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History of Mathematics from Antiquity to 17th Century

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Leibniz versus Newton

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Abstract

Leibniz and Newton are usually considered as the inventors of calculus. During the 17th century, a huge controversy disrupted the scientific world. In fact, for their contemporaries, it was not clear who invented calculus between Leibniz and Newton. And most importantly, the question was "who has copied who". Nowadays, there is no longer dispute about this. Thanks to their manuscripts, we know that their approaches were totally different and that no one has copied the other. But still, this dispute generated great tensions during decades. In this essay we cover their discoveries, compare their dispute with the one between Tartaglia and Cardanno-Ferrari and elucidate an extract from 'The History of Mathematics, a very short introduction' written by Jacqueline Stedall on this topic. We will more about the characteristics of a mathematical dispute and also the role of mathematical historian.

1 Introduction

In the history of mathematics, the question *"Who was first .. ?"* has always been examined with precision. It is very hard to determine exactly who was the first to invent something new or come up with an idea. From algebra to calculus through geometry, numbers of argument broke out between different inventors or between historians themselves. Over time, problems as calculating the lengths, areas or volume but also tangents, normals or curvature have been fundamental research subjects for centuries. With the growth of analytic geometry in the 16th century, the understanding of notion as integration and differentiation was inevitable. It was the beginning of calculus. In this essay we will focus on the famous dispute between Leibniz and Newton about calculus which erupted in the late 17th century. Firstly we will introduce both Leibniz and Newton as well as their work on calculus in order to understand why an argument broke out between those two mathematicians. We will see that with different approaches and notations, they almost led to the same discovery. In a second part, we will elucidate an extract from 'The History of Mathematics, a very short introduction' written by Jacqueline Stedall on this topic (cf. Annex). Finally, we will study the case of Cardano-Ferrari versus Tartaglia in order to determine whether or not the controversy between Leibniz and newton is different or not and in what way. We will alternatively use primary and secondary sources in order to stress our points and being as rigorous as possible in our approach.

2 Leibniz's and Newton's work on calculus

In this section we will present both Leibniz's and Newton's life in details as well as their work on calculus. It is fundamental to know who they are and their discoveries in order to understand the dispute that disrupt the mathematical world for decades. We will observe the difference and the similarities of their researches but also the historical context in which they developed calculus. And to underline the importance of their research, we will quote Betoloni Meli in Domenico [8] :

> The establishment of a new, highly abstract, and general form of algebra, the formulation of analytic geometry, and the creation of a variety of techniques for finding maxima, minima and tangents, paved the way to the great inventions by Newton and Leibniz. Their calculuses can be seen as the starting point of a new phase of mathematical research.

2.1 Gottfried Wilhelm Leibniz

Gottfried Wilhelm Leibniz, allias Leibniz, can be considered as one of the greatest thinker of the 17th and 18th centuries. Born in Leipzig in 1646 and died in 1716, Leibniz was a German philosopher, mathematician, scientific logician and more. His contribution to the field of logic, philosophy, metaphysic, history but also mathematics, physics and geology made him a universal genius of his century. His greatest accomplishment was without any doubts the development of differential and integral calculus. Leibniz was a precocious child and lost his dad when he was 6. He taught himself Latin at the age of 8 and Greek at 12 using his father's library. At the age of 15, Leibniz became a student at the university of Leipzig, his native city. At Leipzig, he received a traditional and conservative education, mostly based on philosophy and religion. After his graduation, he went to Jena in order to improve his mathematical knowledge. He finally went back to Leipzig to concentrate on legal studies and obtain his master degree. In 1667 he became Doctor of Laws at the university of Nuremberg-Altdorf. After declining a professor position at the university of Alrdorf, he accepted a position at the court of appeal of Mainz. It is only in 1672, when moving to Paris, that Leibniz began his mathematical journey. At this time, Paris was considered as the intellectual capital of Europe. During his journey which lasted 4 years, he came in contact with some of the greatest scholars of his time, especially the Dutch scientist Christiaan Huygens. Huygens became the mentor of Leibniz and helped him to improve his mathematical instruction. This is during his journey in Paris than Leibniz developed the principal features and notations of his version of calculus. After his journey in Paris, Leibniz became the librarian and councilor to Duke Johann Friedrich of Hannover from 1676 to 1679 and to Duke Ernst August from 1680 to 1690. He was also a court historian to Duke Ernst August until his death in 1698. From this time, he became historian and librarian to the new Elector, Georg Ludwig, that became King George I of England. Leibniz ends his life in Hannover, where he died in 1716. He never came back to Paris after his 4 years journey between 1672 and 1676 but he continued his work on calculus even after he left and published 3 main articles on calculus that we will cover later in this essay.[10][3]

2.2 Leibniz differential calculus

As mentioned, Leibniz's work on differential calculus is probably the greatest achievement of his life. At that time, some mathematical problems were about determining lengths, areas, volume but also about determining the tangent or normal lines to certain class of curvatures. If different methods were already in place to determine the tangent of curvature, not one was able to determine the equation of the curvature from its tangent. Another problem was computing the area under a curvatures. This problem was the beginning of the invention of calculus by Leibniz. Only 1 year after moving in Paris, in 1673, Leibniz affirmed that the problem of the inverse method of tangents was reducible to quadrature. He first expressed the tangent as a ratio between ordinate and abscissa. His starting point of calculus was based on the fact that a curve could be seen as a polygon with infinitely many sides and he used that to set up computational rules. From this idea he stated that the integral was the sum of an infinite number of rectangle eq. the sum of the ordinates for infinitesimal intervals in the abscissa. It was the first time someone used integral to compute the area under the graph of a function $f(x) = y$. It is from this remark that the problem of the inverse (or differential) became clear to Leibniz. It led Leibniz to a whole new mathematical system and notation. It is in 1675, in Leibniz's notebook, that that we can find his discoveries and experiments about integral and differential. But most importantly this is in this manuscript that we found for the first time the notation of infinitesimal increments for both abscissa and ordinate $dydx$ and the well known notation of integral $\int$. He expressed the magnitude as the composition of infinitesimal parts .The integral notation in form of an 'S' comes from the fact that according to Leibniz, the integral simply refereed to a sum. And as mentioned earlier, the area under a curve could be written as a sum of infinitesimal intervals in the abscissa. Combining both notation he wrote the area under a function $f(x) = y$ as follow : $\int f(x)dx$. Theses symbols are still use nowadays by modern mathematics. The first official publication Leibniz on calculus was published in 1684 and titled "Nova methodus pro maximis et minimis, itemque tangentibus, quae nec fractas nec irrationales quantitates moratur, et singulare pro illis calculi genus", or "A new method for maxima and minima, and for tangents, that is not hindered by fractional or irrational quantities, and a singular kind of calculus for the above mentioned" in English. This article was the definitive exposition of his differential calculus. After this he published 2 other articles about calculus. One titled "De geometria recondite et analysi indivisibilium atque infinitorum" (On a hidden geometry and analysis of indivisibles and infinite)in 1686 and the other "Supplementum geometriae dimensoriae, seu generalissima omnium Tetragonismorum effectio per motum: similiterque multiplex constructio lineae ex data tangentium conditions" (A Supplement to the Geometry of Measurements, or the Most General of all Quadratures to be Effected by a Motion: and likewise the various constructions of a curve from a given condition of the tangent) in 1693. In the first one, Leibniz gave a proof of the Fundamental Theorems of Calculus. This theorem states that if f is continuous on the closed interval $[a, b]$ and F is the indefinite integral of f on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$. Leibniz was also aware, as Newton and Barrow, that integration is the 'inverse' of differentiation : $\frac{d}{dx} \int f(x)dx = f(x)$. In his last publication on calculus he showed that the general problem of quadrature can be reduced to finding a curve that has "a given law of tangency.". The general problem of definite integration can be reduced to finding a function that has a given derivative (that is, finding an anti-derivative function) which is essentially the Fundamental Theorem of Calculus. The wish of Leibniz of integrate rational function (polynomial and quotient) led to the question of factorization of polynomials. It stimulated the study of what became know as the main theorem of algebra.[1][10][3][7][13][12] [8]

2.3 Sir Isaac Newton

As Leibniz, Isaac Newton is considered as a universal genius of his time. Born in 1642 and died in 1727, Newton was an English physicist, mathematician, philosopher, alchemist and more. Newton is well known for having established classical mechanics with concepts such as universal gravitation, the laws of motion and more. He also made huge contributions to the field of optic. For example he built the first reflecting telescope, discovered the composition of white light etc. And of course, he shares credit with Leibniz for his work on calculus. Newton is considered as a great thinker of the 17th century and was one of the most important figure of the Scientific Revolution that took part at the same time. Newton grew up in the country of Lincolnshire, in Woolsthorpe-by-Coldsterwoth. His father died 3 months before his birth. He was the only child of his family. He grew up with his grandmother after his mother remarried and went to live with her new husband. From twelve to seventeen, Newton attended the King's School in Grantham where he was taught Latin and Greek but also basic science as mathematics. After this period, Newton went back to Woolsthorpe and his mother tried to turn him into farmer, which he definitely did not want to. In 1661, Isaac was admitted to Trinity College, Cambridge. This is here, at Trinity, that Newton was taught about mathematics, physics and philosophy. He learned a lot about Kepler's work, Galilelo's work but also more ancient figures of the scientific world as Aristotle. Passionated about modern philosophy, he even wrote some note titled "Quaestiones Quaedam Philosophicae" ("Certain Philosophical Questions") during his journey at Trinity. When the Great Plague shuttered Cambridge in 1965, Isaac Newton decided to go back home. Isolated from almost everything during two years, Newton spent almost all this time developing his feature discoveries on gravity, laws of motion and calculus. He wrote about this period : "All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention and minded Mathematics and Philosophy [physics] more than at any time since.". During the second part of his life, Newton was a Lucasian Professor at Cambridge until 1687. It was Newton's Golden Years. He produced most of his work in mathematics during this time. For example, in 1687 Newton published his most famous book : "Philosophiæ Naturalis Principia Mathematica", usually refereed to as simply the "Principa". This is in this publication that Newton stated some of his most important discoveries on classical mechanic. It is often considered as on of the most important works in the history of science. Finally he ended his career as a government official, and published books on optic and arithmetic. Especially he was the Director of the Mint and the President of the Royal Society for a while. In this essay we focus on his work on calculus. But Newton never completed definitive publication on his fluxional calculus. The book "Method of Fluxions", has been published posthumously in 1736, even if most of the content has been written by Newton in 1671 (even before Leibniz went to Paris). Now that we know more about Sir Isaac Newton,we will focus on Newton's method of fluxions. [5][3]

2.4 Newton's method of fluxions

Newton's fascination about mathematics started while he was studying at Cambridge. Isaac Borrow, a professor from Cambrdige, quickly saw the potential of Newton and helped him catching up the modern theories of mathematics. One year before the beginning of the Great plague, Newton made his first huge contribution to the mathematics by advancing the binomial theorem. In fact, Newton generalized the binomial theorem to allow real exponents other than non-negative integers. It gave him a method to compute derivatives and integrals of powers of a variable, and thus polynomials and infinite series. His reasoning and discovery on calculus started from there. The first written conception of fluxionary calculus was recorder in a paper titled "De Analysi per Aequationes Numero Terminorum Infinitas" written in 1665 but published way later in 1711. At this time, as for Leibniz, Newton was intrigued by the problems of tangent and quadrature. In his paper of 1665, Newton gave a method to determine the area under a curve. Where Leibniz remarked that a curve could be seen as a polygon with infinitely many sides and he uses that to set up computational rules; Newton started by calculating a momentary rate of change and then extrapolating the total area. Using infinitely small triangle - whose area can be compute using y and x (ordinate and abscissa) - where he increased the abscissa value by and infinitesimal, Newton created an expression for the area under a curve by considering a momentary increase at a point. Compared to Leibniz reasoning, Newton's one is way more based on physic's concepts. But at this time the notion of infinitesimal was still unclear and contradicted. Newton knew it and found a way to avoid the use of the infinitesimal by using an analogy with moving object. He used the notion of motion in order to justify variable magnitudes. From this idea he defined the rate of generated change as a fluxion. The notation of a fluxion was represented by a dot over the fluent. The fluent is simply a time-varying quantity or variable. Newton expressed the magnitude as a variable depending on time (σ). So for a fluent x, we can see the fluxion $\dot{x}$ as the speed of the fluent. More over, the second fluxion is denoted with 2 dot as follow : $\ddot{x}$; and could be seen as the acceleration of the fluent. Theses notions could be find into the "De Methodis Serierum et Fluxionum" (On the Methods of Series and Fluxions) that Newton wrote in 1671, arranged until 1676 but finally did not published. It is only in 1736 that the book was published (and translated to English), but the notation did not stand except in England. In reality the method of fluxion was not essentially different from the one used in his first publication on that subject ("De Analysi per Aequationes Numero Terminorum Infinitas"). Newton just changed his approach. He abandoned infinitesimal in favor of the theory of fluxion. Finally, this theory came to rest on prime and ultimate ratio in the last book on Calculus written by Newton : De Quadratura Curvarum (1704). Even if this book was the last one written, it is the first published by Newton. Newton's calculus had several applications already at this time. He found out that the maxima or minima of a function can be fond by finding the time at which the fluxion is equal to 0. He stated : "when a quantity is greatest or least, at that moment its flow neither increases nor decreases,...". Of course it also helped him solving the original problem that led him to this discovery, the tangent problem. As Leibniz, Newton also knew that differentiation and integration are 'inverses', and knew the relation with areas.[5][3][12][13][1][2] [8]

3 Text commentary

In the first part of this essay we learned more about Leibniz's and Newton's life as well as their discoveries on differential calculus and the method fluxion respectively. We will now elucidate an extract from 'The History of Mathematics, a very short introduction' written by Jacqueline Stedall. In order to perform this analysis we will follow the schema learn in class. You can find a copy of this extract in the Annex.

The extract we will elucidate is taken from an handbook titled "The History of Mathematics: A Very Short Introduction" written by Jacqueline Stedall and published by the Oxford University Press in 2012. Jacqueline Stedall is a prolific author and a respected historian. By mean of this book, the author try to 'recognize the richness and diversity of mathematical activity throughout human history', as she mentioned it on the preface of the book. Now, let's investigate in greater details the subject of the extract. As it is mentioned in the first paragraph of the extract, the subject of the text is the invention of Calculus and more precisely 'Who invented calculus?'. And the invention of calculus goes hand in hand with one of the greatest mathematical clash of all time between Newton and Leibniz. In order to integrate the historical background of this text we need to highlight two parts of the clash. The first part of the clash started with the invention of Calculus by Newton and Leibniz in the second part of the 17th century. The second part of the clash is the consequence of this dispute from the end of the 17th century to the beginning of the 19th one. In fact, the clash results in two centuries of confusion for both historians and mathematicians. Note that the invention of calculus took place during the Scientific Revolution which happened in Europe during the 16th and 17th centuries. We now want to elucidate what is the thesis of the text, what does the author wants to demonstrate by mean of this text. In my opinion the author wants to show 3 main ideas in this extract. The first two things are especially linked to the dispute between Newton and Leibniz. She wants to show us that nowadays it is clear to modern historians that there is no dispute about the invention of calculus as we now have all the necessary resources to demonstrate the work of both protagonist in their own way. But she also wants us to understand why this dispute happened, what were the issues at this time that led theses mathematicians to one of the greatest mathematical war of all time. Finally, she also wants us to see the big picture and understand why answering the question of "Who invented first...?" is really problematic and difficult and that the dispute between Newton and Leibniz is not an isolated case.

In the first part of the extract, the author shows us why the dispute between Newton and Leibniz is no longer one nowadays. In fact she informs us that we now considered that both of them invented calculus 'almost simultaneously but independently'. We learn that they both work from different country, Leibniz in Paris and Newton in Cambridge. She also mentions that we now have access to all resources simultaneously and that we can 'see exactly when and in what order their ideas were developed'. Finally she gives us some insight on their approaches (which where different) and also on the vocabularies and notation they used (which were also different).

After that, she explains why it was not clear in the past who invented calculus and why the dispute took place. She states that Newton made his discoveries some years before Leibniz, but did not published them or even did anything with it. To justify Newton's behavior she says that Newton was already in the middle of a controversy with Robert Hooke and he probably did not want to risk having another dispute at the same time. On the other hand, she says that Leibniz worked on the same problem that intrigued Newton and came up with the first publication on this topic in 1684. It is from this publication that the dispute began. She mentions the fact that it was not the two mathematicians but their friends and supporters who actually initiated the dispute and were fighting against each other in order to give credit for the invention of calculus to their respective favorite genius. In fact it was more a dispute between England and the continent than between Newton and Leibniz. She informs us that the main argument of Newton's friends was that Leibniz had seen some Newton's papers during a visit in London and also received letters from himself during the period he was working on differential calculus. Finally she still shows us that Leibniz tried to defend his case by appealing the Royal Society but Newton was the actual President of the Society at this time and so the trial was not fair and Newton took credit of the invention of calculus. She illustrates this using the case of George Peat, almost 100 hundreds after the invention of Calculus, learned a subject called 'fluxions'rather than a subject called 'calculus' at the university of Cumbria, England. She also explains that the main reasons this dispute happened are because no one at this time was in possession of all the resources. Also it was not clear if the dispute was about the whole theory of calculus or just some parts of it that one would have copy from the other. And finally the last reason was that a lot of disagreements were not even related to the invention of calculus itself. She also mentions the fact that some of the most necessary resources needed to determine who invented calculus - the mathematical manuscript - were either not available or not used to solve the dispute. In fact, arguments used related more of what people said and wrote than to the manuscript themselves. She uses the word 'partial' with two meaning, the first one is to demonstrate that what people said was not complete and the second meaning is that their comments were subjective and not impartial.

The second part of the extract tries to map the example of Newton and Leibniz war to a more general opinion. She states 'In mathematics it is not at all uncommon, as in the case of the calculus, for two people to come up with similar ideas at more or less the same time.' and then back ups her words with some examples. She explains that once the groundwork has been done it is very easy for someone to use it and claimed his due too. It leads to an impassibility to decided who really invented what. She even illustrates her opinion by exposing the case of Andrew Wiles, which hid his work on Fermat's Last Theorem for years in order to protect himself from any potential dispute/steal. By using this modern example, she demonstrates that no matter the time, the problem of 'Who invented first ...' is still relevant today in the 21th century. She also takes the example of Bolzano and Cauchy which both came up with similar work at the almost same time during the 19th century. But in this case, due to the lack of documentary evidences for historian to work out, no one could say which of these two mathematicians was the inventor. She demonstrates with 3 cases that took place at 3 different time of history that the question of 'Who invented first...?' went through all ages and portrayed different outcomes depending of the resources available and the comportment of each protagonist. Finally, at the end of the last paragraph she gives us one last general reason of why the question "Who invented first..." is really challenging. The problem comes from the definition of a discovery/invention itself. She states the fact that it is hard in mathematics to determine the precise point in history an invention became formal and complete, and not just a tool for other discoveries. It is especially true in mathematics and basic science where every inventions is based on previous works. That is why she concludes her paragraph telling the task of an mathematical historian is more to understand how mathematical changed over time than when.

To conclude on this extract I would say that in my opinion the author does not try to once more explain the dispute between Newton and Leibniz as it has been done several times by different authors. Instead, I think she uses this dispute in order to illustrate the 'universal' character of their dispute which is just another dispute among others with all the same similar schema. She also highlights the difficulties of the work of an historian and also the importance of understanding why and how, more than when.

4 A remake of the Cardano-Ferrari & Tartaglia dispute?

In this last section we will talk about the dispute between Cardano-Ferrari and Tartaglia. Our main objectives are to first determine in what ways this dispute differs from the one between Newton and Leibniz and secondly observe if we can map the analysis of Jacqueline Stedall about disputes to this one.

We will not go as deep as for Newton and Leibniz but still, we will try to give a brief overview of the historical context and the life of the three protagonist Cardano, Ferrari and Tartaglia

4.1 Cardano, Ferrari and Tartaglia

Gerolamo Cardano born in 1501 and died in 1576 was an Italian mathematician, physician, biologist and more. He is one of the key figure of the field of probability. He also practiced medicine and was recognized for his competence in all the modern world. After a rough life full of pitfalls, he finally received a lifetime annuity in Rome thanks to Pop Gregory XIII. Lodovico Ferrari was also an Italian mathematician. According to the Oxford Dictionary of the Renaissance, Ferrari was the servant of Cardano for a while. Cardano started teaching him mathematics while Ferrari was still a servant. From one hand to another, Ferrari became the student of Cardano. They work together on many themes, especially the one of the dispute : solutions for cubic equation. Thanks to his devoted professor and his intelligence, Ferrari obtained a prestigious teaching post after his own professor resigned from it and refereed him for the post. Finally we will quickly describe Tartaglia. Once more, Niccolò Fontana Tartaglia was an Italian mathematician. Known for his work on ballistic, the Tartaglia's triangle which is the ancestor of Pascal's triangle or in geometry; Tartaglia is considered as a great scientific of the Renaissance. There is one subject who links these 3 famous mathematicians together, cubic equation. [9][6][4][3]

4.2 The dispute

As mentioned, the dispute between Cardano-Ferrari and Tartaglia is related to the solution of cubic equation. As a reminder, a cubic equation is an equation of the form: $ax^3 + bx^2 + cx + d = 0$. The first big step in the resolution of cubic equation has been done by another Italian mathematician named Scipione del Ferro who found a solution for the special cubic equation $x^3 + ax = b$ in 1510-1515. Even if it is a good start, we do not have a general solution and this is why we can not considere Scipione as the inventor of solution for cubic equation. Moreover he did not published about his discovery and even if he entrusted his notes to his son-in-law after his death but we do not have any traces of them. The second step in the resolution of cubic question has been done by Tartaglia. In fact, in the context of a mathematical contest against another mathematician named Fiore, Tartaglia found a general method to solve cubic equation of the form $x^3 + ax = b$ with. Moreover he also found a way to resolve the equation of type $x^3 + ax = b$ and $x^3 = ax + b$. We are closer from the general solution but still, we do not have a complete and general method to resolve all cubic equations. Like Scipione, Tartaglia did not shared his discovery neither published it. Finally the third step has been done by Cardanno and his student Ferrari. After a long negotiation, Tartaglia accepted to give his method to Cardano (in form of an crypted poema) only after he promised to never share it to everyone. Unfortunately, in 1545, Cardano published the book "Ars Magna" which contained Tartaglia's solution of the cubic with a statement that Scipione and Tartaglia had each found solutions by independent research. In this book, Cardano also included his own discovery on the subject, especially the fact that every cubic should have three roots which gave a generalization of the solution for cubic equation. He also added the discovery of his student Ferrari which consisted into finding solution to the biquadratic equation eq. quartic function. The dispute started from there. Tartaglia publicly denounced Cardano and ruined his mathematical credibility. Cardano refused to answer but Ferrari, probably because he was younger, defended his and his master's honor. He even challenged Tartaglia into a mathematical debate but Tartaglia refused for a while. A debate finally took place in Milan in 1548 but it is not clear how it ends, we only know that Tartaglie lost his professor position after that. Finally Tartaglia never published his solution on cubic equation and the solution to solve this type of equation is now know as 'Cardano's Formula'.

We can find some differences and similarities in this dispute with the one between Leibniz and Newton and also with the conclusion of Jacqueline Stedall in the previously elucidated extract. First we can see that both Tartaglia and Scipione del Ferro found a way to solve some cubic equation with different methods. Even if it does not concern Cardano we can see the similarities with Newton and Leibniz who both solved the tangent problem with a different approach and solution. One major difference with the Newton versus Leibniz is that Cardano explicitly took the partial solution of Tartaglia in order to develop a more general solution. Cardano even mentioned it in his book "Ars Magna". So here we are not in the same situation than Newton and Leibniz but more in the situation that Stedall describe, that "Almost all new mathematics is built on previous work, and sometimes on a number of contributory ideas.". And this is exactly what happened here, Cardano used del Ferro's and Tartaglia's work in order to build a more general and complete solution. The dispute between Cardano and Tartaglia also differ in a sense that both dispute did not led to the same conclusion. We have seen that modern historian consider that both Newton and Leibniz invented calculus, but in the case of Cardano-Tartaglia it is only Cardano who got the final credit (especially by having his own formula, the "Cardano's Formula"). Moreover, the dispute for the solution of cubic equation was directly led by the protagonist themselves (Ferrari and Tartaglia, with the debate for example), where Newton's and Leiniz's dispute was mostly due to their friends and supporters. [6][3][11][9]

5 Conclusion

In this essay we firstly determined who are Newton and Leibniz and what did they invented. Then, by elucidating the extract from 'The History of Mathematics, a very short introduction' written by Jacqueline Stedall and published by the Oxford University Press in 2012, we examined the characteristic of the dispute between Newton and Leibniz but also the 'universal' character of their dispute. We also learned more about the reason dispute happened in the mathematical world. Moreover we discovered more about the challenges mathematical historian faced on while doing their researches. It is more important to know how an idea was born and came up to the world than who had this idea, especially in mathematics where ideas are build on top of each others. Finally we analyzed the dispute between Cardano-Ferrari and Tartaglia and compared it to the one between Newton and Leibniz. Also we tried to find a relation between Cardana-Ferrari and Tartaglia's dispute and the thesis exposed by Jacqueline Stedall. We were able to find similarities with the Newton versus Leibniz dispute but also some difference. But most importantly we proved the point of view of Jacqueline Stedall about dispute in the mathematical world and the importance of the role of mathematical historian.

6 Annex

Extract from 'The History of Mathematics, a very short introduction' written by Jacqueline Stedall and published by the Oxford University Press in 2012 :

The question we have just examined, 'Who invented algebra?', is typical of those sometimes asked of historians of mathematics, who are often expected to be able to say who was first to discover or invent certain ideas. Except in the simplest cases such questions can be extraordinarily difficult to answer. Take, for example, the discovery of calculus. This is the branch of mathematics that can be used for describing and predicting change. It is used today in biology, medicine, economics, ecology, meteorology, and every other science that works with complex interactive systems. It is therefore not unreasonable to want to know 'Who invented calculus?'

The short answer is that two people did, almost simultaneously but independently: Isaac Newton working in Cambridge and Gottfried Wilhelm Leibniz working in Paris. To modern historians, there is no longer any dispute about this because we have the manuscripts of both men and can see exactly when and in what order their ideas were developed. We can also see that they approached the work in very different ways, each devising their own vocabulary and notation (Leibniz spoke of 'differentials' while Newton spoke of 'fluxions'; Leibniz invented the now familiar notation $\frac{dx}{df}$ whereas Newton used the now less common $\dot{x}$). For their contemporaries, however, the story was not clear at all. The basic facts are that Newton developed his version of calculus during 1664 and 1665 (before his 23rd birthday) but then did nothing with it. By the early 1670s he had already engaged in an intellectual skirmish with Robert Hooke over his optical findings and was perhaps reluctant to risk another over the calculus. In any case, by that time his interest had shifted to alchemy, which was to preoccupy him for the next decade. In 1673, however, Leibniz, then living in Paris, independently began to work on some of the same problems that had earlier intrigued Newton, and published his first paper on calculus in 1684 followed by others in the 1690s. Newton appears to have taken little notice, probably regarding Leibniz's early work as rather trivial compared with what he himself had been able to achieve. Some of Newton's friends felt differently, however, and around the turn of the century his English supporters began to hint not only that Newton had been first but that Leibniz might actually have stolen the seeds of his ideas from Newton. It did not help Leibniz's case that he had seen some of Newton's papers when he was in London in 1675 and had received letters from Newton in 1676, but what he had learned from them, and how that related to what he had discovered already, no-one but Leibniz really knew.

Both Newton and Leibniz held back from direct confrontation but allowed the battle to be fought out through their henchmen, who were thoroughly belligerent on both sides. Eventually, in 1711, Leibniz appealed to the Royal Society, of which he was a member, to adjudicate in the dispute. Newton, as President of the Society, set up a committee which barely needed to meet because Newton was already busy writing its report. Not surprisingly, it found in Newton's favour. And, also not surprisingly, that was not the end of the matter: the dispute rumbled on until after Leibniz's death in 1716. The dispute explains why in 1809 the English schoolboy George Peat in Cumbria learned a subject called 'fluxions' rather than a subject called 'calculus'.

It is an unedifying story from which no-one comes out well. The point of re-telling it is to emphasize how difficult it was for anyone at the time to get to the bottom of it: no single person was in possession of all the facts; besides, it was difficult to know whether the argument was about the calculus as a whole or about particular aspects of it (Leibniz accused the English of shifting ground on this); and, as can be the way with disputes, several disagreements were dragged in that were never part of the original argument. Another point of the story, however, is that the ultimate evidence for the truth comes not from what people at the time wrote or said, which was almost always partial (in both senses of the word), but from the mathematical manuscripts themselves.

In mathematics it is not at all uncommon, as in the case of the calculus, for two people to come up with similar ideas at more or less the same time. Once the groundwork has been laid, one mathematician can make use of it just as easily as another, and it then becomes very difficult to apportion credit, especially if the two have had some contact with each other. It was for precisely this reason that Wiles shut himself away so carefully during his years of work on Fermat's Last Theorem. In the case of the calculus, there is enough documentary evidence for historians to work out what really happened but this is not always so. Two early 19th-century mathematicians, Bernard Bolzano in Prague and Augustin-Louis Cauchy in Paris, also developed some remarkably similar ideas, Bolzano in 1817, Cauchy in 1821. Did Cauchy 'borrow' from Bolzano or not? Bolzano's work was published in a little-known Bohemian journal which was nevertheless available to Cauchy in Paris. On the other hand, both could have built independently on the earlier work of Lagrange. We might also throw into the assessment circumstantial evidence about Cauchy's way of working, which was very often to pick up good ideas from someone else and develop them at length. In the end, for lack of firm evidence either way, we simply cannot say.

Another problem about saying who was first to make a discovery can be defining what we think the discovery actually consists of. At what precise point in history, for example, can we say we have 'calculus', as opposed to a tangle of related ideas that gradually began to make sense first to Newton and later to Leibniz? It is just as difficult, as we have already seen, to pinpoint where algebra began, or where Pythagoras' Theorem became a formal theorem as opposed to a useful fact known to builders. Almost all new mathematics is built on previous work, and sometimes on a number of contributory ideas. Tracing the antecedents of a particular technique or theorem is one of the tasks of the historian, not in order to say who was first, but to understand more clearly how mathematical ideas have changed over time.

Figure 1: Portrait of Leibniz by Christoph Bernhard Francke

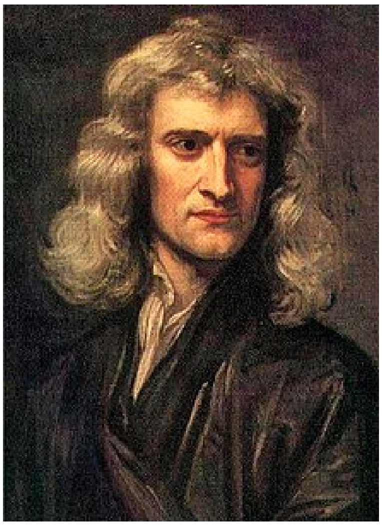

Figure 2: Portrait of Newton by Sir Godfrey Kneller (1689)

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