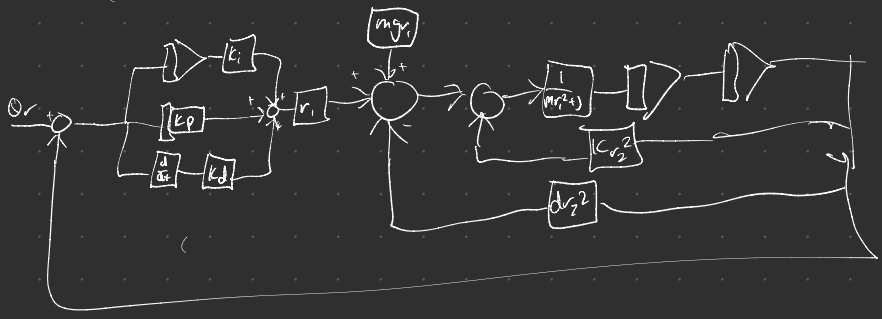
d) $\ddot{\theta} + \frac{dr_2^2}{mr_1^2+J}\dot{\theta} + \frac{kr_2^2}{mr_1^2+J}\theta = \frac{mgr_1}{mr_1^2+J} + \left(\frac{r_1}{mr_1^2+J}\right)(k_p\theta_r - k_p\theta - k_d\dot{\theta})$

$\ddot{\theta} + \left(\frac{dr_2^2 + r_1 k_d}{mr_1^2+J}\right)\dot{\theta} + \left(\frac{kr_2^2 + r_1 k_p}{mr_1^2+J}\right)\theta = \frac{r_1(mg+1)}{mr_1^2+J}$

$\omega_0^2 = \frac{kr_2^2 + r_1 k_p}{mr_1^2+J}$ $\qquad 2\omega_0\zeta = \frac{dr_2^2 + r_1 k_d}{mr_1^2+J}$

$k_p = \frac{\omega_0^2(mr_1^2+J)}{r_1} - \frac{kr_2^2}{r_1}$ $\qquad k_d = \frac{2\omega_0\zeta(mr_1^2+J)}{r_1} - \frac{dr_2^2}{r_1}$

e) $\ddot{\theta} = \frac{1}{mr_1^2+J}\left(-dr_2^2\dot{\theta} - kr_2^2\theta + mgr_1 + r_1(k_p(e) + k_i\int e + k_d\dot{e})\right)$



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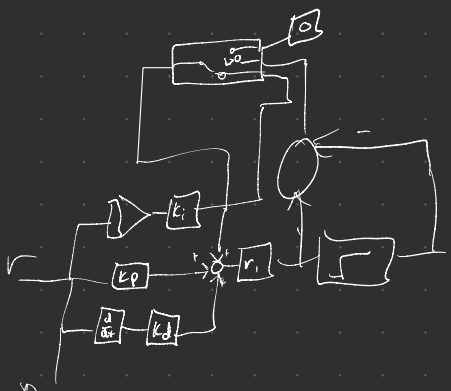


2) a) $x = l(\cos q_1 + \cos(q_2 + q_1) + \cos(q_1 + q_2 + q_3) + \ldots \cos(q_1 + q_2 + q_3 + q_n)$

b) $y = l($

④ a) $\dot{x} = -4x - 0$

$T = -\frac{1}{a} = -\frac{1}{-4} = \frac{1}{4}$

$K = -\frac{b}{a} = -\frac{-1}{-4} = -\frac{1}{4}$

c) $\ddot{x} = -0.6\dot{x} - 0.09x$

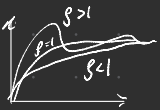
$\ddot{x} + 0.6\dot{x} + 0.09x = 0$

$\omega_0^2 = 0.09 \quad 2\omega_0\zeta = 0.6$

$\omega = 0.3 \quad \zeta = \frac{0.6}{2\omega_0} = \frac{0.6}{2 \cdot 0.3} = 1$

d) kritiskt

e)



⑤ a) $\tau = 1{,}25\,s$

$T = 0{,}5\,s$

$K = 1$

$\zeta = \frac{Ad}{T}$

⑥ $k(x-1)(x-2)(x-3) = 0$

$x=1 \quad x=2 \quad x=3$

$\dot{x} = k(x^2 - 3x + 2)(x-3)$

$(x^3 - 6x^2 + 11x - 6)k$

i) $(3x^2 - 12x + 11)k$

$f'(1) = 3 - 12 + 11 = 2$ ostabilt

$f'(2) = 3 \cdot 4 - 12 \cdot 2 + 11 = -1$ stabilt

$f'(3) = 3 \cdot 9 - 12 \cdot 3 + 11 = 14$ ostabilt

ii) $k = -1$

$f'(1) = -3 + 12 - 11 = -2$ stabilt

$f'(2) = -3 \cdot 4 + 12 \cdot 2 + 1 = 1$ ostabilt

$f'(3) = -3 \cdot 9 + 12 \cdot 3 - 11 = -2$ stabilt

⑦ a) Energibalansen

b) $\dot{T} = -k_1 T + k_2 P$

$P = k_p(T_r - T)$

$\dot{T} = -k_1 T + k_2 k_p T_r - k_2 k_p T$

$T(k_1 + k_2 k_p) = T_r k_2 k_p$

$T_s = \frac{k_2 k_p}{k_1 + k_2 k_p} T_r$

$e_s = (T_r - T_s) = \left(1 - \frac{k_2 k_p}{k_1 + k_2 k_p}\right) T_r = \left(\frac{K_1}{k_1 + k_p k_2}\right) T_r$

c) $\dot{i} = -k_p \dot{T} + k_1 T_r - k_i T$

$\ddot{T} = -k_1 \dot{T} - k_2 k_p \dot{T} + k_2 k_i T_r - k_2 k_i T$

$T_s = T_r$

$e_s = T_r - T_s = T_r - T_r = 0$

$x_1 = \dot{x}$ $\quad u = -k_p x_2 - k_d \dot{x}_1 + r$

$x_2 = x$ B

② Modering

① a) $f = \frac{1}{12 \cdot 10^{-3}} \approx 833\,Hz$

$f_{max} = \frac{f_s}{2} = \frac{833\,Hz}{2} \geq 416{,}5\,Hz$

b)

④ a) Massebalanse

b) $\dot{h}_1 = \frac{k_1}{\rho_1 A_1} h_1 + \frac{\omega_1}{\rho_1 A_1}$

$\dot{h}_2 = -\frac{k_2}{\rho_1 A_2} h_2 + \frac{\omega_1}{\rho_1 A_1}$

$T_1 = -\frac{1}{a} = \frac{\rho_1 A_1}{k_1}$

$T_2 = \frac{\rho_2 A_2}{k_2} \rightarrow \frac{\frac{2}{3}\rho_1 \cdot 3 A_1}{k_1} = \frac{2\rho_1 A_1}{k_1}$

① a) $T = -\frac{1}{-k_2} = \frac{1}{k_2}$

b) $\dot{T} = -k_2 T + k_1 u$

$u = k_p(r - T)$

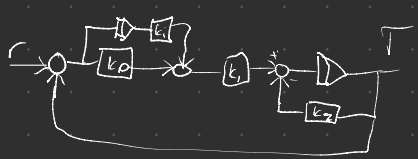
$\dot{T} = -k_2 T - k_1 k_p T + k_1 k_p r$

$\dot{T} = 0 \rightarrow (k_2 + k_1 k_p) T_s = k_1 k_p r$

$T_s = \frac{k_1 k_p}{k_2 + k_1 k_p} r$

$e_s = (r - T_s) = r - \frac{k_1 k_p}{k_2 + k_1 k_p} r = \left(1 - \frac{k_1 k_p}{k_2 + k_1 k_p}\right) r = \left(\frac{k_2}{k_2 + k_1 k_p}\right) r$

c)



d) $x_1 = \int e\,dt$ $\quad \dot{x}_1 = e = r - T$

$x_2 = T$ $\quad \dot{x}_1 = -x_2 + r$

$\dot{x}_2 = -k_2 x_2 + k_1 (k_p x_1 + k_i x_1)$

$\dot{x}_2 = -k_2 x_2 - k_1 k_p x_2 + k_1 k_p r + k_1 k_i x_1$

$\dot{x}_2 = k_1 k_i x_1 - (k_2 + k_1 k_p) x_2 + k_1 k_p r$

e) $0 = -x_2 + r$ $\quad x_2 = r$ $\rightarrow$ $T = r$ treffer stasjonær:

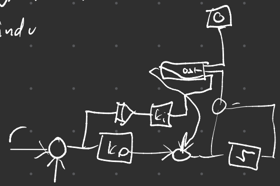
$0 = k_1 k_i x_1 - (k_2 + k_1 k_p) x_2 + k_1 k_p r$

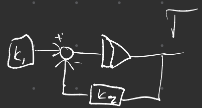
$x_2 = r$

$x_1 = \frac{k_2}{k_1 k_i} r$

f) Metning i pådraget medført av int windup

anti windu





2 a) AV/PÅ

b) Midt av området at den skrus av når temp > 23,5 og på når temp er mindre enn 22,5

c) Endring i temp i rommet, foroverkobling

3 a) $x = \cos(\pi + q_1) + \cos(\pi + q_1 + q_2)$

$y = \sin(\pi + q_1) + \sin(\pi + q_1 + q_2)$

b)