1.

```
import numpy as np

n = 10000
b = 20
x = np.zeros(n+1)
y = np.zeros(n+1)
x[0] = 1
y[0] = 2
h = b/n

for i in range(n):
    x[i+1] = x[i] + h·y[i]
    y[i+1] = y[i] - h·np.sin(x[i])

    x[i+1] = x[i] + (h/2)·(y[i] + y[i+1])
    y[i+1] = y[i] - h·(np.sin(x[i]) + np.sin(x[i+1]))/2
```

2. a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+\sqrt{n}} \geq \underline{\sum_{n=1}^{\infty} \frac{1}{2n}}$

b) $\sum_{n=1}^{\infty} \frac{1}{n3^n}$

bruker

$-\ln(1-x) = \int \frac{1}{1-x} = \int \sum_{n=0}^{\infty} x^n = \sum_{n=1}^{\infty} \frac{x^n}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n3^n} \quad x^n = \left(\frac{1}{3}\right)^n \quad = \sum_{n=1}^{\infty} \frac{\left(\frac{1}{3}\right)^n}{n} = -\ln\left(1-\frac{1}{3}\right) = -\ln\left(\frac{2}{3}\right) = \underline{\underline{\ln\left(\frac{3}{2}\right)}}$

3. $\lim_{x\to 0} \frac{e^{x^3} - \cancel{1}}{\cancel{x} - \sin x}$

$e^{x^3} = \sum_{n=0}^{\infty} \frac{x^{3n}}{n!} = \cancel{1} + x^3 + \frac{x^6}{2}$

$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \cancel{x} - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\lim_{x\to 0} \frac{\left(x^3 + \frac{x^6}{2} + \frac{x^9}{3} \ldots\right)}{+\frac{x^3}{3!} - \frac{x^5}{5!} \ldots} \quad \Big| \div x^3$

$\lim_{x\to 0} \frac{\left(1 + \frac{x^3}{2} + \frac{x^6}{3} \ldots\right)}{\left(\frac{1}{3!} - \frac{x^2}{5!} + \ldots\right)} = \frac{1}{\frac{1}{3!}} = \underline{\underline{6}}$

4. $v_1 = \begin{pmatrix} 3 \\ -3 \\ -6 \end{pmatrix} \quad v_2 = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix} \quad v_3 = \begin{pmatrix} 1 \\ -1 \\ 8 \end{pmatrix}$

$A = \begin{pmatrix} 3 & -2 & 1 \\ -3 & 2 & -1 \\ -6 & 4 & 8 \end{pmatrix}$

$D = 3\begin{vmatrix} 2 & -1 \\ 4 & 8 \end{vmatrix} + 2\begin{vmatrix} -3 & -1 \\ -6 & 8 \end{vmatrix} + \begin{vmatrix} -3 & 2 \\ -6 & 4 \end{vmatrix}$

$D = 3(16+4) + 2(-24-6) + (-12+12)$

$D = 60 - 60 = 0$ lineært avhengig

$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 4 \\ -3 & 2 & -1 & -9 \\ -6 & 4 & 8 & 3 \end{array}\right]$

$\begin{array}{cccc} 3 & -2 & 1 & 4 \\ -3 & 2 & -1 & -9 \\ \hline 0 & 0 & 0 & -5 \end{array}$

$\begin{array}{cccc} 2(3 & -2 & 1 & 4) \\ -6 & 4 & 8 & 3 \\ \hline 0 & 0 & 10 & 11 \end{array}$

$x_3 = \frac{11}{10}$

$x_2 = t$

$x_1 = \frac{4}{3} - \frac{11}{30} + 2t = \frac{29}{30} + 2t$

$\begin{bmatrix} \frac{29}{30} \\ 0 \\ \frac{11}{10} \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ er en lineær kombinasjon som gir b

7. Formel $\Rightarrow \int_a^b \sqrt{1+f'(x)^2}\, dx$

$f(x) = \frac{x^3}{3} + \frac{1}{4x}$

$f'(x) = x^2 - \frac{1}{4x^2}$

$f'(x)^2 = \left(x^2 - \frac{1}{4x^2}\right)^2 = \left(\frac{4x^4 - 1}{4x^2}\right)^2 = \frac{16x^8 - 8x^4 + 1}{16x^4}$

$B = \int_a^b \sqrt{\frac{16x^4}{16x^4} + \frac{16x^8 - 8x^4 + 1}{16x^4}} = \int_a^b \frac{16x^8 + 8x^4 + 1}{16x^4} = \frac{1}{16}\int_a^b \frac{16x^8}{x^4} + \frac{8x^4}{x^4} + \frac{1}{4x^4}$

$B = \left[\frac{16x^5}{5} + 8x - \frac{1}{3x^3}\right]_1^2 = \left(\left(\frac{16\cdot 2^5}{5} + 8\cdot 2 - \frac{1}{3\cdot 2^3}\right) - \left(\frac{16}{5} + 8 - \frac{1}{3}\right)\right) \approx \underline{\underline{107{,}5}}

(8) $\int_0^3 \frac{dx}{(x-2)^{\frac{1}{3}}} = \lim_{b \to 2^-} \int_0^b \frac{dx}{(x-2)^{\frac{1}{3}}} + \lim_{c \to 2^+} \int_c^3 \frac{dx}{(x-2)^{\frac{1}{3}}}$

$\lim_{b \to 2^-} \left[\frac{3}{2}(x-2)^{\frac{2}{3}} \right]_0^b = 0 - \left(\frac{3}{2} \cdot 2^{\frac{2}{3}} \right) = \frac{3}{2} \cdot 2^{\frac{2}{3}}$

$\lim_{c \to 2^+} \left[\frac{3}{3}(x-2)^{\frac{2}{3}} \right]_c^3 = \frac{3}{2} - 0 = \frac{3}{2}$

$\int_0^3 \frac{dx}{(x-2)^{\frac{1}{3}}}$ konvergere mot $\underline{\underline{\frac{3}{2}\left(1 + 2^{\frac{2}{3}}\right)}}$