① $\cos^2\left(\frac{x}{2}\right) \Rightarrow \frac{1+\cos x}{2}$

$f'(x) = \frac{1}{2}x - \frac{\sin x}{2}$

$f''(x) = \frac{1}{4}x^2 - \frac{\cos x}{2}$

$f'''(x) = \frac{1}{12}x^3 + \frac{\sin x}{2}$

$f(x) = f(0) + f'(0)\cdot x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3}x^3 + \dots$

$f(x) = 1 + 0 - \frac{1}{2}x^2 + 0 + \frac{1}{4}x^4 \dots$

$\sum_{n=0}^{\infty} \frac{1}{2\cdot(2n)!} x^{2n}$

② $\lim_{x\to 0} \frac{(e^x-1-x)^2}{x^2 - \ln(1+x^2)} = \lim_{n\to\infty} \frac{\left(\sum_{n=0}^{\infty}\frac{x^n}{n!} \cancel{-1-x}\right)^2}{x^2 - \ln(1+x^2)} = \frac{\left(\sum_{n=2}^{\infty}\frac{x^n}{n!}\right)^2}{\cancel{x^2}\ \sum_{n=1}^{\infty}(-1)^n \frac{x^{2(n+1)}}{n+1}} = \frac{\frac{x^4}{4} + \frac{x^5}{3!} + \dots}{\frac{x^4}{2} - \frac{x^6}{3} + \dots} = \frac{1}{2}$

$\left(\sum_{n=0}^{\infty} = \cancel{1} + \cancel{x} + \frac{x^2}{2} + \dots\right)$

$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2(n+1)}}{n+1} = \cancel{x^2} - \frac{x^4}{2} + \frac{x^6}{3} + \dots$

③
```
import numpy as np
def f(y):
    return y**2 - y**3
b = 200
n = 100000
h = b/n
y = np.zeros(n+1)
y[0] = 0.01

for i in range(n):
    y[i+1] = y[i] + h·(f(y[i]))
    y[i+1] = y[i] + (h/2)(f(y[i]) + f(y[i+1]))
```

④ $\int_1^{\infty} \frac{\cos t}{t^2}\,dt = \lim_{b\to\infty} \int_1^{b} \frac{\cos t}{t^2}\,dt \le \lim_{b\to\infty} \int_1^{b} \frac{1}{t^2} \Rightarrow \lim_{b\to\infty}\left[-\frac{1}{t}\right]_1^{b} = \lim_{b\to\infty}\left(-\frac{1}{b} + \frac{1}{1}\right) = 1$ den konvergere

⑤ $\sum_1^{\infty} \frac{\cos n}{n^2} \le \sum_1^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

⑥ $F(x) = \int_1^x \sqrt{t^3-1}\,dt \qquad F'(x) = \sqrt{x^3-1} - \sqrt{1^3-1} = \sqrt{x^3-1}$

$x^{\frac{3}{2}+\frac{2}{2}=\frac{5}{2}}$

$K = \int_a^b \sqrt{1 + (F'(x))^2}\,dx$

$K = \int_1^2 \sqrt{1+x^3-1}\,dx = \int_1^2 \sqrt{x^3}\,dx = \left[\frac{2}{5}x^{\frac{5}{2}}\right]_1^2 = \frac{2}{5}\left(2^{\frac{5}{2}} - 1^{\frac{5}{2}}\right) = \frac{2}{5}\left(4\sqrt{2} - 1\right)$

⑦ $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 2 \\ 5 & 6 & 2 \end{bmatrix} \quad b = \begin{bmatrix} -5 \\ -17 \\ -29 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 2 \\ 5 & 6 & 4 \end{bmatrix}$ $\begin{array}{r} 3(1\ 2\ 0) \\ -(3\ 4\ 2) \\ \hline 0\ 2\ -2 \end{array}$ $\begin{array}{r} 5(1\ 2\ 0) \\ -(5\ 6\ 4) \\ \hline 0\ 4\ -4 \end{array}$ $= \begin{array}{l} 2(1\ -1) \\ 4(1\ -1) \end{array}$ $\begin{bmatrix} -25 \\ 5 \\ 5 \end{bmatrix} = 5\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

⑦ b)

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & -5 \\ 3 & 4 & 2 & -17 \\ 5 & 6 & 4 & -29 \end{array}\right]$$

$$\begin{array}{l} 3(1\ 2\ 0\ -5) \\ -(3\ 4\ 2\ -17) \\ \hline 0\ 2\ -2\ 2 \end{array} \quad \text{like} \quad \begin{array}{l} 5(1\ 2\ 0\ -5) \\ -(5\ 6\ 4\ -29) \\ \hline 0\ 4\ -4\ 4 \end{array}$$

setter $x_2 = t$

$x_3 = t - 1$

$x_1 = -5 - 2x_2 = -5 - 2t$

$$x = \begin{bmatrix} -5 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

c) $D = (1-\lambda)\begin{vmatrix} 4-\lambda & 2 \\ 6 & 4-\lambda \end{vmatrix} - 2 \begin{vmatrix} 3 & 2 \\ 5 & 4-\lambda \end{vmatrix}$

$D = (1-\lambda)(4 - 8\lambda + \lambda^2) - 2(12 - 3\lambda - 10)$

$D = \cancel{4} - 8\lambda + \lambda^2 - 16\lambda + 8\lambda^2 - \lambda^3 - \cancel{24} + 6\lambda + \cancel{20}$

$D = -\lambda^3 + 9\lambda^2 - 18\lambda = -\lambda(\lambda^2 - 9\lambda + 18) = -\lambda(\lambda-3)(\lambda-6)$

$\lambda = 0$

$\lambda = 3$

$\lambda = 6$

A er diagnoliserbar siden vi får tre ulike λ verdier