① $m\ddot{x} = -c_1\dot{x} - c_2 x + u(t-\tau)$

$x_1 = \dot{x} \quad x_2 = x$

$\dot{x}_1 = \ddot{x}$

$\dot{x}_1 = -\frac{c_1}{m}x_1 - \frac{c_2}{m}x_2 + u(t-\tau)$

$u = -K_p x_2 - K_d x_1$

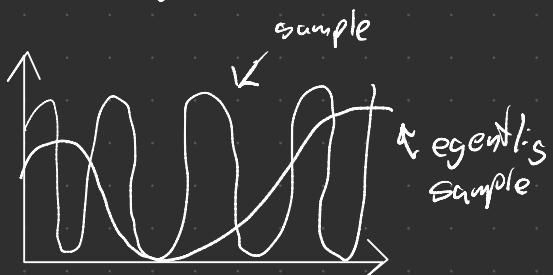
B matche

② Reguleringsfenomenet er måling der det ikke varmeapparatet kan yte mer

③a) $t_s = 12\,ms \quad \frac{1}{12\cdot 10^{-3}} = 83\,Hz$

$f_{max} \leq \frac{1}{2} 83\,Hz = \underline{41{,}5\,Hz}$

b) Nedfolding er at vi sampler med for lav samplingsfrekvens.



④a) masse balanse

b) $\dot{h}_1 = ah_1 + bu$

$T_1 = -\frac{1}{a} = -\frac{1}{-\frac{k_1}{\rho_1 A_1}} = \frac{\rho_1 A_1}{k_1}$

$T_2 = -\frac{1}{a} = -\frac{1}{-\frac{k_2}{\rho_2 A_2}} = \frac{\rho_2 A_2}{k_2} = \frac{\frac{\rho_1}{1{,}5}\cdot 3A_1}{k_1} = 2\frac{\rho_1 A_1}{k_1}$

Tank 2 har større tidskonstant

9 c) $w_{i,1} = k_p(h_r - h)$

$$\dot{h} = -\frac{1}{\rho_1 A_1}(k_1 + k_p)h + \frac{k_p h_r}{\rho_1 A_1}$$

$$T = -\frac{1}{a} = \frac{\rho_1 A_1}{k_1 + k_p}$$

d) $\dot{h} = -\frac{1}{\rho_1 A_1}(k_1 + k_p)h + \frac{k_p h_r}{\rho_1 A_1}$ setter $\dot{h} = 0$

$$0 = -\frac{1}{\rho_1 A_1}(k_1 + k_p)h + \frac{k_p h_r}{\rho_1 A_1} \Rightarrow \frac{1}{\rho_1 A_1}(k_1 + k_p)h = \frac{k_p h_r}{\rho_1 A_1} \Rightarrow h = \frac{k_p h_r}{(k_1 + k_p)}$$

e) PI-regulator $\Rightarrow$ $u_1 = k_p(h_r - h) + k_1 \int (h_r - h)\,dt$

$$\dot{h} = -\frac{k_1}{\rho_1 A_1} h_1 + \frac{k_p}{\rho A} e + \frac{k_i}{\rho A} \int e\,dt$$ derivér uttrykket

$$\ddot{h} = -\frac{k_1}{\rho_1 A_1} \dot{h}_1 + \frac{k_p}{\rho A} \dot{e} + \frac{k_i}{\rho A} e$$ Gjerner alle 0 verdier

$$\frac{k_1}{\rho A} e = 0$$

$$h_r - h_1 = 0 \Rightarrow h_r = h_1$$

f) $w_{v,1} = k_1 h_{r_1}$

$w_{v_2} = k_2 h_{r_2}$

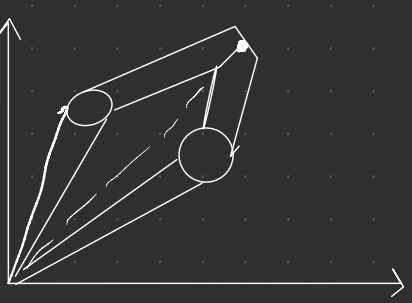
g) Endre ventil åpningen eller referansehøyden

⑤ Vi ser at Lastesteget gir like fortegn i τ_i og i så det er stabilt. Oljetanker gir ulike fortegn og dermed ustabilt

⑥ a) $x = \cos\theta_1 \cdot a_1 + \cos(\theta_1+\theta_2)\cdot a_2$

$y = \sin\theta_1 \cdot a_1 + \sin(\theta_1+\theta_2)\cdot a_2$

b)



⑦ I A er PID brukt

I B er PD brukt

I C er PI brukt

⑧ a) $\dot{x} = -2x + 6$

$T = \frac{1}{a} = -\frac{1}{-2} = \frac{1}{2}$

$K = -\frac{b}{a} = -\frac{1}{-2} = \frac{1}{2}$

b) setter $\dot{x} = 0$

$-2x + 6 = 0$

$x = \frac{6}{2} = \frac{1}{2}$

c) $\frac{dx}{dt} = -2x + 1$

$\int \frac{1}{-2x+1} = \int 1\,dt \qquad u = -2x+1$

$-\frac{1}{2}\int \frac{1}{u}\,du = \int 1\,dt$

$\ln u = -2t + C'$

$u = e^{-2t+C'}$

$-2x+1 = e^{-2t}$

$x = \frac{1}{2}(1 - e^{-2t})$

⑨ d) $\frac{1}{2} - \frac{1}{2}e^{-2t} = \frac{1}{2} \cdot 0{,}95$

$e^{-2t} = 0{,}05$

$t = \frac{-\ln(0{,}05)}{2} = \frac{3}{2}$

$\alpha \cdot T = \frac{3}{2}$

$\alpha \cdot \frac{1}{2} = \frac{3}{2} \Rightarrow \underline{\alpha = 3}$

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⑩ a) $m \cdot \ddot{x} = -kx - d\dot{x}$

$m\ddot{x} + d\dot{x} + kx = 0$

$\ddot{x} + \frac{d}{m}\dot{x} + \frac{k}{m}x = 0$ $\Rightarrow$ $\ddot{x} + p\dot{x} + qx = 0$

$p = \frac{d}{m} = \frac{1{,}5}{1{,}5}$ $\quad q = \frac{6}{1{,}5} = 4$ $\quad \underline{p = 1 \quad q = 4}$

b) $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{6}{1{,}5}} = \underline{2}$

$\zeta = \frac{d}{2\sqrt{km}} = \frac{1{,}5}{2\sqrt{1{,}5 \cdot 6}} = \underline{\frac{1}{4}}$ underdempet siden ζ er < 1

c) $\omega_d = \omega_0\sqrt{1-\zeta^2} = 2\sqrt{1-\frac{1}{16}} = 2 \cdot \frac{\sqrt{15}}{4} = \underline{\frac{\sqrt{15}}{2}}$ frekvensen på svingningen

d) $\delta\left(\frac{1}{4}\right) = e^{\frac{-\frac{1}{4}\pi}{\sqrt{1-\frac{1}{16}}}} = \underline{0{,}44}$