① a) $\dot{x} = ax + b$

$\Rightarrow x = Ce^{at} - \frac{b}{a}$

$\dot{x} - ax = b \quad | \cdot e^{-at}$

$e^{-at}\dot{x} - ae^{-at}x = e^{-at}b$

$\frac{d}{dt}e^{-at}x = e^{-at}b \quad | \int$

$e^{-at}x = -\frac{b}{a}e^{-at} + C$

$x = Ce^{at} - \frac{b}{a}$

b) $Ce^{a\cdot t} - \frac{b}{a} = x_0$

$C - \frac{b}{a} = x_0$

$C = x_0 + \frac{b}{a}$

$x = \left(x_0 + \frac{b}{a}\right)e^{at} - \frac{b}{a} = x_0 e^{at} + \frac{b}{a}e^{at} - \frac{b}{a} = x_0 e^{at} + \frac{b}{a}(e^{at} - 1)$

c) Energibalanse

d) Stasjonær verdi er verdien systemet vil stabilt ligge på. En T verdi som systemet vil stabilisere seg rundt ved $\dot{T} = 0$

$\dot{T} = -\frac{k}{c}T + \frac{1}{c}P - \frac{k}{c}T_{rom}$

$T = -\frac{1}{a} = -\frac{1}{-\frac{k}{c}} = \frac{c}{k} = \frac{4005\,^{\circ}C^{-1}}{2W\,^{\circ}C^{-1}} = 200\frac{s}{W^{\circ}C} = 200s$

$-\frac{k}{c}T + \frac{1}{c}P - \frac{k}{c}T_{rom} = 0$

$T = \frac{P - kT_{rom}}{k} = \frac{500W - 2W^{\circ}C^{-1} \cdot 20^{\circ}C}{2W^{\circ}C^{-1}} = 230^{\circ}C$

e) $\dot{T} + aT = b \qquad b = \frac{1}{c}P - \frac{k}{c}T_{rom}$

$a = \frac{k}{c}$

$T = Ce^{at} - \frac{b}{a}$

$T = Ce^{\frac{k}{c}t} - \frac{P - kT_{rom}}{k}$

$T = Ce^{0,005t} - 230 \Rightarrow C - 230 = 20$

$C = 250$

$T(t) = 250e^{0,005t} - 230$

$T(5) = 250e^{0,005 \cdot 5} - 230 = 26,3\,^{\circ}C$

$T(10) = 250e^{0,005 \cdot 10} - 230 = 32,8\,^{\circ}C$

f) $x_{n+1} = x_n + hf(x_n)$

$x_2 = x(5) = 20 + 5 \cdot \left(-\frac{1}{c}T + \frac{1}{c}(P - kT_{rom})\right)$

$x_2 = 25$

$x_3 = 25,5 + 5 \cdot \left(-\frac{1}{c}T + \frac{1}{c}(P - kT_{rom})\right)$

$x_3 = 30,9$

② a) 

b) Vindelhastighet og vinkel

c) Fordi et kritisk dempet system gir raskest respons uten svingninger

② d) $J\ddot{\theta} = T_m - T_L$

$T_m = -k_p(\theta + T_d\dot{\theta})$

$J\ddot{\theta} = -k_p\theta - T_d\dot{\theta} - T_L$ (0)

$\ddot{\theta} + \frac{T_d}{J}\dot{\theta} + \frac{k_p}{J}\theta = 0$

$\omega_0^2 = \frac{k_p}{J}$

$k_p = \omega_0^2 \cdot J = 1^2 \cdot 1 = \underline{1}$

$2\omega_0\zeta = \frac{T_d}{J}$

$T_d = 2\omega_0\zeta J = 2 \cdot 1 \cdot 1 \cdot 1 = \underline{2}$

e) Foroverkobling

$u = -k_p(\theta + T_d\dot{\theta}) + T_L$

f) integral, oppnå ref verdi, og kompansere for konstant støy
prop, multiplisert med current
derivat, gjevn støy/forstyrrelse, økker demping