1)

```
import numpy as np
def f(x):
    return np.sin(x)/x

n=100000
b=np.pi
int=0
a=0
h=(b-a)/n
for i in range(n):
    x0=a+h*i
    x1=a+h*(i+1)
    int+=(h/2)*(f(x0)+f(x1))

print(int)
```

2) $f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)}$

$$f'(x) = \frac{(\sin x)' \cdot \cos x - \sin x \cdot (\cos(x))'}{(\cos x)^2} = \frac{(\cos x)^2 + (\sin x)^2}{(\cos x)^2} = \frac{1}{\cos^2 x}$$

$f'(x) = \frac{1}{\cos(-\frac{\pi}{2})^2}$ er større enn null mellom $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3a) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

Sjekker om den konvergerer

$$\lim_{n\to\infty} \frac{\frac{x^{2n+3}}{2n+3}}{\frac{x^{2n+1}}{2n+1}} = x^2 \lim_{n\to\infty} \left|\frac{2n+1}{2n+3}\right| = x^2 < 1$$

$$\underline{x < |1|}$$

b) $\frac{1}{1-x} = \sum_{n=0} x^n$

$x - s^2$

$(\arctan x)' = \frac{1}{1-x^2}$

$$\frac{1}{1-s^2} = \sum_{n=0} (-1)^n s^{2n}$$

$$\arctan x = \sum_{n=0} (-1)^n \frac{x^{2n+1}}{2n+1}$$

4) $Ax = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 9 & 3 & 7 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 9 & 3 & 7 & 1 \\ 0 & -6 & 2 & 8 \end{array}\right]$$

$x_2 = t$

$x_3 = -4 + 3t$

$x_1 = 1 - 3(-4+3t) - 7t$

$x_1 = 1 + 12 - 9t - 7t$

$x_1 = 13 - 16t$

Ser at vi får frie variabler som gjør at de er lineært avhengig

5a) $\dot{y} = 1 - y^2 \quad y(0) = 2$

$g(t) = \frac{3e^{2t}+1}{3e^{2t}-1}$

$$g'(t) = \frac{(3e^{2t}+1)'(3e^{2t}-1) - (3e^{2t}+1)(3e^{2t}-1)'}{(3e^{2t}-1)^2} = -\frac{12e^{2t}}{(3e^{2t}-1)^2}$$

$$1 - g^2 = 1 - \left(\frac{3e^{2t}+1}{3e^{2t}-1}\right)^2 = \frac{(3e^{2t}+1)'(3e^{2t}-1) - (3e^{2t}+1)(3e^{2t}-1)'}{(3e^{2t}-1)^2} = -\frac{12e^{2t}}{(3e^{2t}-1)^2}$$

noe som viser at uttrykket stemmer

b) Siden $g(t)$ er kontinuerlig deriverbar så er den entydig

6) $\frac{\pi}{2} - \frac{x}{a} = \arctan x$

$-\frac{x}{a} = \arctan x - \frac{\pi}{2}$

$x = -\arctan x \cdot a + \frac{a\pi}{2}$

```
import numpy as np
def f(x):
    return -np.arctan(x) * a + (a*π/2)

a=213712/10000
x=10000
x_neste=1

while abs(x-x_neste)>10e-10:
    x=x_neste
    x_neste=f(x)
```

7 $f : [0, \infty) \to \mathbb{R}$

$$f(x) = \frac{1}{\sqrt{1+x^2}}$$

$$V = \pi \cdot \int_0^{\infty} \frac{1}{1+x^2}\,dx = \pi \cdot \Big[\arctan x\Big]_0^{\infty} = \pi \cdot \frac{\pi}{2} = \underline{\underline{\frac{\pi^2}{2}}}$$

8 $$\lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = \underline{\underline{1}}$$