Para demais autovalores usa-se método de Deflexão

$B = A - qI$ $\quad \rightarrow M^{-1}M$

$= M^{-1}\Gamma M - q(I)$

$= M^{-1}(\Gamma - qI)M$

Lista de ALC

Igor Tavares
Prof. Ricardo Coelho

1°) Usando o método de Leverrier, encontre os autovalores da matriz

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 3 & 4 \\ 2 & 1 & 2 \end{bmatrix} \qquad A^2 = \begin{bmatrix} 13 & 11 & 17 \\ 20 & 19 & 24 \\ 9 & 9 & 14 \end{bmatrix}$$

$K=1 \therefore S_1 = tr(A)$
$S_1 = 6$

$K=2 \quad S_2 = tr(A^2)$
$S_2 = 46$

$$A^3 = \begin{bmatrix} 80 & 76 & 117 \\ 135 & 126 & 194 \\ 64 & 59 & 91 \end{bmatrix}$$

$K=3 \therefore S_3 = tr(A^3)$
$S_3 = 297$

2) $K=1$

$p_0 = S_1$
$\boxed{p_0 = 6}$

$K=2$

$2 \cdot p_1 = S_2 - p_0 S_1$
$2 \cdot p_1 = 46 - 6 \cdot 6$
$2p_1 = 46 - 36$
$p_1 = 10/2$
$\boxed{p_1 = 5}$

$K=3$

$3p_2 = S_3 - p_0 S_2 - p_1 S_1$
$3p_2 = 297 - 6 \cdot 46 - 5 \cdot 6$
$3p_2 = 297 - 306$
$3p_2 = -9$
$\boxed{p_2 = -3}$

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3º) Passo

$$P(\lambda) = (-1)^n \left[\lambda^n - p_1 \lambda^{n-1} - \dots - p_{n-1}\lambda - p_n \right]$$

$$P(\lambda) = (-1)^3 \left[\lambda^3 - 6\lambda^2 - 5\lambda - (-3) \right]$$

$$P(\lambda) = -\lambda^3 + 6\lambda^2 + 5\lambda - 3$$

$$\boxed{\begin{aligned} \lambda_1 &= 6.6811653 \\ \lambda_2 &= -1.0422606 \\ \lambda_3 &= 0.41110955 \end{aligned}}$$

2º)

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

$k = 1$

$$q_1 = \frac{Tr(A_1)}{1} = 1$$

$$B_1 = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} - 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$$

$$A_2 = A \cdot B_1 = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix}$$

$k = 2$

$$q_2 = \frac{tr(A_2)}{2} = \frac{2}{2} = 2$$

$$A_3 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & -2 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

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$k = 3$

$$A_3 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \qquad tr(A_3) = -6$$

$$q_k = \frac{tr(A_3)}{k} = \frac{-6}{3} = -2$$

$$B_3 = 0_{3\times 3}$$

$q_1 = 1$

$q_2 = 2$

$q_3 = -2$

$$P(\lambda) = (-1)^n [\lambda^n - p_1 \lambda^{n-1} - \dots - p_{n-1}\lambda - p_n]$$

$$P(\lambda) = (-1)^3 [\lambda^3 - 1\lambda^2 - 2\lambda - (-2)]$$

$$P(\lambda) = -1(\lambda^3 - \lambda^2 - 2\lambda + 2)$$

$$P(\lambda) = -\lambda^3 + \lambda^2 + 2\lambda - 2$$

$\lambda_1 = -1.4142136 = -\sqrt{2}$

$\lambda_2 = 1.4142136 = \sqrt{2}$

$\lambda_3 = 1$

$$Q_1 = \lambda_1^{n-1} I + \lambda_1^{n-2} B_1 + \lambda_1^{n-3} B_2$$

$$Q_1 = (-\sqrt{2})^2 I + (-\sqrt{2})^1 B_1 + B_2$$

$$Q_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 0 & \sqrt{2} & -\sqrt{2} \\ 0 & -\sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} & -\sqrt{2} \end{bmatrix} + \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 1 & -\sqrt{2}-1 & \sqrt{2}+1 \\ -1 & 1+\sqrt{2} & -\sqrt{2}-1 \\ \sqrt{2} & -\sqrt{2}-2 & 2-\sqrt{2} \end{bmatrix} \qquad v_1 = \begin{bmatrix} 1 \\ -1 \\ \sqrt{2} \end{bmatrix}$$

Para $\lambda_2 = \sqrt{2}$

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$$Q_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \sqrt{2} & -\sqrt{2} \\ 0 & -\sqrt{2} & \sqrt{2} \\ -\sqrt{2} & \sqrt{2} & -\sqrt{2} \end{bmatrix} + \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 1 & \sqrt{2}-1 & -\sqrt{2}+1 \\ -1 & -\sqrt{2}+1 & \sqrt{2}-1 \\ -\sqrt{2} & \sqrt{2}-2 & -\sqrt{2}+2 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ -1 \\ -\sqrt{2} \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} + \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 0 & -2 & 0 \end{bmatrix}$$

$$Q_3 = \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & 0 \\ -1 & -1 & 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ \sqrt{2} \end{bmatrix} \text{ com } \lambda_1 = \sqrt{2}, \quad v_2 = \begin{bmatrix} 1 \\ -1 \\ -\sqrt{2} \end{bmatrix} \text{ com } \lambda_2 = -\sqrt{2}$$

$$v_3 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, \text{ com } \lambda_3 = 1$$

$$A^{-1} = \frac{1}{\rho_n} \cdot B_{n-1} = \frac{1}{-2} \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

Inversa:

$$A^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$$

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