Math Method
Project

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Background

Cell modeling can be a difficult task to do as there is not a unified approach that works for all cells. Most models come from one of two approaches: Force based coming from newton's Laws or Energy based. Our Model will fall into the later and exploit free energy minimization to achieve our goals. This approach borrows heavily from Condensed Matter Theory and was originally designed for describing the phase transition of superconductors.

Objectives

1. Plot the formation of a cell from a square into a blob (easy)
2. Describe the movement of a cell based on this paper (difficult)

Model

As stated previously this model will be using the Ginzburg-Landau Model for superconductors. The first step of this problem is to describe the Free energy of the system. This is given to us by (for the easy case):

$$\mathcal{F}[\phi.h,\sigma] = \int f(\phi) - \frac{\sigma}{2}|\nabla\phi|^2 d\boldsymbol{r} - h\int g(\phi)d\boldsymbol{r}$$

In this equation, ϕ is a function of x and y and is equal to one if we are in the cell and zero if not, the Γ, h and σ terms are found from experiment. These however do have a physical significance. The Γ value tells us how fast our whole thing is going. The σ value tells us that our system's strength feels the smoothing effect. The h is field strength, or in this case, what the cell is feeling from what it is sitting in. Both $f(\phi)$ and $g(\phi)$ are potential terms based on phase transitions but are usually plugged in and changed depending on the problem. For us the values of f that will be used will change from problem to problem. For the easier case, these will be given:

$$g(\phi) = \phi^3(3\phi(2\phi-5)+10) \implies g'(\phi) = 30(\phi-1)^2\phi^2$$
$$f(\phi) = 4(1-\phi)^2\phi^2 \implies f'(\phi) = 8\phi\left(2\phi^2-3\phi+1\right)$$

From this, we can then now take the Frechet derivative to find the minimum this is tells us that:

$$\frac{\partial\phi}{\partial t} = -\Gamma\frac{\delta\mathcal{F}}{\delta\phi}$$

Doing this functional derivative is not too bad, but it may be better to say that the result is given as:

$$\frac{\partial\phi}{\partial t} = \Gamma\left[\sigma\nabla^2\phi - f'(\phi) - phg'(\phi)\right]$$

and p is a Lagrange multiplier and is obtained by making a quasi-static approximation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int g(\phi)d\boldsymbol{X} = \int g'(\phi)\frac{\mathrm{d}\phi}{\mathrm{d}t}\boldsymbol{X} = 0$$

and then plugging in our value for $\frac{\mathrm{d}\phi}{\mathrm{d}t}$ above to achieve:

$$p = \frac{\int g'(\phi)k\left[\sigma\nabla^2\phi - f'(\phi)\right]d\boldsymbol{X}}{h\int g'(\phi)g'(\phi)d\boldsymbol{X}}$$

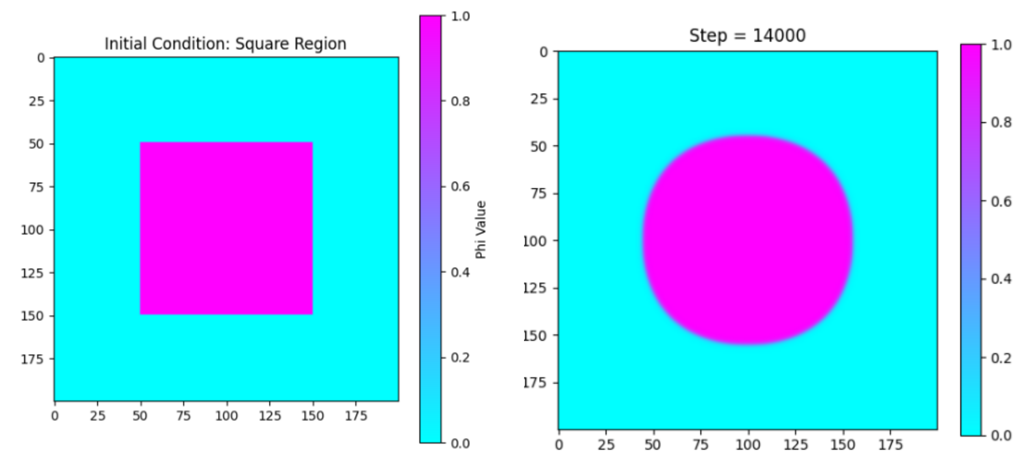
where the integrals of $d\boldsymbol{X}$ are over the whole domain

Method

We will start using the finite difference method, but if that isn't working for more complicated stuff may have to go to some package in Python or Matlab.

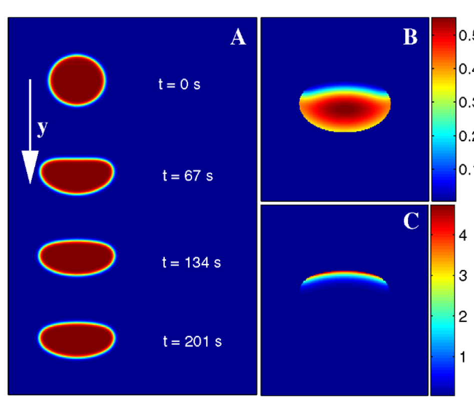
Results

Hopefully, reproduce the formation of a cell as well as see an animation of cell movement.



This is what has been achieved so far in our quest to make a circle starting from a square! Seems close (ish?)

We would also like to reproduce the following from the cited paper:



Citations

Shao, D., Rappel, W.-J. Levine, H. Computational model for cell morphodynamics. Phys. Rev. Lett. 105, 108104 (2010).