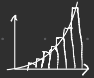
(1) 

$f(x) = x^2$

$\text{bredde}_k = \frac{(6-0)}{n} \cdot k = \frac{6k}{n}$

$$\sum_{k=1}^{n} h \cdot f(x_k) = \sum_{k=1}^{n} \frac{6}{n} f\left(\frac{6k}{n}\right) = \sum_{k=1}^{n} \frac{6}{n}\left(\frac{6k}{n}\right)^2 = \frac{6^3}{n^3} \sum_{k=1}^{n} k^2$$

(4) $S_N = \sum_{n=0}^{N} x^n = 1 + x + x^2 + \dots x^N$

$(1-x) S_N = (1-x) \sum_{n=0}^{N} x^N$

$(1-x) S_N = (1-x)(1 + x + x^2 + \dots x^N)$

$(1-x) S_N = (1 - x + x - x^2 + x^2 - x^2 \dots x^N)$

$(1-x) S_n = 1 - x^{N+1}$

$S_N = \frac{1-x^{N+1}}{1-x}$ sette $N \to \infty$

Dersom $N \to \infty$

$$\sum_{n=0}^{\infty} x \begin{cases} \frac{1}{1-x} & |x| < 1 \\ \infty & |x| > 1 \\ \infty & x = 1 \\ ? & x = -1 \end{cases}$$

(7) $\sum_{n=0}^{\infty} \frac{1}{(1+i)^n} = \sum_{n=0}^{\infty} \left(\frac{1}{1+i}\right)^n \quad \sum_{n=0}^{\infty} x^n \qquad S_n = \frac{1}{1-\frac{1}{1+i}} = \frac{1+i}{i} = -1+i$

$$\sum_{n=1}^{\infty} \frac{1}{(1+i)^n} = \frac{\frac{1}{1+i}}{1-\frac{1}{1+i}} = \frac{\frac{1}{1+i}}{\frac{i}{1+i}} = \frac{1+i}{i-1}$$

$$\sum_{n=10}^{\infty} \frac{1}{(1+i)^n} = -32$$

$\frac{3}{4} \cdot \frac{3}{4} =$

(10) $3 \cdot \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n \qquad 1 + \frac{3}{4} + \frac{9}{16} +$

$$S_n = 3 \cdot \left(\frac{1}{1-\frac{3}{4}}\right) + \frac{9}{4} \cdot \left(\frac{1}{1-\frac{3}{4}}\right) = 3 \cdot (4) + \frac{9}{4} \cdot 4 = 12 + 9 = 21$$

(12) $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} \dots + \frac{1}{n}$

$\geq 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} \dots = \infty$

(13) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \geq \sum_{n=1}^{\infty} \frac{1}{n}$ div

(14) $\sum_{n=1}^{\infty} \frac{1}{n^3} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$ konv

(15) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \frac{1}{n^2+n} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} = $ konv

(16) $\sum_{n=1}^{\infty} \frac{1}{n3^n} \leq \sum_{n=1}^{\infty} \frac{1}{3^n} \Rightarrow$ konv

(17) $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \geq \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} \Rightarrow$ Div

Gamle eksamens oppgaver :

① $\sum_{n=1}^{\infty} \frac{(-3x)^n}{n}$

$\lim_{n\to\infty} \frac{\frac{(-3x)^{n+1}}{n+1}}{\frac{(-3x)^n}{n}} = -3x \lim_{n\to\infty} \frac{n}{n+1} = |-3x \cdot 1|$

$|-3x| < 1$

$x > -\frac{1}{3}$

$3x < 1$

$x < \frac{1}{3}$

$\sum_{n=1}^{\infty} \frac{(3 \cdot \frac{1}{3})^n}{n} = \sum_{n=1}^{\infty} \frac{1^n}{n}$, div

$-\frac{1}{3} < x \leq \frac{1}{3}$

$\sum_{n=1}^{\infty} \frac{(-3 \cdot \frac{1}{3})^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$, konv

② $\sum_{n=3}^{\infty} \frac{1}{n(n-2)} = \sum_{n=3}^{\infty} \frac{A}{n} + \frac{B}{n-2} = \frac{1}{2} \sum_{n=3}^{\infty} \left(\frac{1}{n-2} - \frac{1}{n}\right) \Rightarrow \frac{1}{2}\left(1 - \cancel{\frac{1}{3}} + \frac{1}{2} - \frac{1}{4} + \cancel{\frac{1}{3}} - \frac{1}{5} \dots +\right) = \frac{1}{2}\left(1 + \frac{1}{2}\right) = \frac{1}{2} \cdot \frac{3}{2} = \underline{\underline{\frac{3}{4}}}$

$A(n-2) + B(n) = 1$

$-2A = 1$

$A = -\frac{1}{2}$

$2B = 1$

$B = \frac{1}{2}$