① Setter opp utrykket $\sqrt{2} = \frac{m}{n}$

Vi antar at m og n ikke har noen felles faktorer, siden vi da kunne vi ha forkortet brøken

$2n^2 = m^2$ ser da at m^2 må være et partall, noe som impliserer at m også er et partall. $n^2 = \frac{m^2}{2}$. Siden vi har bevist at m^2 er et partall må også n^2 være et partall.

Dette beviser også at de har en faktor i 2. Noe som går imot antagelsen $\sqrt{2} = \frac{m}{n}$ der m og n er hele tall

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② $\sqrt{-4}\cdot\sqrt{-4} = 2i \cdot 2i = -4$

③ $x^2+2x+2=0$

$$x = \frac{-2 \pm \sqrt{2^2-4\cdot 2}}{2} = \frac{-2\pm\sqrt{-4}}{2}$$

$x = -1 \pm i$

$x_1 = -1+i$

$x_2 = -1-i$

$p(x) = (x+1+i)(x+1-i)$

④ $z = 2+3i$

$w = 4+5i$

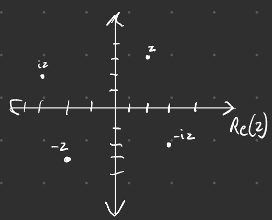
$z+w = 2+4+(3+5)i = 6+8i$

$z-w = 2-4+(3-5)i = -2-2i$

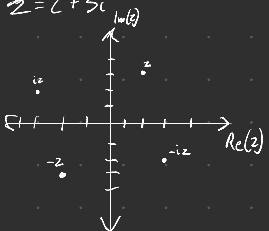
$z\cdot w = 2\cdot 4 - 3\cdot 5 + (3\cdot 4+2\cdot 5)i = -7+22i$

$\frac{z}{w} = \frac{2+3i}{4+5i}\cdot\frac{4-5i}{4-5i} = \frac{8-10i+12i+15}{16-20i+20i+25} = \frac{22}{41} + \frac{2}{41}i$

⑦ $\frac{\bar{z}}{|z|^2} = \frac{\bar{z}}{\sqrt{z\bar{z}}^2} = \frac{\bar{z}}{z\bar{z}} = \frac{1}{z}$



⑧ $z = 2+3i$

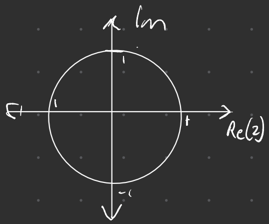


⑨ $z = 1+\sqrt{3}i$

$$\frac{z}{|z|} = \frac{1+\sqrt{3}i}{\sqrt{1^2+\sqrt{3}^2}} = \frac{1+\sqrt{3}i}{2} = \frac{1}{2}+\frac{\sqrt{3}}{2}i$$

$$\theta = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) = \frac{\pi}{3}$$

$$= \left(e^{i\frac{\pi}{3}}\right)^k = e^{i\frac{\pi}{3}k}$$



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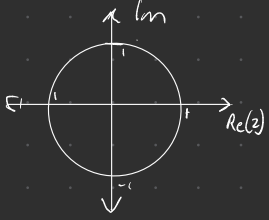
(10) $z = \frac{1}{2} + \frac{\sqrt{3}}{2} i$ $\theta_1 = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$\theta_2 = \tan^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{4}$

$w = 1 + i$

$$z^k w = e^{i\theta_1 k} + e^{i\theta_2} = e^{i\left(\frac{\pi}{3}k + \frac{\pi}{4}\right)}$$



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(16) $|z|^2 = |z^2|$

$$|z|^2 = \sqrt{a^2+b^2}^{\,2} = a^2 + b^2$$

$$|z^2| = \sqrt{(a^2+b^2)^2} = a^2 + b^2$$

(17) $f(\theta) = \frac{\cos\theta + i\sin\theta}{e^{i\theta}}$

$$f'(\theta) = \frac{(\cos\theta + i\sin\theta)' e^{i\theta} - (\cos\theta + i\sin\theta)(e^{i\theta})'}{(e^{i\theta})^2} = \frac{(-\sin\theta + i\cos\theta)\cancel{e^{i\theta}} - (\cos\theta + i\sin\theta)\cancel{(e^{i\theta})} \cdot i}{(e^{i\theta})^{\cancel{2}}}$$

$$\frac{\cancel{-\sin\theta} + \cancel{i\cos\theta} - \cancel{i\cos\theta} + \cancel{\sin\theta}}{e^{i\theta}} = \frac{1}{e^{-i\theta}}$$

(18) $x^2 + x + 1 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-1 \pm \sqrt{3}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

Mer om komplekse tall (Fra 1-4 diff lign)

(1) $z = 1 + i$ $w = 1 + \sqrt{3} i$

$$|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\theta = \arccos\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} \rightarrow \underline{\underline{\sqrt{2} e^{i\frac{\pi}{4}}}}$$

$$|w| = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{4} = 2$$

$$\theta = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3} \rightarrow \underline{\underline{2e^{\frac{\pi}{3} i}}}$$

(2) $z \cdot w = \sqrt{2} e^{i\frac{\pi}{4}} \cdot 2e^{\frac{\pi}{3} i} = \sqrt{2}\, 2e^{i\left(\frac{\pi}{4} + \frac{\pi}{3}\right)} = \underline{\underline{\sqrt{2}\, 2e^{i\frac{7\pi}{12}}}}$

$$\frac{z}{w} = \sqrt{2} e^{i\frac{\pi}{4}} / 2e^{i\frac{\pi}{3}} = \frac{\sqrt{2}}{2} e^{i\left(\frac{\pi}{4} - \frac{\pi}{3}\right)} = \underline{\underline{\frac{\sqrt{2}}{2} e^{-i\frac{\pi}{12}}}}$$