② a) $A = x$

$B = y$

$x' = 0{,}8x + 0{,}15y$

$y' = 0{,}7y + 0{,}10x \Rightarrow 0{,}10x + 0{,}7y$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} 0{,}8 & 0{,}2 \\ 0{,}15 & 0{,}7 \end{pmatrix}}_{M} \cdot \begin{pmatrix} x \\ y \end{pmatrix} \qquad M^2 = \begin{pmatrix} 0{,}8 & 0{,}2 \\ 0{,}15 & 0{,}7 \end{pmatrix} \cdot \begin{pmatrix} 0{,}8 & 0{,}2 \\ 0{,}15 & 0{,}7 \end{pmatrix} =$$

	0,8	0,2
0,15	0,67	0,3
0,7	0,215	0,52

$$M^2 = \begin{pmatrix} 0{,}67 & 0{,}3 \\ 0{,}215 & 0{,}52 \end{pmatrix}$$

b)

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0{,}67 & 0{,}3 \\ 0{,}215 & 0{,}52 \end{pmatrix} \cdot \begin{pmatrix} 100\,000 \\ 100\,000 \end{pmatrix} \Rightarrow$$

	100 000
	100 000
0,67 0,3	97000
0,215 0,52	74500

tot boel mistet

$200\,000 - (97000 + 74500) = 28500$

$A = 97000$

$B = 74500$

③ Formel omdrejningslegme $O_v = \pi \cdot \int_a^b f(x)^2 \, dx$

$y^2 = \left(\frac{1}{\sqrt{1-x^4}}\right)^2 = \frac{1}{1-x^4}$

$O_v = \pi \cdot \int_0^{\frac{\sqrt{2}}{2}} \frac{1}{1-x^4} \, dx$

④ a) $\int \frac{4x+5}{x^2+4x+5} \, dx$

$(x^2 + 4x + 2^2) + 5 - 4$

$(x+2)^2 + 1$

$$\int \frac{4x+5}{(x+2)^2+1} \, dx \qquad \begin{aligned} u &= x+2 \\ du &= dx \\ c^2 &= 1 \\ a &= 1 \\ x &= u-2 \end{aligned}$$

$$\int \frac{4u-8+5}{u^2+1} \, du$$

$$\int \frac{4u-3}{u^2+1} \, du$$

$$\int \frac{4u}{u^2+1} \, du - 3\int \frac{du}{u^2+1} \qquad \begin{aligned} w &= u^2+1 \\ dw &= 2u \, du \\ \frac{dw}{2u} &= du \end{aligned}$$

$$\int \frac{2 \cdot 4u}{w} \cdot \frac{dw}{2u}$$

$$= 2\int \frac{1}{w} dw - 3\int \frac{du}{u^2+1^2}$$

$$= 2 \cdot \ln|w| + C - 3\arctan\left(\frac{u}{a}\right) + C$$

$$= 2 \cdot \ln|u^2+1| + C - 3\arctan\left|\frac{x+2}{1}\right| + C$$

$$= 2 \cdot \ln\left|(x+2)^2+1\right| + C - 3\arctan(x+2) + C$$

$$= 2\ln\left|x^2+4x+5\right| - 3\arctan(x+2) + C$$

b) $2\int x \arctan(x+2) \, dx \qquad u' = 2x, \; v = \arctan(x+2) \qquad v = x^2, \; v' = \frac{1}{1+(x+2)^2} = \frac{1}{x^2+4x+5}$

$$\frac{1}{2}x^2\arctan(x+2) - \frac{1}{2}\int \frac{x^2}{x^2+4x+5}$$

Polynom div.

$$\frac{x^2}{-x^2} : x^2+4x+5 = 1 + \frac{-4x-5}{x^2+4x+5}$$

$$-4x-5$$

$$\int 1 + \frac{-4x-5}{x^2+4x+5}$$

$$\int 1 + \int \frac{-4x-5}{(x+2)^2+1} \, dx \qquad u = x+2, \; du = dx$$

$\int 1 + \int \frac{-4u+3}{u^2+1}$ ser at det er sammensat i stedet men inverterende bortset

$$2\left(\frac{1}{2}x^2\arctan(x+2) - \frac{1}{2}\left(x - 2\ln\left|x^2+4x+5\right| + 3\arctan(x+2) + C\right.\right.$$

$$= x^2\arctan(x+2) - x + 2\ln|x^2+4x+5| - 3\arctan(x+2) + C$$

⑤ $f(x) = \arccos\frac{x}{R}$

$f(f^{-1}(x)) = x$

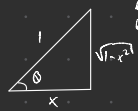
$f'(f^{-1}(x)) \cdot \frac{d}{dx} f^{-1}(x) = 1$

$\frac{d}{dx} f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$

$h^2 = k_1^2 + k_2^2$

$k = \sqrt{h^2 - k_1^2}$

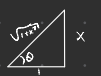
$k = \sqrt{1 - x^2}$



$\frac{d}{dx} \arccos = \frac{1}{-\sin(\arccos(x))}$

$\frac{d}{dx} \arccos = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \arctan(x) = \frac{\cos^2(\arctan(x))}{1}$



$h^2 = k_1^2 + k_2^2$

$h = \sqrt{1+x^2}$

$\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$

$\frac{d}{dx} \arccos\left(\frac{x}{R}\right) = \frac{-1}{\sqrt{1-\left(\frac{x}{R}\right)^2}} =$

$\frac{d}{dx} \arctan\left(\frac{\sqrt{R^2-x^2}}{x}\right) = \frac{1}{1+\frac{R^2-x^2}{x^2}} = 1 + \frac{x^2}{R^2-x^2}$