① $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+\sqrt{n}}$

$$\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}} \geq \sum_{n=1}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+\sqrt{n}} \leq \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} = \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8}\right) \leq \left(1 - \frac{1}{2} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{8}} + \cancel{\frac{1}{8}} + \cancel{\frac{1}{8}}\right) = \left(1 - \frac{1}{2}\right) = \frac{1}{2}$$

② $$\lim_{x \to 0} \frac{e^{x^3} - 1}{x - \sin x} = \frac{\sum_{n=0}^{\infty} \frac{x^{3n}}{n!} - 1}{x - \left(\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}\right)} = \frac{(1 + x^3 + x^6 \ldots) - 1}{x - \left(x - \frac{x^3}{6} + \frac{x^5}{5!}\right)} = \lim_{x \to 0} \frac{x^3 + x^6 \ldots}{\frac{x^3}{6} - \frac{x^5}{5!}} = \lim_{x \to 0} \frac{1 + x^3}{\frac{1}{6} - \frac{x^2}{5!}} = 6$$

②.5 $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{n^2} x^{n+1}$

$$\lim_{n \to \infty} \left| \frac{\left(1 + \frac{1}{n+1}\right)^{(n+1)^2} x^{n+1}}{\left(1 + \frac{1}{n}\right)^{n^2} x^n} \right| = x \lim_{n \to \infty} \left| \frac{\left(1 + \frac{1}{n+1}\right)^{(n+1)^2}}{\left(1 + \frac{1}{n}\right)^{n^2}} \right| = x \cdot \lim_{n \to \infty} \left|\right.$$