① a) $\dot x_1 = x_2$ $\quad x_1 = \psi$ $\quad r = \dot\psi$

$\dot x_2 = -\frac{1}{T}x_2 + \frac{K}{T}u$ $\quad x_2 = r$ $\quad \dot r = \ddot\psi$

$u = -K_p x_1 - K_d x_2$

$$\dot r = -\frac{1}{T}r + \frac{K}{T}\left(-K_p\psi - K_d r\right)$$

$$\ddot\psi = -\frac{1}{T}\dot\psi - \frac{K}{T}K_p\psi - \frac{K}{T}K_d\dot\psi$$

$$\ddot\psi + \left(\frac{1+KK_d}{T}\right)\dot\psi + \frac{KK_p}{T}\psi = 0$$

$\omega_0^2 = \frac{KK_p}{T}$ $\quad 2\omega_0\zeta = \frac{1+KK_d}{T}$

$\omega_0 = \sqrt{\frac{KK_p}{T}}$ $\quad \zeta = \frac{1+KK_d}{2T\cdot\sqrt{\frac{KK_p}{T}}} = \frac{(1+KK_d)\sqrt{T}}{2T\sqrt{KK_p}} \Big| \cdot\sqrt{T} \Rightarrow \underline{\underline{\frac{(1+KK_d)}{2\sqrt{KK_pT}}}}$

b) $\frac{1+KK_d}{T} > 0$ $\quad \frac{KK_p}{T} > 0$

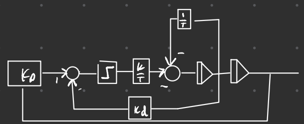
$K_d > -\frac{1}{K}$ $\quad K_p > 0$

c) $K_p = \frac{\omega_0^2 T}{K}$

$K_p = \frac{1^2\cdot 100}{0,2} = \underline{\underline{500}}$ $\quad K_d = \frac{2\omega_0\zeta T - 1}{K} = \frac{2\cdot1\cdot1\cdot100-1}{0,2} = \underline{\underline{995}}$

d) Ustabilitet og dårlig ytelse

e)



f) Kursvinkelen finnes ved int av r
Skaper drift i måling

② a) $\dot x = -bx^2$ $\quad b = 1$

$\dot x = -x^2$

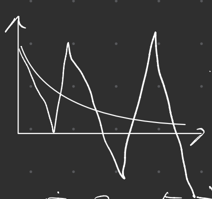
b) $|1+ah| \le 1$

$|1-2h| \le 1 \Rightarrow -1+2h \le 1$, $\underline{\underline{h \le 1}}$

$1-2h \le 1$

$h \ge 0$

c)



③ a) $c\rho V\dot T = P + cw(T_i - T)$

$$\dot T = -\frac{w}{\rho V}T + \frac{P}{c\rho V} + \frac{w}{\rho V}T_i$$

$T = -\frac{1}{a} = -\frac{1}{-\frac{w}{\rho V}} = \frac{\rho V}{w}$

$K = -\frac{b}{a} = -\frac{\frac{1}{c\rho V}}{-\frac{w}{\rho V}} = \frac{1}{c\rho}$

3b) $T = \frac{\rho V}{w} = \frac{kg/m^3 \cdot m^3}{kg/s} = kg \cdot \frac{s}{kg} = \underline{\underline{s}}$

c) $\dot{T} = -\frac{w}{\rho V} T + \frac{P}{c\rho V} + \frac{w}{\rho V} T_i$

$P = u = K_p(T_r - T)$

$\dot{T} = -\frac{w}{\rho V} T + \frac{K_p}{c\rho V} T_r - \frac{K_p}{c\rho V} T + \frac{w}{\rho V} T_i$

$\dot{T} = -\left(\frac{w}{\rho V} + \frac{K_p}{c\rho V}\right) T + \frac{K_p}{c\rho V} T_r + \frac{w}{\rho V} T_i$

$\dot{T} = 0$

$T_s = \frac{K_p}{cw + K_p} T_r + \frac{wc}{cw + K_p} T_i$

$e_s = T_r - T_s = T_r - \frac{K_p T_r}{cw + K_p} - \frac{wc}{cw + K_p} T_i = \left(\frac{cw}{cw + K_p}\right)(T_r - T_i)$

$\lim_{K_p \to \infty} \left(\frac{cw}{cw + K_p}\right)(T_r - T_i) = 0$

d) $u = K_p(T_r - T) + K_i \int_0^t (T_r - T(\tau)) d\tau$

$\dot{T} = -\frac{w}{\rho V} T + \frac{P}{c\rho V} + \frac{w}{\rho V} T_i$

$\dot{u} = -K_p \dot{T} + K_i T_r - K_i T$

derivere utrykket å fjerner $\dot{T}$ $\ddot{T}$ T_i

$\frac{K_i T_r}{c\rho V} - \frac{K_i T_s}{c\rho V} = 0$

$T_s = T_r$

$e_s = T_r - T_s = T_r - T_r = \underline{\underline{0}}$

$u_{(t)} = -cwT_i$

e) $\ddot{T} + \left(\frac{cw + K_p}{c\rho V}\right)\dot{T} + \frac{K_i}{c\rho V} T - \frac{K_i}{c\rho V} T_r = 0$

At forstyrrelsene er konstant og målbare
altså c, w og T_i

f) Monovariabelt siden det kun er en inngang

g) Ulineært

$f = \frac{1}{T} = \frac{1}{1 \cdot 10^{-3}} = 1000$

4a) $T_s \geq 2T_{max}$

$T_{max} \leq \frac{T_s}{2} = 2 \cdot 1kHz = \underline{\underline{2000 Hz}}$

b) Nyquist-Shannon samplings teorem

Teoremet nedfoldings går ut på at du prøver å sample et signal men du har for lav frekvens. Noe som sier at dataen din blir feil fra den reele