

1 Exercício 1

Considere as bases do Espaço vetorial R3, A = {(4, 2, 0),(1, 1, 1),(5, 3, 3)} e B = {(1, 2, 1),(1, 5, 2),(1, 0, 1)}. Exiba as matrizes de mudança de base MB→A e MA→B. Escreva também os vetores abaixo nas bases indicadas:

- v = (0, 1, 2)A em B
- v = (1, 3, 1)B em A

2 Mudança B → A(b1) :

$$x.a_1 + y.a_2 + z.a_3 = b_1$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, -2, 1)$$

$$Início \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & -2 \\ 0 & 1 & 3 & 1 \end{array} \right] l2 \rightarrow l2 + l3 \left[\begin{array}{cccc} \mathbf{4} & \mathbf{1} & \mathbf{5} & \mathbf{1} \\ \mathbf{2} & \mathbf{0} & \mathbf{6} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} & \mathbf{1} \end{array} \right]$$

$$l1 \leftrightarrow l2 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 1 \end{array} \right] l2 \rightarrow l2 - 2.l1 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 0 & 1 & 7 & 3 \\ 0 & 1 & 3 & 1 \end{array} \right]$$

$$l3 \rightarrow l3 - l2 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 0 & 1 & 7 & 3 \\ 0 & 0 & 10 & -2 \end{array} \right] l2 \rightarrow 10.l2 - 7.l3 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array} \right]$$

$$l1 \rightarrow 10.l1 - 6.l3 \left[\begin{array}{cccc} 20 & 0 & 0 & 2 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array} \right] l1 \rightarrow l1/20 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array} \right]$$

$$l2 \rightarrow l2/10 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 1 & 0 & \frac{16}{10} \\ 0 & 0 & 10 & -2 \end{array} \right] l3 \rightarrow l3/10 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 1 & 0 & \frac{16}{10} \\ 0 & 0 & 1 & \frac{-2}{10} \end{array} \right]$$

$$\textbf{Coordenadas } \mathbf{M}_B \rightarrow M_{A(b1)} : a = \frac{1}{10}; \mathbf{b} = \frac{8}{5}; \mathbf{c} = \frac{-1}{5}$$

3 Mudança de MB → MA(b2) :

$$x.a_1 + y.a_2 + z.a_3 = b_2$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, 5, 2)$$

$$Início \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 5 \\ 0 & 1 & 3 & 2 \end{array} \right] l2 \rightarrow l2 + l3 \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 7 \\ 0 & 1 & 3 & 2 \end{array} \right]$$

$$l1 \leftrightarrow l2 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 2 \end{array} \right] l2 \rightarrow l2 - 2.l1 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 1 & 3 & 2 \end{array} \right]$$

$$l3 \rightarrow l3 - l2 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 10 & 15 \end{array} \right] l3 \rightarrow l3/10 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{15}{10} \end{array} \right]$$

$$l3 \rightarrow \textbf{simplicando a fração: dividindo por 5} \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right] l2 \rightarrow l2 + 7.l3 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right]$$

$$l1 \rightarrow l1 - 6.l3 \left[\begin{array}{cccc} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right] l1 \rightarrow l1/2 \left[\begin{array}{cccc} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right]$$

$$\textbf{Coordenadas } \mathbf{M}_B \rightarrow M_{A(b2)} : a = -1; b = \frac{-5}{2}; \mathbf{c} = \frac{3}{2}$$

4 Mudança de MB → MA(b3) :

$$x.a_1 + y.a_2 + z.a_3 = b_3$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (5, 3, 3)$$

$$Início \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 0 \\ 0 & 1 & 3 & 1 \end{array} \right] l2 \rightarrow l2 + l3 \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 1 \\ 0 & 1 & 3 & 1 \end{array} \right]$$

$$l1 \leftrightarrow l2 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 4 & 1 & 5 & -1 \\ 0 & 1 & 3 & 1 \end{array} \right] l2 \rightarrow l2 - 2.l1 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 1 & 3 & 1 \end{array} \right]$$

$$l3 \rightarrow l3 - l2 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 10 & 2 \end{array} \right] l3 \rightarrow l3/10 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{array} \right]$$

$$l1 \rightarrow l1 - 6.l3 \left[\begin{array}{cccc} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{array} \right] l2 \rightarrow l2 + 7.l3 \left[\begin{array}{cccc} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{2}{10} \end{array} \right]$$

$$l3 \rightarrow \textbf{simplicando a fração: dividindo por 2} \left[\begin{array}{cccc} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array} \right] l1 \rightarrow l1/2 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{-1}{10} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array} \right]$$

$$\textbf{Coordenadas } \mathbf{M}_B \rightarrow M_{A(b3)} : a = \frac{-1}{10}; \mathbf{b} = \frac{2}{5}; \mathbf{c} = \frac{1}{5}$$

5 Mudança de A → B(A1)

$x.b_1 + y.b_2 + z.b_3 = a_1$

Início $\begin{bmatrix} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix} l3 \rightarrow 5.l3 - 2.l2 \begin{bmatrix} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix}$

$l1 \rightarrow 5.l1 - l2 \begin{bmatrix} 7 & 0 & 2 & 18 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix} l1 \rightarrow l1 - l3 \begin{bmatrix} -2 & 0 & 0 & 22 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix}$

$l2 \rightarrow l2 - l1 \begin{bmatrix} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 9 & 0 & 5 & -4 \end{bmatrix} l3 \rightarrow 2.l3 + 9.l1 \begin{bmatrix} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{bmatrix}$

$l1 \rightarrow l1 / -2 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{bmatrix} l2 \rightarrow l2 / 5 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 10 & 190 \end{bmatrix}$

$l3 \rightarrow l3 / 10 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 19 \end{bmatrix}$

Coordenadas $M_A \rightarrow M_{B(a1)} : a = -11; b = -4; c = 19$

6 Mudança de A → B(A2)

$x.b_1 + y.b_2 + z.b_3 = a_2$

Início $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 1 & 2 & 1 & 1 \end{bmatrix} l3 \rightarrow l3 - l1 \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

$l2 \leftrightarrow l3 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -2 & 5 & 0 & -1 \end{bmatrix} l1 \leftrightarrow l3 \begin{bmatrix} -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$l1 \rightarrow -1.l1 \begin{bmatrix} 2 & -5 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} l1 \rightarrow l1 + 5.l2 \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$l1 \rightarrow l1 / 2 \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} l3 \rightarrow l3 - l2 \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

$l3 \rightarrow l3 - l1 \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$

Coordenadas $M_A \rightarrow M_{B(a2)} : a = \frac{1}{2}; \mathbf{b} = \mathbf{0}; \mathbf{c} = \frac{1}{2}$

7 Mudança de A → B(A3)

$x.b_1 + y.b_2 + z.b_3 = a_3$

Início $\begin{bmatrix} 1 & 1 & 1 & 5 \\ -2 & 5 & 0 & 3 \\ 1 & 2 & 1 & 3 \end{bmatrix} l3 \rightarrow 5.l3 - 2.l2 \begin{bmatrix} 1 & 1 & 1 & 5 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{bmatrix}$

$l1 \rightarrow 5.l1 - l2 \begin{bmatrix} 7 & 0 & 5 & 22 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{bmatrix} l1 \rightarrow l1 - l3 \begin{bmatrix} -2 & 0 & 0 & 13 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{bmatrix}$

$l2 \rightarrow l2 - l1 \begin{bmatrix} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 9 & 0 & 5 & 9 \end{bmatrix} l3 \rightarrow 2.l3 + 9.l1 \begin{bmatrix} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{bmatrix}$

$l1 \rightarrow l1 / -2 \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{bmatrix} l2 \rightarrow l2 / 5 \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 10 & 135 \end{bmatrix}$

$l3 \rightarrow l3 / 10 \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{135}{10} \end{bmatrix} l3 \rightarrow \text{simplicando a fração: dividindo por 5} \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{27}{2} \end{bmatrix}$

Coordenadas $M_A \rightarrow M_{B(a3)} : a = \frac{-13}{2}; \mathbf{b} = \mathbf{-2}; \mathbf{c} = \frac{27}{2}$

8 v = (1, 3, 1)B em A

$M_B \rightarrow_A: \begin{bmatrix} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{3}{5} \\ \frac{-1}{5} & \frac{3}{5} & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{10}*\mathbf{1} & -1*3 & \frac{-1}{10}* -1 \\ \frac{8}{5}*\mathbf{1} & \frac{-5}{2}*3 & \frac{3}{5}*\mathbf{-1} \\ \frac{-1}{5}*\mathbf{1} & \frac{3}{5}*3 & \frac{1}{5}* -1 \end{bmatrix} = \begin{bmatrix} \frac{-14}{5} \\ \frac{-13}{10} \\ \frac{41}{10} \end{bmatrix}$

9 v = (0, 1, 2)A em B

$M_A \rightarrow_B: \begin{bmatrix} -11 & \frac{1}{2} & \frac{-13}{2} \\ 4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} -11*0 & \frac{1}{2}*1 & \frac{-13}{2}*2 \\ 4*0 & 0*1 & -2*2 \\ 19*0 & \frac{1}{2}*1 & \frac{27}{2}*2 \end{bmatrix} = \begin{bmatrix} \frac{-23}{2} \\ -4 \\ \frac{55}{2} \end{bmatrix}$

Mostre que o conjunto $\{(1, 1, 1, 1, 0, 0, 1, 1), (1, 0, 0, 1, 1, 1, 1, 0), (2, 2, 1, 1, 1, 1, 1), (1, 0, 0, 0, 1, 2, 1, 1), (2, 0, 0, 2, 0, 2, 0, 2), (1, 1, 1, 1, 1, 1, 1, 1), (3, 0, 2, 0, 2, 1, 2)\}$ forma uma base para o Espaço vetorial R^8 . Escreva o vetor $(0, 1, 1, 1, 1, 0, 1)$ nesta base.

$l6 \rightarrow -5/24.$

$$l6 \rightarrow l6 - l5$$

$$l6 \rightarrow -4/5.l6$$

$$l7 \rightarrow l7 - l6$$

$$l6 \rightarrow l6 - (1,$$

$$l4 \rightarrow l4 - (16$$

12→ 12 -

11 → 11 —

13→ *l*3 −

13→ *l*3 –

$$I2 \rightarrow I2 - (4.I3) \left[\begin{array}{c|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} \frac{12a+5e+30b-10g+5c}{25} \\ -480g-1070e-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5e-5c+38a-5b}{25} \\ -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$I1 \rightarrow I1 + (2.I3) \left[\begin{array}{c|c} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} \frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\ -480g-1070e-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5e-5c+38a-5b}{25} \\ -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$I1 \rightarrow -1.I1 \left[\begin{array}{c|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} -\frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\ -480g-1070e-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\ \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5e-5c+38a-5b}{25} \\ -\frac{120e-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

11 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B = $\frac{216}{5}$; -23; 21; $-\frac{241}{5}$; $\frac{217}{10}$; 15; $\frac{19}{5}$