① $f(x) = x^2 - 2x + 1$

$f'(x) = 2x - 2$

$2x - 2 > 2 \quad [2, ?]$

for intervallet $[2,4]$

$x^2 - 2x + 1 = y$

$(x-1)^2 = y$

$x - 1 = \sqrt{y}$

$x = \sqrt{y} + 1$

$f^{-1}(x) = \sqrt{x} + 1$

Punkten

$f(2) = 2^2 - 2 \cdot 2 + 1 = 1$

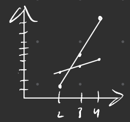
$f(3) = 3^2 - 2 \cdot 3 + 1 = 4$

$f(4) = 4^2 - 4 \cdot 2 + 1 = 9$

$f^{-1}(2) = \sqrt{2} + 1$

$f^{-1}(3) = \sqrt{3} + 1$

$f^{-1}(4) = \sqrt{4} + 1 = 3$



② $f(n) = \dfrac{2 + n^2}{1 + n^2} \rightarrow \lim_{n \to \infty} \dfrac{2 + n^2}{1 + n^2} = \dfrac{1}{1} = \underline{1}$

③ $x^2 = e^x$

$f(x) = x^2 - e^x$

ikke persom lengen

④ $f(x) = x^2 - e^x$

```
import numpy as np
def f(x):
    return x**2 - np.exp(x)
def der(x):
    return 2*x - np.exp(x)
x = -1/2
while abs(x) < 0.0001:
        x = x - f(x)/der(x)
```

⑤ $\ln 2 = \ln(1+x)\Big|^{x=1}$

$(\ln(1+x))' = \dfrac{1}{1+x}$

$\sum_{n=0}^{\infty} x^n = \dfrac{1}{1-x} \rightarrow \sum_{n=0}^{\infty} (-x)^n = \dfrac{1}{1+x}$

$\int_0^x \sum_{n=0}^{\infty} (-x)^n = \ln|x+1|$

$\ln|x+1| = \int_0^x \sum_{n=0}^{\infty} (-1)^n (x)^n = \sum_{n=0}^{\infty} (-1)^n \int_0^x x^n = \sum_{n=0}^{\infty} (-1)^n \left[\dfrac{1}{n+1} x^{n+1}\right]_0^x = \sum_{n=0}^{\infty} \dfrac{(-1)^n \cdot (x)^{n+1}}{n+1} = \underline{\underline{\sum_{n=0}^{\infty} \dfrac{(-1) \cdot \cancel{x}^{n+1}}{n+1}}}$

⑥ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{array}{r} 1\ 1\ 1 \\ -1\ 2\ 3 \\ \hline 0\ -1\ -2 \end{array} \quad \begin{array}{r} 1\ 0\ 0 \\ -0\ 1\ 0 \\ \hline 1\ -1\ 0 \end{array} \qquad \begin{array}{r} 1\ 1\ 1 \\ -1\ 3\ 6 \\ \hline 0\ -2\ -5 \end{array} \quad \begin{array}{r} 1\ 0\ 0 \\ -0\ 0\ 1 \\ \hline 1\ 0\ -1 \end{array}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & -2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

$\begin{array}{r} -2(-1\ -2) \\ -2\ -5 \\ \hline 0\ -1 \\ \cdot(-1) \end{array} \qquad \begin{array}{r} -2(1\ -1\ 0) \\ (1\ 0\ -1) \\ \hline -1\ 2\ -1 \\ \cdot(-1) \end{array}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$

$\begin{array}{r} -(-1\ -2) \\ -2(0\ 1) \\ \hline 1\ 0 \end{array} \qquad \begin{array}{r} -1(1\ -1\ 0) \\ -2(1\ -2\ 1) \\ \hline -3\ 5\ -2 \end{array}$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

$\begin{array}{r} 1 \\ +3 \\ -1 \\ \hline 3 \end{array} \quad \begin{array}{r} 0 \\ -5 \\ +2 \\ \hline -3 \end{array} \quad \begin{array}{r} 0 \\ +2 \\ -1 \\ \hline 1 \end{array}$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & -2 \\ 0 & -2 & -5 \end{bmatrix} \underbrace{\begin{bmatrix} 3 & -3 & 1 \\ -3 & 5 & -2 \\ 1 & -2 & 1 \end{bmatrix}}_{A^{-1}}$

$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 6 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}}_{b}$

$Ax = b$

$A^{-1}Ax = A^{-1}b$

$x = A^{-1}b$

$\begin{array}{r|l} & 1 \\ & 2 \\ & 4 \\ \hline 3\ -3\ 1 & 3 - 6 + 4 \\ -3\ 5\ -2 & -3 + 10 - 8 \\ 1\ -2\ 1 & 1 - 4 + 4 \end{array} = \underline{\underline{\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}}}$

7. $$\int_0^1 \frac{\sin x}{x}\,dx = \int_0^1 \frac{\sum (-1)^n \frac{x^{2n+1}}{(2n+1)!}}{x} = \int_0^1 \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int_0^1 x^{2n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{1}{2n+1}$$

Tar noen av første verdiene i rekken

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{1}{2n+1} = 1 - \frac{1}{18} + \frac{1}{600} \approx \underline{\underline{1}}$$

8. $f : [0, \pi]$ $f(x) = \sin x$

$$V = \pi \cdot \int_0^{\pi} \sin^2 x\,dx = \pi \cdot \int_0^{\pi} 1 - \cos^2 x = \pi \cdot \int_0^{\pi} 1 - \frac{1}{2} - \frac{\cos 2x}{2} = \frac{\pi}{2} \int_0^{\pi} 1 - \cos 2x = \frac{\pi}{2}\Big[x - \sin 2x\Big]_0^{\pi} = \frac{\pi}{2}\Big(\pi - \sin 2\pi - (0 - \sin 2 \cdot 0)\Big)$$

$$= \underline{\underline{\frac{\pi^2}{2}}}$$

9.
```
import numpy as np

y = 1
a = 0
b = 1
n = 10000
h = (b-a)/n
for i in range(n):
    y = y - h np.sqrt(y)
```