① $\int_0^{\pi} \frac{\sin x}{x} dx$

```
import numpy as np
def f(x):
    return np.sin(x)/x


a=0
b=np.pi
n=100000
h=(b-a)/n
int=0
for i in range(n):
    x0= a+ h·i
    x1= a+ h·(1+i)
    int += h/2 (f(x0) + f(x1))
```

② $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$

$f(x) = \tan x$

$f'(x) = \left(\frac{\sin x}{\cos x}\right)' = \frac{(\sin x)' \cdot \cos x - \sin x \cdot (\cos x)'}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

Vi ser at $f'(x)$ er positiv i hele intervallet noe som gjør at den er injektiv i intervallet

$f^{-1}(x) = \arctan(x)$

③ a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \to \left| \lim_{n\to\infty} \frac{\frac{x^{2n+3}}{2n+3}}{\frac{x^{2n+1}}{2n+1}} \right| = x^2 \lim_{n\to\infty} \frac{2n+1}{2n+3} = |x^2| < 1$

$x < 1$
$x > -1$

$\sum_{n=0}^{\infty} (-1)^n \frac{1}{2n+1}$ konv

$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{(-1)^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} \frac{(-1)^{3n+1}}{2n+1}$ konv

$-1 \leq x \leq 1$

b) $(\arctan x)' = \frac{1}{1+x^2}$

$\sum x^n = \frac{1}{1-x}$

$\sum (-1)^n \cdot a^{2n} = \frac{1}{1+a^2}$

$\int_0^x \frac{1}{1+a^2} = \int_0^x \sum_{n=0}^{\infty} (-1)^n \cdot a^{2n}$

$\arctan x = \sum_{n=0}^{\infty} (-1)^n \cdot \int_0^x a^{2n} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

④ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cccc} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right]$

$x_1 + 2x_2 + 3x_3 = 1$
$x_2 + 2x_3 = 0$
$x_3 = s \quad x_2 = -2s$

$x_1 = 1 + s$

$x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ Får vendelig mange løsninger som gjør at systemet e lineært avhengig

⑤ a) $\dot{y} = 1 - y^2 \quad y(0) = 2$

$g'(t) = 1 - g(t)^2$

$g'(t) = \left(\frac{3e^{2t}+1}{3e^{2t}-1}\right)' = \frac{(3e^{2t}+1)'(3e^{2t}-1) - (3e^{2t}+1)(3e^{2t}-1)'}{(3e^{2t}-1)^2} = \frac{6e^{2t}(3e^{2t}-1) - (3e^{2t}+1)6e^{2t}}{(3e^{2t}-1)^2} = -\frac{12e^{2t}}{(3e^{2t}-1)^2}$

$1 - \left(\frac{3e^{2t}+1}{3e^{2t}-1}\right)^2 = 1 - \frac{9e^{4t} + 6e^{2t} + 1}{(3e^{2t}-1)^2} = \frac{9e^{4t} - 6e^{2t} + 1 - (9e^{4t} - 6e^{2t} + 1)}{(3e^{2t}-1)^2} = -\frac{12e^{2t}}{(3e^{2t}-1)^2}$

⑤ b) Siden $1-y^2$ er kontinuerlig deri verbar er løsninger entydig

⑥

```
import numpy as np

def f(x):
    return -(10123 / 10000) * (np.arctan(x) - np.pi/2)

x = 1
for i in range(100):
    x = f(x)
```

⑦ $f: [0, \infty) \to \mathbb{R}$

$f(x) = \frac{1}{\sqrt{1+x^2}}$

$$V = \pi \cdot \int_0^\infty f(x)^2 \, dx = \pi \cdot \int_0^\infty \frac{1}{1+x^2} = \pi \cdot \Big[\arctan x \Big]_0^\infty = \pi \cdot \lim_{b \to \infty} \left(\arctan b - \arctan 0 \right) = \pi \cdot \frac{\pi}{2} = \frac{\pi^2}{4}$$

⑧ $$\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\sum_{n=0}^\infty (-1)^n \frac{x^{2n+1}}{(2n+1)!}}{x \cdot \sum_{n=0}^\infty (-1)^n \frac{x^{2n}}{(2n)!}} = \frac{x - \frac{x^3}{5!} \dots}{x\left(1 - \frac{x^2}{2!}\right.} = \frac{1 - \frac{x^2}{5!} \dots}{1\left(1 - \frac{x}{2!}\right.} = \frac{1 - 0 \dots}{1(1 - 1 \dots} = \underline{\underline{1}}$$