

1 Exercício 1

Considere as bases do Espaço vetorial R^3 , $A = \{(4, 2, 0), (1, 1, 1), (5, 3, 3)\}$ e $B = \{(1, 2, 1), (1, 5, 2), (1, 0, 1)\}$. Exiba as matrizes de mudança de base $MB \rightarrow A$ e $MA \rightarrow B$. Escreva também os vetores abaixo nas bases indicadas:

- $v = (0, 1, 2)$ A em B
- $v = (1, 3, 1)$ B em A

2 Mudança $B \rightarrow A(b1)$:

$$x.a_1 + y.a_2 + z.a_3 = b_1$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, -2, 1)$$

$$Início \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & -2 \\ 0 & 1 & 3 & 1 \end{array} \right] I2 \rightarrow I2 + I3 \left[\begin{array}{cccc} \mathbf{4} & \mathbf{1} & \mathbf{5} & \mathbf{1} \\ \mathbf{2} & \mathbf{0} & \mathbf{6} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{3} & \mathbf{1} \end{array} \right]$$

$$I1 \leftrightarrow I2 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 1 \end{array} \right] I2 \rightarrow I2 - 2.I1 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 0 & 1 & 7 & 3 \\ 0 & 1 & 3 & 1 \end{array} \right]$$

$$I3 \rightarrow I3 - I2 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 0 & 1 & 7 & 3 \\ 0 & 0 & 10 & -2 \end{array} \right] I2 \rightarrow 10.I2 - 7.I3 \left[\begin{array}{cccc} 2 & 0 & 6 & -1 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array} \right]$$

$$I1 \rightarrow 10.I1 - 6.I3 \left[\begin{array}{cccc} 20 & 0 & 0 & 2 \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array} \right] I1 \rightarrow I1/20 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 10 & 0 & 16 \\ 0 & 0 & 10 & -2 \end{array} \right]$$

$$I2 \rightarrow I2/10 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 1 & 0 & \frac{16}{10} \\ 0 & 0 & 10 & -2 \end{array} \right] I3 \rightarrow I3/10 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{2}{20} \\ 0 & 1 & 0 & \frac{16}{10} \\ 0 & 0 & 1 & \frac{-2}{10} \end{array} \right]$$

$$\textbf{Coordenadas } M_B \rightarrow M_{A(b1)} : a = \frac{1}{10}; \textbf{ b} = \frac{8}{5}; \textbf{ c} = \frac{-1}{5}$$

3 Mudança de $M_B \rightarrow M_{A(b2)}$:

$$x.a_1 + y.a_2 + z.a_3 = b_2$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (1, 5, 2)$$

$$Início \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 5 \\ 0 & 1 & 3 & 2 \end{array} \right] I2 \rightarrow I2 + I3 \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 7 \\ 0 & 1 & 3 & 2 \end{array} \right]$$

$$I1 \leftrightarrow I2 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 4 & 1 & 5 & 1 \\ 0 & 1 & 3 & 2 \end{array} \right] I2 \rightarrow I2 - 2.I1 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 1 & 3 & 2 \end{array} \right]$$

$$I3 \rightarrow I3 - I2 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 10 & 15 \end{array} \right] I3 \rightarrow I3/10 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{15}{10} \end{array} \right]$$

$$I3 \rightarrow \textbf{simplicando a fração: dividindo por 5} \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & -7 & -13 \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right] I2 \rightarrow I2 + 7.I3 \left[\begin{array}{cccc} 2 & 0 & 6 & 7 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right]$$

$$I1 \rightarrow I1 - 6.I3 \left[\begin{array}{cccc} 2 & 0 & 0 & -2 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right] I1 \rightarrow I1/2 \left[\begin{array}{cccc} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 & \frac{3}{2} \end{array} \right]$$

$$\textbf{Coordenadas } M_B \rightarrow M_{A(b2)} : a = -1; b = \frac{-5}{2}; \textbf{ c} = \frac{3}{2}$$

4 Mudança de $M_B \rightarrow M_{A(b3)}$:

$$x.a_1 + y.a_2 + z.a_3 = b_3$$

$$x.(4, 2, 0) + y.(1, -1, 1) + z.(5, 3, 3) = (5, 3, 3)$$

$$Início \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & -1 & 3 & 0 \\ 0 & 1 & 3 & 1 \end{array} \right] I2 \rightarrow I2 + I3 \left[\begin{array}{cccc} 4 & 1 & 5 & 1 \\ 2 & 0 & 6 & 1 \\ 0 & 1 & 3 & 1 \end{array} \right]$$

$$I1 \leftrightarrow I2 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 4 & 1 & 5 & -1 \\ 0 & 1 & 3 & 1 \end{array} \right] I2 \rightarrow I2 - 2.I1 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 1 & 3 & 1 \end{array} \right]$$

$$I3 \rightarrow I3 - I2 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 10 & 2 \end{array} \right] I3 \rightarrow I3/10 \left[\begin{array}{cccc} 2 & 0 & 6 & 1 \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{array} \right]$$

$$I1 \rightarrow I1 - 6.I3 \left[\begin{array}{cccc} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & -7 & -1 \\ 0 & 0 & 1 & \frac{2}{10} \end{array} \right] I2 \rightarrow I2 + 7.I3 \left[\begin{array}{cccc} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{2}{10} \end{array} \right]$$

$$I3 \rightarrow \textbf{simplicando a fração: dividindo por 2} \left[\begin{array}{cccc} 2 & 0 & 0 & \frac{-1}{5} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array} \right] I1 \rightarrow I1/2 \left[\begin{array}{cccc} 1 & 0 & 0 & \frac{-1}{10} \\ 0 & 1 & 0 & \frac{2}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array} \right]$$

$$\textbf{Coordenadas } M_B \rightarrow M_{A(b3)} : a = \frac{-1}{10}; \textbf{ b} = \frac{2}{5}; \textbf{ c} = \frac{1}{5}$$

5 Mudança de A → B(A1)

$x.b_1 + y.b_2 + z.b_3 = a_1$

Início $\begin{bmatrix} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 1 & 2 & 1 & 0 \end{bmatrix} l3 \rightarrow 5.l3 - 2.l2 \begin{bmatrix} 1 & 1 & 1 & 4 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix}$

$l1 \rightarrow 5.l1 - l2 \begin{bmatrix} 7 & 0 & 2 & 18 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix} l1 \rightarrow l1 - l3 \begin{bmatrix} -2 & 0 & 0 & 22 \\ -2 & 5 & 0 & 2 \\ 9 & 0 & 5 & -4 \end{bmatrix}$

$l2 \rightarrow l2 - l1 \begin{bmatrix} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 9 & 0 & 5 & -4 \end{bmatrix} l3 \rightarrow 2.l3 + 9.l1 \begin{bmatrix} -2 & 0 & 0 & 22 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{bmatrix}$

$l1 \rightarrow l1 / -2 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 5 & 0 & -20 \\ 0 & 0 & 10 & 190 \end{bmatrix} l2 \rightarrow l2 / 5 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 10 & 190 \end{bmatrix}$

$l3 \rightarrow l3 / 10 \begin{bmatrix} 1 & 0 & 0 & -11 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 19 \end{bmatrix}$

Coordenadas $M_A \rightarrow M_{B(a1)} : a = -11; b = -4; c = 19$

6 Mudança de A → B(A2)

$x.b_1 + y.b_2 + z.b_3 = a_2$

Início $\begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 1 & 2 & 1 & 1 \end{bmatrix} l3 \rightarrow l3 - l1 \begin{bmatrix} 1 & 1 & 1 & 1 \\ -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

$l2 \leftrightarrow l3 \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ -2 & 5 & 0 & -1 \end{bmatrix} l1 \leftrightarrow l3 \begin{bmatrix} -2 & 5 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$l1 \rightarrow -1.l1 \begin{bmatrix} 2 & -5 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} l1 \rightarrow l1 + 5.l2 \begin{bmatrix} 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$l1 \rightarrow l1 / 2 \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} l3 \rightarrow l3 - l2 \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

$l3 \rightarrow l3 - l1 \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$

Coordenadas $M_A \rightarrow M_{B(a2)} : a = \frac{1}{2}; \mathbf{b} = \mathbf{0}; \mathbf{c} = \frac{1}{2}$

7 Mudança de A → B(A3)

$x.b_1 + y.b_2 + z.b_3 = a_3$

Início $\begin{bmatrix} 1 & 1 & 1 & 5 \\ -2 & 5 & 0 & 3 \\ 1 & 2 & 1 & 3 \end{bmatrix} l3 \rightarrow 5.l3 - 2.l2 \begin{bmatrix} 1 & 1 & 1 & 5 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{bmatrix}$

$l1 \rightarrow 5.l1 - l2 \begin{bmatrix} 7 & 0 & 5 & 22 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{bmatrix} l1 \rightarrow l1 - l3 \begin{bmatrix} -2 & 0 & 0 & 13 \\ -2 & 5 & 0 & 3 \\ 9 & 0 & 5 & 9 \end{bmatrix}$

$l2 \rightarrow l2 - l1 \begin{bmatrix} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 9 & 0 & 5 & 9 \end{bmatrix} l3 \rightarrow 2.l3 + 9.l1 \begin{bmatrix} -2 & 0 & 0 & 13 \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{bmatrix}$

$l1 \rightarrow l1 / -2 \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 5 & 0 & -10 \\ 0 & 0 & 10 & 135 \end{bmatrix} l2 \rightarrow l2 / 5 \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 10 & 135 \end{bmatrix}$

$l3 \rightarrow l3 / 10 \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{135}{10} \end{bmatrix} l3 \rightarrow \text{simplicando a fração: dividindo por 5} \begin{bmatrix} 1 & 0 & 0 & \frac{-13}{2} \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & \frac{27}{2} \end{bmatrix}$

Coordenadas $M_A \rightarrow M_{B(a3)} : a = \frac{-13}{2}; \mathbf{b} = \mathbf{-2}; \mathbf{c} = \frac{27}{2}$

8 v = (1, 3, 1)B em A

$M_B \rightarrow_A: \begin{bmatrix} \frac{1}{10} & -1 & \frac{-1}{10} \\ \frac{8}{5} & \frac{-5}{2} & \frac{3}{5} \\ \frac{-1}{5} & \frac{3}{5} & \frac{1}{5} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} \rightarrow \begin{bmatrix} \frac{1}{10}*\mathbf{1} & -1*3 & \frac{-1}{10}* -1 \\ \frac{8}{5}*\mathbf{1} & \frac{-5}{2}*3 & \frac{3}{5}*\mathbf{-1} \\ \frac{-1}{5}*\mathbf{1} & \frac{3}{5}*3 & \frac{1}{5}* -1 \end{bmatrix} = \begin{bmatrix} \frac{-14}{5} \\ \frac{-13}{10} \\ \frac{41}{10} \end{bmatrix}$

9 v = (0, 1, 2)A em B

$M_A \rightarrow_B: \begin{bmatrix} -11 & \frac{1}{2} & \frac{-13}{2} \\ 4 & 0 & -2 \\ 19 & \frac{1}{2} & \frac{27}{2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} -11*0 & \frac{1}{2}*1 & \frac{-13}{2}*2 \\ 4*0 & 0*1 & -2*2 \\ 19*0 & \frac{1}{2}*1 & \frac{27}{2}*2 \end{bmatrix} = \begin{bmatrix} \frac{-23}{2} \\ -4 \\ \frac{55}{2} \end{bmatrix}$

10 Exercício 2

Considere o conjunto $S = \{(1, 1, 1, 1, 1), (2, 0, 1, 1, 3), (3, 1, 0, 2, 4), (2, 2, 5, 8, 1), (0, 1, 0, 2, 3)\}$.

- S é LI ou LD?

$$S = \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 1 & 0 & 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right] \xrightarrow{l_2 \leftarrow l_2 - 1 * l_1} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 5 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right]$$

$$\xrightarrow{l_3 \rightarrow l_3 - 1 * l_1} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 1 & 1 & 2 & 8 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right] \xrightarrow{l_4 \rightarrow l_4 - 1 * l_1} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 1 & 3 & 4 & -1 & 3 & 0 \end{array} \right]$$

$$\xrightarrow{l_5 \rightarrow l_5 - 1 * l_1} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & -3 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array} \right] \xrightarrow{l_3 \rightarrow l_3 - (3/2) * l_2} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & -1 & -1 & 6 & 2 & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array} \right]$$

$$\xrightarrow{l_4 \rightarrow l_4 - (1/2) * l_2} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (3/2) & 0 \\ 0 & 1 & 1 & -3 & 3 & 0 \end{array} \right] \xrightarrow{l_5 \rightarrow l_5 - (-1/2) * l_2} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 6 & (3/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array} \right]$$

$$\xrightarrow{l_4 \rightarrow l_4 - 2 * l_3} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & -3 & (7/2) & 0 \end{array} \right] \xrightarrow{l_5 \rightarrow l_5 - (-1) * l_3} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \end{array} \right]$$

$$\xrightarrow{l_5 \rightarrow l_5 - (-4/9) * l_4} \left[\begin{array}{ccccc|c} 1 & 2 & 3 & 2 & 0 & 0 \\ 0 & -2 & -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 & (-3/2) & 0 \\ 0 & 0 & 0 & 0 & (9/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

R : Portanto, é LD

Forma base do R-espaço vetorial R^5 ?
 R : Como S é ld, ele não forma uma base de R^5

11 Exercício 3

Considere o conjunto $W = \{(x, y, z, w, t, u) \mid x, y, z, w, t, u \in R \wedge x + y + w + z + t + u = 0 \wedge y - w - z = 0 \wedge w + t - x = 0\} \subseteq R^6$.
Mostre que conjunto W é um subespaço do R -espaço vetorial R^6 .

$t - x = 0$
 $t = x$
 $y - w - z = 0$
 $y = w + z$
 $x + y + w + z + t + u = 0 \rightarrow x + w + z + w + z + x + u = 0$
 $u = -x - y - w - z - t \rightarrow u = -2x - 2w - 2z$
 $W = \{(x, w + z, z, w, x, -x - w - z - w - z - x)\} \rightarrow$
 $W = \{(x, w + z, z, w, x, -2x - 2w - 2z) \mid x, z, w \in R\}$

I) $0 \in W$
 $para x = 0 z = 0 w = 0$
 $(w, w, w, w, w, -w) \rightarrow (x, w + z, z, w, x, -2x - 2w - 2z)$
 $= (0, 0, 0, 0, 0, -0)$
 $= 0$
Logo, $0 \in W$

II) $u, z \in W \rightarrow u + z \in W$, sendo :
 $u = (u1, u2, u3, u4, u5, -u6) \rightarrow (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$
 $z = (z1, z2, z3, z4, z5, -z6) \rightarrow (x_2, w_2 + z_2, z_2, w_2, x_2, -2x_2 - 2w_2 - 2z_2)$
 $u + z = (x_1 + x_2, (w_1 + z_1) + (w_2 + z_2), z_1 + z_2, w_1 + w_2, x_1 + x_2, (-2x_1 - 2w_1 - 2z_2) + (-2x_2 - 2w_2 - 2z_2))$
 $u + z = (x_1 + x_2, w_1 + z_1 + w_2 + z_2, z_1 + z_2, w_1 + w_2, x_1 + x_2, -2x_1 - 2x_2 - 2w_1 - 2w_2 - 2z_1 - 2z_2)$
Logo, $u + z \in W$

III) $a \in R, v \in W \rightarrow av \in W$, sendo :
 $v = (v1, v2, v3, v4, v5, -v6) \rightarrow (x, w + z, z, w, x, -2x - 2w - 2z)$
 $av = a \cdot (x_1, w_1 + z_1, z_1, w_1, x_1, -2x_1 - 2w_1 - 2z_1)$
 $av = (a \cdot x_1, a \cdot w_1 + z_1, a \cdot z_1, a \cdot w_1, a \cdot x_1, a \cdot -2x_1 - 2w_1 - 2z_1)$
 $av = (ax_1, aw_1w_2, az_1, aw_1, ax_1, a - 2x_1 - 2w_1 - 2z_1)$
Logo, $av \in W$.

Logo W é subespaço vetorial de R^6 .

• O conjunto $W = \{(x, y, z) \mid x, y, z \in R \wedge x - z = 1 \wedge y + x = 0\}$
é um subespaço vetorial de R^3 ? Esboce graficamente W .

 $x - z = 1 \rightarrow x = 1 + z$
 $y + x = 0 \rightarrow y + 1 + z = 0 \rightarrow y = -1 - z$.
 $W = \{(1 + z, -1 - z, z)\}$

I) $0 \in W$,
 $para z = 0 \rightarrow (1 + 0, -1 - 0, 0) = (1, -1, 0)$.
Logo 0 NÃO pertence a W para $z = 0$. Portanto, W NÃO é subespaço vetorial.

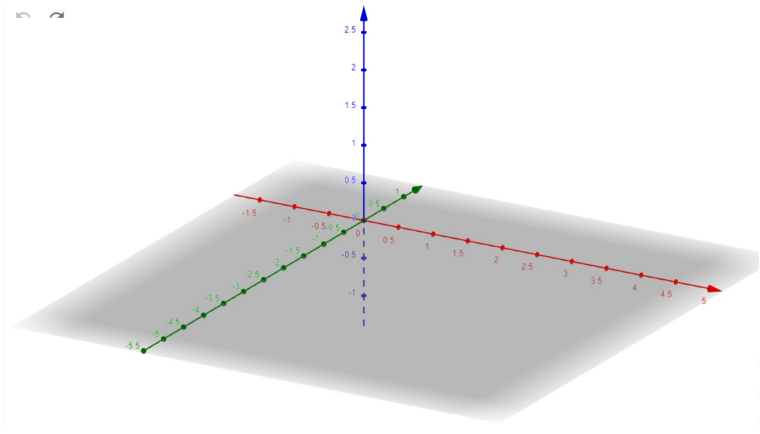


Figure 1: Representação gráfica.

• Invente seu subespaço vetorial em qualquer R n com n maior igual a 2. Mostre que o conjunto apresentado é de fato um subespaço vetorial. Não vale usar nenhum exemplo da aula ou da prova

 $W = \{(x, y, z) \mid x + y - 2z = 0\}$
 $x + y - 2z = 0$
 $x = -y + 2z$
 $W = \{-y + 2z, y, z\} \mid y, z \in R\}$

I) $(0,0,0) \in W$,
 $pois : y, z = 0$

II) $v, w, x \in W \rightarrow v + w + x \in W$, sendo :
 $v = (-y1 + 2z1, y1, z1)$
 $w = (-y2 + 2z2, y2, z2)$
 $z = (-y3 + 2z3, y3, z3)$
 $v + w + z = (-y1 + 2z, y1, -2y1) + (-y2 + 2z2, y2, z2) + (-y3 + 2z3, y3, z3)$
 $u + w + z = (-y1+2z1-y2+2z2-y3+2z3,y1+y2+y3,-2y1+z2+z3)$
Logo, $u + w + z \in W$

III) $a \in R, v \in Z \rightarrow av \in W$.Sendo :
 $v = (v1,v2,v3) \rightarrow (-y1 + 2z1, y1, z1)$
 $a.v = a \cdot (-y1 + 2z1, y1, z1)$
 $a.v = (a \cdot (-y1 + 2z1), a \cdot y1, a \cdot z1)$
 $a.v = (-ay1 + a2z1, ay1, -az1)$
Logo, $av \in W$

Logo W é subespaço vetorial de R^3

$l6 \rightarrow -5/24.i$

$$l6 \rightarrow l6 - l5$$

$l6 \rightarrow -4/5.l6$

$$17 \rightarrow 17 - 16$$

$$l6 \rightarrow l6 - (1,$$

14 → 14 -

12→ 12 -

11 → 11 —

13→ l3 -

13→ l3 -

$$I2 \rightarrow I2 - (4.I3) \left[\begin{array}{c|c} -1 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} \frac{12a+5c+30b-10g+5c}{25} \\ -480g-1070c-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

$$I1 \rightarrow I1 + (2.I3) \left[\begin{array}{c|c} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} \frac{330g+655c-6048a+3080b-1295c+470d-90f}{75} \\ -480g-1070c-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$

13 Coordenadas

Portanto o conjunto forma base para o espaço vetorial R7 e as coordenadas são B = $\frac{216}{5}$; -23; 21; $-\frac{241}{5}$; $\frac{217}{10}$; 15; $\frac{19}{5}$

$$I1 \rightarrow -1.I1 \left[\begin{array}{c|c} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \middle| \begin{array}{l} -\frac{330g+655c-6048a+3080b-1295c+470d-90f}{75} \\ -480g-1070c-3070b+5883a+1330c+1565d+195f \\ \frac{180g+320c-655c-3042a+1495b+235d-45f}{75} \\ \frac{16g+60c-302d+161a-25b+10c-16f}{5} \\ \frac{60c-1061a+685b-315c+575d+155g-75f}{50} \\ \frac{10g-5c-5c+38a-5b}{25} \\ -\frac{120c-422a+270b-130c-225d+60g-25f}{25} \end{array} \right]$$