Transportation model

There are i can factories and j consumption markets. Each factory has a maximum capacity of a_i cases, and each market demands a quantity of b_j cases (it is assumed that the total production capacity is greater than the total market demand for the problem to be feasible). The transportation cost between each factory i and each market j for each case is c_{ij}. The demand must be satisfied at a minimum cost.

The decision variables of the problem will be cases transported between each factory i and each market j, x_{ij}.

My first GAMS transportation model

https://github.com/IIT-EnergySystemModels/Fixed-Charge-Transportation-Problem-Benders-Decomposition/blob/main/TransportModel.gms

```
sets
   I origins      / VIGO, ALGECIRAS /
   J destinations / MADRID, BARCELONA, VALENCIA /

parameters
   pA(i) origin capacity
       / VIGO      350
         ALGECIRAS 700 /

   pB(j) destination demand
       / MADRID    400
         BARCELONA 450
         VALENCIA  150 /

table pC(i,j) per unit transportation cost
          MADRID BARCELONA VALENCIA
VIGO        0.06     0.12     0.09
ALGECIRAS   0.05     0.15     0.11

variables
   vX(i,j) units transported
   vCost   transportation cost

positive variable vX

equations
   eCost          transportation cost
   eCapacity(i) maximum capacity of each origin
   eDemand   (j) demand supply at destination ;

eCost        .. sum[(i,j), pC(i,j) * vX(i,j)] =e= vCost ;
eCapacity(i) .. sum[   j ,              vX(i,j)] =l= pA(i) ;
eDemand   (j) .. sum[ i   ,              vX(i,j)] =g= pB(j) ;

model mTransport / all /
solve mTransport using LP minimizing vCost
```

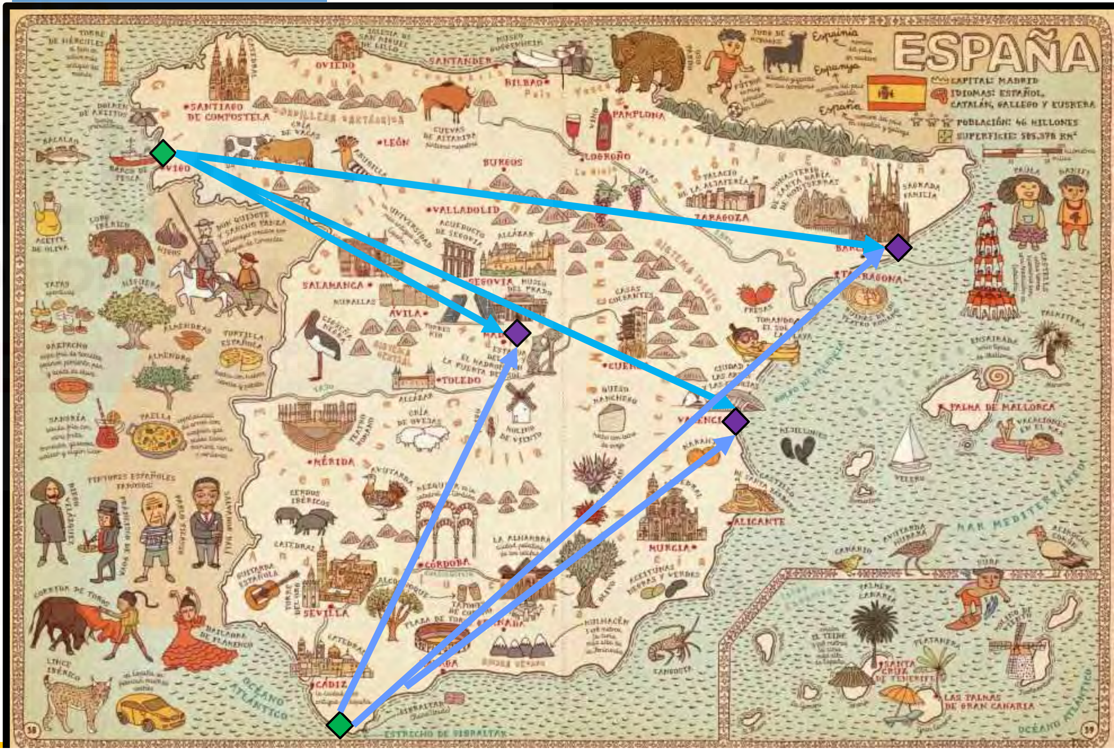
$$\min_{x_{ij}} \sum_{ij} c_{ij} x_{ij}$$

$$\sum_{j} x_{ij} \le a_i \quad \forall i$$

$$\sum_{i} x_{ij} \ge b_j \quad \forall j$$

$$x_{ij} \ge 0$$

A. Mizielinska y D. Mizielinski *Atlas del mundo: Un insólito viaje por las mil curiosidades y maravillas del mundo* Ed. Maeva 2015



My first Pyomo transportation model

https://github.com/IIT-EnergySystemModels/Fixed-Charge-Transportation-Problem-Benders-Decomposition/blob/main/TransportModel.py

```
import pyomo.environ as pyo
from   pyomo.environ import ConcreteModel, Set, Param, Var, NonNegativeReals, Constraint, Objective, minimize, Suffix
from   pyomo.opt     import SolverFactory

mTransport = ConcreteModel('Transportation Problem')

mTransport.i = Set(initialize=['Vigo',   'Algeciras'             ], doc='origins'     )
mTransport.j = Set(initialize=['Madrid', 'Barcelona', 'Valencia'], doc='destinations')

mTransport.pA = Param(mTransport.i, initialize={'Vigo'  : 350, 'Algeciras': 700                 }, doc='origin capacity'   )
mTransport.pB = Param(mTransport.j, initialize={'Madrid': 400, 'Barcelona': 450, 'Valencia': 150}, doc='destination demand')

TransportationCost = {
    ('Vigo',      'Madrid'   ): 0.06,
    ('Vigo',      'Barcelona'): 0.12,
    ('Vigo',      'Valencia' ): 0.09,
    ('Algeciras', 'Madrid'   ): 0.05,
    ('Algeciras', 'Barcelona'): 0.15,
    ('Algeciras', 'Valencia' ): 0.11,
    }

mTransport.pC = Param(mTransport.i, mTransport.j, initialize=TransportationCost, doc='per unit transportation cost')

mTransport.vX = Var  (mTransport.i, mTransport.j, bounds=(0.0,None), doc='units transported', within=NonNegativeReals)

def eCapacity(mTransport, i):
    return sum(mTransport.vX[i,j] for j in mTransport.j) <= mTransport.pA[i]
mTransport.eCapacity = Constraint(mTransport.i, rule=eCapacity, doc='maximum capacity of each origin')

def eDemand  (mTransport, j):
    return sum(mTransport.vX[i,j] for i in mTransport.i) >= mTransport.pB[j]
mTransport.eDemand   = Constraint(mTransport.j, rule=eDemand,   doc='demand supply at destination'   )

def eCost(mTransport):
    return sum(mTransport.pC[i,j]*mTransport.vX[i,j] for i,j in mTransport.i*mTransport.j)
mTransport.eCost = Objective(rule=eCost, sense=minimize, doc='transportation cost')

mTransport.write('mTransport.lp', io_options={'symbolic_solver_labels': True})

mTransport.dual = Suffix(direction=Suffix.IMPORT)

Solver = SolverFactory('gurobi')
Solver.options['LogFile'] = 'mTransport.log'
SolverResults = Solver.solve(mTransport, tee=True)

SolverResults.write()
mTransport.pprint()

mTransport.vX.display()
for j in mTransport.j:
    print(mTransport.dual[mTransport.eDemand[j]])
```

$$\min_{x_{ij}} \sum_{ij} c_{ij} x_{ij}$$

$$\sum_{j} x_{ij} \leq a_i \quad \forall i$$

$$\sum_{i} x_{ij} \geq b_j \quad \forall j$$

$$x_{ij} \geq 0$$

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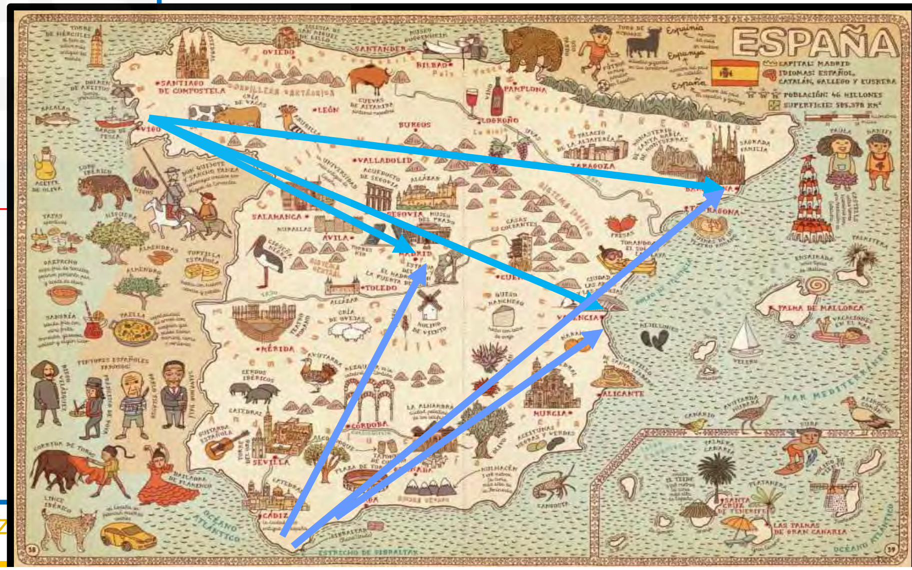

LP File: `mTransport.write('mTransport.lp', io_options={'symbolic_solver_labels': True})`

```
\* Source Pyomo model name=unknown *\

min
eCost:
+0.14999999999999999 vX(Algeciras_Barcelona)
+0.050000000000000003 vX(Algeciras_Madrid)
+0.11 vX(Algeciras_Valencia)
+0.12 vX(Vigo_Barcelona)
+0.059999999999999998 vX(Vigo_Madrid)
+0.089999999999999997 vX(Vigo_Valencia)

s.t.

c_u_eCapacity(Algeciras)_:
+1 vX(Algeciras_Barcelona)
+1 vX(Algeciras_Madrid)
+1 vX(Algeciras_Valencia)
<= 700

c_u_eCapacity(Vigo)_:
+1 vX(Vigo_Barcelona)
+1 vX(Vigo_Madrid)
+1 vX(Vigo_Valencia)
<= 350
```

```
c_l_eDemand(Barcelona)_:
+1 vX(Algeciras_Barcelona)
+1 vX(Vigo_Barcelona)
>= 450

c_l_eDemand(Madrid)_:
+1 vX(Algeciras_Madrid)
+1 vX(Vigo_Madrid)
>= 400

c_l_eDemand(Valencia)_:
+1 vX(Algeciras_Valencia)
+1 vX(Vigo_Valencia)
>= 150

c_e_ONE_VAR_CONSTANT:
ONE_VAR_CONSTANT = 1.0

bounds
   0 <= vX(Algeciras_Barcelona) <= +inf
   0 <= vX(Algeciras_Madrid) <= +inf
   0 <= vX(Algeciras_Valencia) <= +inf
   0 <= vX(Vigo_Barcelona) <= +inf
   0 <= vX(Vigo_Madrid) <= +inf
   0 <= vX(Vigo_Valencia) <= +inf
end
```

Problem summary: `SolverResults.write()`

```
# ==========================================================
# = Solver Results                                         =
# ==========================================================
# ----------------------------------------------------------
#   Problem Information
# ----------------------------------------------------------
Problem:
- Name: x7
  Lower bound: 93.5
  Upper bound: 93.5
  Number of objectives: 1
  Number of constraints: 6
  Number of variables: 7
  Number of binary variables: 0
  Number of integer variables: 0
  Number of continuous variables: 7
  Number of nonzeros: 13
  Sense: minimize
# ----------------------------------------------------------
#   Solver Information
# ----------------------------------------------------------
Solver:
- Status: ok
  Return code: 0
  Message: Model was solved to optimality (subject to tolerances), and an optimal solution is available.
  Termination condition: optimal
  Termination message: Model was solved to optimality (subject to tolerances), and an optimal solution is available.
  Wall time: 0.020067214965820312
  Error rc: 0
  Time: 0.30008649826049805
# ----------------------------------------------------------
#   Solution Information
# ----------------------------------------------------------
Solution:
- number of solutions: 0
  number of solutions displayed: 0
```

Optimal results: `mTransport.pprint()`

```
4 Set Declarations
    i : origins
        Dim=0, Dimen=1, Size=2, Domain=None, Ordered=False, Bounds=None
        ['Algecirass', 'Vigo']
    j : destinations
        Dim=0, Dimen=1, Size=3, Domain=None, Ordered=False, Bounds=None
        ['Barcelona', 'Madrid', 'Valencia']
    pC_index : Dim=0, Dimen=2, Size=6, Domain=None, Ordered=False, Bounds=None
        Virtual
    vX_index : Dim=0, Dimen=2, Size=6, Domain=None, Ordered=False, Bounds=None
        Virtual
3 Param Declarations
    pA : origin capacity
        Size=2, Index=i, Domain=Any, Default=None, Mutable=False
        Key       : Value
        Algeciras :   700
             Vigo :   350
    pB : destination demand
        Size=3, Index=j, Domain=Any, Default=None, Mutable=False
        Key       : Value
        Barcelona :   450
           Madrid :   400
         Valencia :   150
    pC : per unit transportation cost
        Size=6, Index=pC_index, Domain=Any, Default=None, Mutable=False
        Key                        : Value
        ('Algeciras', 'Barcelona') :  0.15
           ('Algeciras', 'Madrid') :  0.05
         ('Algeciras', 'Valencia') :  0.11
             ('Vigo', 'Barcelona') :  0.12
                ('Vigo', 'Madrid') :  0.06
              ('Vigo', 'Valencia') :  0.09
```

```
1 Var Declarations
    vX : units transported
        Size=6, Index=vX_index
        Key                        : Lower : Value : Upper : Fixed : Stale : Domain
        ('Algeciras', 'Barcelona') :   0.0 : 100.0 :  None : False : False :  Reals
           ('Algeciras', 'Madrid') :   0.0 : 400.0 :  None : False : False :  Reals
         ('Algeciras', 'Valencia') :   0.0 : 150.0 :  None : False : False :  Reals
             ('Vigo', 'Barcelona') :   0.0 : 350.0 :  None : False : False :  Reals
                ('Vigo', 'Madrid') :   0.0 :   0.0 :  None : False : False :  Reals
              ('Vigo', 'Valencia') :   0.0 :   0.0 :  None : False : False :  Reals
1 Objective Declarations
    eCost : transportation cost
        Size=1, Index=None, Active=True
        Key  : Active : Sense    : Expression
        None :   True : minimize : 0.06*vX[Vigo,Madrid] + 0.12*vX[Vigo,Barcelona] + 0.09*vX[Vigo,Valencia] +
0.05*vX[Algeciras,Madrid] + 0.15*vX[Algeciras,Barcelona] + 0.11*vX[Algeciras,Valencia]
2 Constraint Declarations
    eCapacity : maximum capacity of each origin
        Size=2, Index=i, Active=True
        Key       : Lower : Body                                                                      : Upper : Active
        Algeciras :  -Inf : vX[Algeciras,Madrid] + vX[Algeciras,Barcelona] + vX[Algeciras,Valencia] : 700.0 :   True
             Vigo :  -Inf :                     vX[Vigo,Madrid] + vX[Vigo,Barcelona] + vX[Vigo,Valencia] : 350.0 :   True
    eDemand : demand supply at destination
        Size=3, Index=j, Active=True
        Key       : Lower : Body                                        : Upper : Active
        Barcelona : 450.0 : vX[Vigo,Barcelona] + vX[Algeciras,Barcelona] :  +Inf :   True
           Madrid : 400.0 :       vX[Vigo,Madrid] + vX[Algeciras,Madrid] :  +Inf :   True
         Valencia : 150.0 :   vX[Vigo,Valencia] + vX[Algeciras,Valencia] :  +Inf :   True
11 Declarations: j pA pB pC_index pC vX_index vX eCapacity eDemand eCost i
```