① Gjort før

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③ $w = c_1 v_1 + c_2 v_2 + \dots c_n v_n$ må $c_1 = c_2 = c_n = 0$

antar

$w = d_1 v_1 + d_2 v_2 + \dots d_n v_n$

$w - w = 0 = (c_1 - d_1) v_1 + (c_2 - d_2) v_2 + \dots (c_n - d_n) v_n$

$c_1 - d_1 = c_2 - d_2 = c_n - d_n = 0$ så er c_n entydig bestemt dersom v_n er lineært uavhengig

④ a) $D = \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 9 - 6\lambda + \lambda^2 - 1 = \lambda^2 - 6\lambda + 8 = (\lambda-2)(\lambda-4)$

$\lambda = 2 \quad \lambda = 4$

$(3-2)x_1 + x_2 = 0$
$x_1 + (3-2)x_2 = 0$
$\Rightarrow$
$x_1 + x_2 = 0$
$x_1 + x_2 = 0$
$x_1 = s \quad x_2 = -s$
$s \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 4$

$(3-4)x_1 + x_2 = 0$
$x_1 + (3-4)x_2 = 0$
$\Rightarrow$
$-x_1 + x_2 = 0$
$x_1 - x_2 = 0$
$\Rightarrow s \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$

b) $\dot{x} = Ax$

$\dot{x} = C_1 e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$x = e^{2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

⑤ $f(x) = \cos x$

$u = x \quad v' = 1$
$u' = \cos x \quad v = \sin x$

$V = 2\pi \int_0^{\frac{\pi}{2}} x \cdot \cos x \, dx = 2\pi \Big|_0^{\frac{\pi}{2}} x \sin x - \int \sin x = 2\pi \Big|_0^{\frac{\pi}{2}} x \sin x - \sin x \Big| = 2\pi \cdot \left(\frac{\pi}{2} - 1\right) = \pi(\pi - 2)$

⑥ a) $f(x) = 1 + \tan^{-1}(x)$

$f'(x) = \left|\frac{1}{1+x^2}\right| \leq 1$ for alle x verdier

b)

```
import numpy as np
x=1
for i in range(100):
    x= 1+ np.arctan(x)
```

⑦ $\int \frac{1}{x^4 - 4x^2} = \int \frac{A}{x^2} + \frac{B}{x^2-4} = \frac{A}{x^2} + \frac{B}{x+2} + \frac{C}{x-2}$

$A(x^2-4) + Bx(x-2) + Cx(x+2) = 1$

$-4A = 1 \quad A = -\frac{1}{4}$

$8B = 1 \quad B = \frac{1}{8}$

$8C = 1 \quad C = \frac{1}{8}$

$\int -\frac{1}{4x^2} + \frac{1}{8(x+2)} + \frac{1}{8(x-2)} \, dx = \frac{1}{4}\left(\frac{1}{x} + \frac{1}{2}\ln|x-2| + \frac{1}{2}\ln|x+2|\right) + C$

⑧ $f: [1, \infty) \Rightarrow \mathbb{R}$

$f(x) = \frac{1}{x} \qquad f'(x) = -\frac{1}{x^2}$

$A = 2\pi \cdot \int^b f(x) \sqrt{1 + f'(x)^2}$

$A = 2\pi \cdot \int_1^{\infty} \frac{1}{x} \sqrt{1 + \frac{1}{x^4}} = 2\pi \cdot \int_1^{\infty} \frac{1}{x} \cdot \frac{\sqrt{x^4+1}}{x^2} = 2\pi \int_1^{\infty} \frac{\sqrt{x^4+1}}{x^3} \geq 2\pi \int_1^{\infty} \frac{x^2}{x^3} = 2\pi \int_1^{\infty} \frac{1}{x} = 2\pi \lim_{b \to \infty} (\ln b - \ln 1) = \infty$