/ /

2: Mostre que:

$$\|A\|_F^2 = \sigma_1^2 + \sigma_2^2 + \dots + \sigma_p^2$$

onde σ_i, $i = 1, \dots, p$ são ~~...~~ os valores singulares de A

$A = U \Sigma V^t$ U e V ortogonais

Logo, $\|A\|_F = \|U\Sigma V^t\|_F$:

(I) $\|\underbrace{U\Sigma V^t}_{B}\|_F \leq \|U\| \cdot \|\Sigma V^t\|_F$

$\|U\Sigma V^t\| \leq \|\Sigma V^t\| = \|(V\Sigma^t)^t\|$

$\|U\Sigma V^t\|_F \leq \|V\Sigma^t\|_F \leq \|V\|_F \cdot \|\Sigma^t\|_F$

$\|U\Sigma V^t\| \leq \|\Sigma^t\|_F$

(II) $\|U^t U\Sigma V^t\|_F \leq \|U^t\|_F \cdot \|U\Sigma V^t\|_F$

$\|\Sigma V^t\|_F \leq \|U\Sigma V^t\|_F$

$\|\Sigma V^t V\|_F \leq \|U\Sigma V^t\|_F \|V\|_F$

$\|\Sigma\|_F \leq \|U\Sigma V^t\|_F$ como: em (I) temos $\|U\Sigma V^t\|_F \leq \|\Sigma^t\|_F$

$\therefore$ em (II) $\|\Sigma\|_F \leq \|U\Sigma V^t\|_F$ logo são iguais