① $\dot{y} + \sqrt{y} = 0$

$\dot{y} = -\sqrt{y}$

```
import numpy as np

yneste=0
a=0
b=1
n=10000
h=(b-a)/n

for i in range(n):
    y= yneste
    yneste = y+ h·np.sqrt(y)
    yneste = y_(h/2)(np.sqrt(y) + np.sqrt(yneste))
```

② $\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \sum_{n=1}^{\infty} \frac{A}{n} + \frac{B}{n+2} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2} \to \frac{1}{2}\left(1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \frac{1}{5} + \cancel{\frac{1}{4}} - \frac{1}{6}\right.$

ser at alle verdiene vil gå mot hverandre utenom $\log \frac{1}{2}$

$A(n+2) + Bn = 1$

setter inn -2

$-2B = 1$

$B = \frac{-1}{2}$

setter inn 0

$2A = 1$

$A = \frac{1}{2}$

$= \frac{1}{2}\left(1 + \frac{1}{2}\right) = \underline{\underline{\frac{3}{4}}}$

$x_3 = 2$

③ a) $\begin{bmatrix} 2 & 4 & 6 & 0 \\ 3 & 5 & 7 & 0 \\ 2 & 4 & 8 & 4 \\ 1 & 1 & 1 & 0 \end{bmatrix} \sim$

$\begin{array}{r} 2460 \\ -2484 \\ \hline 00\text{-}2\text{-}4 \end{array}$

$\begin{array}{r} 3(2460) \\ -2(3570) \\ \hline 0240 \end{array}$

$x_2 = -2x_3 = -4$

$x_1 = -x_2 - x_3 = 4 - 2 = 2$

entydig løsning

noe som sir

lineært uavhengig vektorer

$\begin{bmatrix} 2 & 4 & 6 & 0 \\ 3 & 5 & 7 & 0 \\ 2 & 4 & 8 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$\begin{array}{r} 2460 \\ -(2481) \\ \hline 00\text{-}2\text{-}1 \end{array}$

$\begin{array}{r} 3(2460) \\ -2(3570) \\ \hline 0240 \end{array}$

$x_3 = \frac{1}{2}$

$x_2 = -2x_3 = -1$

$x_1 = 1 - x_2 - x_3 = 1 + 1 - \frac{1}{2} = \frac{3}{2}$

$2x_1 + 4x_2 + 6x_3 = 0$

$2 \cdot \frac{3}{2} + 4 \cdot (-1) + 6 \cdot \frac{1}{2} =$

$3 - 4 + 3 = 0$

$2 \neq 0$

ikke løselig

④ $\cos x \le 1$

$\int_0^1 \frac{\cos x}{\sqrt{x}} dx \le \int_0^1 \frac{1}{\sqrt{x}} dx = \left[2\sqrt{x}\right]_0^1 = 2\sqrt{1} - 2\sqrt{0} = \underline{\underline{2}}$

⑤ $\dot{y} + y = \cos t + \sin t$

setter $\cos t + \sin t = f(t)$

$\dot{y} + y = f(t)$

$\frac{d}{dt} e^t y = e^t(f(t))$

$\int \frac{d}{dt} e^t y = \int e^t f(t)$

$e^t y = \int e^t f(x)$

$y = e^{-t}\left(\int e^t \cos t + \int e^t \sin t\right)$

⑥ $f: \mathbb{R} \to \mathbb{R} \quad f(x) = |x|$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{|x+h| - x}{h} = \lim_{h \to 0^+} \frac{|h|}{h} = \underline{\underline{1}}$

$\lim_{\to 0^-} \frac{|h|}{h} = \underline{\underline{-1}}$

⑦
```
import numpy as np
def f(x):
    return np.sqrt(1 + (np.cos(x))**2)

a=0
b=np.pi
n=100000
h=(b-a)/n
summen = 0
for i in range(n):
    x= a+h*i
    x2= a+h*(i+1)
    summen += (h/2) (f(x)+f(x2))
```

⑧ $f(x) = x^5 + x^3 + x^2$

$f'(x) = 5x^4 + 3x^2 + 2x$

$5x^4 + 3x^2 + 2x > 0$; intervallet

$g'(3) \Rightarrow f'(x) = 3$

$5x^4 + 3x^2 + 2x = 3$

$f(1) = 3$

$f^{-1}(3) = 1$

$(f(f^{-1}(x)))' = (x)'$

$f'(1) = 5 + 3 + 2 = 10$

$f'(f^{-1}(x)) \cdot (f^{-1}(x))' = 1$

$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$

$(f^{-1}(3))' = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(1)} = \underline{\underline{\frac{1}{10}}}$

⑨ $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \geq \sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow$ Divergerer