Study of phononic crystals and band gaps

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Introduction

Waves propagating in phononic crystals would experience attenuation at certain frequencies. This occurs when scattering waves create destructive interference. In this context, this study aims to highlight the existence of band gaps for acoustic waves. We will be working with square matrices composed of steel cylinders immersed in water. By comparing our results with predictions made by the Bloch theorem, we are able to evaluate the crystal's band gaps by observing the minima of the acoustic transmission through the crystal.

Phononic crystal and band gaps

A phononic crystal is an artificial periodic structure (denoted Ω in the figure) in which acoustic waves propagate. It is a human-made creation where cylinders (often made of metal) are periodically spaced. We call "band gaps" a region in which certain frequencies are greatly attenuated. As suggested by this figure, we worked with a crystal featuring a square lattice. Given that the lattice is squared, it is sufficient to study the primitive cell between 0° and 45° to fully evaluate the crystal's behavior.

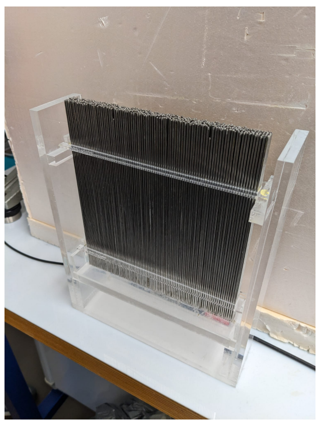

Figure 1: Studied crystal

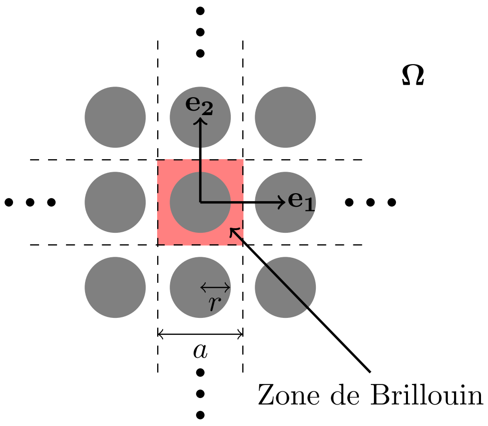

Figure 2: Periodicity of the crystal

We predict these "band gaps" by using Bloch's theorem, which essentially enables us to determine how acoustic waves behave in a periodic medium. By solving the wave equation with specified boundary conditions, we can use PDE solvers (such as *FreeFEM*) to get different dispersion relations. These relations give us the couples' frequency/wavelength, for which the solution to the Bloch theorem exists. As depicted below, the solution obtained is for a square lattice with a side length of $a = 2\ mm$ and cylinders of radius $r = 0.75\ mm$. In this plot, the gray rectangles correspond to a "band gap". We observe that the most significant forbidden bands are located between $[0; 1\ MHz]$. We will experimentally determine the forbidden bands in the interval $[0; 2\ MHz]$ for the same crystal.

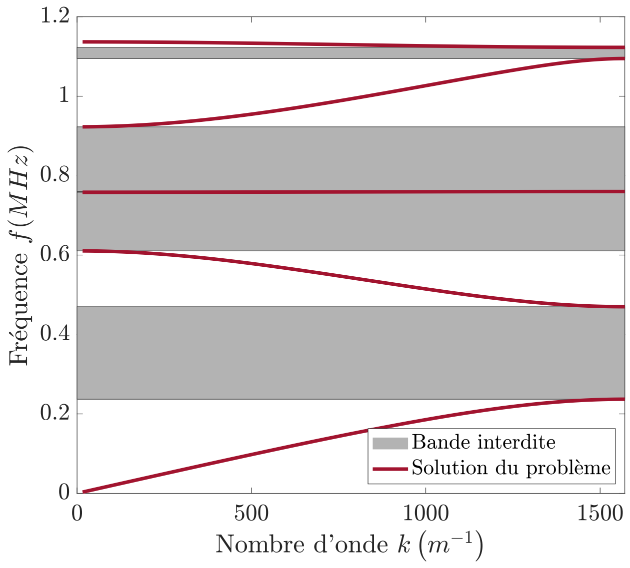

Figure 3: Theoretical band gaps for a square lattice: $a = 2\ mm$, $r = 0.75\ mm$ and $\theta = 0°$

Experimental measurements and results

The "band gaps" of a crystal are identified by the minima in acoustic transmission : we send an acoustic wave in an aquarium (as shown in figure 6), and we evaluate the ratio between a situation with the crystal, and one without the crystal. This will give us the transfer function depicted as $|\tilde{h}_c|$.

We have worked with transducers to generate and receive our acoustic wave. The signal generated is a sinusoidal function modulated with a Gaussian function to be able to cover a wide range of frequencies. For the receiving signal, we simply applied a *tukey window* to limit the echos and smooth out the FFT process.

In the graphs below, the theoretically obtained "band gaps" are superimposed with the crystal's transfer function. Firstly, we observe that the results obtained are pretty much consistent with the theoretical model: a severe attenuation of the acoustic wave for frequencies in the predicted regions. For high frequencies above 1 Mhz, the signal received by the receiver is likely too weak, leading it to mix with noise. This underscores the importance of choosing the sensor bandwidth wisely.

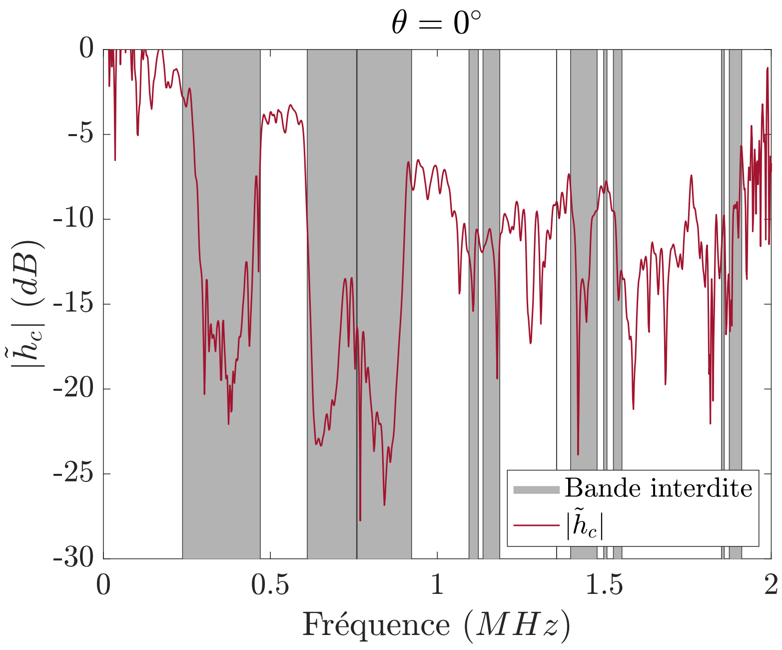

Figure 4: Experimental band gaps with $\theta = 0°$

For wider angles, "band gaps" tend to reduce in size. We notice that, for example with $\theta = 20°$, the first "band gap" remains well-positioned, while the subsequent ones seem to shift from the theoretical values.

One of the main errors for this mismatch occurred during angle measurements: we read the crystal orientation angle through water. Secondly, we approximated our experimental crystal as an infinite medium to enable comparison. Thirdly, our crystal is not perfect (not perfectly rigid nor periodic).

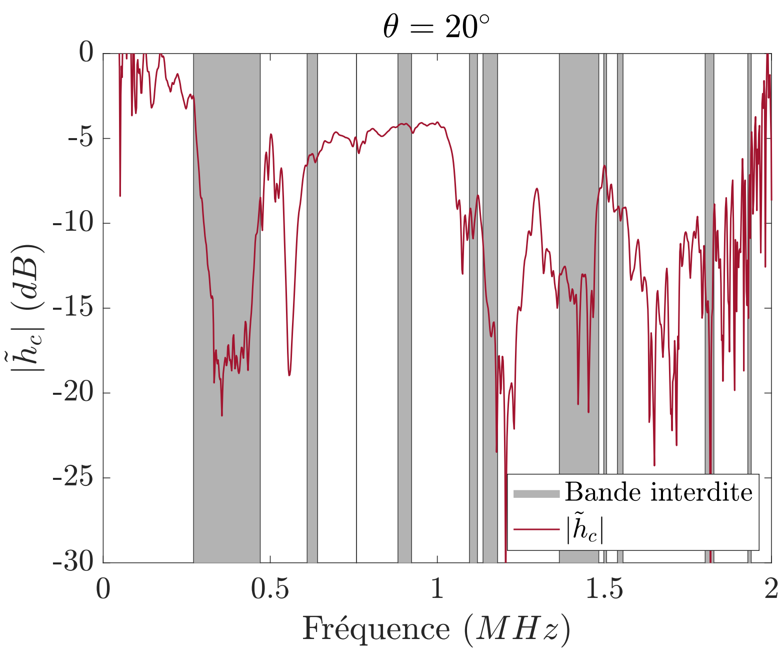

Figure 5: Experimental band gaps with $\theta = 20°$

Finally, we also implement a numerical simulation of the acoustic wave propagating in the crystal by using the finite difference method. However, we have not managed to implement the correct non-penetration condition on the cylinders. This simulation is available **here**.

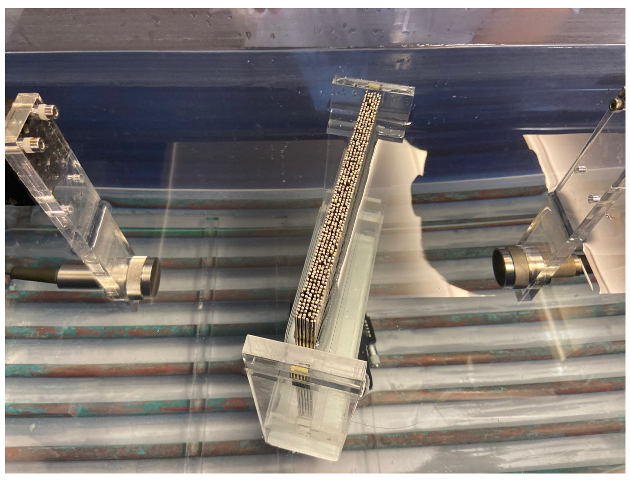

Figure 6: Experimental image