① a) $L_a \frac{d}{dt} i_a = -R_a i_a - K_E \omega_m + u_a$

$J_m \dot{\omega}_m = K_M i_a - M_L$

$\dot{\theta}_m = \omega_m$

(1) Kirchoffs første lov

(2) moment balanse

b) Mono, ett inngangsignal

c) $L_a \dot{i}_a = -R_a i - K_E \omega_m + u_a$

$J_m \dot{\omega}_m = K_M i_a - M_L$

$\dot{\theta}_m = \omega_m$

$L_a \ddot{i}_a = -R_a \dot{i} - K_E \dot{\omega}_m + \dot{u}_a$

$\dot{\omega}_m = \frac{K_m}{J_m} i_a - \frac{M_L}{J_m}$

$$L_a \ddot{i}_a = -R_a \dot{i} - \frac{K_E K_m}{J_m} i_a + \frac{K_E M_L}{J_m} - K_p \dot{i}_a$$

$$\ddot{i}_a + \left(\frac{R_a + K_p}{L_a}\right)\dot{i}_a + \frac{K_E K_m}{L_a J_m} i_a - \frac{K_E M_L}{J_m} = 0$$

d) $\omega_0^2 = \frac{K_E K_m}{J_m}$

$\omega_0 = \underline{\underline{\sqrt{\frac{K_E K_M}{L_a J_m}}}}$

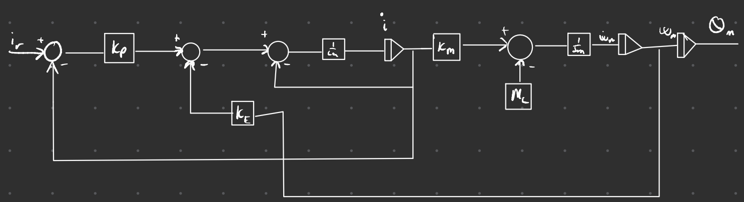
$2\omega_0 \zeta = \frac{R_a + K_p}{L_a}$

$\zeta = \frac{R_a + K_p}{2\omega_0 L_a J_m} = \underline{\underline{\frac{R_a + K_p}{2\sqrt{\frac{K_E K_m}{L_a J_m}} L_a}}}$

e) $2\omega_0 \zeta = \frac{R_a + K_p}{L_a}$

$K_p = 2\omega_0 \zeta L_a - R_a = 2 \cdot \sqrt{\frac{1 \cdot 1}{1 \cdot 0{,}01}} \cdot 1 \cdot 1 - 10 = 10$

f)



g) $K_{pK} = 20 \quad T_K = 2$

$K_p = 0{,}6 \cdot 20 = 12$

$T_i = 0{,}5 \cdot 2 = 1$

$T_K = 0{,}125 \cdot 2 = 0{,}250$

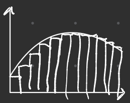
$K_i = \frac{K_p}{T_i} = \frac{12}{1} = 12$

$K_d = 12 \cdot 0{,}25 = 3$

h) $f = \frac{1}{2s} = 0{,}5\,Hz$

$f_s \geq 2 f_{max} = 2 \cdot 0{,}5 = \underline{\underline{1\,Hz}}$

② a)



$t = \frac{T}{2}$

T er samptid

b) $x(t) = x_0\left(1 - e^{-\frac{t}{T}}\right)$

$t = T$

$x(t) = x_0(1 - e^{-1}) = \underline{\underline{x_0 \cdot 0{,}63}}$

c) Nei, siden Voltaire eller Van der pols har ustabile likevekt punkter som ikke går mot uendelig

③ a) $\dot{x} + k_1 x = k_2 u \Rightarrow \dot{x} = -k_1 x + k_2 u$

$T = -\frac{1}{a} = -\frac{1}{-k_1} = \underline{\underline{\frac{1}{k_1}}}$

$K = -\frac{b}{a} = -\frac{k_2}{-k_1} = \underline{\underline{\frac{k_2}{k_1}}}$

b) $\dot{x} = -k_1 x + k_2(k_p(x_r - x))$

$\dot{x} = -k_1 x + k_2 k_p x_r - k_2 k_p x$

$(k_1 + k_2 k_p) x_s = k_2 k_p x_r$

$\underline{\underline{x_s = \frac{k_2 k_p}{k_1 + k_2 k_p} x_r}}$

c) For at systemet skal nå ref verdi må $e_s = 0$ Ser at hvis $k_p \Rightarrow \infty$ så blir $e_s = 0$

$e_s = (x_r - x_s) = x_r - \frac{k_2 k_p}{k_1 + k_2 k_p} x_r = \left(1 - \frac{k_2 k_p}{k_1 + k_2 k_p}\right) x_r = \frac{k_1}{k_1 + k_2 k_p} x_r$ + fretter metning i systemet om det er implementert

Probemet er at høy k_p fører til overregulering og et svært overdempet system

d) $\dot{x} = k_1 x + k_2 k_p (x_r - x) + k_2 k_i \int (x_r - x)$

Deriverer uttrykket

$\ddot{x} = k_1 \dot{x} - k_2 k_p \dot{x} + k_2 k_i x_r - k_2 k_i x$

setter $\ddot{x} = 0$ og $\dot{x} = 0$

$k_2 k_i x_r - k_2 k_i x = 0$

$x_s = x_r$

$e_s = (x_r - x_s) = (x_r - x_r) = \underline{0}$

e) $u_f = \frac{v(t)}{k_2}$