① $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$$D = (1-\lambda)\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 \qquad \lambda = 1$$

$x_2 = 0$
$x_3 = 0 \Rightarrow s\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ siden matrisen kun har en egenvektor er den ikke diagonaliserbar
$x_1 = s$

② $\dot{x}_1 = x_1 + x_2$, $x_1(0) = 1$
$\dot{x}_2 = x_2 + x_3 \Rightarrow x_2(0) = 0$
$\dot{x}_3 = x_3$, $x_3(0) = 0$

ser at
$\Rightarrow \dot{x}_3 = \dot{x}_2 = x_2 = x_3 = 0$ for alltid
$\dot{x}_1 = 1 + 0$
$\dot{x}_2 = x_1 + 0 = 1 + 0$

$$x = e^t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} e^t \\ 0 \\ 0 \end{bmatrix}$$

③ $\dot{x} + ax = 0$

$e^{at}(\dot{x} + ax) = 0$

$\frac{d}{dt} e^{at} x = 0$

$\int \frac{d}{dt} e^{at} x = \int 0 \, dt$

$e^{at} x = c$

$x = c e^{-at}$

$x(0) = c e^{-at} = x_0 \Rightarrow c = x_0$

$\underline{x = x_0 e^{-at}}$

④ $$P_n(x) = 1 \cdot \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)} + 2\,\frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)} + 3 \cdot \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)} + 5 \cdot \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}$$

$(x^2 - 3x + 2)(x-3)$

$$\frac{x^3 - 6x^2 + 11x - 6}{-6} + 2\,\frac{x^3 - 5x^2 + 6x}{2} + 3 \cdot \frac{x^3 - 4x^2 + 3x}{-2} + 5 \cdot \frac{x^3 - 3x^2 + 2x}{6}$$

$$= \frac{-x^3 + 6x^2 - 11x + 6 + 6x^3 - 30x^2 + 36x - 9x^3 + 36x^2 - 27x + 5x^3 - 15x^2 + 10x}{6} = \frac{x^3 - 3x^2 + 8x + 6}{6} = \frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{4}{3}x + 1$$

⑤ $\sqrt{2} = \frac{n}{m}$

$m^2 = \frac{n^2}{2}$ ser at m^2 må være et partall siden det er n^2 delelig på 2 og M må da være et partall for at m^2 er partall

$n^2 = (2)m^2$ se at n^2 er partall · partall. Noe som gjør at n også er partall

begge tallene har en felles vektor så $\frac{n}{m}$ er delelig og som gjør at $\frac{n}{m}$ alltid har en felles faktor i 2 som motstrider antagelsen $\underline{\sqrt{2} = \frac{n}{m}}$

⑥ $2\sin 2\theta + \pi - 4\theta = 0$

$\theta = \frac{1}{2}\sin 2\theta + \frac{\pi}{4}$

```
import numpy as np

def f(x):
    return (1/2) np.sin(2·x) + np.pi/4

x = 1
for i in range(100):
    x = f(x)
```

⑦ gjort før

8) $\int_{-\pi}^{\pi} \cos(2\theta)\cos\theta \, d\theta$

$u = \sin\theta$
$u' = \cos\theta$

$\int_{-\pi}^{\pi} (1-2\sin^2\theta)\cos\theta \, d\theta = \Big|_{-\pi}^{\pi} \int \cos\theta - 2\int \sin\theta\cos\theta = \Big|_{-\pi}^{\pi} \sin\theta - 2\left(\int u \, du\right)$

$\Big|_{-\pi}^{\pi} \sin\theta - 2\frac{1}{2}u^2 = \Big|_{-\pi}^{\pi} \sin\theta - u^2 = \Big|_{-\pi}^{\pi} \sin\theta - (\sin\theta)^2 = \underline{\underline{0}}$

9) <u>Gjort før</u>

7) $\sum_{n=3}^{\infty} \frac{1}{n(n-2)} = \sum_{n=3}^{\infty} \frac{A}{n} + \frac{B}{n-2}$

$A(n-2) + B(n) = 1$

$2B = 1$
$B = \frac{1}{2}$

$-2A = 1$
$A = -\frac{1}{2}$

$\frac{1}{2}\sum_{n=3}^{\infty} \frac{1}{n-2} - \frac{1}{n} = \frac{1}{2}\left(1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \frac{1}{5} + \cancel{\frac{1}{4}} - \frac{1}{6} \ldots\right)$

$\frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{1}{2} \cdot \frac{3}{2} = \underline{\frac{3}{4}}$