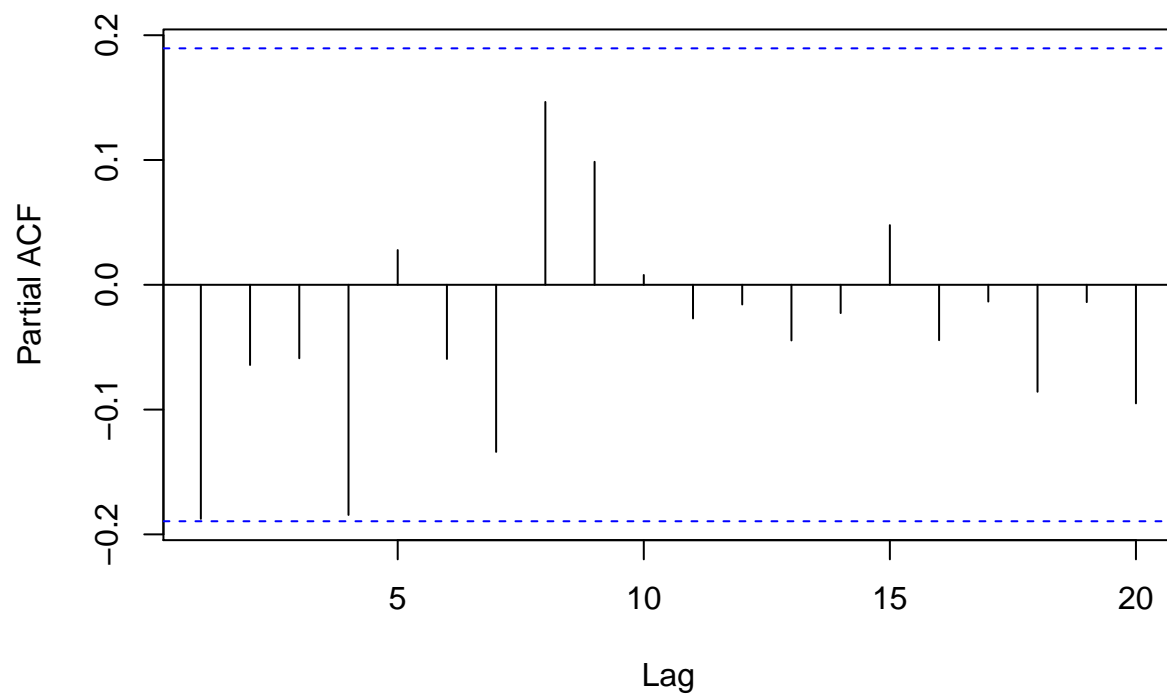
The model is stationary after differencing it once.

```
acf(SandPdiff)
```



```
pacf(SandPdiff)
```

Series SandPdiff

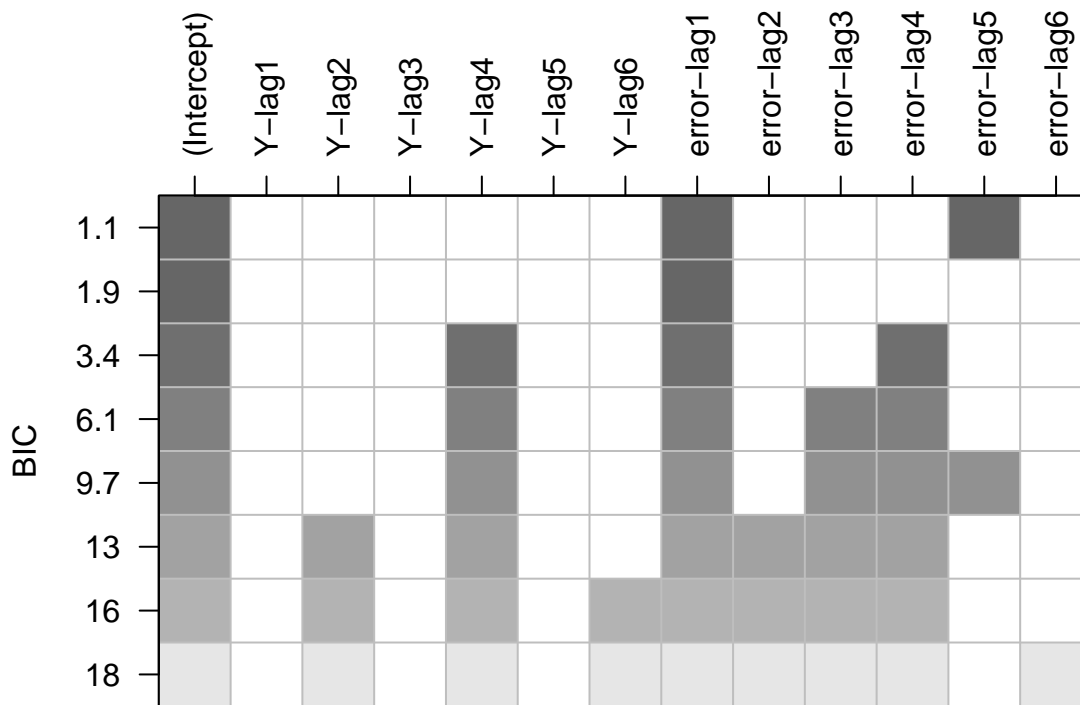


```
eacf(SandPdiff)
```

```
AR/MA
  0 1 2 3 4 5 6 7 8 9 10 11 12 13
0 o o o o o o o x o o o o o o
1 x o o o o o o o o o o o o
2 x o o o o o o o o o o o o
3 x x x o o o o o o o o o o
4 o x x x o o o o o o o o o
5 x o x o o o o o o o o o o
6 x x o x o o o o o o o o o
7 x x x x x o o o o o o o o
```

The partial acf output does not have any lags outside of the expected errors.

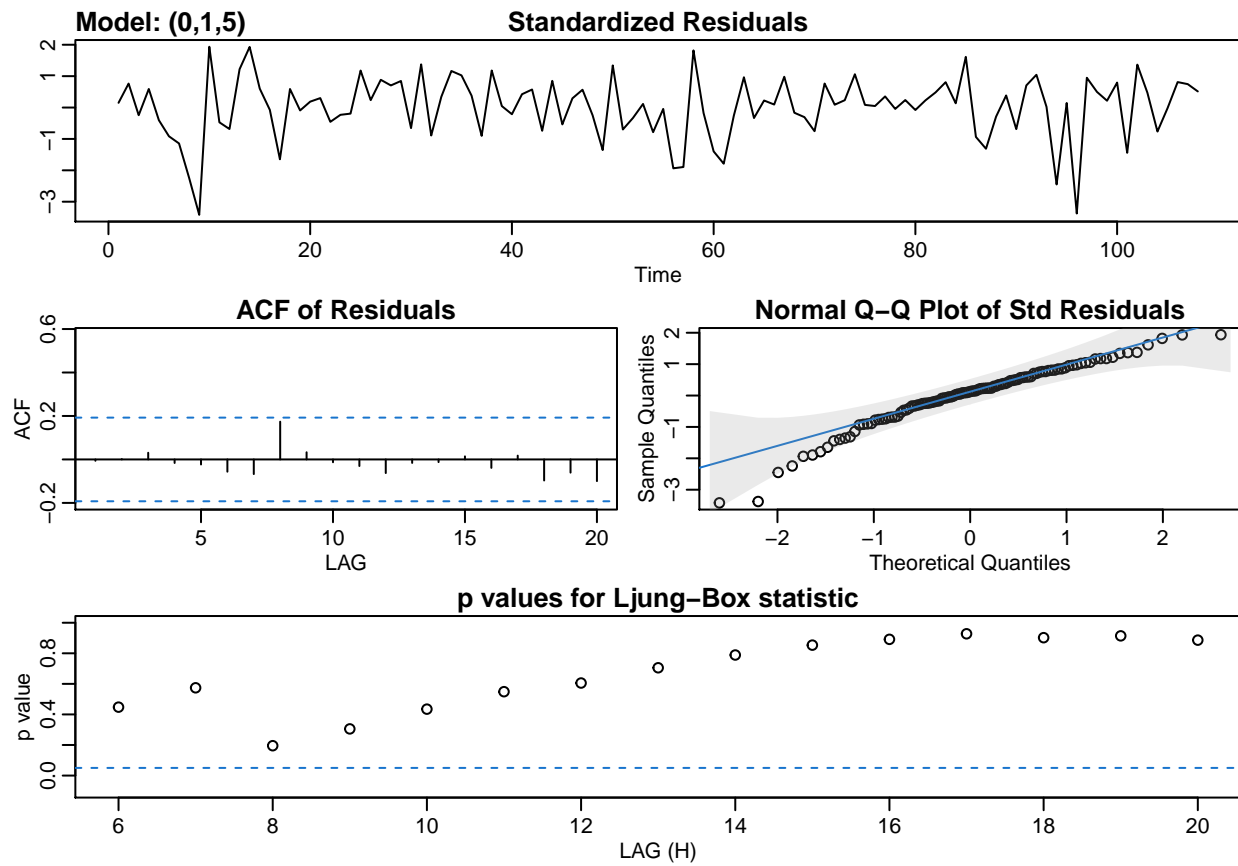
```
plot(armasubsets(SandPdiff, nar = 6, nma = 6))
```



There are several models that may be appropriate.

```
sarima(SandPTS, 0, 1, 5)
```

```
initial value -3.389534
iter 2 value -3.426072
iter 3 value -3.429313
iter 4 value -3.432067
iter 5 value -3.432534
iter 6 value -3.432565
iter 7 value -3.432568
iter 8 value -3.432569
iter 8 value -3.432569
iter 8 value -3.432569
final value -3.432569
converged
initial value -3.431690
iter 2 value -3.431702
iter 3 value -3.431739
iter 4 value -3.431739
iter 5 value -3.431739
iter 5 value -3.431739
iter 5 value -3.431739
final value -3.431739
converged
```



```
$fit
```

```
Call:
```

```
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
  Q), period = S), xreg = constant, transform.pars = trans, fixed = fixed,
  optim.control = list(trace = trc, REPORT = 1, reltol = tol))
```

```
Coefficients:
```

	ma1	ma2	ma3	ma4	ma5	constant
	-0.2029	-0.0512	-0.1419	-0.1098	0.1505	0.0083
s.e.	0.0957	0.0948	0.1067	0.0873	0.1120	0.0020

```
sigma^2 estimated as 0.001043: log likelihood = 215.37, aic = -416.74
```

```
$degrees_of_freedom
```

```
[1] 101
```

```
$ttable
```

	Estimate	SE	t.value	p.value
ma1	-0.2029	0.0957	-2.1208	0.0364
ma2	-0.0512	0.0948	-0.5405	0.5900
ma3	-0.1419	0.1067	-1.3299	0.1866
ma4	-0.1098	0.0873	-1.2578	0.2114
ma5	0.1505	0.1120	1.3435	0.1821
constant	0.0083	0.0020	4.0915	0.0001

```
$AIC  
[1] -3.894761
```

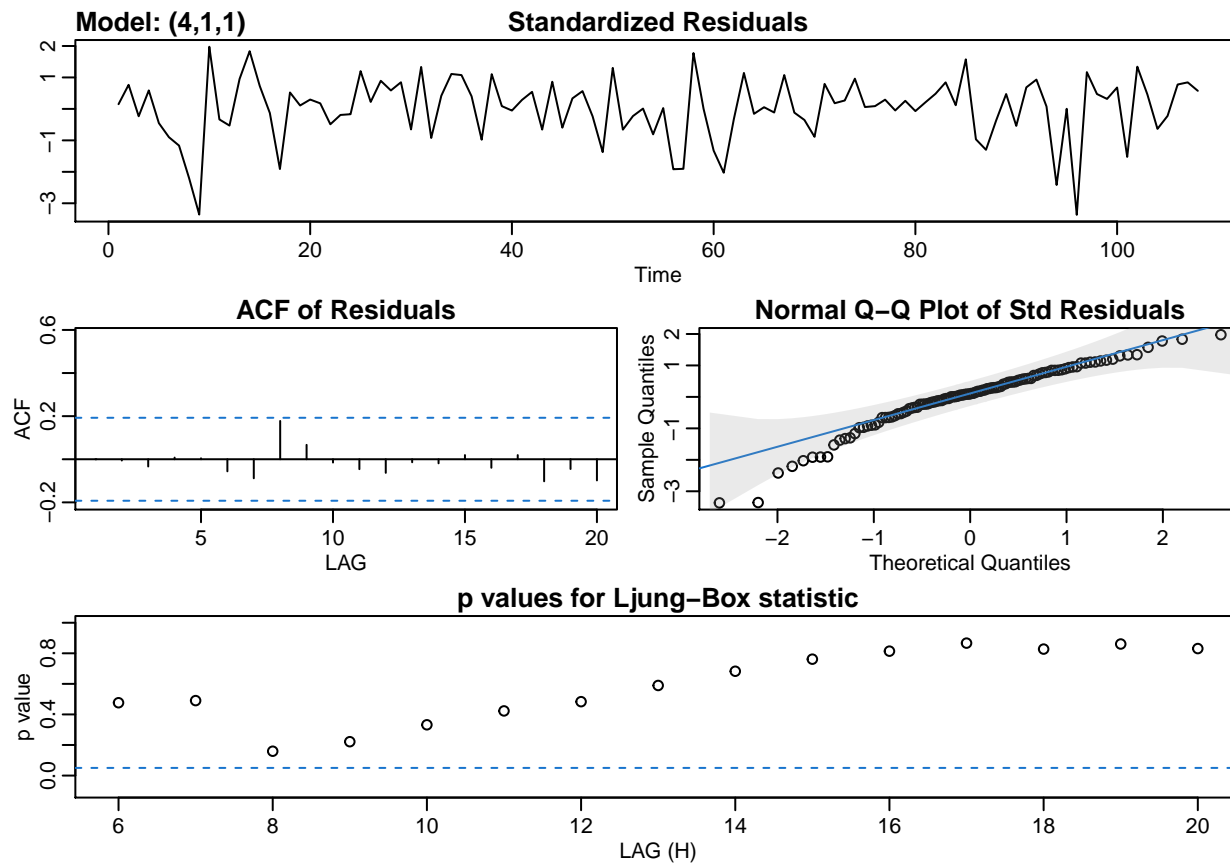
```
$AICc  
[1] -3.88691
```

```
$BIC  
[1] -3.719903
```

```
SandP015 <- arima(SandPTS, order = c(0, 1, 5))
```

```
sarima(SandPTS, 4, 1, 1)
```

```
initial value -3.377146  
iter 2 value -3.401971  
iter 3 value -3.415597  
iter 4 value -3.415927  
iter 5 value -3.415928  
iter 6 value -3.415966  
iter 7 value -3.415967  
iter 8 value -3.415967  
iter 9 value -3.415971  
iter 10 value -3.415984  
iter 11 value -3.416001  
iter 12 value -3.416015  
iter 13 value -3.416035  
iter 14 value -3.416078  
iter 15 value -3.416173  
iter 16 value -3.416186  
iter 17 value -3.416193  
iter 18 value -3.416194  
iter 19 value -3.416195  
iter 20 value -3.416197  
iter 21 value -3.416204  
iter 22 value -3.416212  
iter 23 value -3.416216  
iter 24 value -3.416217  
iter 24 value -3.416217  
iter 24 value -3.416217  
final value -3.416217  
converged  
initial value -3.428989  
iter 2 value -3.429007  
iter 3 value -3.429041  
iter 4 value -3.429048  
iter 5 value -3.429057  
iter 6 value -3.429133  
iter 7 value -3.429154  
iter 8 value -3.429160  
iter 9 value -3.429161  
iter 9 value -3.429161  
iter 9 value -3.429161  
final value -3.429161  
converged
```

\$fit

Call:

```
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
  Q), period = S), xreg = constant, transform.pars = trans, fixed = fixed,
  optim.control = list(trace = trc, REPORT = 1, reltol = tol))
```

Coefficients:

	ar1	ar2	ar3	ar4	ma1	constant
	-0.3799	-0.1272	-0.1120	-0.1933	0.1705	0.0084
s.e.	0.4225	0.1312	0.1052	0.0948	0.4274	0.0020

sigma² estimated as 0.001049: log likelihood = 215.09, aic = -416.19

\$degrees_of_freedom

[1] 101

\$ttable

	Estimate	SE	t.value	p.value
ar1	-0.3799	0.4225	-0.8991	0.3707
ar2	-0.1272	0.1312	-0.9700	0.3343
ar3	-0.1120	0.1052	-1.0653	0.2893
ar4	-0.1933	0.0948	-2.0393	0.0440
ma1	0.1705	0.4274	0.3989	0.6908
constant	0.0084	0.0020	4.0944	0.0001

```
$AIC  
[1] -3.889603
```

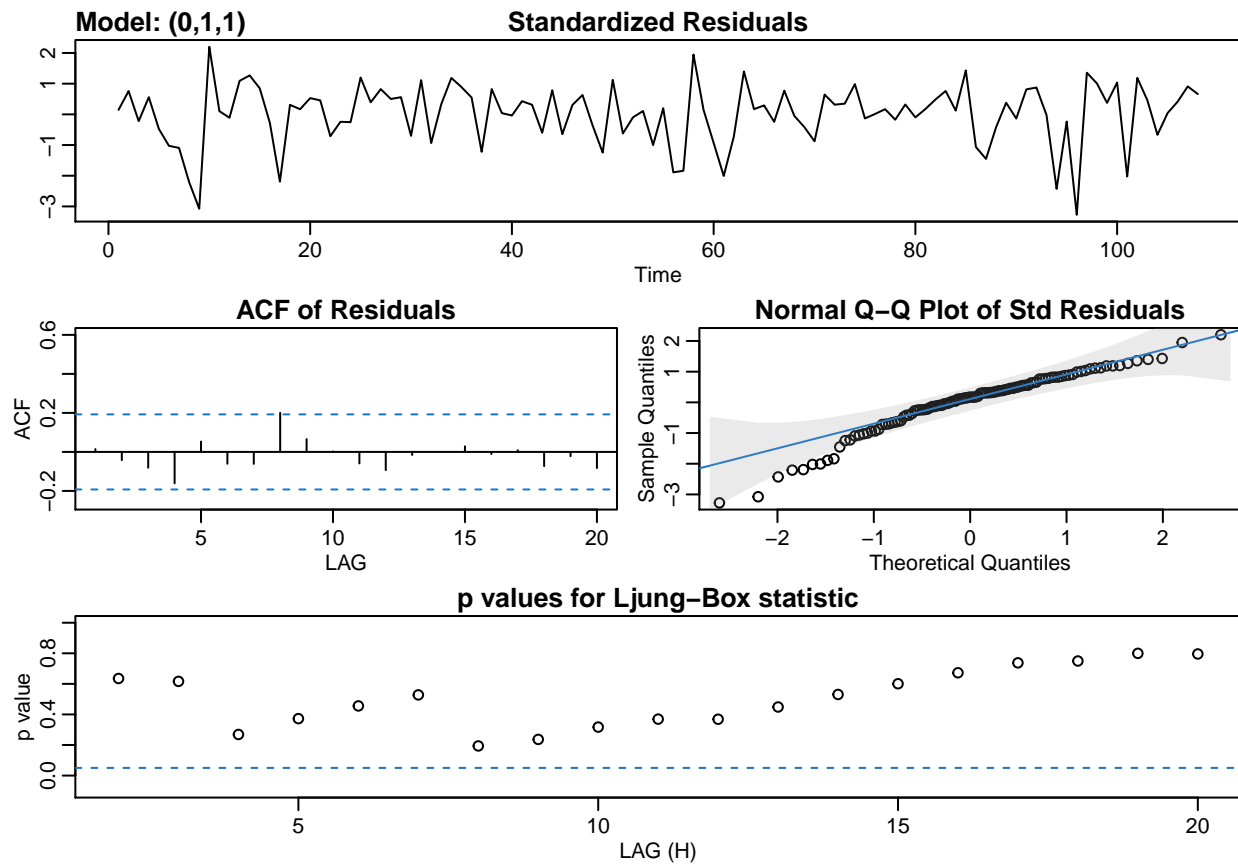
```
$AICc  
[1] -3.881753
```

```
$BIC  
[1] -3.714745
```

```
SandP411 <- arima(SandPTS, order = c(4, 1, 1))
```

```
sarima(SandPTS, 0, 1, 1)
```

```
initial value -3.389534  
iter 2 value -3.409647  
iter 3 value -3.410121  
iter 4 value -3.410121  
iter 5 value -3.410122  
iter 5 value -3.410122  
iter 5 value -3.410122  
final value -3.410122  
converged  
initial value -3.410009  
iter 2 value -3.410010  
iter 2 value -3.410010  
iter 2 value -3.410010  
final value -3.410010  
converged
```



\$fit

Call:

```
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
Q), period = S), xreg = constant, transform.pars = trans, fixed = fixed,
optim.control = list(trace = trc, REPORT = 1, reltol = tol))
```

Coefficients:

	ma1	constant
	-0.2196	0.0085
s.e.	0.1049	0.0025

sigma^2 estimated as 0.001091: log likelihood = 213.04, aic = -420.09

\$degrees_of_freedom

[1] 105

\$ttable

	Estimate	SE	t.value	p.value
ma1	-0.2196	0.1049	-2.0935	0.0387
constant	0.0085	0.0025	3.3897	0.0010

\$AIC

[1] -3.926068

\$AICc

```
[1] -3.92499
```

```
$BIC
```

```
[1] -3.851129
```

```
SandP011 <- arima(SandPTS, order = c(0, 1, 1))
```

```
sarima(SandPTS, 4, 1, 4)
```

```
initial value -3.377146
iter 2 value -3.381571
iter 3 value -3.414100
iter 4 value -3.415456
iter 5 value -3.416942
iter 6 value -3.421574
iter 7 value -3.423061
iter 8 value -3.423075
iter 9 value -3.430900
iter 10 value -3.433793
iter 11 value -3.437280
iter 12 value -3.442074
iter 13 value -3.455092
iter 14 value -3.469776
iter 15 value -3.476636
iter 16 value -3.477753
iter 17 value -3.483887
iter 18 value -3.498283
iter 19 value -3.511488
iter 20 value -3.515535
iter 21 value -3.520911
iter 22 value -3.526419
iter 23 value -3.530148
iter 24 value -3.532614
iter 25 value -3.537018
iter 26 value -3.537705
iter 27 value -3.541374
iter 28 value -3.544623
iter 29 value -3.550804
iter 30 value -3.557485
iter 31 value -3.558324
iter 32 value -3.558635
iter 33 value -3.569101
iter 34 value -3.572167
iter 35 value -3.577111
iter 36 value -3.580384
iter 37 value -3.582056
iter 38 value -3.585357
iter 39 value -3.587814
iter 40 value -3.591282
iter 41 value -3.596998
iter 42 value -3.601076
iter 43 value -3.606648
iter 44 value -3.610394
```

```
iter 45 value -3.615493
iter 46 value -3.618377
iter 47 value -3.630736
iter 48 value -3.633882
iter 49 value -3.634357
iter 50 value -3.635496
iter 51 value -3.635593
iter 52 value -3.635651
iter 53 value -3.635652
iter 54 value -3.637554
iter 55 value -3.644007
iter 56 value -3.644471
iter 57 value -3.644512
iter 58 value -3.644518
iter 59 value -3.650902
iter 60 value -3.650996
iter 61 value -3.651405
iter 62 value -3.651796
iter 63 value -3.651842
iter 64 value -3.651868
iter 65 value -3.651881
iter 66 value -3.652588
iter 67 value -3.652590
iter 68 value -3.652855
iter 69 value -3.654111
iter 69 value -3.654111
iter 70 value -3.656558
iter 71 value -3.656705
iter 72 value -3.656707
iter 73 value -3.656715
iter 74 value -3.657192
iter 75 value -3.657278
iter 76 value -3.657455
iter 76 value -3.657455
iter 77 value -3.658695
iter 78 value -3.659087
iter 79 value -3.659157
iter 80 value -3.659158
iter 81 value -3.659410
iter 81 value -3.659410
iter 82 value -3.660128
iter 83 value -3.660188
iter 84 value -3.660195
iter 85 value -3.660196
iter 86 value -3.660242
iter 86 value -3.660242
iter 87 value -3.660979
iter 88 value -3.661056
iter 89 value -3.661067
iter 90 value -3.661067
iter 91 value -3.661121
iter 92 value -3.661121
iter 92 value -3.661121
iter 93 value -3.661652
```

```
iter 94 value -3.661706
iter 95 value -3.661709
iter 96 value -3.661710
iter 97 value -3.661773
iter 98 value -3.661776
iter 98 value -3.661776
iter 99 value -3.662000
iter 100 value -3.662013
final value -3.662013
stopped after 100 iterations
initial value -3.389534
iter 2 value -3.394042
iter 3 value -3.425211
iter 4 value -3.426235
iter 5 value -3.428060
iter 6 value -3.428752
iter 7 value -3.430533
iter 8 value -3.436020
iter 9 value -3.437143
iter 10 value -3.439016
iter 11 value -3.439701
iter 12 value -3.440176
iter 13 value -3.440580
iter 14 value -3.441016
iter 15 value -3.441175
iter 16 value -3.441473
iter 17 value -3.441661
iter 18 value -3.441818
iter 19 value -3.441889
iter 20 value -3.441954
iter 21 value -3.441964
iter 22 value -3.441972
iter 23 value -3.441978
iter 24 value -3.441990
iter 25 value -3.442009
iter 26 value -3.442046
iter 27 value -3.442086
iter 28 value -3.442120
iter 29 value -3.442150
iter 30 value -3.442166
iter 31 value -3.442170
iter 32 value -3.442171
iter 33 value -3.442172
iter 34 value -3.442190
iter 35 value -3.442202
iter 36 value -3.442220
iter 37 value -3.442232
iter 38 value -3.442251
iter 39 value -3.442267
iter 40 value -3.442270
iter 41 value -3.442276
iter 42 value -3.442288
iter 43 value -3.442413
iter 44 value -3.442700
```

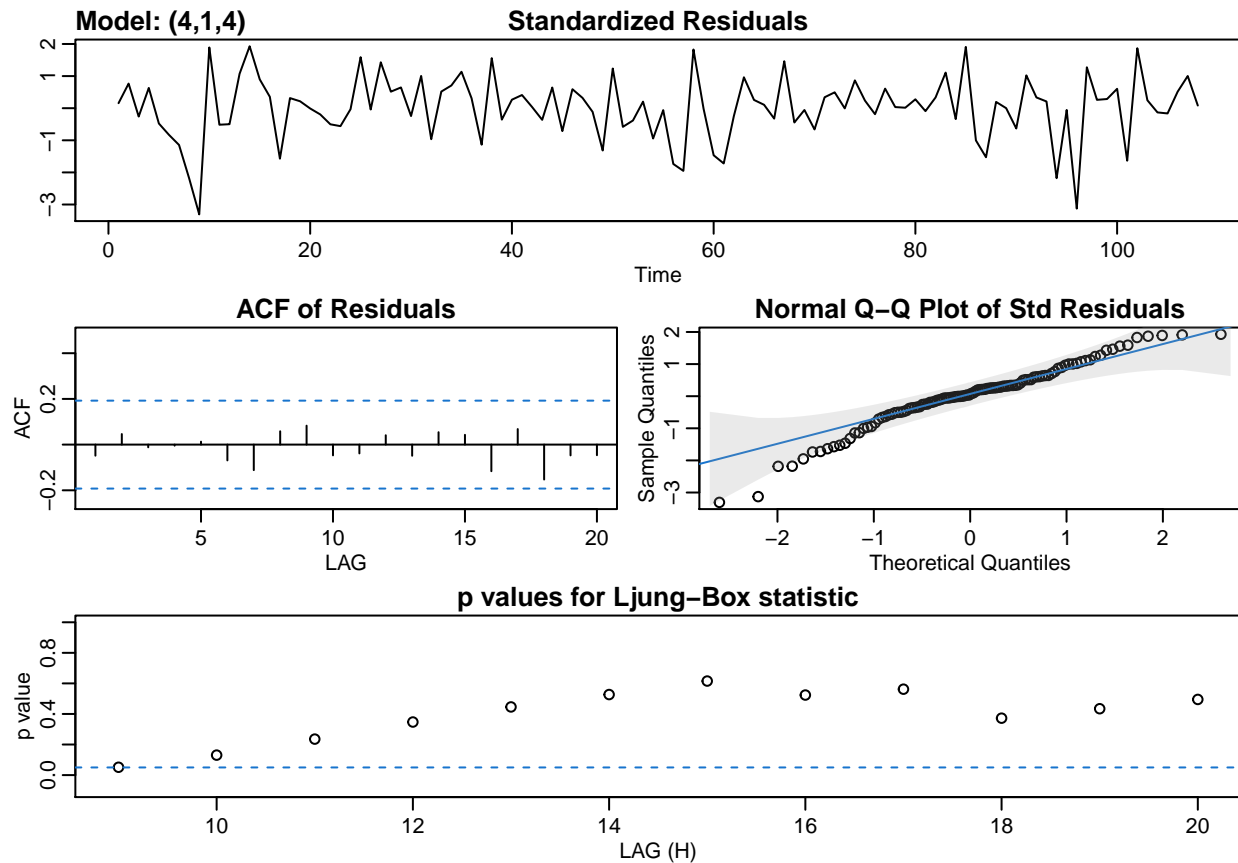
```
iter 45 value -3.442761
iter 46 value -3.443164
iter 47 value -3.443769
iter 48 value -3.444018
iter 49 value -3.444088
iter 50 value -3.444520
iter 51 value -3.444760
iter 52 value -3.445327
iter 53 value -3.445645
iter 54 value -3.445967
iter 55 value -3.446487
iter 56 value -3.446544
iter 57 value -3.447217
iter 58 value -3.447483
iter 59 value -3.447605
iter 60 value -3.447714
iter 61 value -3.447756
iter 62 value -3.447791
iter 63 value -3.448078
iter 64 value -3.448301
iter 65 value -3.448597
iter 66 value -3.448911
iter 67 value -3.449349
iter 68 value -3.449793
iter 69 value -3.449857
iter 70 value -3.450223
iter 71 value -3.450564
iter 72 value -3.450818
iter 73 value -3.451043
iter 74 value -3.451534
iter 75 value -3.451791
iter 76 value -3.451842
iter 77 value -3.451985
iter 78 value -3.452023
iter 79 value -3.452047
iter 80 value -3.452079
iter 81 value -3.452112
iter 82 value -3.452205
iter 83 value -3.452303
iter 84 value -3.452367
iter 85 value -3.452401
iter 86 value -3.452466
iter 87 value -3.452588
iter 88 value -3.452854
iter 89 value -3.452974
iter 90 value -3.453064
iter 91 value -3.453137
iter 92 value -3.453283
iter 93 value -3.453428
iter 94 value -3.453483
iter 95 value -3.453932
iter 96 value -3.454026
iter 97 value -3.454042
iter 98 value -3.454093
```

```

iter 99 value -3.454116
iter 100 value -3.454122
final value -3.454122
stopped after 100 iterations

```

Warning in stats::arima(xdata, order = c(p, d, q), seasonal = list(order = c(P, : possible convergence problem: optim gave code = 1



\$fit

Call:

```

stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
Q), period = S), xreg = constant, transform.pars = trans, fixed = fixed,
optim.control = list(trace = trc, REPORT = 1, reltol = tol))

```

Coefficients:

	ar1	ar2	ar3	ar4	ma1	ma2	ma3	ma4
	-0.0872	0.2124	0.2970	-0.7305	-0.0333	-0.3422	-0.4045	0.7385
s.e.	0.2373	0.3354	0.4684	0.3945	0.2392	0.3539	0.4179	0.3117
constant								
	0.0082							
s.e.	0.0022							

sigma^2 estimated as 0.0009612: log likelihood = 217.76, aic = -415.53


```
$degrees_of_freedom  
[1] 98
```

```
$ttable
```

	Estimate	SE	t.value	p.value
ar1	-0.0872	0.2373	-0.3675	0.7141
ar2	0.2124	0.3354	0.6333	0.5280
ar3	0.2970	0.4684	0.6342	0.5274
ar4	-0.7305	0.3945	-1.8517	0.0671
ma1	-0.0333	0.2392	-0.1391	0.8897
ma2	-0.3422	0.3539	-0.9669	0.3360
ma3	-0.4045	0.4179	-0.9681	0.3354
ma4	0.7385	0.3117	2.3695	0.0198
constant	0.0082	0.0022	3.7213	0.0003

```
$AIC
```

```
[1] -3.88345
```

```
$AICc
```

```
[1] -3.866107
```

```
$BIC
```

```
[1] -3.633653
```

```
SandP414 <- arima(SandPTS, order = c(4, 1, 4), fixed = c(0, 0, NA, NA, 0, 0, NA, NA))
```

```
Warning in stats::arima(x = x, order = order, seasonal = seasonal, xreg =  
xreg, : some AR parameters were fixed: setting transform.pars = FALSE
```

The Lyung-Box statistic is the best for $\text{arima}(0, 1, 5)$, but all of the MAs are insignificant except for the first one, so we decided to use $\text{arima}(0, 1,)$

```
shapiro.test(rstandard(SandP011))
```

Shapiro-Wilk normality test

```
data: rstandard(SandP011)  
W = 0.9633, p-value = 0.004484
```

The residuals do not appear to be normal because of a few outliers on the lower end on the qqnorm plot.

```
runs(rstandard(SandP011))
```

```
$pvalue
```

```
[1] 0.389
```

```
$observed.runs
```

```
[1] 41
```

```
$expected.runs
```

```
[1] 45.2037
```

```
$n1
```

```
[1] 31
```

```
$n2
```

```
[1] 77
```

```
$k
```

```
[1] 0
```

There is no statistical evidence that the observations are not independent.

$$\hat{Y}_t = Y_{t-1} + .2196e_{t-1}$$

```
SandP.predict <- predict(SandP011, n.ahead=11)
```

```
lower.end <- (SandP.predict$pred - qnorm(0.975,0,1) * SandP.predict$se)
```

```
upper.end <- (SandP.predict$pred + qnorm(0.975,0,1) * SandP.predict$se)
```

```
predictsand <- data.frame(time = 108:118, lower.end, upper.end, actual = log(SandP2$Close))
```

```
lowerp <- cbind(predictsand$time, predictsand$lower.end)
```

```
upperp <- cbind(predictsand$time, predictsand$upper.end)
```

```
actualp <- cbind(predictsand$time, predictsand$actual)
```

```
predicts <- as.data.frame(rbind(lowerp, upperp, actualp))
```

```
colnames(predicts) <- c("time", "value")
```

```
predicts$type <- c(rep("lower", 11), rep("upper", 11), rep("actual", 11))
```

```
predicts
```

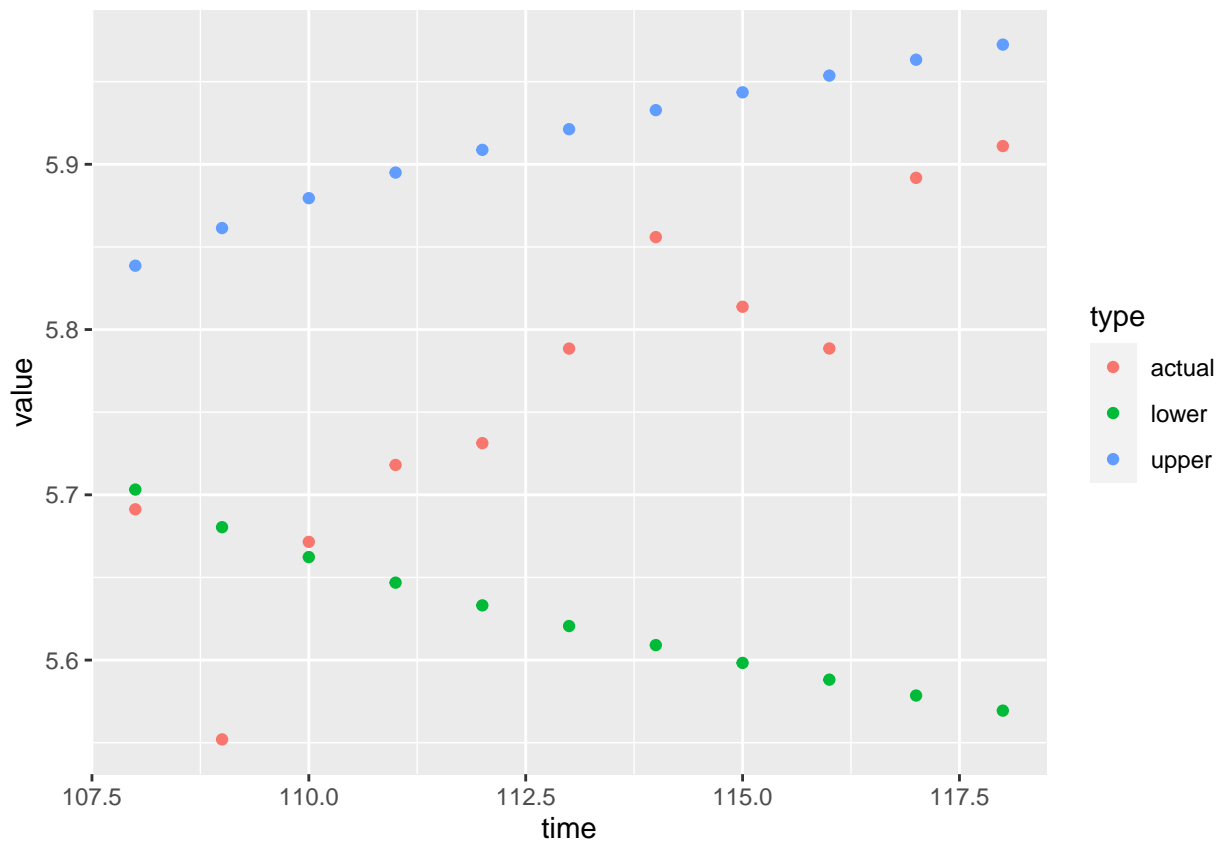
	time	value	type
1	108	5.703185	lower
2	109	5.680421	lower
3	110	5.662329	lower
4	111	5.646847	lower
5	112	5.633094	lower
6	113	5.620594	lower
7	114	5.609056	lower
8	115	5.598288	lower
9	116	5.588153	lower
10	117	5.578552	lower
11	118	5.569407	lower
12	108	5.838648	upper
13	109	5.861412	upper
14	110	5.879504	upper
15	111	5.894985	upper
16	112	5.908738	upper
17	113	5.921239	upper
18	114	5.932776	upper
19	115	5.943544	upper
20	116	5.953679	upper
21	117	5.963281	upper
22	118	5.972426	upper
23	108	5.691237	actual
24	109	5.551990	actual

```

25 110 5.671535 actual
26 111 5.718080 actual
27 112 5.731268 actual
28 113 5.788491 actual
29 114 5.855960 actual
30 115 5.813802 actual
31 116 5.788552 actual
32 117 5.891810 actual
33 118 5.911041 actual

```

```
ggplot(predicts, aes(time, value)) + geom_point(aes(color = type))
```



Only the first two predictions missed the forecast, and the first barely missed it. The forecasting was actually fairly good for the model.