① $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 1-\lambda & 1 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix}$

$D = (1-\lambda)\cdot\begin{pmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 1-\lambda \end{pmatrix} + 0\begin{pmatrix} 0 & 1-\lambda \\ 0 & 0 \end{pmatrix}$

$0 = (1-\lambda)\cdot(1-\lambda-\lambda+\lambda^2) = (1-\lambda)(1-2\lambda+\lambda^2) = (1-\lambda)^3$

$1-\lambda = 0$
$\lambda = 1$

$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow$ $x_1 = t$, $x_2 = 0$, $x_3 = 0$ $\quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

② $\dot{x}_1 = x_1 + x_2$ $\qquad x_1(0) = 1$
$\dot{x}_2 = x_2 + x_3$ $\qquad x_2(0) = 0$
$\dot{x}_3 = x_3$ $\qquad x_3(0) = 0$

$x_1 = e^{at}$

$x_3 = 0$
$x_2 = 0$ $\qquad \dot{x}_1 = 1$

$x_2 = 0$
$x_3 = 0$

③ $\dot{x} + ax = 0$

$\int \dot{x}\, dt = \int -ax\, dt$

$\int \frac{1}{x} dx = -at$

$\ln x = -at + c$

$x = e^{-at+c} = e^{-at} \cdot e^{c}$ $\nearrow$ $e^c = C$

$x = Ce^{-at}$

$x(t) = Ce^{-at}$

$x(0) = Ce^{-a\cdot 0} = x_0 \Rightarrow C = x_0$

$x(t) = x_0 e^{-at}$

④ 0 0 0 1 1 ser at $c=1$
fjerne c fra resterende rekker ved å ta -1 fra begge sider i alle ligninger

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 8 & 4 & 2 & 2 \\ 27 & 9 & 3 & 4 \end{array}\right]$

$27(1\ 1\ 1\ 1) - (27\ 9\ 3\ 4) = 0\ 18\ 24\ 23$

$8(1\ 1\ 1\ 1) - (8\ 4\ 2\ 2) = 0\ 4\ 6\ 6$

$2(18\ 24\ 23) - 9(4\ 6\ 6) = 0\ {-6}\ {-8}$

$-6x_3 = -8$
$x_3 = \frac{4}{3}$
$4x_2 + 6\cdot\frac{4}{3} = 6$
$4x_2 = \frac{18}{3} - \frac{24}{3}$
$x_2 = -\frac{1}{2}$

$x_1 - \frac{1}{2} + \frac{4}{3} = 1$
$x_1 = \frac{6}{6} + \frac{3}{6} - \frac{8}{6} = \frac{1}{6}$

$p(x) = \frac{1}{6}x^3 - \frac{1}{2}x^2 + \frac{4}{3}x + 1$

⑤ $\sqrt{2} = \frac{m}{n}$

Vi skal vise at $\sqrt{2}$ ikke kan skrives som en brøk
tar alt i andre
$2 = \frac{m}{n} \Rightarrow 2n = m$, vi ser at i dette tilfellet må m være et partall siden det er det dobbelte av n.
$n = \frac{m}{2} \Rightarrow$ vi ser at n må være et partall så lenge m oppfyller kravet om å være et heltall.
Dette ~~fungerer~~ som en motsigelse der både m og n er heltall og har en felles faktor.
Noe som viser at $\sqrt{2} \neq \frac{m}{n}$

⑥ $2\sin 2\theta + \pi - 4\theta = 0$

$\theta = \frac{1}{2}\sin(2\theta) + \frac{1}{4}\pi$

```
import numpy as np

def f(x):
    return (1/2)·np.sin(2·x) + (1/4)·np.pi

x=0
x_neste = 1
runde =0
while abs(x_neste-x) ≥ 10e(-10):
    x = x_neste
    x_neste = f(x)
    runde += 1
```

⑦ $\sum_{n=3}^{\infty} \frac{1}{n(n-2)} = \frac{A}{n} + \frac{B}{n-2}$ $\quad 1 = A(n-2)+Bn$

setter inn $n=0$

$1 = A(0-2) + B\cdot 0$

$1 = -2A \Rightarrow A = -\frac{1}{2}$

setter inn $n=1$

$1 = -A + B$

$B = 1 - \frac{1}{2} = \frac{1}{2}$

$$\sum_{n=3}^{\infty}\left(\frac{1}{2(n-2)} - \frac{1}{2n}\right) = \frac{1}{2}\sum_{n=3}^{\infty}\left(\frac{1}{n-2} - \frac{1}{n}\right) = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{2} - \frac{1}{4} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} - \frac{1}{6}\right)$$

$= \frac{1}{2}\left(1 + \frac{1}{2}\right) = \underline{\frac{3}{4}}$

⑧ $\int_{-\pi}^{\pi} \cos(2\theta)\cos\theta \, d\theta$ $\quad \cos(2\theta) = 1 - 2\sin^2(\theta)$

$\int_{-\pi}^{\pi} (1-2\sin^2(\theta))\cos\theta \, d\theta$

$\int_{-\pi}^{\pi} \cos\theta \, d\theta - 2\int_{-\pi}^{\pi} \sin^2\theta\cos\theta \, d\theta$ $\quad u = \sin\theta$

$\left[\sin\theta\right]_{-\pi}^{\pi} - 2\int_{-\pi}^{\pi} \sin^2\theta\cos\theta \, d\theta$ $\quad u' = \cos\theta$

0

$\int_{-\pi}^{\pi} u^2 \cancel{\cos\theta} \frac{du}{\cancel{\cos\theta}} = \left[\frac{1}{3}\sin^3\theta\right]_{-\pi}^{\pi} = \underline{\underline{0}}$

⑨ $f(x) = \sqrt{R^2 - (x-R)^2}$

$V(L) = \pi\int_{L}^{2R} f(x)^2 \, dx$

$= \pi \cdot \int_{L}^{2R} (R^2 - (x-R)^2) \, dx$

$= \pi \left[2Rx - \frac{1}{3}x^3\right]_{L}^{2R} dx$

$= \pi\left(\left(4R^2 - \frac{8}{3}R^3\right) - \left(RL^2 + \frac{1}{3}L^3\right)\right) = \underline{\underline{\pi\left(\frac{4}{3}R^3 - RL^2 + \frac{1}{3}L^3\right)}}$

⑩ $v(t) = k$

$i(t) = \frac{V}{R}\left(1 - e^{-\frac{tR}{L}}\right)$

$V = \frac{I}{R} \Rightarrow \frac{V}{R^2}\left(1 - e^{-\frac{tR}{L}}\right)$