This assignment consists of two parts, which are described in more detail below. Please write a detailed report, addressing the questions posed below. **Make sure that you interpret all of your results.** Upload your report (in PDF format!) to Canvas before the deadline (which can be found in Canvas) and include your source code as a separate attachment.

In Canvas you can find a helpful document containing more detailed information about what we expect from your report.

Part 1: Verification of some theoretical results

In this part, we ask you to write R programs to answer the following questions.

1. An important result that we need quite regularly in this course, is the following:

$$\sum_{k=0}^{\infty} p^k = \frac{1}{1-p}, \qquad \text{if } |p| < 1. \tag{1}$$

In this exercise, we are interested in the quantity

$$S_m(p) = \sum_{k=0}^{m} p^k,$$

and in particular how well it approximates $\frac{1}{1-p}$. From Equation (1), we can infer that $S_m(p) \to \frac{1}{1-p}$ for $m \to \infty$, given that $|p| < 1$. But how quickly does $S_m(p)$ converge to this value? And does this depend on p? What happens to $S_m(p)$ if $|p| > 1$? Write an R program to answer these questions. In more detail:

- Take several values for p. Be sure to include negative and positive values, inside and outside the interval $(-1, 1)$. Take several values for m and make a table of $S_m(p)$, for each combination of the chosen values for m and p. Include the value of $\frac{1}{1-p}$ in this table and compute the difference. Be sure to include this table, somewhere in your report:

$S_m(p)$	$p=-2$	$p=-1$	$p=-0.5$	$p=0.5$	$p=0.99$	$p=1.01$	$p=2$
$m=0$							
$m=10$							
$m=100$							
$m=1000$							

Table 1: $S_m(p)$ for several values of m and p.

- Include figures where you plot $S_m(p)$ versus m, for several values of p. Include a line indicating the value of $\frac{1}{1-p}$, to get an impression of the speed of convergence.

2. Use *stochastic simulation* to verify the following theoretical results, all of which are described in more detail in the lecture notes “Stochastics for Finance”:

Let $X \sim \mathsf{Pois}(\lambda)$. Verify that

$$\mathbb{P}(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad k \in \{0, 1, \dots, \infty\}, \ \lambda > 0, \tag{2}$$

and $\mathbb{E}[X] = \lambda$, $\mathsf{Var}[X] = \lambda$.

You are *not* allowed to use the functions `ppois`, `dpois`, or `qpois`. Only `rpois` is allowed.

Part 2: Old exam questions

In Canvas you can find old exams for the courses 2DF20. Many questions in these exams can also be answered using stochastic simulation. The following exercise is taken from the exam of October 2018 (and has been slightly modified). Answer these questions *using stochastic simulation*. This means that you are *not* allowed to use functions like `punif`, `dunif` and `qunif` (also for the binomial distribution). You are only allowed to use `runif` and `rbinom`.

3. Let X_i be continuous uniform random variables on $[0, 1]$; $X_i \sim \text{Unif}(0, 1)$.
   (a) Show that $\text{Var}(X_i) = 1/12$.
   (b) Let $n = 1200$. Compute $\mathbb{P}(580 \leq \sum_{i=1}^{n} X_i \leq 620)$.
   (c) Let $N \sim \text{Bin}(3600, 1/3)$. Find the distribution of $S_N = \sum_{i=1}^{N} X_i$.
   (d) Now compute $\mathbb{P}(580 \leq S_N \leq 620)$. Is this probability greater or less than the answer you found in (b)?

Remark: whenever a distribution is asked, please draw a histogram, and provide the mean and variance.