① $f(x) = \cos^2\left(\frac{x}{2}\right)$

$$f(x) = \frac{1}{2} + \frac{\cos\left(2\cdot\frac{x}{2}\right)}{2} = \frac{1}{2} + \frac{\cos(x)}{2}$$

$$\frac{1+\cos(x)}{2} = \frac{1+\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}}{2} = \underline{\frac{1}{2} + \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{2(2n)!}}$$

② $$\lim_{x\to 0}\frac{(e^x-1-x)^2}{x^2-\ln(1+x^2)} = \frac{\left(\sum_{n=0}^{\infty}\frac{x^n}{n!}-1-x\right)^2}{x^2-\sum_{n=1}^{\infty}\frac{(-1)^n}{n}(x^2)^n} = \frac{\left(\cancel{1}+\cancel{x}+\frac{x^2}{2}\dots\right)^2-\cancel{1}-\cancel{x}}{\cancel{x^2}-\left(-\cancel{x^2}+\frac{x^4}{2}-\frac{x^6}{3}\dots\right)} = \frac{\frac{x^4}{4}+\dots}{\frac{x^4}{2}-\frac{x^6}{3}\dots} = \lim_{x\to 0}\frac{\frac{1}{4}}{\frac{1}{2}-\frac{x^2}{3}\dots}$$

$$= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4}\cdot\frac{2}{1} = \underline{\underline{\frac{1}{2}}}$$

③ $\dot{y} = y^2 - y^3$

```
import numpy as np
import matplotlib.pyplot as plt

def f(y):
    return y**2 - y**3
a=0
b=200
n= 1000
h =(b-a)/n
y= np.zeros(n+1)
y[0] = 0.001
t = np.array
for i in range(y):
    y[i+1] = y[i] + h·f(y[i])
    y[i+1]= y[i] +(h/2)·(f(y[i]) + f(y[i+1]))
    t[i]=i
plt.plot (t,y)
plt.show()
```

④ $$\int_1^{\infty}\frac{\cos t}{t^2}\,dt \le \int_1^{\infty}\frac{1}{t^2}\,dt = \left[-\frac{1}{t}\right]_1^{\infty} = \lim_{b\to\infty}\left(-\frac{1}{b}+\frac{1}{1}\right) = 1$$

⑤ $$\sum_1^{\infty}\frac{\cos n}{n^2} \le \sum_1^{\infty}\frac{1}{n^2} = \underline{\underline{\frac{\pi^2}{6}}}$$

⑥ $$f(x) = \int_1^x\sqrt{t^3-1} \rightarrow f'(x) = \left(\int_1^x g(t)\right)' = \left(G(x)-G(1)\right)' = g(x) = \sqrt{x^3-1}$$

$$f'(x)^2 = x^3-1$$

$$S = \int_1^2\sqrt{1+f'(x)^2}\,dx$$

$$S = \int_1^2\sqrt{1+x^3-1}\,dx = \int_1^2\sqrt{x^3}\,dx = \int_1^2 x^{\frac{3}{2}}\,dx = \left[\frac{2}{5}x^{\frac{5}{2}}\right]_1^2 = \frac{2}{5}\cdot\left(2^{\frac{5}{2}}-1^{\frac{5}{2}}\right) = \underline{\underline{\frac{2}{5}\cdot(4\cdot\sqrt{2}-1)}}$$

⑦ a) $$\begin{bmatrix}1&2&0\\3&4&2\\5&6&4\end{bmatrix} \sim \left[\begin{array}{ccc|c}1&2&0&0\\0&2&-2&0\\0&4&-4&0\end{array}\right]$$

$x_1 = -2s$
$x_2 = s$
$x_3 = s$

$s\begin{bmatrix}-2\\1\\1\end{bmatrix}$ linjært avhengig siden det finnes uendelig mange løsninger og $\begin{bmatrix}0\\0\\0\end{bmatrix}$ er ikke eneste løsning

b) $$\begin{bmatrix}1&2&0&-5\\3&4&2&-17\\5&6&4&-29\end{bmatrix} \sim \begin{bmatrix}1&2&0&-5\\0&2&-2&2\\0&4&-4&4\end{bmatrix} \sim \begin{bmatrix}1&2&0&-5\\0&2&-2&2\\0&0&0&0\end{bmatrix}$$

$x_1 = -5-2s$
$x_2 = s$
$-2x_3 = 2-2s$
$x_3 = s-1$

$$\underline{\underline{\begin{bmatrix}-5\\0\\-1\end{bmatrix} + s\begin{bmatrix}-2\\1\\1\end{bmatrix}}}$$

7 c)

$$D = (1-\lambda)\begin{vmatrix} 4-\lambda & 2 \\ 6 & 4-\lambda \end{vmatrix} + 2\begin{vmatrix} 3 & 2 \\ 5 & 4-\lambda \end{vmatrix}$$

$$D = (1-\lambda)(\lambda^2 - 8\lambda + 16 - 12) + 2(12 - 3\lambda - 10)$$

$$D = (1-\lambda)(\lambda^2 - 8\lambda + 4) - 2(-3\lambda + 2)$$

$$D = \lambda^2 - 8\lambda + \cancel{4} - \lambda^3 + 8\lambda^2 - 4\lambda + 6\lambda - \cancel{4} = -\lambda^3 + 9\lambda^2 - 6\lambda = -\lambda(\lambda^2 - 9\lambda + 6) = 0$$

$$\lambda = 0$$

$$\lambda = \frac{9 \pm \sqrt{9^2 - 4 \cdot 1 \cdot 6}}{2} = \frac{9 \pm \sqrt{57}}{2}$$

får 3 egenvektorer som gir
at matrisen er diagonaliserbar

8 $f(x) = x^2$
$g(x) = \sqrt{x}$

$$V = 2\pi\left(\int_0^1 x^3 - \int_0^1 x^{\frac{3}{2}}\right) = 2\pi \cdot \left(\left[\frac{1}{4}x^4\right]_0^1 - \left[\frac{2}{5}x^{\frac{5}{2}}\right]_0^1\right) = 2\pi \cdot \left(\frac{1}{4} - \frac{2}{5}\right) = 2\pi \cdot -\frac{3}{20} = \left|-\frac{3\pi}{10}\right| = \underline{\underline{\frac{3\pi}{10}}}$$