1) slide 07

1) $\mathrm{cond}_F(A) = \|A\|_F \cdot \|A^{-1}\|_F$

$= tr(AA^t) \cdot tr(A^{-1}(A^{-1})^t)$

$= (\sigma_1^2 + \sigma_2^2 + \sigma_n^2)\left(\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \dots + \frac{1}{\sigma_n^2}\right)$

$= \sum_{j=1}^{n}\left(\frac{\sum_{i=1}^{n} \sigma_i}{\sigma_j}\right)$

1) slide 08

$P = I - \frac{2}{v^t v} v v^t$ e $Q_K = \begin{pmatrix} I & 0 \\ 0 & P \end{pmatrix}$

$Q_K Q_K^t = I$

$\begin{bmatrix} I & 0 \\ 0 & P \end{bmatrix} \begin{bmatrix} I^t & 0 \\ 0 & P^t \end{bmatrix} = I$

$\begin{bmatrix} I I^t & 0 \\ 0 & P P^t \end{bmatrix} = I$

$\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} = I$, logo é ortogonal

$P P^t = I$

$P = \left(I - \frac{2}{v^t v} v v^t\right)$

$\left(I - \frac{2}{v^t v} v v^t\right)\left(I - \frac{2}{v^t v} v v^t\right) = I$

$\left(I - \alpha v v^t\right)\left(I^t - \left[\alpha v v^t\right]^t\right) = I$

$\left(I - \alpha v v^t\right)\left(I - \alpha v v^t\right) = I$

$I - \frac{2}{v^t v} v v^t - \frac{2}{v^t v} v v^t + \left(\frac{2}{v v^t}\right)^2 v v^t = I$

$I - \frac{4}{v^t v} v v^t - \frac{4}{v v^t} v v^t = I$

$I = I$

$Q = \begin{bmatrix} I & 0 \\ 0 & P \end{bmatrix}$ é simétrica

$Q = Q^t$

$\begin{bmatrix} I & 0 \\ 0 & P \end{bmatrix} = \begin{bmatrix} I^t & 0 \\ 0 & P^t \end{bmatrix} = I = I^t$

$P = P^t$

$P^t = \left(I^t - \left(\frac{2}{v v^t} v v^t\right)\right)^t$

$P^t = \left(I - \left(\frac{2}{v^t v} v v^t\right)\right)^t$

$P^t = \left(I^t - (\alpha v v^t)^t\right.$ $\quad (v v^t)^t = (v^t)^t v^t = v v^t$

$P^t = \left(I - \alpha v v^t\right) = I - \frac{2}{v^t v} v v^t = P$

2)

$$A=\begin{pmatrix}-4 & 2 & 3\\ 4 & 3 & 0\\ 2 & 4 & 2\end{pmatrix}$$

$v=(-4,4,2)\rightarrow \|v\|=6$

$w=(6,0,0)$, $a=v-w=(-10,4,2)$ $aa^t=\|a\|=120$

$$H=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}-\frac{1}{60}\begin{bmatrix}100&-40&-20\\-40&16&8\\-20&8&4\end{bmatrix}=\begin{bmatrix}-2/3&2/3&1/3\\2/3&11/15&-2/15\\1/3&-2/15&14/15\end{bmatrix}=Q_1$$

$$\underset{Q_1}{\begin{bmatrix}-2/3&2/3&1/3\\2/3&11/15&-2/15\\1/3&-2/15&14/15\end{bmatrix}}\cdot\underset{A}{\begin{bmatrix}-4&2&3\\4&3&0\\2&4&2\end{bmatrix}}=\begin{bmatrix}6&2&-4/3\\0&3&26/15\\0&4&43/15\end{bmatrix}$$

$v=[3,4]\rightarrow\|v\|=5$

$w=[5,0]$

$a=[3,4]-[5,0]=[-2,4]$

$\|a\|^2=20$

$$H=\begin{bmatrix}1&0\\0&1\end{bmatrix}-\frac{1}{10}\begin{bmatrix}4&-8\\-8&16\end{bmatrix}=\begin{bmatrix}3/5&4/5\\4/5&-3/5\end{bmatrix}$$

$$Q_2=\begin{bmatrix}1&0&0\\0&3/5&4/5\\0&4/5&-3/5\end{bmatrix}$$

$$Q_2Q_1A=\begin{bmatrix}1&0&0\\0&3/5&4/5\\0&4/5&-3/5\end{bmatrix}\begin{bmatrix}6&2&-4/3\\0&3&26/15\\0&4&43/15\end{bmatrix}=\begin{bmatrix}6&2&-4/3\\0&5&10/3\\0&0&-1/3\end{bmatrix}$$

3) $x = \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix}$ $i=1,\ j=2,\ x_i=4,\ x_j=4,\ \operatorname{sen}\theta = \frac{4}{\sqrt{32}},\ \cos\theta = \frac{4}{\sqrt{32}}$

G_1

$$\begin{bmatrix} \frac{4}{\sqrt{32}} & \frac{4}{\sqrt{32}} & 0 \\ -\frac{4}{\sqrt{32}} & \frac{4}{\sqrt{32}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} \sqrt{32} \\ 0 \\ 2 \end{bmatrix}$$

$i=1,\ j=3,\ x_i=\sqrt{32},\ x_j=2$

$\operatorname{sen}\theta = \frac{2}{6}\quad \cos\theta = \frac{\sqrt{32}}{6}$

G_2

$$\begin{bmatrix} \frac{\sqrt{32}}{6} & 0 & 2/6 \\ 0 & 1 & 0 \\ -2/6 & 0 & \frac{\sqrt{32}}{6} \end{bmatrix} \begin{bmatrix} \sqrt{32} \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

Slide 04

2) $A_{m\times n}$, $posto(A) = m$

$$\underset{n\times m}{A^t} = QR$$

$$A = R^t Q^t \qquad R^t = \bar{R},\ Q^t = \bar{Q}$$

$A = \bar{R}\bar{Q}$ sendo $\bar{R}$ triângular inferior

3)

Dado $A \in M(\mathbb{R})_{m\times n}$, então $\exists$ matrizes ortogonais $Q_{m\times m}$, $W_{m\times n}$ e $B_{m\times n}$ bidiagonal inferior.

$$A = Q B W^t$$

Dem: Seja $p = \min\{m, n\}$, então existe $W_1 \in M(\mathbb{R})_{n\times n}$ uma matriz ortogonal que zera a primeira linha de A com excessão do 1º elemento. Então, $A_1 = A \cdot W_1 = \begin{bmatrix} \bar{a}_{11} & 0 \\ \bar{a}_{2:m} & A_1 \end{bmatrix}$

$Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & \bar{Q}_1 \end{bmatrix}$ $\quad \bar{Q}_{1\ m-1\times m-1}$ é uma matriz ortogonal que zera os elementos do vetor $\bar{a}_{2:m}$ exceto o primeiro elemento. Então:

$$A_2 = Q_1 A_1 = \begin{bmatrix} 1 & 0 \\ 0 & \bar{Q}_1 \end{bmatrix} \cdot \begin{bmatrix} \bar{a}_{11} & 0 \\ \bar{a}_{2:m} & \bar{A}_1 \end{bmatrix}$$

$$= \begin{bmatrix} \bar{a}_{11} & 0 \\ \bar{Q}_1 \bar{a}_{2:m} & \bar{Q}_1 \bar{A}_1 \end{bmatrix}$$

$$= \begin{bmatrix} \bar{a}_{11} & 0 & 0 \\ \bar{a}_{12} & \bar{a}_{22} & 0 \\ 0 & \bar{a}_{3m} & \bar{A}_2 \end{bmatrix}$$

4) slide 8

$$\underbrace{\begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}}_{R} \underbrace{\begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix}}_{R^t} \overset{?}{=} I$$

$$\begin{bmatrix} \cos^2\theta + \operatorname{sen}^2\theta & -\operatorname{sen}\theta\cos\theta + \operatorname{sen}\theta\cos\theta \\ -\operatorname{sen}\theta\cos\theta + \operatorname{sen}\theta\cos\theta & \cos^2\theta + \operatorname{sen}^2\theta \end{bmatrix} \overset{?}{=} I$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Para o caso geral

$$\begin{bmatrix} I & & 0 & & 0 \\ & \cos\theta & & \operatorname{sen}\theta & \\ & & I & & 0 \\ & -\operatorname{sen}\theta & & \cos\theta & \\ 0 & & 0 & & I \end{bmatrix} \begin{bmatrix} I & & 0 & & 0 \\ & \cos\theta & & -\operatorname{sen}\theta & \\ 0 & & I & & 0 \\ & \operatorname{sen}\theta & & \cos\theta & \\ 0 & & 0 & & I \end{bmatrix} =$$

$$= \begin{bmatrix} I & & & & 0 \\ & I & & & \\ & & I & & \\ & & & I & \\ 0 & & & & I \end{bmatrix}$$