① import numpy as np

```
b=10
n=10000
y=np.zeros(n+1)
x=np.zeros(n+1)
y[0]=1
x[0]=1
h=b/n
for i in range
    x[i+1] = x[i] + h·y[i]
    y[i+1] = y[i+1] - h·np.sin(x[i])
    x[i+1] = x[i] + h·(y[i] + y[i+1])/2
    y[i+1] = y[i+1] - h·(np.sin(x[i]) + np.sin(x[i+1]))/2
```

② a) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n+\sqrt{n}} \leq \sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ $\qquad \lim_{n\to\infty} \frac{1}{n+\sqrt{n}} = \underline{\underline{0}}$

b) $\sum_{n=1}^{\infty} \frac{1}{n3^n} = \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{3}\right)^n$

$\int \sum_{n=0}^{\infty} x^n = \int \frac{1}{1-x}$

$\sum_{n=0}^{\infty} \frac{1}{n} x^{n+1} = \ln|1-x|$

$\sum_{n=1}^{\infty} \frac{1}{n} x^n = \ln|1-x|$

$\sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{3}\right) = \ln|1-\frac{1}{3}| = \underline{\ln\frac{2}{3}}$

③ $\lim_{x\to 0} \frac{e^{x^3}-1}{x-\sin x} = \frac{\sum_{n=0}^{\infty} \frac{x^{3n}}{n!} - 1}{x - \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}} = \lim_{x\to 0} \frac{\left(1+x^3+\frac{x^6}{2}\right)-1}{x-\left(x+\frac{x^3}{6}\ldots\right)} = \lim_{x\to 0} \frac{x^3+\frac{x^6}{2}\ldots}{\frac{x^3}{6}+\ldots} \Big| \frac{1}{1}x^3 = \frac{1+\ldots}{-\frac{1}{6}+\ldots} = \underline{\underline{-6}}$

④ $\begin{bmatrix} 3 & -2 & 1 & 0 \\ -3 & 2 & -1 & 0 \\ -6 & 4 & 8 & 0 \end{bmatrix}$

3 -2 1 0
-3 2 -1 0
0 0 0 0

-6 4 8 0
-2(-3 2 -1 0)
0 0 10 0

$\to \begin{bmatrix} 3 & -2 & 1 \\ -6 & 4 & 8 \end{bmatrix}$ er lineært uavhengig

$\begin{bmatrix} 3 & -2 & 1 & 4 \\ -3 & 2 & -1 & -9 \\ -6 & 4 & 8 & 3 \end{bmatrix}$ er ikke løselig

⑤ $\int_0^1 \frac{e^{-x^2}-1}{x^2}\,dx = \int_0^1 \frac{\sum_{n=1}^{\infty} \frac{x^{2n}}{n!} + 1 - 1}{x^2} = \int_0^1 \sum_{n=1}^{\infty} \frac{x^{2n-2}}{n!} = \sum_{n=1}^{\infty} \frac{1}{n!}\left[\frac{1}{2n-1}x^{2n-1}\right]_0^1 = \underline{\underline{\sum_{n=1}^{\infty} \frac{1}{n!(2n-1)}}}$

$x_1 = -\frac{7}{3}s + t$
$x_2 = s$
$x_3 = s$
$x_4 = t$

⑥ $A = \begin{bmatrix} 9 & -3 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix} \begin{bmatrix} a & -3 \\ -3 & 2 \end{bmatrix}$

		x_1 x_2 x_3 x_4	
a	-3	$9x_1-3x_3$	$9x_2-3x_4$
-3	2	$-3x_1+2x_3$	$-3x_2+2x_4$

		a -3 -3 2	
x_1	x_2	$9x_1-3x_2$	$-3x_1+2x_2$
x_3	x_4	$9x_3-3x_4$	$-3x_3+2x_4$

$9x_1-3x_3-9x_1+3x_2=0$
$9x_2-3x_4+3x_1-2x_2=0$
$-3x_1+2x_3-9x_3+3x_4=0$
$-3x_2+2x_4+3x_3-2x_4=0$

$\to$

$3x_2-3x_3=0$
$3x_1+7x_2-3x_4=0$
$-3x_1-7x_2+3x_4=0$
$-3x_2+3x_3=0$

$\to \begin{bmatrix} 0 & 3 & -3 & 0 \\ 3 & 7 & 0 & -3 \\ -3 & -7 & 3 & 0 \\ 0 & -3 & 3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{7}{3} & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}$

7. $f(x) = \frac{x^3}{3} + \frac{1}{4x} \quad f: [1,2]$

$$f'(x) = x^2 - \frac{1}{4x^2}$$

$$f'(x) = \frac{4x^4 - 1}{4x^2}$$

$$S = \int_a^b \sqrt{1 + f'(x)^2}$$

$$S = \int_1^2 \sqrt{1 + \left(x^2 - \frac{1}{4x^2}\right)^2} = \int_1^2 \sqrt{1 + \left(\frac{4x^4 - 1}{4x}\right)^2} = \int_1^2 \sqrt{1 + \frac{(4x^4-1)^2}{16x^4}} = \int_1^2 \sqrt{\frac{16x^2 + 16x^4 - 8x^2 + 1}{16x^4}} = \int_1^2 \sqrt{\frac{16x^4 + 8x^2 + 1}{16x^4}} = \int_1^2 \sqrt{\frac{(4x^4+1)^2}{16x^2}} = \int_1^2 \frac{4x^4 + 1}{4x^2} = \int_1^2 x^2 + \frac{1}{4x^2}$$

$$= \left[\frac{1}{3}x^3 - \frac{1}{4x}\right]_1^2 = \left(\frac{1}{3} \cdot 2^3 - \frac{1}{4 \cdot 2} - \left(\frac{1}{3} \cdot 1^3 - \frac{1}{4}\right)\right) = \frac{8}{3} - \frac{1}{8} - \frac{1}{3} + \frac{1}{4} = \frac{7}{3} + \frac{1}{8} = \frac{56}{24} + \frac{3}{24} = \underline{\underline{\frac{59}{24}}}$$

8. $\int_0^3 \frac{dx}{(x-2)^{\frac{1}{3}}} \geq \int_0^3 \frac{1}{x-2}\,dx = \Big[\ln|x-2|\Big]_0^3 = \ln 1 - \ln|-2| = 0 - \ln 2 = -\ln 2$ konvergeva

9. $f(x) = \ln(\arcsin(x) + 2) \quad f: [-1,1]$

$$f'(x) = \frac{1}{\arcsin(x) + 2} \cdot \frac{1}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2} \cdot \arcsin(x) + 2} > 0$$

$$\ln(\arcsin(x) + 2) = y$$

$$\arcsin(x) + 2 = e^y$$

$$\arcsin(x) = e^y - 2$$

$$x = \sin(e^y - 2)$$