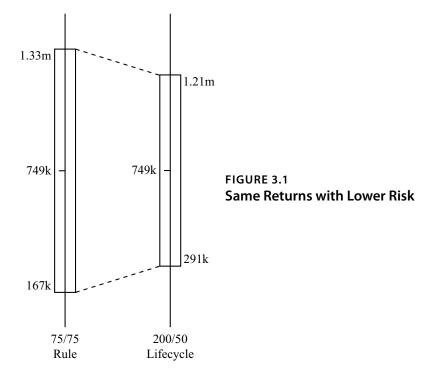
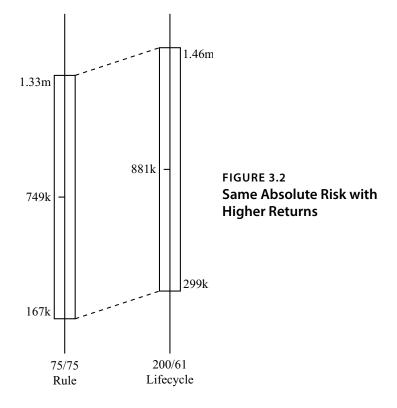


FIGURE 2.1 Phases of Lifecycle Strategy





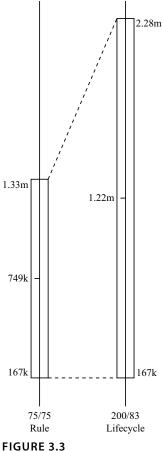


FIGURE 3.3
Same Worst Case with
Higher Returns

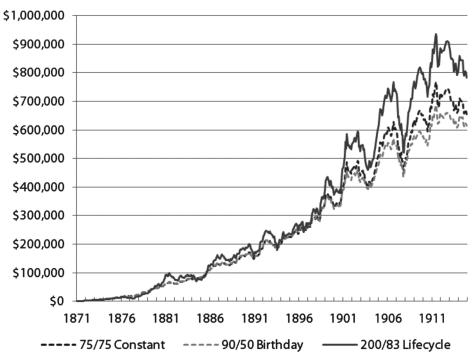


FIGURE 3.4 Zachary's Savings Under Different Strategies from 1871 to 1914

Diversifying *Improvement Improvement* Birthday Constant % Lifecycle over Stocks Rule Strategy Birthday Rule Constant %

75

75

\$748,839

\$308,726

\$449,266

\$561,032

\$691,427

\$922,028

\$1,152,276

\$1,252,684

90

50

\$646,575

\$290,310

\$416,253

\$539,343

\$641,555

\$779,044

\$870,921

\$1,026,903

TABLE 3.1

Max. % Inv.

Min. % Inv.

Mean Result

Min. Result

10th pct.

25th pct.

Median

75th pct.

90th pct.

Max. Result

Results from 96 Simulated Investors: 1871–2009

200

83

89.2%

33.4%

68.6%

63.9% 78.8%

95.5%

121.6%

112.0%

\$1,223,105

\$387,172

\$701,834

\$884,138

\$1,146,812

\$1,522,653

\$1,929,577

\$2,177,424

over

63.3%

25.4%

56.2% 57.6%

65.9%

65.1%

67.5%

73.8%

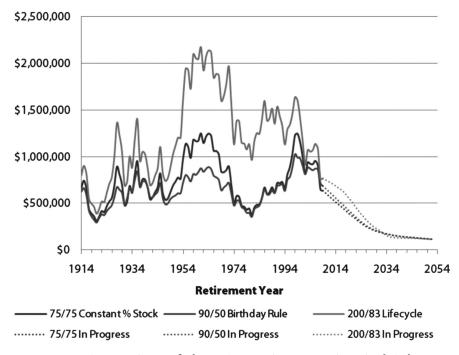
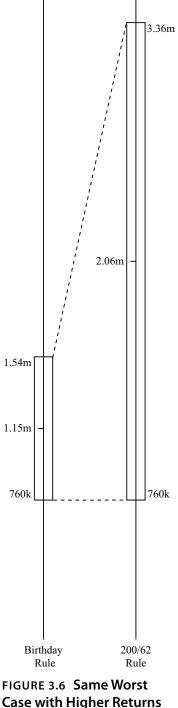


FIGURE 3.5 Comparison of Three Strategies over Historical Cohorts and Cohorts in Progress



Case with Higher Returns (Social Security Included)

30 FTSE All-Shares Cohorts 96 U.S. Cohorts (1871–2009) (1937–2009)

**TABLE 4.1 Extension of Lifecycle Results to Foreign Markets** 

90th pct.

Max. Result

\$1,152,276

\$1,252,684

\$1,929,577

\$2,177,424

67.5%

73.8%

	Constant % (75/75)	Diversifying (200/83%)	Improvement over	Constant % (75/75)	Diversifying (200/83%)	Improvement over	Constant % (75/75)	Diversifying (200/83%)	Improvement over
	Strategy	Strategy	Constant %	Strategy	Strategy	Constant %	Strategy	Strategy	Constant %
Max. % Inv.	75	200		75	200		75	200	
Min. % Inv.	75	83		75	83		75	83	
Mean Result	\$748,839	\$1,223,105	63.3%	£538,863	£870,619	61.6%	¥51,120,760	¥96,864,911	89.5%
Min. Result	\$308,726	\$387,172	25.4%	£258,337	£492,356	90.6%	¥24,626,938	¥32,181,435	30.7%
10th pct.	\$449,266	\$701,834	56.2%	£328,675	£637,066	93.8%	¥27,480,810	¥37,304,150	35.7%
25th pct.	\$561,032	\$884,138	57.6%	£457,336	£742,526	62.4%	¥33,088,478	¥47,345,900	43.1%
Median	\$691,427	\$1,146,812	65.9%	£546,441	£873,400	59.8%	¥42,556,445	¥62,026,139	45.8%
75th pct.	\$922,028	\$1,522,653	65.1%	£649,572	£1,006,454	54.9%	¥58,724,153	¥107,895,235	83.7%

£1,078,129

£1,203,285

47.5%

42.2%

¥89,853,314

¥100,875,394

£730,990

£846,333

17 Nikkei 225 Cohorts (1950–2009)

¥207,352,643

¥290,197,957

130.8%

187.7%



FIGURE 4.1 Performance of the Nikkei 225 (1950–2009)



FIGURE 4.2 Final Retirement Accumulation by Retirement Year for Japanese Cohorts

TABLE 4.2	<b>Years Spent in Each Phase</b>	of Investment
	200/83 Strategy	200/50 Strategy

12.8 years

**14.2** years

17.0 years

8.3 years

7.1 years

28.6 years

Phase 1: Fully leveraged
Phase 2: Partially leveraged

Phase 3: Unleveraged

10,000 Monte Carlo Draws from a Lognormal Stock Distribution Constant % Lifecycle *Improvement* 

TABLE 4.3 Constant % Stock Strategy vs. Mean-Preserving Lifecycle

Stock Strategy over Constant Max. % Inv. 50.0 200.0

\$490,450

\$856,652

\$1,112,747

\$3,523,088

25th pct.

75th pct.

90th pct.

Max. Result

Stock Distribution

Min. % Inv. 50.0 32.1

Mean Result \$711,746 \$711,746 0.0%

St. Dev. \$320,699 \$276,903 -13.7%Min. Result \$130,575 \$159,394 22.1% 10th pct. \$384,025 \$412,317 7.4%

\$516,189

\$849,440

\$1,067,025

\$3,084,903

Mean

St. Dev.

5.3%

-0.8%

-4.1%

-12.4%

6.1%

17.3%

TABLE 4.4 Constant % Stock Strategy vs. Mean-Preserving Lifecycle 10,000 Monte Carlo Draws from a Lognormal Stock Distribution with a Lower Mean

Lower Mean	Cario Draws Irom a	a Lognormai 5to	ck distribution with a
	Constant %	Lifecycle	Improvement
	Stock	Strategy	over Constant
Max. % Inv.	50.0	200.0	
Min. % Inv.	50.0	31.6	
Mean Result	\$544,785	\$544,785	0.0%
St. Dev.	\$234,763	\$204,868	-12.7%
Min. Result	\$109,267	\$126,532	15.8%
10th pct.	\$302,382	\$322,261	6.6%

\$382,266

\$652,709

\$838,548

\$2,521,100

\$399,713

\$648,084

\$810,276

Mean St. Dev.

\$2,258,514

4.6%

-0.7%

-3.4%

-10.4%

4.3%

17.3%

25th pct.

75th pct.

90th pct.

Max. Result

Stock Distribution

TABLE 4.5 Constant % Stock Strategy vs. Mean-Preserving Ramp Down 10,000 Monte Carlo Draws from a Lognormal Stock Distribution with Increased Volatility

Constant % Lifecycle Improvement

	Constant % Stock	Lifecycle Strategy	Improvement over Constant	
Max. % Inv.	50.0	200.0		Ī
Min. % Inv.	50.0	33.0		

\$911,087

\$564,139

\$92,893

\$169,983

\$236,705

\$385,503

\$535,653

\$1,122,598

\$1,589,371

\$7,921,964

Mean

St. Dev.

0.0%

-15.6%

15.2%

10.8%

12.9%

10.8%

8.9%

0.5%

-3.5%

-19.7%

6.1% 25.0%

\$911,087

\$668,540

\$80,625

\$153,436

\$209,694

\$348,005

\$491,757

\$1,116,997

\$1,647,642

\$9,865,224

Mean

St. Dev.

Min.

0.1 pct.

1st pct.

10th pct.

25th pct.

75th pct.

90th pct.

Stock Distribution

Max.

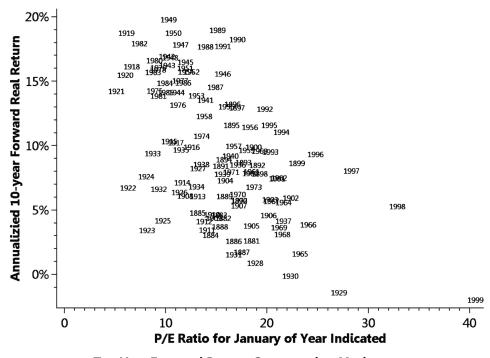


FIGURE 4.3 Ten-Year Forward Return Compared to Market

 	, _ : - : - : - : - : - : - : - : - : - :	
P/E 10	Samuelson Share	

TABLE 4.6 Market P/F 10 and the Associated Samuelson Share

163%

118%

83%

57% 33%

12%

Unadjusted Adjusted *Improvement* Lifecycle Lifecycle Birthday Constant over

Strategy

200

\$1,204,712

\$1,587,704

\$1,939,646

\$2,177,424

TABLE 4.7 Results from 86 Simulated Investors: 1881–2009

% Stock

75

\$711,777

\$944,026

\$1,154,342

\$1,286,285

Rule

90

\$670,886

\$793,996

\$872,311

\$1.026.903

Max. % Inv.

Median

75th pct.

90th pct.

Max. Result

Min. % Inv.	50	75	83	0	_	_	_
Mean Result	\$671,239	\$771,214	\$1,296,998	\$1,639,374	144.2%	112.6%	26.4%
Min. Result	\$351,550	\$390,988	\$611,774	\$611,515	73.9%	56.4%	0.0%
10th pct.	\$465,921	\$476,942	\$768,404	\$903,884	94.0%	89.5%	17.6%
25th pct.	\$564,012	\$587,319	\$1,010,702	\$1,086,915	92.7%	85.1%	7.5%

Strategy

200

\$1,365,441

\$1,963,838

\$3,205,714

\$3,522,336

Birthday

103.5%

147.3%

267.5%

243.0%

*Improvement* 

over

Constant

91.8%

108.0%

177.7%

173.8%

*Improvement* 

over Unadjusted

13.3%

23.7%

65.3%

61.8%

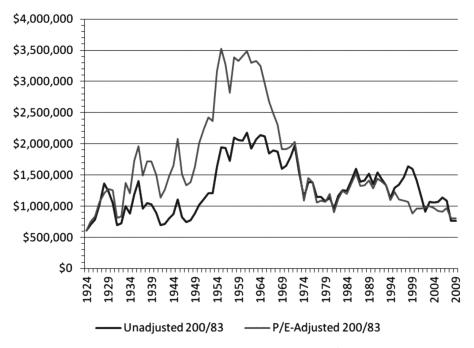


FIGURE 4.4 Final Retirement Accumulation by Year for P/E-Adjusted vs. Unadjusted Strategies

TABLE 5.1 Switching from Birthday Rule to Lifecycle Strategy Later in Work Life

irthday Life Rule Str :46,575 \$1,22 :90,310 \$38 :16,253 \$70 :39,343 \$88 :41,555 \$1,14 :79,044 \$1,52 :70,921 \$1,92	ecycle o ategy Birth 23,105 88,37,172 3,01,834 6,34,138 6,812 7,22,653 9,29,577 12	3.4% 8.6% 3.9% 8.8% \$ 5.5% \$ 1.1.6% \$	Lifecycle for Last 40 Years 51,200,075 \$355,710 \$696,726 \$869,564 51,139,359 51,443,402 51,887,323	Improvement over Birthday  85.6% 22.5% 67.4% 61.2% 77.6% 85.3% 116.7%	Lifecycle for Last 35 Years \$1,128,916 \$368,754 \$719,691 \$825,371 \$1,064,119 \$1,381,705 \$1,771,132	Improvement over Birthday 74.6% 27.0% 72.9% 53.0% 65.9% 77.4% 103.4%
Rule         Str           46,575         \$1,22           90,310         \$38           16,253         \$70           39,343         \$88           41,555         \$1,14           79,044         \$1,52           470,921         \$1,92	rategy     Birt       23,105     8       37,172     3       31,834     6       34,138     6       46,812     7       22,653     9       29,577     12	thday \$ 9.2% \$ 3.4% 88.6% 83.9% \$ 5.5% \$ 11.6% \$	40 Years 51,200,075 \$355,710 \$696,726 \$869,564 51,139,359 51,443,402 51,887,323	85.6% 22.5% 67.4% 61.2% 77.6% 85.3%	35 Years \$1,128,916 \$368,754 \$719,691 \$825,371 \$1,064,119 \$1,381,705	74.6% 27.0% 72.9% 53.0% 65.9% 77.4%
\$1,22 90,310 \$38 16,253 \$70 39,343 \$88 41,555 \$1,14 79,044 \$1,52 70,921 \$1,92	23,105 8 37,172 3 01,834 6 34,138 6 46,812 7 22,653 9 29,577 12	9.2% \$ 3.4% 8.6% 93.9% 88.8% \$ 95.5% \$ 11.6% \$	\$1,200,075 \$355,710 \$696,726 \$869,564 \$1,139,359 \$1,443,402 \$1,887,323	85.6% 22.5% 67.4% 61.2% 77.6% 85.3%	\$1,128,916 \$368,754 \$719,691 \$825,371 \$1,064,119 \$1,381,705	74.6% 27.0% 72.9% 53.0% 65.9% 77.4%
90,310 \$38 16,253 \$70 39,343 \$88 41,555 \$1,14 79,044 \$1,52 70,921 \$1,92	37,172 3 01,834 6 34,138 6 46,812 7 22,653 9 29,577 12	3.4% 8.6% 3.9% 8.8% \$ 5.5% \$ 1.1.6% \$	\$355,710 \$696,726 \$869,564 51,139,359 51,443,402 51,887,323	22.5% 67.4% 61.2% 77.6% 85.3%	\$368,754 \$719,691 \$825,371 \$1,064,119 \$1,381,705	27.0% 72.9% 53.0% 65.9% 77.4%
\$16,253 \$70 \$39,343 \$88 \$41,555 \$1,14 \$79,044 \$1,52 \$70,921 \$1,92	01,834 6 84,138 6 86,812 7 22,653 9 29,577 12	88.6% 63.9% 68.8% \$ 15.5% \$ 11.6% \$	\$696,726 \$869,564 51,139,359 51,443,402 51,887,323	67.4% 61.2% 77.6% 85.3%	\$719,691 \$825,371 \$1,064,119 \$1,381,705	72.9% 53.0% 65.9% 77.4%
39,343 \$88 41,555 \$1,14 79,044 \$1,52 70,921 \$1,92	34,138 6 46,812 7 22,653 9 29,577 12	33.9% (8.8% \$ (5.5% \$ (1.6% \$	\$869,564 51,139,359 51,443,402 51,887,323	61.2% 77.6% 85.3%	\$825,371 \$1,064,119 \$1,381,705	53.0% 65.9% 77.4%
\$1,14 79,044 \$1,52 \$70,921 \$1,92	16,812 7 22,653 9 29,577 12	8.8% \$ 95.5% \$ 11.6% \$	51,139,359 51,443,402 51,887,323	77.6% 85.3%	\$1,064,119 \$1,381,705	65.9% 77.4%
79,044 \$1,52 70,921 \$1,92	22,653 9 29,577 12	5.5% \$ 1.6% \$	51,443,402 51,887,323	85.3%	\$1,381,705	77.4%
70,921 \$1,92	29,577 12	1.6% \$	51,887,323			
				116.7%	\$1,771,132	103.4%
26,903 \$2,17	77,424 11	2.0% \$	2 227 200			
		,	52,237,399	117.9%	\$2,378,396	131.6%
	200		200		200	
Swi	itch to		Switch to		Switch to	
Life	ecycle Impro	vement	Lifecycle	Improvement	Lifecycle	Improvement
foi	r Last o	ver	for Last	over	for Last	over
30	Years Birt	thday	25 Years	Birthday	20 Years	Birthday
\$1,04	14,939 6	1.6%	\$972,221	50.4%	\$898,173	38.9%
\$38	38,864 3	4.0%	\$399,330	37.6%	\$336,199	15.8%
\$63	5,924 5	2.8%	\$563,407	35.4%	\$497,530	19.5%
\$77	78,659 4	4.4%	\$676,065	25.4%	\$591,064	9.6%
	\$38 \$63	\$388,864 3 \$635,924 5	\$388,864 34.0% \$635,924 52.8%	\$388,864 34.0% \$399,330 \$635,924 52.8% \$563,407	\$388,864 34.0% \$399,330 37.6% \$635,924 52.8% \$563,407 35.4%	\$388,864       34.0%       \$399,330       37.6%       \$336,199         \$635,924       52.8%       \$563,407       35.4%       \$497,530

Median	\$909,276	41.7%	\$797,963	24.4%	\$811,275	26.5%
75th pct.	\$1,180,967	51.6%	\$1,175,825	50.9%	\$1,086,152	39.4%
90th pct.	\$1,676,114	92.5%	\$1,691,259	94.2%	\$1,541,586	77.0%
Max. Result	\$2,338,557	127.7%	\$2,129,773	107.4%	\$1,943,502	89.3%
Avg. % Inv. at Switch	191		152		121	
	Switch to Lifecycle for Last 15 Years	Improvement over Birthday	Switch to Lifecycle for Last 10 Years	Improvement over Birthday	Switch to Lifecycle for Last 5 Years	Improvement over Birthday
Mean Result						
	\$825,553	27.7%	\$755,535	16.9%	\$698,956	8.1%
Min. Result	\$825,553 \$299,142	27.7% 3.0%	\$755,535 \$296,963	16.9% 2.3%	\$698,956 \$287,949	8.1% -0.8%
Min. Result 10th pct.						
	\$299,142	3.0%	\$296,963	2.3%	\$287,949	-0.8%
10th pct.	\$299,142 \$454,665	3.0% 9.2%	\$296,963 \$431,217	2.3% 3.6%	\$287,949 \$418,161	-0.8% 0.5%
10th pct. 25th pct.	\$299,142 \$454,665 \$571,529	3.0% 9.2% 6.0%	\$296,963 \$431,217 \$564,212	2.3% 3.6% 4.6%	\$287,949 \$418,161 \$525,010	-0.8% 0.5% -2.7%
10th pct. 25th pct. Median	\$299,142 \$454,665 \$571,529 \$751,761	3.0% 9.2% 6.0% 17.2%	\$296,963 \$431,217 \$564,212 \$712,307	2.3% 3.6% 4.6% 11.0%	\$287,949 \$418,161 \$525,010 \$679,321	-0.8% 0.5% -2.7% 5.9%
10th pct. 25th pct. Median 75th pct.	\$299,142 \$454,665 \$571,529 \$751,761 \$989,416	3.0% 9.2% 6.0% 17.2% 27.0%	\$296,963 \$431,217 \$564,212 \$712,307 \$917,150	2.3% 3.6% 4.6% 11.0% 17.7%	\$287,949 \$418,161 \$525,010 \$679,321 \$848,698	-0.8% 0.5% -2.7% 5.9% 8.9%

200% Stock, 100% Stock, and 100% Bonds from Birth to Age 22 and Following a Diversifying 200/83 Strategy Thereafter

\$500 \$500 \$500

Inheritance Inheritance Improvement Inheritance Improvement

TABLE 5.2 Impact of \$500, \$1,000, and \$5,000 Inheritance Invested at

	\$500	\$500		\$500	
	Inheritance at 100%	Inheritance at 100%	Improvement over 100%	Inheritance at 200%	Improvement over 100%
	Bonds	Stock	Bonds	Stock	Bonds
Mean Result	\$1,378,031	\$1,410,573	2.4%	\$1,490,899	8.2%
Min. Result	\$727,570	\$736,572	1.2%	\$733,416	0.8%
10th pct.	\$844,958	\$859,045	1.7%	\$939,261	11.2%
25th pct.	\$1,087,263	\$1,106,721	1.8%	\$1,161,063	6.8%
Median	\$1,327,069	\$1,352,427	1.9%	\$1,457,633	9.8%
75th pct.	\$1,655,024	\$1,729,245	4.5%	\$1,803,963	9.0%
90th pct.	\$1,968,761	\$1,981,700	0.7%	\$2,167,269	10.1%
Max. Result	\$2,220,573	\$2,265,918	2.0%	\$2,991,801	34.7%
	\$1,000 Inheritance	\$1,000 Inheritance	Improvement	\$1,000 Inheritance	Improvement
	at 100%	at 100%	over 100%	at 200%	over 100%
	Bonds	Stock	Bonds	Stock	Bonds
Mean Result	\$1,400,825	\$1,464,596	4.6%	\$1,620,486	15.7%
Min. Result	\$760,323	\$773,853	1.8%	\$769,655	1.2%

\$904,163

\$1,152,625

\$1,424,139

\$1,802,864

\$2,039,670

\$2,345,004

\$5,000

Inheritance

at 100%

Stock

\$1,852,215

\$1,133,555

\$1,419,360

\$1,705,009

\$2,284,061

\$2,597,872

\$3,237,877

\$913,078

5.2%

5.8%

4.7%

8.6%

3.2%

3.9%

*Improvement* 

over 100%

Bonds

18.1%

17.5%

16.3%

20.6%

8.6%

19.5%

17.0%

28.8%

\$859,708

\$1,089,626

\$1,360,025

\$1,659,814

\$1,976,648

\$2,256,302

\$5,000

Inheritance

at 100%

Bonds

\$1,568,369

\$776,811

\$974,506

\$1,177,094

\$1,570,632

\$1,911,256

\$2,220,743

\$2,514,668

\$994,446

\$1,183,069

\$1,552,222

\$1,949,127

\$2,297,024

\$3,902,278

\$5,000

Inheritance

at 200%

Stock

\$2,445,979

\$1,240,896

\$1,473,687

\$2,086,440

\$2,768,188

\$4,688,804

\$9,945,576

\$889,470

15.7%

8.6%

14.1%

17.4%

16.2%

73.0%

*Improvement* 

over 100%

Bonds 56.0%

14.5%

27.3%

25.2%

32.8%

44.8%

111.1%

295.5%

10th pct.

25th pct.

Median

75th pct.

90th pct.

Max. Result

Mean Result

Min. Result

10th pct.

25th pct.

Median

75th pct.

90th pct.

Max. Result



FIGURE 5.1 Optimal Stock Allocation by Age from Gomes, Kotlikoff, and Viceira

Ford

MIT

Yale

Rockefeller\*\*

2008
Endowment
Value

\$10.87

\$10.07

\$22.87

\*Assumed annual risk-free interest rate of 2.56%.

\$4.10

TABLE 5.3 Endowments (in Billions)

2008

Contributions

and Pledges

\$0.00

\$0.00

\$0.39

\$0.37

\*\*2008 endowment value was unavailable for Rockefeller; \$4.1 billion is the 2007 value.

Present Value

of Future

Contributions

and Pledges\*

\$0.00

\$0.00

\$15.08

\$14.45

2008

Present &

Future

Endowment

\$10.87

\$4.10

\$25.15

\$37.32

Stock

Allocation

(50%

Target)

50.0%

50.0%

124.9%

81.6%

%

Increase

0.0

0.0

149.7

63.2

Stock

Allocation

(83%

Target)

83.0%

83.0%

219.8%

143.6%

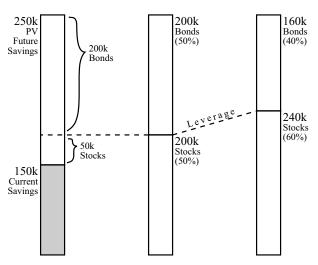


FIGURE 6.1 Adjusting Leverage for Human Capital

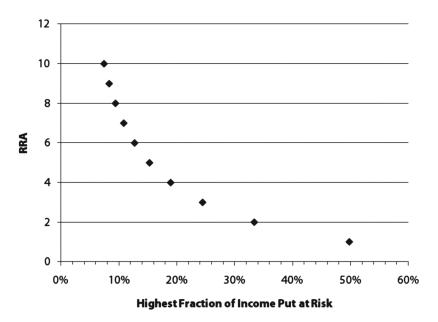


FIGURE 7.1 RRA as a Function of Willingness to Put Income at Risk

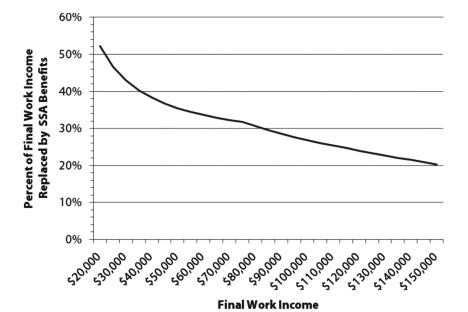


FIGURE 7.2 Social Security Benefits as a Percentage of Final Income

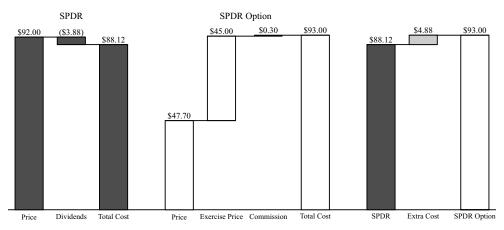


FIGURE 8.1 Andrew's Implied Cost of Borrowing

A corrected version of this figure is available here.