$$A = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 2 & 2 \\ 4 & 2 & 5 \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

$$L = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 2 & 0 \end{pmatrix} \qquad U = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Gauss-seidel

$M = L + D \qquad N = -U$

$$M = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 2 & 5 \end{bmatrix} \qquad N = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$M^{-1} * N = -(L+U)^{-1} * U$

$$M^{-1} * N = \begin{bmatrix} 0 & 0 & -1/3 \\ 0 & 0 & -2/3 \\ 0 & 0 & 8/15 \end{bmatrix}$$

$\rho(M^{-1}N) = 8/15 = 0,5333\ldots$

Jacobi

$M = D \qquad N = -(L+U)$

$$M = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{pmatrix} \qquad N = \begin{pmatrix} 0 & 0 & -1 \\ -2 & 0 & -2 \\ -4 & -2 & 0 \end{pmatrix}$$

$$M^{-1}N = \begin{pmatrix} 0 & 0 & -1/3 \\ -1 & 0 & -1 \\ -4/5 & -2/5 & 0 \end{pmatrix}$$

$\rho(M^{-1}N) = 0,9024$

SOR ($\omega = 1.2$)

$$M = L + D/\omega = \begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 2 & 0 \end{pmatrix} + \begin{pmatrix} 5/2 & 0 & 0 \\ 0 & 5/3 & 0 \\ 0 & 0 & 25/6 \end{pmatrix}$$

$$= \begin{pmatrix} 5/2 & 0 & 0 \\ 2 & 5/3 & 0 \\ 4 & 2 & 25/6 \end{pmatrix}$$

$$N = \frac{1-\omega}{\omega} D - U = \begin{pmatrix} -1/2 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -5/6 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -1/2 & 0 & -1 \\ 0 & -1/3 & -2 \\ 0 & 0 & -5/6 \end{pmatrix}$$

$$M^{-1}N = \begin{pmatrix} -0,2 & 0 & -0,4 \\ 0,24 & -0,2 & -0,72 \\ 0,0768 & 0,096 & 0,5296 \end{pmatrix}$$

$$\rho(M^{-1}N) = 0,2839$$

Gradiente

$\alpha = 0,24$

$$M = \frac{1}{\alpha} \cdot I_n = \begin{pmatrix} 4,166 & 0 & 0 \\ 0 & 4,166 & 0 \\ 0 & 0 & 4,166 \end{pmatrix}$$

$$N = \frac{1}{\alpha} \cdot I_n - A = \begin{pmatrix} 1,166 & 0 & -1 \\ -2 & 2,166 & -2 \\ -4 & -2 & -0,8333 \end{pmatrix}$$

$$M^{-1}N = I_n - \alpha A = \begin{pmatrix} 0,24 & 0 & -0,24 \\ -0,48 & 0,52 & -0,48 \\ -0,96 & -0,48 & -0,2 \end{pmatrix} \qquad \rho(M^{-1}N) = 0,76$$

$\alpha = 0,25$

$$M = \frac{1}{\alpha} I_n = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$N = \frac{1}{\alpha} I_n - A = \begin{pmatrix} 1 & 0 & -1 \\ -2 & 2 & -2 \\ -4 & -2 & -1 \end{pmatrix}$$

$$M^{-1}N = I_n - \alpha A = \begin{pmatrix} 0,25 & 0 & -0,25 \\ -0,5 & 0,5 & -0,5 \\ -1 & -0,5 & -0,25 \end{pmatrix} \qquad \rho(M^{-1}N) = 0,75$$

$\alpha = 0{,}26$

$$M = \frac{1}{\alpha} I_n = \begin{pmatrix} 3{,}8461 & 0 & 0 \\ 0 & 3{,}8461 & 0 \\ 0 & 0 & 3{,}8461 \end{pmatrix}$$

$$N = \frac{1}{\alpha} I_n - A = \begin{pmatrix} 0{,}8461 & 0 & -1 \\ -2 & 1{,}8461 & -2 \\ -4 & -2 & -1{,}1538 \end{pmatrix}$$

$$M^{-1}N = I_n - \alpha A = \begin{pmatrix} 0{,}22 & 0 & -0{,}26 \\ -0{,}52 & 0{,}48 & -0{,}152 \\ -1{,}04 & -0{,}52 & -0{,}3 \end{pmatrix} \quad \rho(M^{-1}N) = 0{,}82$$