Reprova Álgebra Linear - Exercícios 2 e 3

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1 Exercício 1

Considere as bases do Respaço vetorial R3, A = {(4, 2, 0),(1, 1, 1),(5, 3, 3)} e B = {(1, 2, 1),(1, 5, 2),(1, 0, 1)}. Exiba as matrizes de mudança de base MB→A e MA→B. Escreva também os vetores abaixo nas bases indicadas:
• v = (0, 1, 2)A em B
• v = (1, 3, 1)B em A

2 Mudança B $\rightarrow A(b1)$:

$x.a_1 + y.a_2 + z.a_3 = b_1$

x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, -2, 1)

$$Inicio \begin{bmatrix} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & -2 \\ 0 & 1 & 3 & 1 \end{bmatrix} \mathbf{l2} \rightarrow l2 + l3 \begin{bmatrix} \mathbf{4} & \mathbf{1} & \mathbf{5} & \mathbf{1} \\ \mathbf{2} & \mathbf{0} & \mathbf{6} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} & \mathbf{1} \end{bmatrix}$$

$$l1 \leftrightarrow l2 \begin{bmatrix} 2 & 0 & 6 & -1 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix} l2 \rightarrow l2 - 2.l1 \begin{bmatrix} 2 & 0 & 6 & -1 \\ 0 & 1 & 7 & 3 \\ 0 & 1 & 3 & 1 \end{bmatrix}$$

$$l3 \rightarrow l3 - l2 \begin{bmatrix} 2 & 0 & 6 & -1 \\ 0 & 1 & 7 & 3 \\ 0 & 0 & 10 & -2 \end{bmatrix} l2 \rightarrow 10.l2 - 7l3 \begin{bmatrix} 2 & 0 & 6 & -1 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{bmatrix}$$

$$l1 \rightarrow 10.l1 - 6.l3 \begin{bmatrix} 20 & 0 & 0 & 2 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{bmatrix} l1 \rightarrow l1/20 \begin{bmatrix} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{bmatrix}$$

$$l2 \rightarrow l2/10 \begin{bmatrix} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 1 & 0 & \frac{16}{10} \\ 0 & 0 & 10 & -2 \end{bmatrix} l3 \rightarrow l3/10 \begin{bmatrix} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 1 & 0 & \frac{16}{10} \\ 0 & 0 & 1 & \frac{-2}{10} \end{bmatrix}$$

Coordenadas M$_B \rightarrow M_{A(b1)} : a = \frac{1}{10}$**; b =** $\frac{8}{5}$**; c =** $\frac{-1}{5}$

3 Mudança de M$_B \rightarrow M_{A(b2)}$:

$x.a_1 + y.a_2 + z.a_3 = b_2$

x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, 5, 2)

$$\text{Início} \begin{bmatrix} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix} \text{l2}\rightarrow l2 + l3 \begin{bmatrix} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 7 \\ 0 & 1 & 3 & 2 \end{bmatrix}$$

$$\text{l1}\leftrightarrow \text{12} \begin{bmatrix} 2 & 0 & 6 & 7 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 2 \end{bmatrix} l2 \rightarrow l2 - 2.l1 \begin{bmatrix} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 1 & 3 & 2 \end{bmatrix}$$

$$l3 \rightarrow 13 - l2 \begin{bmatrix} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 10 & 15 \end{bmatrix} l3 \rightarrow l3/10 \begin{bmatrix} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{15}{10} \end{bmatrix}$$

$$l3 \rightarrow \textbf{simplificando a fração: dividindo por 5} \begin{bmatrix} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{3}{2} \end{bmatrix} l2 \rightarrow l2 + 7.l3 \begin{bmatrix} 2 & 0 & 6 & 7 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{bmatrix}$$

$$l1 \rightarrow l1 - 6.l3 \begin{bmatrix} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{bmatrix} l1 \rightarrow l1/2 \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{bmatrix}$$

Coordenadas M$_B \rightarrow M_{A(b2)} : a = -1; b = \frac{-5}{2}$**; c =** $\frac{3}{2}$

4 Mudança de M$_B \rightarrow M_{A(b3)}$:

$x.a_1 + y.a_2 + z.a_3 = b_3$

x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (5, 3, 3)

$$\text{Inicio} \begin{bmatrix} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix} l2 \rightarrow l2 + l3 \begin{bmatrix} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 1 \\ 0 & 1 & 3 & 1 \end{bmatrix}$$

$$\text{l1} \leftrightarrow l2 \begin{bmatrix} 2 & 0 & 6 & 1 \\ 4 & 1 & 5 & -1 \\ 0 & 1 & 3 & 1 \end{bmatrix} l2 \rightarrow l2 - 2.l1 \begin{bmatrix} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 1 & 3 & 1 \end{bmatrix}$$

$$\text{l3} \rightarrow l3 - l2 \begin{bmatrix} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 10 & 2 \end{bmatrix} l3 \rightarrow l3/10 \begin{bmatrix} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{bmatrix}$$

$$\text{l1} \rightarrow l1 - 6.l3 \begin{bmatrix} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{bmatrix} l2 \rightarrow l2 + 7.l3 \begin{bmatrix} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{2}{10} \end{bmatrix}$$

$$\text{l3} \rightarrow \textbf{simplificando a fração: dividindo por 2} \begin{bmatrix} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{bmatrix} l1 \rightarrow l1/2 \begin{bmatrix} 1 & 0 & 0 & \frac{-1}{10} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{bmatrix}$$

Coordenadas M$_B \rightarrow M_{A(b3)} : a = \frac{-1}{10}$**; b =** $\frac{2}{5}$**; c =** $\frac{1}{5}$

5 Mudança de A $\to B_{(A1)}$

$x.b_1 + y.b_2 + z.b_3 = a_1$

$$\text{Início} \begin{bmatrix} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix} l3 \to 5.l3 - 2.l2 \begin{bmatrix} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix}$$

$$\text{l1} \to 5.l1 - l2 \begin{bmatrix} 7 & 0 & 2 & 18 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix} l1 \to l1 - l3 \begin{bmatrix} -2 & 0 & 0 & 22 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix}$$

$$\text{l2} \to l2 - l1 \begin{bmatrix} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 9 & 0 & 5 & -4 \end{bmatrix} l3 \to 2.l3 + 9.l1 \begin{bmatrix} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{bmatrix}$$

$$\text{l1} \to l1/-2 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{bmatrix} l2 \to l2/5 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 10 & 190 \end{bmatrix}$$

$$\text{l3} \to l3/10 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 19 \end{bmatrix}$$

Coordenadas $\mathbf{M}_A \to M_{B(a1)} : a = -11; b = -4; c = 19$

6 Mudança de A $\to B_{(A2)}$

$x.b_1 + y.b_2 + z.b_3 = a_2$

$$\textbf{Início} \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{1} \end{bmatrix} l3 \to l3 - l1 \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\textbf{l2} \leftrightarrow l3 \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{-1} \end{bmatrix} l1 \leftrightarrow l3 \begin{bmatrix} \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

$$\textbf{l1} \to -1.l1 \begin{bmatrix} \mathbf{2} & \mathbf{-5} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix} l1 \to l1 + 5.l2 \begin{bmatrix} \mathbf{2} & \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

$$\textbf{l1} \to l1/2 \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix} l3 \to l3 - l2 \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{1} \end{bmatrix}$$

$$\textbf{l3} \to l3 - l1 \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{1}{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{1}{2} \end{bmatrix}$$

Coordenadas $\mathbf{M}_A \to M_{B(a2)} : a = \frac{1}{2}$**; b = 0; c =** $\frac{1}{2}$

7 Mudança de A $\to B_{(A3)}$

$x.b_1 + y.b_2 + z.b_3 = a_3$

$$\textbf{Início} \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{5} \\ \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{3} \\ \mathbf{1} & \mathbf{2} & \mathbf{1} & \mathbf{3} \end{bmatrix} l3 \to 5.l3 - 2.l2 \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{5} \\ \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{3} \\ \mathbf{9} & \mathbf{0} & \mathbf{5} & \mathbf{9} \end{bmatrix}$$

$$\textbf{l1} \to 5.l1 - l2 \begin{bmatrix} \mathbf{7} & \mathbf{0} & \mathbf{5} & \mathbf{22} \\ \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{3} \\ \mathbf{9} & \mathbf{0} & \mathbf{5} & \mathbf{9} \end{bmatrix} l1 \to l1 - l3 \begin{bmatrix} \mathbf{-2} & \mathbf{0} & \mathbf{0} & \mathbf{13} \\ \mathbf{-2} & \mathbf{5} & \mathbf{0} & \mathbf{3} \\ \mathbf{9} & \mathbf{0} & \mathbf{5} & \mathbf{9} \end{bmatrix}$$

$$\textbf{l2} \to l2 - l1 \begin{bmatrix} \mathbf{-2} & \mathbf{0} & \mathbf{0} & \mathbf{13} \\ \mathbf{0} & \mathbf{5} & \mathbf{0} & \mathbf{-10} \\ \mathbf{9} & \mathbf{0} & \mathbf{5} & \mathbf{9} \end{bmatrix} l3 \to 2.l3 + 9.l1 \begin{bmatrix} \mathbf{-2} & \mathbf{0} & \mathbf{0} & \mathbf{13} \\ \mathbf{0} & \mathbf{5} & \mathbf{0} & \mathbf{-10} \\ \mathbf{0} & \mathbf{0} & \mathbf{10} & \mathbf{135} \end{bmatrix}$$

$$\textbf{l1} \to l1/-2 \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{-13}{2} \\ \mathbf{0} & \mathbf{5} & \mathbf{0} & \mathbf{-10} \\ \mathbf{0} & \mathbf{0} & \mathbf{10} & \mathbf{135} \end{bmatrix} l2 \to l2/5 \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{-13}{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{-2} \\ \mathbf{0} & \mathbf{0} & \mathbf{10} & \mathbf{135} \end{bmatrix}$$

$$\textbf{l3} \to l3/10 \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{-13}{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{-2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{135}{10} \end{bmatrix} l3 \to \textbf{simplificando a fração: dividindo por 5} \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} & \frac{-13}{2} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{-2} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \frac{27}{2} \end{bmatrix}$$

Coordenadas $\mathbf{M}_A \to M_{B(a3)} : a = \frac{-13}{2}$**; b = -2; c =** $\frac{27}{2}$

8 v = (1, 3, 1)B em A

$$\mathbf{M_B} \to_{\mathbf{A}}: \begin{bmatrix} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{2}{5}) \\ \frac{-1}{5} & \frac{3}{2} & \frac{1}{5} \end{bmatrix} . \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \to \begin{bmatrix} \frac{1}{10}\mathbf{*1} & -1 * 3 & \frac{-1}{10} * -1 \\ \frac{8}{5}\mathbf{*1} & \frac{-5}{2}\mathbf{*3} & \frac{2}{5}\mathbf{*\text{-}1} \\ \frac{-1}{5}\mathbf{*1} & \frac{3}{2} * 3 & \frac{1}{5} * -1 \end{bmatrix} = \begin{bmatrix} \frac{-14}{5} \\ \frac{-63}{10} \\ \frac{41}{10} \end{bmatrix}$$

9 v = (0, 1, 2)A em B

$$\mathbf{M}_A \to_B: \begin{bmatrix} -11 & \frac{1}{2} & \frac{-13}{2} \\ 4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix} . \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \to \begin{bmatrix} -11 * 0 & \frac{1}{2} * 1 & \frac{-13}{2} * 2 \\ 4 * 0 & 0 * 1 & -2 * 2 \\ 19 * 0 & \frac{1}{2} * 1 & \frac{27}{2} * 2 \end{bmatrix} = \begin{bmatrix} \frac{-25}{2} \\ -4 \\ \frac{55}{2} \end{bmatrix}$$

10 Exercício 2

Considere o conjunto S = {(1, 1, 1, 1, 1),(2, 0, 1, 1, 3),(3, 1, 0, 2, 4),(2, 2, 5, 8, 1),(0, 1, 0, 2, 3)}.

- **S é LI ou LD?**

$$\mathbf{S} = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] \mathbf{l2}\leftarrow l2 - 1 * l1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right]$$

$$\mathbf{l3}\rightarrow l3 - 1 * l1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right] l4 \rightarrow l4 - 1 * l1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array}\right]$$

$$l5 \rightarrow l5 - 1 * l1 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] l3 \rightarrow l3 - (3/2) * l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right]$$

$$l4 \rightarrow l4 - (1/2) * l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (-3/2) & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array}\right] l5 \rightarrow l5 - (-1/2) * l2 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (3/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array}\right]$$

$$l4 \rightarrow l4 - 2 * l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array}\right] l5 \rightarrow l5 - (-1) * l3 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{array}\right]$$

$$l5 \rightarrow l5 - (-4/9) * l4 \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

R: Portanto, é LD

Forma base do R-espaço vetorial R5?
R:Como S é ld, ele não forma uma base de R^5

11 Exercício 3

Considere o conjunto W = {(x, y, z, w, t, u) | $x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R^6$.
Mostre que conjunto W é um subespaço do R-espaço vetorial R^6.

t – x = 0
t = x
y – w – z = 0
y = w + z
x + y + w + z + t + u = 0 $\rightarrow x + w + z + w + z + x + u = 0$
u = -x -y -w -z -t $\rightarrow u = -2x - 2w - 2z$
W = {(x, w + z, z, w, x, - x – w – z – w – z – x)} $\rightarrow$
W = {(x, w + z, z, w, x, - 2x – 2w – 2z)| $x, z, w \in R\}$

I) $0 \in W para x = 0 z = 0 w = 0$
(w, w, w, w, w, -w) $\rightarrow (x, w + z, z, w, x, -2x–2w–2z)$
= (0, 0, 0, 0, 0, -0)
= 0
Logo, $0 \in W$

II) u, z $\in W \rightarrow u + z \in W, sendo:$
u = (u1, u2, u3, u4, u5, -u6) $\rightarrow (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1–2w_1–2z_1)$
z = (z1, z2, z3, z4, z5, -z6) $\rightarrow (x_2, w_2 + z_2, z_2, w_2, x_2, -2x_2–2w_2–2z_2)$
u + z = ($x_1 + x_2, (w_1 + z_1) + (w_2 + z2), z_1 + z_2, w_1 + w_2, x_1 + x_2, (-2x_1 - 2w_1 - 2z_2) + (-2x_2 - 2w_2 - 2z_2))$
u + z= ($x_1 + x_2, w_1 + z_1 + w_2 + z_2, z_1 + z_2, w_1 + w_2, x_1 + x_2, -2x_1 - 2x_2, -2w_1 - 2w_2, -2z_1 - 2z_2)$
Logo, u + z $\in W$

III) a $\in R, v \in W \rightarrow av \in W, sendo:$
v = (v1, v2, v3, v4, v5, -v6) $\rightarrow (x, w + z, z, w, x, -2x–2w–2z)$
av = a . ($x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$
av = (a. $x_1, a.w_1 + z_1, a.z_1, a.w_1, a.x_1, a. - 2x_1 - 2w_1 - 2z_1)$
av = (a$x_1, aw_1w_2, az_1, aw_1, ax_1, a - 2x_1 - 2w_1 - 2z_1)$
Logo, av $\in W$.

Logo W é subespaço vetorial de R6.

- O conjunto W = {(x, y, z) | $x, y, z \in R \wedge x - z = 1 \wedge y + x = 0\}$
é um subsespaço vetorial de R3? Esboce graficamente W.

x – z = 1 $\rightarrow x = 1 + z$
y + x = 0 $\rightarrow y + 1 + z = 0 \rightarrow y = -1–z.$
W = {(1 + z, -1 - z, z)}

I) $0 \in W, para z = 0$
(1 + z, - 1 – z, z) $\rightarrow (1 + 0, -1–0, 0) = (1, -1, 0).$
Logo 0 NÃO pertence a W para z = 0. Portanto, W NÃO é subespaço vetorial.

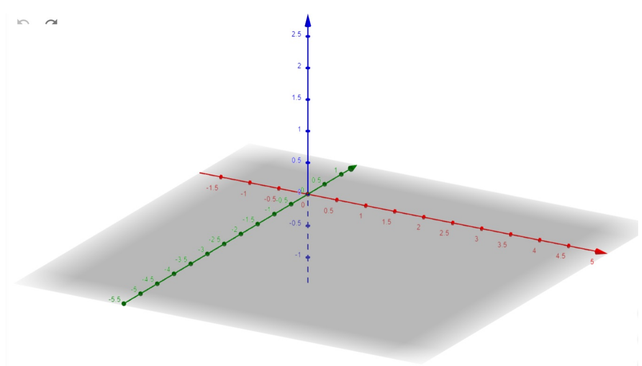

Figure 1: Representação gráfica.

- Invente seu subespaço vetorial em qualquer R n com n maior igual a 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova

W = {(x, y, z) | $x + y - 2z = 0\}$
x + y -2z = 0
x = -y + 2z
W= (-y + 2z, y, z) | $y, z \in R\}$

I) (0,0,0) $\in W, pois: y, z = 0$

II) v, w, x $\in W \rightarrow v + w + x \in W, sendo:$
v = (-y1 + 2z1, y1, z1)
w = (-y2 + 2z2, y2, z2)
z = (-y3 + 2z3, y3, z3)
v + w + z = (-y1 + 2z, y1, -2y1) + (-y2 + 2z2, y2, z2) + (-y3 + 2z3, y3, z3)
u + w + z = (-y1+2z1-y2+2z2-y3+2z3,y1+y2+y3,-2y1+z2+z3)
Logo, u + w + z $\in W$

III) a $\in R, v \in Z \rightarrow av \in W. Sendo:$
v = (v1,v2,v3) $\rightarrow (-y1 + 2z1, y1, z1)$
a.v = a . (-y1 + 2z1, y1, z1)
a.v = (a . (-y1 +2z1), a . y1, a . z1)
a.v = (-ay1 + a2z1, ay1, -az1)
Logo, av $\in W$

Logo W é subespaço vetorial de R3

12 Exercício 4

Mostre que o conjunto $\{(1, 1, 1, 1, 0, 1, 1),(1, 0, 1, 1, 1, 1, 0),(2, 2, 1, 1, 1, 1, 1), (1, 0, 0, 1, 2, 1, 1),(2, 0, 2, 0, 2, 0, 2),(1, 1, 1, 1, 1, 1, 1),(3, 0, 2, 0, 2, 1, 2)\}$ forma uma base para o Respaço vetorial R7. Escreva o vetor (0, 1, 1, 1, 1, 0, 1) nesta base.

$$\text{Início}\left[\begin{array}{ccccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 1 & 0 & 2 & 0 & 0 & -1 & 0 & b \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 & c \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & d \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & f \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & g \end{array}\right]$$

$$l2 \to l2 + l1\left[\begin{array}{ccccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 1 & -1 & 1 & 0 & 2 & -1 & 2 & c \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & d \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & f \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & g \end{array}\right]$$

$$\text{l3}\to l3 + l1\left[\begin{array}{ccccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 1 & 1 & 1 & 1 & 0 & -1 & 0 & d \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & f \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & g \end{array}\right]$$

$$l4 \to l4 + l1\left[\begin{array}{ccccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & d+a \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 1 & -1 & 1 & 1 & 0 & -1 & -1 & f \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & g \end{array}\right]$$

$$\text{l6}\to l6 + l1\left[\begin{array}{ccccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & d+a \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 1 & 0 & 1 & 1 & 2 & 1 & 2 & g \end{array}\right] l7 \to l7 + l1\left[\begin{array}{ccccccc|c} -1 & 1 & 2 & 1 & 2 & 1 & 3 & a \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & d+a \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{array}\right]$$

$$l1 \to l1 - l2\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 2 & 3 & 2 & 2 & 0 & 3 & d+a \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{array}\right] l4 \to l4/2\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 1 & 3/2 & 0 & 0 & 0 & 3/2 & d+a/2 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{array}\right]$$

$$l4 \to l4 - l2\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/2 \\ 0 & 1 & -1 & 2 & 2 & -1 & 2 & e \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{array}\right] l5 \to l5 - l2\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/2 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 1 & 3 & 2 & 4 & 2 & 5 & g+a \end{array}\right]$$

$$l7 \to l7 - l2\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 3 & 1 & 4 & 0 & 5 & c+a \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/2 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right] l3 \to l3/3\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & -5/2 & 0 & -1 & 0 & -3/2 & d+3a-2b/5 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right]$$

$$l4 \to -2.5.l4\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 1 & 0 & 2/5 & 0 & 3/5 & -d+3a-2b/5 \\ 0 & 0 & -5 & 1 & 0 & -1 & -1 & e-b+a/5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & f+a \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right] l5 \to l5/5\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 1 & 0 & 2/5 & 0 & 3/5 & -d+3a-2b/5 \\ 0 & 0 & -1 & 1/5 & 0 & -1/5 & -1/5 & e-b+a/5 \\ 0 & 0 & 3 & 2 & 2 & 0 & 2 & b+a/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right]$$

$$l6 \to l6/3\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 1 & 0 & 2/5 & 0 & 3/5 & \frac{-3d-14a+6b-2c}{15} \\ 0 & 0 & -1 & 1/5 & 0 & -1/5 & -1/5 & e-b+a/5 \\ 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 & f+a/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right] l4 \to l4 - l3\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 & -3d-14a+6b-3c \\ 0 & 0 & -1 & 1/5 & 0 & -1/5 & -1/5 & e-b+a/5 \\ 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 & f+a/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right]$$

$$l5 \to l5 + l3\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 & \frac{-3d-14a+6b-3c}{15} \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 & \frac{8a+3e-3b+3c}{15} \\ 0 & 0 & 1 & 2/3 & 2/3 & 0 & 2/3 & f+a/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right] l6 \to l6 - l3\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 & \frac{-3d-14a+6b-3c}{15} \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 & \frac{8a+3e-3b+3c}{15} \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 & f-c/3 \\ 0 & 0 & -1 & 1 & 2 & 2 & 2 & g+2a-b \end{array}\right]$$

$$l7 \to l7 + l3\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & -1/3 & -14/15 & 0 & 16/15 & \frac{-3d-14a+6b-3c}{15} \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 & \frac{8a+3e-3b+3c}{15} \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 & f-c/3 \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{array}\right] l4 \to 3.l4\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-3c}{5} \\ 0 & 0 & 0 & 8/15 & 4/3 & -1/5 & 22/15 & \frac{8a+3e-3b+3c}{15} \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 & f-c/3 \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{array}\right]$$

$$l5 \to 15/8.l5\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-3c}{5} \\ 0 & 0 & 0 & 1 & 5/2 & -3/8 & 11/4 & \frac{8a+3e-3b+3c}{8} \\ 0 & 0 & 0 & 1/3 & -2/3 & 0 & -1 & f-c/3 \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{array}\right] l6 \to 3.l6\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-}{5} \\ 0 & 0 & 0 & 1 & 5/2 & -3/8 & 11/4 & \frac{8a+3e-3b+3c}{8} \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & f-c \\ 0 & 0 & 0 & 4/3 & 10/3 & 2 & 11/3 & \frac{3g+7a-3b+c}{3} \end{array}\right]$$

$$l5 \to l5 - l4\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-}{5} \\ 0 & 0 & 0 & 0 & -3/10 & -3/8 & -9/20 & \frac{8a+3e-3b+3c}{8} \\ 0 & 0 & 0 & 1 & -2 & 0 & -3 & f-c \\ 0 & 0 & 0 & 1 & 5/2 & 3/2 & 11/4 & \frac{3g+7a-3b+c}{4} \end{array}\right] l6 \to l6 - l4\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & c+a/3 \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & \frac{-3d-14a+6b-}{5} \\ 0 & 0 & 0 & 0 & -3/10 & -3/8 & -9/20 & \frac{-72a+13e-+55b+15c-24d}{40} \\ 0 & 0 & 0 & 0 & -24/5 & 0 & -31/5 & f-c \\ 0 & 0 & 0 & 1 & 5/2 & 3/2 & 11/4 & \frac{3g+7a-3b+c}{4} \end{array}\right]$$

$$l7 \to l7 - l4\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 0 & -3/10 & -3/8 & -9/20 & \frac{-72a+15e+33b-15c-24d}{40} \\ 0 & 0 & 0 & 0 & -24/5 & 0 & -31/5 & \frac{5f-10c-3d-14a+6b}{5} \\ 0 & 0 & 0 & 0 & -3/10 & 3/2 & -9/20 & \frac{15g-21a+9b-15c-12d}{20} \end{array}\right] l5 \to -10/3.l4\left[\begin{array}{ccccccc|c} -1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\ 0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\ 0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\ 0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\ 0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\ 0 & 0 & 0 & 0 & -24/5 & 0 & -31/5 & \frac{5f-10c-3d-14a+6b}{5} \\ 0 & 0 & 0 & 0 & -3/10 & 3/2 & -9/20 & \frac{15g-21a+9b-15c-12d}{20} \end{array}\right]$$

$$l6 \to -5/24.l6 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 1 & 0 & 31/24 & -\frac{5f-10c-3d-14a+6b}{24} \\
0 & 0 & 0 & 0 & -3/10 & 3/2 & -9/20 & \frac{15g-21a+9b-15c-12d}{20}
\end{array}\right] l7 \to -10/3.l7 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 1 & 0 & 31/24 & -\frac{5f-10c-3d-14a+6b}{24} \\
0 & 0 & 0 & 0 & 1 & -5 & 3/2 & -\frac{5g+7a+3b-5c-4d}{2}
\end{array}\right]$$

$$l6 \to l6 - l5 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 0 & -5/4 & -5/24 & \frac{-20c+30e-5f-45d-130a+60b}{24} \\
0 & 0 & 0 & 0 & 1 & -5 & 3/2 & -\frac{5g+7a+3b-5c-4d}{2}
\end{array}\right] l7 \to l7 - l5 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 0 & -5/4 & -5/24 & \frac{-20c+30e-5f-45d-130a+60b}{24} \\
0 & 0 & 0 & 0 & 0 & -25/4 & 0 & \frac{-38a+5e-10g+5b+5c}{4}
\end{array}\right]$$

$$l6 \to -4/5.l6 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 0 & 1 & 1/6 & -\frac{-4c+6e-f-9d-26a+12b}{6} \\
0 & 0 & 0 & 0 & 0 & -25/4 & 0 & \frac{-38a+5e-10g+5b+5c}{4}
\end{array}\right] l7 \to -4/25.l7 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 0 & 1 & 1/6 & -\frac{-4c+6e-f-9d-26a+12b}{6} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{-38a+5e-10g+5b+5c}{4}
\end{array}\right]$$

$$l7 \to l7 - l6 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 0 & 1 & 1/6 & -\frac{-4c+6e-f-9d-26a+12b}{6} \\
0 & 0 & 0 & 0 & 0 & 0 & -1/6 & \frac{120e-422a+270b-130c-225d+60g-25f}{150}
\end{array}\right] l7 \to -6.l7 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & -\frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 0 & 1 & 1/6 & -\frac{-4c+6e-f-9d-26a+12b}{6} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$l6 \to l6 - (1/6.l7) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 3/2 & \frac{-24a+5e+11b-5c-8d}{4} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right] l5 \to l5 - (3/2.l7) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 16/5 & -\frac{-3d-14a+6b-5c}{5} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 0 & \frac{595e-1932a+1345b-655c+1150d+360g-150f}{100} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$l4 \to l4 - (16/5.l7) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 5/3 & \frac{c+a}{3} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 0 & \frac{1920e-3525d-3402a+4170b-1955c+960g-400f}{125} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 0 & \frac{595e-1932a+1345b-655c+1150d+360g-150f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right] l3 \to l3 - (5/3.l7) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 3 & b+a \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 0 & \frac{120e-125c-417a+270b-255d+60g-25f}{15} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 0 & \frac{1920e-3525d-3402a+4170b-1955c+960g-400f}{125} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 0 & \frac{595e-1932a+1345b-655c+1150d+360g-150f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$l2 \to l2 - (3.l7) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 0 & \frac{360e+835b-1214a-390c-675d+180g-75f}{25} \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 0 & \frac{120e-125c-417a+270b-255d+60g-25f}{15} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 0 & \frac{1920e-3525d-3402a+4170b-1955c+960g-400f}{125} \\
0 & 0 & 0 & 0 & 1 & 5/4 & 0 & \frac{595e-1932a+1345b-655c+1150d+360g-150f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right] l5 \to l5 - (5/4.l6) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 1 & 0 & 2a+b \\
0 & 1 & 4 & 1 & 2 & 0 & 0 & \frac{360e+835b-1214a-390c-675d+180g-75f}{25} \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 0 & \frac{120e-125c-417a+270b-255d+60g-25f}{15} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 0 & \frac{1920e-3525d-3402a+4170b-1955c+960g-400f}{125} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$l1 \to l1 - l6 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\
0 & 1 & 4 & 1 & 2 & 0 & 0 & \frac{360e+835b-1214a-390c-675d+180g-75f}{25} \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 0 & \frac{120e-125c-417a+270b-255d+60g-25f}{15} \\
0 & 0 & 0 & 1 & 14/5 & 0 & 0 & \frac{1920e-3525d-3402a+4170b-1955c+960g-400f}{125} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right] l4 \to l4 - (14/5.l5) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\
0 & 1 & 4 & 1 & 2 & 0 & 0 & \frac{360e+835b-1214a-390c-675d+180g-75f}{25} \\
0 & 0 & 1 & 1/3 & 4/3 & 0 & 0 & \frac{120e-125c-417a+270b-255d+60g-25f}{15} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$l3 \to l3 - (4/3.l5) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\
0 & 1 & 4 & 1 & 2 & 0 & 0 & \frac{360e+835b-1214a-390c-675d+180g-75f}{25} \\
0 & 0 & 1 & 1/3 & 0 & 0 & 0 & \frac{260g+620e-605c-2237a+1370b-1275d-125f}{75} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right] l2 \to l2 - (2.l5) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\
0 & 1 & 4 & 1 & 0 & 0 & 0 & \frac{74e+169b-258a-76c-135d+32g-15f}{5} \\
0 & 0 & 1 & 1/3 & 0 & 0 & 0 & \frac{260g+620e-605c-2237a+1370b-1275d-125f}{75} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$l3 \to l3 - (1/3.l4) \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\
0 & 1 & 4 & 1 & 0 & 0 & 0 & \frac{74e+169b-258a-76c-135d+32g-15f}{5} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right] l2 \to l2 - l4 \left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\
0 & 1 & 4 & 0 & 0 & 0 & 0 & \frac{16g+14e+194b-419a-86c+167d+f}{5} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$\text{l2}\to l2-(4.l3)\left[\begin{array}{ccccccc|c}
-1 & 0 & -2 & 0 & 0 & 0 & 0 & \frac{12a+5e+30b-10g+5c}{25} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{-480g-1070e-3070b+5883a+1330c+1565d+195f}{75} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right] l1\to l1+(2.l3)\left[\begin{array}{ccccccc|c}
-1 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{-480g-1070e-3070b+5883a+1330c+1565d+195f}{75} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

$$l1\to -1.l1\left[\begin{array}{ccccccc|c}
1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{330g+655e-6048a+3080b-1295c+470d-90f}{75} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{-480g-1070e-3070b+5883a+1330c+1565d+195f}{75} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & \frac{180g+320e-655c-3042a+1495b+235d-45f}{75} \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & \frac{16g+60e-302d+161a-25b+10c-16f}{5} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{60e-1061a+685b-315c+575d+155g-75f}{50} \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{10g-5e-5c+38a-5b}{25} \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{120e-422a+270b-130c-225d+60g-25f}{25}
\end{array}\right]$$

13 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B $= \frac{216}{5}$; -23; 21; $\frac{-241}{5}$; $\frac{217}{10}$; 15; $\frac{19}{5}$