① $Ax = \begin{bmatrix}1\\2\\3\\4\end{bmatrix}$ $Ay = \begin{bmatrix}5\\6\\7\\8\end{bmatrix}$

$$A(2x+3y) = A\left(2\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + 3\begin{bmatrix}5\\6\\7\\8\end{bmatrix}\right) = A\begin{bmatrix}2\cdot1+3\cdot5\\2\cdot2+3\cdot6\\2\cdot3+3\cdot7\\2\cdot4+3\cdot8\end{bmatrix} = A\begin{bmatrix}17\\22\\27\\32\end{bmatrix}$$

② $\int_0^1 \frac{\cos x}{x}dx \Rightarrow \cos x \geq \cos(1)$

$$\int_0^1 \frac{\cos x}{x}dx \geq \int_0^1 \frac{\cos 1}{x}dx \Rightarrow \cos(1)\int_0^1 \frac{1}{x}dx = \cos(1)\Big[\ln x\Big]_0^1 = \cos(1)\lim_{a\to0^+}(\ln 1 - \ln 0) = \infty \text{ Divergent}$$

③ a) $S_n = \sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3\ldots$

$(1-x)S_n = (1-x)\sum_{n=0}^{\infty}x^n$

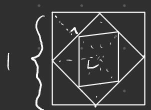
$(1-x)S_n = (1-x)(1+x+x^2+\ldots x^N)$

$(1-x)S_n = 1-x+x-x^2\ldots x^N$

$(1-x)S_n = 1+x^{N+1}$

$S_N = \frac{1+x^{N+1}}{1-x}$ setter $N\to\infty$

$$S_n \begin{cases} \frac{1}{1-x} & \text{når } |x|<1 \\ \infty & \text{når } x>1 \\ -\infty & \text{når } x<-1 \\ ? & \text{når } x=-1 \end{cases}$$



$h^2 = \frac{1^2}{2} + \frac{1^2}{2}$

$h = \sqrt{\frac{1}{4}} = \frac{1}{\sqrt{2}}$

$h = \sqrt{\frac{1^2}{2\sqrt{2}} + \frac{1^2}{2\sqrt{2}}} = \sqrt{\frac{1}{8}+\frac{1}{8}} = \sqrt{\frac{2}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$h = \sqrt{\frac{1^2}{4}+\frac{1^2}{4}} = \sqrt{\frac{2}{16}} = \frac{1}{\sqrt{8}} = \frac{1}{2\sqrt{2}}$

b) $O_{m_1} = 1+1+1+1 = 4$

$O_{m_2} = \frac{1}{2\sqrt{2}}\cdot 4 = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$O_{m_3} = \frac{1}{2}\cdot 4 = 1$

$4\left(1+\frac{1}{\sqrt{2}}+\frac{1}{2}+\frac{1}{2\sqrt{2}}\right)$

$$4\cdot\sum_{n=0}^{\infty}\frac{1}{(\sqrt{2})^n} = 4\cdot\frac{1}{1-\frac{1}{\sqrt{2}}} = \frac{4}{\frac{\sqrt{2}-1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1}$$

④
```
2 4 6 -2
3 5 7 -2
2 4 8 0
1 1 1 0
```

$\sim$
```
  2 4 6 -2
- (2 4 8 0)
  0 0 -2 -2
```
$z=1$

```
  3(2 4 6 -2)
- 2(3 5 7 -2)
  0 2 4 -2
```

$y = -1-2 = -3$

$x = -y-z = -1-(-3) = 2$

⑤ $x = \frac{1}{\ln x}$

```
import numpy as np

x = 1,2
for i in range(20):
    x = 1/np.log(x)
```

⑥ $f(x)\begin{cases} x\sin\frac{1}{x} & x\neq0 \\ 0 & x=0\end{cases}$

$$f'(x) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x) = \lim_{h\to0}\frac{(x+h)\cdot\sin\frac{1}{x+h} - x\cdot\sin\frac{1}{x}}{h} = \frac{x\left(\sin\frac{1}{x+h}-\sin\frac{1}{x}\right)+h\cdot\sin\frac{1}{x+h}}{h} = \frac{\cancel{h}\sin\frac{1}{h}}{\cancel{h}}$$

$$\lim_{h\to0}\sin\frac{1}{h} \approx 0$$

⑧ $f:[1,2]$ $f(x)=\frac{1}{x}$

$$V=\pi\cdot\int_1^2\left(\frac{1}{x}\right)^2dx=\pi\cdot\int_1^2\frac{1}{x^2}=\pi\cdot\left[-\frac{1}{x}\right]_1^2=\pi\cdot\left(-\frac{1}{2}+\frac{1}{1}\right)=\underline{\underline{\frac{\pi}{2}}}$$

⑨ $F'(x)=\cos^2x=\frac{1+\cos 2x}{2}$

$$\int\frac{1}{2}+\frac{\cos 2x}{2}=\frac{1}{2}\int 1+\cos 2x=\underline{\underline{\frac{1}{2}x+\frac{1}{2}\sin 2x+C}}$$