1) Mostre que $AA'A = A$

$\boxed{A = U\Sigma V^+}$ $\boxed{A' = V\Sigma' U^+}$

U, V são ortogonais

$$\begin{cases} V^+ = V^{-1} & (1) \\ U^+ = U^{-1} & (2) \end{cases}$$

$$\Sigma^* \Sigma' = I \quad (3)$$

$$A * A'^* A = A$$

$$(U\Sigma V^+)^*(V\Sigma' U^+)^*(U\Sigma V^+) \stackrel{?}{=} A$$

(1) I

$$I^*(\Sigma V^+)^*(V\Sigma')^*(U\Sigma V^+) \stackrel{?}{=} A$$

(2) I

$$I^* I^*(\Sigma)^*(\Sigma')^*(U\Sigma V^+) \stackrel{?}{=}$$

(3) I

$$I^* I^* I^*(U\Sigma V^+) \stackrel{?}{=} A$$

$$(U\Sigma V^+) \stackrel{?}{=} A$$

$$A = A$$

2) Mostre que $A'*A*A'=A'$ $\quad A'=V\Sigma'U^+ \quad A=U\Sigma V^+$

(1) $V^+=V^{-1}$

(2) $U^+=U^{-1}$

(3) $\Sigma*\Sigma'=I$

$$A'*A*A'=A'$$

$$(U^+\Sigma' V)*(U\Sigma V^+)*(U^+\Sigma' V)=A'$$

(1)

I

$$I*(U^+\Sigma')*(U\Sigma)*(U^+\Sigma' V)=A'$$

(3)

I

$$I*(U^+\Sigma')*U*I*(U^+V)=A'$$

(2)

I

$$I*I*I*(U^+\Sigma' V)=A'$$

$$(U^+\Sigma' V)=A'$$

$$A'=A'$$

3) Se $m=n$ e $\sigma_i \neq 0$ $(i=1,\dots,n)$, então $A' = A^{-1}$.

Dado que $m=n$, $\Sigma' = \begin{bmatrix} 1/\sigma_1 & & & 0 \\ & 1/\sigma_2 & & \\ & & \ddots & \\ 0 & & & 1/\sigma_n \end{bmatrix}_{n\times n}$

Fazendo $\Sigma^* \Sigma'$, obtemos

$$\Sigma^* \Sigma' = \begin{bmatrix} 1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ 0 & & & & 1 \end{bmatrix}_{n\times n}$$

Concluímos que:

$$\Sigma^* \Sigma' = I \text{, então:}$$

$$\Sigma' = \Sigma^{-1} \quad (*)$$

Tomando $A = U \Sigma V^+$ e tirando sua inversa, temos:

$$A^{-1} = U^+ \Sigma^{-1} V$$

Agora dado o item $(*)$, temos

$$A^{-1} = U^+ \Sigma^{-1} V = U^+ \Sigma' V = A'$$

finalmente,

$$A^{-1} = A'$$

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