6.1.1

a) $\sum_{n=0}^{\infty}\left(\frac{4}{5}\right)^n$ $\quad |x|<1$, $\frac{4}{5}<1$ Konv $\quad S_n=\frac{1}{1-\frac{4}{5}}=\frac{1}{\frac{1}{5}}=\underline{\underline{5}}$

b) $\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^{n+1}$ $\quad |-\frac{1}{2}|<1$ Konv $\quad S_n=\frac{\left(-\frac{1}{2}\right)^2}{1+\frac{1}{2}}=\frac{\frac{1}{4}}{\frac{3}{2}}=\frac{1}{4}\cdot\frac{2}{3}=\underline{\underline{\frac{1}{6}}}$

c) $\sum_{n=0}^{\infty}\left(\frac{-3}{2}\right)^n$ $\quad |-\frac{3}{2}|\not<1$ $\underline{\text{Divergens}}$

d) $\sum_{n=0}^{\infty}\frac{(-3)^n}{7^n}\rightarrow\sum_{n=0}^{\infty}\left(-\frac{3}{7}\right)^n$ $\quad |-\frac{3}{7}|<1$ Konv $\quad S_n=\frac{1}{1-(-\frac{3}{7})}=\frac{1}{\frac{10}{7}}=\frac{7}{10}$

e) $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}\dots=\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{2}\right)^n=-1\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^n$ $\quad |-\frac{1}{2}|<1$ Kon $\quad S_n=\frac{\frac{1}{2}}{1+\frac{1}{2}}=\frac{\frac{1}{2}}{\frac{3}{2}}=\frac{2}{6}=\underline{\underline{\frac{1}{3}}}$

6.1.2

$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\dots$ $\quad \sum_{n=1}^{\infty}\frac{1}{n}$

6.1.3 $\sum_{n=0}^{\infty}\frac{1}{12^n}\left(6^n+(-4^n)-3^n\right)=\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n+\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n-\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$

$S_{\frac{1}{2}}=\frac{1}{1-\frac{1}{2}}=2$

$S_{-\frac{1}{3}}=\frac{1}{1+\frac{1}{3}}=\frac{3}{4}$

$S_{\frac{1}{4}}=\frac{1}{1-\frac{1}{4}}=\frac{4}{3}$

$S_{tot}=2+\frac{3}{4}-\frac{4}{3}=\frac{24}{12}+\frac{9}{12}-\frac{16}{12}=\underline{\underline{\frac{17}{12}}}$

6.2.1 a) $\sum_{n=1}^{\infty}\frac{n^2}{2n^2+3n+1}$

$\lim_{n\to\infty}\frac{n^2}{2n^2+3n+1}=\underline{\underline{\frac{1}{2}}}$

b) $\sum_{n=1}^{\infty}2ne^{-n^2}=\int_1^{\infty}2xe^{-x^2}$ $\quad u=-x^2$, $u'=-2x$

$\int_1^{\infty}2xe^{u}\frac{du}{-2x}=-\left[e^{-x^2}\right]_1^{\infty}=-\lim_{b\to\infty}\left(-e^{-b^2}+e^{1}\right)=\underline{\underline{-e}}$ Konv

c) $\sum_{n=2}^{\infty}\frac{1}{n-1}\geq\sum_{n=2}^{\infty}\frac{1}{n}\Rightarrow\underline{\underline{\text{Div}}}$

d) $\sum_{n=1}^{\infty}\frac{n+5}{7n^2-3n-2}$ $\quad B=\sum_{n=1}^{\infty}\frac{1}{n^2}$

$\lim_{n\to\infty}\frac{\frac{n+5}{7n^2-3n-2}}{\frac{1}{n^2}}=\lim_{n\to\infty}\frac{n^3+5n^2}{7n^2-3n-2}=\infty$ $\underline{\underline{\text{Divergens}}}$

e) $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\leq\sum_{n=1}^{\infty}\frac{1}{n^2}\Rightarrow\lim_{n\to\infty}\frac{1}{n^2}=\underline{\underline{0}}$ Konv

f) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{5n-4}}\Rightarrow\lim_{n\to\infty}\frac{1}{\sqrt{5n-4}}=0$

$f(x)=\frac{1}{\sqrt{5x-4}}\Rightarrow f'(x)=-\frac{5}{2\sqrt{5x-4}}$ synkende for all $x>1$ $\underline{\underline{\text{Konv}}}$

g) $\sum_{n=1}^{\infty}\frac{n+1}{n!}$ $\quad \lim_{n\to\infty}\left|\frac{\frac{n+2}{(n+1)!}}{\frac{n+1}{n!}}\right|=\lim_{n\to\infty}\left|\frac{(n+2)n!}{(n+1)!\cdot(n+1)}\right|=\lim_{n\to\infty}\left|\frac{(n+2)}{(n+1)(n+1)}\right|=\underline{\underline{0}}$ Konv

h) $\sum_{n=1}^{\infty}\frac{(-1)^n n^{2n}}{(n^3+1)^n}$

$\lim_{n\to\infty}\frac{n^2}{n^3+1}=\underline{\underline{0}}$ Konv

6.3.3 a) $f(x) = e^{2x}$ $f(0) = 1$

$f'(x) = 2e^{2x}$ $f'(0) = 2$

$f''(x) = 4e^{2x}$ $f''(0) = 4$

$f'''(x) = 8e^{2x}$ $f'''(0) = 8$

$T_4(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4$

6.4.1 a) $\sum_{n=0}^{\infty} \frac{(-1)^n n^2}{10^n}(x-3)^n$

$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2 (x-3)^{n+1}}{10^{n+1}}}{\frac{n^2 (x-3)^n}{10^n}} \right| = \frac{(x-3)}{10} \lim_{n\to\infty} \frac{(n+1)^2}{n^2} = \left| \frac{(x-3)}{10} \cdot 1 \right| < 1$

$|x-3| < 10$

$x_1 < 13$

$-x_2 < 7$

$x_2 > -7$

b) $\sum_{n=0}^{\infty} \frac{n!\,x^n}{1000^n}$

$\left| \lim_{n\to\infty} \frac{\frac{(n+1)!\,x^{n+1}}{1000^{n+1}}}{\frac{n!\,x^n}{1000^n}} \right| = \left| \frac{x}{1000} \lim_{n\to\infty} \frac{(n+1)!}{n!} \right|$

$\left| \frac{x \cdot \infty}{1000} \right| < 1$ $\underline{x = 0}$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}(x-2)^n$

$\lim_{n\to\infty} \frac{\frac{(x-2)^{n+1}}{n+2}}{\frac{(x-2)^n}{n+1}} = (x-2) \lim_{n\to\infty} \left| \frac{n+2}{n+1} \right|$ $|x-2| < 1$

$x < 3$

$-x < -1$

$x > 1$

d) $\sum_{n=1}^{\infty} \frac{1}{5^n (n+2)^2}(x+1)^n$

$\lim_{n\to\infty} \frac{\frac{(x+1)^{n+1}}{5^{n+1}(n+3)^2}}{\frac{x+1}{5^n (n+2)^2}} = \left| \frac{x+1}{5} \lim_{n\to\infty} \frac{(n+2)^2}{(n+3)^2} \right| \to$ $\left| \frac{x+1}{5} \right| < 1$

$(x+1) < 5$

$x < 5-1$

$x < 4$

$-x < 5+1$

$x > -6$