



$$27.1) A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \\ 1 & -12 \end{bmatrix} \cdot X = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{matrix} 2 \\ 3 \\ -1 \end{matrix} \\ \hline \begin{matrix} 2 & 1 \\ 2 & 3 \\ 1 & -12 \end{matrix} & \begin{matrix} 4+3-1 \\ 4+9-1 \\ 2-3-2 \end{matrix} \end{array} = \begin{bmatrix} 6 \\ 9 \\ -3 \end{bmatrix} = 3 \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$

$$27.2 a) 0 = (8-\lambda) \cdot (3-\lambda) - (-2 \cdot 9) = 24 - 8\lambda - 3\lambda + \lambda^2 - 6 = \lambda^2 - 11\lambda + 18 = (\lambda-2)(\lambda-9)$$

$\lambda=2$   
 $\lambda=9$

$$\begin{aligned} (8-2)x_1 - 3x_2 &= 0 & \rightarrow 6x_1 - 3x_2 &= 0 & x_1 &= s \\ -2x_1 + (3-2)x_2 &= 0 & \rightarrow -2x_1 + x_2 &= 0 & x_2 &= 2x_1 \\ & & & & x_2 &= 2s \end{aligned} \quad \rightarrow \underline{\underline{\begin{bmatrix} 1 \\ 2 \end{bmatrix}}}$$

$$(8-9)x_1 - 3x_2 = 0 \Rightarrow -x_1 - 3x_2 = 0 \Rightarrow -2x_1 - 6x_2 = 0$$

$$x_1 = s \rightarrow \underline{\underline{\begin{bmatrix} 1 \\ 3 \end{bmatrix}}}$$

$$b) \begin{bmatrix} 2 & 7 \\ 0 & 1 \end{bmatrix} \\ 0 = (2-\lambda)(1-\lambda) - (7 \cdot 0) = (2-\lambda)(1-\lambda)$$

$$\begin{aligned} (2-\lambda)x_1 + x_2 &= 0 & \lambda=2 & & (2-1)x_1 + x_2 &= 0 & x_1 &= -x_2 \\ (1-\lambda)x_2 &= 0 & \lambda=1 & & (1-1)x_2 &= 0 & x_2 &= s \end{aligned} \quad \rightarrow \underline{\underline{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}}$$

$$c) \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \quad \lambda=3$$

$$0 = (3-\lambda)(1-\lambda) \quad \lambda=1$$

$$\begin{aligned} (3-1)x_1 + 0x_2 &= 0 & \lambda=1 & & (3-1)x_1 + 0x_2 &= 0 \\ 5x_1 + (1-1)x_2 &= 0 & & & 5x_1 - 2x_2 &= 0 \end{aligned} \quad \rightarrow \underline{\underline{\begin{bmatrix} 1 \\ 5/2 \end{bmatrix}}}$$

$$d) \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ (2-\lambda)(2-\lambda) = 0$$

$$\lambda=2 \quad \begin{bmatrix} s \\ k \end{bmatrix} \text{ beliebig oder beide}$$

$$27.3 a) \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} 0 = (1-\lambda) \cdot \begin{vmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} 2 & 0 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 - 4(1-\lambda) = (1-\lambda)((1-\lambda)^2 - 4) = (1-\lambda)(\lambda^2 - 2\lambda - 3) = (1-\lambda)(\lambda-3)(\lambda+1)$$

$$\begin{aligned} \lambda=1 & & \lambda=3 & & \lambda=3 & & x_1=s \\ 2x_2=0 & & 2x_1+2x_2=0 & & -2x_1+2x_2=0 & & \\ 2x_1=0 & & 2x_1+2x_2=0 & & 2x_1-2x_2=0 & & \\ & & 2x_3=0 & & -2x_3=0 & & \end{aligned} \quad \rightarrow \underline{\underline{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}}}$$

$$b) \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} 0 = (1-\lambda) \begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} - 2 \begin{vmatrix} 2-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 \\ 0 & 4 \end{vmatrix}$$

$$0 = (1-\lambda)(2-\lambda)^2$$

$\lambda=1 \quad \lambda=2$

$$\begin{aligned} \lambda=1 & & \lambda=2 & & \lambda=2 & & x_1=s \\ 2x_2+x_3=0 & & -x_1+2x_2+x_3=0 & & -x_1+2x_2+x_3=0 & & \\ x_2=0 & & x_2=0 & & x_2=0 & & \\ 4x_2+x_3=0 & & x_3=0 & & x_3=0 & & \\ x_1=s & & & & & & \end{aligned} \quad \rightarrow \underline{\underline{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}}}$$

$$7.73c) \begin{bmatrix} -5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} 0 = (-5-\lambda) \begin{vmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} \\ 0 = (-5-\lambda)(1-\lambda)^2 \end{array}$$

$$\lambda = -5 \quad \lambda = 1$$

$$\lambda = 1: \quad \begin{array}{l} -6x_1 = 0 \\ x_1 = 0 \end{array} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{array}{l} \lambda = -5: \\ 6x_2 = 0 \\ 6x_3 = 0 \\ x_3 = 5 \end{array} \quad S \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$d) \begin{bmatrix} -5 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 1 \end{bmatrix} \quad 0 = (-5-\lambda) \begin{vmatrix} 1-\lambda & 0 \\ 4 & 1-\lambda \end{vmatrix}$$

$$\lambda = -5 \quad \lambda = 1$$

$$\lambda = -5: \quad \begin{array}{l} x_2 = 5 \\ x_1 = -6x_2 \end{array} \quad \begin{array}{l} -6x_1 = 0 \quad x_1 = x_2 = 0 \\ x_1 = 0 \quad x_3 = 5 \end{array} \quad S \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{l} x_1 + 6x_2 = 0 \\ -x_1 + 4x_2 + 6x_3 = 0 \\ x_3 = -\frac{5}{3} \end{array} \quad S \begin{bmatrix} 6 \\ 1 \\ -\frac{5}{3} \end{bmatrix}$$