1)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 4 \\ 2 & 1 & 2 \end{pmatrix} \quad A^2 = \begin{pmatrix} 13 & 11 & 17 \\ 20 & 19 & 29 \\ 9 & 9 & 14 \end{pmatrix} \quad A^3 = \begin{pmatrix} 80 & 76 & 117 \\ 135 & 126 & 194 \\ 64 & 59 & 91 \end{pmatrix}$$

$S_1 = Tr(A)$ $\quad S_2 = Tr(A^2)$ $\quad S_3 = Tr(A^3) =$

$S_1 = 6$ $\quad S_2 = 46$ $\quad S_3 = 297$

$1 \cdot p_1 = S_1$

$p_1 = 6$

$2 p_2 = S_2 - p_1 S_1$

$2 \cdot p_2 = 46 - 6 \cdot 6$

$p_2 = \frac{10}{2}$

$p_2 = 5$

$3 \cdot p_3 = S_3 - p_1 S_2 - p_2 S_1$

$3 \cdot p_3 = 297 - 6 \cdot 46 - 5 \cdot 6$

$3 \cdot p_3 = -9$

$p_3 = -3$

$$P(\lambda) = (-1)^3 (\lambda^3 - p_1\lambda^2 - p_2\lambda - p_3)$$

$$= (-1)^3 (\lambda^3 - 6\lambda - 5\lambda + 3)$$

$$= -\lambda^3 + 6\lambda^2 + 5\lambda - 3$$

$\lambda_1 \cong -1{,}0923$

$\lambda_2 \cong 0{,}41110$

$\lambda_3 \cong 6{,}6812$

2)

$$A_1 = A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$q_1 = Tr(A_1) = 1$$

$$B_1 = A_1 - q_1 I = \begin{pmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

$$p(\lambda) = (-1)^3 \left[\lambda^3 - p_1 \lambda^2 - p_2 \lambda - p_3\right]$$
$$= -1\left[\lambda^3 - \lambda^2 - 2\lambda + 2\right]$$
$$= -\lambda^3 + \lambda^2 + 2\lambda - 2$$

$$\lambda' = 1 \quad \lambda'' = -\sqrt{2} \quad \lambda''' = \sqrt{2}$$

$$1/\lambda' = 1 \quad 1/\lambda'' = -0{,}7071068 \quad 1/\lambda''' = 0{,}7071068$$

$$A_2 = AB_1 = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & -2 & 2 \end{pmatrix}$$

$$q_2 = \frac{Tr(A_2)}{2} = \frac{4}{2}$$

$$B_2 = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 0 & -2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 0 & -2 & 0 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}\begin{pmatrix} -1 & -1 & 1 \\ -1 & -1 & -1 \\ 0 & -2 & 0 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$q_3 = \frac{Tr(A_3)}{3} = \frac{-6}{2} = -2$$

$$Q_1 = \lambda_1^2 I + \lambda_1^1 B_1 + \lambda_1^0 B_2 = \begin{pmatrix} 1 & -2{,}4142 & 2{,}41421 \\ -1 & 241421 & -2{,}41421 \\ 1{,}41421 & -3{,}4142 & 3{,}41421 \end{pmatrix}$$

$$Q_2 = \lambda_2^2 I + \lambda_2 B_1 + B_2 = \begin{pmatrix} 1 & 0{,}41421 & -0{,}41421 \\ -1 & -0{,}41421 & 0{,}41421 \\ -1{,}4142 & -0{,}58578 & 0{,}58578 \end{pmatrix}$$

$$Q_3 = \lambda_3^2 I + \lambda_3 B_1 + B_2 = \begin{pmatrix} 0 & 0 & 0 \\ -1 & -1 & 0 \\ -1 & -1 & 0 \end{pmatrix}$$