(1)
$$\begin{cases} 0{,}25x - 0{,}2y = 0{,}45 \\ 4y + 6z = -4 \\ -0{,}2x + 5y = -5{,}2 \end{cases}$$

(a) Determine factorización L.U Doolittle, Crout

Doolittle

$$A = \begin{pmatrix} \frac{1}{4} & -1/5 & 0 \\ 0 & 4 & 6 \\ -1/5 & 5 & 0 \end{pmatrix} \quad F_3 \leftarrow F_3 + \frac{4}{5} F_1 \qquad \begin{pmatrix} \frac{1}{4} & -1/5 & 0 \\ 0 & 4 & 6 \\ 0 & 4{,}84 & 0 \end{pmatrix}$$

$$\begin{array}{ccc} -1/5 & 5 & 0 \\ 1/5 & -4/25 & 0 \\ \hline 0 & 4{,}84 & 0 \end{array}$$

$F_3 \leftarrow F_3 - \frac{4{,}84}{4} F_2$

$$\rightarrow \begin{pmatrix} \frac{1}{4} & -\frac{1}{5} & 0 \\ 0 & 4 & 6 \\ 0 & 0 & -7{,}26 \end{pmatrix} = U \qquad L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4/5 & \frac{4{,}84}{4} & 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4/5 & \frac{4{,}84}{4} & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{4} & -1/5 & 0 \\ 0 & 4 & 6 \\ 0 & 0 & -7{,}26 \end{pmatrix}$$

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Crout

$$A = \begin{pmatrix} 0{,}25 & -0{,}2 & 0 \\ 0 & 4 & 6 \\ -0{,}2 & 5 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{pmatrix}$$

$l_{11} = 0{,}25$ $\quad l_{11} \cdot U_{12} = -0{,}2 \rightarrow 0{,}25 \cdot U_{12} = -0{,}2 \quad U_{12} = \frac{-0{,}2}{0{,}25}$

$l_{21} = 0$ $\quad l_{21} U_{12} + l_{22} = 4 \quad l_{22} = 4 \quad l_{31} = -0{,}2$

$l_{31} \cdot U_{12} + l_{32} = 5 \rightarrow \frac{-0{,}2}{0{,}25} \cdot -0{,}2 + l_{32} = 5 \quad l_{32} = 4{,}84$

$l_{31} U_{13} + l_{32} \cdot U_{23} + l_{33} = 0 \rightarrow U_{23} = 3/2$

$0 U_{13} + 4 \cdot U_{23} = 6 \quad U_{23} = 3/2 \qquad 0{,}25 \cdot U_{13} = 0 \quad U_{13} = 0$

$4{,}8 \cdot \frac{3}{2} + l_{33} = 0 \quad l_{33} = -7{,}26$

$$A = \begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 4 & 0 \\ -1/5 & \frac{121}{25} & \frac{-363}{50} \end{pmatrix} \begin{pmatrix} 1 & -4/5 & 0 \\ 0 & 1 & 1.5 \\ 0 & 0 & 1 \end{pmatrix}$$

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(b) $Ly = b$

$$\begin{pmatrix} 1/4 & 0 & 0 \\ 0 & 4 & 0 \\ -1/5 & 121/25 & -\frac{363}{50} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 9/20 \\ -4 \\ -5,2 \end{pmatrix}$$

$\frac{1}{4} y_1 = \frac{9}{20} \quad y_1 = 0,45$

$4y_2 = -4$

$y_2 = -1$

$-\frac{1}{5} y_1 + \frac{121}{25} y_2 - \frac{313}{50} y_3 = -5,2$

$-\frac{1}{5} \frac{9}{5} - \frac{121}{25} - \frac{313}{50} y_3 = -5,2$

$y_3 = 6$

$Ux = y$

$$\begin{pmatrix} 1 & -4/5 & 0 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0,45 \\ -1 \\ 0 \end{pmatrix}$$

$\boxed{z = 0}$

$y + \frac{3}{2} z = -1$

$\boxed{y = -1}$

$x + \frac{4}{5} = \frac{9}{5} \quad \boxed{x = 1}$

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② Cholesky:

$$\begin{cases} 2x - y + z = 1 \\ -x + 3y - 2z = 2 \\ x - 2y + 4z = 0 \end{cases}$$

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 4 \end{pmatrix}$$

$$l_{KK} = \sqrt{a_{KK} - \sum_{j=1}^{K-1} l_{Kj}^2}$$

$$l_{Ki} = \frac{a_{Ki} - \sum_{j=1}^{i-1} l_{ij} l_{Kj}}{l_{ii}}$$

$$A = L \cdot L^t$$

$$l_{11} = \sqrt{a_{11}} = \sqrt{2} \qquad l_{21} = \frac{a_{Ki}}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$l_{22} = \sqrt{3 - l_{21}^2} = \sqrt{3 - \frac{1}{2}} = \sqrt{5/2} \qquad l_{31} = \frac{1-0}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$l_{32} = \frac{-2 - \sum_{j=1}^{1} l_{21} l_{31}}{\sqrt{5/2}} = \frac{-2 - \left(\frac{-1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)}{\sqrt{5/2}} = \frac{-2 + \frac{1}{2}}{\sqrt{5/2}}$$

$$= \frac{\frac{-3}{2}}{\sqrt{\frac{5}{2}}} = \frac{-3\sqrt{10}}{10} \qquad l_{33} = \sqrt{4 - l_{31}^2 - l_{32}^2}$$

$$l_{33} = \sqrt{4 - \frac{1}{2} - \frac{90}{100}} \qquad l_{33} = \frac{\sqrt{65}}{5}$$

$$A = \underset{L}{\begin{pmatrix} \sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & \sqrt{\frac{5}{2}} & 0 \\ 1/\sqrt{2} & -\frac{3\sqrt{10}}{10} & \frac{\sqrt{65}}{5} \end{pmatrix}} \underset{U}{\begin{pmatrix} \sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} \\ 0 & \sqrt{5/2} & -\frac{3\sqrt{10}}{10} \\ 0 & 0 & \frac{\sqrt{65}}{5} \end{pmatrix}}$$

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b) $LY = B$

$$\begin{pmatrix} \sqrt{2} & 0 & 0 \\ -\frac{1}{\sqrt{2}} & \sqrt{\frac{5}{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{3\sqrt{10}}{10} & \frac{\sqrt{65}}{5} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

$\sqrt{2}\, y_1 = 1$

$\boxed{y_1 = \frac{1}{\sqrt{2}}}$

$-\frac{1}{\sqrt{2}} y_1 + \frac{\sqrt{5}}{\sqrt{2}} y_2 = 2$

$\left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \frac{\sqrt{5}}{\sqrt{2}} y_2 = 2$

$-\frac{1}{2} + \sqrt{\frac{5}{2}}\, y_2 = 2$

$y_2 = \frac{5}{2} \cdot \frac{\sqrt{2}}{\sqrt{5}}$

$\frac{1}{\sqrt{2}} \cdot y_1 - \frac{3\sqrt{10}}{10} y_2 + \frac{\sqrt{65}}{5} y_3 = 0$

$\frac{1}{2} - \frac{3\sqrt{10}}{\cancel{10}} \frac{5}{2} \frac{\sqrt{2}}{\sqrt{5}} + \frac{\sqrt{65}}{5} y_3 = 0$

$\frac{1}{2} - \frac{3}{\cancel{2}} \frac{2}{2} + \frac{\sqrt{65}}{5} \cdot y_3 = 0 \quad y_3 = \frac{\sqrt{5}}{\sqrt{13}}$

$UX = Y$

$$\begin{pmatrix} \sqrt{2} & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{5}}{\sqrt{2}} & -\frac{3\sqrt{5}}{5\sqrt{2}} \\ 0 & 0 & \frac{\sqrt{13}}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{5}{2} \cdot \frac{\sqrt{2}}{\sqrt{5}} \\ \frac{\sqrt{5}}{\sqrt{13}} \end{pmatrix}$$

$\frac{\sqrt{13}}{\sqrt{5}} z = \frac{\sqrt{5}}{\sqrt{13}}$

$\boxed{z = \frac{5}{13}}$

$\frac{\sqrt{5}}{\sqrt{2}} y - \frac{3\sqrt{5}}{5\sqrt{2}} z = \frac{5}{2} \frac{\sqrt{2}}{\sqrt{5}} \Rightarrow \frac{\sqrt{5}}{\sqrt{2}} y - \frac{3\sqrt{5}}{5\sqrt{2}} \frac{\cancel{5}}{13} = \frac{5}{2} \frac{\sqrt{2}}{\sqrt{5}}$

$\frac{\sqrt{5}}{\sqrt{2}} y = \frac{5\sqrt{2}}{2\sqrt{5}} - \frac{3\sqrt{5}}{13\sqrt{2}} \Rightarrow \frac{\sqrt{5}}{\sqrt{2}} y = \frac{13\sqrt{2}.5 - 3.5.\sqrt{5}}{\sqrt{2}.\sqrt{5}.13} = \boxed{y = \frac{16}{13}}$

$\boxed{x = \frac{12}{13}} \Leftarrow x_1 = \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} + \frac{8\sqrt{2}}{13} - \frac{5\sqrt{2}}{26} \right)$

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③ $\begin{cases} 2x - y + z = 1 \\ -x + 3y - 2z = 2 \\ x - 2y + 4z = 0 \end{cases}$

$\left.\begin{matrix} |2| = |-1| + |1| \\ |3| = |-1| + |-2| \\ |4| > |-2| + |1| \end{matrix}\right\}$ No es diagonal dominante

Jacobi:

$$x = \frac{1 + y - z}{2} \qquad z = \frac{2y - x}{4}$$

$$y = \frac{2 + x + 2z}{3}$$

$I=1$: $X_0 = (0, 0, 0)$ $\quad x = \frac{1}{2} \quad y = \frac{2}{3} \quad z = 0$

$I_2 = X_1 (1/2, 2/3, 0)$

$$x = \frac{1 + \frac{2}{3} - 0}{2} = \frac{5}{6} \qquad y = \frac{2 + \frac{1}{2} + 0}{3} = \frac{5}{6} \qquad z = \frac{2\left(\frac{2}{3}\right) - \frac{1}{2}}{4} = \frac{5}{24}$$

$I_3 = X_2 (5/6, 5/6, 5/24)$

$$x = \frac{1 + \frac{5}{6} - \frac{5}{24}}{2} = \frac{13}{16} \qquad y = \frac{2 + \frac{5}{6} + \frac{5}{12}}{3} = \frac{13}{12} \qquad z = \frac{\frac{5}{3} - \frac{5}{6}}{4} = \frac{5}{24}$$

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$I_4 = X_3\left(\frac{13}{16} ; \frac{13}{12} ; \frac{5}{24}\right.$

$$X = \frac{1 + \frac{13}{12} - \frac{5}{24}}{2} = \frac{15}{16} \qquad Y = \frac{2 + \frac{13}{16} + \frac{5}{12}}{3} = \frac{155}{144}$$

$$Z = \frac{2\left(\frac{13}{12}\right) - \frac{13}{16}}{4} = \frac{65}{192}$$

$I_5 = X_4 = \left(\frac{15}{16} ; \frac{155}{144} ; \frac{65}{192}\right)$

$$X = \frac{1 + \frac{155}{144} - \frac{65}{192}}{2} = \frac{1001}{1152} \qquad Y = \frac{2 + \frac{15}{16} + 2\,\frac{65}{192}}{3} = \frac{347}{288}$$

$$Z = \frac{2\left(\frac{155}{144}\right) - \frac{15}{16}}{4} = \frac{175}{576}$$

$I_6 : X_4 = \left(\frac{1001}{1152} ; \frac{347}{288} ; \frac{175}{576}\right)$

$$X = \frac{1 + \frac{347}{288} - \frac{175}{576}}{2} = \frac{365}{384} \qquad Y = \frac{2 + \frac{1001}{1152} + 2\left(\frac{175}{576}\right)}{3} = \frac{448}{384}$$

$$Z = \frac{2\left(\frac{347}{288}\right) - \frac{1001}{1152}}{4} = \frac{1175}{4608}$$

(b) Gauss $\Rightarrow$ Excel

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	0	1	2	3	4	5	6
x	0	0.5	0.77083333	0.87586806	0.91019242	0.92003189	0.92250124
y	0	0.83333333	1.11805556	1.20283565	1.22503135	1.22998901	1.23082477
z	0	0.29166667	0.36631944	0.38245081	0.38496757	0.38498653	0.38478707

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(4)

Y	X_1	X_2	$X_1 . X_2$
3	-1	2	-2
5	2	3	6
7	3	4	12
4	5	6	36

$$\beta = (X^t . X)^{-1} X^t . Y$$

$$X = \begin{pmatrix} 1 & -1 & 2 & -2 \\ 1 & 2 & 3 & 6 \\ 1 & 3 & 4 & 12 \\ 1 & 5 & 6 & 30 \end{pmatrix} \quad X^t = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 2 & 3 & 5 \\ 2 & 6 & 4 & 6 \\ -2 & 6 & 12 & 30 \end{pmatrix}$$

$$X^t . X = \begin{pmatrix} 4 & 9 & 15 & 46 \\ 9 & 39 & 46 & 200 \\ 15 & 46 & 65 & 242 \\ 46 & 200 & 242 & 1084 \end{pmatrix} \quad (X^t . X)^{-1} = \begin{pmatrix} \frac{2075}{18} & \frac{103}{18} & \frac{-626}{9} & \frac{277}{36} \\ \frac{103}{18} & \frac{13}{18} & \frac{-23}{9} & \frac{7}{36} \\ \frac{-626}{9} & \frac{-23}{9} & \frac{547}{18} & \frac{-121}{36} \\ \frac{277}{36} & \frac{7}{36} & \frac{-121}{36} & \frac{7}{18} \end{pmatrix}$$

$$X^t . Y = \begin{pmatrix} 1 & 1 & 1 & 1 \\ -1 & 2 & 3 & 5 \\ 2 & 3 & 4 & 6 \\ -2 & 6 & 12 & 36 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \\ 7 \\ 4 \end{pmatrix} = \begin{pmatrix} 19 \\ 48 \\ 73 \\ 228 \end{pmatrix}$$

$$(X^t . X)^{-1} X^t . Y = \begin{pmatrix} -\frac{83}{6} \\ \frac{7}{6} \\ \frac{47}{6} \\ -\frac{7}{6} \end{pmatrix} \quad Y = -\frac{83}{6} + \frac{7}{6} X_1 + \frac{47}{6} X_2 - \frac{7}{6} X_1 X_2$$

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(9) $Y = \beta_0 X_1^{\beta_1} X_2^{\beta_2}$

$$\ln(Y) = \ln\left(\beta_0 X_1^{\beta_1} X_2^{\beta_2}\right)$$

$$\ln(Y) = \ln(\beta_0) + \beta_1 \ln(X_1) + \beta_2 \ln(X_2)$$

$$Y^* = \beta_0^* + \beta_1 X_1^* + \beta_2 X_2^*$$

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