

### 1 Exercício 1

Considere as bases do Espaço vetorial R3, A = {(4, 2, 0),(1, 1, 1),(5, 3, 3)} e B = {(1, 2, 1),(1, 5, 2),(1, 0, 1)}. Exiba as matrizes de mudança de base MB→A e MA↔MB. *Escrevatambém os vetores abaixo nas bases indicadas :*

- **v = (0, 1, 2)A em B**
- **v = (1, 3, 1)B em A**

### 2 Mudança MB → M<sub>A(b1)</sub> :

$$x.a_1 + y.a_2 + z.a_3 = b_1$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.( 5, 3, 3) = (1, -2, 1)$$

$$Início \left[ \begin{array}{ccc} 4 & 1 & 5 \\ 2 & -1 & 3 \\ 0 & 1 & 3 \end{array} \right] l2 \rightarrow l2 + l3 \left[ \begin{array}{ccc} 4 & 1 & 5 \\ 2 & 0 & 6 \\ 0 & 1 & 3 \end{array} \right]$$

$$l1 \leftrightarrow l2 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 4 & 1 & 5 \\ 0 & 1 & 3 \end{array} \right] l2 \rightarrow l2 - 2.l1 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & 7 \\ 0 & 1 & 3 \end{array} \right]$$

$$l3 \rightarrow l3 - l2 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & 7 \\ 0 & 0 & 10 \end{array} \right] l2 \rightarrow 10.l2 - 7.l3 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{array} \right]$$

$$l1 \rightarrow 10.l1 - 6.l3 \left[ \begin{array}{ccc} 20 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{array} \right] l1 \rightarrow l1/20 \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{array} \right]$$

$$l2 \rightarrow l2/10 \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{array} \right] l3 \rightarrow l3/10 \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\textbf{Coordenadas M}_B \rightarrow M_{A(b1)} : a = \frac{1}{10}; \textbf{b} = \frac{8}{5}; \textbf{c} = \frac{-1}{5}$$

### 3 Mudança de MB → M<sub>A(b2)</sub> :

$$x.a_1 + y.a_2 + z.a_3 = b_2$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.( 5, 3, 3) = (1, 5, 2)$$

$$Início \left[ \begin{array}{ccc} 4 & 1 & 5 \\ 2 & -1 & 3 \\ 0 & 1 & 3 \end{array} \right] l2 \rightarrow l2 + l3 \left[ \begin{array}{ccc} 4 & 1 & 5 \\ 2 & 0 & 6 \\ 0 & 1 & 3 \end{array} \right]$$

$$l1 \leftrightarrow l2 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 4 & 1 & 5 \\ 0 & 1 & 3 \end{array} \right] l2 \rightarrow l2 - 2.l1 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 1 & 3 \end{array} \right]$$

$$l3 \rightarrow l3 - l2 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 10 \end{array} \right] l3 \rightarrow l3/10 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{array} \right]$$

$$l3 \rightarrow \textbf{simplicando a fração: dividindo por 5} \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{array} \right] l2 \rightarrow l2 + 7.l3 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$l1 \rightarrow l1 - 6.l3 \left[ \begin{array}{ccc} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] l1 \rightarrow l1/2 \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\textbf{Coordenadas M}_B \rightarrow M_{A(b2)} : a = -1; b = \frac{-5}{2}; \textbf{c} = \frac{3}{2}$$

### 4 Mudança de MB → M<sub>A(b3)</sub> :

$$x.a_1 + y.a_2 + z.a_3 = b_3$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.( 5, 3, 3) = (1, 0, 1)$$

$$Início \left[ \begin{array}{ccc} 4 & 1 & 5 \\ 2 & -1 & 3 \\ 0 & 1 & 3 \end{array} \right] l2 \rightarrow l2 + l3 \left[ \begin{array}{ccc} 4 & 1 & 5 \\ 2 & 0 & 6 \\ 0 & 1 & 3 \end{array} \right]$$

$$l1 \leftrightarrow l2 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 4 & 1 & 5 \\ 0 & 1 & 3 \end{array} \right] l2 \rightarrow l2 - 2.l1 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 1 & 3 \end{array} \right]$$

$$l3 \rightarrow l3 - l2 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 10 \end{array} \right] l3 \rightarrow l3/10 \left[ \begin{array}{ccc} 2 & 0 & 6 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{array} \right]$$

$$l1 \rightarrow l1 - 6.l3 \left[ \begin{array}{ccc} 2 & 0 & 0 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{array} \right] l2 \rightarrow l2 + 7.l3 \left[ \begin{array}{ccc} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$l3 \rightarrow \textbf{simplicando a fração: dividindo por 2} \left[ \begin{array}{ccc} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right] l1 \rightarrow l1/2 \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$$

$$\textbf{Coordenadas M}_B \rightarrow M_{A(b3)} : a = \frac{-1}{10}; \textbf{b} = \frac{2}{5}; \textbf{c} = \frac{1}{5}$$

5 Mudança de  $\mathbf{M}_A \rightarrow M_{B(A1)}$

$x.b_1 + y.b_2 + z.b_3 = a_1$

$\mathbf{x}.(1, -2, 1) + \mathbf{y}.(1, 5, 2) + \mathbf{z}.(1, 0, 1) = (4, 2, 0)$

Início  $\left[\begin{array}{rrr} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{array}\right] l3 \rightarrow 5.l3 - 2.l2 \left[\begin{array}{rrr} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{array}\right]$

$l1 \rightarrow 5.l1 - l2 \left[\begin{array}{rrr} 7 & 0 & 2 & 18 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{array}\right] l1 \rightarrow l1 - l3 \left[\begin{array}{rrr} -2 & 0 & 0 & 22 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{array}\right]$

$l2 \rightarrow l2 - l1 \left[\begin{array}{rrr} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 9 & 0 & 5 & -4 \end{array}\right] l3 \rightarrow 2.l3 + 9.l1 \left[\begin{array}{rrr} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{array}\right]$

$l1 \rightarrow l1 / -2 \left[\begin{array}{rrr} 1 & 0 & 0 & -11 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{array}\right] l2 \rightarrow l2/5 \left[\begin{array}{rrr} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 10 & 190 \end{array}\right]$

$l3 \rightarrow l3/10 \left[\begin{array}{rrr} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 19 \end{array}\right]$

Coordenadas  $\mathbf{M}_A \rightarrow M_{B(a1)} : a = -11; b = -4; c = 19$

6 Mudança de  $\mathbf{M}_A \rightarrow M_{B(A2)}$

$x.b_1 + y.b_2 + z.b_3 = a_2$

$\mathbf{x}.(1, -2, 1) + \mathbf{y}.(1, 5, 2) + \mathbf{z}.(1, 0, 1) = (1, -1, 1)$

Início  $\left[\begin{array}{rrr} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 1 & 2 & 1 & 1 \end{array}\right] l3 \rightarrow l3 - l1 \left[\begin{array}{rrr} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{array}\right]$

$l2 \leftrightarrow l3 \left[\begin{array}{rrr} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -2 & 5 & 0 & -1 \end{array}\right] l1 \leftrightarrow l3 \left[\begin{array}{rrr} -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right]$

$l1 \rightarrow -1.l1 \left[\begin{array}{rrr} 2 & -5 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] l1 \rightarrow l1 + 5.l2 \left[\begin{array}{rrr} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right]$

$l1 \rightarrow l1/2 \left[\begin{array}{rrr} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] l3 \rightarrow l3 - l2 \left[\begin{array}{rrr} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{array}\right]$

$l3 \rightarrow l3 - l1 \left[\begin{array}{rrr} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$

Coordenadas  $\mathbf{M}_A \rightarrow M_{B(a2)} : a = \frac{1}{2}; \mathbf{b} = \mathbf{0}; \mathbf{c} = \frac{1}{2}$

7 Mudança de  $\mathbf{M}_A \rightarrow M_{B(A3)}$

$x.b_1 + y.b_2 + z.b_3 = a_3$

$\mathbf{x}.(1, -2, 1) + \mathbf{y}.(1, 5, 2) + \mathbf{z}.(1, 0, 1) = (5, 3, 3)$

Início  $\left[\begin{array}{rrr} 1 & 1 & 1 & 5 \\ -2 & 5 & 0 & 3 \\ 1 & 2 & 1 & 3 \end{array}\right] l3 \rightarrow 5.l3 - 2.l2 \left[\begin{array}{rrr} 1 & 1 & 1 & 5 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{array}\right]$

$l1 \rightarrow 5.l1 - l2 \left[\begin{array}{rrr} 7 & 0 & 5 & 22 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{array}\right] l1 \rightarrow l1 - l3 \left[\begin{array}{rrr} -2 & 0 & 0 & 13 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{array}\right]$

$l2 \rightarrow l2 - l1 \left[\begin{array}{rrr} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 9 & 0 & 5 & 9 \end{array}\right] l3 \rightarrow 2.l3 + 9.l1 \left[\begin{array}{rrr} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{array}\right]$

$l1 \rightarrow l1 / -2 \left[\begin{array}{rrr} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{array}\right] l2 \rightarrow l2/5 \left[\begin{array}{rrr} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 10 & 135 \end{array}\right]$

$l3 \rightarrow l3/10 \left[\begin{array}{rrr} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{135}{10} \end{array}\right] l3 \rightarrow \text{simplicando a fração: dividindo por } 5 \left[\begin{array}{rrr} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{27}{2} \end{array}\right]$

Coordenadas  $\mathbf{M}_A \rightarrow M_{B(a3)} : a = \frac{-13}{2}; \mathbf{b} = -2; \mathbf{c} = \frac{27}{2}$

8  $\mathbf{v} = (1, 3, 1)\mathbf{B}$  em  $\mathbf{A}$

$\mathbf{M_B} \rightarrow_{\mathbf{A}}: \left[\begin{array}{rrr} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{2}{5} \\ \frac{-1}{5} & \frac{3}{2} & \frac{1}{5} \end{array}\right] \cdot \left[\begin{array}{r} 1 \\ 3 \\ -1 \end{array}\right] \rightarrow \left[\begin{array}{rrr} \frac{1}{10}*\mathbf{1} & -1*3 & \frac{-1}{10}* -1 \\ \frac{8}{5}*\mathbf{1} & \frac{-5}{2}*3 & \frac{2}{5}* -1 \\ \frac{-1}{5}*\mathbf{1} & \frac{3}{2}*3 & \frac{1}{5}* -1 \end{array}\right] = \left[\begin{array}{r} \frac{-14}{5} \\ \frac{-63}{10} \\ \frac{41}{10} \end{array}\right]$

9  $\mathbf{v} = (0, 1, 2)\mathbf{A}$  em  $\mathbf{B}$

$\mathbf{M_A} \rightarrow_{\mathbf{B}}: \left[\begin{array}{rrr} -11 & \frac{1}{2} & \frac{-13}{2} \\ 4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{array}\right] \cdot \left[\begin{array}{r} 0 \\ 1 \\ 2 \end{array}\right] \rightarrow \left[\begin{array}{rrr} -11*0 & \frac{1}{2}*1 & \frac{-13}{2}*2 \\ 4*0 & 0*1 & -2*2 \\ 19*0 & \frac{1}{2}*1 & \frac{27}{2}*2 \end{array}\right] = \left[\begin{array}{r} \frac{-26}{2} \\ -4 \\ \frac{55}{2} \end{array}\right]$

10 Exercício 2

Considere o conjunto  $S = \{(1, 1, 1, 1, 1), (2, 0, 1, 1, 3), (3, 1, 0, 2, 4), (2, 2, 5, 8, 1), (0, 1, 0, 2, 3)\}$ .

- $S$  é LI ou LD?

$$S = \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right] \xrightarrow{l_2 \leftarrow l_2 - 1 * l_1} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right]$$

$$\xrightarrow{l_3 \rightarrow l_3 - 1 * l_1} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right] \xrightarrow{l_4 \rightarrow l_4 - 1 * l_1} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right]$$

$$\xrightarrow{l_5 \rightarrow l_5 - 1 * l_1} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array} \right] \xrightarrow{l_3 \rightarrow l_3 - (3/2) * l_2} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array} \right]$$

$$\xrightarrow{l_4 \rightarrow l_4 - (1/2) * l_2} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (3/2) & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array} \right] \xrightarrow{l_5 \rightarrow l_5 - (-1/2) * l_2} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (3/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array} \right]$$

$$\xrightarrow{l_4 \rightarrow l_4 - 2 * l_3} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array} \right] \xrightarrow{l_5 \rightarrow l_5 - (-1) * l_3} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{array} \right]$$

$$\xrightarrow{l_5 \rightarrow l_5 - (-4/9) * l_4} \left[ \begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$R$ : Portanto, é LD

Forma base do R-espaço vetorial  $R^5$ ?  
 $R$ : Como  $S$  é ld, ele não forma uma base de  $R^5$

11 Exercício 3

Considere o conjunto  $W = \{(x, y, z, w, t, u) \mid x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R^6$ .  
Mostre que conjunto  $W$  é um subespaço do  $R$ -espaço vetorial  $R^6$ .

$t - x = 0$   
 $t = x$   
 $y - w - z = 0$   
 $y = w + z$   
 $x + y + w + z + t + u = 0 \rightarrow x + w + z + w + z + x + u = 0$   
 $u = -x - y - w - z - t \rightarrow u = -2x - 2w - 2z$   
 $W = \{(x, w + z, z, w, x, -x - w - z - w - z - x)\} \rightarrow$   
 $W = \{(x, w + z, z, w, x, -2x - 2w - 2z) \mid x, z, w \in R\}$   
  
I)  $0 \in W$   
 $para x = 0 z = 0 w = 0$   
 $(w, w, w, w, w, -w) \rightarrow (x, w + z, z, w, x, -2x - 2w - 2z)$   
 $= (0, 0, 0, 0, 0, -0)$   
 $= 0$   
Logo,  $0 \in W$   
  
II)  $u, z \in W \rightarrow u + z \in W$ , sendo :  
 $u = (u1, u2, u3, u4, u5, -u6) \rightarrow (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$   
 $z = (z1, z2, z3, z4, z5, -z6) \rightarrow (x_2, w_2 + z_2, z_2, w_2, x_2, -2x_2 - 2w_2 - 2z_2)$   
 $u + z = (x_1 + x_2, (w_1 + z_1) + (w_2 + z_2), z_1 + z_2, w_1 + w_2, x_1 + x_2, (-2x_1 - 2w_1 - 2z_2) + (-2x_2 - 2w_2 - 2z_2))$   
 $u + z = (x_1 + x_2, w_1 + z_1 + w_2 + z_2, z_1 + z_2, w_1 + w_2, x_1 + x_2, -2x_1 - 2x_2 - 2w_1 - 2w_2, -2z_1 - 2z_2)$   
Logo,  $u + z \in W$   
  
III)  $a \in R, v \in W \rightarrow av \in W$ , sendo :  
 $v = (v1, v2, v3, v4, v5, -v6) \rightarrow (x, w + z, z, w, x, -2x - 2w - 2z)$   
 $av = a \cdot (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$   
 $av = (a \cdot x_1, a \cdot w_1 + z_1, a \cdot z_1, a \cdot w_1, a \cdot x_1, a \cdot -2x_1 - 2w_1 - 2z_1)$   
 $av = (ax_1, aw_1w_2, az_1, aw_1, ax_1, a - 2x_1 - 2w_1 - 2z_1)$   
Logo,  $av \in W$ .

Logo  $W$  é subespaço vetorial de  $R^6$ .

• O conjunto  $W = \{(x, y, z) \mid x, y, z \in R \wedge x - z = 1 \wedge y + x = 0\}$   
é um subespaço vetorial de  $R^3$ ? Esboce graficamente  $W$ .  
  
 $x - z = 1 \rightarrow x = 1 + z$   
 $y + x = 0 \rightarrow y + 1 + z = 0 \rightarrow y = -1 - z$ .  
 $W = \{(1 + z, -1 - z, z)\}$   
  
I)  $0 \in W$ ,  
 $para z = 0 \rightarrow (1 + 0, -1 - 0, 0) = (1, -1, 0)$ .  
Logo  $0$  NÃO pertence a  $W$  para  $z = 0$ . Portanto,  $W$  NÃO é subespaço vetorial.

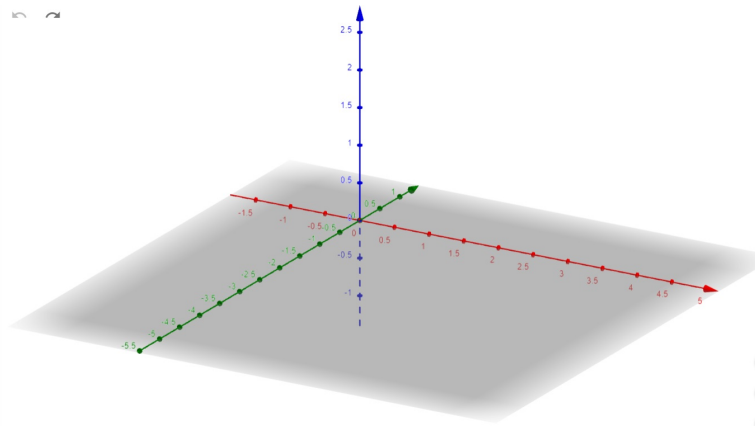


Figure 1: Representação gráfica.

• Invente seu subespaço vetorial em qualquer  $R$  n com n maior igual a 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova  
  
 $W = \{(x, y, z) \mid x + y - 2z = 0\}$   
 $x + y - 2z = 0$   
 $x = -y + 2z$   
 $W = \{-y + 2z, y, z\} \mid y, z \in R\}$   
  
I)  $(0,0,0) \in W$ ,  
 $pois : y, z = 0$   
  
II)  $v, w, x \in W \rightarrow v + w + x \in W$ , sendo :  
 $v = (-y1 + 2z1, y1, z1)$   
 $w = (-y2 + 2z2, y2, z2)$   
 $z = (-y3 + 2z3, y3, z3)$   
 $v + w + z = (-y1 + 2z, y1, -2y1) + (-y2 + 2z2, y2, z2) + (-y3 + 2z3, y3, z3)$   
 $u + w + z = (-y1+2z1-y2+2z2-y3+2z3,y1+y2+y3,-2y1+z2+z3)$   
Logo,  $u + w + z \in W$   
  
III)  $a \in R, v \in Z \rightarrow av \in W$ .Sendo :  
 $v = (v1,v2,v3) \rightarrow (-y1 + 2z1, y1, z1)$   
 $a.v = a \cdot (-y1 + 2z1, y1, z1)$   
 $a.v = (a \cdot (-y1 + 2z1), a \cdot y1, a \cdot z1)$   
 $a.v = (-ay1 + a2z1, ay1, -az1)$   
Logo,  $av \in W$   
  
Logo  $W$  é subespaço vetorial de  $R^3$



$l6 \rightarrow -5/24.$

$$l6 \rightarrow l6 - l5$$

$l6 \rightarrow -4/5.l6$

$$l7 \rightarrow l7 - l6$$

$$l6 \rightarrow l6 - (1,$$

14 → 14 -

12→ 12 -

11 → 11 —

13→ *l*3 –

13→ l3 -

$$I2 \rightarrow I2 - (4.I3) \left[ \begin{array}{c|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} \frac{12a+5c+30b-10g+5c}{25} \\ -480g-1070c-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$I1 \rightarrow I1 + (2.I3) \left[ \begin{array}{c|c} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} \frac{330g+655c-6048a+3080b-1295c+470d-90f}{75} \\ -480g-1070c-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

### 13 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B =  $\frac{216}{5}$ ; -23; 21;  $-\frac{241}{5}$ ;  $\frac{217}{10}$ ; 15;  $\frac{19}{5}$

$$I1 \rightarrow -1.I1 \left[ \begin{array}{c|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} -\frac{330g+655c-6048a+3080b-1295c+470d-90f}{75} \\ -480g-1070c-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$