Reprova Álgebra Linear - Exercícios 2 e 3

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29 de outubro de 2022

1 Exercício 2

Considere o conjunto S = {(1, 1, 1, 1, 1),(2, 0, 1, 1, 3),(3, 1, 0, 2, 4),(2, 2, 5, 8, 1),(0, 1, 0, 2, 3)}.

- S é LI ou LD?

$$\text{S} = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & \text{-1} & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right] \mathbf{l2}\leftarrow l2 - 1 * l1 \left[\begin{array}{ccccc|c} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{-2} & \mathbf{-2} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{-1} & \mathbf{0} & \mathbf{5} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{2} & \mathbf{8} & \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{3} & \mathbf{4} & \mathbf{-1} & \mathbf{3} & \mathbf{0} \end{array}\right]$$

$$\text{l3}\rightarrow l3 - 1 * l1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & \text{-2} & \text{-2} & 0 & 1 & 0 \\ 0 & \text{-3} & \text{-3} & 3 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right] l4 \rightarrow l4 - 1 * l1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & \text{-2} & \text{-2} & 0 & 1 & 0 \\ 0 & \text{-3} & \text{-3} & 3 & 0 & 0 \\ 0 & \text{-1} & \text{-1} & 6 & 2 & 0 \\ 1 & 3 & 4 & \text{-1} & 3 & 0 \end{array}\right]$$

$$l5 \rightarrow l5 - 1 * l1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & \text{-2} & \text{-2} & 0 & 1 & 0 \\ 0 & \text{-3} & \text{-3} & 3 & 0 & 0 \\ 0 & \text{-1} & \text{-1} & 6 & 2 & 0 \\ 0 & 1 & 1 & \text{-3} & 3 & 0 \end{array}\right] l3 \rightarrow l3 - (3/2) * l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & \text{-2} & \text{-2} & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & \text{(-3/2)} & 0 \\ 0 & \text{-1} & \text{-1} & 6 & 2 & 0 \\ 0 & 1 & 1 & \text{-3} & 3 & 0 \end{array}\right]$$

$$l4 \to l4 - (1/2) * l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (-3/2) & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] l5 \to l5 - (-1/2) * l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (3/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array}\right]$$

$$l4 \to l4 - 2 * l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array}\right] l5 \to l5 - (-1) * l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{array}\right]$$

$$l5 \to l5 - (-4/9) * l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

R: Portanto, é LD

Forma base do R-espaço vetorial R5?
R:Como S é ld, ele não forma uma base de R^5

2 Exercício 3

Considere o conjunto W = {(x, y, z, w, t, u) | $x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R^6$.

Mostre que conjunto W é um subespaço do R-espaço vetorial R^6.

t − x = 0
t = x
y − w − z = 0
y = w + z
x + y + w + z + t + u = 0 $\rightarrow x + w + z + w + z + x + u = 0$
u = -x -y -w -z -t $\rightarrow u = -2x - 2w - 2z$
W = {(x, w + z, z, w, x, - x − w − z − w − z − x)} $\rightarrow$
W = {(x, w + z, z, w, x, - 2x − 2w − 2z)| $x, z, w \in R\}$

I) 0 $\in W para x = 0 z = 0 w = 0$
(w, w, w, w, w, -w) $\rightarrow (x, w + z, z, w, x, -2x–2w–2z)$
= (0, 0, 0, 0, 0, -0)
= 0
Logo, 0 $\in W$

II) u, z $\in W \rightarrow u + z \in W, sendo:$
u = (u1, u2, u3, u4, u5, -u6) $\rightarrow (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1–2w_1–2z_1)$
z = (z1, z2, z3, z4, z5, -z6) $\rightarrow (x_2, w_2 + z_2, z_2, w_2, x_2, -2x_2–2w_2–2z_2)$
u + z = ($x_1 + x_2, (w_1 + z_1) + (w_2 + z2), z_1 + z_2, w_1 + w_2, x_1 + x_2, (-2x_1 - 2w_1 - 2z_2) + (-2x_2 - 2w_2 - 2z_2)$)
u + z= ($x_1 + x_2, w_1 + z_1 + w_2 + z_2, z_1 + z_2, w_1 + w_2, x_1 + x_2, -2x_1 - 2x_2, -2w_1 - 2w_2, -2z_1 - 2z_2$)
Logo, u + z $\in W$

III) a $\in R, v \in W \rightarrow av \in W, sendo:$
v = (v1, v2, v3, v4, v5, -v6) $\rightarrow (x, w + z, z, w, x, -2x–2w–2z)$
av = a . ($x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1$)
av = (a. $x_1, a.w_1 + z_1, a.z_1, a.w_1, a.x_1, a. - 2x_1 - 2w_1 - 2z_1$)
av = ($ax_1, aw_1w_2, az_1, aw_1, ax_1, a - 2x_1 - 2w_1 - 2z_1$)
Logo, av $\in W$.

Logo W é subespaço vetorial de R6.

- O conjunto W = {(x, y, z) | $x, y, z \in R \wedge x - z = 1 \wedge y + x = 0\}$

é um subsespaço vetorial de R3? Esboce graficamente W.

x − z = 1 $\rightarrow x = 1 + z$
y + x = 0 $\rightarrow y + 1 + z = 0 \rightarrow y = -1–z.$
W = {(1 + z, -1 - z, z)}

I) 0 $\in W, para z = 0$

(1 + z, - 1 − z, z) $\rightarrow (1+0,-1-0,0)=(1,-1,0)$.
Logo 0 NÃO pertence a W para z = 0. Portanto, W NÃO é subespaço vetorial.

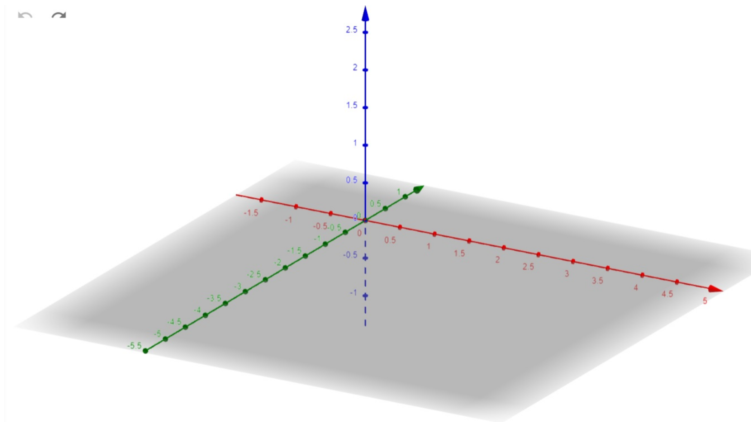

Figure 1: Representação gráfica.

- Invente seu subespaço vetorial em qualquer R n com n maior igual a 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova

W = {(x, y, z) | $x+y-2z=0\}$
x + y -2z = 0
x = -y + 2z
W= (-y + 2z, y, z) | $y,z \in R\}$

I) (0,0,0) $\in W, pois: y,z=0$

II) v, w, x $\in W \rightarrow v+w+x \in W, sendo:$
v = (-y1 + 2z1, y1, z1)
w = (-y2 + 2z2, y2, z2)
z = (-y3 + 2z3, y3, z3)
v + w + z = (-y1 + 2z, y1, -2y1) + (-y2 + 2z2, y2, z2) + (-y3 + 2z3, y3, z3)
u + w + z = (-y1+2z1-y2+2z2-y3+2z3,y1+y2+y3,-2y1+z2+z3)
Logo, u + w + z $\in W$

III) a $\in R, v \in Z \rightarrow av \in W. Sendo:$
v = (v1,v2,v3) $\rightarrow (-y1+2z1, y1, z1)$
a.v = a . (-y1 + 2z1, y1, z1)
a.v = (a . (-y1 +2z1), a . y1, a . z1)
a.v = (-ay1 + a2z1, ay1, -az1)
Logo, av $\in W$

Logo W é subespaço vetorial de R3