**Homework 2, LINMA 2450**
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# 1 Pricing a (non-convex) unit commitment problem

One of the most famous problem in power system operations is the so-called *unit commitment* problem. It expresses the problem of fulfilling the demand $D_t$ for each period $t$ of a given horizon $\mathcal{T}$ (typically the 24 hours of the next day) given a set of generators $\mathcal{G}$. It aims at minimizing the production cost while making sure that the generators constraints are not violated. A basic version of the unit commitment problem can be formulated as follows.

$$\min \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left[C_g^P p_{g,t} + C_g^{NL} u_{g,t} + C_g^{SU} v_{g,t}\right] \tag{1a}$$

$$(\pi_t)\quad D_t = \sum_{g\in\mathcal{G}} p_{g,t} \qquad \forall t\in\mathcal{T} \tag{1b}$$

$$u_{g,t}P_g^{min} \le p_{g,t} \le u_{g,t}P_g^{max} \qquad \forall t\in\mathcal{T}, g\in\mathcal{G} \tag{1c}$$

$$u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t} \qquad \forall t\in[2,T], g\in\mathcal{G} \tag{1d}$$

$$\sum_{i=t-TU_g+1}^{t} v_{g,i} \le u_{g,t} \qquad \forall g\in\mathcal{G}, t\in[TU_g,T] \tag{1e}$$

$$\sum_{i=t-TD_g+1}^{t} w_{g,i} \le 1 - u_{g,t} \qquad \forall g\in\mathcal{G}, t\in[TD_g,T] \tag{1f}$$

$$p_{g,t}\ge 0\ ;\ u_{g,t}, v_{g,t}, w_{g,t}\in\{0,1\} \qquad \forall t\in\mathcal{T}, g\in\mathcal{G} \tag{1g}$$

Each generator has four variables associated to each time period: a certain power output $p_{g,t}$ which is continuous between $P_g^{min}$ (the minimum output power of the generator) and $P_g^{max}$ (the maximum output power of the generator) ; a commitment decision $u_{g,t}$ telling whether the generator $g$ is *on* or *off* at period $t$; a start-up decision $v_{g,t}$ telling whether the generator $g$ has started-up at period $t$ and a shut-down decision $w_{g,t}$ telling whether the generator $g$ has shut down at period $t$. The unit commitment problem aims at minimizing the total cost, which includes the variable production cost $C_g^P$, the start-up cost $C_g^{SU}$ and the no-load cost (the fix cost of being *on*) $C_g^{NL}$. Constraint (1b) is the so-called market clearing constraint which enforces the total production power to match the demand at each period of the day. Constraint (1d) is a logical constraint connecting the variables $u$, $v$ and $w$ together. The constraints (1e) and (1f) are the so-called minimum up and down time constraints, which ensure that the unit $g$ remains at least $TU_g$ hours *on* after it started up, and at least $TD_g$ hours *off* after it shut down. Variables $u$, $v$ and $w$ are binary variables, which means the problem is non-convex (MIP).

If the problem was convex, the dual variable associated to the market clearing constraint (1b) would have the interpretation of the market clearing price: solving problem (1) would then provide both (i) the dispatch instructions $p_{g,t}$ for each generator and for each period of the next day and (ii) the market clearing price $\pi_t$ for each hour of the next day. Since the problem (1) is actually *non-convex*, the duality theory does not hold and the question of what would be an appropriate market clearing price remains open. One somehow natural possibility is to define the price as the Lagrangian dual multiplier $\pi_t$, associated the market clearing constraints, in the Lagrangian dual problem where only the market clearing constraint is relaxed. In this homework, you will study this Lagrangian dual problem and implement an algorithm to solve it efficiently.

### 1.1 Part I: analysis of a basic unit commitment problem

**Question 1.1** Write the Lagrangian dual problem that corresponds to relaxing the market clearing constraint (1b) in problem (1). Characterize the Lagrangian function $L(\pi)$ in terms of *convexity* and *differentiability.*

**Question 1.2** Provide a reason why it is computationally interesting to relax this constraint in particular? Make it explicit in the expression of the Lagrangian dual problem.

**Question 1.3** Provide the analytical expression of the *subgradient* of the Lagrangian function $L(\pi)$.

**Question 1.4** Write the pseudo-code of a *subgradient algorithm* that seek to optimize the Lagrangian function and would allow you to compute the optimum Lagrangian multiplier $\pi_t^*$.

**Question 1.5** Prove for the case where $TU = TD = 1$ that $conv\{(u, v, w)| (1d), (1e), (1f), u_{g,t}, v_{g,t}, w_{g,t} \in \{0, 1\}\} = \{(u, v, w)|(1d), (1e), (1f), 0 \leq u_{g,t}, v_{g,t}, w_{g,t} \leq 1\}$.

**Question 1.6** It is actually possible to extend this result to any $TU$ and $TD$ and to constraint (1c): $conv\{(p, u, v, w)|(1c), (1d), (1e), (1f), u_{g,t}, v_{g,t}, w_{g,t} \in \{0, 1\}\} = \{(p, u, v, w)|(1c), (1d), (1e), (1f), 0 \leq u_{g,t}, v_{g,t}, w_{g,t} \leq 1\}$. Based on this result, explain how to compute the optimum Lagrangian multipliers $\pi_t^*$ straightforwardly.

## 1.2 Part II: Dantzig-Wolfe column generation algorithm

Let's now consider a more *advanced* version of the above unit commitment problem, provided in appendix of this assignment (*model.pdf*). It essentially extends problem (1) by adding (i) ramp constraints between the time periods (limiting the change in production from one period to another), (ii) a piecewise linear production cost function and (iii) a time-dependent start-up cost.

Unlike the previously discussed *basic* unit commitment problem, the convex hull of the generators' constraints in the *advanced* problem is not tractable. The subgradient algorithm turns out to be ineffective for solving this Lagrangian relaxation. Instead, we propose to use the *column generation Dantzig-Wolfe* (D-W) algorithm.

**Question 2.1** Write the full *Master Problem*[1] of D-W applied to problem (1). Explain why it amounts to solving the Lagrangian relaxation we are interested in.

**Question 2.2** Provide the expression of the *reduced costs*, the *Restricted Master Problem* and the *subproblems*[2]. Give the pseudo-code of D-W algorithm.

**Question 2.3** Let's now consider the comprehensive unit commitment model of *model.pdf*. We shall consider the data "2015-06-01_hw.json" that come from the Federal Energy Regulatory Commission (FERC) — the US regulatory authority for energy — and PJM Interconnection. PJM is an independent system operator (ISO), in charge of one of the biggest markets in the United States[3]. An ISO such as PJM essentially solves a unit commitment problem, as the one of this assignment[4], every day in order to compute the market clearing prices and the dispatch instructions towards all the generators for the next operation day.

You are asked to implement a D-W algorithm that computes the optimum Lagrangian multipliers associated to the market cleating constraint (i.e. the market clearing price) for the provided instance "2015-06-01_hw.json". The code should be in julia. You should report at least (i) the objective of the master program (i.e. which is a *lower bound* of the dispatch cost), (ii) the market clearing prices (a vector of 24 hourly prices), (iii) the number of iterations required for D-W to converge and (iv) D-W run time.

[1] i.e. the version of problem (8.7) in the syllabus when applied to problem (1) of this homework.

[2] The subproblems are the optimization programs you need to solve in order to evaluate the reduced costs. Hint: in this case, the subproblems should have the interpretation of the *profit maximization program* of each generator.

[3] It is part of the Eastern Interconnection grid operating an electric transmission system serving Delaware, Illinois, Indiana, Kentucky, Maryland, Michigan, New Jersey, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia, West Virginia, and the District of Columbia.

[4] The main missing requirement, that we don't consider in the assignment, is the power network model (i.e. a linear model of the electric grid, with buses and electric lines).

**Remark 1:** Regarding the model provided in *model.pdf*, you should ignore: the variable $p_w(t)$ (you can ignore it in equation (2) and simply drop equation (24)) and the reserve variable $r_g(t)$ (you can ignore it in equations (8), (17)-(18)-(19) and simply drop equation (3))

**Remark 2:** The julia code implementing the *model.pdf* is provided in the file *model.jl*. This should help you in your implementation of the Dantzig-Wolfe algorithm.

# 2 Submission

This homework should be performed by groups of two students. You should submit both code and report in a zip file until 23:59 10/12/2021. No late acceptance. The report and the code should be the *personal* work of each group.

- Code (in julia)
- Report (.pdf file within 6 pages)
- File Name (include FirstName LastName for each group members)