SIMULATION OF ACOUSTIC WAVES PROPAGATION IN CLOSED ROOMS

Augustin Carteron
CMI (Formation Cursus de Master en Ingénierie, Mechanical Engineering) - Second Year - Sorbonne Université

May 10, 2023

1 Introduction

The subject of acoustics is a field in mechanical engineering that I find particularly interesting. Concerning one of the courses that focuses on numerical simulation, I wanted to mix passion and interest: sound and physics. At Sorbonne Université, with certain subjects, we tend to mix theory with practice. In this case, we combine programming (C), mathematics and physics, to simulate theories or experiments. I decided to take interest in the simulation of acoustic wave propagation in a room. We have all been once in a room with bad acoustics, lecture halls especially. We tend to blame noisy students, or shy professors, but we know that it's just the design of the acoustics. But what could be the solution to that problem? I mainly focused on three aspects: the material of the walls, the dimension of the room and the position of the receptor(human or not).

2 Used equations

Let's first explain the different theories and equations that we need to understand using them in our context. First, the conservation of energy. As one of the most important law in physics, we use it here to get the acoustic intensity of a sound after bouncing of a wall. By admitting that the intensity is proportional to the square of the amplitude of that sound, we can establish equation (1), where I_i, R_r, I_r, A_r are the acoustic intensity and amplitude of the incident and reflected wave, while I_t is the acoustic intensity of the transmitted wave (see figure 1):

$$I_i = I_r + I_t \Rightarrow A_r = \sqrt{A_i^2(1-\alpha)} \quad (1)$$

Note that α is the absorption coefficient, where it will have a specific value depending on the frequency of a material, as shown bellow on the right (figure 1[1]).

Secondly, knowing that sound is not generally composed by one frequency, we need to use a Fourier Transform. In summary, it's a tool that helps us deconstruct a complicated function into a sum of sine functions(see figure 4). Let $s(t)$ be our function that represents our sound, A_0 the amplitude of the fundamental, A_k the amplitude of the harmonics with k the rank of the harmonic, ω the pulsation and θ_k the phase shift:

$$s(t) = A_0 + \sum_{k=1}^{\infty} [A_k \sin(k\omega t + \theta_k)] \quad (2)$$

Finally, we need to regroup the amplitude of our fundamental and our harmonics to obtain A_r in equation (1) by using Parseval's identity. The equality is pretty straight forward, where P is the power of the sound :

$$P = \langle s(t)^2 \rangle \Rightarrow P = A_0^2 + \frac{1}{2}\sum_{k=1}^{\infty} A_k^2 \quad (3)$$

Lets summarize: we're interested on how much sound is absorbed when hitting a wall. By using the conservation of energy, we realized that a wall softens differently the certain frequencies that make up a sound (our coefficient α). To counter this problem, we use a Fourier transform to know the spectrum of our sound (our function $s(t)$). In the end, we rebuild our sound by using Parseval's identity to obtain the power of our sound after bouncing the wall. We'll represent our results in decibels, using the final equation, with P_0 our reverence point:

$$L_P = 10\log_{10}(\frac{P}{P_0}) \quad (4)$$

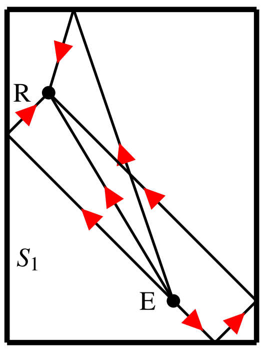

Figure 1: Schematic of the problem

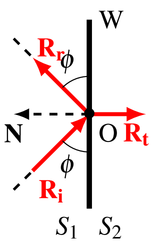

Figure 2: Réflection and absorption

Here, W symbolise the wall between two different environments S_1 and S_2, **N** the normal vector and ϕ the reflected angle.

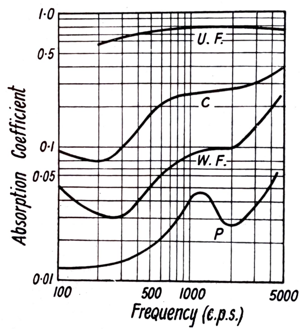

Figure 3: Coefficient absorption in terms of frequency

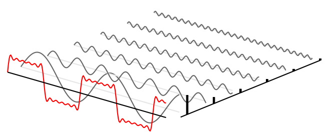

Figure 4: Example of a Fourier Transform

3 Some results

After discussing the mathematics and the physics, let's see some results that have been simulated in C (the output is done in Python with MatPlotLib). By using the information about how the absorption coefficient evolves depending on the frequency (figure 1), we can simulate how the composition of different walls play a key role in our study. On that note, the following schematics show how sound is soften when the walls are made out of plaster, wooden furniture, carpet and upholstered furniture, where the blue circle is the transmitter, whilst the black circle is the receptor.

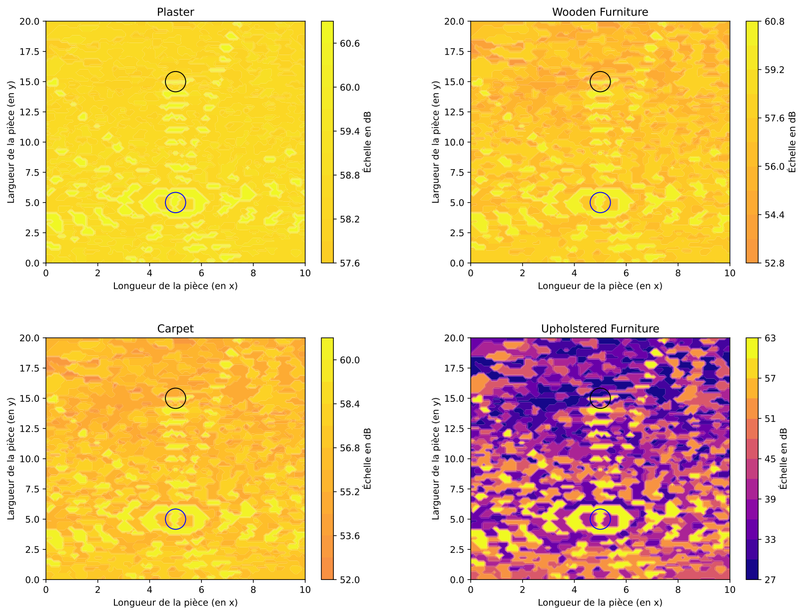

Figure 5: Comparing the material of the walls

The results are pretty obvious, more the composition of the walls are thickened and dense, more the sound is absorbed. But this coherence will help us solve some specific problems later on. Now, by taking interest on the position of the receptor, we get and interesting result. We realize that we have almost the same trajectory of sound for each case. This result repeats its self for any combination in this specific room. The result is interesting because we can play on where the most trajectories hit walls and place specific objects to soften the least possible our sound.

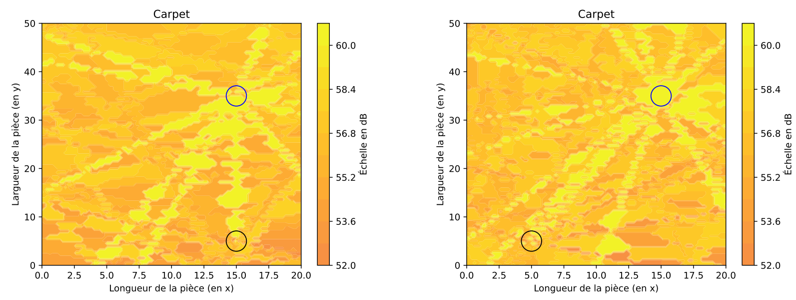

Figure 6: Comparing the position of the receptor

Finally, let's now fix both positions of the transmitter and receptor but enlarge the room. We observe new trajectories that go to right of the room. These trajectories will arrive late compared to those arriving directly on the left: this will cause echo and reverb, the most common problem in most lecture halls. But, by analyzing our previous results, we can solve this issue. A first solution would to change the composition of the right wall to one made out of upholstered furniture, so that the sound would hardly be softened. Or, a second solution is to place a beam in the middle of the room to reflect the trajectory directly on the receptor.

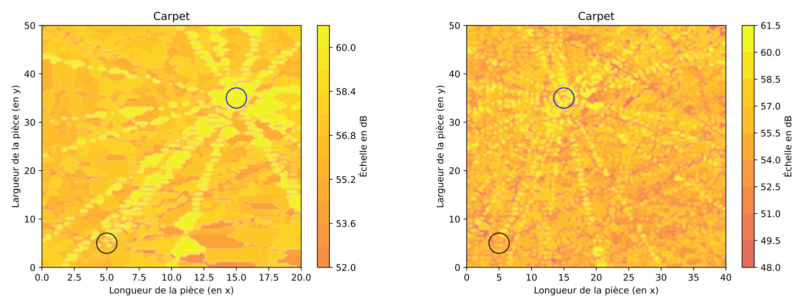

Figure 7: Comparing the dimension of the room

4 Conclusion

My simulation proves that using the perfect material and the choice of the right dimensions help to overcome these phenomenons: both solutions mentioned above are highly used in many places. Its very intuitive yet rooms or halls still suffer from echo and reverb. In depth, my code is far from being perfect: if we look closely on the images, we see our straight trajectories being superposed by late arriving ones. Also, I have not yet managed to place a certain object in the room, nor to change the composition of a single wall, to prove the previous point mentioned for figure 7. I hope that this article has opened your eye on the problem of acoustics in general. With basic knowledge in physics and programming, it is easy to explore the world of science by simulating real life phenomenons. With that in mind, a definite improvement can be made overall to take this problem to a next level to explore new fields in acoustics.

[1] **alma991000079749806616**.