① $f(x) = \sqrt{2x+1}$ $\quad f: [-\frac{1}{2}, \infty) \to \mathbb{R}$

$f'(x) = \frac{1}{\sqrt{2x+1}}$ Vi ser at den holder seg mellom $[0, \infty)$

$\sqrt{2x+1} = y$ $\quad g(y) = \frac{1}{2}(y^2-1)$

$2x+1 = y^2$

$x = \frac{1}{2}(y^2-1)$

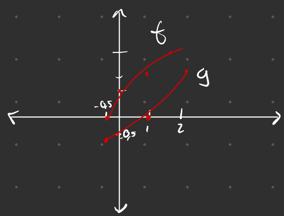
setter inn for $-\frac{1}{2}, 0, 1, 2$

$f(-\frac{1}{2}) = 0$ $\quad g(-\frac{1}{2}) = -\frac{3}{8}$

$f(0) = 1$ $\quad g(0) = -\frac{1}{2}$

$f(1) = \sqrt{3}$ $\quad g(1) = 0$

$f(2) = \sqrt{5}$ $\quad g(2) = \frac{3}{2}$



② $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ bruker integraltesten

$$\int_1^{\infty} \frac{1}{\sqrt{n}} = \lim_{b\to\infty} \left[2\sqrt{n}\right]_1^{\infty} = \lim_{b\to\infty} 2\sqrt{b} - 2\sqrt{1} = \underline{\infty} \text{ divergerer}$$

③ $w = c_1 v_1 + c_2 v_2 + \dots c_n v_n$

Vi vet at hvis $c_1 v_1 + c_2 v_2 + \dots c_n v_n = 0$ er de lineært uavhengig

og at

$c_1 = c_2 \dots = c_n = 0$

$c_1 v_1 + c_2 v_2 + \dots c_n v_n = w$

$d_1 v_1 + d_2 v_2 + \dots d_n v_n = w$

$w - w = 0$

$(c_1 - d_1) v_1 + (c_2 - d_2) v_2 \dots (c_n - d_n) v_n = 0$

da må

$c_1 - d_1 = c_2 - d_2 = c_n - d_n = 0$

som gjør at

$c_1 = d_1$, $c_2 = d_2$, $c_3 = d_3$. Noe som beviser at c_n må være entydig bestemt

④ a) $A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$

$$D = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 9 - 6\lambda + \lambda^2 - 1 = \lambda^2 - 6\lambda + 8 = (\lambda-2)(\lambda-4)$$

$\lambda = 2$ $\quad x_1 + x_2 = 0$ $\quad x_1 = t$ $\quad t\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 4$ $\quad x_1 + x_2 = 0$

$P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$-x_1 + x_2 = 0$ $\to$ $x_1 = x_2$ $\quad \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x_1 - x_2 = 0$

b) $\dot{x} = Ax$

$$x = c_1 e^{\lambda_1 t} v_1 + c_2 e^{\lambda_2 t} = c_1 e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{2t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

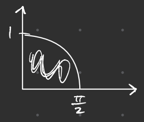
$$x(0) = c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$c_1 = c_2 = 1$

$$\underline{\dot{x} = e^{4t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{2t}\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

(5) $f: [0, \frac{\pi}{2}] \to \mathbb{R}$

$f(x) = \cos x$



$V = 2\pi \cdot \int_0^{\frac{\pi}{2}} x \cdot \cos x \, dx$

$u = x \quad u' = 1$
$v' = \cos x \quad v = \sin x$

$x \sin x - \int \cos x = x \sin x - \sin x$

$V = 2\pi \cdot \left[x \sin x - \sin x \right]_0^{\frac{\pi}{2}} = \underline{2\pi \cdot \left(\frac{\pi}{2} - 1 \right)}$

(6) a) $x_{n+1} = 1 + \arctan x_n$

$f'(x) = \frac{1}{1+x^2}$ vil konvegere for alle x verdier utenom 0

b)
```
import numpy as np
def f(x):
    return 1+np.arctan(x)

x = 1
x_neste = 2

while abs(x-x_neste) ≥ 10e-6:
    x = x_neste
    x_neste = f(x)

print(x_neste)
```

(7) $\int \frac{1}{x^4 - 4x^2} \, dx \Rightarrow \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2} + \frac{D}{x+2}$

$1 = Ax(x^2-4) + B(x^2-4) + Cx^2(x+2) + Dx^2(x-2)$

Aner ikke veien videre

(8) $f: [1, \infty) \to R$

$2\pi \int_a^b \frac{1}{x}\sqrt{1+\frac{1}{x^4}} \geq 2\pi \cdot \int_a^b \frac{1}{x} = 2\pi \cdot \lim_{b \to \infty} \left[\ln x \right]_1^b = \underline{\underline{\infty}}$